Entropy Accumulation in Device-independent Protocols

QIP17

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Outline

1. Introduction to device-independence

2. The difficulty of proving security

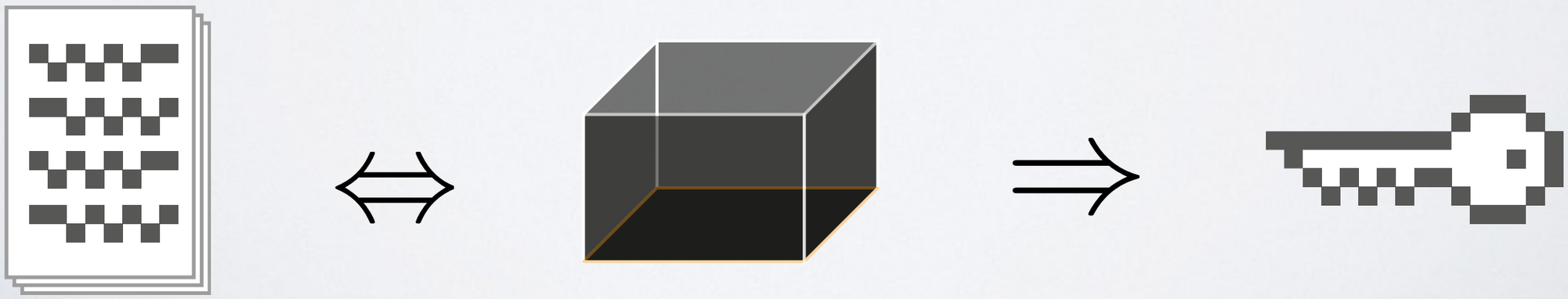
3. Overview …

Brief introduction to

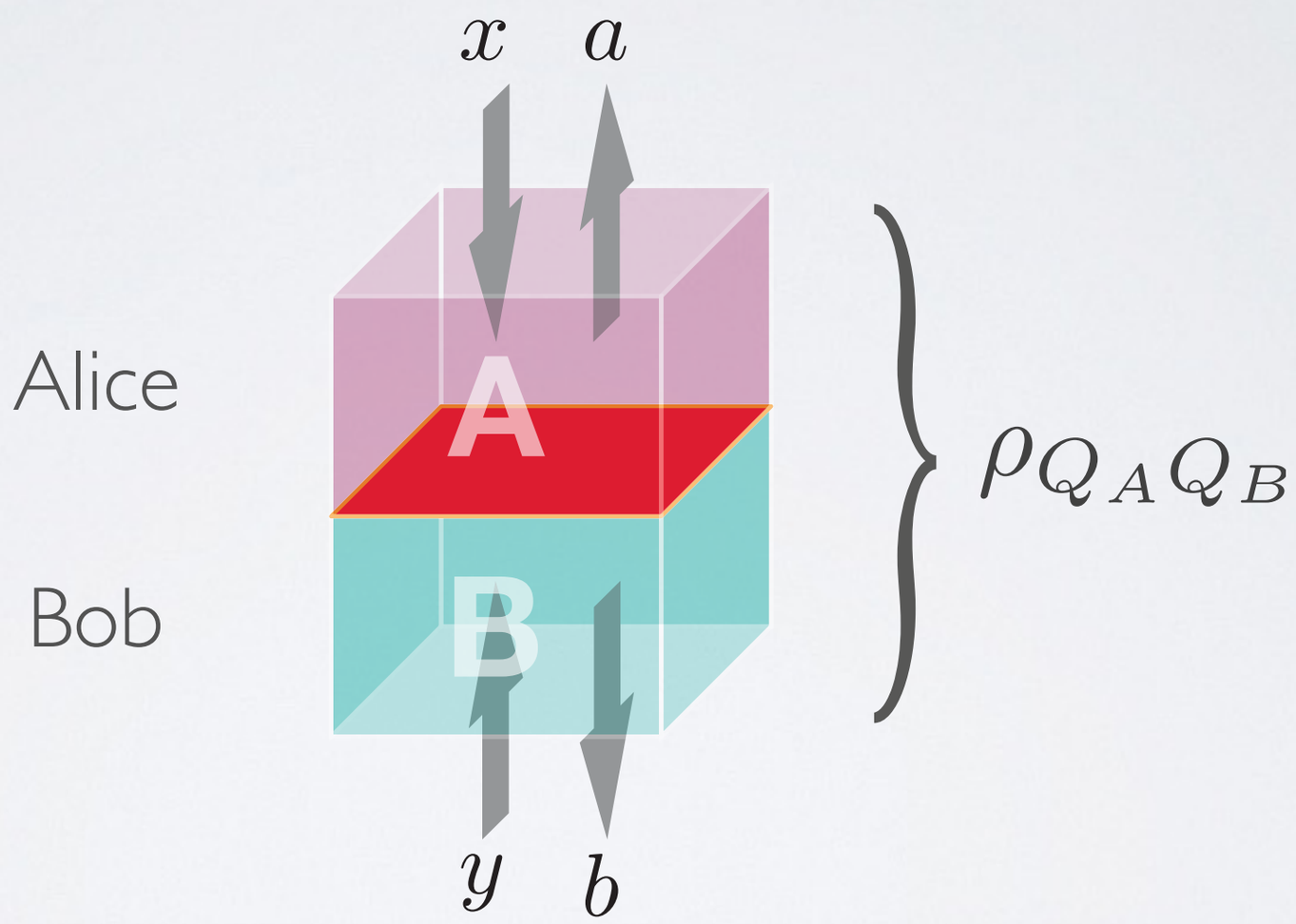
Device-independent Cryptography

The concept of DI

- Alice and Bob share an uncharacterised device
- They interact with it according to some known protocol (e.g., DI quantum key distribution protocol)
- They either abort or accomplish their task (e.g., output a good key)

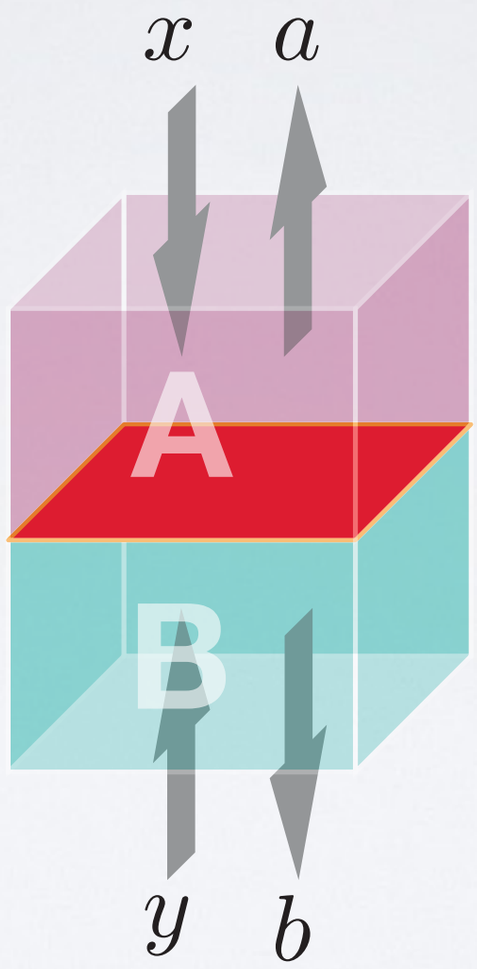


Bell inequality / game



Bell inequality / game

No communication



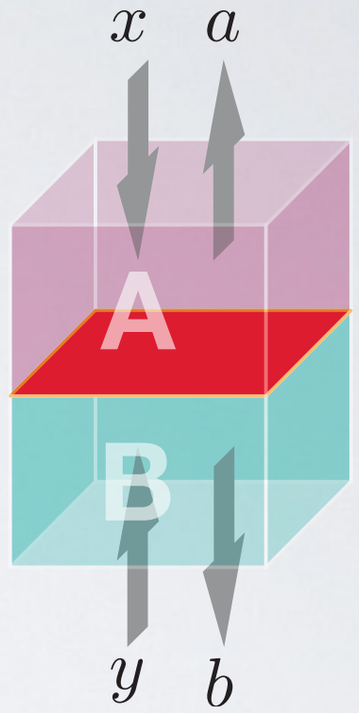
Winning condition: $\omega(a, b, x, y) \in \{0, 1\}$

Bell inequality / game

- Winning prob. of the device: $\omega \in [0, 1]$
- Bell inequality: $\forall \omega_c \quad \omega_c \leq I$
- Quantum advantage (violation): $\exists \omega_q \quad \omega_q > I$
- $\Rightarrow$ some secret randomness in the outputs with respect to an adversary holding a purification of $\rho_{Q_A Q_B}$

Example: the CHSH game

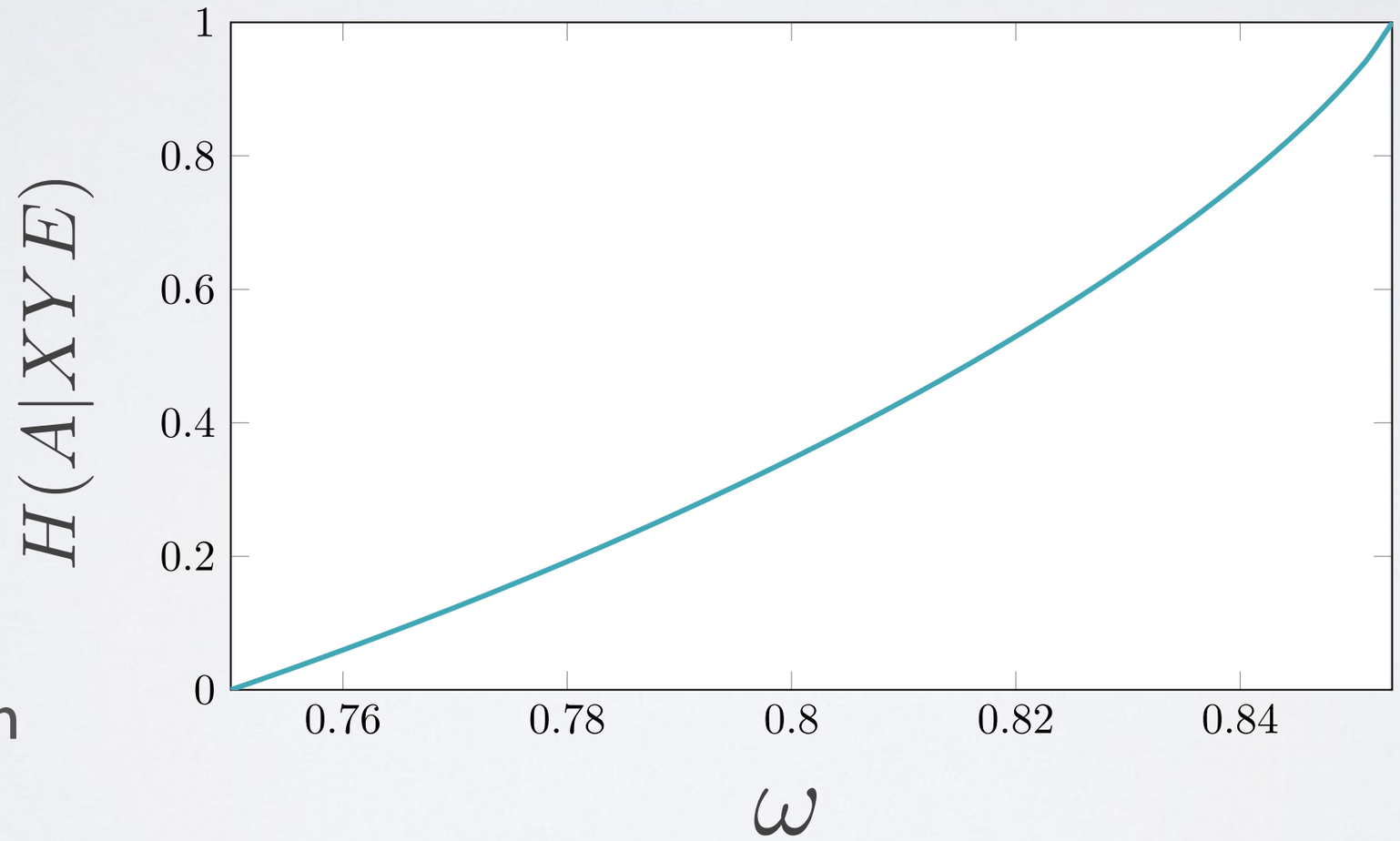
Alice:	Input	$x \in \{0, 1\}$
	Output	$a \in \{0, 1\}$
Bob:	Input	$y \in \{0, 1\}$
	Output	$b \in \{0, 1\}$
Win:		$a \oplus b = x \cdot y$



- Best classical strategy: 75% winning
- Best quantum strategy: ~85% winning
- Quantum advantage

Example: the CHSH game

- Quantum advantage implies secret randomness:

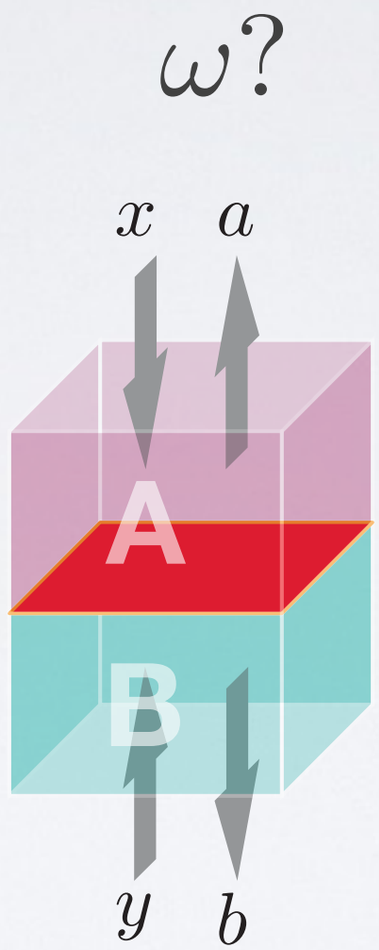


How **random** A is from **Eve**'s point of view

[Pironio, Acìn, Brunner *et al.*, 09]

The Difficulty of Proving Security

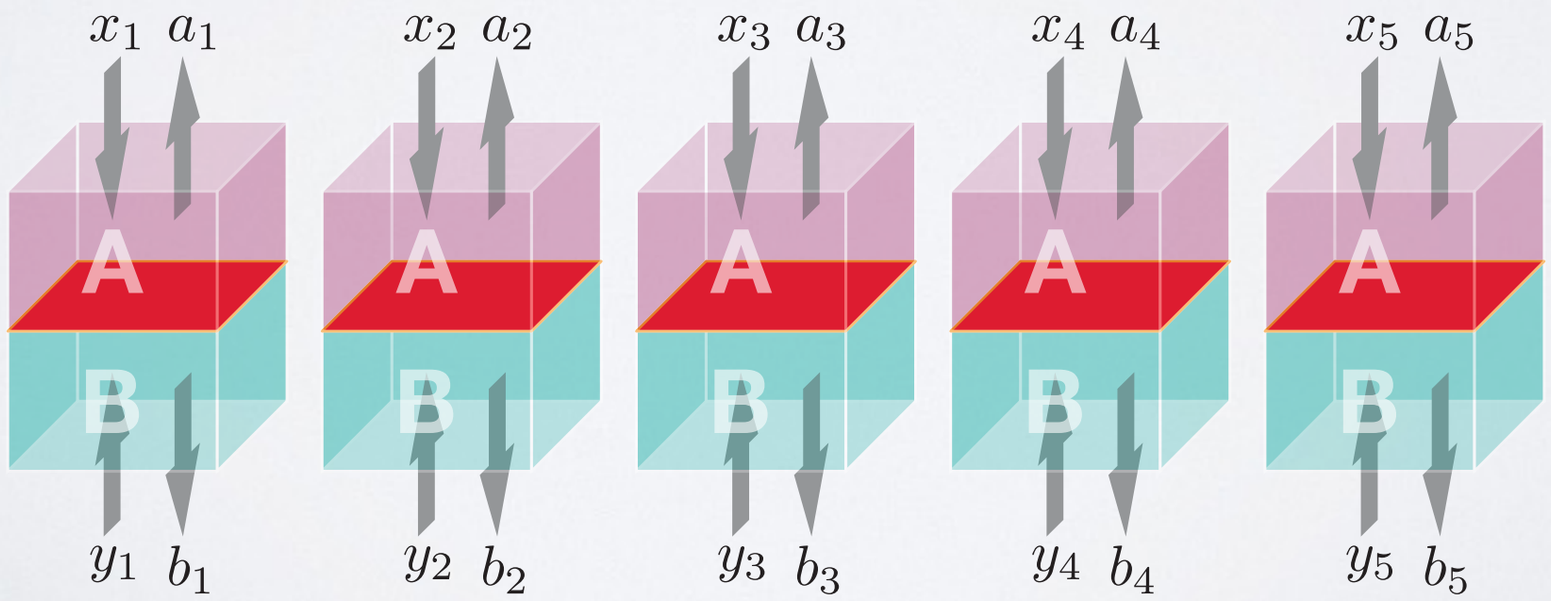
The difficulty of proving security



The IID assumption

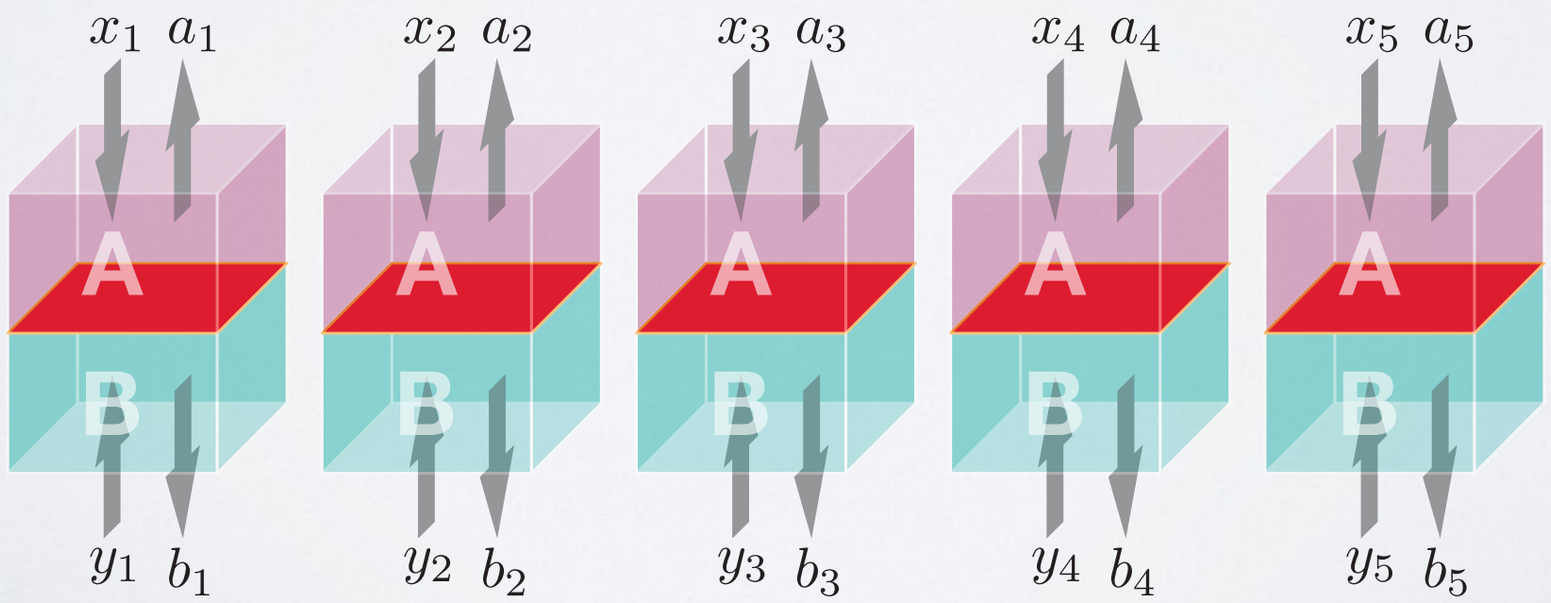
- Play the game many times independently and identically
- Estimate the winning probability in one device
- The total amount of entropy is *roughly* the number of games × entropy in one game

Simple! ✔



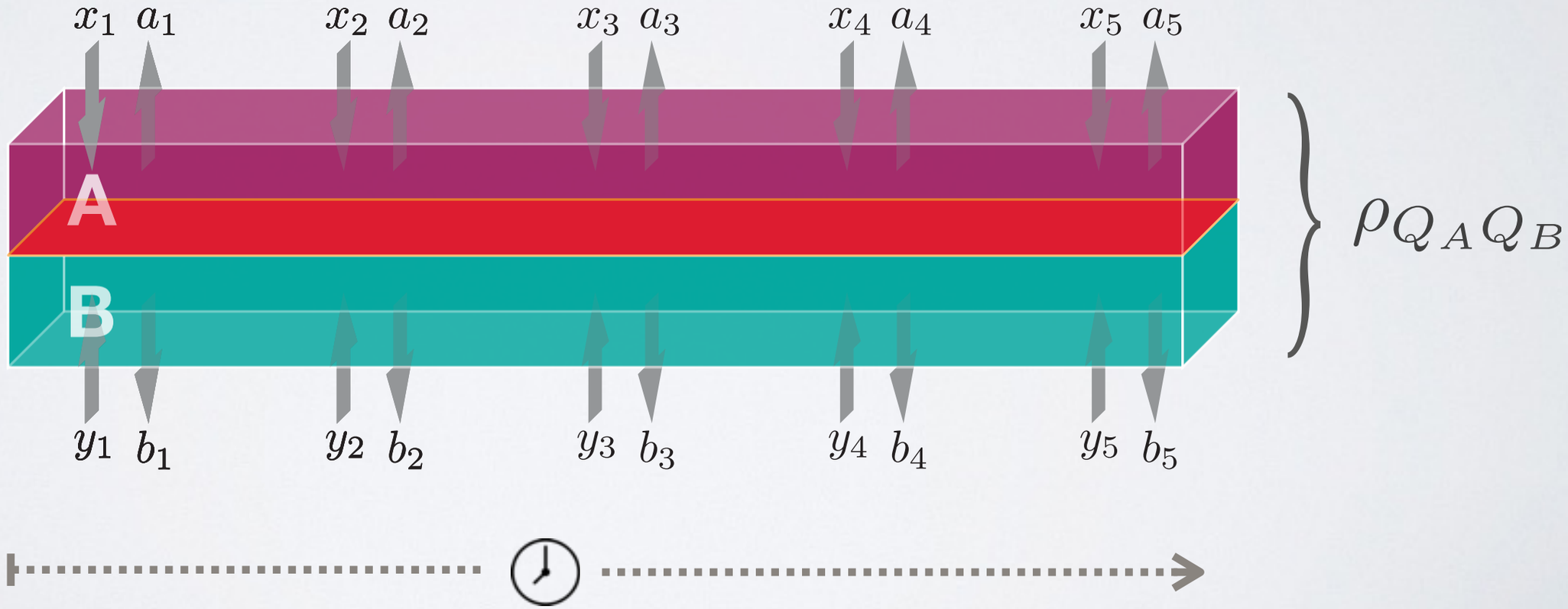
The IID assumption

- IID is a strong assumption! (e.g., no memory at all)
- Cannot use de Finetti theorems (in contrast to standard QKD for example)



The general case

- One component to each party
- Sequential interaction with Alice and Bob's components



Previous DIQKD works

[Ekert, 91]
[Mayers and Yao, 98]

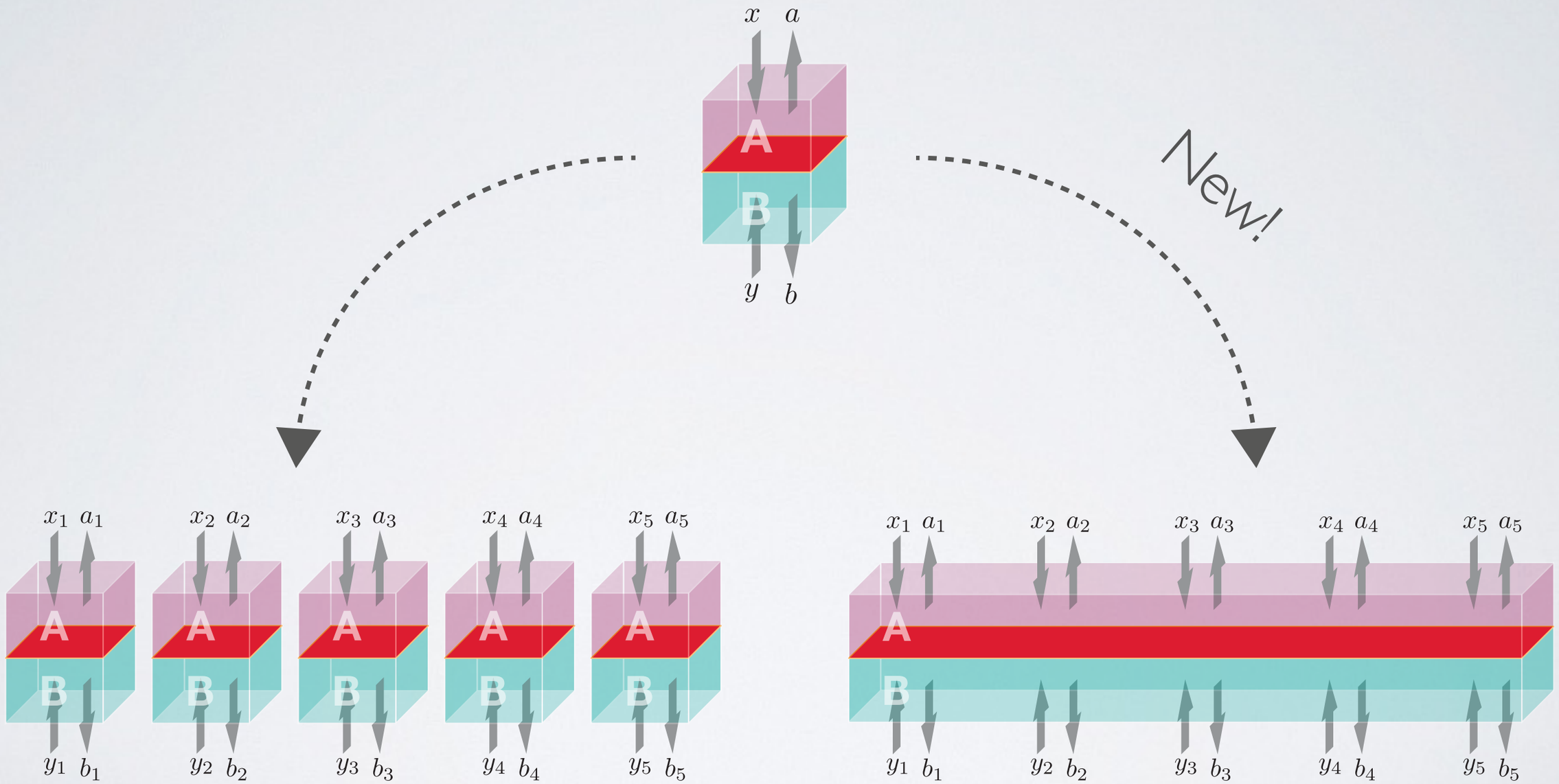
[Barrett, Hardy, and Kent, 05]
Proof of concept

[Pironio, Acìn, Brunner *et al.*, 09]
IID + asymptotic
Optimal rates! ✔

General security
[Reichardt, Unger, and Vazirani, 13]
[Vazirani and Vidick, 14]
[Miller and Shi, 14]

Overview

Overview


x a
A
B
y b
New!
x_1 a_1 x_2 a_2 x_3 a_3 x_4 a_4 x_5 a_5
A
B
y_1 b_1 y_2 b_2 y_3 b_3 y_4 b_4 y_5 b_5
IID
x_1 a_1 x_2 a_2 x_3 a_3 x_4 a_4 x_5 a_5
A
B
y_1 b_1 y_2 b_2 y_3 b_3 y_4 b_4 y_5 b_5
Sequential

Outline of the rest of talk

4. Security under the IID assumption

5. General security proof

   - New tool: the Entropy Accumulation Theorem
   - Application: new results for DI cryptography

6. Summary and open questions

Security Proof under the IID Assumption

Proving security

- Main task: lower-bounding the smooth min-entropy

$$H_{\min}^{\varepsilon}(K|E)$$

where K is the raw data, E the quantum side-information belonging to the adversary, and ε a security parameter

- Tightly determines the maximal length of an extractable secret key

Security — IID

- $K = K_1 \dots K_n$ IID random variables
- $E = E_1 \dots E_n$ IID quantum side-information

- For the **von-Neumann entropy**:

$$\begin{aligned} H(K_1 \dots K_n | E_1 \dots E_n) &= \sum_i H(K_i | E_1 \dots E_n K_1 \dots K_{i-1}) \\ &= \sum_i H(K_i | E_i) \\ &= n H(K_1 | E_1) \end{aligned}$$

Security — IID

- $K = K_1 \dots K_n$ IID random variables
- $E = E_1 \dots E_n$ IID quantum side-information
- For the **smooth min-entropy**:

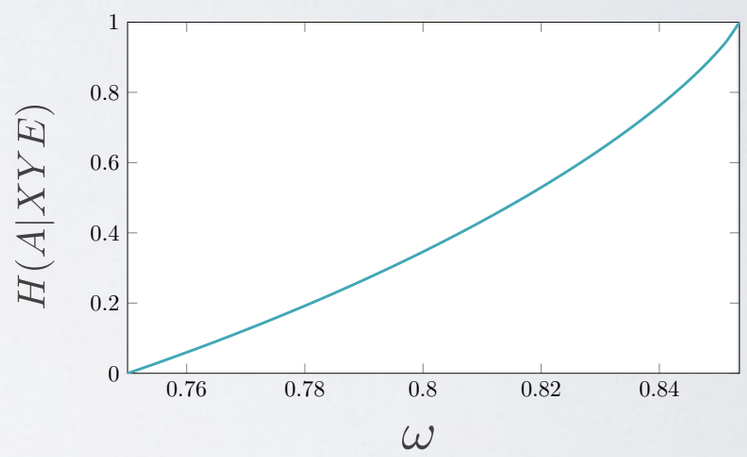
$$H_{\min}^{\varepsilon}(K|E) \geq nH(K_1|E_1) - c_\varepsilon \sqrt{n}$$

Quantum Asymptotic Equipartition Property
[Tomamichel, Colbeck, and Renner, 09]

Security — IID

1. Play the game many times and calculate the average winning probability

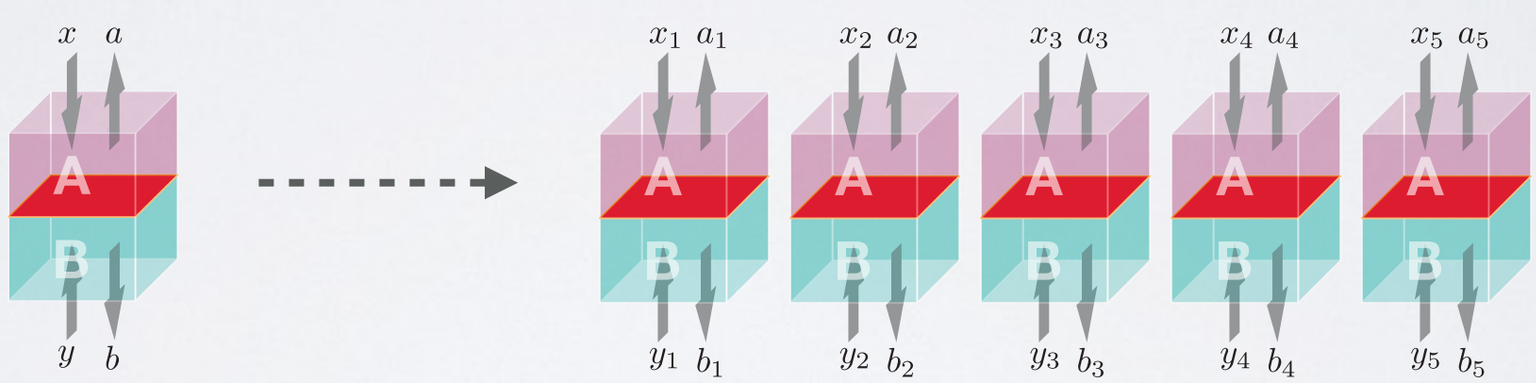
2. Use the single-round relation between the winning probability and the von-Neumann entropy



3. Plug into the quantum AEP: total smooth min-entropy is $nH(K_1|E_1)$ in first order

Security — IID (remarks)

- Need to understand only the physics of a single-round Simple! ✔



- The von-Neumann entropy is the relevant single-round quantity Tight! ✔

Security — IID

1. Play the game many times and calculate the average winning probability

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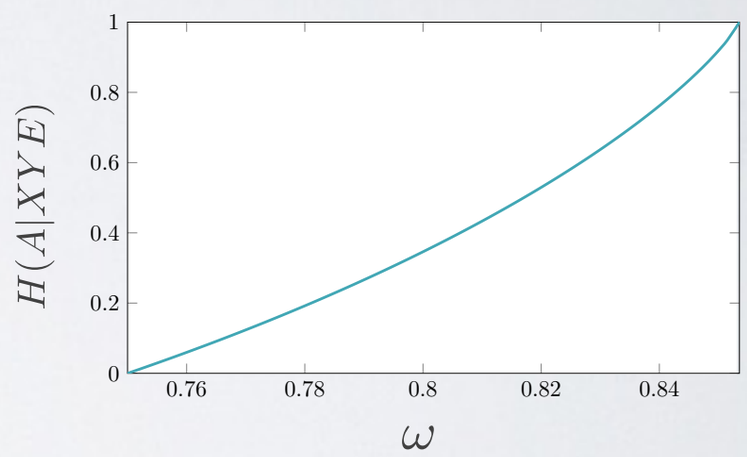


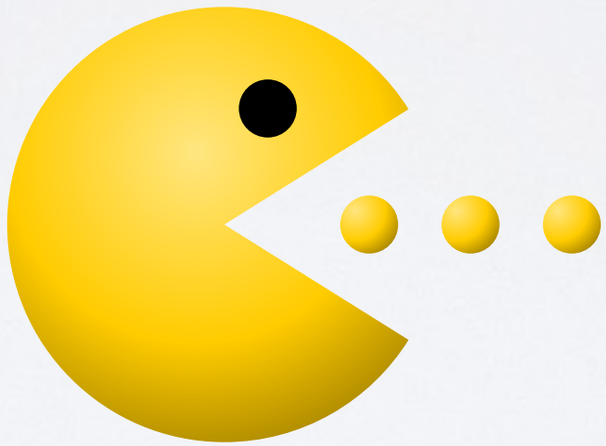
3. Plug into the quantum AEP: total smooth min-entropy is $nH(K_1|E_1)$ in first order

General Security Proof

General security

- Still need to lower-bound $H_{\min}^{\varepsilon}(K|E)$
- Instead of IID behaviour of the device, consider more general sequential processes
- "Extend" the quantum AEP to the sequential scenario $\Rightarrow$ The Entropy Accumulation Theorem

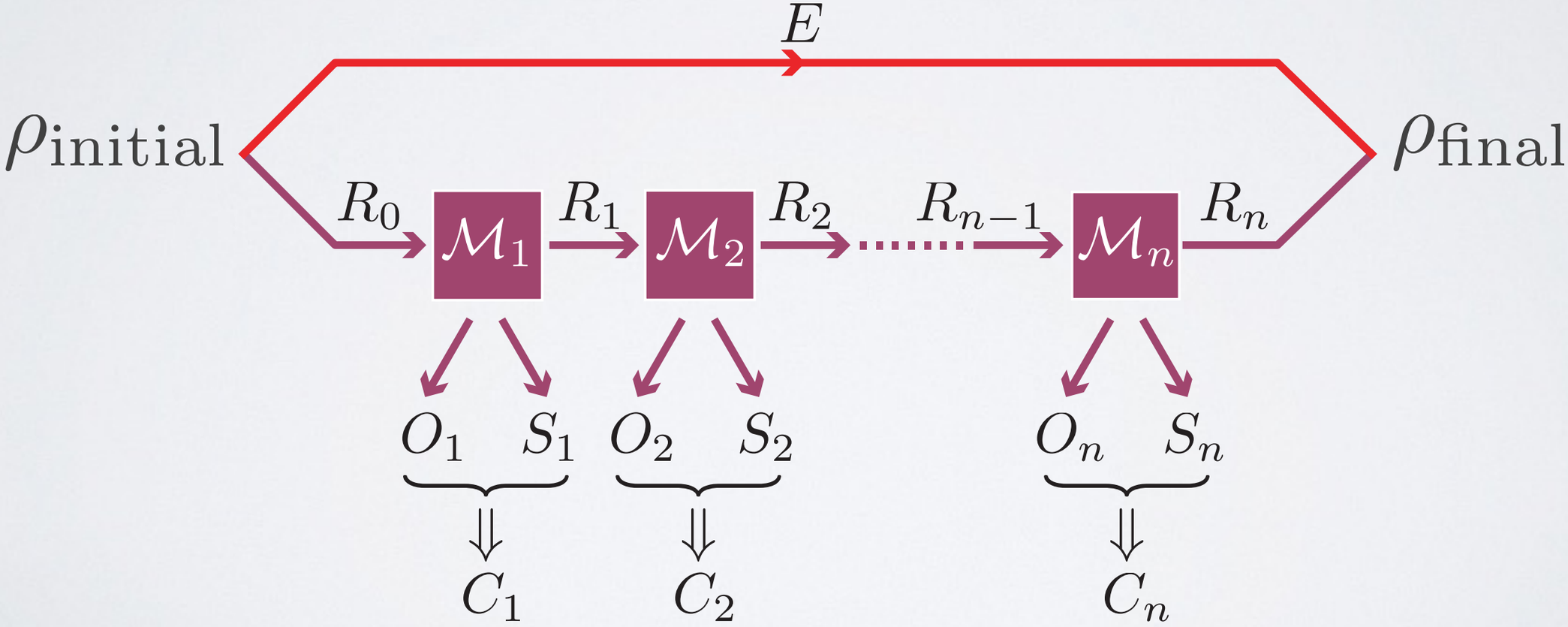
The EAT



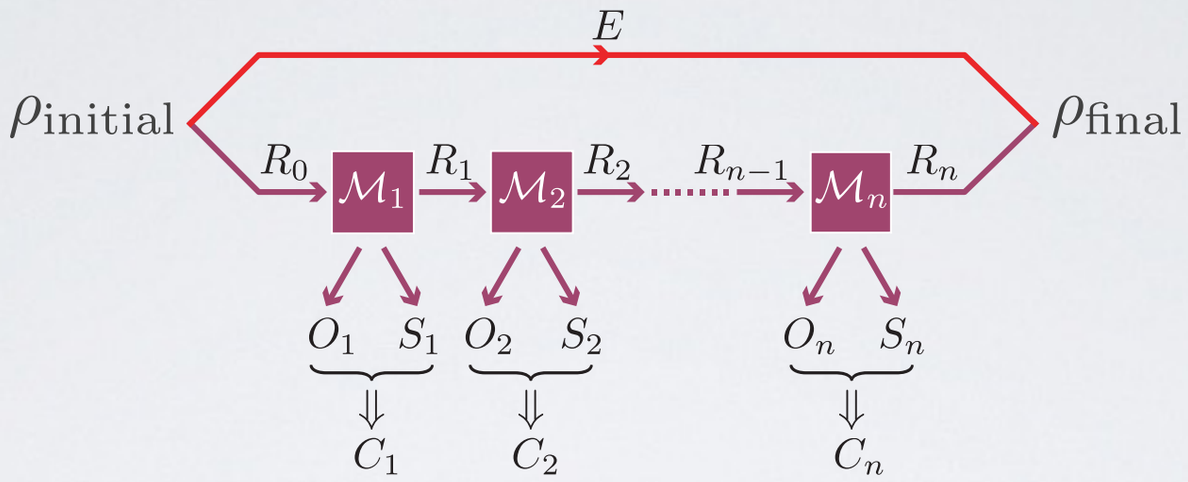
Sequential process

- Model of a sequential process:

$H^{\varepsilon}_{\min}(O|SE)?$



EAT channels



- Assumptions on the channels: $\forall i$

1. O_i finite dimensional with dimension d_{O_i}

2. C_i is a classical register that can be measured from $\rho_{O_i S_i}$ without changing the state

3. For any initial state, the final state fulfils the Markov-chain condition: $O_{1\ldots i-1} \leftrightarrow S_{1\ldots i-1} E \leftrightarrow S_i$

Empirical statistics

$$c_1 \dots c_n = 0100110000000010$$

- Frequencies from the observed data:

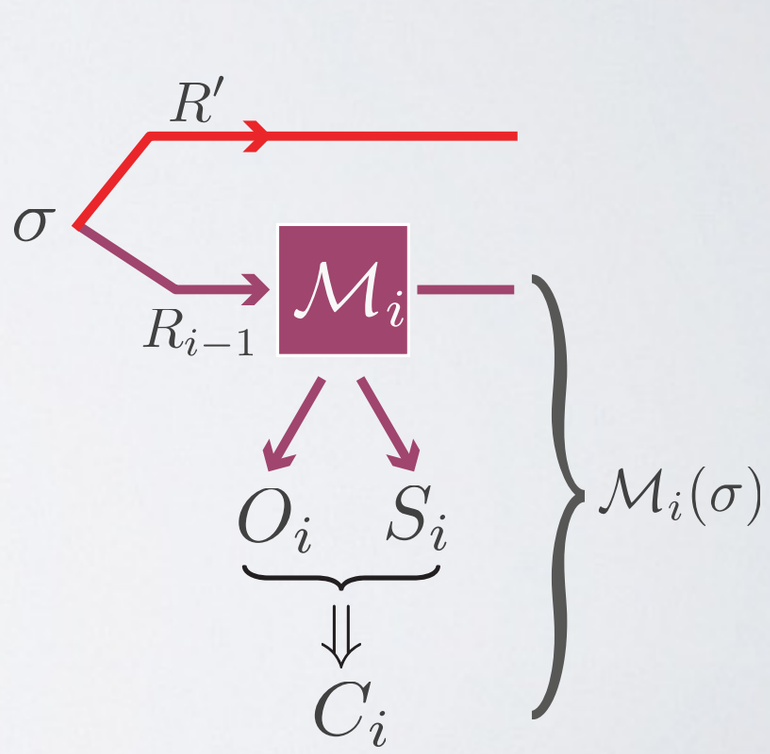
$$\mathsf{freq}(c_1 \dots c_n)(0) = \frac{3}{4} \qquad \mathsf{freq}(c_1 \dots c_n)(1) = \frac{1}{4}$$

- $\mathsf{freq}(c_1 \dots c_n)$ is a probability distribution over $\mathcal{C}$

Min-tradeoff function

- Min-tradeoff function $f_{\min}$ — worst-case von-Neumann entropy in a single-round

$$f_{\min}(\mathsf{freq}(c_1 \dots c_n)) \leq \inf H\left(O_i | S_i R'\right)_{\mathcal{M}_i(\sigma)}$$



- The infimum is over states $\sigma_{R_{i-1}R'}$ such that

$$\mathcal{M}_i(\sigma)_{C_i} = \mathsf{freq}(c_1 \dots c_n)$$

Entropy accumulation theorem

- Event depending on the frequencies $\Omega \subseteq \mathcal{C}^{\otimes n}$
- $\rho_{|\Omega}$ the final state conditioned on Ω
- $t \in \mathbb{R}$ such that $f_{min}(\mathsf{freq}(c_1 \dots c_n)) \geq t$ for all $c_1 \dots c_n \in \Omega$

Entropy accumulation theorem

- $f_{min}(\mathsf{freq}(c_1 \dots c_n)) \geq t$ for all $c_1 \dots c_n \in \Omega$
- EAT:

$$H^{\varepsilon}_{\min}(O|SE)_{\rho_{|\Omega}} > nt - v\sqrt{n}$$

where v depends on $\|\nabla f_{\min}\|_{\infty},\ \varepsilon,\ p_{\Omega},\ d_{O_i}$

- Similar statement for the smooth max-entropy

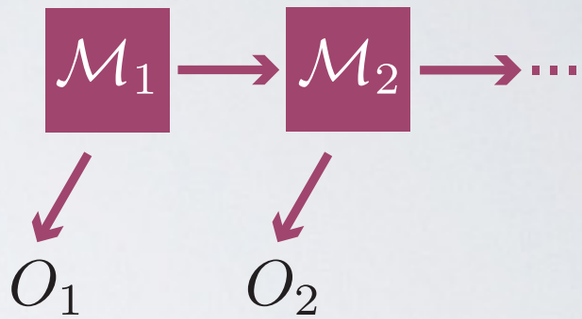
Main ingredients in the proof

- Heavily relies on the sandwiched relative Rényi entropies introduced in [Wilde, Winter, and Yang, 14] and [Müller-Lennert, Dupuis, Szehr, *et al.*, 13]

- A new chain rule for the sandwiched relative Rényi entropies was developed to prove the EAT

Main ingredients in the proof

- "Classical version of the min-tradeoff function":
  Seq. proc. creates $O_1, O_2, \ldots$
  How much can we extract from O_2 after we use O_1?



$$H(O_2|O_1) = \mathbb{E}_{o_1,o_2}\left[-\log \Pr(o_2|o_1)\right]$$

Too optimistic

$$H^{w.c.}_{\min}(O_2|O_1) = \min_{o_1,o_2}\left[-\log \Pr(o_2|o_1)\right]$$

Too pessimistic

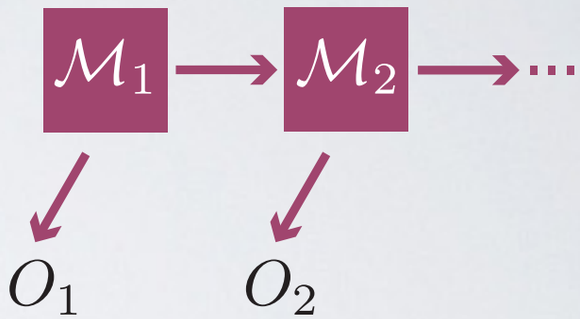
$O_1 = 0 \Rightarrow O_2 = 0$

$O_1 = 1 \Rightarrow O_2$ uniform

O_1 & O_2 independent

Main ingredients in the proof

- "Classical version of the min-tradeoff function":
  Seq. proc. creates $O_1, O_2, \ldots$
  How much can we extract from O_2 after we use O_1?



$$H(O_2|O_1) = \mathbb{E}_{o_1,o_2}\left[-\log \Pr(o_2|o_1)\right]$$

Too optimistic

$$H^{w.c.}_{\min}(O_2|O_1) = \min_{o_1,o_2}\left[-\log \Pr(o_2|o_1)\right]$$

Too pessimistic

Intermediate:

$$\min_{o_1} \mathbb{E}_{o_2} - \log \Pr[o_2|o_1]$$

the min-tradeoff function is the "quantum version" of this

Finally, we are ready!

Applying the EAT to DI Cryptography

DI entropy accumulation pro.

- Main building block in DI cryptographic protocols

DI Entropy Accumulation Protocol

Arguments:

G – two-player non-local game
$\mathcal{X}, \mathcal{Y}$ – possible inputs for Alice Bob
D – untrusted device of two components that can play G repeatedly
$n \in \mathbb{N}_+$ – number of rounds
ω_{exp} – expected winning prob. for an honest (noisy) implementation
$\delta_{\text{est}} \in (0, 1)$ – width of the statistical confidence interval

1: For every round $i \in [n]$ do Steps 2-4:
2: Alice and Bob choose inputs $X_i \in \mathcal{X}$ and $Y_i \in \mathcal{Y}$ respectively.
3: They use D with X_i, Y_i and record the outputs A_i and B_i respectively.
4: They set $C_i = w(A_i, B_i, X_i, Y_i)$.
5: Alice and Bob abort if $\sum_j C_j < (\omega_{\text{exp}} - \delta_{\text{est}}) \cdot n$.

DI entropy accumulation pro.

- Channels — the behaviour of Alice and Bob + uncharacterised device in each round

- C_i — win or lose in game i

- Event Ω — the protocol not aborting

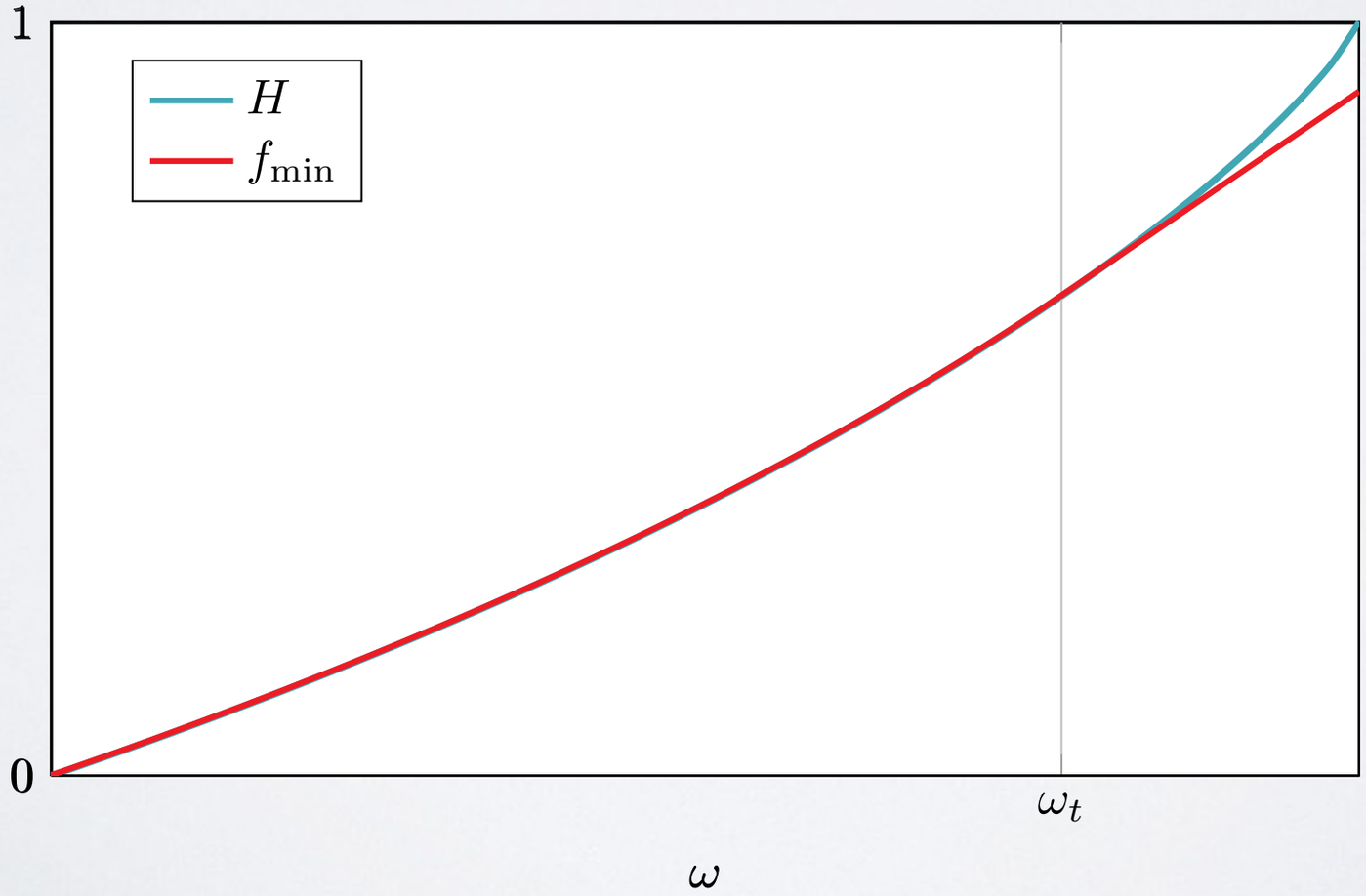
$$\Omega = \{c | \sum_i c_i \geq (\omega_{\text{exp}} - \delta_{\text{est}}) \cdot n\}$$

- $\rho_{|\Omega}$ — final state conditioned on not aborting

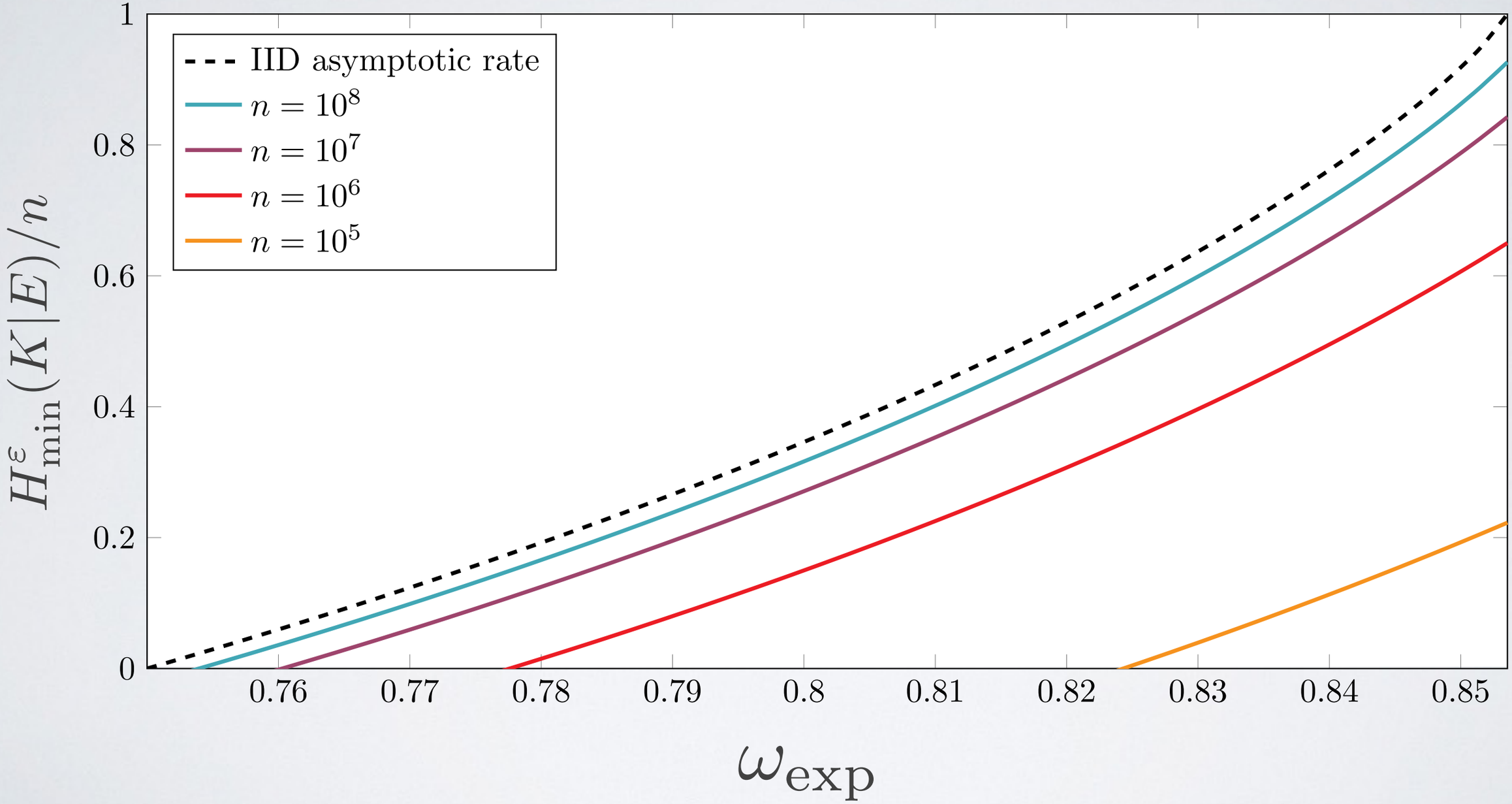
- We lower-bound $H^{\varepsilon}_{\min}(A|XYE)_{\rho_{|\Omega}}$

Min-tradeoff function

$$f_{\min}(\omega) \leq \inf_{\substack{\sigma \text{ with winning} \\ \text{prob.} \geq \omega_{\exp} - \delta_{\text{est}}}} H(A_1|X_1 Y_1 E)_{\mathcal{M}_i(\sigma)}$$



Entropy rate (CHSH)



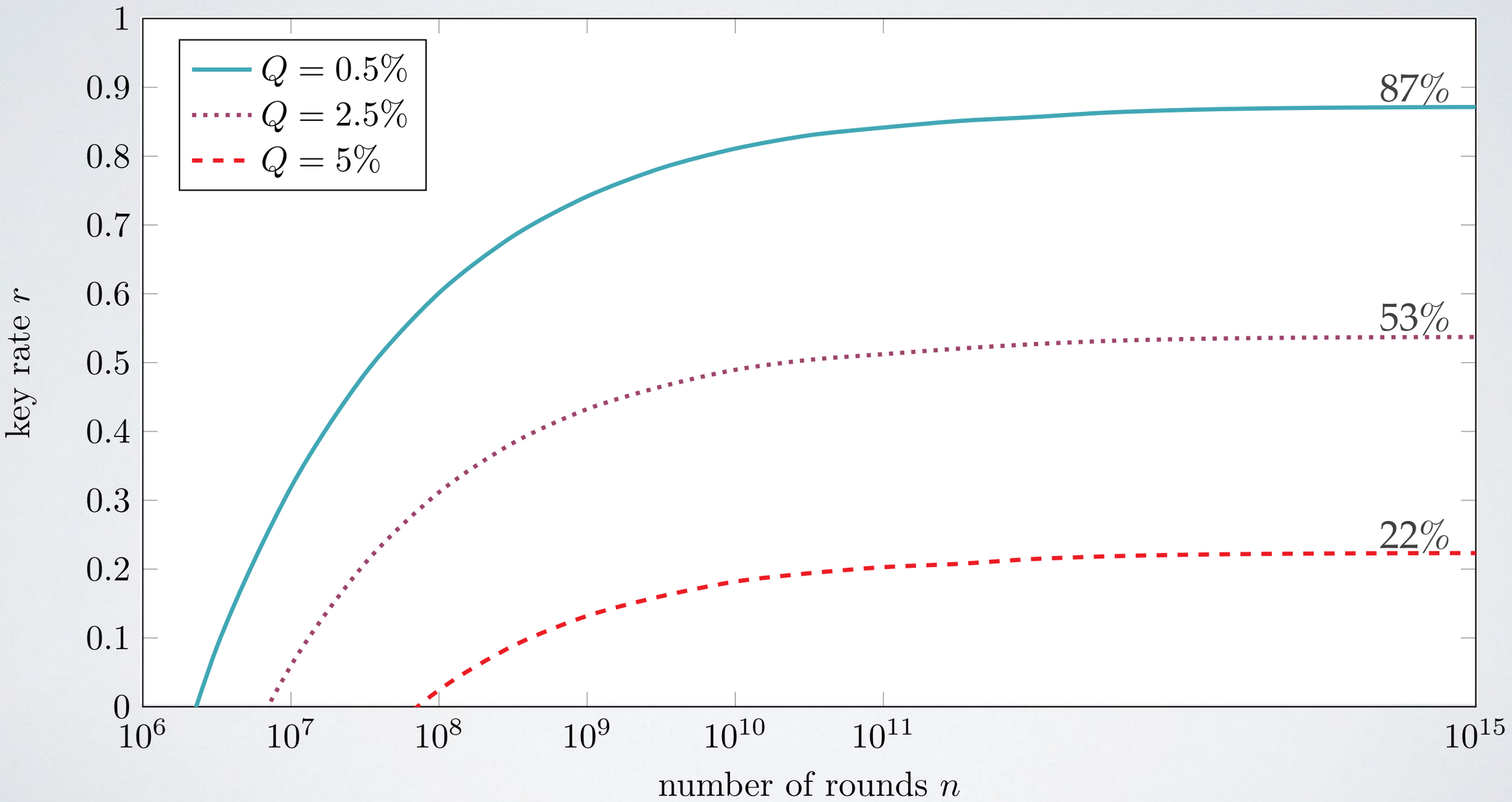
DIQKD

- Based on the Entropy Accumulation protocol
- Classical-post processing on top:
  - Error correction
  - Privacy amplification

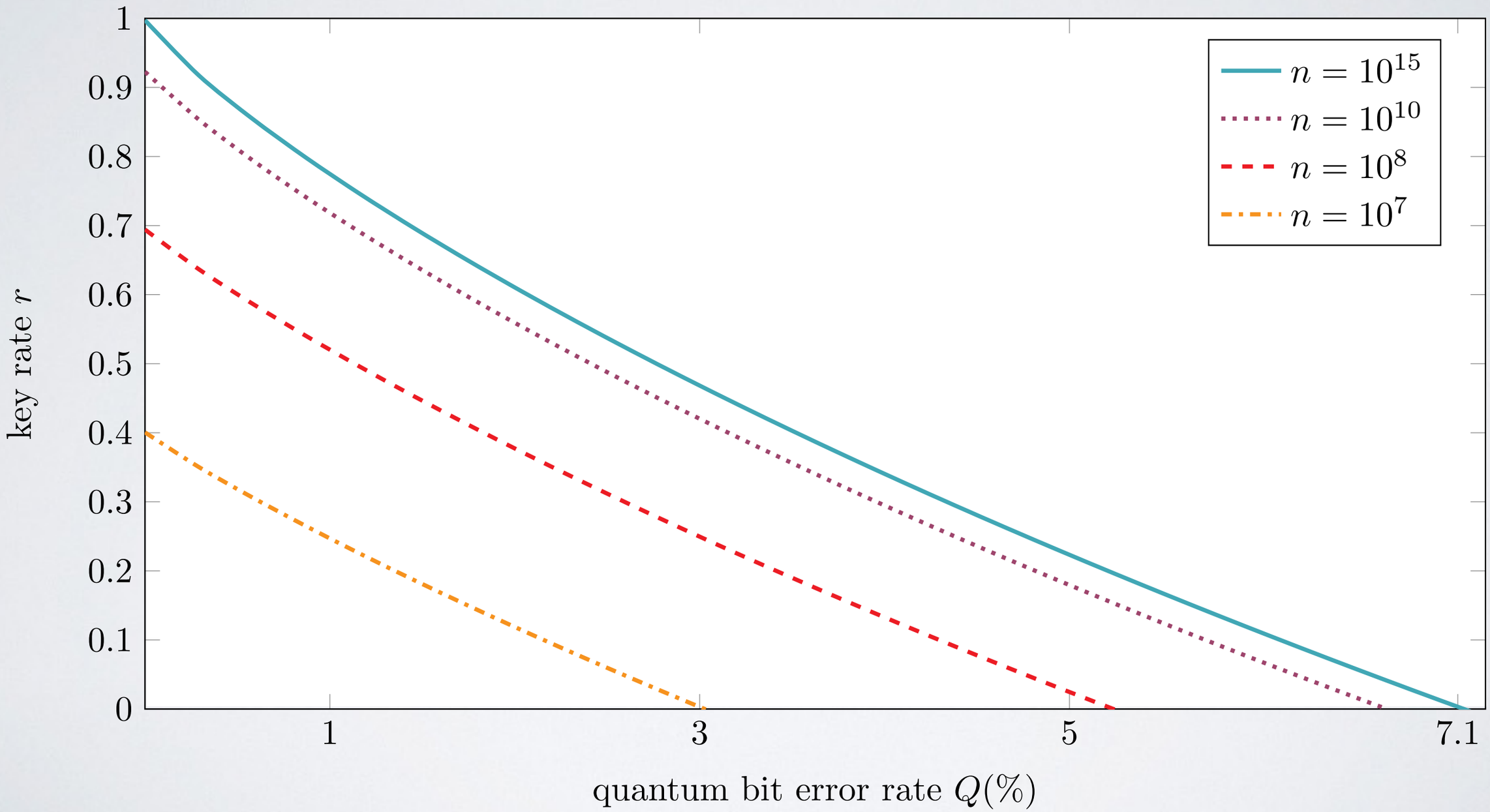
DIQKD — The setting

- Standard assumptions:
  - Alice and Bob's physical locations are secure (unwanted information cannot leak outside to Eve or between their devices)
  - Trusted random number generator
  - Trusted classical post-processing units
  - Authenticated, but public, classical channel
  - Quantum physics is correct (and complete)
- Communication is allowed between Alice and Bob, and from Eve to Alice and Bob, between the rounds of the game (can create "entanglement on the fly")

DIQKD

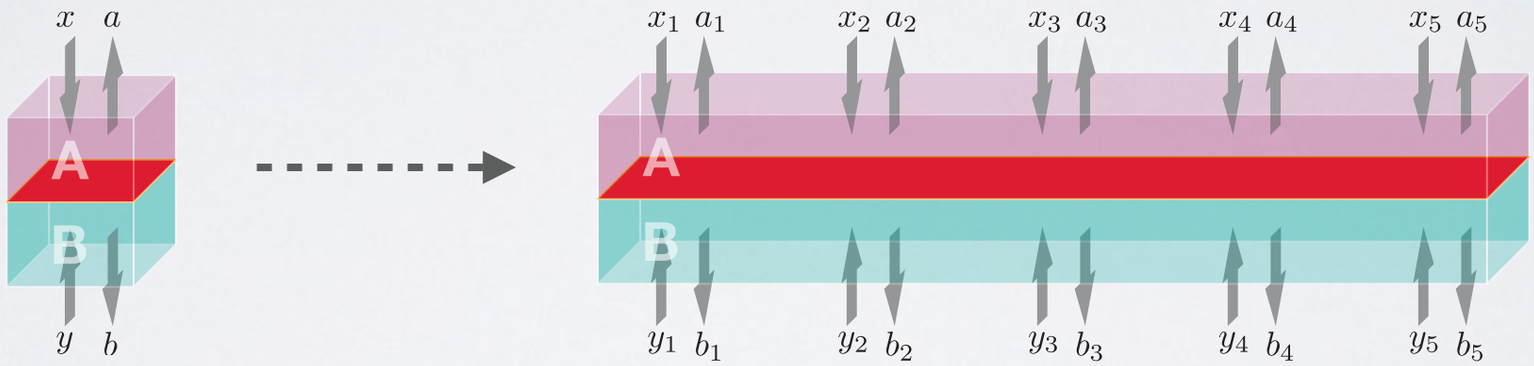

$Q = 0.5\%$
$Q = 2.5\%$
$Q = 5\%$
87%
53%
22%
key rate r
number of rounds n
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{15}

DIQKD


key rate r
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
3
5
7.1
quantum bit error rate $Q(\%)$
$n = 10^{15}$
$n = 10^{10}$
$n = 10^{8}$
$n = 10^{7}$

General security (remarks)

- Need to understand only the physics of a single-round Simple! ✔



- The von-Neumann entropy is the relevant single-round quantity Tight! ✔

- The optimal attack is the IID attack in first order

Summary

Summary

1. New information-theoretic tool: the EAT

- Describes how entropy accumulates in sequential quantum processes
- The von-Neumann entropy is the relevant single-round quantity

2. New framework to prove security of DI protocols

- Modular, simple, and tight security proof
- Concrete examples: DIQKD and randomness expansion based on CHSH
- In essence, the best adversarial attack is the IID attack also in the DI scenario

What's next?

1. Apply the EAT and our framework to other protocols and scenarios
   - Example: two-party DI crypto [Ribeiro, Murta, and Wehner, 16]
   - Also relevant for device dependent cryptography, instead of de Finetti thm.
2. DIQKD:
   - Apply with different Bell inequalities & classical post-processing
   - Experiment: detection efficiencies should be relatively high for a positive key rate with the current protocol
3. Is there a general technique to bound the conditional von-Neumann entropy $H(K_1|E_1)$ given the Bell violation?

Thank you!

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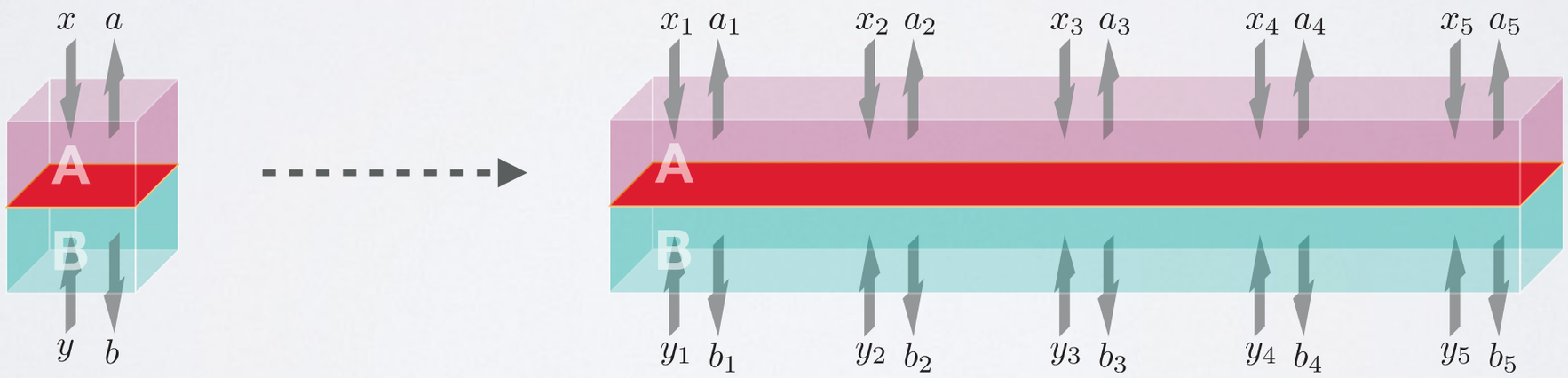

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