DESIGN AND ANALYSIS OF ALGORITHMS

ASSIGNMENT 1

To:-

Amina Siddique

From:-

Abdul Sami Qasim
22i-1725
CY-D

Q3)

```
person ← 0
count ← 0
  while (line)                               // Keeps going until line is empty
    person ← ID from person in line
    ~~count~~
    if ID == person
        increment count by 1
    if ID =/= person
        decrement count by 1
  end while loop
count ← 0
totalplates ← total plates served
  while (line)
      if ID== person
      increment count by 1
  end ~~to~~ while loop

if count > totalplates/2
        print("ID of person who ate more than half plates": person)
~~else~~
else
    print("No one at more than half plates")
```

Q1) A)

$$f(n) = c_1 + c_2 + c_3(n+1) + n(c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10} + c_{11} + c_{12})$$

$$f(n) = c_1 + c_2 + \cancel{n} \, c_3 \cdot n + c_3 + n(c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10} + c_{11} + c_{12})$$

$$f(n) = n(c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10} + c_{11} + c_{12}) + (c_1 + c_2 + c_3)$$

$$\underline{f(n) = O(n)}$$

B)

outer loop = $\sum_{i=1}^{n} i$

inner loop = $\sum_{j=1}^{2i+1} j$

$$f(n) = \sum_{i=1}^{n} \sum_{j=1}^{2i+1} j$$

$$= \sum_{i=1}^{n} 2i+1$$

$$= \sum_{i=1}^{n} 2i + \sum_{i=1}^{n} 1$$

$$= \cancel{2} \cdot \frac{n(n+1)}{\cancel{2}} + n$$

$$\boxed{f(n) = n^2+2n}$$

$$\boxed{f(n) = O(n^2)}$$

C)

$$f(n) = \sum_{i=1}^{n^2} \sum_{j=1}^{i} j$$

$$= \sum_{i=1}^{n^2} \frac{i(i+1)}{2}$$

$$= \frac{n^2(n^2+1)}{2}$$

$$\boxed{f(n) = \frac{n^4+n^2}{2}}$$

$$\boxed{f(n) = O(n^4)}$$

D)

$\left.\begin{array}{l} \text{outer loop} = m+1 \text{ times} \\ \text{inner loop} = \log_2 m \text{ times} \end{array}\right\}$ seperately

$$f(n) = m+1 + c_1 m + m \cdot \log_2 m + c_2 \cdot m \log_2 m + c_3 \cdot m \log_2 m$$

$$= m \cdot \log_2 m (c_2 + c_3 + 1) + m(c_1 + 1) + 1$$

$$\boxed{f(n) = O(m \cdot \log_2 m)}$$

E)

$$f(n) = \sum_{i=1}^{2n} \sum_{j=1}^{i^2} \sum_{k=1}^{j} 1$$

$$= \sum_{i=1}^{2n} \sum_{j=1}^{i^2} j$$

$$= \sum_{i=1}^{2n} \frac{i^2(i^2+1)}{2}$$

$$= \frac{(2n)^2((2n)^2+1)}{2}$$

$$= \frac{{}^{2}\cancel{4}n^2(4n^2+1)}{\cancel{2}}$$

$$= 8n^4+2n^2$$

$$\boxed{f(n) = O(n^4)}$$

F)

$$f(n) = n+1 + n(n+1) + n^2 + n+1 + n(n+1) + n^2(n+1) + n^3$$

$$= 2n^3 + 4n^2 + 4n + 2$$

$$\boxed{f(n) = O(n^3)}$$

Q2)a)

$$100n+5 \geq c.n^2$$

let $n=1$,

$$100n+5 \leq 100n+5n = 105n$$

$$105n \geq c.n^2$$

$$105 \geq c.n$$

$$n \leq \frac{105}{c}$$

which is not true as value of n keeps increasing

Hence proved

b) $f(n) = \frac{n^2}{2} - \frac{n}{2}$

$c_1 . g(n) \le f(n) \le c_2 . g(n)$

$c_1 . n^2 \le \frac{n^2}{2} - \frac{n}{2} \le c_2 . n^2$ (Assuming n^2 as $g(n)$ because it has exponential change upon different value of n as compared to n)

let $c_1 = \frac{1}{4}$

$\frac{n^2}{4} \le \frac{n^2}{2} - \frac{n}{2}$

$\frac{n^2}{2} \le n^2 - n$

$n^2 \le 2n(n-1)$

$n \le 2n-2$

$0 \le n-2$

$2 \le n$

$\boxed{n \ge 2}$ so, $f(n) = \Omega(n^2)$

Now,

$\frac{n^2}{2} - \frac{n}{2} \le c_2 . n^2$

let $c_2 = \frac{1}{2}$

$\frac{n^2}{2} - \frac{n}{2} \le \frac{n^2}{2}$

$n^2 - n \le n^2$

$n - 1 \le n$

which is true for all values where $n \ge 1$

so, $f(n) = O(n^2)$

as $\Omega(n^2) \le f(n) \le O(n^2)$

then theta bound for $f(n)$ is also n^2

$\boxed{f(n) = \Theta(n^2)}$

c)

$c_1 . n^2 \leq 6n^3 \leq c_2 . n^2$

$c_1 . n^2 \leq 6n^3$

$c_1 \leq 6n$

which is true because, as value of n gets bigger, it will eventually grow larger than the constant

$6n^3 \leq c_2 . n^2$

$6n \leq c_2$

while is false because at some point, L.H.S will become larger.

As the function is not upper bound by n^2,

$$6n^3 \neq \Theta(n^2)$$

d)

$f(n) = n$

$\Theta(\log n)$

$c_1 . \log n \leq n \leq c_2 . \log n$

$c_1 . \log n \leq n$

$c_1 \leq \frac{n}{\log n}$

which is true for larger values because n is always larger than log n so there wont be much difference.

$n \leq c_2 . \log n$

$\cancel{\frac{n}{\log n} \leq c_2}$

which becomes false later on, consider taking n=3 as an example

$3 \leq c_2 . \log(3)$

$3 \leq c_2 . (1.58)$

and the difference only becomes larger as value of n increases.

e) $f(n) = \log_2(n^3)$

$O(\log_2 n)$

$\log_2 n^3 \leq c . \log_2 n$

$3 \log_2 n \leq c . \log_2 n$ —— (i)

$\boxed{c \geq 3}$

The statement (i) is equal for any value of n when $\boxed{c \geq 3}$.

$f(n) = \log_2(n^3)$ belongs to $O \log_2(n)$