

Design and Analysis of Algorithms (CS2009)

Course Instructor(s):

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Sessional-I Exam

Total Time (Hrs): 1

Total Marks: 50

Total Questions: 6

Date: Sep 23, 2024

Roll No

Course Section

Student Signature

Do not write below this line.

Attempt all the questions.

[CLO 2. Analyze the time and space complexity of different algorithms by using standard asymptotic notations for recursive and non recursive algorithms.]

Q1: Write time complexity of following code in terms of big-Oh (O).

[2 x 5 = 10 marks]

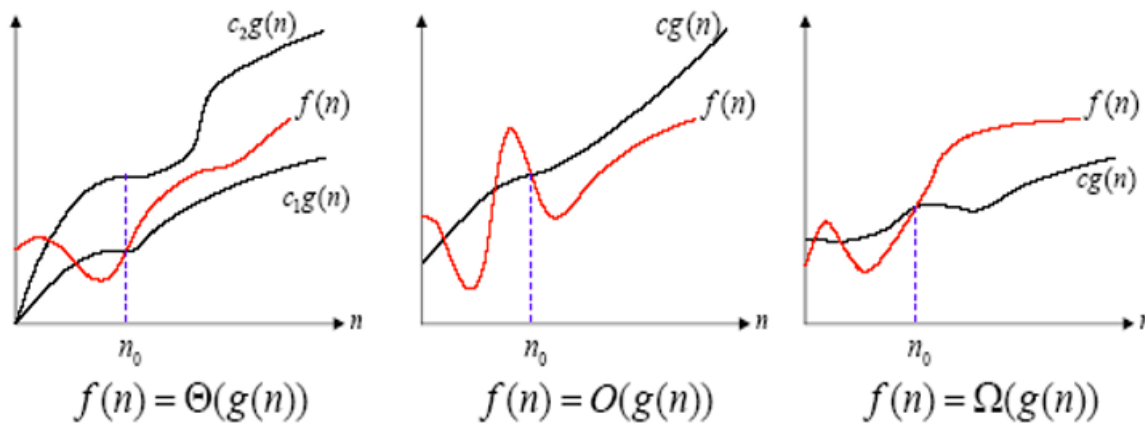
A	<pre>if(n==1) return 1; else { int p=0; for(int i=1;p<=n;i++) { p=p+i; } }</pre> <p>Ans: \sqrt{n}</p>
B	<pre>for (int i = n*n; i >= 1; i = i / 5) { for (int j = 0; j < i; j*=2) { cout << j; } }</pre> <p>Ans: $O(\infty)$</p>

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C	<pre> int fun(int n) { int sum = 0; int outer = 0, inner = 0; for (int i = 1; i < n; i=i*2) { outer++; cout << endl<<"Outer : " <<i<<" cycle no. " << outer << endl; inner = 0; for (int j = 1; j <= i; j *= 2) { inner++; cout<< inner << endl; sum++; } } cout << endl << sum << endl; return 0; } </pre> <p>Ans: $O(\log n \times \log n)$ or $O(\log i \times \log n)$</p>
D	<pre> int fun(int n) { int count = 0; for (int i = 0; i < n; i++) for (int j = i; j > 0; j--) count = count + 1; return count; } </pre> <p>Ans: $O(n(n+1)/2)$ $O(n^2)$</p>
E	<pre> int fun(int n) { int count= 0 for (int j = 0; i < n*n ; i++) count++; if(j%2 == 0) break; return count; } </pre> <p>Ans: $O(1)$</p>

Q2. Explain mathematical function model of Big-Oh, Big Omega and Theta with the help of the graph. **[3x2 = 6 marks]**

Asymptotic notation



Equations too.

[CLO 2. Analyze the time and space complexity of different algorithms by using standard asymptotic notations for recursive and non recursive algorithms.]

Q3: Insertion sort is an iterative sorting algorithm.

[2+4 = 6 marks]

```
void insertionSort(int arr[], int n)
{
    int i, key, j;
    for (i = 1; i < n; i++) {
        key = arr[i];
        j = i - 1;

        // Move elements of arr[0..i-1],
        // that are greater than key, to one
        // position ahead of their
        // current position
        while (j >= 0 && arr[j] > key) {
            arr[j + 1] = arr[j];
            j = j - 1;
        }
        arr[j + 1] = key;
    }
}
```

a) Can we convert it into recursive function? If yes, mention base and recursive case?

Ans: Yes

Link: <https://www.geeksforgeeks.org/recursive-insertion-sort/>

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b) Write recursive function of Insertion Sort?

Link: <https://www.geeksforgeeks.org/recursive-insertion-sort/>

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Q4: Write time complexity of following recurrence equation using Master Theorem.[5 x 2 = 10 marks]

a) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Ans: $O(n^2 \log(n))$

b) $T(n) = 2T\left(\frac{n}{2}\right) + n\sqrt{\log n}$

Ans: $O(n(\log^{3/2} n))$

c) $T(n) = 5T\left(\frac{n}{3}\right) + n^{\frac{3}{2}}$

Ans: $O(n^{3/2})$

d) $T(n) = 2T\left(\frac{n}{4}\right) + \frac{n}{\log(n)}$

Ans: $O(n)$

e) $T(n) = 6T\left(\frac{n}{3}\right) + n^{\frac{5}{4}}$

Ans: $O(n^{1.63})$

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Q5: Solve the following using iteration method (substitution method):

[8 marks]

$$T(n) = T(n-1) + \log(n), n > 0$$

$$T(1) = 1, n = 0$$

$$\begin{aligned}
 T(n) &= T(n-1) + \log n \\
 T(n-1) &= T(n-2) + \log(n-1) \quad T(n) = T(n-2) + \log(n-1) + \log n \\
 T(n-2) &= T(n-3) + \log(n-2) \quad T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n \\
 &\vdots \\
 T(n) &= T(n-k) + \log(n-k+1) + \dots + \log(n-1) + \log(n) \\
 \text{for base case } T(1) &= 1 \\
 n-k &= 1 \\
 n &= 1+k \\
 &= T(1+k-k) + \log(1+k-k+1) + \dots + \log(n-1) + \log(n) \\
 &= T(1) + \log(2) + \log(3) + \dots + \log(n-1) + \log(n) \\
 &= 1 + \log(n!) \\
 &= O(\log n!)
 \end{aligned}$$

$\sum_{n=1}^N \log n = \log N!$

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Q6: Solve the following recurrence using recurrence tree method:

[10 marks]

$$T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + n, \quad n > 1$$

$$T(1) = 1, \quad n = 1$$

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$$T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + n$$

$$T(1) = 1$$

$$T\left(\frac{n}{10}\right) = T\left(\frac{n}{100}\right) + T\left(\frac{9n}{100}\right) + \frac{n}{10}$$

$$T\left(\frac{9n}{10}\right) = T\left(\frac{9n}{100}\right) + T\left(\frac{81n}{100}\right) + \frac{9n}{10}$$

$$T\left(\left(\frac{9}{10}\right)^k n\right) = T(1) \Rightarrow \left(\frac{9}{10}\right)^k n = 1 \Rightarrow n = \left(\frac{10}{9}\right)^k$$

$$\Rightarrow k = \log_{10/9} n$$


```

graph TD
    n[n] --> n10[n/10]
    n --> n9_10[9n/10]
    n10 --> n100[n/100]
    n10 --> n9_100[9n/100]
    n9_10 --> n9_100
    n9_10 --> n81_100[81n/100]
    n100 --> T1_100[T(1)]
    n9_100 --> T1_9_100[T(1)]
    n9_100 --> T1_81_100[T(1)]
    n81_100 --> T1_81_100
  
```


Leaf nodes cost = $2^k = 2^{\log_{10/9} n}$

Internal nodes cost = $k n$

$$= n (\log_{10/9} n)$$

Total cost = $2^{\log_{10/9} n} + n \log_{10/9} n$

$$O(n \log_{10/9} n)$$