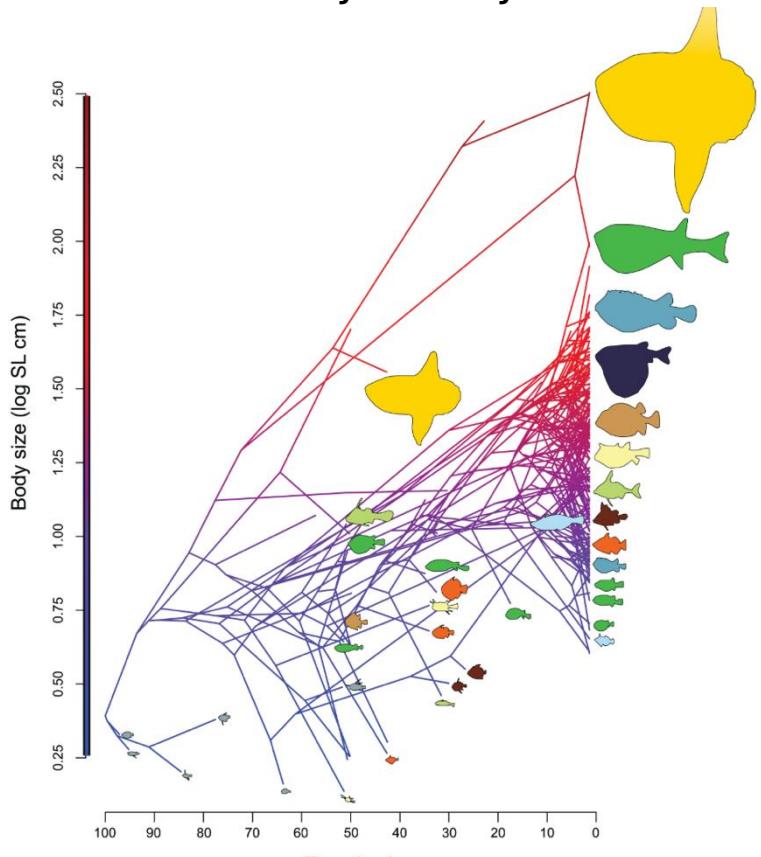


# Introduction to Modeling Continuous Traits

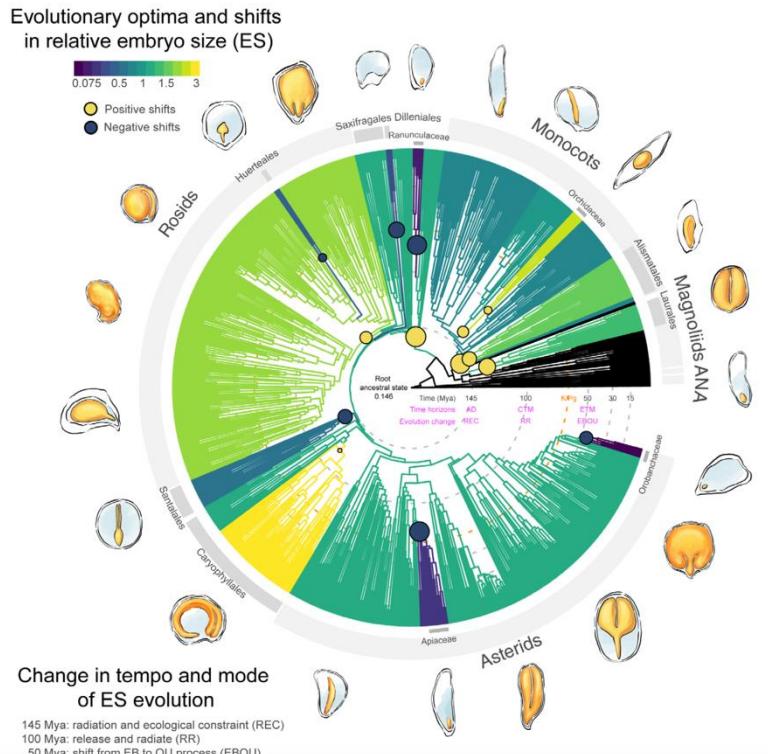
# Continuous traits examples

Diversity in body size



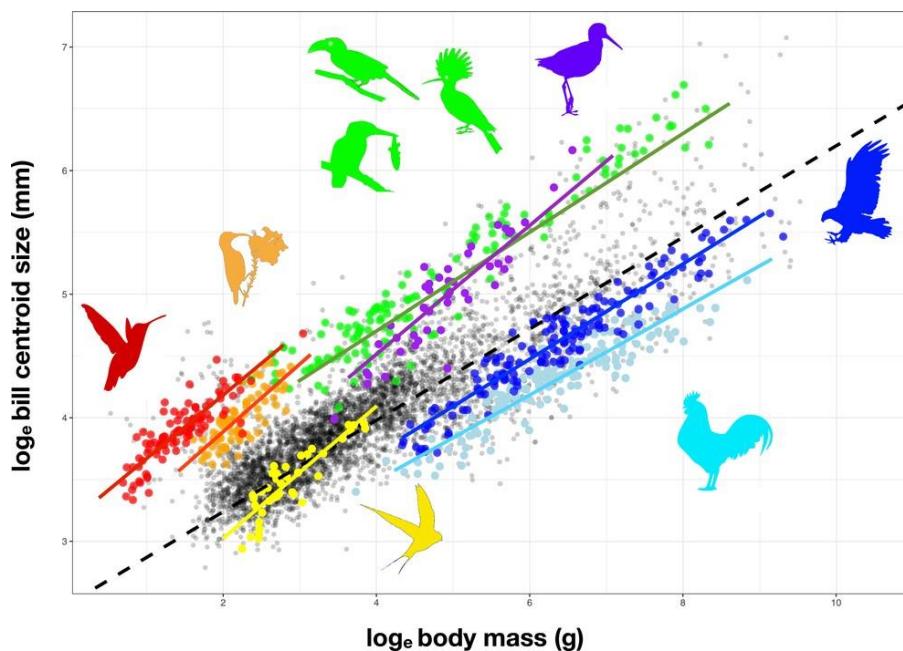
Troyer et al. 2023

Shifts in embryo size



Vandeloek and Carta. 2025

Scaling relationship beak vs. body mass

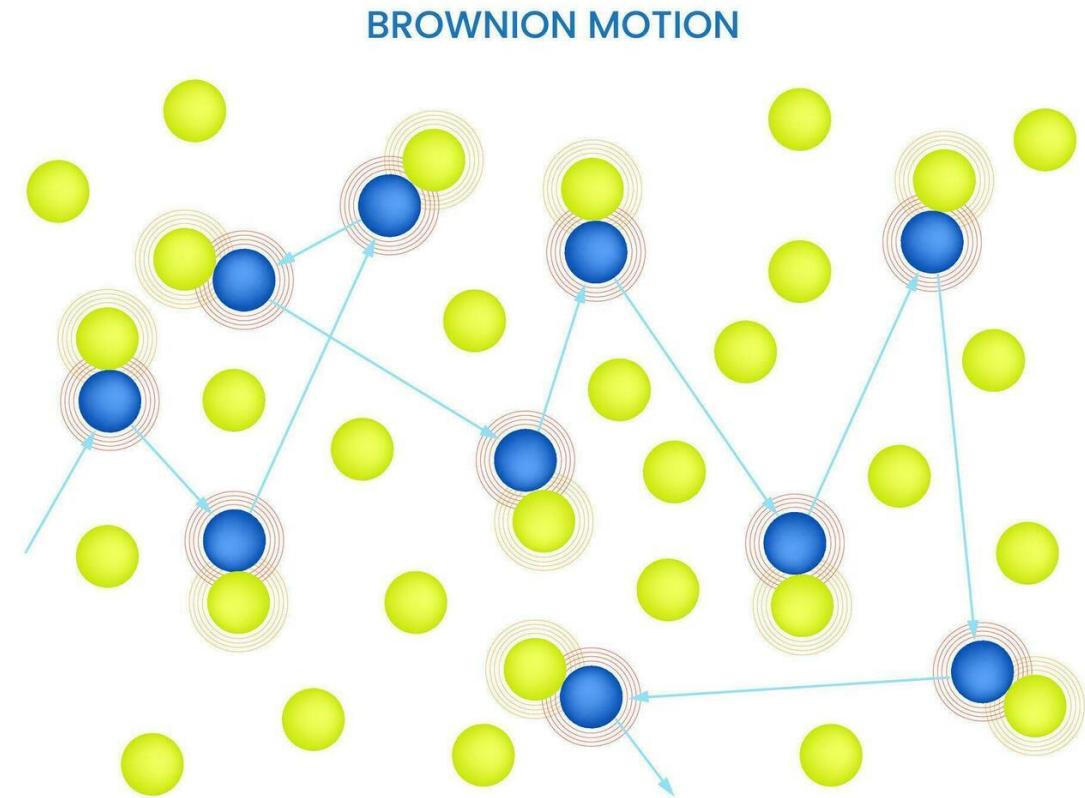


Rombaut et al. 2022

# The “simplest” model- Brownian motion

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- Brownian motion models the evolution of a trait through time.
- Trait value changes randomly up and down in value over time- Sometimes called a “random walk” model
- The statistical process of Brownian motion was originally invented to describe the motion of particles suspended in a fluid.



The motion of the object is due to the sum of a large number of very small random forces

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- Brownian motion is a popular model in comparative biology
- It captures the way traits might evolve under a reasonably wide range of scenarios.
- Brownian motion has some very convenient statistical properties that allow relatively simple analyses and calculations on phylogenetic trees
- Brownian motion **IS NOT** genetic drift



# Mathematics of Brownian motion

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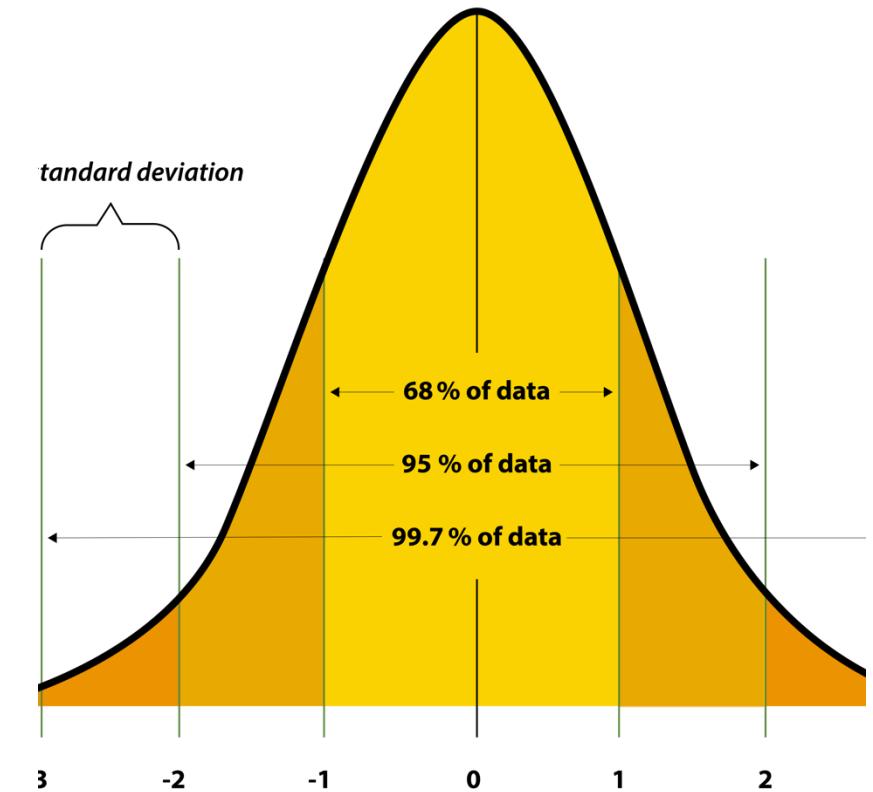
Mean value:

The mean value is the value at the root of our phylogeny (or start of the simulation) before any trait change happens

Evolutionary rate parameter:

We call this parameter “drift”, but it is not drift simply a variance on how much a trait can change.

Under Brownian motion, changes in trait values over any interval of time are always drawn from a normal distribution with mean  $\theta$  and variance  $\sigma^2$



# Brownian motion properties

When  $X(t)$  is the value of the continuous trait at time  $t$  we have that

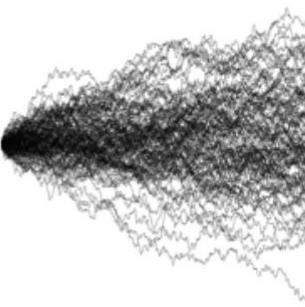
1.  $E[X(t)] = \theta$
2. Each successive interval of the “walk” is independent
3.  $X(t) \sim N(\theta, \sigma^2 t)$  the value of the trait is distributed Normal

Simulations of Brownian Motion  
(without phylogenetic trees!)

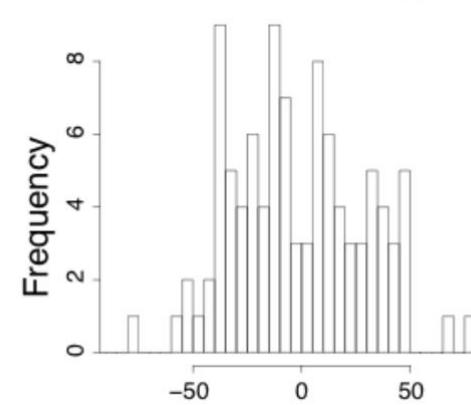
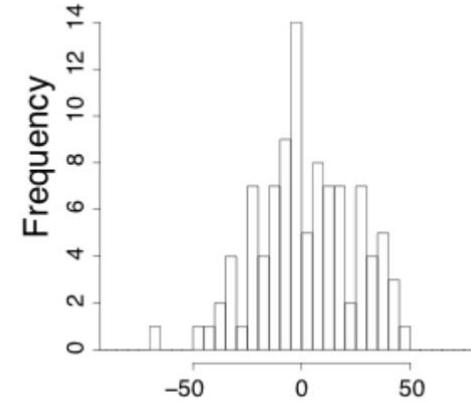
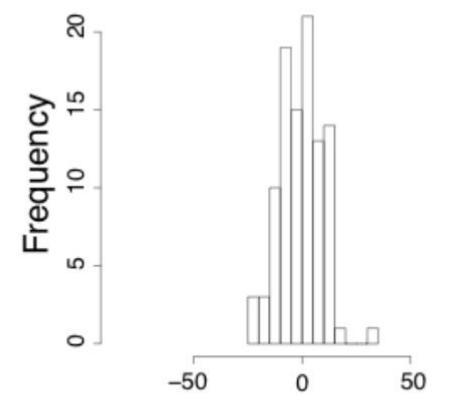
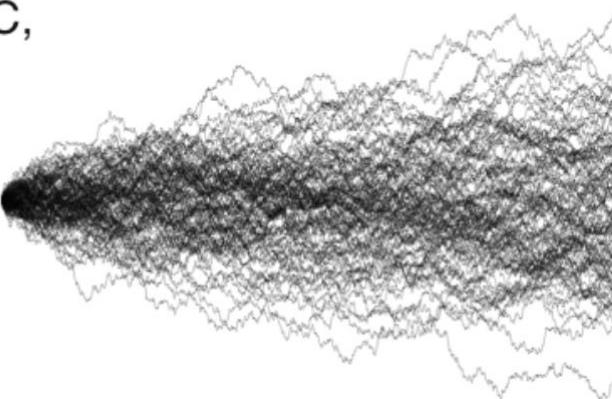
A,



B,



C,



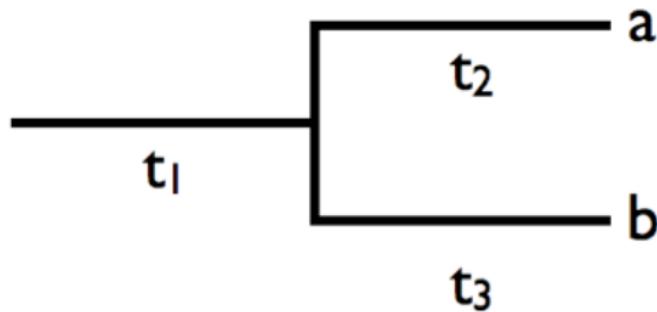
# Brownian motion over phylogenetic trees

- Evolution of a trait along a single branch with length  $t_1$

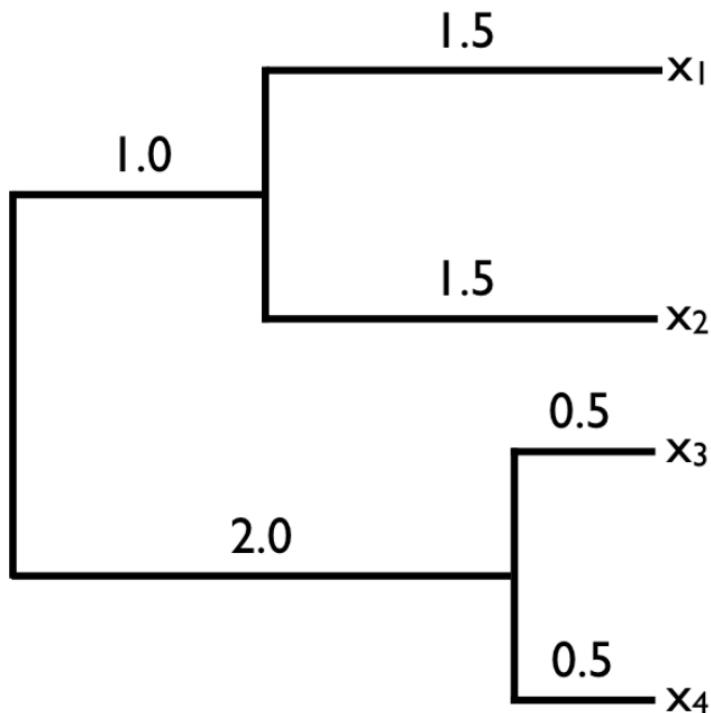
$$\begin{array}{ccc} X(0) & \xrightarrow{\hspace{2cm}} & X(t_1) \\ & t_1 & \end{array}$$

# Brownian motion over phylogenetic trees

- a and b are not independent of each other. Instead, the two species share one branch in common .
- Each tip trait value can be thought of as an ancestral value plus the sum of two evolutionary changes: one (from branch 1) that is shared between the two species and one that is unique.



# Variance-Covariance matrix



# Simulating Brownian motion

Let's try it with the  
phytools tutorial!

