Exercise 6

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Question 1

If you select 2 random integers between 1 and 10 without replacement, what is the probability that the second integer is larger than the first? Write the R code that calculates the probability through a Monte Carlo simulation of 100 runs.Also calculate the exact probability answer: P(second integer is larger than the first) = 45/90 = 0.5

```
n<-100
count<-0
for(i in 1:n){
  x<-sample(1:10,size=2,replace=FALSE)
  if(x[2]>x[1]){
    count<-count+1
  }
}
prob<-count/n
prob
```

```
## [1] 0.42
```

Question 2

If you select a random integer between 1 and 100, what is the probability that it is divisible by either 2 or 3? Calculate the exact probability using the inclusion-exclusion principle and write a Monte Carlo simulation that provides an approximation to the probability. answer:P(div by 2)=50, P(div by 3)=33, P(div by 6(by 2 and 3))=16. there for P(div by 2 or 3)=50+33-16=67 67/100=0.67(it's the probability)

```
n<-10000
count<-0
for(i in 1:n){
  x<-sample(1:100,size=1,replace=TRUE)
  if(x %% 2 == 0 | x %% 3 == 0){
    count<-count+1
  }
}
prob<-count/n
prob
```

```
## [1] 0.6644
```

Question 3

If you roll two fair six-sided dice, what is the probability that the sum of the two numbers is 7? Write the exact probability and provide a Monte Carlo simulation that approximates the probability. answer: there are 6 outcomes that will give us the sum 7, overall there are 36 outcome for two dice. we will do 6/36 = 1/6=0.1667.

```
n<-10000
count<-0
for(i in 1:n){
  x<-sample(1:6, size = 2, replace = TRUE)
  if(sum(x) == 7){
    count<-count+1
  }
}
prob<-count/n
prob
```

```
## [1] 0.1606
```

Question 4

Define the following events for a random experiment where a fair dice with four black wigs and two white wigs is tossed twice: A = In the first toss, a white wig was obtained B = In the first and second toss, wigs of different colors were obtained. Are A and B dependent? Provide reasoning to support your answer answer: P(A)=2/6 P(B)= 2/6 * 4/6 + 4/6 * 2/6 = 8/36 + 8/36 = 16/36 = 4/9 P(A and B)= 2/6 * 4/6= 8/36 P(A)*P(B)= 2/6 4/9 = 4/27 P(A and B) != P(A)*P(B) there for A and B dependent

Question 5

A group of people are interviewed for a position in a desirable company. Among the candidates, 40% are Spanish speakers, and 60% of the Spanish speakers have an academic degree. It is known that half of the academic degree holders are Spanish speakers.

5.1 What is the probability of a random candidate having an academic degree? answer: 0.4*0.6=0.24 Spanish speakers with academic degree. 0.24*2=0.48 half of the academic degree holders are Spanish speakers. the probability of a random candidate having an academic degree is 0.48.

5.2 Are the events "a candidate with an academic degree" and "a Spanish-speaking candidate" dependent or not? Explain. answer: we will define 2 events: A= a Spanish-speaking candidate. B= a candidate with an academic degree. now we check if A and B dependent. P(A)=0.4, P(B)=0.48. P(A)*P(B)= 0.192 P(A and B)=0.24 P(A and B) != P(A)*P(B) there for A and B dependent.

5.3 The company invites for a second interview only the candidates who speak Spanish or have an academic degree. What percentage of the candidates will be invited for a second interview? answer: P(A or B)=P(A)+P(B)-P(A and B)=0.4+0.48-0.24=0.64 the percentage of the candidates will be invited for a second interview is 64%.

Question 6

A pair of dices are thrown over and over again until you get a sum of 10 or a sum of 8 or a sum of 3 and then you stop. What is the probability that the game stops and on the last throw you got a sum of 10? (Calculate the exact probability and a Monte Carlo simulation that approximates the probability.) answer: P(A)=P(sum 3)=2/36 P(B)=P(sum 8)=5/36 P(C)=P(sum 10)=3/36 P(D)=P(A or B or C)=10/36 P(C|D)= (3/36) / (10/36) = 3/10

```
n <- 10000
count <- 0
for (i in 1:n) {
  sum <- 0
  while (sum != 10 && sum != 8 && sum != 3) {
    x <- sample(1:6, size = 2, replace = TRUE)
    sum <- sum(x)
  }
  if (sum == 10) {
    count <- count + 1
  }
}
prob <- count/n
prob
```

```
## [1] 0.2961
```

Good luck!