

Bush 631-603: Quantitative Methods

Lecture 13 (04.18.2023): Uncertainty vol. III

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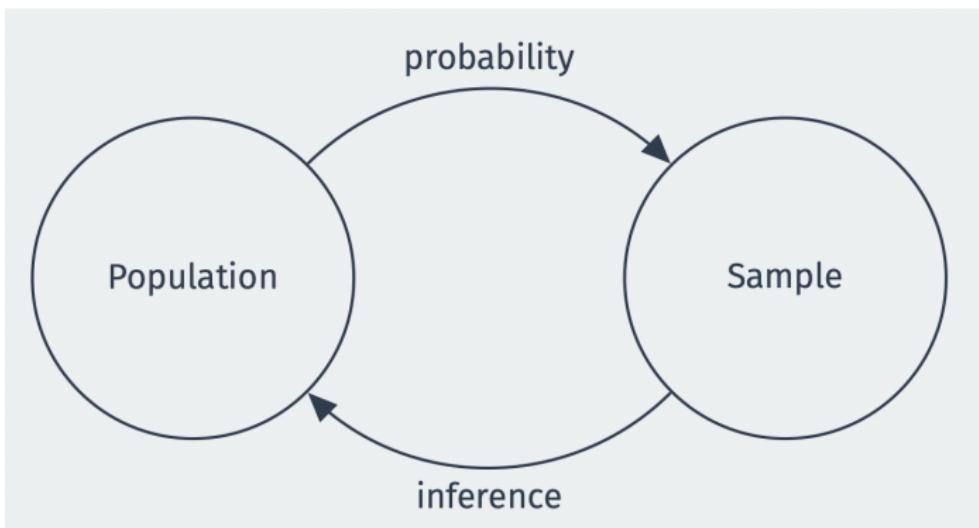
Spring 2023

What is today's plan?

- ▶ Calculating uncertainty: the full package.
- ▶ Linear regression model estimator.
- ▶ Assumptions for OLS estimators.
- ▶ Bivariate and multivariate models.
- ▶ R work: `lm()`, `summary(lm())`

Our data - our research interests

- ▶ Making inferences from data to population



Statistical hypothesis testing

- ▶ Probabilistic *proof by contradiction*
- ▶ Assume the contrast to our expectations is not possible.
- ▶ Assume → difference (sample and analyst) are zero.
- ▶ Incorrect? → differences exist.
- ▶ Senior analyst may have been wrong.
- ▶ We can never **fully** reject a hypothesis (no 100% certainty).

Procedure for hypothesis tests

- ▶ Steps for testing:
 1. Define null and alternative hyps ($H_0; H_1$).
 2. Select *test statistic* and level of test (α).
 3. Derive reference distribution.
 4. Calculate p-values.
 5. Make a decision: reject/retain.
- ▶ **Decision rule:**
 - ▶ **Reject null** if p-value is *below* α
 - ▶ Otherwise **retain the null or fail to reject**.
- ▶ Common thresholds:
 - ▶ $p \geq 0.1$: “not statistically significant”.
 - ▶ $p < 0.05$: “statistically significant”.
 - ▶ $p < 0.01$: “highly significant”.

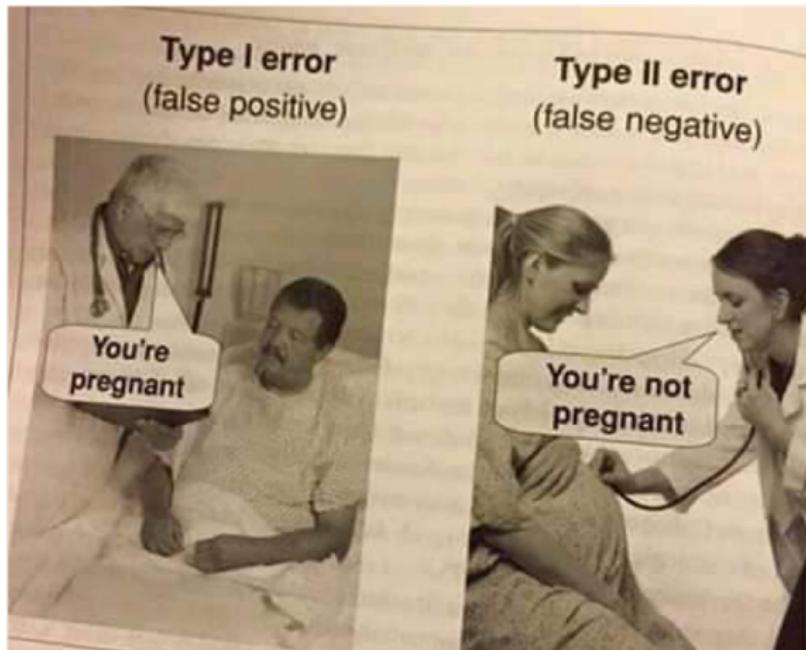
Test errors

- ▶ $p = 0.05 \rightarrow$ extreme data only happen in 5% of repeated samples (if null is true).
- ▶ \rightsquigarrow 5% of time we reject null that is true!
- ▶ Types of errors:

	H_0 True	H_0 False
Retain H_0	Awesome!	Type II error
Reject H_0	Type I error	Good stuff!

Test errors

- ▶ What does these errors mean?



One sample test

- ▶ The z-statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Or:

$$Z = \frac{\text{observed} - \text{null}}{SE}$$

- ▶ How many SEs away from the null guess is the sample mean?
- ▶ **Small samples problem:** uncertainty about \bar{X} distribution.
- ▶ Find t-statistic instead:

$$T = \frac{\bar{X} - \mu}{\hat{SE}} \approx t_{n-1}$$

Two sample tests

- ▶ Goal: learn about population difference in means.
- ▶ Compare differences b-w multiple groups: same testing procedures.
- ▶ Define:
 - ▶ Null PATE: $H_0 : \mu_T - \mu_C = 0$
 - ▶ Alt. PATE: $H_1 : \mu_T - \mu_C \neq 0$
 - ▶ Test statistic: diff-in-means estimator.
 - ▶ z-score for *two sample z-test*.
- ▶ Are the differences in sample means just random chance?

Two sample test

- ▶ Run a **two sample t-test** → `t.test()`

```
t.test(exp.dat$cont_cor1[exp.dat$trt1 == 0],  
       exp.dat$cont_cor1[exp.dat$trt1 == 1])
```

```
##  
## Welch Two Sample t-test  
##  
## data: exp.dat$cont_cor1[exp.dat$trt1 == 0] and exp.dat$cont_cor1[exp.dat$trt1 == 1]  
## t = -13.697, df = 993.53, p-value < 2.2e-16  
## alternative hypothesis: true difference in means is not equal to 0  
## 95 percent confidence interval:  
## -23.59653 -17.68267  
## sample estimates:  
## mean of x mean of y  
## 1489.333 1509.973
```

What we did? and next...

- ▶ So far, we covered uncertainty in:
 - ▶ Sample proportions (Trump vs. the polls).
 - ▶ Sample means (Israel thermometer scores).
 - ▶ Differences in sample means (experimental data, leaders' type).
- ▶ What about our regression estimates?
- ▶ Much uncertainty about them too!

Least squared

- ▶ Assumption: model \rightsquigarrow Data generation process (DGS)
- ▶ **Parameters/coefficients** (α, β) : true values unknown.
- ▶ Use data to estimate $\alpha, \beta \implies \hat{\alpha}, \hat{\beta}$
- ▶ Predictions:
 - ▶ Use the *regression line*.
 - ▶ Calculate *fitted value* (\neq observed value)

$$\hat{Y} = \hat{\alpha} + \hat{\beta} * x$$

Linear model elements

- ▶ *Residual/prediction error:* the difference b-w fitted and observed values.
- ▶ Real error is unknown $\Rightarrow \hat{\epsilon}$

$$\hat{\epsilon} = Y - \hat{Y}$$

Linear model estimation

Least squared:

- ▶ A method to estimate the regression line.
- ▶ Use data (values of Y & X_i).
- ▶ 'Select' $\hat{\alpha}, \hat{\beta}$ to minimize SSR.

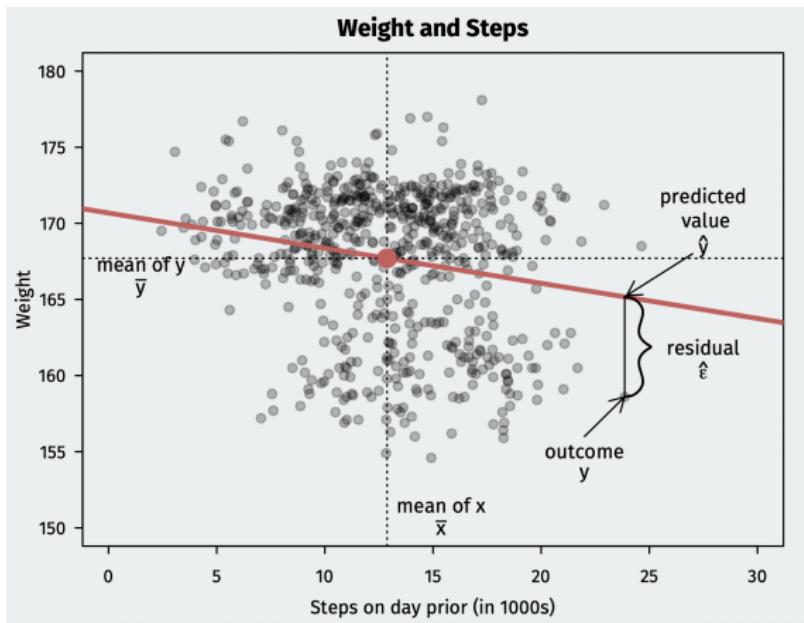
$$SSR = \sum_{i=1}^n \hat{\epsilon}^2 = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n (Y_i - \hat{\alpha} - \hat{\beta} * X_i)^2$$

Linear regression in R

Fit the model

- ▶ Syntax: `lm(Y ~ x, data = mydata)`
- ▶ Y = dependent variable; x = independent variable(s).

How does it look like?



Linear models in RCT

Binary independent variable:

- ▶ Slope coefficient (β) = diff-in-means estimator.
- ▶ $\hat{\beta}$: estimated average treatment effect.

- ▶ Why works?
 - ▶ Randomization \rightarrow causal interpretation
 - ▶ Slope (β): the average change in Y when X increases by 1 unit.

When X is binary:

- ▶ Treatment: yes or no.
- ▶ X change by 1 unit \rightarrow no to yes.
- ▶ Y changes as well.

Building linear models

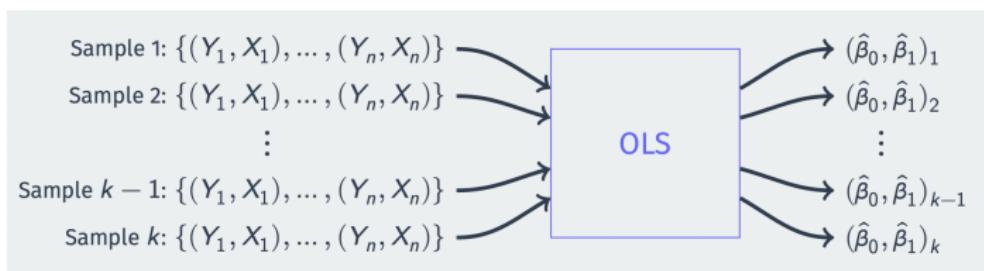
- ▶ Leader background and nuclear technology pursuit (2015)
 - ▶ Rebel or not?
 - ▶ Our model → rebel exp. & nukes technology.
-
- ▶ $Y_i = \beta_0 + \beta_1 * RebelExp_i + \epsilon_i$
 - ▶ P(Nukes) = rebel experience and ϵ (error).

Uncertainty in regression

- ▶ Quantify uncertainty in linear models
- ▶ Model parameters - estimators
- ▶ What estimator? **least squared.**

Least squared estimator

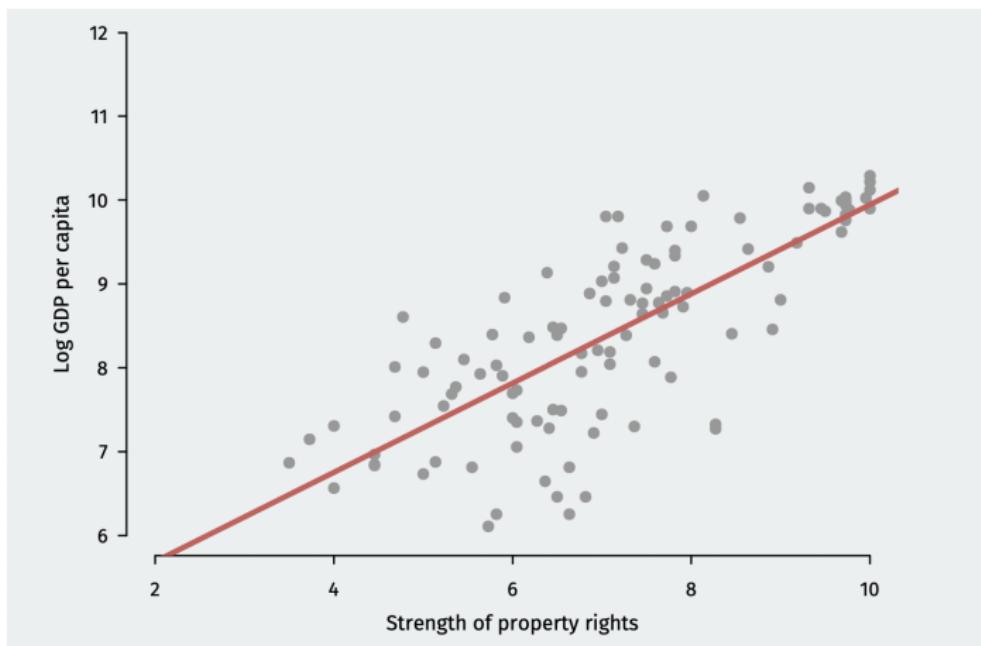
- ▶ We ‘plug-in’ data and get estimates.



- ▶ Estimators values are uncertain.

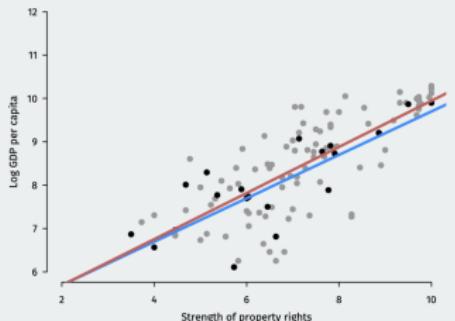
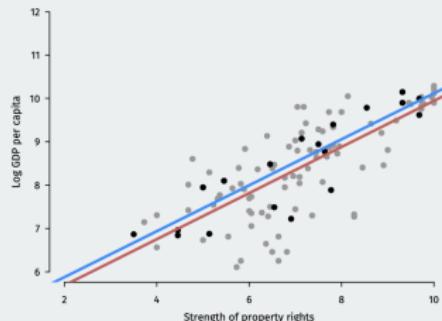
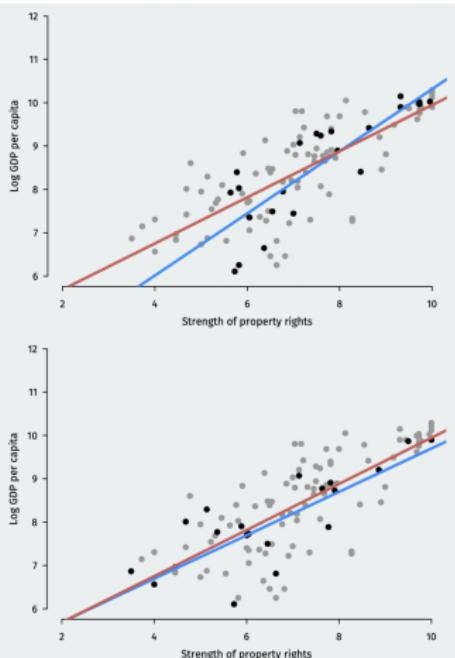
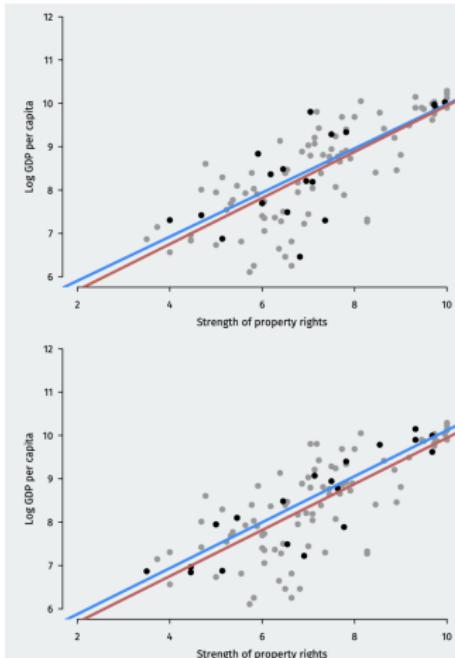
Uncertainty of least squared estimators

- ▶ Data: Relationship between strength of property rights and GDP.



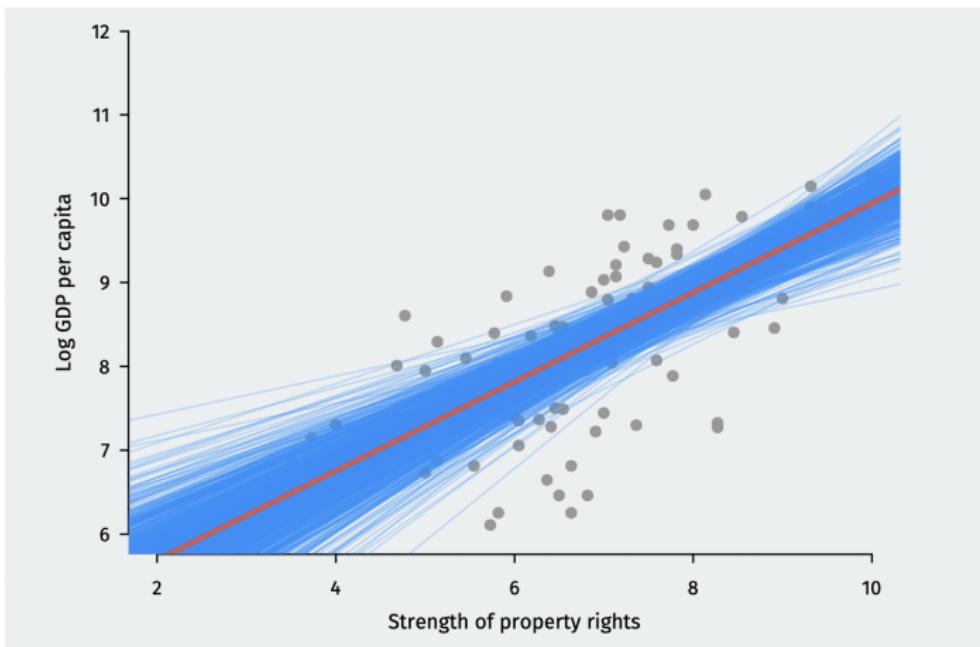
Simulation Again?

- ▶ Sample 30 countries and calculate $\text{Im}(\text{GDP} \sim \text{Property.rights})$



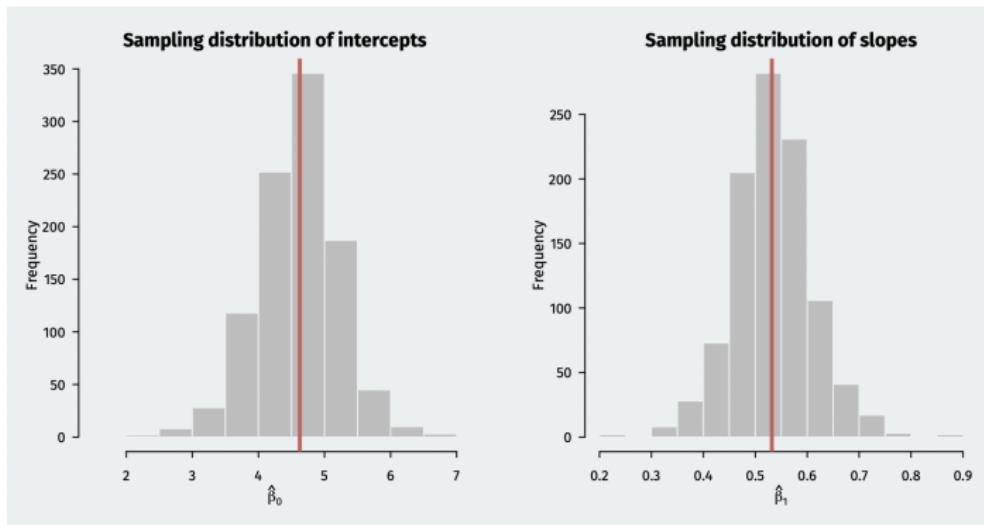
Simulation Again?

- ▶ Multiple iterations of the model within the data.



OLS sampling distributions

- ▶ Variations of intercept ($\hat{\beta}_0$) and slope ($\hat{\beta}_1$)



Least squared estimator

- ▶ Uncertainty in *least squared* estimator:
 - ▶ Generate reference distribution.
 - ▶ Calculate SEs.
 - ▶ Construct 95% CIs.
 - ▶ Run hypotheses tests.
 - ▶ Results are ‘statistically significant’, or not.
 - ▶ Is our estimator different than zero? (reject the null)

Assumptions

- ▶ Assumptions for regression estimates:

(1) **Exogeneity**: mean of ϵ_i does not depend on X_i

$$E(\epsilon_i | X_i) = E(\epsilon_i) = 0$$

(2) **Homoskedasticity**: variance of ϵ_i does not depend on X_i

$$V(\epsilon_i | X_i) = V(\epsilon_i) = \sigma^2$$

Problem of exogenous factors

- ▶ Confounders between X_i and Y_i
 - ▶ Factors in ϵ_i that are related to X_i
 - ▶ Why?
-
- ▶ Business background (X_i) → defense spending (Y_i)
 - ▶ Socioeconomic background → ϵ_i
 - ▶ But Socioeconomic background → Business experience, so . . .
 - ▶ Is Y_i due to business experience?

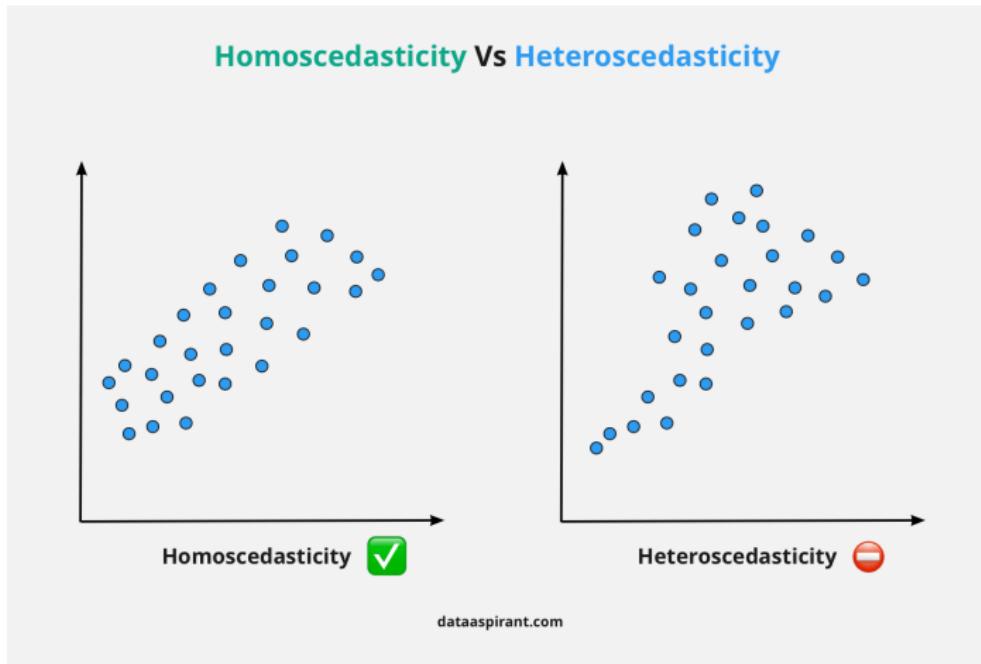
Problem of exogenous factors

- ▶ RCTs → **no exogeneity** problem.
- ▶ Randomized treatments!

- ▶ Severe issue for observational studies.
- ▶ Rebel background → nuclear weapons pursuit.
- ▶ Perhaps more conflicts → pursue advanced technology.

Homoskedas... what?

- When spread of Y_i depends on X_i



OLS properties

$$Y_i = \beta_0 + \beta_1 * X_i + \epsilon_i$$

- ▶ Our estimates: $\hat{\beta}_0, \hat{\beta}_1$ are r.v.s.
- ▶ Equal to true value? (population parameters)
- ▶ How spread are they around their center?
- ▶ Estimate the SE $\rightarrow \hat{SE}(\hat{\beta}_1)$
- ▶ Next? construct CIs. . .
- ▶ Run hypotheses tests.

Putting everything together

- ▶ Hypotheses:
 - ▶ $H_0 : \beta_1 = 0$
 - ▶ $H_a : \beta_1 \neq 0$
- ▶ Our estimators: $\hat{\beta}_0, \hat{\beta}_1$
- ▶ SE and CIs:
 - ▶ $\hat{\beta}_0 \pm 1.96 * \hat{SE}(\hat{\beta}_0)$
 - ▶ $\hat{\beta}_1 \pm 1.96 * \hat{SE}(\hat{\beta}_1)$
- ▶ Hypotheses test:
 - ▶ Test statistic: $\frac{\hat{\beta}_1 - \beta_1^*}{\hat{SE}(\hat{\beta}_1)} \sim N(0,1)$
 - ▶ $\hat{\beta}_1$ is **statistically significant** if $p < 0.05$ (reject null H_0).

Now with data

- ▶ Rebel experience and pursuit of nuclear tech (2015)

```
head(nukes, n=9)
```

```
## # A tibble: 9 x 76
##   ccode idacr year leadid30 leader~1 startdate    inday inmonth inyear starty
##   <dbl> <chr> <dbl> <chr>   <chr>     <date>    <dbl>   <dbl>   <dbl> <date>
## 1     2 USA     1945 A2.9-43 Roosevelt 1933-03-04      4       3 1933 1945-0
## 2     2 USA     1945 A2.9-46 Truman   1945-04-12     12       4 1945 1945-0
## 3     2 USA     1946 A2.9-46 Truman   1945-04-12     12       4 1945 1946-0
## 4     2 USA     1947 A2.9-46 Truman   1945-04-12     12       4 1945 1947-0
## 5     2 USA     1948 A2.9-46 Truman   1945-04-12     12       4 1945 1948-0
## 6     2 USA     1949 A2.9-46 Truman   1945-04-12     12       4 1945 1949-0
## 7     2 USA     1950 A2.9-46 Truman   1945-04-12     12       4 1945 1950-0
## 8     2 USA     1951 A2.9-46 Truman   1945-04-12     12       4 1945 1951-0
## 9     2 USA     1952 A2.9-46 Truman   1945-04-12     12       4 1945 1952-0
## # ... with 66 more variables: enddate <date>, outday <dbl>, outmonth <dbl>,
## #   outyear <dbl>, yearlyduration <dbl>, entry <dbl+lbl>, exit <dbl+lbl>,
## #   pursuit <dbl>, initiation <dbl>, explore <dbl>, bombprgm <dbl>,
## #   pursuitjg <dbl>, pursuitswh <dbl>, rebel <dbl>, milservice <dbl>,
## #   jcrevolutionary <dbl>, revolutionaryleader <dbl>, irregular <dbl>,
## #   fiveyear <dbl>, polity2 <dbl>, total <dbl>, spally <dbl>, NCA67 <dbl>,
## #   gdpcap <dbl>, lndgdpca <dbl>, npt <dbl>, openness <dbl>, rivalry <dbl>,
```

Rebels and Nukes (2015)

- ▶ OLS regression models in R

```
lm(pursuit ~ rebel, data = nukes)

## 
## Call:
## lm(formula = pursuit ~ rebel, data = nukes)
## 
## Coefficients:
## (Intercept)      rebel
##       0.01051     0.03767
```

Rebels and Nukes (2015)

- ▶ Simple/bivariate regression

```
summary(lm(pursuit ~ rebel, data = nukes))

##
## Call:
## lm(formula = pursuit ~ rebel, data = nukes)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.04819 -0.04819 -0.01051 -0.01051  0.98949
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 0.010513  0.002295  4.582 4.68e-06 ***
## rebel       0.037673  0.003513 10.725 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1598 on 8460 degrees of freedom
##   (390 observations deleted due to missingness)
## Multiple R-squared:  0.01341,    Adjusted R-squared:  0.0133 
## F-statistic: 115 on 1 and 8460 DF,  p-value: < 2.2e-16
```

Rebels and Nukes (2015)

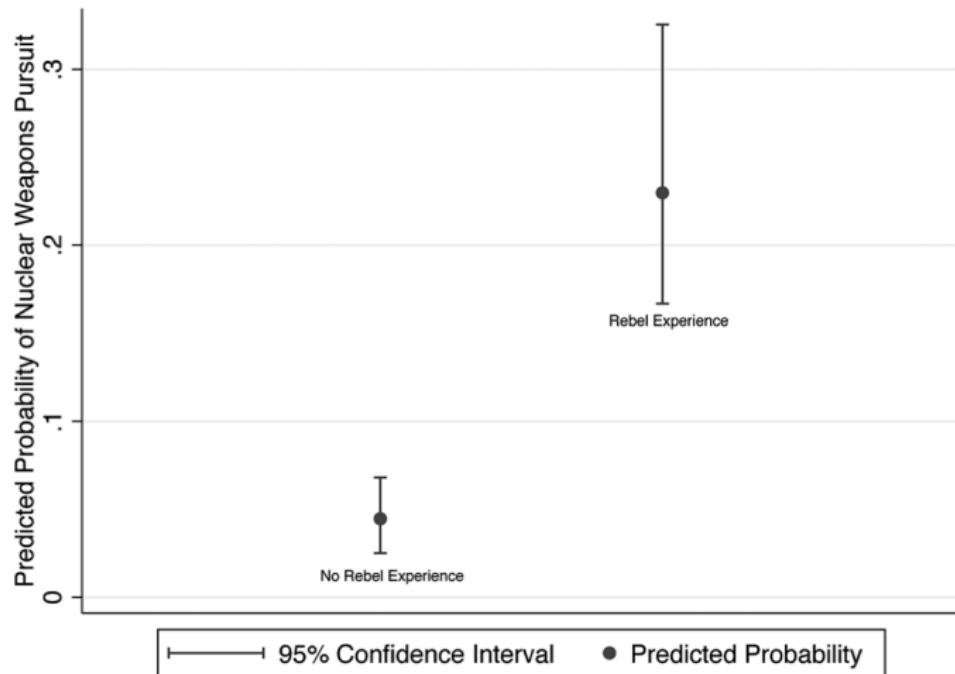
- ▶ Multivariate regression: account for confounders

```
summary(lm(pursuit ~ rebel + milservice + polity2, data = nukes))

##
## Call:
## lm(formula = pursuit ~ rebel + milservice + polity2, data = nukes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -0.06587 -0.04408 -0.02544 -0.01020  0.99682 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 0.0073899  0.0027782   2.660  0.00783 ** 
## rebel       0.0320096  0.0044238   7.236 5.08e-13 *** 
## milservice  0.0217914  0.0045106   4.831 1.38e-06 *** 
## polity2     0.0004679  0.0002801   1.670  0.09489 .  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1672 on 7684 degrees of freedom
##   (1164 observations deleted due to missingness)
## Multiple R-squared:  0.01596,    Adjusted R-squared:  0.01558 
## F-statistic: 41.54 on 3 and 7684 DF,  p-value: < 2.2e-16
```

OLS coefficient interpretation

- Rebel experience and nuclear technology (2015)



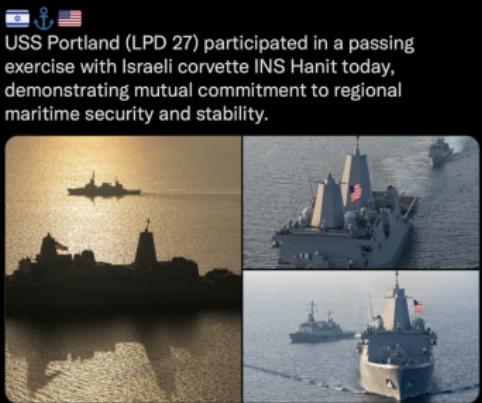
OLS Multivariate regression

- ▶ **Remember:** correlation does not mean causation.
- ▶ Multiple confounders → same process:
 - ▶ CIs are constructed the same for all $\hat{\beta}_j$.
 - ▶ Hypothesis tests also run the same for all $\hat{\beta}_j$.
 - ▶ p-values have the same interpretation.
- ▶ Interpretation of $\hat{\beta}_j$:
 - ▶ A change in Y_i is associated with a one-unit increase in X_i when...
 - ▶ All other variables are held constant (at mean value, usually).

OLS regression models: FP research

- ▶ Joint military exercises and conflict (2021)

 U.S. 5th Fleet 
@US5thFleet

...


USS Portland (LPD 27) participated in a passing exercise with Israeli corvette INS Hanit today, demonstrating mutual commitment to regional maritime security and stability.

11:00 AM · Nov 15, 2021 · Twitter Web App

41 Retweets 6 Quote Tweets 169 Likes

Flashpoints

China, Russia launch joint naval drills in Russian Far East

By The Associated Press

Friday, Oct 15



The Liaoning aircraft carrier is accompanied by frigates and submarines on April 12, 2018, conducting exercises in the South China Sea. (Li Gang/Xinhua via AP)

JME and conflict

- ▶ Under what conditions violence is more likely? who will initiate?
- ▶ Outcome conditioned by alliance partnership.
- ▶ Use two-stage model:
 1. Selection into conflict.
 2. Effects of JMEs.
- ▶ Data: directed dyad-year (1973-2003).

JME and military conflict

Table 2. Main Results for the Effects of JMEs and Alliances on Conflict Escalation.

	Targets		Participants	
	Model 1:	Model 2:	Model 3:	Model 4:
JME	-0.311*** (0.100)		-0.573*** (0.101)	
Non-Ally JME		-0.050 (0.146)		-0.148 (0.141)
Ally JME		-0.443*** (0.117)		-0.823*** (0.124)
Alliances	0.013* (0.007)	0.016** (0.007)	-0.009 (0.008)	-0.004 (0.008)
Joint Democracy	-0.753*** (0.092)	-0.745*** (0.092)	-0.730*** (0.089)	-0.720*** (0.089)
CINC	9.042*** (1.114)	8.901*** (1.114)	10.800*** (1.063)	10.597*** (1.063)
UNGA	-0.055 (0.045)	-0.050 (0.045)	-0.047 (0.044)	-0.041 (0.044)
Trade	0.00001 (0.00000)	0.00001 (0.00000)	0.00001 (0.00000)	0.00001 (0.00000)
Lagged DV	6.631*** (0.092)	6.623*** (0.092)	6.171*** (0.092)	6.159*** (0.092)
Constant	-6.970*** (0.272)	-6.984*** (0.272)	-6.945*** (0.271)	-6.967*** (0.271)
N	541,920	541,920	541,920	541,920
AIC	7,757.394	7,753.953	8,415.870	8,402.368
Log Likelihood	-3,839.697	-3,836.977	-4,168.935	-4,161.184

Note: Coefficients Represent Logistic Regression Coefficients.

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Targeting the stock market

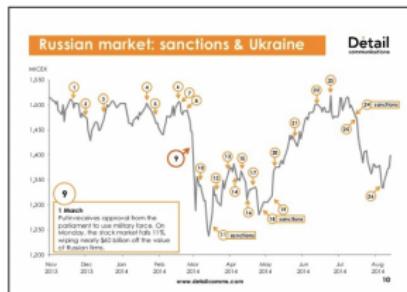
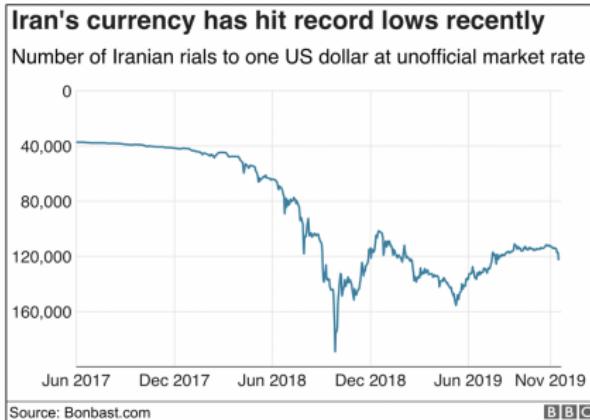
WORLD MARKETS

Iran's stock market roars as sanctions go away

PUBLISHED WED, JAN 20 2016 11:11 AM EST | UPDATED WED, JAN 20 2016 11:35 AM EST

SHARES

Jonas Grotzke



Targeting the stock market

- ▶ How sanctions affect stock markets' in targeted countries (2021).
- ▶ Imposing costs on stock market → behavior change.
- ▶ Account for types of sanctions.
- ▶ The cumulative effects of sanctions over time.
- ▶ Data: monthly stock market values for 66 countries (1990-2005)

Targeting the stock market

- ▶ Types of sanctions matter:
 - ▶ Import: restrict access to global markets and reduce firm revenues.
 - ▶ Also harm exporters: investment shifts away from losing firms.
 - ▶ Export: limits on exports thus loss of hard currency.
 - ▶ Less efficient as import firms make-up for lost capital and goods.
- ▶ Example: Iraqi oil boycott (1990).
- ▶ Cumulative sanctions regime:
 - ▶ More is better.
 - ▶ But decreasing marginal effect.
 - ▶ Initial sanctions are more useful
 - ▶ Target adjusts to additional restrictions.

Targeting the stock market

- ▶ Empirical analysis:
 - ▶ OLS regression models.
 - ▶ ADL: account for time lags.
- ▶ Results:
 - ▶ Negative effect on stocks.
 - ▶ Type matters, as well as number of sanctions.
 - ▶ Sender state also matters.
- ▶ Models 1&2: full and reduced set of controls.
- ▶ Models 3-5: sanctions types.
- ▶ Models 6&7: Comparing G20 to non-G20 countries.

International Aid and civilian casualties



Apr 13, 2016

Balochistan: Pakistan Army Kills Over 35 Civilians and Carries Out Mass Abductions

International Aid and civilian casualties

- ▶ Are civilians facing risks due to aid distribution?
- ▶ Two mechanisms:
 1. Persuasion: reduce incentives to target civilians (military).
 2. Predation: adverse incentives for resource capturing and extended collective violence (development).
- ▶ Data: military and ODA flows in 135 countries (1989-2011).

Military and development aid flows

Variables	1(a) U.S. military aid	1(b) Development aid	1(c) Full model	1(d) Lagged DV	1(e) Excluding outliers
OSV (t-1)				0.000** (0.000)	0.0148** (0.00666)
U.S. military aid (logged, lagged)	-0.338*** (0.109)		-0.368*** (0.097)	-0.348*** (0.101)	-0.187** (0.090)
Development aid (logged, lagged)		0.237** (0.117)	0.366*** (0.135)	0.371*** (0.136)	0.269** (0.130)
State strength	-0.000 (0.001)	-0.002*** (0.001)	0.000 (0.001)	-0.000 (0.001)	0.000 (0.001)
Polity2	-0.256*** (0.075)	-0.117* (0.069)	-0.167** (0.079)	-0.151* (0.079)	-0.009 (0.045)
Rebel OSV (lag)	-0.000 (0.001)	0.001 (0.002)	-0.000 (0.000)	-0.001 (0.001)	-0.001 (0.001)
Intrastate conflict	4.717*** (0.646)	4.858*** (0.634)	5.230*** (0.709)	5.463*** (0.816)	3.653*** (0.621)
Trade openness	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Previous regime change	2.107*** (0.380)	2.071*** (0.520)	2.036*** (0.359)	2.002*** (0.368)	2.045*** (0.382)
Oil production	-0.299*** (0.087)	-0.237*** (0.074)	-0.223** (0.091)	-0.235*** (0.090)	-0.109 (0.097)
Ethnic exclusion	0.754*** (0.210)	0.776*** (0.202)	0.765*** (0.215)	0.724*** (0.224)	0.263 (0.172)
Ethnic fractionalization	0.624 (0.799)	0.362 (0.852)	-0.201 (0.814)	-0.163 (0.817)	-0.286 (0.901)
Constant	4.112** (1.759)	-2.553*** (0.975)	2.301 (1.791)	1.965 (1.831)	-0.300 (1.523)
Observations	2,032	2,791	2,032	2,032	2,005

Note. Robust standard errors in parentheses.

*** $p < .01$, ** $p < .05$, * $p < .1$

What to do with reg models?

- ▶ Regression models:
 - ▶ Useful tool to assess causality.
 - ▶ Pack **a lot** of information.
 - ▶ Can be hard to interpret.
- ▶ So, what to do?
 - ▶ Substantive results.
 - ▶ Predictions!!
 - ▶ Sub-groups and effects by types.

Show meaningful results!

Reg models to presentations

- ▶ Predictions → quantity of interest

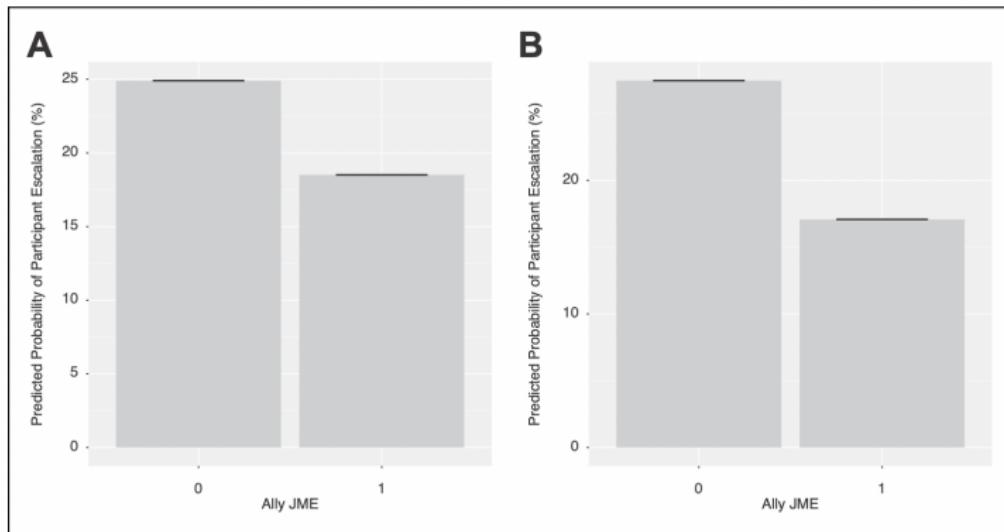
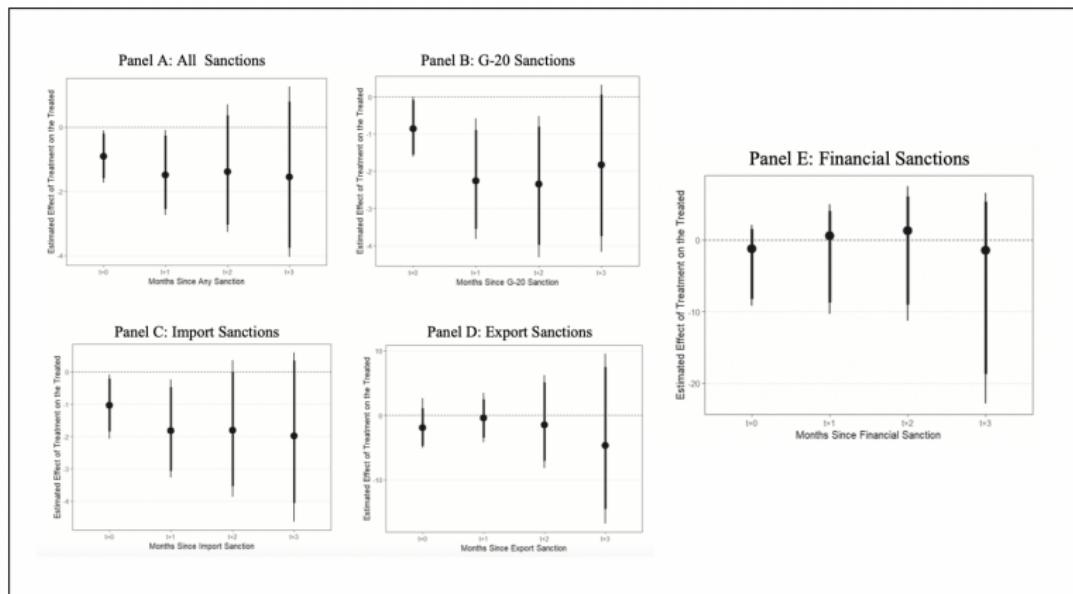


Figure 3. Predicted probability of Escalation as a function of Ally JME, with 95 percent confidence intervals. Results obtained from a Heckman selection model and are conditional upon conflict onset. (A) Targets. (B) Participants.

Reg models to presentations

- Predicting sanction types effectiveness



Wrapping up Week 13

- ▶ Summary:
 - ▶ Testing uncertainty: the full package.
 - ▶ Linear regression model estimator.
 - ▶ Assumptions for OLS estimators.
 - ▶ Bivariate and multivariate models.
 - ▶ Interpretation of β coefficient.
 - ▶ Reading a regression table.