

(1)

(a) Boolean

	a	b	c	d	e	f	g	h
A	1	1	0	1	1	0	1	1
B	0	1	1	1	1	1	1	0
C	1	0	1	1	0	1	1	1

Jaccard distance

$$AB = \frac{4}{8} = \frac{1}{2}$$

$$AC = \frac{4}{8} = \frac{1}{2}$$

$$BC = \frac{4}{8} = \frac{1}{2}$$

Cosine similarity

$$AB = \frac{4}{\sqrt{6}\sqrt{6}} = \frac{4}{6} = \frac{2}{3}$$

$$AC = \frac{4}{\sqrt{6}\sqrt{6}} = \frac{2}{3}$$

$$BC = \frac{4}{\sqrt{6}\sqrt{6}} = \frac{2}{3}$$

distance

$$1 - \frac{2}{3} = \frac{1}{3}$$

$$1 - \frac{2}{3} = \frac{1}{3}$$

$$1 - \frac{2}{3} = \frac{1}{3}$$

(B)

Let 3, 4, 5 \Rightarrow 12, 1, blank \Rightarrow 0

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

Jaccard distance

$$AB = \frac{3}{5}$$

$$AC = \frac{4}{6} = \frac{2}{3}$$

$$BC = \frac{5}{6}$$

Cosine similarity

$$AB = \frac{3}{\sqrt{4}\sqrt{5}} = \frac{3}{2\sqrt{5}} = \frac{\sqrt{3}}{3}$$

$$AC = \frac{4}{\sqrt{4}\sqrt{4}} = \frac{4}{4} = 1$$

$$BC = \frac{5}{\sqrt{3}\sqrt{4}} = \frac{5}{\sqrt{12}} = \frac{5\sqrt{3}}{6}$$

distance

$$1 - \frac{\sqrt{3}}{3}$$

$$1 - 1 = 0$$

$$1 - \frac{5\sqrt{3}}{6}$$

(c)

$$\text{Avg } A = \frac{4+5+5+1+3+2}{6} = \frac{20}{6} = \frac{10}{3}$$

$$\text{Avg } B = \frac{3+4+3+1+2+1}{6} = \frac{14}{6} = \frac{7}{3}$$

$$\text{Avg } C = \frac{2+1+3+4+5+3}{6} = \frac{18}{6} = 3$$

1) (C)

Normalized matrix

	a	b	c	d	e	f	g	h
A	$\frac{2}{3}$	$\frac{5}{3}$		$\frac{5}{3}$	$-\frac{7}{3}$		$-\frac{1}{3}$	$-\frac{4}{3}$
B		$\frac{2}{3}$	$\frac{5}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$	
C	-1		-2	0		1	2	0

$$\text{Avg } A = \frac{10}{3} = 3\frac{1}{3}$$

$$B = \frac{7}{3} = 2\frac{1}{3}$$

$$C = 3$$

Cosine similarity

$$\begin{aligned}
 AB &= \frac{\frac{5}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{5}{3} + (-\frac{4}{3})(-\frac{7}{3}) + \frac{4}{9}}{\sqrt{\frac{4}{9} + \frac{25}{9} + \frac{25}{9} + \frac{49}{9} + \frac{1}{9} + \frac{16}{9}} \sqrt{\frac{4}{9} + \frac{25}{9} + \frac{4}{9} + \frac{16}{9} + \frac{1}{9} + \frac{16}{9}}} \\
 &= \frac{1}{\sqrt{\frac{120}{9}}} \sqrt{\frac{66}{9}} \\
 &= \frac{52}{2\sqrt{30 \times 66}} = \frac{52}{12\sqrt{55}} = \frac{13}{3\sqrt{55}} = 0.584
 \end{aligned}$$

$$\begin{aligned}
 AC &= \frac{-\frac{2}{3} + -\frac{2}{3}}{\sqrt{\frac{120}{9}} + \sqrt{1+4+1+4}} = \frac{-\frac{4}{3}}{\frac{1}{3}\sqrt{1200}} = \frac{-4}{20\sqrt{3}} = \frac{-\sqrt{3}}{15} = -0.12
 \end{aligned}$$

$$\begin{aligned}
 BC &= \frac{-\frac{10}{3} - \frac{1}{3} - \frac{8}{3}}{\sqrt{\frac{66}{9}} \sqrt{10}} = \frac{-19}{2\sqrt{165}} = -0.7395
 \end{aligned}$$

distance:

$$AB = 1 - 0.584 = 0.416$$

$$AC = 1 - (-0.12) = 1.12$$

$$BC = 1 - (-0.7395) = 1.7395$$