Economics 403A: Homework 3
Fall 2018, UCLA
Instructor: Dr. Rojas

Due Date: Oct 18, 2018

All problems require detailed worked out solutions to receive full credit. All problems are worth the same.

1. An article in The Engineer ("Redesign for Suspect Wiring," June 1990) reported the results of an investigation into wiring errors on commercial transport aircraft that may produce faulty information to the flight crew. Such a wiring error may have been responsible for the crash of a British Midland Airways aircraft in January 1989 by causing the pilot to shut down the wrong engine. Of 1600 randomly selected aircraft, eight were found to have wiring errors that could display incorrect information to the flight crew.

   (a) Find a 99% confidence interval on the proportion of aircraft that have such wiring errors.

   (b) Suppose we use the information in this example to provide a preliminary estimate of p. How large a sample would be required to produce an estimate of p that we are 99% confident differs from the true value by at most 0.008?

   (c) Suppose we did not have a preliminary estimate of p. How large a sample would be required if we wanted to be at least 99% confident that the sample proportion differs from the true proportion by at most 0.008 regardless of the true value of p?

   (d) Comment on the usefulness of preliminary information in computing the needed sample size.

2. The proportion of residents in Phoenix favoring the building of toll roads to complete the freeway system is believed to be $p = 0.3$. If a random sample of 10 residents shows that 1 or fewer favor this proposal, we will conclude that $p < 0.3$.

   (a) Find the probability of type I error if the true proportion is $p = 0.3$.

   (b) Find the probability of committing a typeII error with this procedure if $p = 0.2$.

   (c) What is the power of this procedure if the true proportion is $p = 0.2$?

3. Assume that $X_1, \ldots, X_9$ are i.i.d. having Bernoulli distribution with parameter p. Suppose that we wish to test the hypotheses

$$H_0 : p = 0.4$$
$$H_1 : p \neq 0.4$$

   Let $Y = \sum_{i=1}^{9} X_i$.

   (a) Find c_1 and c_2 such that $P(Y \leq c_1 | p = 0.4) + P(Y \geq c_2 | p = 0.4)$ is as close as possible to 0.1 without being larger than 0.1.

(b) Let δ be he test that rejects H_0 if either $Y \leq c_1$ or $Y \geq c_2$. What is the size of the test δ_c?

(c) Draw a graph of the power function δ_c.

4. The file '`Prob4_data.txt`' contains 30 observations from a Gamma distribution with unknown parameters α and β.

   (a) Plot a histogram of the data and overlay the respective density curve.

   (b) Compute the Method of Moments estimates of the parameters, $\widehat{\alpha}$ and $\widehat{\beta}$.

   (c) Generate 1000 new samples from your data and compute the Bootstrap Mean, standard errors, and 95% confidence intervals of the parameters and compare them against your results from part (b).

5. A 1992 article in the Journal of the *American Medical Association* ("A Critical Appraisal of 98.6 Degrees F, the Upper Limit of the Normal Body Temperature, and Other Legacies of Carl Reinhold August Wunderlich") reported body temperature, gender, and heart rate for a number of subjects. The body temperatures for 25 female subjects follow: 97.8, 97.2, 97.4, 97.6, 97.8, 97.9, 98.0, 98.0, 98.0, 98.1, 98.2, 98.3, 98.3, 98.4, 98.4, 98.4, 98.5, 98.6, 98.6, 98.7, 98.8, 98.8, 98.9, 98.9, and 99.0.

   (a) Test the hypothesis $H_0 : \mu = 98.6$ versus $H_1 : \mu \neq 98.6$, using $\alpha = 0.05$. Find the P-value.

   (b) Check the assumption that female body temperature is normally distributed.

   (c) Compute the power of the test if the true mean female body temperature is as low as 98.0.

   (d) What sample size would be required to detect a true mean female body temperature as low as 98.2 if we wanted the power of the test to be at least 0.9?

   (e) Explain how the question in part (a) could be answered by constructing a two-sided confidence interval on the mean female body temperature.

6. Suppose a sample of size 1 is taken from the pdf $f_Y(y) = (1/\lambda)e^{-y/\lambda}$, $y > 0$, for the purpose of testing

$$H_0 : \lambda = 1$$
$$H_1 : \lambda > 1$$

   The null hypothesis will be rejected if $y \geq 3.20$.

   (a) Calculate the probability of committing a Type I error.

   (b) Calculate the probability of committing a Type II error when $\lambda = 4/3$.

   (c) Draw a diagram that shows the α and β calculated in parts (a) and (b).

7. Evans & Rosenthal: 5.4.12

8. Evans & Rosenthal: 5.5.18
9. Evans & Rosenthal: 6.2.17
10. Evans & Rosenthal: 6.2.25
11. Evans & Rosenthal: 6.3.22
12. Evans & Rosenthal: 6.4.17