Discrete trait models

Correlation of traits

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Correlating two discrete traits

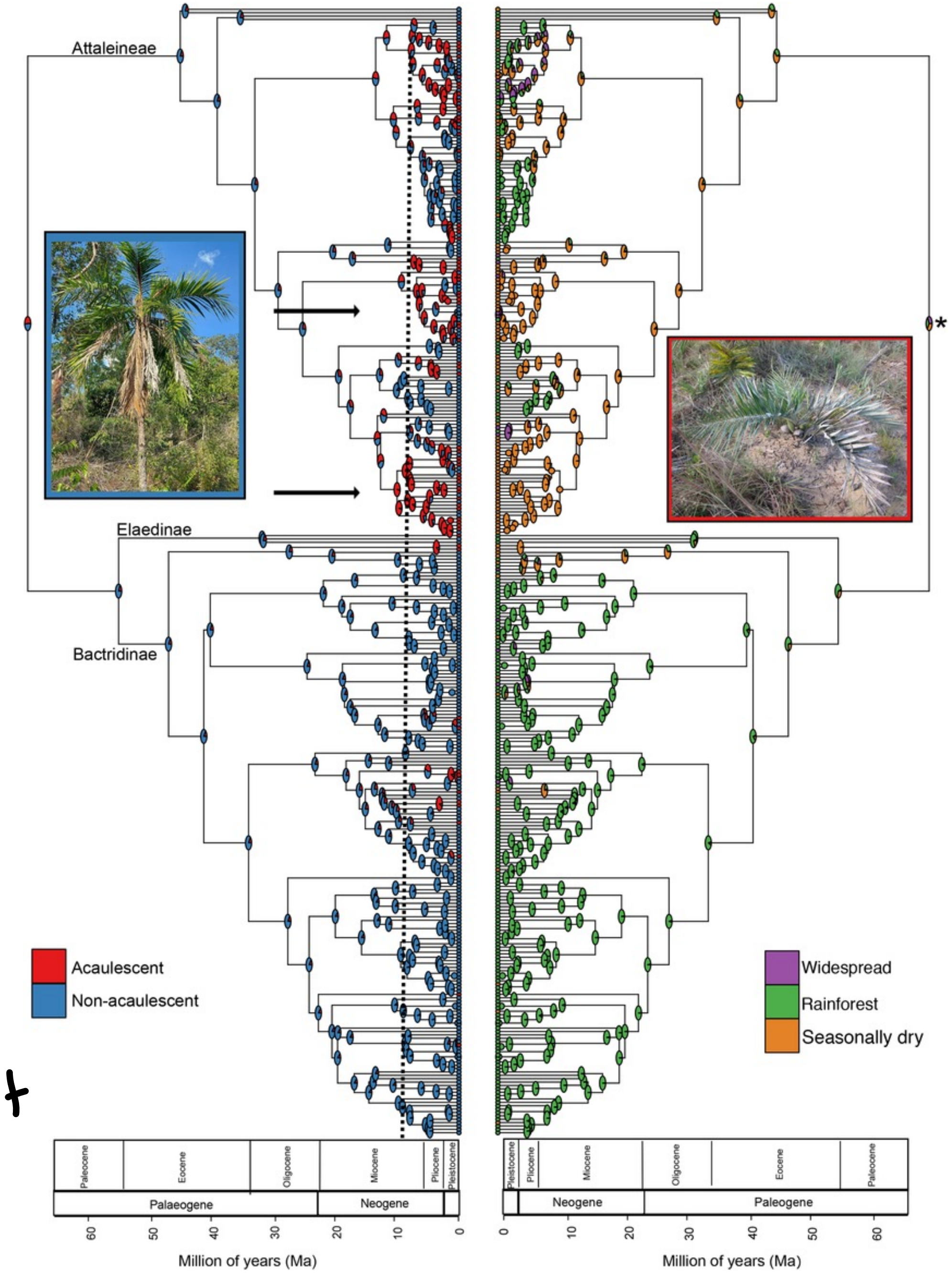
Mark Pagel -1994

Two traits

Acaulescent: $X_1 = \begin{cases} 0 - \text{non acaulescent} \\ 1 - \text{acaulescent} \end{cases}$

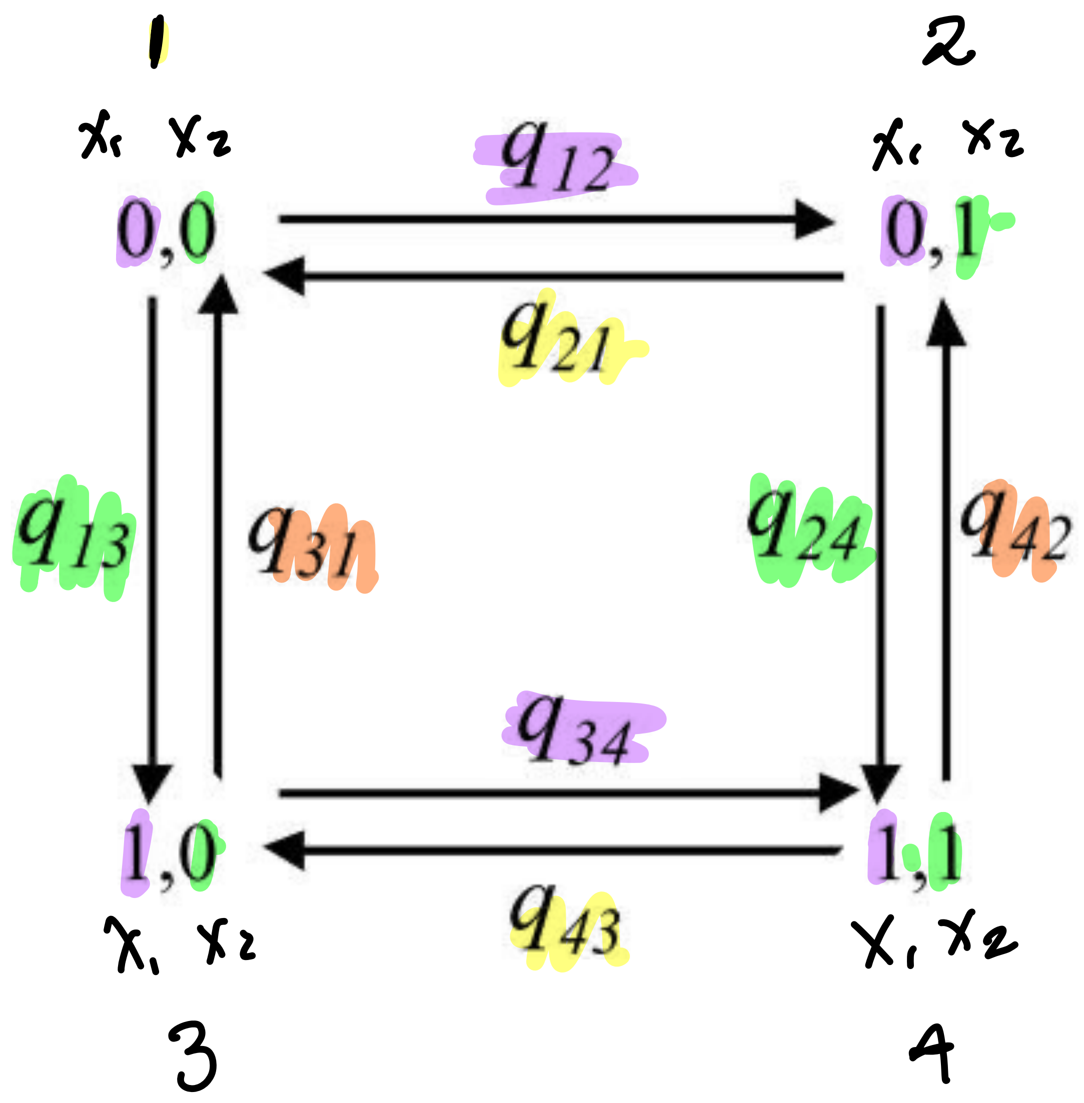
Water Availability: $X_2 = \begin{cases} 0 - \text{dry} \\ 1 - \text{Rainforest} \end{cases}$

Cassia Silva et al. Ecography. 2022



Acaulescent: No visible above-ground stem

Correlated model



- No double transitions

NO $0,0 \rightleftarrows 1,1$

NO $0,1 \rightleftarrows 1,0$

- 8 rates

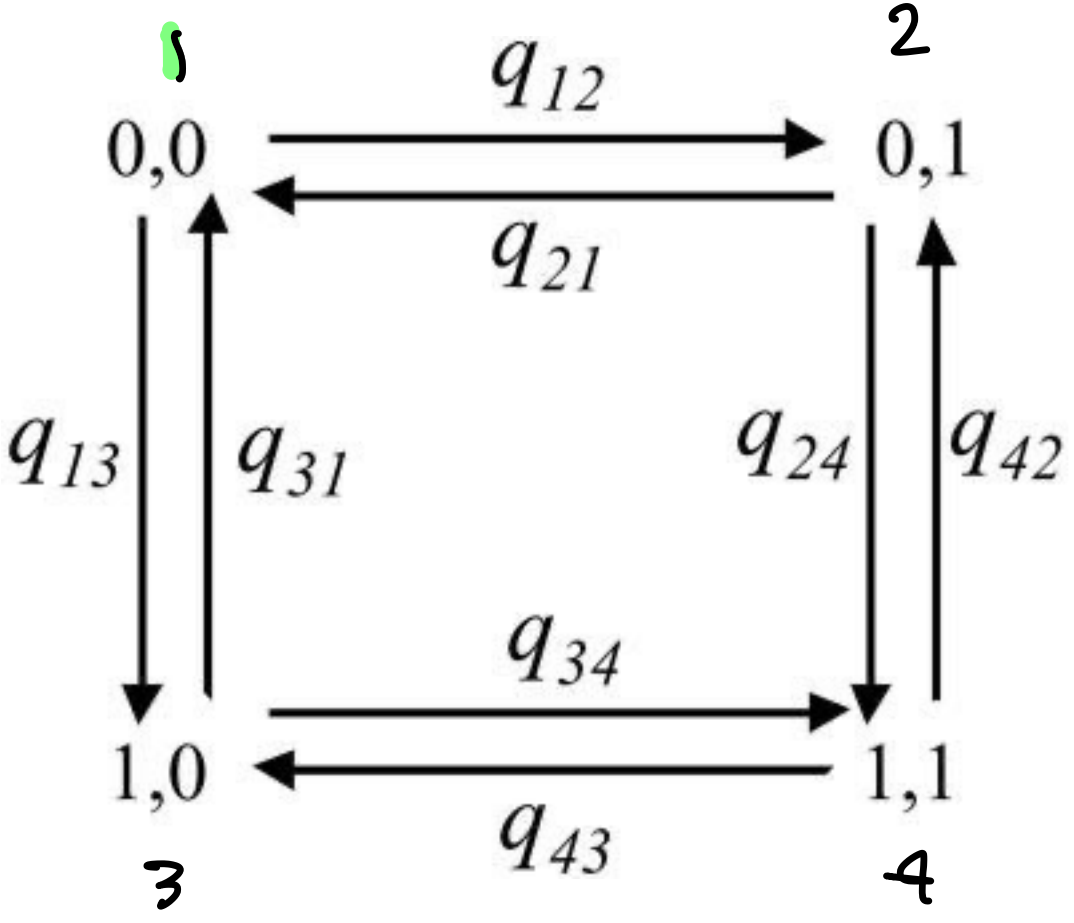
$q_{13} \neq q_{24}$

$q_{12} \neq q_{34}$

$q_{31} \neq q_{42}$

$q_{21} \neq q_{43}$

Correlated model



$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} -(q_{12}+q_{13}) & q_{12} & q_{13} & 0 \\ q_{21} & - & 0 & q_{24} \\ q_{31} & 0 & - & q_{34} \\ 0 & q_{42} & q_{43} & - \end{pmatrix} \end{array}$$

Uncorrelated model

x_1 X_2

(α, β) are from original pagel and Madison's approach

$\alpha_1 = q_{12} = q_{34} = q_1 \quad q_{13} = q_{24} = \alpha_2 = q_3$

$\beta_1 = q_{21} = q_{43} = q_2 \quad q_{31} = q_{42} = \beta_2 = q_4$

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} - & q_1 & q_3 & 0 \\ q_2 & - & 0 & q_3 \\ q_4 & 0 & - & q_1 \\ 0 & q_4 & q_2 & - \end{pmatrix} \end{array}$$

Comparing nested models

- Likelihood Ratio Test

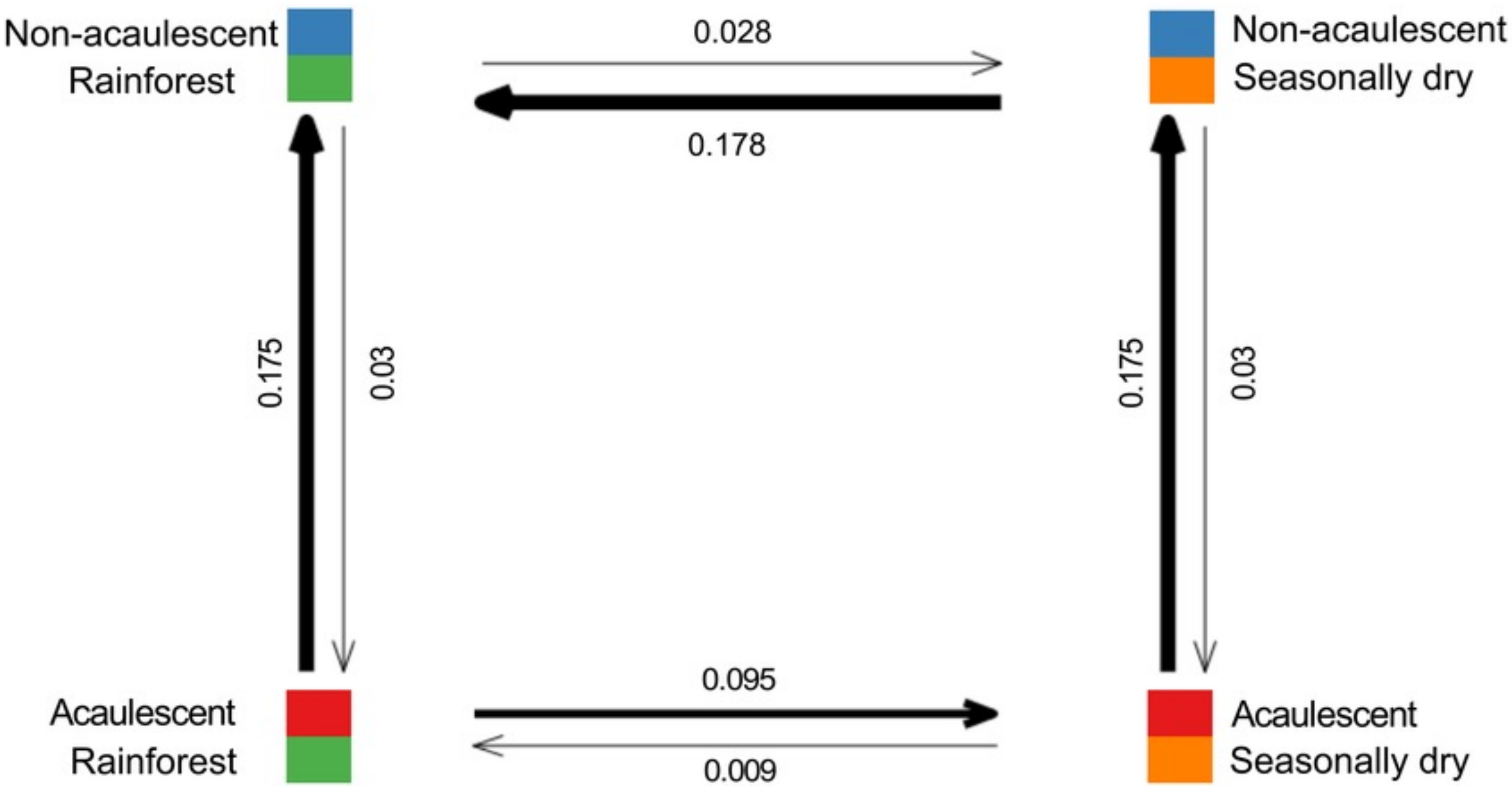
- Null hypothesis:

- Test statistic: $LRT = -2\times \log(\frac{Likelihood\ (Uncorrelated\ model)}{Likelihood\ (Correlated\ model)})$

4 parameters

8 parameters

- $LTR \sim \chi^2$ with degrees of freedom k= #parameters full model-#parameters reduced model $= 8 - 4 = 4$



Non-acaulescent
Rainforest
0.028
0.178
Non-acaulescent
Seasonally dry
0.175
0.03
0.175
0.03
0.095
0.009
Acaulescent
Rainforest
Acaulescent
Seasonally dry