Cosc 4370 Homework 1 September 2022

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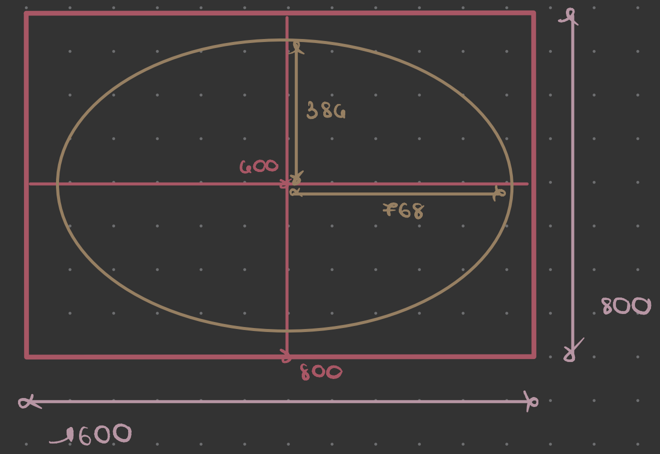


1) Problem: The assignment requires to rasterize the ellipse:

$$\left(\frac{x}{12}\right)^2 + \left(\frac{y}{6}\right)^2 = 64^2 \text{ where } x \geq 0$$

→ The shorter radius is $6 \times 64 = 384$

longer radius is $12 \times 64 = 768$

2) Method: Midpoint Ellipse

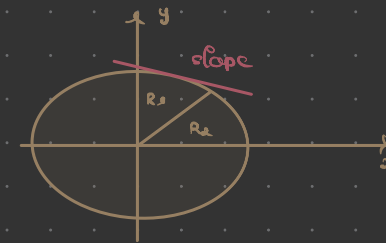
$$\left(\frac{x}{12}\right)^2 + \left(\frac{y}{6}\right)^2 = 64^2 \rightarrow \left(\frac{x}{768}\right)^2 + \left(\frac{y}{384}\right)^2 = 1$$

→ $f\text{ellipse}(x,y) = x^2 \times 384^2 + y^2 \times 768^2 - 768^2 \times 384^2$

→ $b^2x^2 + a^2y^2 - a^2 \times b^2$

Solve for the point on the first quadrant then other points on the remain quadrants will be mirrored to solve.

- for any tangents in the first quadrant, slope $m = dy/dx < 1$



- regarding region 1, the slope $m < -1$ and $m > -1$ for region 2

- take partial derivation of $f(x, y)$ with respect to $x \rightarrow f_x = 2 \times b^2 x$
  $y \rightarrow f_y = 2 \times a^2 y$

$\hookrightarrow dy/dx = -f_x / f_y = \dfrac{-2 \times b^2 x}{2 \times a^2 y} \rightarrow$ when $dy/dx > -1$ we finish region 1 and continue with region 2

for region 1

$$\begin{cases} x_{k+1} = x_k + 1 \\ y_{k+1} = y_k \text{ or } y_k - 1 \end{cases} \rightarrow \text{next point coordinate } (x_k + 1, y_k) \text{ or } (x_k + 1, y_k - 1)$$

$\hookrightarrow$ midpoint

$$\begin{cases} x_m = \dfrac{(x_k + 1) + (x_k + 1)}{2} = x_k + 1 \\ y_m = \dfrac{y_k + y_k - 1}{2} = y_k - 1/2 \end{cases} \rightarrow (x_m, y_m) = (x_k + 1, y_k - 1/2)$$

Substitute to the ellipse equation to get the decision parameter

$$P_{1k} \text{ (region 1)} = b^2 (x_k + 1)^2 + a^2 (y_k - 1/2)^2 - a^2 b^2$$

$$\hookrightarrow P_{1k+1} = b^2 [(x_k + 1) + 1]^2 + a^2 [(y_{k+1}) - 1/2]^2 - a^2 b^2$$

$$p_{1k+1} = b^2(x_k+1)^2 + 2 \cdot b^2(x_k+1) + b^2 + a^2(y_{k+1})^2 - a^2(y_{k+1}) + a^2 \cdot \frac{1}{4} - a^2b^2$$

Then $p_{1k+1} - p_{1k} = b^2(\cancel{x_k+1})^2 + 2 b^2(x_k+1) + b^2 + a^2(y_{k+1})^2 - a^2(y_{k+1}) + \cancel{a^2 \cdot \frac{1}{4}} - \cancel{a^2} \cdot b^2$

$$- \left[b^2(x_k+1)^2 + a^2(y_k - 1/2)^2 - a^2b^2 \right]$$

$$= -\{ b^2(\cancel{x_k+1})^2 + a^2 y_k^2 - a^2 y_k + \cancel{\tfrac{1}{4}a^2} - \cancel{a^2b^2}$$

$$p_{1k+1} - p_{1k} = 2b^2(x_k+1) + b^2 + a^2\{(y_{k+1})^2 - y_k^2\} - a^2\{(y_{k+1}) - y_k\}$$

$$\Rightarrow p_{1k+1} = p_{1k} + 2b^2(x_k+1) + b^2 + a^2\{(y_{k+1})^2 - y_k^2\} - a^2\{y_{k+1} - y_k\}$$

$$= p_{1k} + 2b^2(x_k+1) + b^2 + a^2(y_{k+1} - y_k) + a^2(y_{k+1} + y_k) - a^2(y_{k+1} - y_k)$$

$$= p_{1k} + 2b^2(x_k+1) + b^2 + a^2(y_{k+1} + y_k)$$

For region 1, the ellipse starts from $(x = 0, y = b)$ and substitute in P_{1k} we get

$$p_{1k}(0,b) = b^2(0+1)^2 + a^2(b - 1/2)^2 - a^2b^2$$

$$= b^2 + a^2b^2 - a^2b + \frac{1}{4}a^2 - a^2b^2$$

$$= b^2 + \tfrac{1}{4}a^2 - a^2b$$ → Initial Decision parameter for 1st region

Then for every point in region 1: if $P_{1k} < 0$ then the next coordinate is (x_k+1, y_k)

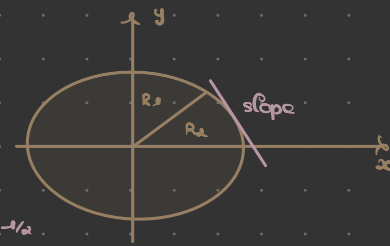
if $P_{1k} \geq 0$ then the next coordinate is (x_k+1, y_k-1)

for the region 2 The slope of tangent $m > 1$

- Since y decrease → $y_{k+1} = y_k - 1$
- x value → x_k or $x_k + 1$

↳ $(x_k, y_k - 1)$ or $(x_k + 1, y_k - 1)$

↳ Midpoint (x_m, y_m) = $\begin{cases} x_m = \dfrac{x_k + (x_k + 1)}{2} = x_k + 1/2 \\ y_m = \dfrac{(y_k - 1) + (y_k - 1)}{2} = y_k - 1 \end{cases}$



Substitute to ellipse equation to get P_{2k}

$$P_{2k} = (x_k + 1/2)^2 b^2 + (y_k - 1)^2 a^2 - a^2 b^2$$

$$P_{2k+1} = (x_{k+1} + 1/2)^2 b^2 + (y_{k+1} - 1)^2 a^2 - a^2 b^2$$

$$= (x_{k+1} + 1/2)^2 b^2 + [(y_k - 1) - 1]^2 a^2 - a^2 b^2$$

Then

$$P_{2k+1} - P_{2k} = \frac{[(x_{k+1})^2 + x_{k+1} + \frac{1}{4}]\, b^2 + a^2 [\cancel{(y_k - 1)^2} - 2(y_k - 1) + 1] - \cancel{a^2 b^2}}{- b^2 (x_k^2 + x_k + 1/4) + a^2 \cancel{(y_k - 1)^2} - \cancel{a^2 b^2}}$$

$$\frac{(x_{k+1})^2 b^2 + (x_{k+1}) b^2 - 2a^2(y_k - 1) + a^2}{- b^2 x_k^2 - b^2 x_k - b^2 1/4}$$

↳ $P_{2k+1} = P_{2k} + a^2 - 2a^2 (y_k - 1) + b^2 [(x_{k+1})^2 - (x_k)^2] + b^2 [x_{k+1} - x_k]$

for region 2, the ellipse starts when region 1 end.

Therefore,

$$\begin{cases} P_{2k} \geq 0 \text{ then the next coordinate is } (x_k, y_k - 1) \\ P_{2k} < 0 \text{ then the next coordinate is } (x_k + 1, y_k - 1) \end{cases}$$

Now since we get decision parameter for 2 regions in first quadrants we can use symmetric property of a point on the ellipse to get the other coordinate in the other 3 quadrants.

$(-x + center\_x, y + center\_y)$	$(x + center\_x, y + center\_y)$
$(-x + center\_x, -y + center\_y)$	$(x + center\_x, -y + center\_y)$

Scanning method

running x and y coordinates in nested for loop, checking if any pairs of x and y fit in the ellipse equation.

Note that I add a offset to the function to create the thickness for the edge

$$\begin{cases} f(x,y)\ \text{ellipse} < 0 \rightarrow \text{inside} \\ f(x,y)\ \text{ellipse} > 0 \rightarrow \text{outsi} \end{cases}$$

$$(x/12)^2 + (y/6)^2 \leq (64 + 0.1)^2$$

$$(x/12)^2 + (y/6)^2 \geq (64 - 0.1)^2$$