Visualizing flow fields using fractal dimensions

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Abstract— Streamlines are a popular way of visualizing flow in vector fields. A major challenge in streamline visualization is selecting the streamlines for visualization. Rendering too many streamlines clutters the visualization and makes features of the field difficult to identify. Rending too few streamlines causes viewers to completely miss features of the flow field not rendered.
The fractal dimension of a streamline represents its space-filling properties. To identify complex or interesting streamlines, we build a regular grid of scalar values which represent the fractal dimension of streamlines around each grid vertex. High fractal dimension indicates vortices or turbulent regions. We use this scalar grid both to filter streamlines by fractal dimension and to identify and visualize regions containing vortices and turbulence. We describe an interactive tool which allows for quick streamline selection and visualization of regions containing vortices and turbulence.

Index Terms—Streamlines, fractal dimension.

1 Introduction

2 Related Work

3 Box Counting Ratio

Explain definition of fractal dimension?
Explain defintion of box counting dimension?
Show derivation and motivation of box counting formula?

4 Streamline Complexity Grid

For each regular point p in a vector field $f : \mathbb{R}^3 \Rightarrow \mathbb{R}^3$, let ζ_p be the unique streamline passing through p. Define $\phi_w(p)$ as the local box counting ratio of ζ_p in a $w \times w \times w$ region around p, where boxes have edge lengths 1 and 2. Function ϕ_w defines a scalar field on the regular points of f. The scalar $\phi_w(p)$ represents the complexity of the streamline through p in a $w \times w \times w$ neighborhood of p. We call the scalar field ϕ_w, the streamline complexity field of f.

To construct a scalar grid representing the streamline complexity field of f, we compute a set of streamlines so that every voxel is intersected by at least one streamline. We then compute the box counting ratio around each voxel v, by selecting a streamline ζ_v intersecting the voxel v and computing the local box counting ratio of ζ_v in a $w \times w \times w$ region centered at v. We call the resulting scalar grid, the streamline complexity grid of f.

To compute a set of streamlines intersecting every voxel, we...

To compute the local box counting ratio of ζ_v in a $w \times w \times w$ region centered at v, we...

A parameter w defines the width of the region around each voxel. From each vertex of the scalar grid, a streamline is seeded and a local box counting ratio is calculated from each voxel that the streamline intersects. In a local box counting ratio calculation, only a portion of the streamline from some distance to the vertex is considered to exclusively capture and record the behavior of the streamline and flow field around the grid vertices. A vertex's final scalar value will be the greatest local box counting ratio computed from that box's vertex. Once the scalar grid calculations are complete, visualization techniques using the scalar field and box counting ratios can be applied to the collection of streamlines generated to allow users to identify turbulent regions.

5 Visualization Techniques

Using the streamline complexity grid, a variety of different visualization techniques can be applied to allow the user to identify complex or turbulent regions of streamlines. Since the scalar values of the grid quanitfy the complexity of the streamlines in that region of the flow field, this provides a simple way to identify the complex streamlines and significant regions of the flow field.

Local maximums: To filter the amount of streamlines shown in the visualization, only the streamline near local maximums of the streamline complexity grid can be shown. This allows for the visualization to show links representing different regions of complexity while limiting the amount of streamlines shown to reduce clutter.

Colored Plane: A plane colored by the values on streamline complexity grid can be moved through the field to identify turbulent regions of flow. The streamlines can then be shown that have been seeded from these turbulent regions to see the behavior of the flow field in that region.

Isosurfaces: An isosurface, which is defined by

$$\{x \mid \phi_w(x) = \sigma\} \tag{1}$$

where σ is the isovalue and ϕ_w is the streamline complexity field. This isosurface will enclose the regions of streamlines with a fractal dimension higher than the (sigma) value. At high (sigma) values, the isosurface encloses regions of the field with high complexity.

Gradient Magnitudes: The magnitude of gradients of the streamline complexity field can be computed and stored as another scalar field. The isosurface of the gradient magnitudes constructed at high isovalues identifies regions of the flow field with high change in complexity or isolated regions of turbulence.

6 Examples

7 Evaluation

8 Summary and Future Work

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