Tutorial:

# Device-Independent Quantum Key Distribution

## Security Proofs and Practical Challenges

QCrypt | August 2019 | Montreal, Canada

**Rotem Arnon-Friedman** | UC Berkeley

# Outlook

- Introduction
- Protocol
- Proof techniques
- Challenges and open questions

1. What: Task

2. Why: Motivation

3. How: Intuition

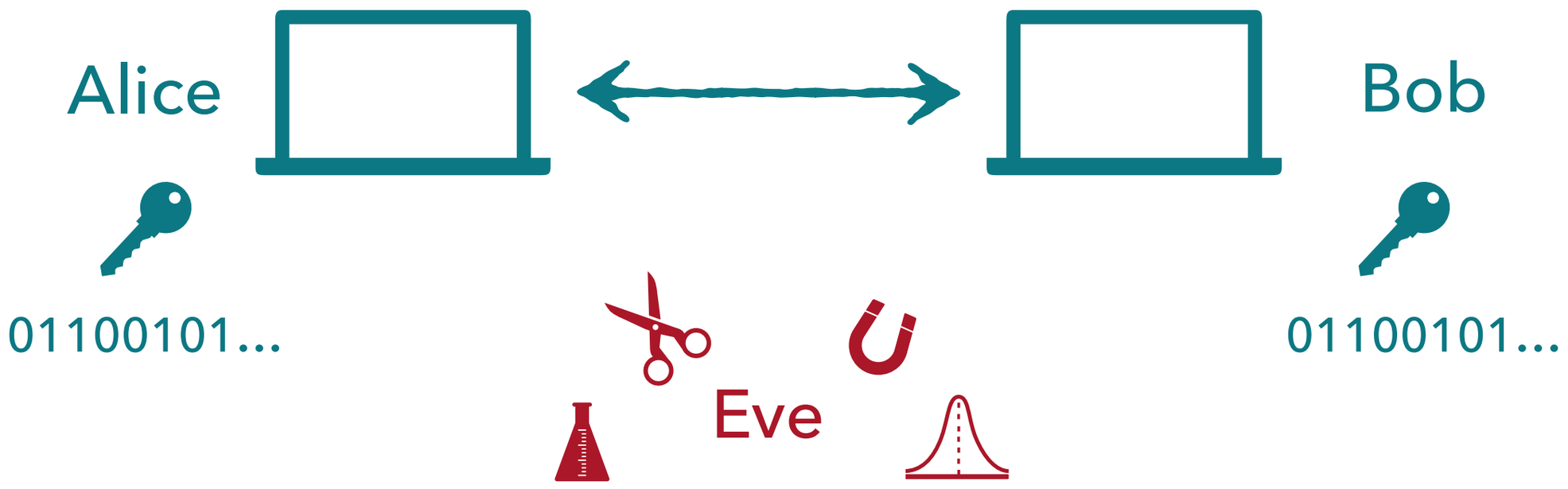
# Introduction

What?

# 1. The Task

# The Task: Key Distribution

- Two honest parties: Alice and Bob
  - Alice and Bob's goal: share a private random key
- One dishonest party: Eve
  - Eve's goal: gain information about the key




(Whatever Norbert said...)

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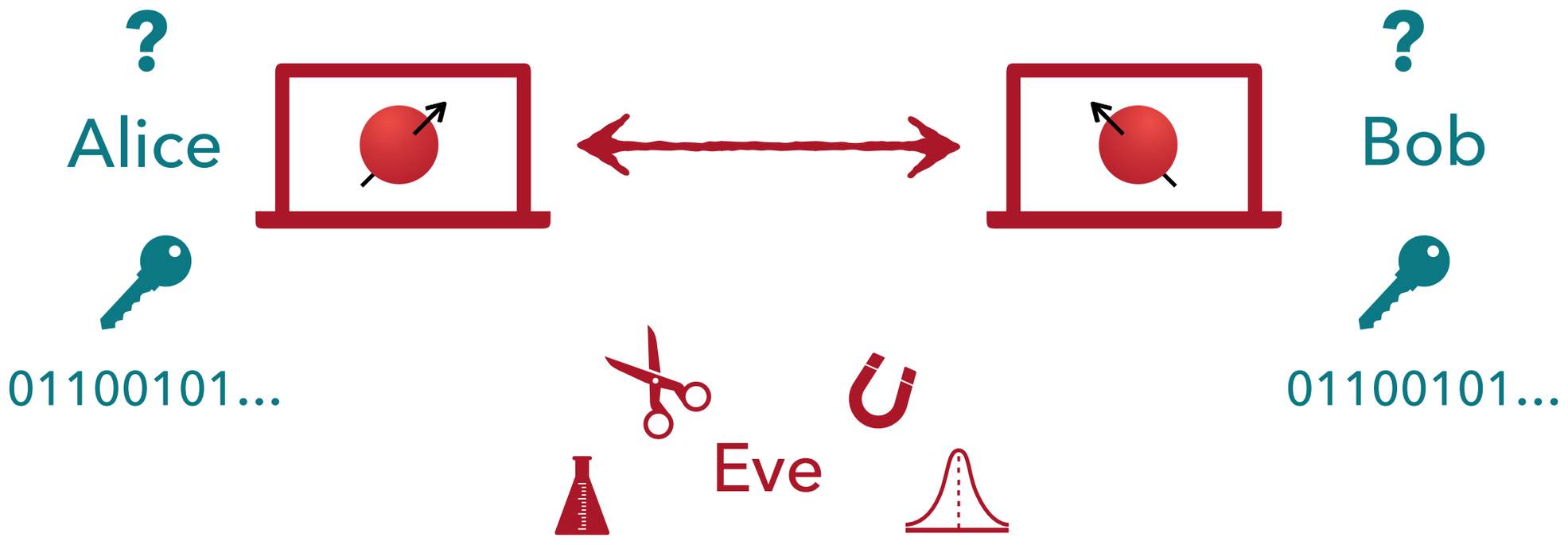


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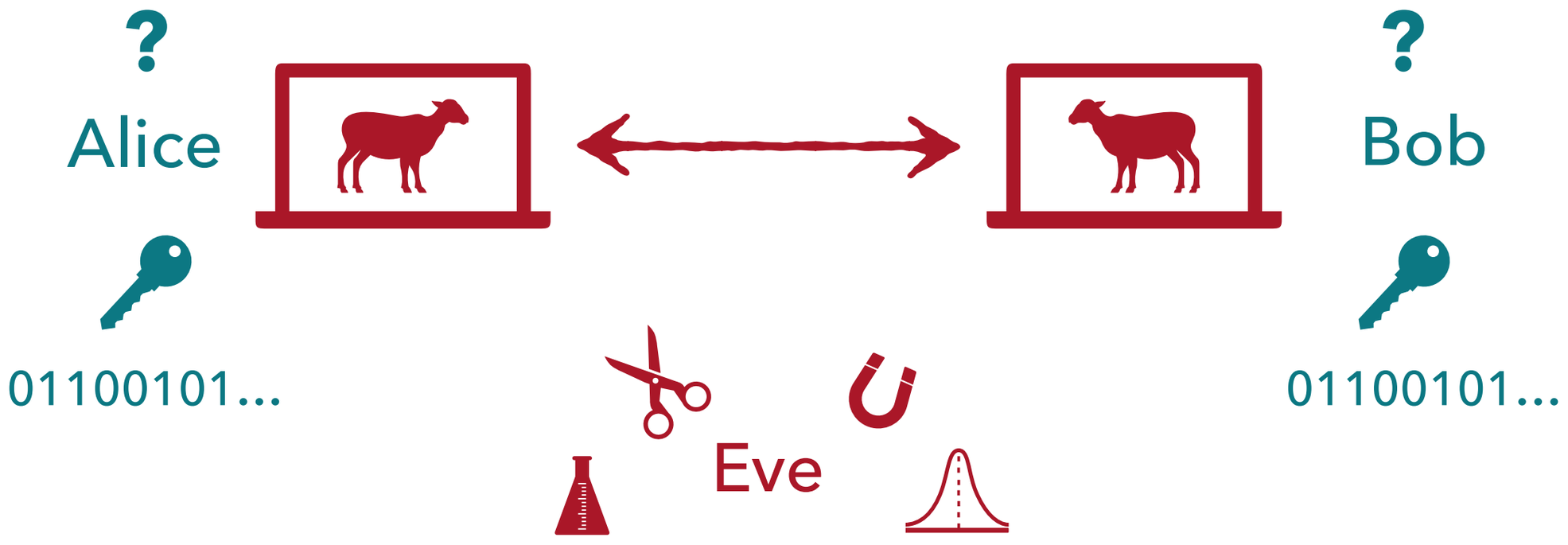


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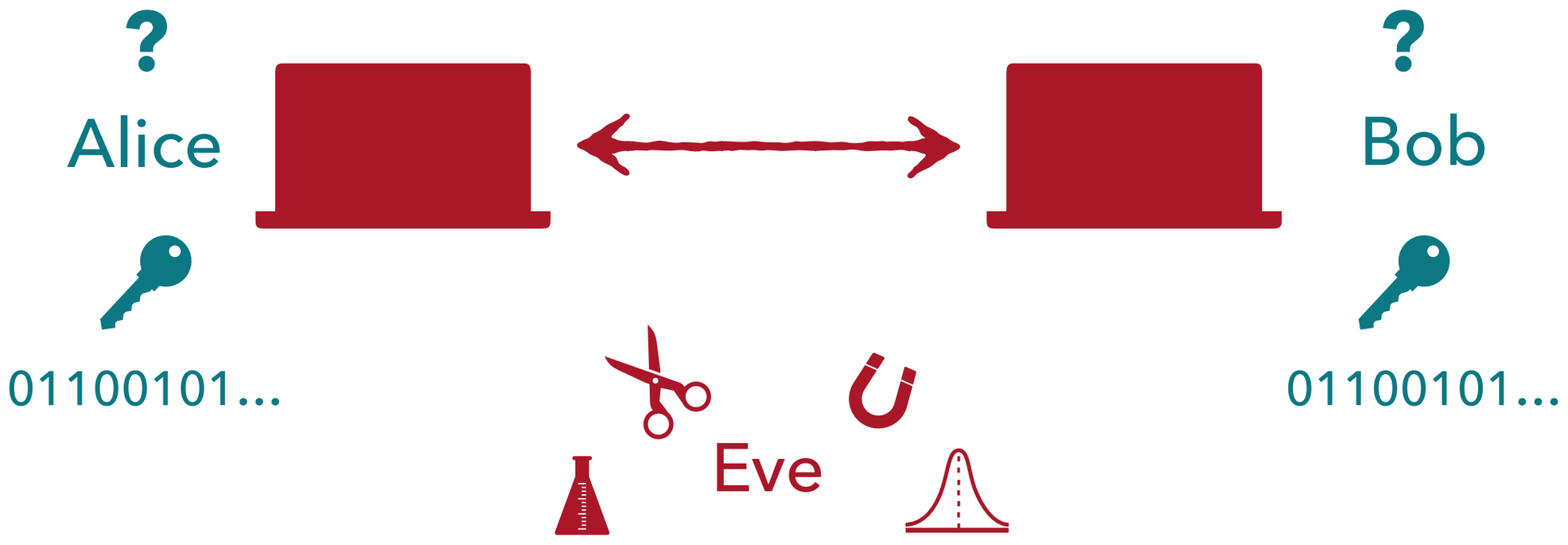



(Whatever Norbert said...)

# 2. Motivation

# Motivation

- Device-independence = the security of the produced key is **independent** of the actual **implementation** of the physical devices

- Why?
  - Paranoid: Manufacturer may be **malicious**
  - Realistic: Physical devices are **imperfect** in ways that we cannot fully characterize (“device vs. model”)
  - Scientist: It’s interesting!
    - Fundamental aspects of quantum physics
    - Pushing cryptography to its limits
    - Motivates new models, e.g., semi-DI cryptography

How?

# 3. Intuition (As far as "quantum intuition" goes...)

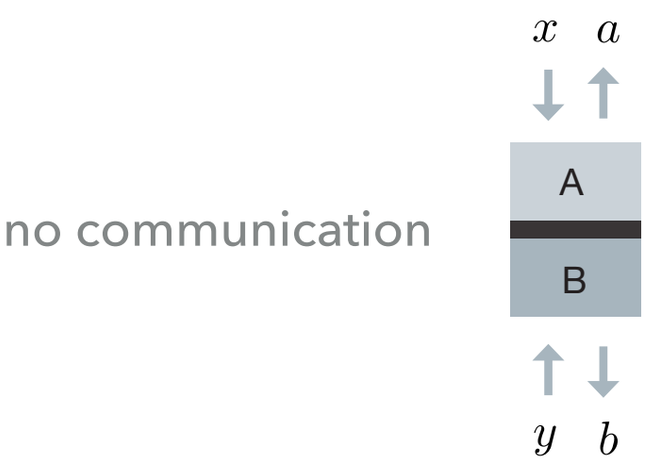
# Intuition

How can we do something useful?

# Intuition

How can we distinguish the two?

Play a non-local game!

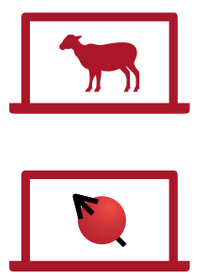



no communication

CHSH Game:

| | | |
|---|---|---|
| Alice: | Input | $x \in \{0, 1\}$ |
| | Output | $a \in \{0, 1\}$ |
| Bob: | Input | $y \in \{0, 1\}$ |
| | Output | $b \in \{0, 1\}$ |
| Win: | $a \oplus b = x \cdot y$ | |

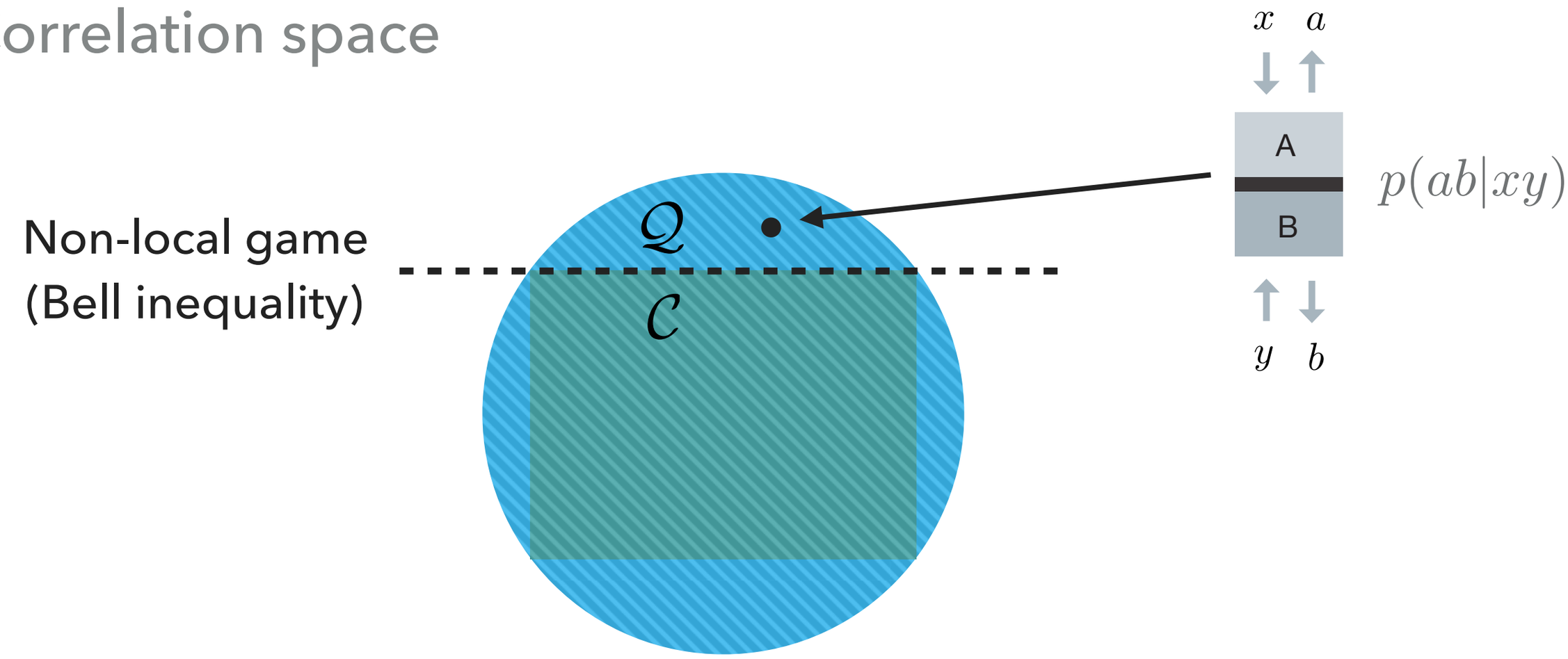
- Best classical strategy: 75% winning probability
- Best quantum strategy: ~85% winning probability



} Quantum advantage

# Intuition

- Correlation space



# Intuition

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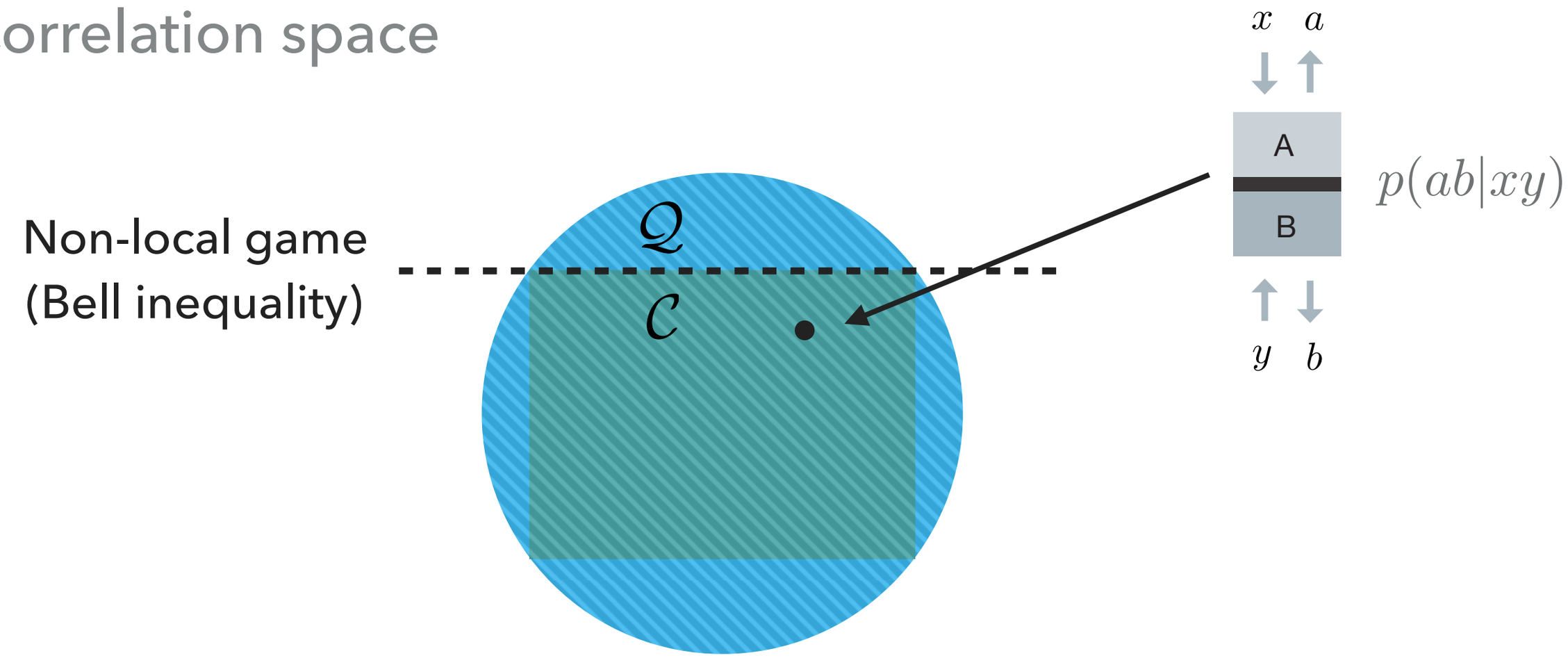
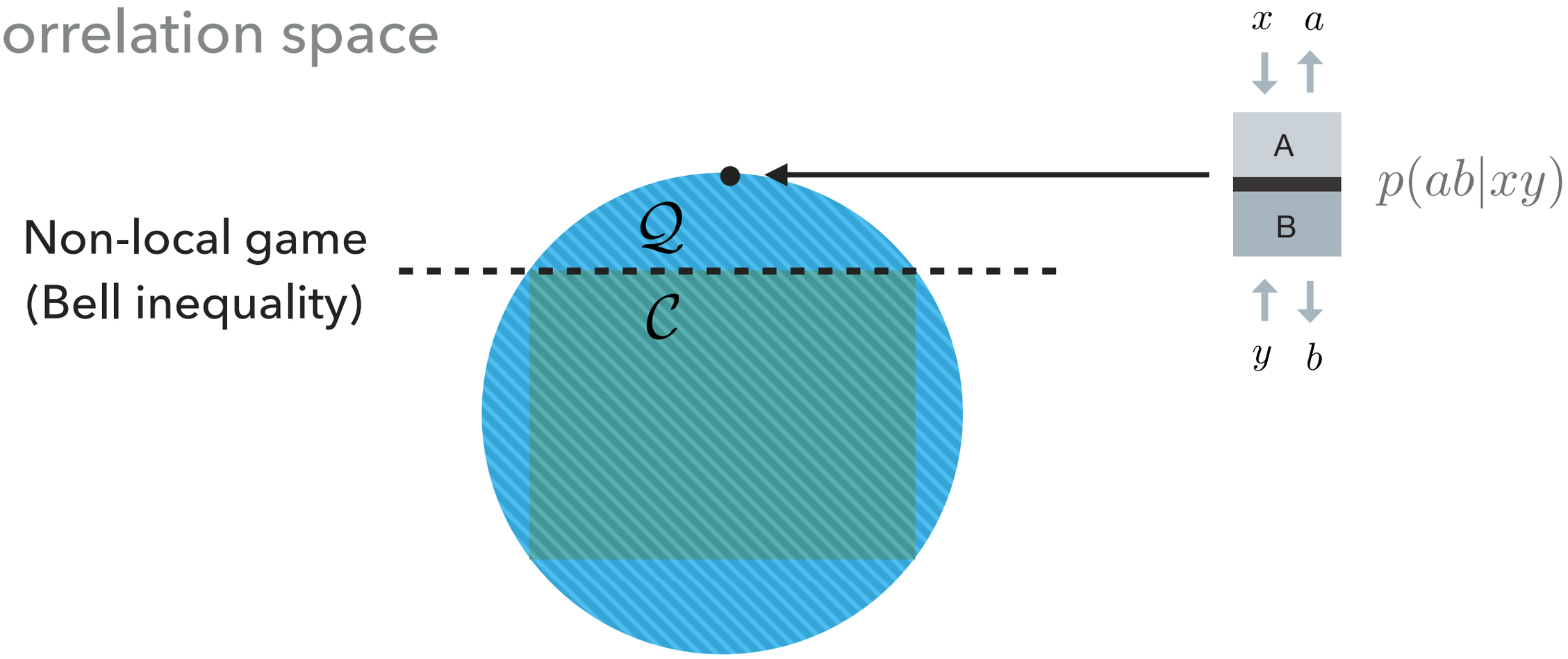



- Two cases:
  - Classical strategy: $p(ab|xy) = \sum_{\lambda} p(\lambda)p(a|x\lambda)p(b|y\lambda)$

# Intuition

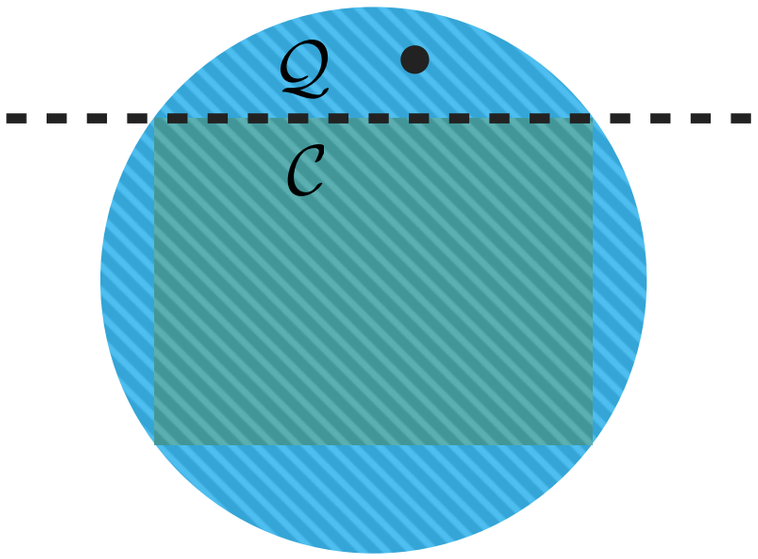
- Correlation space



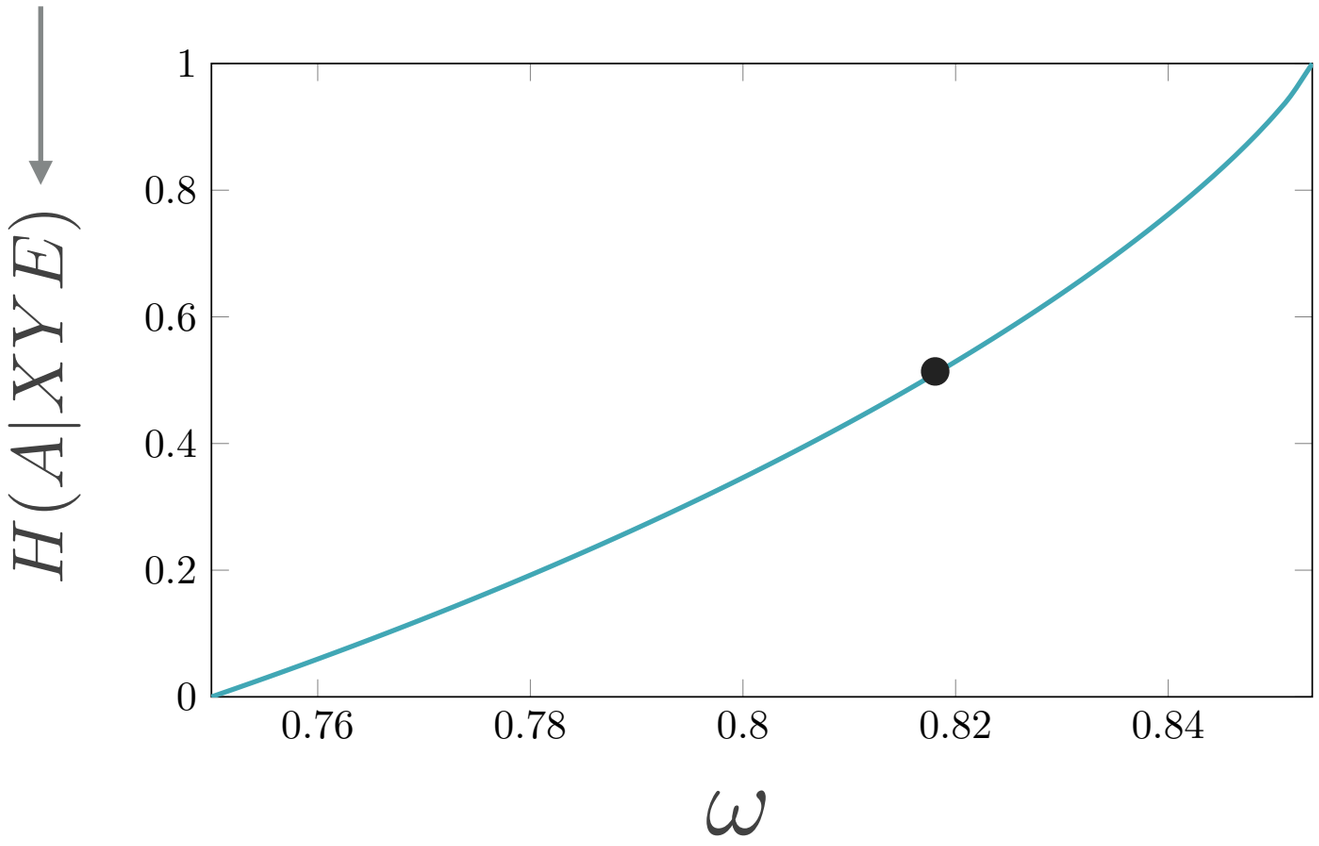

- Two cases:
  - Classical strategy: $p(ab|xy) = \sum_{\lambda} p(\lambda)p(a|x\lambda)p(b|y\lambda)$
  - Optimal CHSH quantum strategy: measuring $\Phi^{+}_{AB}$

# Intuition

- Correlation space




How random A is from Eve's point of view




- Two extreme cases:
  - Classical strategy: $p(ab|xy) = \sum_{\lambda} p(\lambda)p(a|x\lambda)p(b|y\lambda)$
  - Optimal CHSH quantum strategy: measuring $\Phi^{+}_{AB} \otimes \rho_E$

1. Assumptions

2. First step: Generating the raw data

3. Second step: Classical processing

# Protocol

# 1. Assumptions

# Assumptions

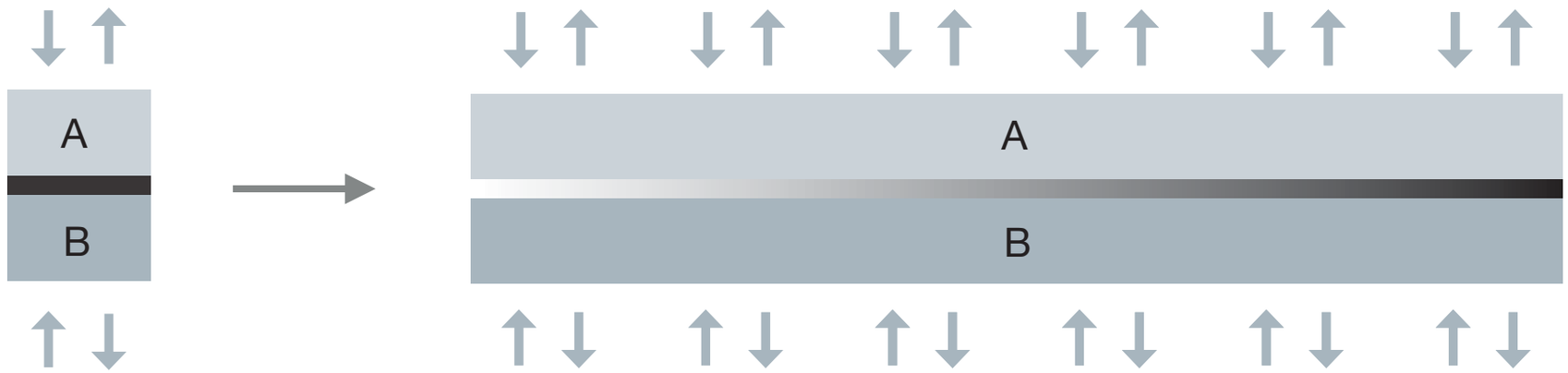
- Standard assumptions:
  - Alice and Bob's physical locations are secure (unwanted information cannot leak outside to Eve or between their devices)
  - Trusted random number generator
  - Trusted classical post-processing units
  - Authenticated, but public, classical channel
  - Quantum physics is correct (and complete)
- Communication is allowed between Alice and Bob, and from Eve to Alice and Bob, between the rounds of the protocol (can create "entanglement on the fly")

First Step

# 2. Data Generation

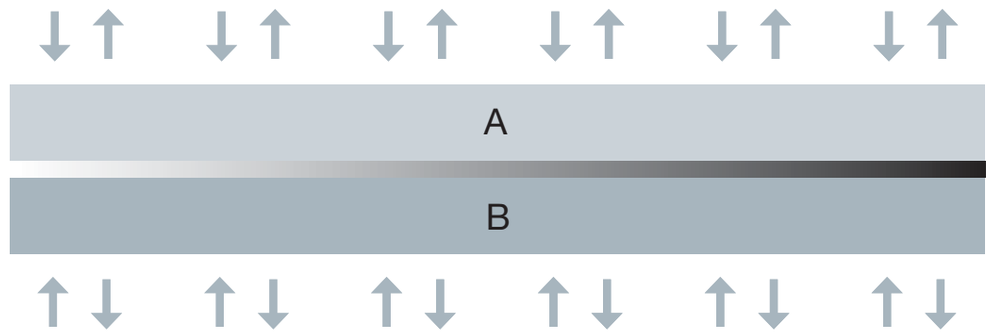
# DIQKD Protocol

- To generate a key playing one game with the device is not enough
- Many rounds of the game with one (big) device




- The generated data is used to **test** the device and to generate a key

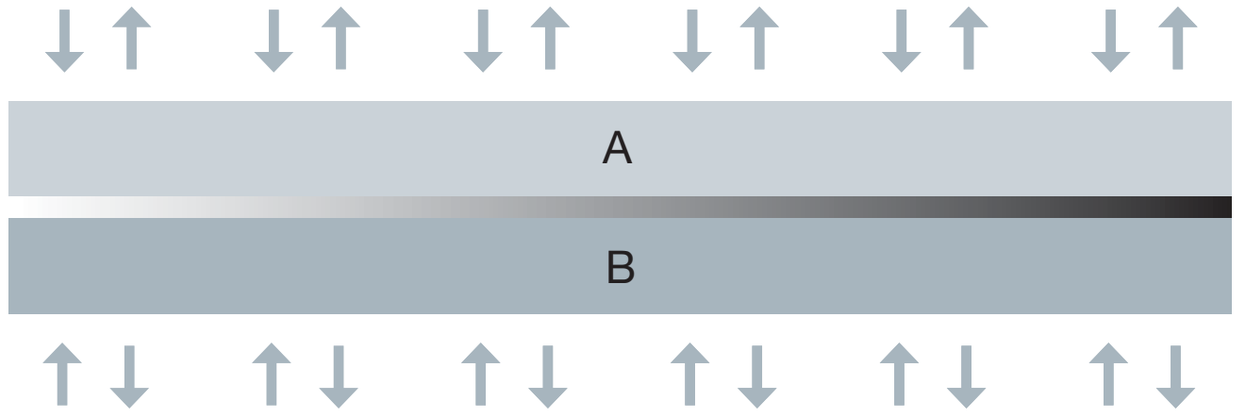
# DIQKD Protocol: Data Generation




- Step 1: Generate data by interacting classically with the physical devices
  - Randomly choose test/generation rounds
    - **Test round:**
      - Play the non-local game with the device (random inputs)
      - Recored the outputs for parameter estimation
    - **Generation round:**
      - Give the devices a pre-defined fixed input pair
      - Recored the outputs as the raw data for the key

Parallel / sequential interaction

# DIQKD Protocol: Data Generation




| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$: | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| $y$: | 2 | 0 | 1 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 2 | 2 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 0 |
| $a$: | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| $b$: | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |

- The crucial part: When $X = 0$, Alice's device cannot distinguish a test round from a generation round!

Second Step

# 3. Classical Processing

# DIQKD Protocol: Parameter Estimation

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| y: | 2 | 0 | 1 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 2 | 2 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 0 |
| a: | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| b: | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| | | ✓ | ✓ | | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ | | | ✓ | ✓ | ✗ | ✓ | | ✓ | | ✓ | ✗ | | | ✓ | ✓ |

Parameter estimation: # of ✓ > threshold?

Yes– continue

No– abort

# DIQKD Protocol: Parameter Estimation

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| y: | 2 | 0 | 1 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 2 | 2 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 2 | 1 | 1 | 2 | 2 | 0 | 0 |
| a: | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| b: | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| | | ✓ | ✓ | | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ | | | | ✓ | ✓ | ✗ | ✓ | | ✓ | | ✓ | ✗ | | | ✓ | ✓ |

Parameter estimation: # of ✓ > threshold?

# of ✓ > threshold?

Yes– continue

No– abort

# DIQKD Protocol: Classical Processing

- Step 2: Classical post-processing
  - **Parameter estimation**: Check whether sufficiently many of the games (in the test rounds) are won
    - Yes– continue
    - No– abort
  - **Error correction**
    - Error correction protocol raises a flag– abort
    - Otherwise– continue
  - **Privacy amplification**

One-way communication

1. History

2. Security definition

3. Main task

4. Entropy accumulation in DIQKD

5. Classical post-processing

# Proof Techniques

# 1. History

# History

[Ekert, 91]
[Mayers & Yao, 98]
Main ideas

→ [Barrett, Hardy & Kent, 05]
Proof of concept

→ [Pironio *et al.*, 09]
IID + asymptotic:
tight rates & noise tolerance

↓

[Reichardt, Unger & Vazirani, 13]
[Vazirani & Vidick, 14]
[Miller & Shi, 14]
General security

← [AF, Renner & Vidick, 16]
General security:
tight rates & noise tolerance

[Dupuis, Fawzi & Renner, 16]
[Dupuis & Fawzi, 18]
Entropy accumulation theorem

Sequential protocols

[Knill, Zhang & Bierhorst, 17]
Classical adversary

← [Knill, Zhang & Fu, 18]
General security:
tight rates & noise tolerance

Parallel

[Jain, Miller & Shi, 17]
Introduced

→ [Vidick, 17]
Simplified

The Most Important Thing

# 2. Security Definition

# Security Definition

- What does it mean to prove security?
  - If the device is sufficiently good– we would like to get a key:
    - Identical keys for Alice and Bob — Correctness
    - Unknown to Eve — Secrecy
  - If the device is not good– we would like to detect it and abort — Soundness
- There exists a sufficiently good device — **Completeness** (noise-tolerance)

# Security Definition

- **Def. [Security]:** A protocol is $\left(\varepsilon^{s}_{\mathrm{QKD}}, \varepsilon^{c}_{\mathrm{QKD}}, \ell\right)$-secure if:
  - **Soundness:** For any device $\mathrm{D}$, the protocol is $\varepsilon^{s}_{\mathrm{QKD}}$-sound, with $\varepsilon_{\mathrm{corr}} + \varepsilon_{\mathrm{sec}} = \varepsilon^{s}_{\mathrm{QKD}}$, if

    1. Correctness: $\Pr\left(K_A \neq K_B\right) \leq \varepsilon_{\mathrm{corr}}$

    2. Secrecy: $\left(1 - \Pr(\mathrm{abort})\right) \left\| \rho_{K_A E} - \rho_{U_\ell} \otimes \rho_E \right\| \leq \varepsilon_{\mathrm{sec}}$

  - **Completeness** (noise-tolerance): There exists a (noisy) device $\mathrm{D}$ such that the protocol aborts with probability at most $\varepsilon^{c}_{\mathrm{QKD}}$.

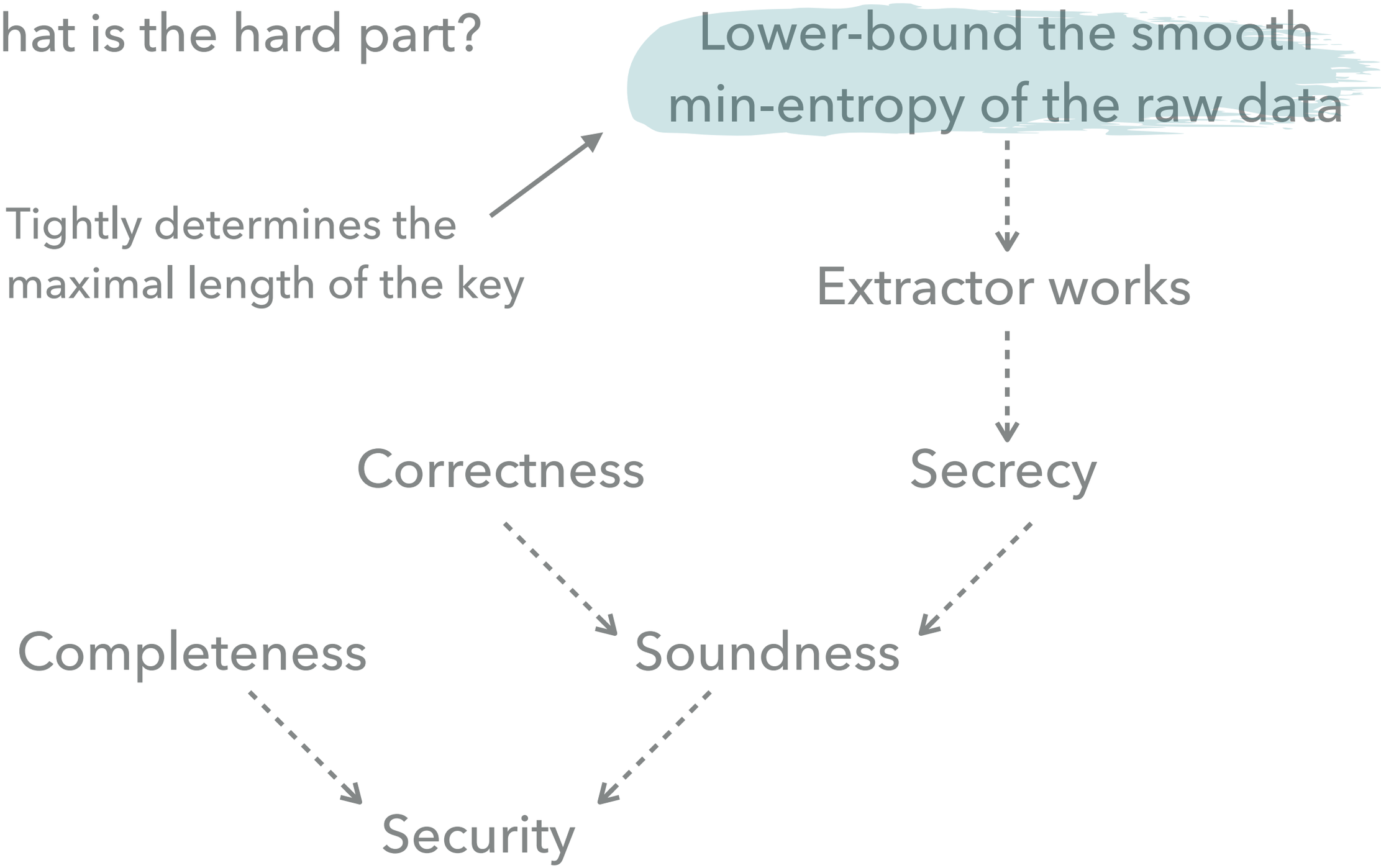
**For DIQKD: Not (fully) composable!**

- Security definitions do not fall from the sky… See Renato Renner's tutorial (QCrypt18) and Christopher Portmann's tutorial (QCrypt17).

# 3. Main Task When Proving Security

# Main Task When Proving Security

- What is the hard part?



# Main Task When Proving Security

Goal: Lower-bound the smooth min-entropy of the raw data

- Two (most recent & tight) approaches:
  - Entropy accumulation → Analytically oriented
    [AF, Dupuis, Fawzi, Renner, & Vidick, 16]
  - Probability estimators →Numerically oriented
    [Knill, Zhang, & Fu, 18]
    See talk on Thursday!

Also for other QKD models
and other protocols

Best Practices

→ RAF's PhD thesis | arXiv: 1812.10922

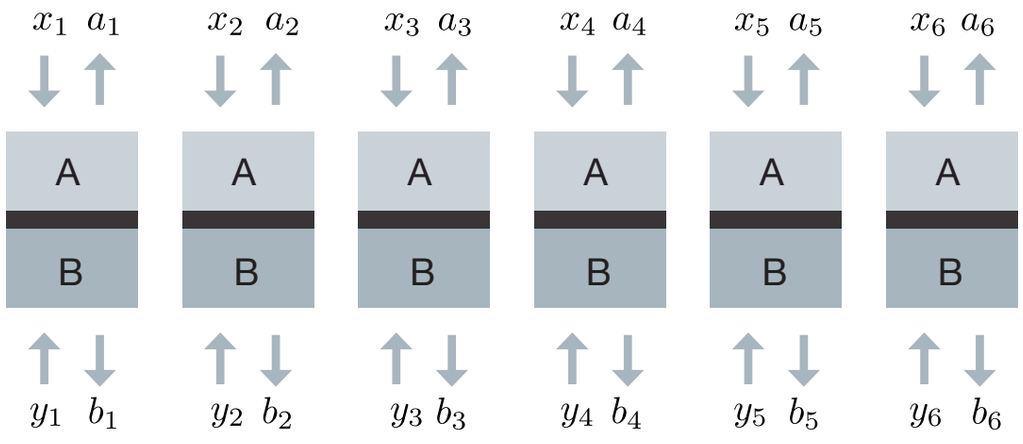
# 4. Entropy Accumulation in DIQKD

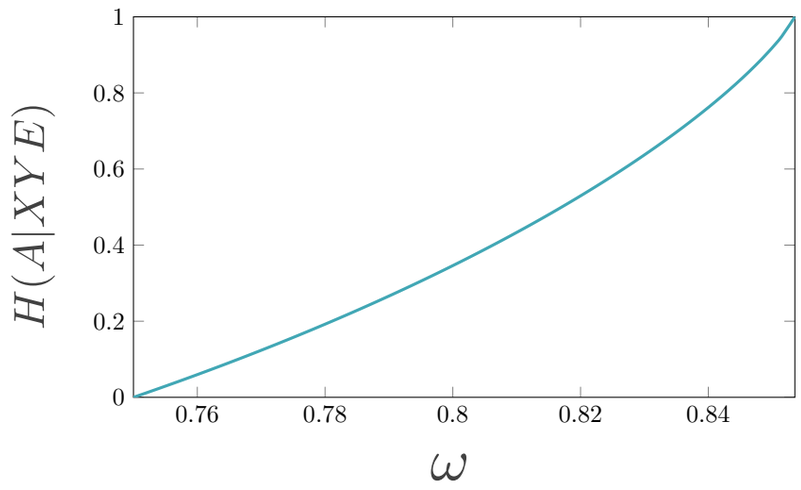
# Working Under the IID Assumption

- Security proof under the independent and identically distributed [IID] assumption (roughly):
  - Play the game many times independently and identically
  - Use the data to estimate the winning probability in a single game
  - Quantum asymptotic equipartition property [Tomamichel, Colbeck & Renner, 09]:

The total amount of entropy is roughly
the number games $\times$ entropy in one game

$$H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{XYE}) \geq nH(A|XYE) - c_{\varepsilon}\sqrt{n}$$



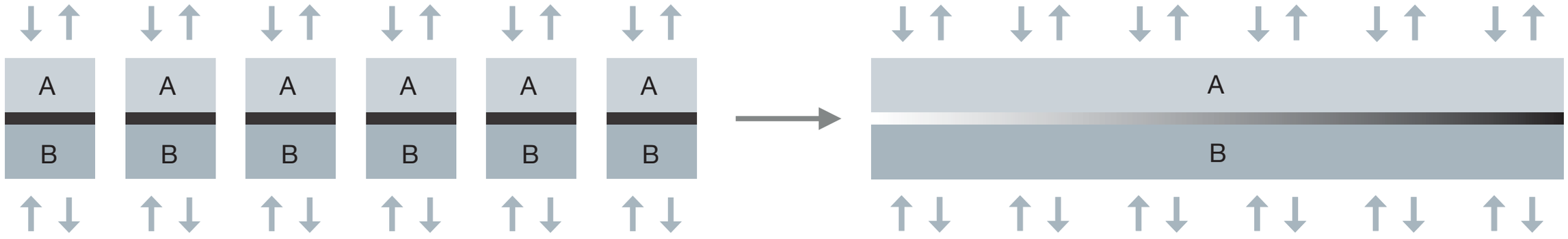



# Entropy Accumulation in DIQKD

- Moving on from the IID case: need to replace the QAEP; this is where the EAT enters [Dupuis, Fawzi, & Renner, 16]
- Goal:

$$H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{XYE}) \geq n\, t - c_{\varepsilon}\sqrt{n}$$

What is this?



# Entropy Accumulation in DIQKD

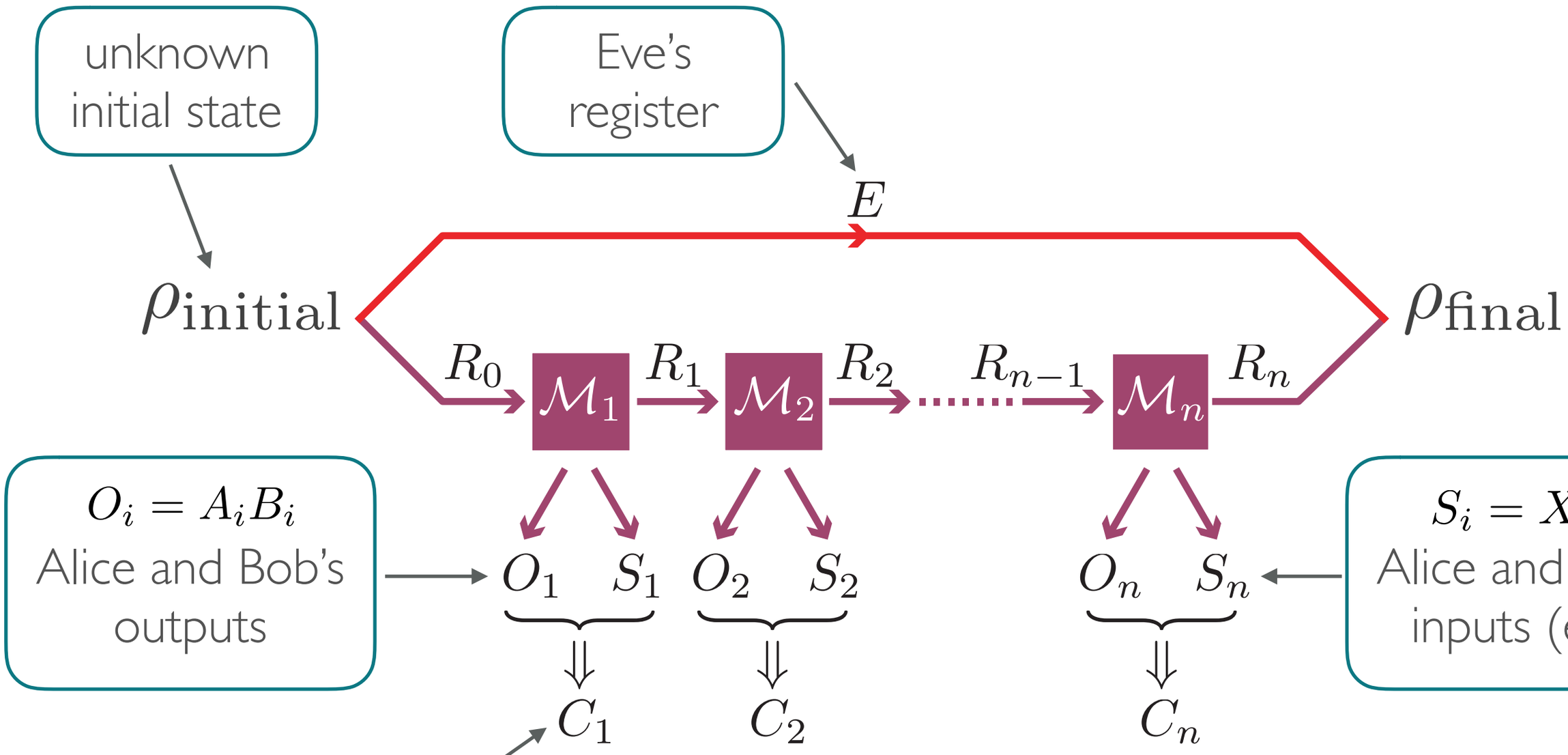
- EAT: a tool that can be used, **in some sense**, to reduce the analysis of a **sequential** device to that of an IID device

**Not a "black box reduction"!**
We cannot just analyze the IID case and be done

# Entropy Accumulation in DIQKD

- EAT: a tool that can be used, **in some sense**, to reduce the analysis of a **sequential** device to that of an IID device
- Before we can apply the EAT we need to define and construct certain objects and make sure that certain conditions hold:
  1. Sequential process: EAT channels
  2. Markov-chain conditions
  3. Min-tradeoff function
- Start by modeling the process: protocol + device

# Sequential Process: EAT Channels




**Goal:**

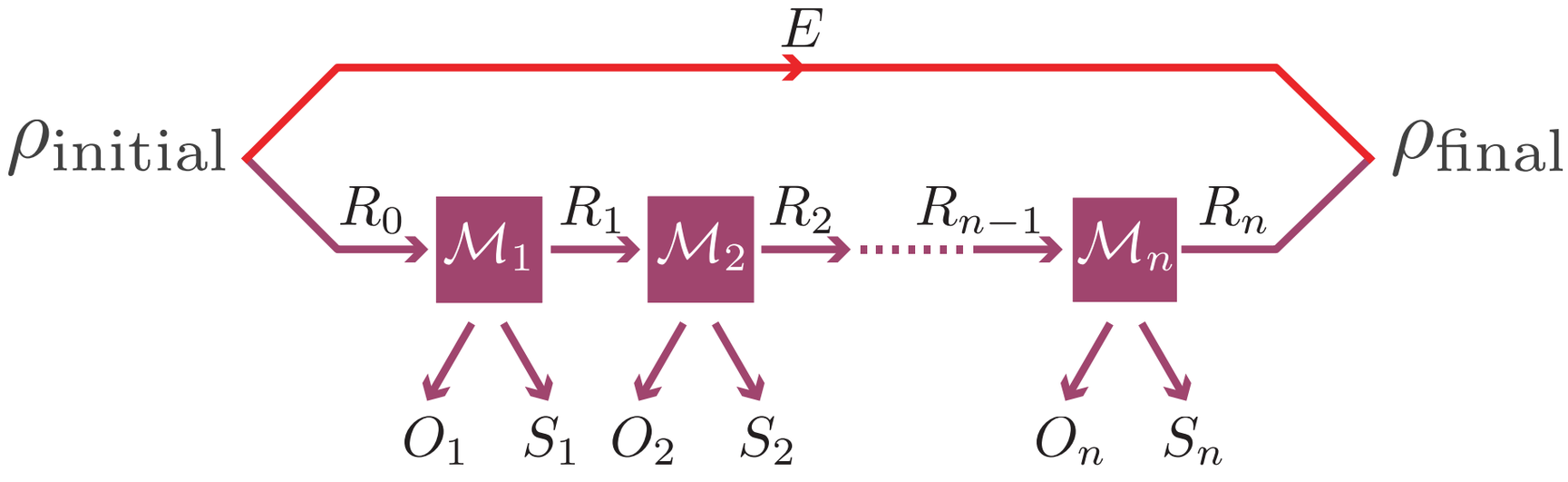
$$H_{\min}^{\varepsilon}(O|SE) = H_{\min}^{\varepsilon}(AB|XYE) = ?$$

# EAT Channels

- The EAT channels should fulfill certain conditions;
  Most important one:
  - For **any** initial state, the **final** state fulfills the Markov-chain conditions: for any $i$,

$$O_{1,\dots,i-1} \leftrightarrow S_{1,\dots,i-1} E \leftrightarrow S_i$$

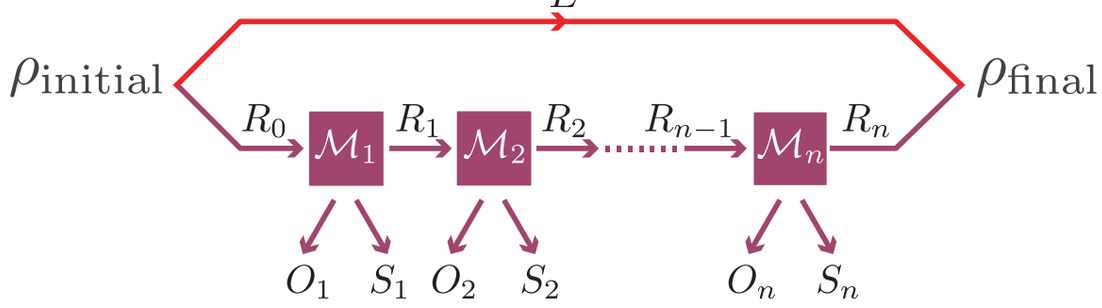
$$(AB)_{1,\dots,i-1} \leftrightarrow (XY)_{1,\dots,i-1} E \leftrightarrow (XY)_i$$



# Markov-Chain Conditions

- Markov-chain conditions make sure that entropy accumulates during the sequential process: Future steps do not reduce the entropy accumulated in the past

$$O_{1,\cdots,i-1} \leftrightarrow S_{1,\cdots,i-1}E \leftrightarrow S_i$$




- The **choice** of the sequential process may require some creativity:
  - The model affects the way we can use the EAT
  - Not trivial for all sequential processes (see later today)

# Min-Tradeoff Function

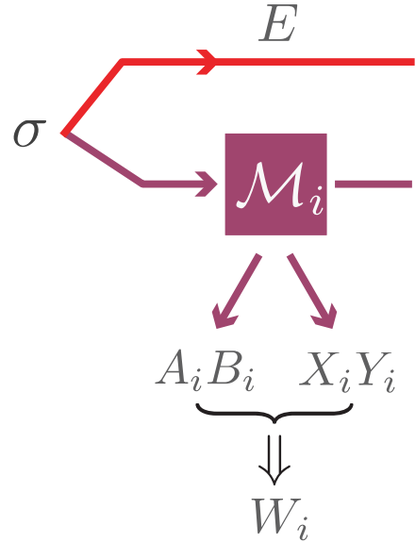
$$H_{\min}^{\varepsilon}\left(\mathbf{AB}|\mathbf{XYE}\right)_{\rho_{|\Omega}} \geq nt - \nu\sqrt{n}$$

What is this?

AEP (IID case):

$$t = H(AB|XYE)$$

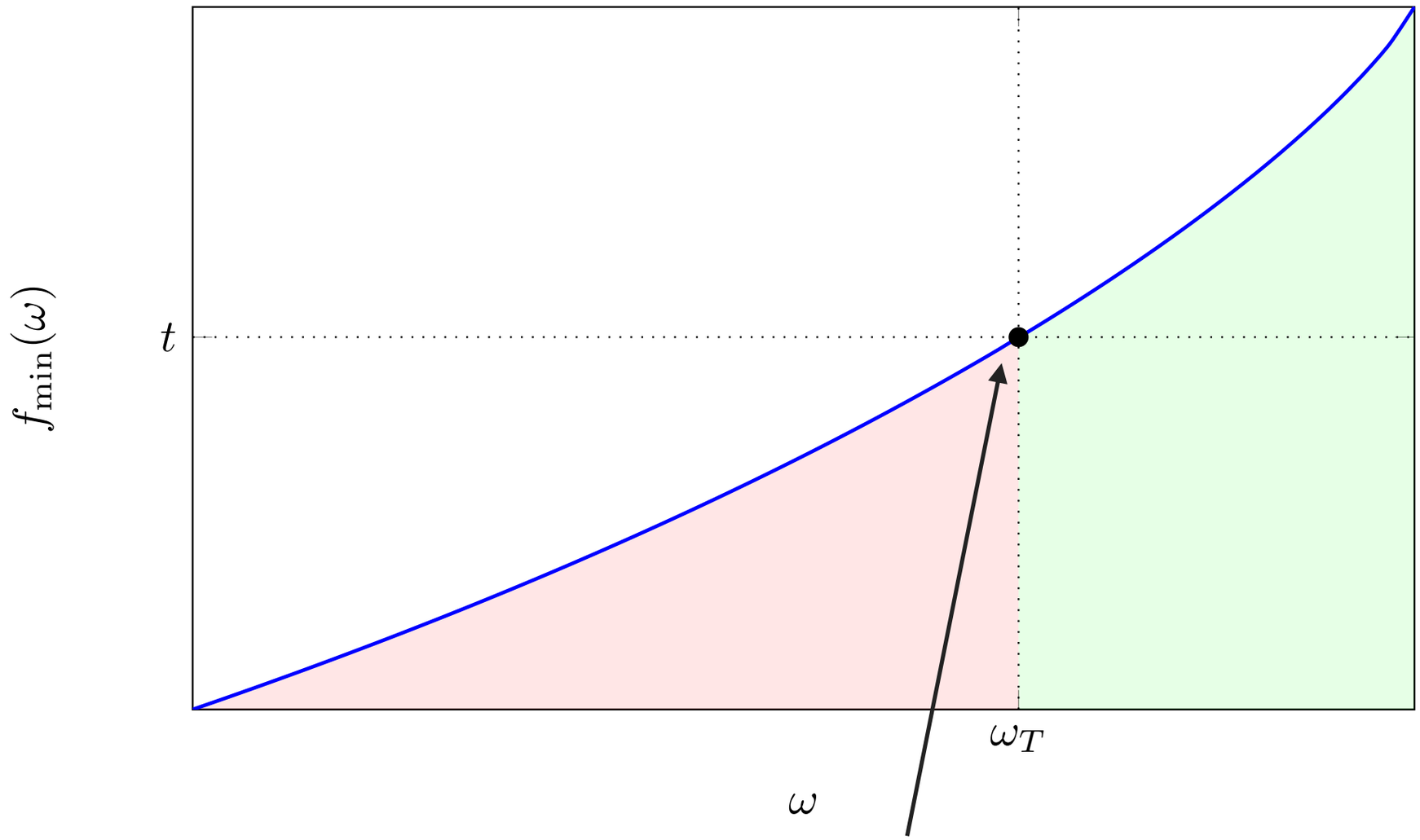
- Min-tradeoff function will allow us to get the right value of $t$
- $f_{\min}$ : The worst-case von Neumann entropy in a single round, restricted to the "correct average winning probability"




$$f_{\min}(\omega) \leq \inf_{\substack{\sigma \text{ with winning} \\ \text{prob.} \geq \omega}} H(AB|XYE)_{\mathcal{M}_i(\sigma)}$$

# Min-Tradeoff Function

$$f_{\min}(\omega) \leq \inf_{\substack{\sigma \text{ with winning} \\ \text{prob.} \geq \omega}} H(AB|XYE)_{\mathcal{M}_i(\sigma)}$$




$$H^{\varepsilon}_{\min}(\mathbf{AB}|\mathbf{XYE})_{\rho_{|\Omega}} \geq nt - \nu\sqrt{n}$$

# Entropy Accumulation

- The EAT gives us:

total smooth min-entropy

first and second order terms

$$H_{\min}^{\varepsilon}(\mathbf{AB}|\mathbf{XYE})_{\rho_{|\Omega}} \geq nt - \nu\sqrt{n}$$

final state conditioned on not aborting the protocol

the value given by the min-tradeoff function (for the observed statistics)

# 5. Classical Post-Processing

# Error Correction

- The hard part is done: we have a lower-bound on the smooth min-entropy of the raw keys


- Error correction: Alice sends classical information to Bob over a **public** authenticated channel

- Information is leaked to Eve $\Rightarrow$ we lose some of the entropy

$$H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{XYE}) \longrightarrow H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{XYE}) - \mathrm{leak}_{\mathrm{EC}}$$

- Positive entropy rate does not imply positive key rate

# Privacy Amplification

$\mathbf{a}$: 0 1 1 0 1 0 0 0 1

$\mathbf{b}$: 0 1 1 0 1 0 0 0 1

Privacy amplification →

$K_A$: 0 1 0 1 1 0

$K_B$: 0 1 0 1 1 0

- Removes the remaining correlations with Eve at the cost of reducing the key length
- Privacy amplification: Apply a **quantum-proof** randomness extractor $\mathbf{A} \xrightarrow{\text{EXT}} K_A$

$$H_{min}^{\varepsilon}(\mathbf{A}|\mathbf{XY}E) \geq \kappa \quad \Rightarrow \quad \left\|\rho_{K_A\mathbf{XY}E} - \rho_{U_\ell} \otimes \rho_{\mathbf{XY}E}\right\| \leq \varepsilon_{\text{sec}}$$

- See Amnon Ta-Shma's tutorial (QCrypt 13)

# “Closing the Loop”

$$H^{\varepsilon}_{min}(\mathbf{A}|\mathbf{XY}E) \geq \kappa$$

$$H^{\varepsilon}_{min}(\mathbf{A}|\mathbf{XY}E) \geq \kappa \;\; \Rightarrow$$

$$\left\|\rho_{K_A\mathbf{XY}SE} - \rho_{U_\ell} \otimes \rho_{\mathbf{XY}SE}\right\| \leq \varepsilon_{\text{sec}}$$

$$(1 - \Pr(\text{abort}))\left\|\rho_{K_AE} - \rho_{U_\ell} \otimes \rho_E\right\| \leq \varepsilon_{\text{sec}}$$

Lower-bound the smooth min-entropy of the raw data

Extractor works

Secrecy

Correctness

Soundness

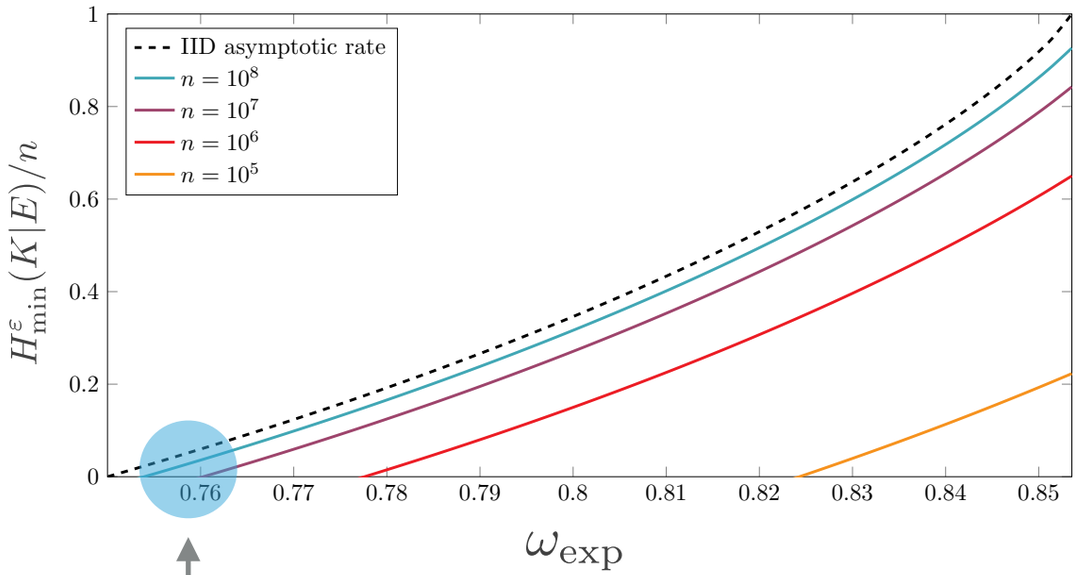
Completeness

Security

# Challenges

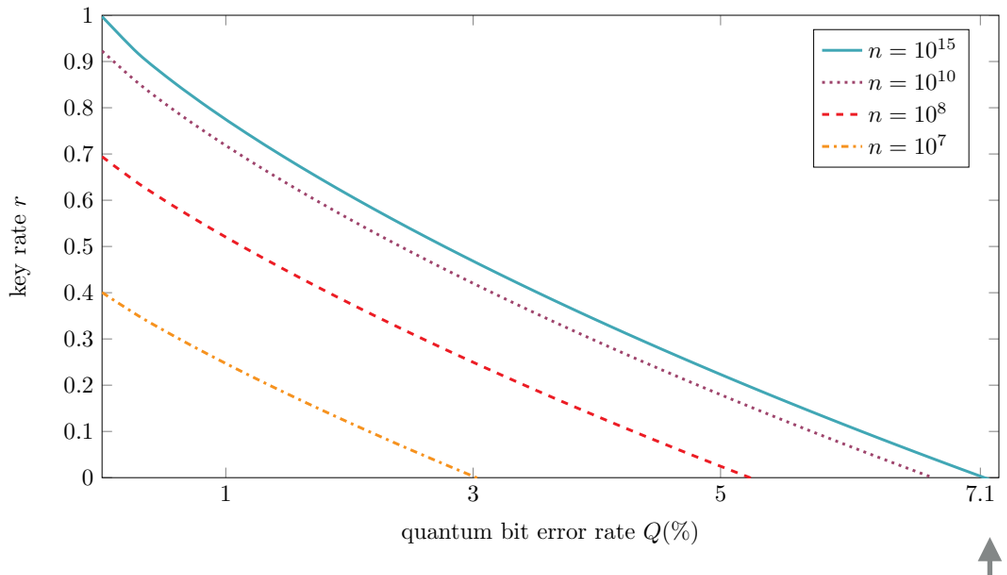
# Theory–Experiment Gap

## Entropy rate




current experiments

## DIQKD key rate

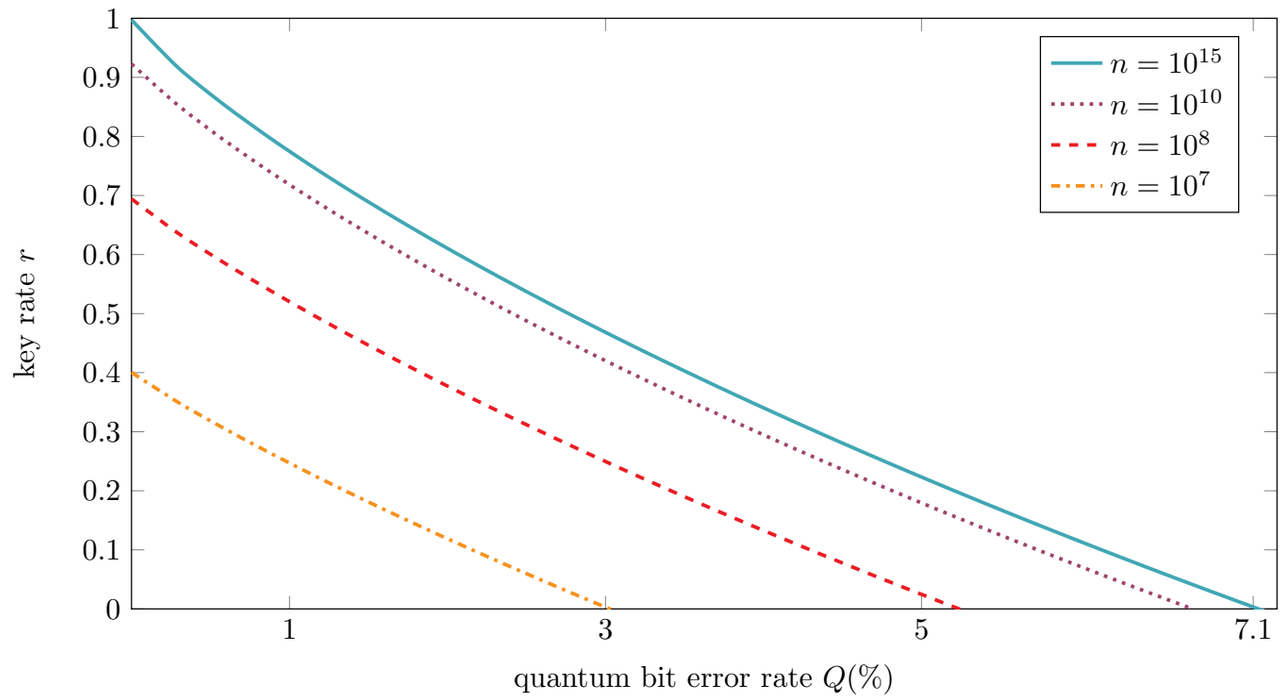



$\omega_{\exp} \approx 0.8$

- Still far from experimental abilities
- “Standard” DIQKD protocol key rates are asymptotically tight

# Theory–Experiment Gap

- How can we bridge the gap?
  - Better experiments :)
  - Better theory? What can we try to improve?



# The Quest for Better DIQKD Protocols

1. **Classical post-processing**: optimality of the rate is known only for protocols based on one-way communication post-processing.
    - Main challenge: Look for two-way advantage distillation protocols and prove their security

2. **Non-local game**: Is there a non-local game that can certify more key rate compared to the CHSH game?
    - Main challenge: Constructing a good min-tradeoff function (bound $H$, not $H_{\min}$)
        - Can numerical techniques help with that?

Stay for the next session!

# The Quest for Better DIQKD Protocols

3. Analysis of no-click event

- In photonics experiments, the Bell violation is low due to the effect of non-click events
- Cannot just drop them– that would open a “loophole”
- Can we take no-click events into account in a security proof in a more refined way?
  - E.g., modify the protocol to “announce” no-click events and incorporate this information to the min-tradeoff function

# The Quest for Better DIQKD Protocols

Maybe there are not (much) better protocols? :(

4. Look for **upper bounds** on DIQKD key rates

   - Main challenge: Argument should hold for any possible protocol (or family of protocols)
   - See the next talk by Eneet Kaur for a step in that direction

5. Consider **semi**-DIQKD protocols instead

   - Which model is relevant? Talk yesterday & on Thursday!
   - Perform tight non-asymptotic analysis: Can we really do better in the relevant regime of parameters?

# Device-Independent Quantum Key Distribution: Security Proofs and Challenges

# Thank You!

QCrypt | August 2019 | Montreal, Canada

**Rotem Arnon-Friedman** | UC Berkeley

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