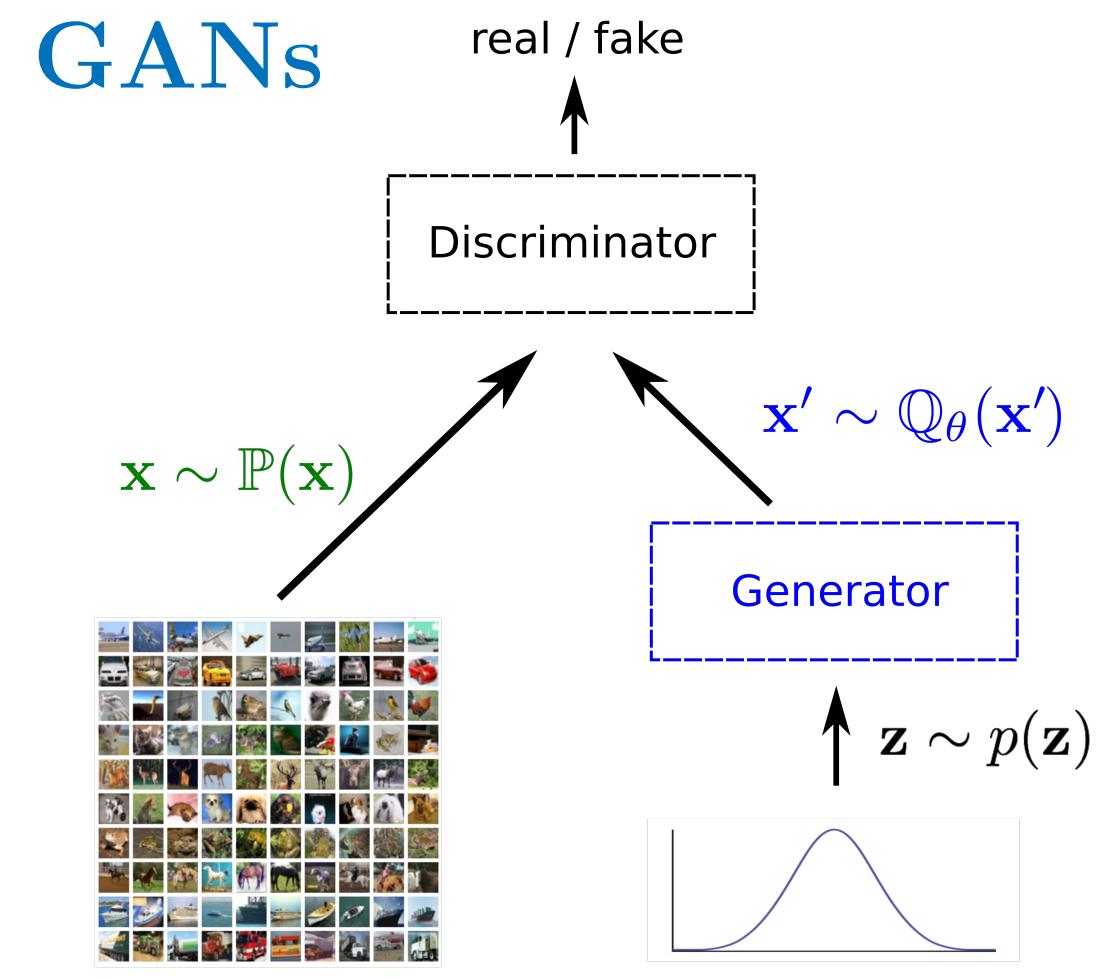


Stabilizing Training of Generative Adversarial Networks through Regularization

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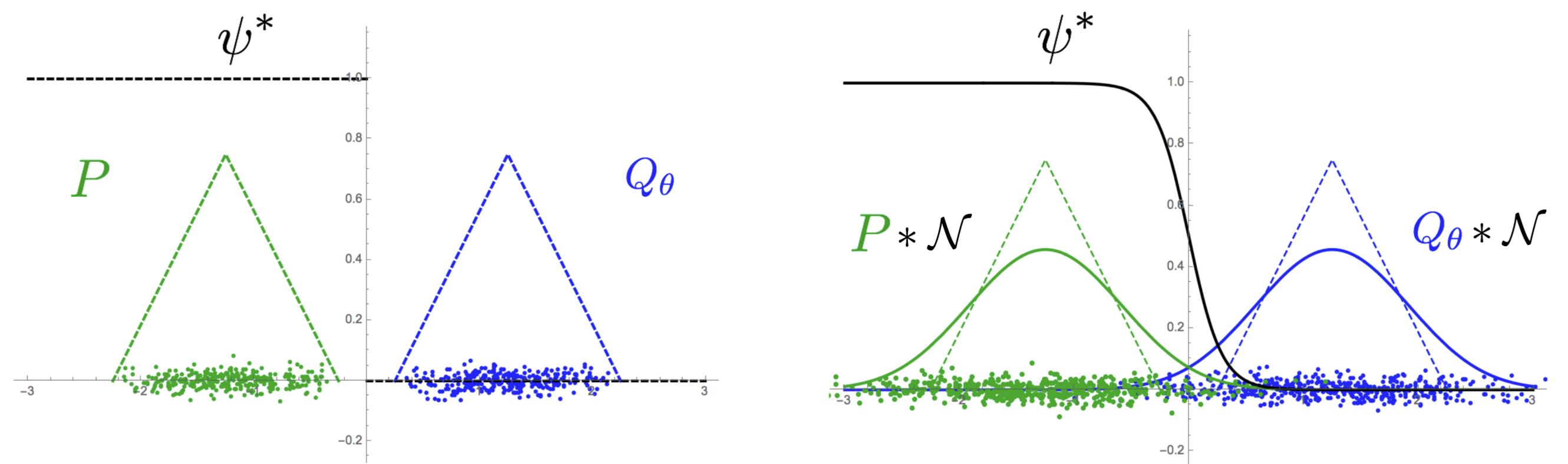
GANs are exciting, but ...
they're also notoriously hard to train!

GAN objective [3]

$$\begin{aligned} & \min_G \max_{\varphi} \mathbf{E}_P[\ln \varphi(x)] + \mathbf{E}_{p(z)}[\ln(1 - \varphi(G(z)))] \\ & = \min_G D_{\text{JS}}(P||Q_\theta) \text{ for Bayes-optimal } \varphi^*(x) \end{aligned}$$

Problem: dim. mismatch
or $\text{supp}(P) \cap \text{supp}(Q_\theta) = \emptyset$

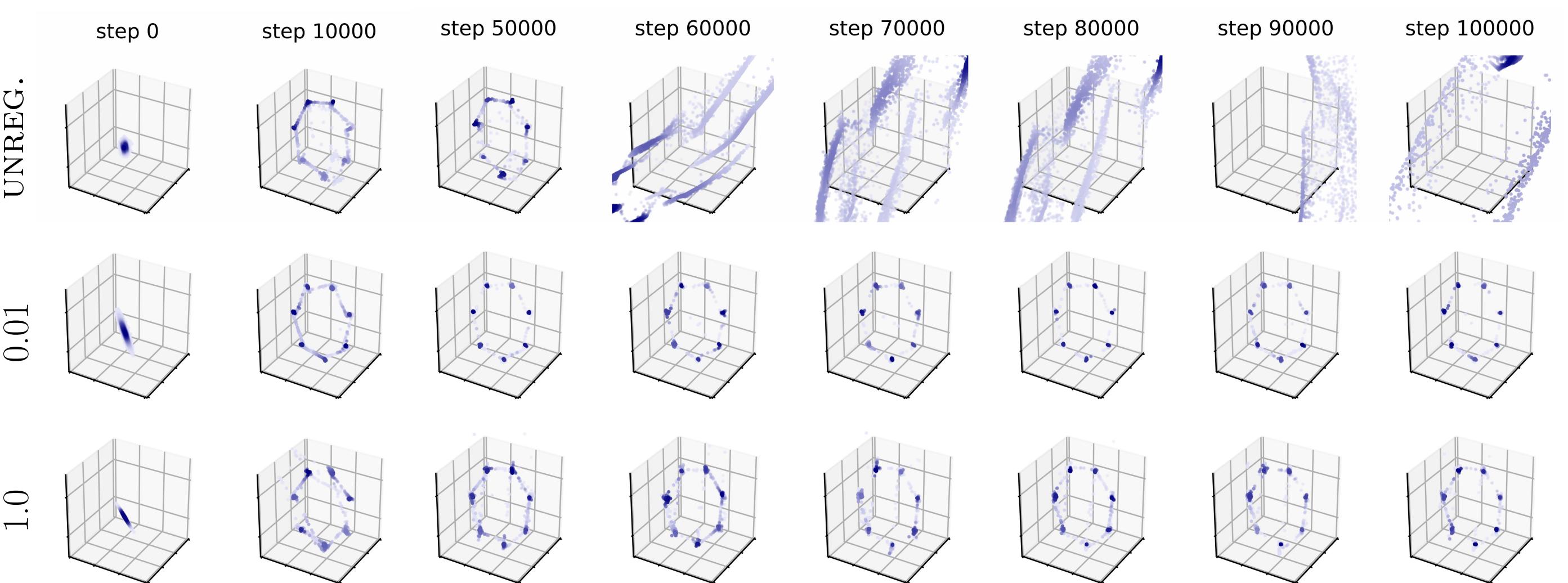
Solution: Adding Noise
(Convolving Densities)



=> undef. f-div. $D_f(P||Q_\theta) = ???$

=> Regularizing Discriminator

Dimensionally Misspecified Submanifold Mixture



Unstable unregularized GAN vs. stable regularized GANs for different levels of γ .
The regularized GAN can essentially be trained indefinitely without collapse.

References

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Training with Noise

From f -divergences to f -GAN objectives [5]

$$D_f(P||Q) \geq \sup_{\psi \in \Psi} \left[F(\mathbb{P}, \mathbb{Q}; \psi) := \mathbf{E}_{\mathbb{P}}[\psi] - \mathbf{E}_{\mathbb{Q}}[f^c \circ \psi] \right]$$

- Practitioner: Explicitly adding noise $\xi \sim \Lambda_\gamma \equiv \mathcal{N}(0, \gamma \mathbb{I})$ to $x \sim \mathbb{P}, Q$
- Theory: Convolving Distributions

$$\mathbf{E}_{\mathbb{P}} \mathbf{E}_{\Lambda} [h(\psi(x + \xi))] = \int h(\psi(x)) \int p(x - \xi) \lambda(\xi) d\xi dx = \mathbf{E}_{\mathbb{P} * \Lambda} [h \circ \psi]$$

=> Convolved f -GAN Objective

$$F(\mathbb{P} * \Lambda_\gamma, \mathbb{Q} * \Lambda_\gamma; \psi) = \mathbf{E}_{\mathbb{P} * \Lambda_\gamma} [\psi] - \mathbf{E}_{\mathbb{Q} * \Lambda_\gamma} [f^c \circ \psi]$$

Analytic Approximation

- For small noise variance γ we can Taylor expand ψ around $\xi = 0$ [2]:

$$\psi(x + \xi) = \psi(x) + [\nabla \psi(x)] \xi + \frac{1}{2} \xi^T [\nabla^2 \psi(x)] \xi + \mathcal{O}(\xi^3)$$

- Third-order approximation (in ξ) of F_γ via $F = F_0$ plus a correction, i.e.

$$F_\gamma(\mathbb{P}, \mathbb{Q}; \psi) = F(\mathbb{P}, \mathbb{Q}; \psi) + \frac{\gamma}{2} \{ \mathbf{E}_{\mathbb{P}}[\Delta \psi] - \mathbf{E}_{\mathbb{Q}}[\Delta(f^c \circ \psi)] \} + \mathcal{O}(\gamma^2)$$

- Interpretation: Laplace $\Delta = \text{Tr}(\nabla^2)$ measures how much ψ and $f^c \circ \psi$ differ from their local average

Efficient Gradient-Based Regularization

- Chain-rule: $\Delta(f^c \circ \psi) = (f^{c''} \circ \psi) \cdot \|\nabla \psi\|^2 + (f^{c'} \circ \psi) \Delta \psi$
- Property of optimal discriminant ψ^* [4]: $(f^{c'} \circ \psi^*) d\mathbb{Q} = d\mathbb{P}$

=> Convenient cancellation at $\psi = \psi^* + \mathcal{O}(\gamma)$ [2]:

$$\mathbf{E}_{\mathbb{P}}[\Delta \psi^*] - \mathbf{E}_{\mathbb{Q}}[\Delta(f^c \circ \psi^*)] = -\mathbf{E}_{\mathbb{Q}}[(f^{c''} \circ \psi^*) \cdot \|\nabla \psi^*\|^2]$$

=> Tractable regularization which avoids (i) detrimental sampling variance, (ii) explicitly convolving the distributions, and (iii) the computation of Laplacians

Regularized f -GAN

$$F_\gamma(\mathbb{P}, \mathbb{Q}; \psi) = F(\mathbb{P}, \mathbb{Q}; \psi) - \frac{\gamma}{2} \Omega_f(\mathbb{Q}; \psi), \quad \Omega_f(\mathbb{Q}; \psi) := \mathbf{E}_{\mathbb{Q}}[(f^{c''} \circ \psi) \cdot \|\nabla \psi\|^2]$$

$$\begin{aligned} F_{\text{JS}}(\mathbb{P}, \mathbb{Q}; \varphi) &= \mathbf{E}_{\mathbb{P}}[\ln(\varphi)] + \mathbf{E}_{\mathbb{Q}}[\ln(1 - \varphi)] - \frac{\gamma}{2} \Omega_{\text{JS}}(\mathbb{P}, \mathbb{Q}; \varphi) \\ \Omega_{\text{JS}}(\mathbb{P}, \mathbb{Q}; \varphi) &:= \mathbf{E}_{\mathbb{P}}[(1 - \varphi)^2 \|\nabla \varphi\|^2] + \mathbf{E}_{\mathbb{Q}}[\varphi^2 \|\nabla \varphi\|^2] \end{aligned}$$

- “soft Lipschitz” constraint, non-negative weighting function $f^{c''} \geq 0$
- lower variance compared to explicitly adding noise
- easy to implement & computationally cheap
- can train indefinitely without collapse!

Algorithm 1 Regularized f -GAN. Default values: $\gamma_0 = 2.0$, $\alpha = 0.01$ (with annealing), $\gamma = 0.1$ (without annealing), $n_\psi = 1$

Require: Initial noise variance γ_0 , annealing decay factor α , number of discriminator update steps n_ψ per generator iteration, minibatch size m , number of training iterations T
Require: Initial discriminator parameters ω_0 , initial generator parameters θ_0
for $t = 1, \dots, T$ **do**
 $\gamma \leftarrow \gamma_0 \cdot \alpha^{t/T}$ # annealing
 for $1, \dots, n_\psi$ **do**
 Sample minibatch of real data $\{x^{(1)}, \dots, x^{(m)}\} \sim \mathbb{P}$.
 Sample minibatch of latent variables from prior $\{z^{(1)}, \dots, z^{(m)}\} \sim p(z)$.
 $\omega \leftarrow \omega + \nabla_\omega (F(\omega, \theta) - \frac{\gamma}{2} \Omega_f(\omega, \theta))$ # gradient ascent
 end for
 Sample minibatch of latent variables from prior $\{z^{(1)}, \dots, z^{(m)}\} \sim p(z)$.
 $\theta \leftarrow \theta - \nabla_\theta F(\omega, \theta)$ # gradient descent
end for

Cross-Testing Protocol

Regularized $\gamma = 0.1$

True Cond.	
Pos.	Neg.
0.9688	0.0002
0.0312	0.9998

Pred.

Unregularized

True Cond.	
Pos.	Neg.
1.0	0.0013
0.0	0.9987

Pred.

Cross-Testing:
Classify 10k samples generated by the regularized GAN with the discriminator of the unregularized GAN and vice versa.

Cross-testing: FP: 0.0

Cross-testing: FP: 1.0

=> Regularized GAN generalizes better!

Stability across Architectures



Sample Quality and Diversity



tf code → github.com/rothk

```
# ----- JS-Regularizer -----
# -----
def Discriminator_Regularizer(self, D1, D1_logits, D1_arg, D2, D2_logits, D2_arg):
    grad_D1_logits = tf.gradients(D1_logits, D1_arg)[0]
    grad_D2_logits = tf.gradients(D2_logits, D2_arg)[0]
    grad_D1_logits_norm = tf.norm(tf.reshape(grad_D1_logits, [self.batch_size,-1]), axis=1, keep_dims=True)
    grad_D2_logits_norm = tf.norm(tf.reshape(grad_D2_logits, [self.batch_size,-1]), axis=1, keep_dims=True)
    reg_D1 = tf.multiply(tf.square(1.0-D1), tf.square(grad_D1_logits_norm))
    reg_D2 = tf.multiply(tf.square(D2), tf.square(grad_D2_logits_norm))
    return tf.reduce_mean(reg_D1 + reg_D2)
```