Exercise 8

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Question 1

1.1 Suppose a basketball player has a 70% free throw success rate. What is the probability that she makes exactly 2 out of 3 free throws?

answer:

P(x=2)=choose(3,2)x0.7^2x0.3=0.441

1.2 Suppose a basketball player has a 33% free throw success rate. What is the probability that she will make as least 10 out of 30 free throws?

answer:

$$P(x => 10) = \sum_{k=10}^{20} \binom{30}{k} \times (\frac{1}{3})^k \times (\frac{2}{3})^{30-k} = 1 - P(x <= 9) = 1 - (P(x = 0) + P(x = 1) + ... + P(x = 9))$$

P(x=>10)=

```
1-pbinom(9,30,1/3)
```

```
## [1] 0.5682556
```

Question 2

2.1 In a group of 20 people, 8 are left-handed. If 4 people are selected at random from the group, what is the probability that exactly 2 of them are left-handed?

answer:

P(x=2)=choose(4,2)x0.4^2 x0.6^2=0.3456

2.2 A software company finds that 20% of its customers renew their annual subscription. If the company has 1000 customers, what is the probability that between 190 and 220 of them renew their subscription (inclusive)?

answer:

$$P(190 <= x <= 220) = P(x <= 220) - P(x < 190) = \sum_{k=0}^{220} \binom{1000}{k} \times 0.2^k \times 0.8^{1000-k} - \sum_{k=0}^{189} \binom{1000}{k} \times 0.2^k \times 0.8^{1000-k}$$

P(190<=x<=220)=

```
pbinom(220,1000,0.2)-pbinom(189,1000,0.2)
```

```
## [1] 0.7422088
```

2.3 A survey of 500 people found that 45% of them own a car. What is the probability that in a group of 8 people chosen at random, at most 3 (not inclusive) own a car?

answer:

P(x<3)=P(x=0)+P(x=1)+P(x=2)=choose(8,0)x0.45^0 x0.55^8 +choose(8,1)x0.45^1 x0.55^7 +choose(8,2)x0.45^2 x0.55^6=0.22

Question 3

3.1 What is the 90th percentile of the number of successes in 20 independent Bernoulli trials with a probability of success of 0.3?

answer:

```
n <- 20
p <- 0.3
percentile <- 0.9

k <- qbinom(percentile, size = n, prob = p, lower.tail = TRUE)

k
```

```
## [1] 9
```

3.2 If a fair coin is flipped 10 times, what is the maximum number of heads that can be obtained so that the probability of getting at least that many heads is less than or equal to 0.05?

answer:

```
n_flips <- 10
alpha <- 0.05

n_heads <- 0:n_flips
a<-dbinom(n_heads, n_flips, 0.5)
x <- pbinom(n_heads, n_flips, 0.5)
max_heads <- max(n_heads[x <= 1 - alpha])
max_heads
```

```
## [1] 7
```

Question 4

A study done on the AmaZona website found that the chance that an Israeli user who enters the website will also buy a product is 0.2. Calculate:

4.1 The probability that after 8 logins a random Israeli user bought 3 products

answer:

P(x=3)=choose(8,3)x0.2^3 x0.8^5=0.146

4.2 The expected value (E[X]) and variance of the number of items bought by an Israeli user after 12 visits to the site?

answer:

E[x]=12x0.2=2.4

Var[X] = 12x0.2x0.8=1.92

4.3 What is the probability that an Israeli user bought his third item exactly on his sixth visit to the site?

answer:

P(x=6)=choose(5,2)x 0.2^2 x0.8^3 x0.2=0.04096

Question 5

Suppose there are four white balls and four black balls in an urn. You randomly select four balls without replacement. If the selection contains one white ball and three black balls, you stop. Otherwise, you return the balls to the pool and repeat the process. What is the probability that it will take more than n attempts to stop?

answer:

A-1 white and 3 black

x- number of trials until first succcess

P(A)= (choose(4,1)x choose(4,3))/choose(8,4) = 8/35

$$P(x > n) = 1 - P(x <= n) = 1 - \sum_{k=0}^{n} \left(1 - \tfrac{8}{35}\right)^{k-1} \times \frac{8}{35}$$

Question 6

Matilda is teaching Jorgina how to play ping-pong. Jorgina has a 60% chance of scoring a point against Matilda. They will play until Jorgina scores 5 points and then stop. Assuming that the points are independent:

6.1 What is the probability that they will play 10 games?

answer:

we define 2 situations:

A= Jorgina scores 4 points in first 9 games.

B=Jorgina scores 5th point in 10th game.

C= 10 games

P(A)=P(X=4) = choose(9, 4)x0.6^4 x0.4^5 =0.167

P(B)=0.6

P(C)=0.167x0.6=0.1

6.2 Find the expected value and variance of the number of games played until they stop.

answer:

E[x]=5/0.6=25/3

Var[x]= 5x0.4/0.36=50/9

Good luck!