Stabilizing Training of Generative Adversarial Networks through Regularization

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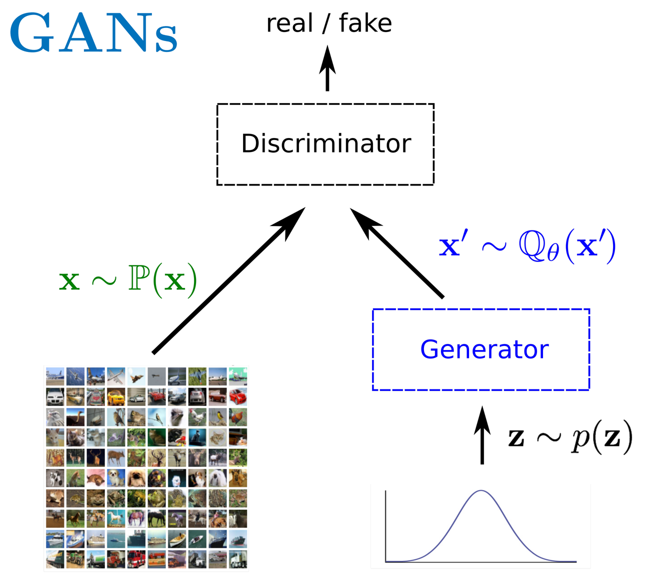
GANs

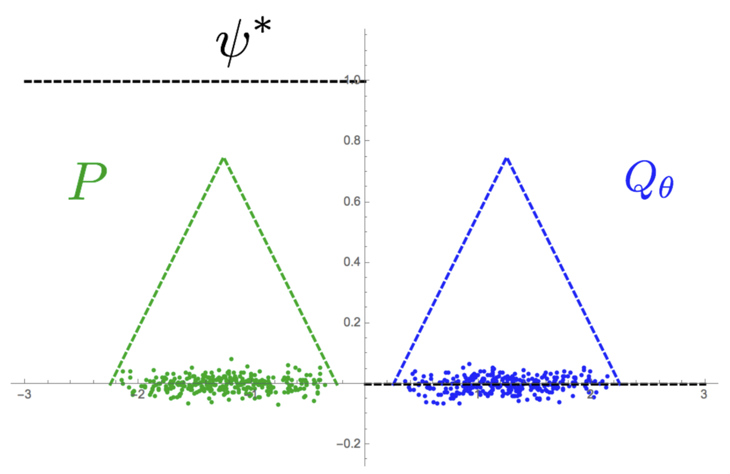


GANs are exciting, but ...
they're also notoriously hard to train!

GAN objective [3]

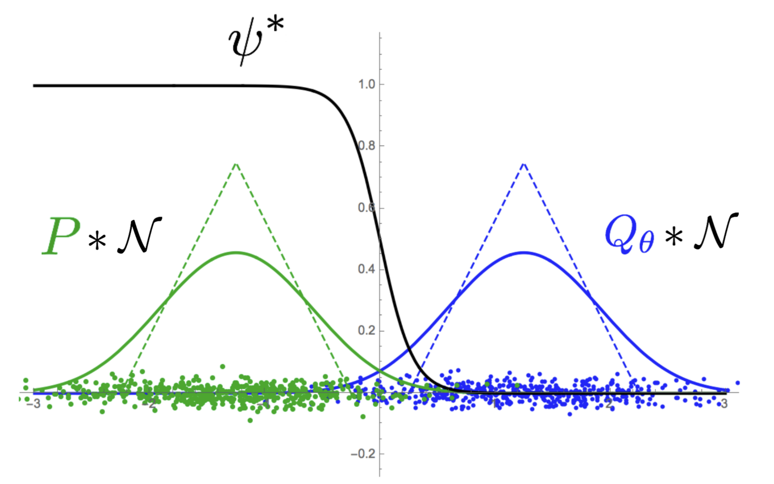
$$\min_G \max_\varphi \ \mathbf{E}_P[\ln \varphi(x)] + \mathbf{E}_{p(z)}[\ln(1-\varphi(G(z)))]$$
$$= \min_G D_{\mathrm{JS}}(P\|Q_\theta) \quad \text{for Bayes-optimal } \varphi^*(x)$$

Problem: *dim. mismatch* or supp(P) $\cap$ supp(Q_θ) $= \varnothing$



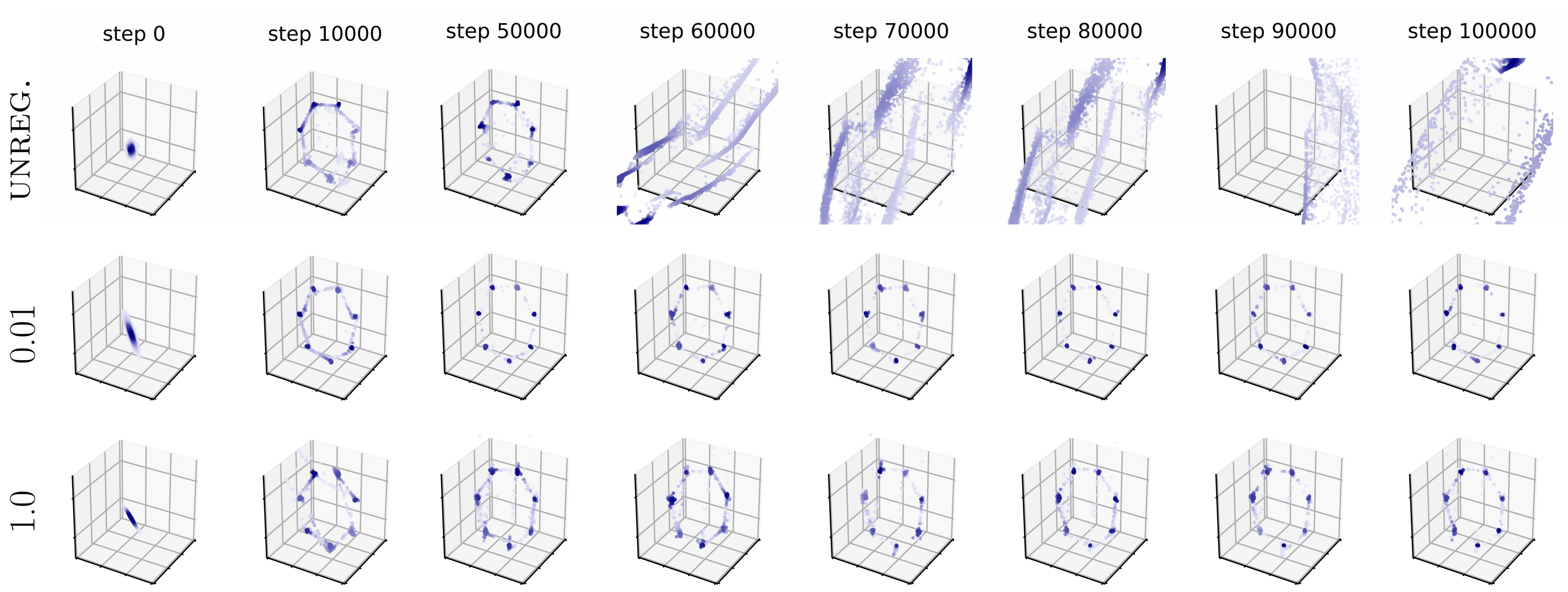
=> undef. f-div. $D_f(P\|Q_\theta)$ =???

Solution: Adding Noise (Convolving Densities)



=> Regularizing Discriminator

Dimensionally Misspecified Submanifold Mixture



Unstable unregularized GAN vs. stable regularized GANs for different levels of γ.
The regularized GAN can essentially be trained indefinitely without collapse.

References

[1] M. Arjovsky and L. Bottou. Towards principled methods for training generative adversarial networks. In *NIPS 2016 Workshop on Adversarial Training. In review for ICLR*, volume 2016, 2017.

[2] C. M. Bishop. Training with noise is equivalent to tikhonov regularization. *Neural computation*, 7(1):108–116, 1995.

[3] I. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative Adversarial Networks. *NIPS*, 2014.

[4] X. Nguyen, M. J. Wainwright, and M. I. Jordan. Estimating divergence functionals and the likelihood ratio by convex risk minimization. *IEEE Transactions on Information Theory*, 56(11):5847–5861, 2010.

[5] S. Nowozin, B. Cseke, and R. Tomioka. f-GAN: Training generative neural samplers using variational divergence minimization. In *Advances in Neural Information Processing Systems*, pages 271–279, 2016.

[6] C. K. Sønderby, J. Caballero, L. Theis, W. Shi, and F. Huszár. Amortised map inference for image super-resolution. *arXiv preprint arXiv:1610.04490*, 2016.

Training with Noise

From f-divergences to f-GAN objectives [5]

$$D_f(P\|Q) \geq \sup_{\psi\in\Psi}\left[F(\mathbb{P},\mathbb{Q};\psi) := \mathbf{E}_{\mathbb{P}}[\psi] - \mathbf{E}_{\mathbb{Q}}[f^c\circ\psi]\right]$$

- Practitioner: **Explicitly adding noise** $\boldsymbol{\xi} \sim \Lambda_\gamma \equiv \mathcal{N}(0,\gamma\mathbb{I})$ to $\mathbf{x}\sim\mathbb{P},\mathbb{Q}$
- Theory: **Convolving Distributions**

$$\mathbf{E}_{\mathbb{P}}\mathbf{E}_{\Lambda}[h(\psi(\mathbf{x}+\boldsymbol{\xi}))] = \int h(\psi(\mathbf{x}))\int p(\mathbf{x}-\boldsymbol{\xi})\lambda(\boldsymbol{\xi})d\boldsymbol{\xi}\,d\mathbf{x} = \mathbf{E}_{\mathbb{P}*\Lambda}[h\circ\psi]$$

=> **Convolved f-GAN Objective**

$$F(\mathbb{P}*\Lambda_\gamma,\mathbb{Q}*\Lambda_\gamma;\psi) = \mathbf{E}_{\mathbb{P}*\Lambda_\gamma}[\psi] - \mathbf{E}_{\mathbb{Q}*\Lambda_\gamma}[f^c\circ\psi]$$

Analytic Approximation

- For small noise variance γ we can Taylor expand ψ around $\boldsymbol{\xi}=0$ [2]:

$$\psi(\mathbf{x}+\boldsymbol{\xi}) = \psi(\mathbf{x}) + [\nabla\psi(\mathbf{x})]\,\boldsymbol{\xi} + \frac{1}{2}\boldsymbol{\xi}^T[\nabla^2\psi(\mathbf{x})]\,\boldsymbol{\xi} + \mathcal{O}(\boldsymbol{\xi}^3)$$

- Third-order approximation (in $\boldsymbol{\xi}$) of F_γ via $F=F_0$ plus a correction, i.e.

$$F_\gamma(\mathbb{P},\mathbb{Q};\psi) = F(\mathbb{P},\mathbb{Q};\psi) + \frac{\gamma}{2}\left\{\mathbf{E}_{\mathbb{P}}[\triangle\psi] - \mathbf{E}_{\mathbb{Q}}[\triangle(f^c\circ\psi)]\right\} + \mathcal{O}(\gamma^2)$$

- Interpretation: Laplace $\triangle = \mathrm{Tr}(\nabla^2)$ measures how much ψ and $f^c\circ\psi$ differ from their local average

Efficient Gradient-Based Regularization

- Chain-rule: $\triangle(f^c\circ\psi) = (f^{c\prime\prime}\circ\psi)\cdot\|\nabla\psi\|^2 + (f^{c\prime}\circ\psi)\,\triangle\psi$
- Property of optimal discriminant ψ^* [4]: $(f^{c\prime}\circ\psi^*)\,d\mathbb{Q} = d\mathbb{P}$

=> **Convenient cancellation** at $\psi = \psi^* + \mathcal{O}(\gamma)$ [2]:

$$\mathbf{E}_{\mathbb{P}}[\triangle\psi^*] - \mathbf{E}_{\mathbb{Q}}[\triangle(f^c\circ\psi^*)] = -\mathbf{E}_{\mathbb{Q}}[(f^{c\prime\prime}\circ\psi^*)\cdot\|\nabla\psi^*\|^2]$$

=> Tractable regularization which avoids (i) detrimental sampling variance, (ii) explicitly convolving the distributions, and (iii) the computation of Laplacians

Regularized f-GAN

$$F_\gamma(\mathbb{P},\mathbb{Q};\psi) = F(\mathbb{P},\mathbb{Q};\psi) - \frac{\gamma}{2}\Omega_f(\mathbb{Q};\psi), \quad \Omega_f(\mathbb{Q};\psi) := \mathbf{E}_{\mathbb{Q}}\left[(f^{c\prime\prime}\circ\psi)\cdot\|\nabla\psi\|^2\right]$$

$$F_{\mathrm{JS}_\gamma}(\mathbb{P},\mathbb{Q};\varphi) = \mathbf{E}_{\mathbb{P}}[\ln(\varphi)] + \mathbf{E}_{\mathbb{Q}}[\ln(1-\varphi)] - \frac{\gamma}{2}\Omega_{\mathrm{JS}}(\mathbb{P},\mathbb{Q};\varphi)$$
$$\Omega_{\mathrm{JS}}(\mathbb{P},\mathbb{Q};\varphi) := \mathbf{E}_{\mathbb{P}}\left[(1-\varphi)^2\|\nabla\phi\|^2\right] + \mathbf{E}_{\mathbb{Q}}\left[\varphi^2\|\nabla\phi\|^2\right]$$

- "soft Lipschitz" constraint, non-negative weighting function $f^{c\prime\prime}\geq 0$
- lower variance compared to explicitly adding noise
- **easy to implement & computationally cheap**
- **can train indefinitely without collapse!**

Algorithm 1 **Regularized f-GAN.** Default values: $\gamma_0 = 2.0$, $\alpha = 0.01$ (with annealing), $\gamma = 0.1$ (without annealing), $n_\varphi = 1$

Require: Initial noise variance γ_0, annealing decay factor α, number of discriminator update steps n_φ per generator iteration, minibatch size m, number of training iterations T
Require: Initial discriminator parameters ω_0, initial generator parameters θ_0
for $t = 1,\ldots,T$ **do**
  $\gamma \leftarrow \gamma_0\cdot\alpha^{t/T}$ # annealing
  for $1,\ldots,n_\varphi$ **do**
    Sample minibatch of real data $\{\mathbf{x}^{(1)},\ldots,\mathbf{x}^{(m)}\}\sim\mathbb{P}$.
    Sample minibatch of latent variables from prior $\{\mathbf{z}^{(1)},\ldots,\mathbf{z}^{(m)}\}\sim p(\mathbf{z})$.
    $\omega \leftarrow \omega + \nabla_\omega\left(F(\omega,\theta) - \frac{\gamma}{2}\Omega_f(\omega,\theta)\right)$ # gradient ascent
  end for
  Sample minibatch of latent variables from prior $\{\mathbf{z}^{(1)},\ldots,\mathbf{z}^{(m)}\}\sim p(\mathbf{z})$.
  $\theta \leftarrow \theta - \nabla_\theta F(\omega,\theta)$ # gradient descent
end for

Cross-Testing Protocol

Regularized $\gamma = 0.1$

True Cond.

Pred.	Pos.	Neg.
Pos.	0.9688	0.0002
Neg.	0.0312	0.9998

Cross-testing: FP: 0.0

Unregularized

True Cond.

Pred.	Pos.	Neg.
Pos.	1.0	0.0013
Neg.	0.0	0.9987

Cross-testing: FP: 1.0

Cross-Testing:
Classify 10k samples generated by the regularized GAN with the discriminator of the unregularized GAN and vice versa.

=> **Regularized GAN generalizes better!**

Stability across Architectures

ResNet DCGAN No Normalization tanh

Sample Quality and Diversity

UNREG. 0.5 - ANNEALING - 2.0

tf code → github.com/rothk

```python
# ---------------------------------------------------------------------------------
#     JS-Regularizer
# ---------------------------------------------------------------------------------

 def Discriminator_Regularizer(self, D1, D1_logits, D1_arg, D2, D2_logits, D2_arg):
     grad_D1_logits = tf.gradients(D1_logits, D1_arg)[0]
     grad_D2_logits = tf.gradients(D2_logits, D2_arg)[0]
     grad_D1_logits_norm = tf.norm(tf.reshape(grad_D1_logits, [self.batch_size,-1]), axis=1, keep_dims=True)
     grad_D2_logits_norm = tf.norm(tf.reshape(grad_D2_logits, [self.batch_size,-1]), axis=1, keep_dims=True)
     reg_D1 = tf.multiply(tf.square(1.0-D1), tf.square(grad_D1_logits_norm))
     reg_D2 = tf.multiply(tf.square(D2), tf.square(grad_D2_logits_norm))
     return tf.reduce_mean(reg_D1 + reg_D2)
```

References

[1] M. Arjovsky and L. Bottou. Towards principled methods for training generative adversarial networks. In *NIPS 2016 Workshop on Adversarial Training. In review for ICLR*, volume 2016, 2017.

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