Stabilizing Training of Generative Adversarial Networks through Regularization

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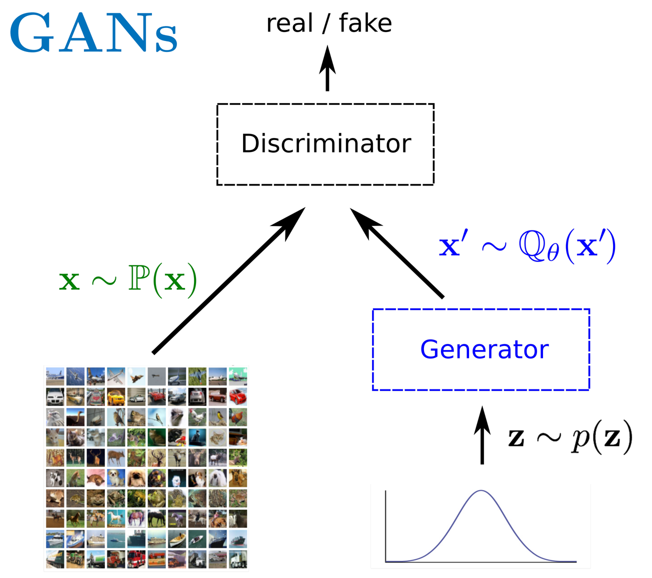
GANs



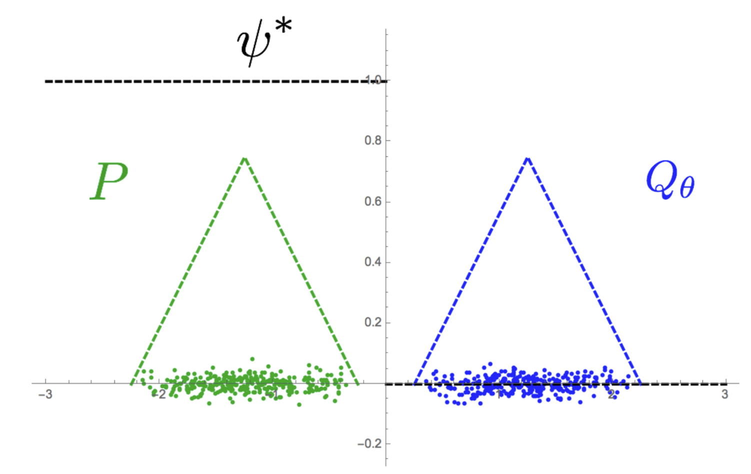
GANs are exciting, but ...
they're also notoriously hard to train!

GAN objective [2]

$$\min_G \max_\varphi \mathbf{E}_{\mathbb{P}}[\ln \varphi(x)] + \mathbf{E}_{p(z)}[\ln(1-\varphi(G(z)))]$$

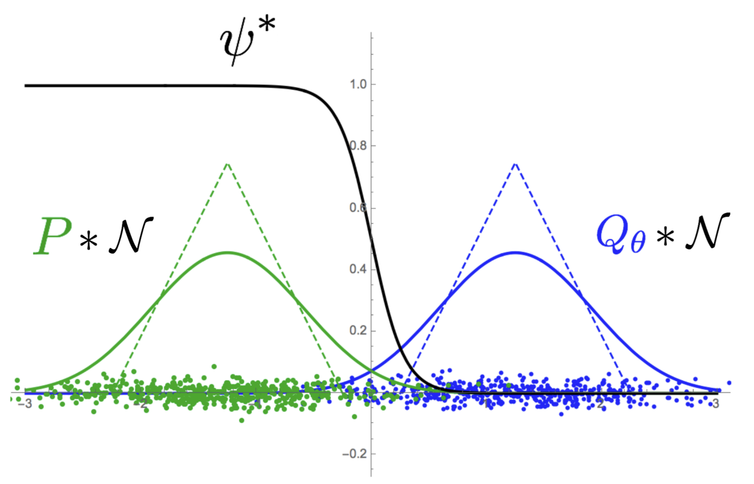
$$= \min_G D_{\mathrm{JS}}(\mathbb{P}\|\mathbb{Q}_\theta) \quad \text{for Bayes-optimal } \varphi^*(x)$$

Problem: Dim. Mismatch or $\mathrm{supp}(\mathbb{P}) \cap \mathrm{supp}(\mathbb{Q}_\theta) = \varnothing$



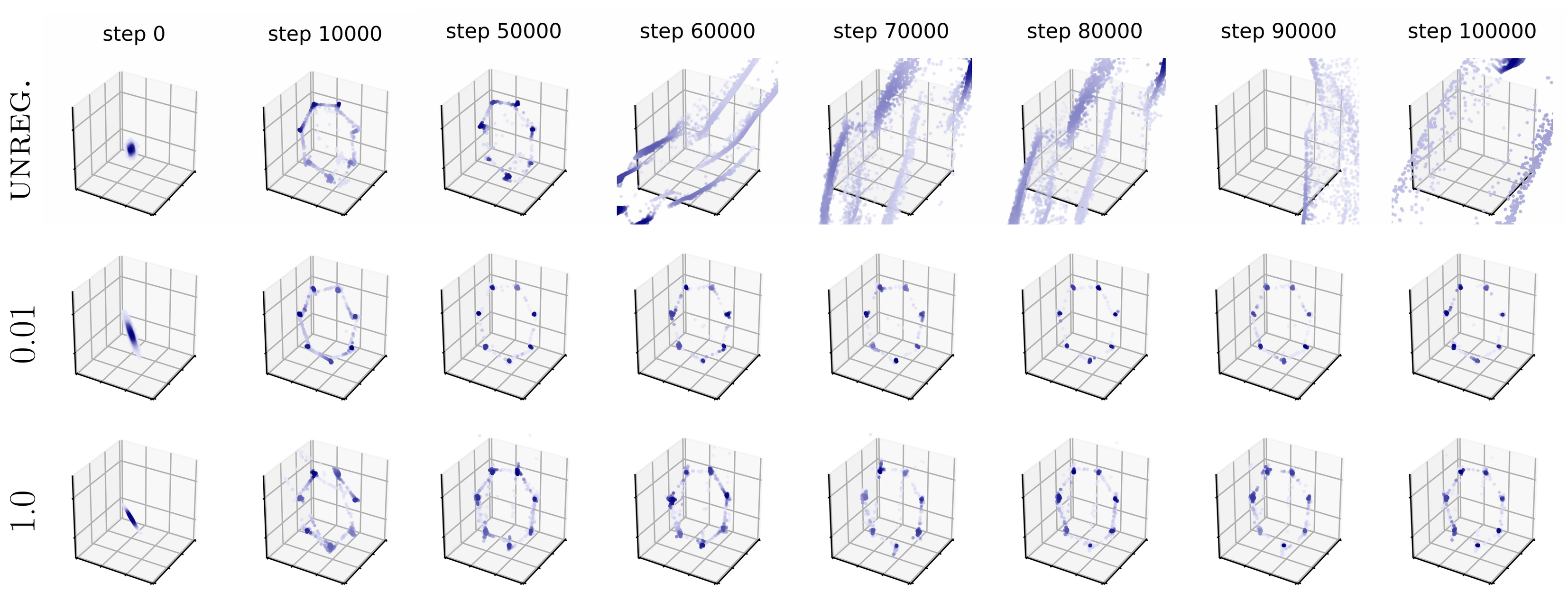
=> Undef. f-div. $D_f(\mathbb{P}\|\mathbb{Q}_\theta)$ =???

Solution: Adding Noise (Convolving Densities)



=> Regularizing Discriminator

Dimensionally Misspecified Submanifold Mixture



Unstable unregularized GAN vs. stable regularized GANs for different levels of γ.
The regularized GAN can essentially be trained indefinitely without collapse.

References

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Training with Noise

From f-divergences to f-GAN objectives [4]

$$D_f(\mathbb{P}\|\mathbb{Q}) \geq \sup_{\psi \in \Psi} \left[F(\mathbb{P},\mathbb{Q};\psi) := \mathbf{E}_{\mathbb{P}}[\psi] - \mathbf{E}_{\mathbb{Q}}[f^c \circ \psi] \right]$$

- Practitioner: **Explicitly adding noise** $\boldsymbol{\xi} \sim \Lambda_\gamma \equiv \mathcal{N}(0, \gamma \mathbb{I})$ to $\mathbf{x} \sim \mathbb{P}, \mathbb{Q}$
- Theory: **Convolving Distributions**

$$\mathbf{E}_{\mathbb{P}}\mathbf{E}_{\Lambda}[h(\psi(\mathbf{x}+\boldsymbol{\xi}))] = \int h(\psi(\mathbf{x})) \int p(\mathbf{x}-\boldsymbol{\xi})\lambda(\boldsymbol{\xi}) d\boldsymbol{\xi}\, d\mathbf{x} = \mathbf{E}_{\mathbb{P}*\Lambda}[h \circ \psi]$$

=> **Convolved f-GAN Objective**

$$F(\mathbb{P} * \Lambda_\gamma, \mathbb{Q} * \Lambda_\gamma; \psi) = \mathbf{E}_{\mathbb{P}*\Lambda_\gamma}[\psi] - \mathbf{E}_{\mathbb{Q}*\Lambda_\gamma}[f^c \circ \psi]$$

Analytic Approximation

- For small noise variance γ we can Taylor expand ψ around $\boldsymbol{\xi} = 0$ [1]:

$$\psi(\mathbf{x}+\boldsymbol{\xi}) = \psi(\mathbf{x}) + [\nabla \psi(\mathbf{x})]^T \boldsymbol{\xi} + \frac{1}{2}\boldsymbol{\xi}^T [\nabla^2 \psi(\mathbf{x})]\, \boldsymbol{\xi} + \mathcal{O}(\boldsymbol{\xi}^3)$$

- Third-order approximation (in $\boldsymbol{\xi}$) of F_γ via $F = F_0$ plus a correction, i.e.

$$F_\gamma(\mathbb{P},\mathbb{Q};\psi) = F(\mathbb{P},\mathbb{Q};\psi) + \frac{\gamma}{2}\left\{ \mathbf{E}_{\mathbb{P}}[\triangle \psi] - \mathbf{E}_{\mathbb{Q}}[\triangle(f^c \circ \psi)] \right\} + \mathcal{O}(\gamma^2)$$

- Interpretation: Laplace $\triangle = \mathrm{Tr}(\nabla^2)$ measures how much ψ and $f^c \circ \psi$ differ from their local average

Efficient Gradient-Based Regularization

- Chain-rule: $\quad \triangle(f^c \circ \psi) = \left(f^{c\prime\prime} \circ \psi\right) \cdot \|\nabla \psi\|^2 + \left(f^{c\prime} \circ \psi\right) \triangle \psi$
- Property of optimal discriminant ψ^* [3]: $\quad (f^{c\prime} \circ \psi^*)\, d\mathbb{Q} = d\mathbb{P}$

=> **Convenient cancellation** at $\psi = \psi^* + \mathcal{O}(\gamma)$ [1]:

$$\mathbf{E}_{\mathbb{P}}[\triangle \psi^*] - \mathbf{E}_{\mathbb{Q}}[\triangle(f^c \circ \psi^*)] = -\mathbf{E}_{\mathbb{Q}}[(f^{c\prime\prime} \circ \psi^*) \cdot \|\nabla \psi^*\|^2]$$

=> Tractable regularization which avoids (i) detrimental sampling variance, (ii) explicitly convolving the distributions, and (iii) the computation of Laplacians

Regularized f-GAN

$$F_\gamma(\mathbb{P},\mathbb{Q};\psi) = F(\mathbb{P},\mathbb{Q};\psi) - \frac{\gamma}{2}\Omega_f(\mathbb{Q};\psi), \quad \Omega_f(\mathbb{Q};\psi) := \mathbf{E}_{\mathbb{Q}}\left[(f^{c\prime\prime} \circ \psi) \cdot \|\nabla \psi\|^2 \right]$$

$$F_{\mathrm{JS}_\gamma}(\mathbb{P},\mathbb{Q};\varphi) = \mathbf{E}_{\mathbb{P}}[\ln(\varphi)] + \mathbf{E}_{\mathbb{Q}}[\ln(1-\varphi)] - \frac{\gamma}{2}\Omega_{\mathrm{JS}}(\mathbb{P},\mathbb{Q};\varphi)$$

$$\Omega_{\mathrm{JS}}(\mathbb{P},\mathbb{Q};\varphi) := \mathbf{E}_{\mathbb{P}}\left[(1-\varphi)^2 \|\nabla \phi\|^2\right] + \mathbf{E}_{\mathbb{Q}}\left[\varphi^2 \|\nabla \phi\|^2\right]$$

- "soft Lipschitz" constraint, non-negative weighting function $f^{c\prime\prime} \geq 0$
- lower variance compared to explicitly adding noise
- **easy to implement & computationally cheap**
- **can train indefinitely without collapse!**

Algorithm 1 Regularized f-GAN. Default values: $\gamma_0 = 2.0$, $\alpha = 0.01$ (with annealing), $\gamma = 0.1$ (without annealing), $n_\psi = 1$

Require: Initial noise variance γ_0, annealing decay factor α, number of discriminator update steps n_ψ per generator iteration, minibatch size m, number of training iterations T
Require: Initial discriminator parameters ω_0, initial generator parameters θ_0
  for $t = 1, ..., T$ **do**
    $\gamma \leftarrow \gamma_0 \cdot \alpha^{t/T}$ # annealing
    for $1, ..., n_\psi$ **do**
      Sample minibatch of real data $\{\mathbf{x}^{(1)}, ..., \mathbf{x}^{(m)}\} \sim \mathbb{P}$.
      Sample minibatch of latent variables from prior $\{\mathbf{z}^{(1)}, ..., \mathbf{z}^{(m)}\} \sim p(\mathbf{z})$.
      $\omega \leftarrow \omega + \nabla_\omega \left(F(\omega,\theta) - \frac{\gamma}{2}\Omega_f(\omega,\theta) \right)$ # gradient ascent
    end for
    Sample minibatch of latent variables from prior $\{\mathbf{z}^{(1)}, ..., \mathbf{z}^{(m)}\} \sim p(\mathbf{z})$.
    $\theta \leftarrow \theta - \nabla_\theta F(\omega,\theta)$ # gradient descent
  end for

Cross-Testing Protocol

Regularized $\gamma = 0.1$

Pred. \ True Cond.	Pos.	Neg.
Pos.	0.9688	0.0002
Neg.	0.0312	0.9998

Cross-testing: FP: 0.0

Unregularized

Pred. \ True Cond.	Pos.	Neg.
Pos.	1.0	0.0013
Neg.	0.0	0.9987

Cross-testing: FP: 1.0

Cross-Testing:
Classify 10k samples generated by the regularized GAN with the discriminator of the unregularized GAN and vice versa.

=> **Regularized GAN generalizes better!**

Stability across Architectures

RESNET   DCGAN   NO NORMALIZATION   TANH

Sample Quality and Diversity

UNREG.   0.5   ANNEALING   2.0

tf code → github.com/rothk

```
# ---------------------------------------------------------------------------
#     JS-Regularizer
# ---------------------------------------------------------------------------

  def Discriminator_Regularizer(self, D1, D1_logits, D1_arg, D2, D2_logits, D2_arg):
      grad_D1_logits = tf.gradients(D1_logits, D1_arg)[0]
      grad_D2_logits = tf.gradients(D2_logits, D2_arg)[0]
      grad_D1_logits_norm = tf.norm(tf.reshape(grad_D1_logits, [self.batch_size,-1]), axis=1, keep_dims=True)
      grad_D2_logits_norm = tf.norm(tf.reshape(grad_D2_logits, [self.batch_size,-1]), axis=1, keep_dims=True)
      reg_D1 = tf.multiply(tf.square(1.0-D1), tf.square(grad_D1_logits_norm))
      reg_D2 = tf.multiply(tf.square(D2), tf.square(grad_D2_logits_norm))
      return tf.reduce_mean(reg_D1 + reg_D2)
```