

Cosc 4370 Homework 1 September 2022

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17. Problem:

The assignment requires to rasterize the ellipse.

$$\left(\frac{x}{12}\right)^2 + \left(\frac{y}{6}\right)^2 = 64^2 \text{ where } x \geq 0$$

Major radius is $6 \times 64 = 384$

Minor radius is $12 \times 64 = 768$

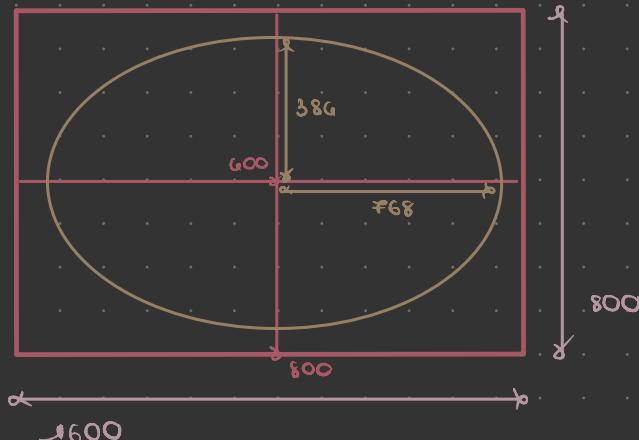
Method: Midpoint Ellipse

$$\left(\frac{x}{12}\right)^2 + \left(\frac{y}{6}\right)^2 = 64^2 \Rightarrow \left(\frac{x}{768}\right)^2 + \left(\frac{y}{384}\right)^2 = 1$$

$$\text{So if ellipse } (x, y) : x^2/384^2 + y^2/768^2 = 768^2/384^2$$

$$\therefore b^2x^2 + a^2y^2 = a^2 \times b^2$$

Solve for the point on the first quadrant then other points on the remain quadrants will be mirrored to solve.



Reference <https://www.geeksforgeeks.org/midpoint-ellipse-drawing-algorithm/>

for any tangents in the first quadrant, slope $m = \frac{dy}{dx} < 1$

regarding region 1, the slope $m < 1$ and $m > -1$ for region 2

Take partial derivation of $f(x, y)$ with respect to $x \rightarrow f_x = 2x b^2 x$

$$y \rightarrow f_y = 2x a^2 y$$

$\therefore \frac{dy}{dx} = -f_x/f_y = \frac{-2x b^2 x}{2x a^2 y} \Rightarrow$ when $\frac{dy}{dx} > -1$ we finish region 1 and continue with region 2

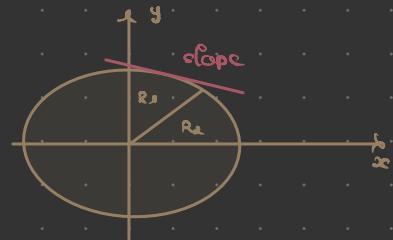
for region 1 $\left\{ \begin{array}{l} x_{k+1} = x_k + 1 \\ y_{k+1} = y_k \text{ or } y_k - 1 \end{array} \right\} \rightarrow$ next point coordinate (x_{k+1}, y_k) or $(x_k + 1, y_{k-1})$

midpoint $\left\{ \begin{array}{l} x_m = \frac{(x_k + 1) + (x_k + 1)}{2} = x_k + 1 \\ y_m = \frac{y_k + y_{k-1}}{2} = y_k - 1/2 \end{array} \right\} \rightarrow (x_m, y_m) = (x_k + 1, y_{k-1/2})$

Substitute to the ellipse equation to get the decision parameter

$$P_{1k}(\text{region 1}) = b^2 (x_{k+1})^2 + a^2 (y_k - 1/2)^2 - a^2 b^2$$

$$\therefore P_{1k+1} = b^2 [(x_k + 1)^2] + a^2 [(y_{k-1}) - 1/2]^2 - a^2 b^2$$



$$P_{x_{k+1}} = b^\alpha (x_{k+1})^\alpha + \alpha^\alpha b^\alpha (x_{k+1})^\alpha + b^\alpha + a^\alpha (y_{k+1})^\alpha - \alpha^\alpha (y_{k+1})^\alpha + a^\alpha \frac{1}{\alpha} - \alpha^2 b^2$$

$$\begin{aligned} \text{Then } P_{x_{k+1}} - P_{x_k} &= b^\alpha (x_{k+1})^\alpha + \alpha^\alpha b^\alpha (x_{k+1})^\alpha + b^\alpha + a^\alpha (y_{k+1})^\alpha - \alpha^\alpha (y_{k+1})^\alpha + a^\alpha \frac{1}{\alpha} - \cancel{\alpha^2 b^2} \\ &\quad - \cancel{\left[b^\alpha (x_k)^\alpha + a^\alpha (y_k - 1/\alpha)^\alpha - \alpha^2 b^2 \right]} \\ &= \cancel{\left[b^\alpha (x_{k+1})^\alpha + a^\alpha y_k^\alpha - a^\alpha y_k + \frac{1}{\alpha} \alpha^2 - \alpha^2 b^2 \right]} \end{aligned}$$

$$P_{x_{k+1}} - P_{x_k} = \cancel{\alpha^\alpha b^\alpha (x_{k+1})^\alpha} + b^\alpha + a^\alpha \cancel{\left[(y_{k+1})^\alpha - y_k^\alpha \right]} - a^\alpha \cancel{\left[(y_{k+1})^\alpha - y_k^\alpha \right]}$$

$$\begin{aligned} \therefore P_{x_{k+1}} &= P_{x_k} + \cancel{\alpha^\alpha b^\alpha (x_{k+1})^\alpha} + b^\alpha + a^\alpha \cancel{\left[(y_{k+1})^\alpha - y_k^\alpha \right]} - a^\alpha \cancel{\left[(y_{k+1})^\alpha - y_k^\alpha \right]} \\ &= P_{x_k} + \cancel{\alpha^\alpha b^\alpha (x_k)^\alpha} + b^\alpha + a^\alpha (y_{k+1} - y_k) + a^\alpha (y_{k+1} + y_k) - a^\alpha (y_{k+1} - y_k) \\ &= P_{x_k} + \cancel{\alpha^\alpha b^\alpha (x_k)^\alpha} + b^\alpha + a^\alpha (y_{k+1} + y_k) \end{aligned}$$

for region 1, the ellipse starts from $(x=0, y=b)$ and substitute in P_{x_k} we get

$$\begin{aligned} P_{x_k}(0, b) &= b^\alpha (0 + 1)^\alpha + a^\alpha (b - 1/\alpha)^\alpha - a^\alpha b^\alpha \\ &= b^\alpha + a^\alpha b^\alpha - a^\alpha b + \frac{1}{\alpha} a^\alpha - a^\alpha b^2 \end{aligned}$$

$= b^\alpha + \frac{1}{\alpha} a^\alpha - a^\alpha b \rightarrow$ initial condition parameter for 1st region

Then for every point in region 1. if $P_{x_k} < 0$ then the next coordinate is (x_{k+1}, y_k) .
 if $P_{x_k} > 0$. then the next coordinate is $(x_k + 1, y_{k+1})$

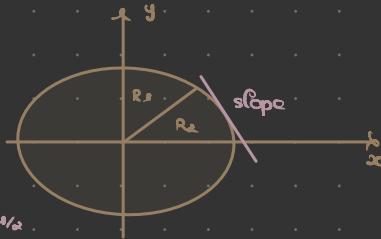
for the region 2. The slope of tangent in R_2

α Since y decreases $\rightarrow y_{k+1} = y_k - 1$.

α x value $\rightarrow x_k$ on x_{k+1} .

$\hookrightarrow (x_k, y_k - 1)$ on (x_{k+1}, y_{k+1}) .

$$\hookrightarrow \text{Midpoint } (x_m, y_m) = \begin{cases} x_m = \frac{x_k + (x_{k+1})}{2} = x_k + \frac{-1}{2} \\ y_m = \frac{(y_k - 1) + (y_{k+1})}{2} = y_k - 1 \end{cases}$$



Substitute to ellipse equation to get P_{2k}

$$P_{2k} = (x_k)^2/a^2 + (y_k)^2/b^2 - 1/a^2$$

$$P_{2k+1} = (x_{k+1})^2/a^2 + (y_{k+1})^2/b^2 - 1/a^2$$

$$= (x_{k+1})^2/a^2 + [(y_k - 1)]^2/b^2 - 1/a^2$$

$$\text{Then } P_{2k+1} - P_{2k} = \frac{[(x_{k+1})^2/a^2 + x_{k+1} \cdot \frac{1}{a} b^2 + a^2(y_k - 1)^2/b^2 - 2(y_k - 1) + 1] - a^2/b^2}{b^2(x_k^2/a^2 + x_k \cdot -1/a) + a^2(y_k - 1)^2/b^2 - a^2/b^2}$$

$$= \frac{(x_{k+1})^2/b^2 + (x_{k+1})b^2 - 2a^2(y_k - 1) + a^2}{b^2x_k^2/a^2 - b^2x_k - b^2/a^2}$$

$$\hookrightarrow P_{2k+1} = P_{2k} + a^2 - 2a^2(y_k - 1) + b^2[(x_{k+1})^2/a^2 - (x_k)^2/a^2] + b^2[x_{k+1} - x_k]$$

for region 2, the ellipse starts when region 1 end.

Therefore,

- $P_{xk} > 0$ then the next coordinate is $(x_k, y_k - 1)$
- $P_{xk} < 0$ then the next coordinate is $(x_k + 1, y_k - 1)$

Now since we get decision parameter for 2 regions in first quadrants, we can use symmetric property of a point on the ellipse to get the other coordinate in the other 3 quadrants.

$$(-x + \text{center}_x, y + \text{center}_y) \rightarrow$$

$$(x + \text{center}_x, y + \text{center}_y)$$

$$(-x + \text{center}_x, -y + \text{center}_y) \rightarrow$$

$$(x + \text{center}_x, -y + \text{center}_y)$$

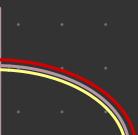
Scanning method

running x and y coordinates in nested for loop, checking if any pairs of x and y fit in the ellipse equation.

Note that I add a offset to the function to create the thickness for the edge of $f(x,y)$ ellipse $< 0 \rightarrow$ inside

$$(x/12)^2 + (y/6)^2 \leq (64 + 0.1)^2$$

$$(x/12)^2 + (y/6)^2 \geq (64 - 0.1)^2$$



$f(x,y)$ ellipse $> 0 \rightarrow$ outside

