Exercise 7

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Question 1

For each one of the questions below, calculate: the possible values of X (Rx), the probability function (P(X)) and the expected value (E[X]). In addition, through simulation, create a barplot showing an approximation to P(x), when simulating the experiment 10000 times. a) In a box there are 8 balls, 3 of them are red and 5 are black. Three balls are drawn without replacement. Let X be the number of black balls drawn. b) A experiment is performed with a success probability of 0.3, repeatedly until 5 successes are obtained, and then the experiment stops. Let X be the number of trials performed. c) An urn contains 20 numbered balls (1,. . . 20). A player draws two balls at random from the urn. If the two numbers drawn add up to 20, the player wins 10 shekels. If the two numbers are consecutive (i.e., their absolute difference is 1), the player wins 20 shekels. Otherwise, the player loses 5 shekels. Let X be the player's profit in the game.

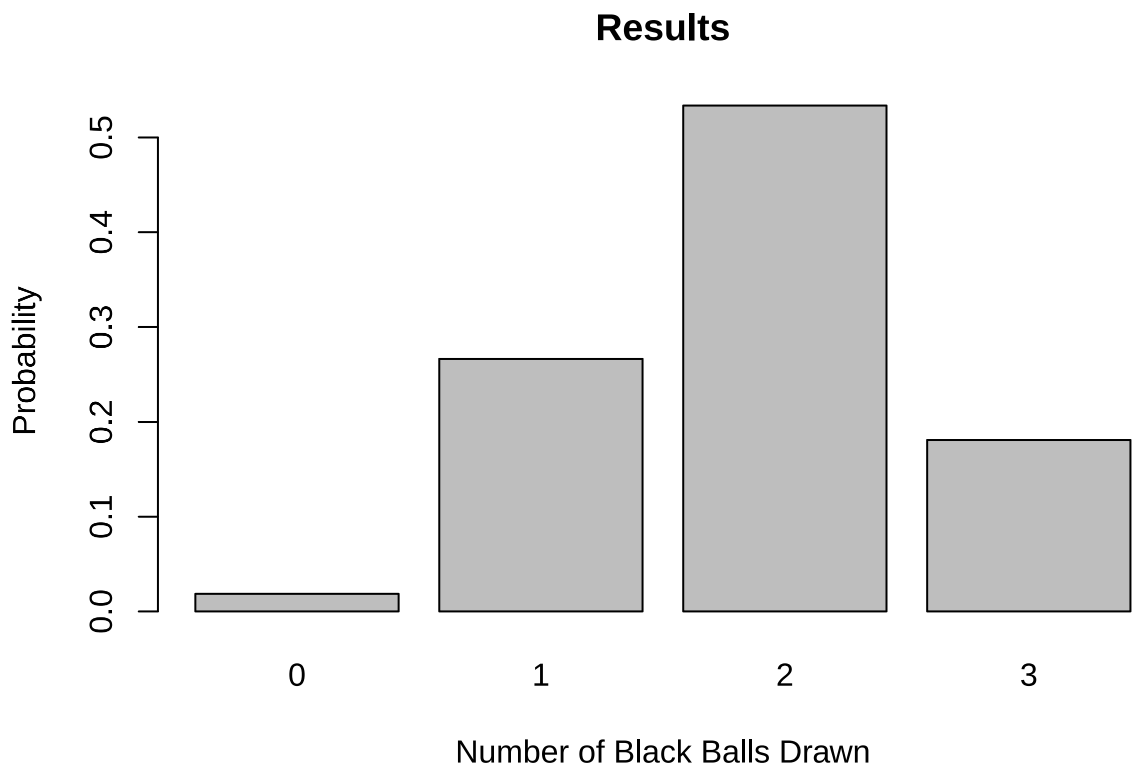
1.1 answer:

Rx={0,1,2,3}

P(X)=(choose(3,5)xchoose(0,3))/choose(3,8)=5/28=0.178

E[X]=0xP(X=0)+1xP(X=1)+2xP(X=2)+3xP(X=3)= 0x1/56+1x15/56+2x45/112+3x5/28=45/28=1.607

```r
balls <- c(rep("black", 5), rep("red", 3))
simulations <- 10000
results <- numeric(simulations)

for (i in 1:simulations) {
  drawn_balls <- sample(balls, 3, replace = FALSE)
  black_balls <- sum(drawn_balls == "black")
  results[i] <- black_balls
}

probabilities <- table(results) / simulations
barplot(probabilities, xlab = "Number of Black Balls Drawn",
        ylab = "Probability",main = "Results")
```



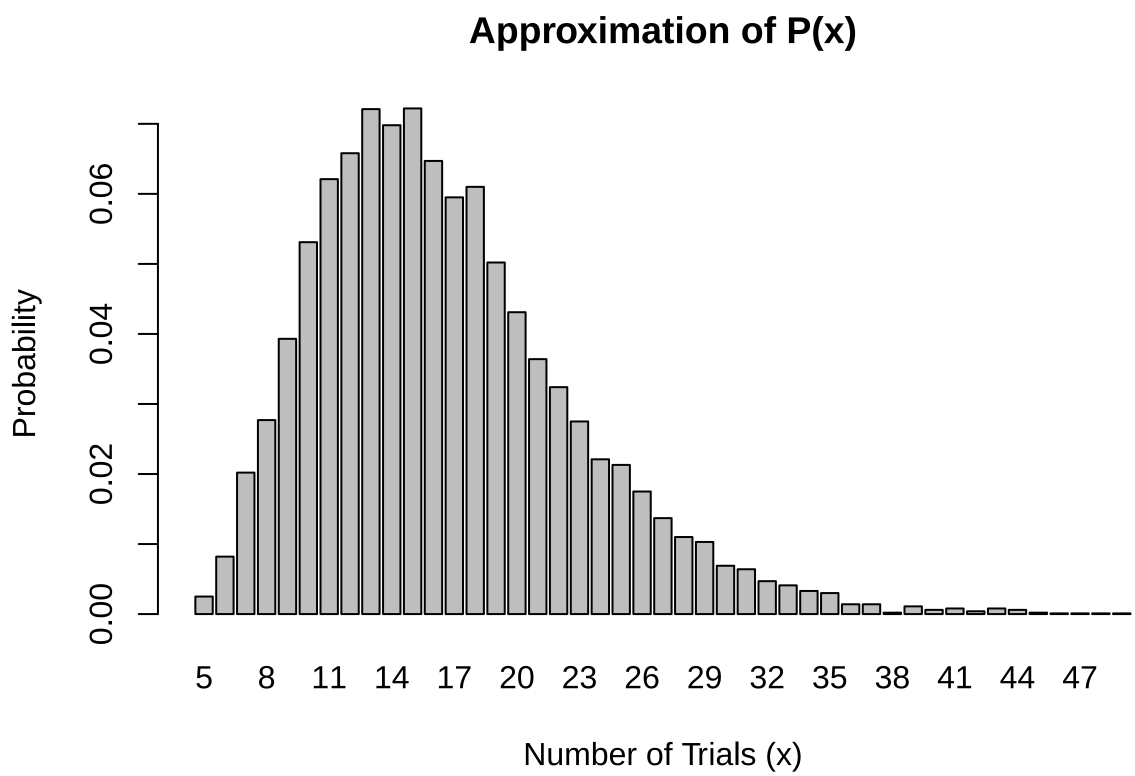
1.2 answer:

Rx={5,6,7,...}

P(x)=choose(k-1,4))x0.3^5 x(1-0.3)^(k-5)

```
success_prob <- 0.3
num_successes <- 5
num_simulations <- 10000

simulate_experiment <- function() {
  num_trials <- 0
  num_success <- 0

  while (num_success < num_successes) {
    num_trials <- num_trials + 1

    if (runif(1) < success_prob) {
      num_success <- num_success + 1
    }
  }

  return(num_trials)
}

sim_results <- replicate(num_simulations, simulate_experiment())
```

```r
trial_counts <- table(sim_results)

probabilities <- trial_counts / num_simulations

mean_trials <- sum(sim_results) / num_simulations

barplot(probabilities, main = "Approximation of P(x)", xlab = "Number of Trials (x)", ylab = "Probabili
```




```r
cat("Mean number of trials:", mean_trials)
```

```
## Mean number of trials: 16.5678
```

1.3 answer:

Rx={-5,10,20}

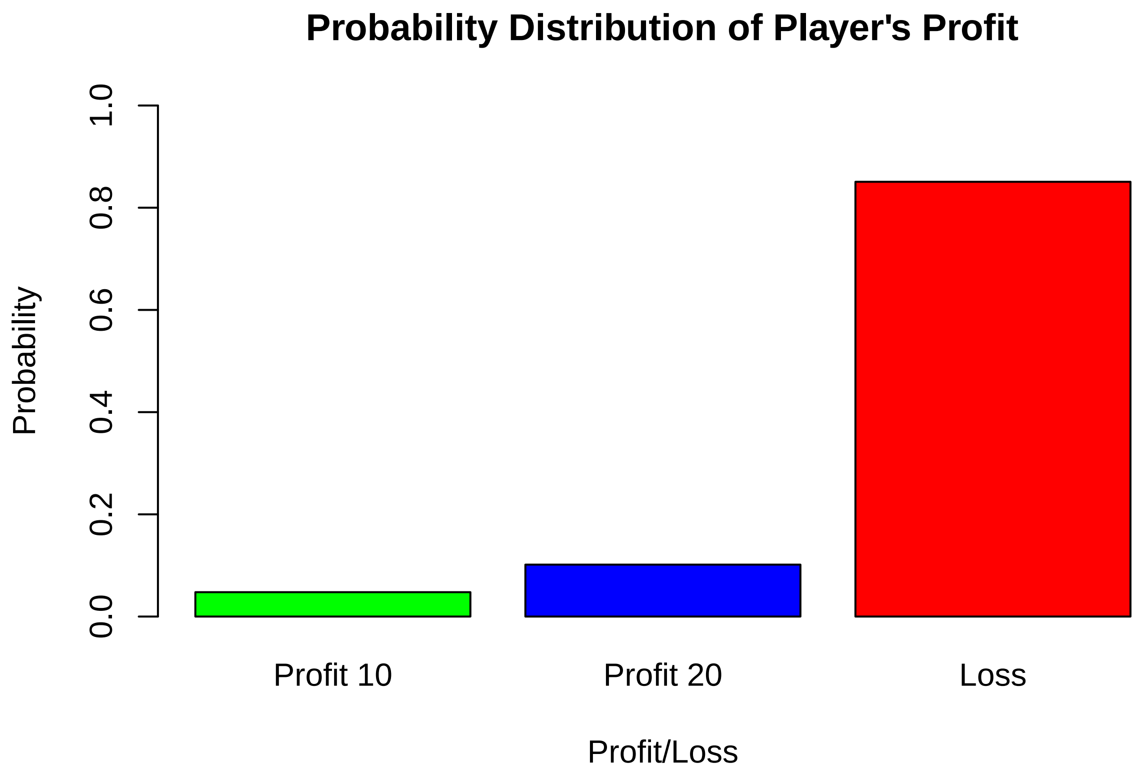
P(x=10)=19/choose(20,2)=1/10=0.1

P(x=20)=38/choose(20,2)=1/5=0.2

P(x=-5)=1-P(x=10)-P(x=20)=1-0.1-0.2=7/10=0.7

E[x]=-5xP(x=-5)+10xP(x=10)+20xP(x=20)= -5x0.7+10x0.1+20x0.2 =3/2=1.5

```r
n_simulations <- 10000

profit_10 <- 0
profit_20 <- 0
loss <- 0

for (i in 1:n_simulations) {
  ball1 <- sample(1:20, 1)
  ball2 <- sample(1:20, 1)

  if (ball1 + ball2 == 20) {
    profit_10 <- profit_10 + 10
  } else if (abs(ball1 - ball2) == 1) {
    profit_20 <- profit_20 + 20
  } else {
    loss <- loss + 5
  }
}

prob_10 <- profit_10 / (n_simulations * 10)
prob_20 <- profit_20 / (n_simulations * 20)
prob_loss <- loss / (n_simulations * 5)

barplot(c(prob_10, prob_20, prob_loss), names.arg = c("Profit 10", "Profit 20",
                                                      "Loss"),
        xlab = "Profit/Loss", ylab = "Probability",
        main = "Probability Distribution of Player's Profit",
        col = c("green", "blue", "red"),ylim=c(0,1))
```



Question 2

The time t (in minutes) required to assemble a Logi robot is a random variable with the following distribution:
t P(t) 2 0.1 3 0.1 4 0.3 5 0.2 6 0.2 7 0.1

2.1 answer:

E[x]=2x0.1+3x0.1+4x0.3+5x0.2+6x0.2+7x0.1=23/5=4.6

2.2 answer:

Y={2,2,2+0.5x1,2+0.5x2,2+0.5x3,2+0.5x4}={2,2.5,3,3.5,4}

Fy(2)=P(Y<=2)=P(t=6)+P(t=7)=0.2+0.1=0.3

Fy(2.5)=P(Y<=2.5)=F(2)+P(Y=2.5)=0.3+0.2=0.5

Fy(3)=P(Y<=3)=F(2.5)+P(Y=3)=0.5+0.3=0.8

Fy(3.5)=P(Y<=3.5)=F(3)+P(Y=3.5)=0.8+0.1=0.9

Fy(4)=P(Y<=4)=F(3.5)+P(Y=4)=0.9+0.1=1

E[Y]=2x0.1+2x0.2+2.5x0.2+3x0.3+3.5x0.1+4x0.1=2.75

E[Y^2]=4x0.1+4x0.2+6.25x0.2+ 9x0.3+12.25x0.1+16x0.1=7.975

Var(Y)=E[Y^2]-E[Y]^2 =7.975-(2.75^2)=0.4125

Question 3

The Probability function of a random variable X is given: $P(x) = w/x$ It is also given that $Rx=\{1,3,5,7\}$.

3.1 answer:

P(x)=(w/1)+(w/3)+(w/5)+(w/7)=1

P(x)=(105w+35w+21w+15w)/105=1

P(x)=176w/105=1

w=105/176

3.2 answer:

P(2<=x<=6)=P(x=3)+P(x=5)=(w/3)+(w/5)=((105/176)/3)+((105/176)/5)=7/22

3.3 answer:

F(x)=P(x<=5)=P(x=1)+P(x=3)+P(x=5)=((105/176)/1)+((105/176)/3)+((105/176)/5)= 161/176

Question 4

In an experiment we roll 2 dice. Let X = the sum of the 2 cubes and Y = the difference (in absolute value) between the results.

4.1 answer:

P(x=2)=1/36

P(x=3)=2/36

P(x=4)=3/36

p(x=5)=4/36

P(x=6)=5/36

P(x=7)=6/36

P(x=8)=5/36

P(x=9)=4/36

P(x=10)=3/36

P(x=11)=2/36

P(x=12)=1/36

4.2 answer:

y={0,1,2,3,4,5}

P(y=0)=6/36

P(y=1)=10/36

P(y=2)=8/36

P(y=3)=6/36

P(y=4)=4/36

p(y=5)=2/36

Fy(a<0) = 0

Fy(0<=a<1)=6/36

Fy(1<=a<2)=6/36+10/36=4/9

Fy(2<=a<3)=4/9+8/36=2/3

Fy(3<=a<4)=2/3+6/36=5/6

Fy(4<=a<5)=5/6+4/36=17/18

Fy(5<=a)=17/18+2/36=1

4.3 answer:

P(2<=Y<=5 and Y>=1)=P(y=2)+P(y=3)+P(y=4)+P(y=5)=8/36+6/36+4/36+2/36=5/9

P(Y>=1)=P(y=1)+P(y=2)+P(y=3)+P(y=4)+P(y=5)=10/36+8/36+6/36+4/36+2/36=5/6

P(2 <= Y <= 5 | Y >= 1)=P(2<=Y<=5 and Y>=1)/P(Y>=1)=(5/9)/(5/6)=2/3

Good luck!