Exercise

(week 7)

Submit by 10 May 2023

Instructions: Submit your exercise in both PDF and RMD formats, including the R code for the questions when relevant.

Question 1

For each one of the questions below, calculate: the possible values of X (Rx), the probability function (P(X)) and the expected value (E[X]). In addition, through simulation, create a barplot showing an approximation to P(x), when simulating the experiment 10000 times.

a) In a box there are 8 balls, 3 of them are red and 5 are black. Three balls are drawn without replacement. Let X be the number of black balls drawn.

b) A experiment is performed with a success probability of 0.3, repeatedly until 5 successes are obtained, and then the experiment stops. Let X be the number of trials performed.

c) An urn contains 20 numbered balls (1,... 20). A player draws two balls at random from the urn. If the two numbers drawn add up to 20, the player wins 10 shekels. If the two numbers are consecutive (i.e., their absolute difference is 1), the player wins 20 shekels. Otherwise, the player loses 5 shekels. Let X be the player's profit in the game.

Question 2

The time t (in minutes) required to assemble a Logi robot is a random variable with the following distribution:

t	P(t)
2	0.1
3	0.1
4	0.3
5	0.2
6	0.2
7	0.1

a) Calculate the expected value of X.

b) For each assembled robot, the worker earns a fixed amount of 2 shekels If he assembles the robot in less than six minutes, he earns an additional 0.50 shekel for each minute saved. For example, if the worker assembles the robot in four minutes, he saves 2 minutes (6-4) and therefore receives a total of 2 + 0.5*2 = 3 shekels. Find the distribution, expected value, and variance of Y - the amount in shekels that the worker receives for each robot.

Question 3

The Probability function of a random variable X is given:

$P(x) = w/x$

It is also given that Rx={1,3,5,7}

a) Find w.
b) Calculate $P(2 \leq X \leq 6)$
c) Calculate F(5)

Question 4

In an experiment we roll 2 dice.

Let X = the sum of the 2 cubes and Y = the difference (in absolute value) between the results.

a) Find the probability function of X
b) Find the probability function of Y and its cumulative distribution function.
c) Calculate the probability of the event: $P(2 \leq Y \leq 5 \mid Y \geq 1)$

Good luck !