Economics 403A

Statistical Inference

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Today's Class

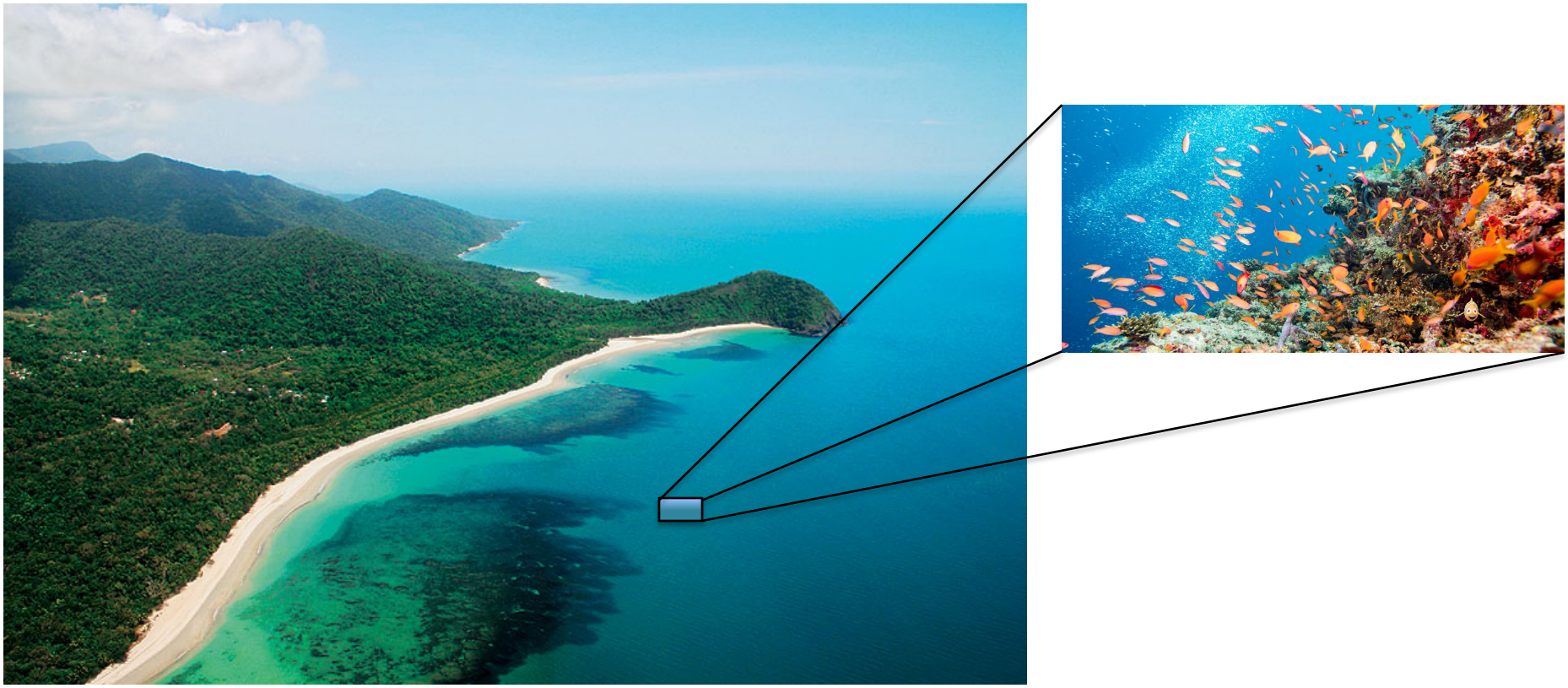
- Introduction to Inference
- Inference using Probability
- Statistical Models
- Data Collection
- Basic Inferences

Introduction to Inference

Statistical Inference

- Def: Statistical inference is the process of using data analysis to deduce properties of an underlying probability distribution.
  - Inference requires assumptions about the data, where different sets of assumptions represent the different models we would consider.
- Types of Models:
  a) Fully Parametric: A PDF is assumed for the DGP
  b) Non-Parametric: Very limited assumptions made about the DGP
  c) Semi-Parametric: These models are between a) and c)

Introduction to Inference



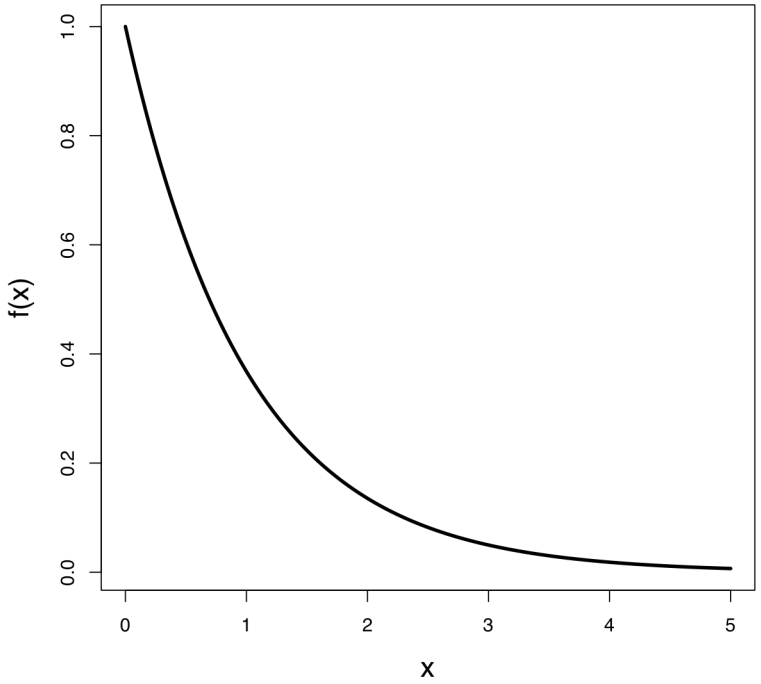
Inference using Probability (Fully Parametric)

- Example (1)*: Should I purchase an extended warranty for my new washing machine? Assume you want to keep it for 5 – 6 years.
  - Washing machine costs $650 and the extended warranty is $125
  - Manufacturer's warranty is good for 3 years
  - Extended warranty covers an added 5 years
- Q: How can we make an informed decision given the amount of uncertainty we face?

*Note: See problem 5.2.1 from Evans & Rosenthal

Inference using Probability (Fully Parametric)

- Solution: Assume the lifelength X in years of the washing machine follows an *exp($\lambda = 1$)* distribution.



- According to our model, the expected lifelength for a new machine would be $E[X] = 1$ yr.

- The smallest interval containing 95% of the probability for *X* is (0, *c*), where *c* satisfies

$$0.95 = \int_0^c e^{-x} dx = 1 - e^{-c} \rightarrow c \approx 3$$

is equal to [0,3].

- How likely is it that it will still work after 5 yrs?

$$P(X \geq 5) = \int_5^\infty e^{-x} dx = 0.0067$$

Better get that warranty! 😬

Inference using Probability (Fully Parametric)

- Example (1): Suppose you didn't get the extended warranty, and a year has past but so far its working fine. Would your previous results change?

$$E(X \mid X > 1) = \int_1^\infty x e^{-(x-1)}\, dx = -x e^{-(x-1)}\Big|_1^\infty + \int_1^\infty e^{-(x-1)}\, dx = 2$$

$$0.95 = \int_1^c e^{-(x-1)}\, dx = e(e^{-1} - e^{-c}) \rightarrow c = -\ln\left(e^{-1} - 0.95e^{-1}\right) = 3.9957$$

$$P(X > 5 \mid X > 1) = \int_5^\infty e^{-(x-1)}\, dx = e^{-4} = 0.0183$$

Statistical Models

- Example: Suppose that you work for an insurance company and historically they found that the number of claims they receive can be described by a Poisson distribution but the parameter value (i.e., θ) is unknown.
  - Recall that: $P(X = x|\theta) = \frac{\theta^x e^{-\theta}}{x!}$
- They assign you the task of figuring out the probability that 6 claims will be made tomorrow given that today 4 were received.
- Q: How should you proceed?

Statistical Models

- Solution:
  1. Try a 'brut force approach' by considering all the values θ can take, e.g., θ=1, θ=2,…, and for each one, computing the respective PMF.

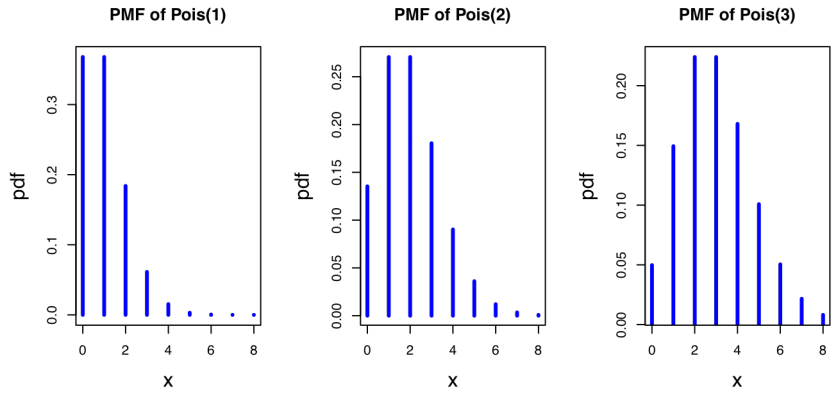
  2. From the family of PMFs, identify the one(s) most 'consistent' with the data.

  3. Based on your PMF(s) from Step 2, compute the desired probability $P(X = 6|\theta)$.

Statistical Models

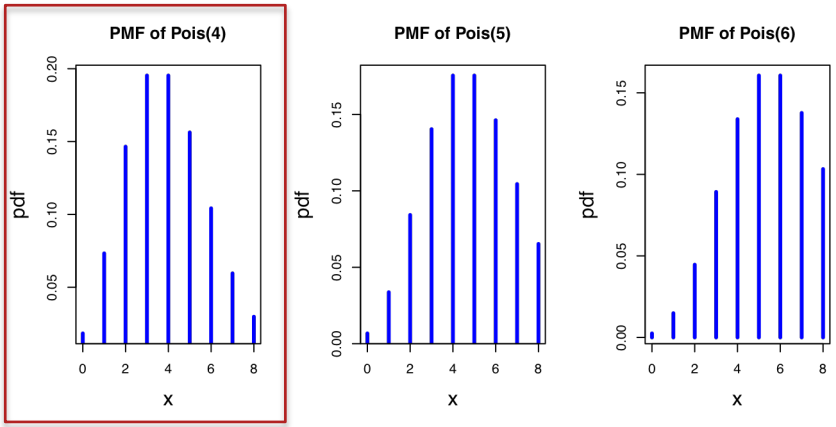
- Step 1:

  Try a 'brut force approach' by considering all the values θ can take, e.g., θ=1, θ=2,…, and for each one, computing the respective PMF.



- Step 2:

Data: {X =4}
→P(X=4|θ=4)≈0.2



Statistical Models

- Step 3:

  Based on your PMF(s) from Step 2, compute the desired probability $P(X = 6|\theta)$.

  From the figures we can try finding P(X=6) using the parameter value θ=4, i.e., P(X=6|θ=4):

  →P(X=6| θ=4) = 0.10.

Statistical Models

Definitions

- Parameter Space: $\Omega = \{\theta_1, \theta_2, \ldots., \theta_n\}$

  Is the set of all possible values of the parameter θ.

- Data: Observations ($= x$) obtained from a random mechanism ($= P$ = 'probability measure') assumed to have generated them.

- Statistical Model: Represents our choice of probability measure from the many plausible ones, such that given $\theta \in \Omega$, P_θ is the true probability measure.

Goal: We observe the data but not P, yet we want to make inferences about P

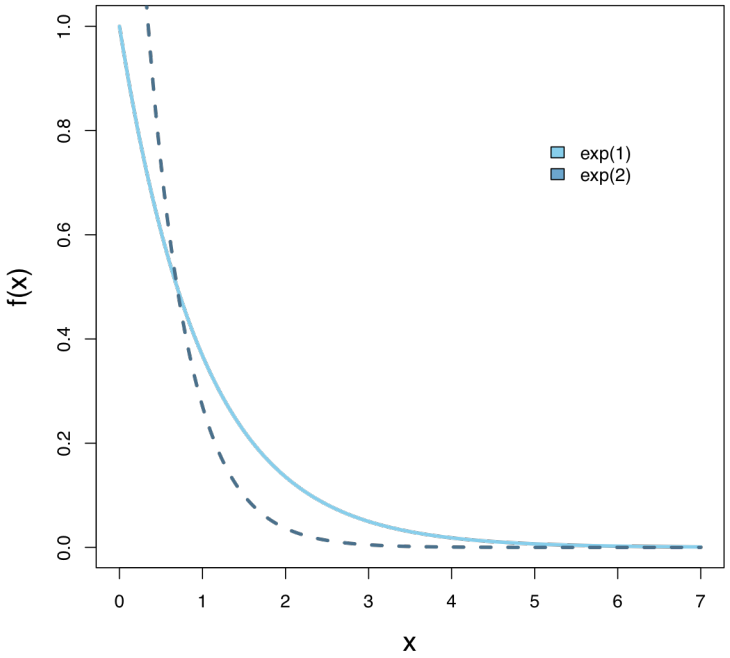
Statistical Models

- In practice, we often have more than just one observation, and the distribution may depend on more than one parameter.

- The previous formalism can be easily generalized to a vector of parameters $\vec{\theta}$ and set of observations $(x_1, \ldots, x_n)$.

Statistical Models

- Example (see 5.3.2): Suppose there are two manufacturing plants for machines. It is known that machines built by the first plant have lifelengths distributed Exponential(1), while machines manufactured by the second plant have lifelengths distributed Exponential(2).
- You have purchased five of these machines knowing that all five came from the same plant, but you do not know which plant.



Statistical Model: $\{P_1, P_2\}$

Parameter Space: $\Omega = \{1,2\}$

If we observe $(x_1,\ldots x_5)$ =

- (5.0, 3.5, 3.3, 4.1, 2.8) → $\theta = 2$
- (2.0, 2.5, 3.0, 3.1, 1.8) → $\theta = 1$

Data Collection

- If the sample is large enough and representative of the degree of variation of the population, you do not need to use the entire population.

- There are 4* common forms of 'Probability Sampling' from a population:
  a) Simple Random Sampling
  b) Stratified Sampling (R package = splitstackshape)
  c) Cluster & Multistage Sampling (R library = survey)
  d) Systematic Sampling

*Note: See De Veaux, Velleman & Bock, Intro Stats (Ch 12) for more details

Data Collection

(A) Simple Random Sampling (SRS)

- We draw samples because we can't work with the entire population.
  - We need to be sure that the statistics we compute from the sample reflect the corresponding parameters accurately.
  - A sample that does this is said to be **representative**.

- Example: The IRS may audit a random sample of 500 tax returns from a small county with 20,000 people.
  - This is financially and practically more feasible to do than to inspect all 20,000 returns

Data Collection

(B) Stratified Sampling

- Simple random sampling is not the only fair way to sample.
- More complicated designs may save time or money or help avoid sampling problems.
- All statistical sampling designs have in common the idea that chance, rather than human choice, is used to select the sample.
- Designs used to sample from large populations are often more complicated than simple random samples.

Data Collection

(B) Stratified Sampling

- Sometimes the population is first sliced into homogeneous groups, called **strata**, before the sample is selected.
- Then simple random sampling is used within each stratum before the results are combined.
- This common sampling design is called **stratified random sampling**.
- Example: Suppose Boeing wants to determine the level of compliance with their no-drugs consumption policy. Would an SRS of 50 employees out of 2000 be appropriate?

→ No! Some departments are much larger than others, and therefore more likely to be included in the sample. Instead use Stratified Sampling!

Data Collection

(C) Cluster and Multistage Sampling

- Sometimes stratifying is not practical and simple random sampling is difficult (e.g., to expensive to include everyone).
- Splitting the population into similar parts or **clusters** can make sampling more practical.
  - Then we could select one or a few clusters at random and perform a census within each of them.
  - This sampling design is called **cluster sampling**.
  - If each cluster fairly represents the full population, cluster sampling will give us an unbiased sample.
- Example: In the US, are auto shops charging women more than men for the same car repairs?

Data Collection

(C) Cluster and Multistage Sampling

- Sometimes we use a variety of sampling methods together.
- Sampling schemes that combine several methods are called **multistage samples**.
- Most surveys conducted by professional polling organizations use some combination of stratified and cluster sampling as well as simple random sampling.

Data Collection

(D) Systematic Sampling

- Sometimes we draw a sample by selecting individuals systematically.
  - For example, you might survey every 10th person on an alphabetical list of students.
- To make it random, you must still start the systematic selection from a randomly selected individual.
- When there is no reason to believe that the order of the list could be associated in any way with the responses sought, **systematic sampling** can give a representative sample.

Basic Inferences

Descriptive Statistics

- Def: *pth* quantile (or 100 *pth* percentile) = π_p

$$p = \int_{-\infty}^{\pi_p} f(x)dx = F(\pi_p)$$

where F = CDF and $f(x)$ = PDF.

- Five Number Summary
  - Consists of the min, Q_1, Q_2 (median), IQR, Q_3 and max, where $Q_1 = \pi_{0.25}$, median = $\pi_{0.50}$, $Q_3 = \pi_{0.75}$, and IQR = $Q_3 - Q_1$

Basic Inferences

Descriptive Statistics

- Example: Given $f(x) = 2e^{-2x}, x > 0$, find $\pi_{0.5}$.

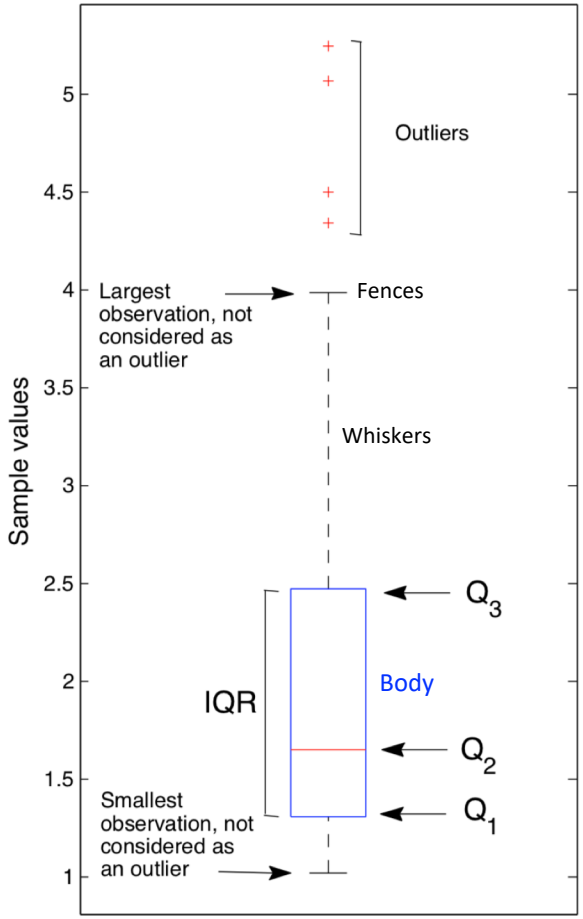
- Solve for π_p from: $$0.5 = \int_0^{\pi_{0.5}} 2e^{-2x}dx = F(\pi_{0.5})$$

$$\rightarrow \pi_{0.5} = \frac{1}{2}\ln(2)$$

Basic Inferences

Descriptive Statistics

- Boxplot: Graphical representation of the 5 number summary.
- Useful for comparing groups
- It consists of:
  - Body: Q_1, Q_2, Q_3
  - Fences: Lower = Q_1 - 1.5IQR
  - Upper = Q_3 + 1.5IQR
  - Whiskers: dashed lines



Basic Inferences

Descriptive Statistics

- Histogram: Graphical representation a quantitative variable's density distribution.
- Q: How many bins should be used?
- A: For a dataset with n observations, a nearly optimal number is $k = 1 + \log_2(n)$