



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:
Modelling and Simulating Social Systems with MATLAB

Project Report

**Swiss Rail Network Formation with
Physarum Polycephalum**

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Agreement for free-download

We hereby agree to make our source code for this project freely available for download from the web pages of the SOMS chair. Furthermore, we assure that all source code is written by ourselves and is not violating any copyright restrictions.

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1. Abstract

The process of continuous urbanization is noticeable in Switzerland like in the most other countries.¹ Besides the benefits of combining educational, medical, cultural and industrial centers in a small area, there are also some problems arising. The cities are growing, whereupon this growth in population causes an increasing use of the public transport system, which has to be adapted to the new conditions. The main goal of this project is the simulation of the Swiss rail network depending on population growth. The network is simulated with a biological inspired model based on *Physarum polycephalum*. This slime mold is a large single-celled amoeboid organism that forages for food sources. To maximize the searched area, it explores its environment with a relatively continuous foraging margin. It is forming different junctions and nodes to reduce the overall length of the connecting network [5]. This principle is adapted to the main railroads in Switzerland.

2. Individual contributions

- Lucas Böttcher
- Simon Roth
- Gabriela Schär

3. Introduction and Motivations

Everyday thousands of commuters use the Swiss rail network to reach their workplace, school or university, mainly the same railroad is used. Seeing these facts we are going to ask, whether the actual situation of the network is the most efficient one. And related to that question, whether different connections between cities have maybe a higher performance.

We assume, that the rail network is developed in a way that the connections between cities are the most efficient. Certain biological organisms connect their food sources in a similar way. In this project we simulate a biological organism creating its own efficient connections between the cities. This approach leads to the following main questions:

1. Is the biological model a good approximation to simulate the network compared to reality?

¹see the autumn sunday lectures at ETH Science City (Die Stadt der Zukunft - die Zukunft der Stadt)

2. Are there any new built or destroyed connections in the network because of the future population growth?

To answer these questions, the results of the simulations are compared with the Swiss rail network. For this comparison geodata from geodata.ethz.ch [3] are used. This data are processed with *Arcgis 10.1* to get the basis for the simulation.

Cities are chosen based on the numbers of inhabitants (more than 10000) in 2011 and the presence of a railway station. Additional to these some cities with less than 10000 inhabitants are added because of actual importance for the Swiss rail network [1].

To answer the second question, we are going to manipulate parameters in Physarum network, like described in 5.

We expect that the most efficient network created by the simulation is a good approximation of the actual Swiss railroad network. We also expect that the network won't change even when the population is growing because a network changes the most at the beginning of developing. And the Swiss railroad network reaches the end of development already.

4. Description of the Model

The model is inspired of the physarum polycephalum and the way this single-celled fungus searching for food. The organism have been subjected to successive rounds of evolutionary selection and have found an appropriate balance between efficiency, cost and resilience. The plasmodium contains a network of tubes, which enables chemical signals and nutrients to circulate through the organism. If some of the tubes have found food sources, this implies a positive feedback to the system. This tubes are getting thicker so the flux of nutrients increase. Other tubes which don't connect to food sources are shrinking and tend to disappear, because there is no flux available. Experiments showed two empirical rules. **1)** Tubes with no connection to a source disappear and **2)** if there are two tubes connecting the same source, the longer one disappears. This led to useful approaches to problem-solving like optimization of railroad systems or networks in general.

The mathematical model is based on *Physarum solver: A biologically inspired method of road-network navigation* [4]. Figure 2 shows the concept of the mathematical model. There are different nodes. The first two nodes corresponding to the food sources (N_1 and N_2) and other nodes ($N_3, N_4 \dots N_j$). The junction between the node N_i and N_j is denoted as M_{ij} . In the model we connect the single nodes with

circular tubes and want to measure their inner flux (Figure 1). Therefore the fluid dynamics prepare us with the law of Hagen-Poiseuille, which allows us to calculate the flux from the difference in pressure, the tube length and some constants. Starting from the fluid velocity in a cylindric tube:

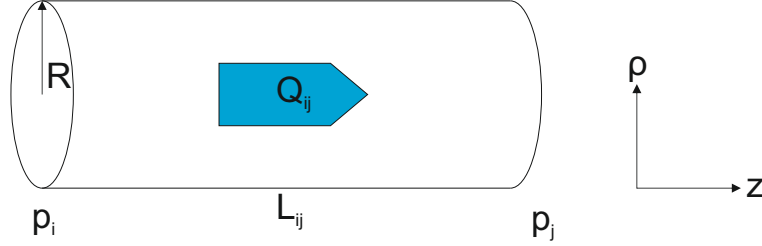


Figure 1: Illustration of the Hagen-Poiseuille law for a circular tube, which connects the single nodes in our model.

$$u(z) = -\frac{1}{4\eta} \frac{dp}{dz} (R^2 - \rho^2) \quad (1)$$

, where R is the radius of the cylinder and η the viscosity. Now we can integrate that expression over a circle, and normalize to get a mean velocity:

$$\overline{u(z)} = -\frac{1}{4\pi\eta R^2} \frac{dp}{dz} \int_0^{2\pi} \int_0^R (R^2 - \rho^2) \rho d\rho d\phi = -\frac{1}{8\eta} \frac{dp}{dz} R^2 \quad (2)$$

We can assume that the pressure gradient is uniform and express the mean velocity by the flux divided by its cross section:

$$Q_{ij} = \frac{D_{ij}}{L_{ij}} \Delta p_{ij} \quad (3)$$

, where $D_{ij} = -\frac{\pi R^4}{8\eta}$ is the conductance. So we see that the Hagen-Poiseuille law is related to Ohms law, where we have a similar expression.

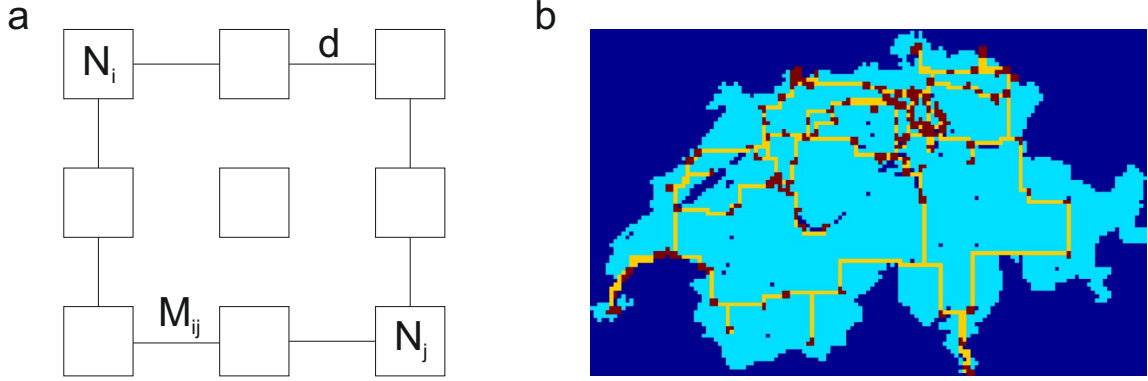


Figure 2: **a)** Concept of the mathematical model with a source and sink node. The distance between the given nodes is a constant d . The centered node is not reachable. **b)** Any square represents a node with red sources, yellow transport junctions (cyan is not used) and not reachable blue terrain.

The variable Q_{ij} stands for the flux through the junction M_{ij} from N_i to N_j . The flux Q_{ij} is given by an approximately Poiseuille flow where p_i is a pressure at the node N_i , L_{ij} is the length and D_{ij} is the conductivity of the junction M_{ij} :

$$Q_{ij} = \frac{D_{ij}}{L_{ij}} (p_i - p_j) \quad (4)$$

By considering Kirchhoff's law at each node, there is:

$$\sum_i Q_{ij} = 0 \quad \text{if } (j \neq 1, 2) \quad (5)$$

N_1 is assumed as a source node and N_2 as a sink and I_0 represent the flux from the source node and is in this model constant. It follows:

$$\sum_i Q_{i1} + I_0 = 0, \quad \sum_i Q_{i2} - I_0 = 0 \quad (6)$$

To describe the thickness of the junctions we assumed that the conductivity D_{ij} changes in time according to the flux Q_{ij} :

$$\frac{dD_{ij}}{dt} = f(|Q_{ij}|) - D_{ij} \quad (7)$$

where $f(|Q|)$ is a increasing function with $f(0) = 0$. Here $f(|Q|)$ is given by:

$$f(|Q|) = \frac{|Q|^\gamma}{1 + |Q|^\gamma} \quad (8)$$

The network Poisson equation for the pressure is derived from the equations (4), (5) and (6) as followed:

$$\sum_i \frac{D_{ij}}{L_{ij}} (p_i - p_j) = \begin{cases} -I_0 & \text{for } j = 1, \\ I_0 & \text{for } j = 2, \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

All p_i 's can be determined by solving the equation system (9) wenn setting $p_2 = 0$ as a basic pressure level. With this also each Q_{ij} is obtained. They're definend bay the D_{ij} 's and L_{ij} 's at each time step. Conductivity is closely related to the thickness of the junctions and so when a conductivity of a junction is zero, it disappears.

5. Implementation

To get the basis matrix for the simulation, Swiss geodata [3] are processed with *Arcgis 10.1*. Therefor the rasterdata are distinguished in different classes as in Table 1 are described.

Table 1: classes of data

Indentification	Class	Description
0	Ghostland	Cells in which the slime mold isn't allowed to be (foreign coutries, lakes)
1	Freeland	Cells in which the slime mold is allowed to grow
2	Junction	Cells in which the slime mold is located, processed in MATLAB
3	City	Cells which represents food sources

All the geodata are in vector format. So all lakes and the foreign countries can be erased. It would be possible to erase more types of covering (eg. rivers, rocks) where a train can't ride. The slope is in this case also neglected. So the mountains aren't considered in the simulation. Then all the choosen cities are imported to *Arcgis*. Therefor the coordinates (Y,X) [2] for each city are searched and a buffer of 2500m is layed on them. In this case no city will disapear when the vectordata gets converted to rasterdata with a cellsize of 2500m. So after converting in rasterdata with the classes above, the basis matrix is exported as ASCII-File which can now used in MATLAB as simulation surface.

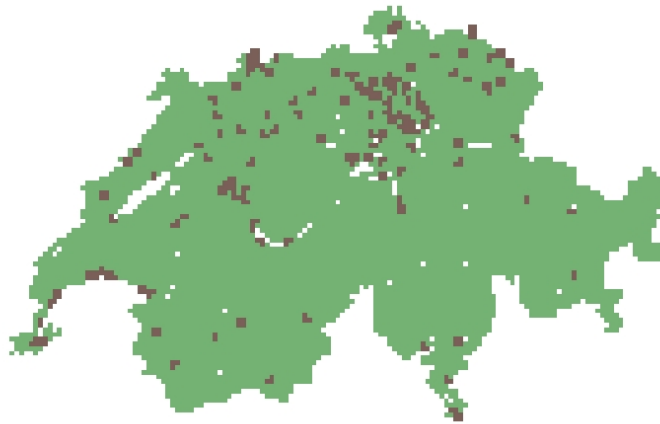


Figure 3: Rastardata from Arcgis with a gridsize of 2500m - white: Ghostland, green: Freeland, brown: City

6. Simulation Results and Discussion

7. Summary and Outlook

A. Matlab-Code

Matlab-Code

B. Declaration of Originality

References

- [1] Bilanz der ständigen Wohnbevölkerung (Total) nach Bezirken und Gemeinden. <http://www.bfs.admin.ch/bfs/portal/de/index/themen/01/02/blank/data/01.html>, october 2012.
- [2] Koordinator. <http://tools.retorte.ch/map/>, october 2012.
- [3] GeoVITe ETH Geodata Portal. <http://www.geodata.ethz.ch>, october 2012.

- [4] Atsushi Tero, Ryo Kobayashi, and Toshiyuki Nakagaki. Physarum solver: A biologically inspired method of road-network navigation. *Physica A Statistical Mechanics and its Applications*, 2006.
- [5] Atsushi Tero, Seiji Takagi, Tetsu Saigusa, Kentaro Ito, Dan P. Bebber, Mark D. Fricker, Kenjo Yumiki, Ryo Kobayashi, and Toshiyuki Nakagaki. Rules for biologically inspired adaptive network design. *Science Magazine*, 2010.