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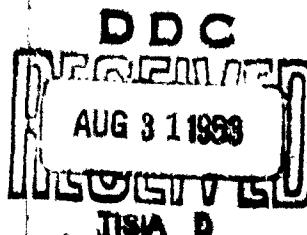
TECHNICAL NOTE R-28

A FORTRAN PROGRAM TO CALCULATE  
A BALLISTIC MISSILE TRAJECTORY  
FROM BURN OUT TO IMPACT

Prepared By

Charles F. Ostner

November, 1962



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A FORTRAN PROGRAM TO CALCULATE  
A BALLISTIC MISSILE TRAJECTORY  
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November, 1962

Prepared For

DIRECTORATE OF MISSILE INTELLIGENCE  
ARMY MISSILE COMMAND

By

SCIENTIFIC RESEARCH STAFF  
BROWN ENGINEERING COMPANY, INC.

Contract No. DA-01-009-ORD-1068

Prepared By:

Charles F. Ostner  
Senior Project Engineer

## ABSTRACT

This computer program computes a ballistic trajectory from burn out or any point thereafter to impact. It assumes point mass and a spherical earth. The program takes into account air resistance, the variation of the gravitational and centrifugal field with altitude, and earth's rotation. It uses a system of co-ordinates (X, Y, and Z) rigidly connected with the rotating earth.

The computer program is written in FORTRAN II and an autocoder deck has been compiled. The program is run on the IBM 1410 computer. It is being adapted for use on the IBM 1620 computer for use by the Army. This program may be obtained from the Scientific Programming Section; Program No. SP9. It was programmed by W. B. Warren of the Scientific Research Staff.

Approved By:

  
Harry C. Crews, Jr.  
Director, Special Projects Office

## SYMBOLS

<u>Symbol</u>	<u>Computer Symbol</u>	
$\beta$	BETA	Bearing angle - Clockwise horizontal angle from north to direction of flight
$\delta$	DELTA	Flight path angle - Angle between missile axis and local horizontal. Angle is positive when missile is climbing.
$\phi$	PHI	Latitude - Positive in northern hemisphere, negative in southern
$\theta$	THE T A	Longitude - Increases to the east. When firing east, $\theta_0 = 0$ , to the west, $\theta_0 = \pi/2$
X	X	See Figure 1 - Co-ordinate system with center of earth as origin. Z is the polar axis. X and Y are in the plane of the Equator and are $90^\circ$ apart. The X axis corresponds to the point where $\theta = 0$ . All are positive as shown.
Y	Y	
Z	Z	
$\gamma$	GAMMA	Range angle - Vertex at center of earth measured from the point where data were put into the program to the position attained along the trajectory.
$A_x$	AX	Acceleration in the X direction
$A_y$	AY	Acceleration in the Y direction
$A_z$	AZ	Acceleration in the Z direction
$A_t$	AT	Acceleration along the trajectory
$V_x$	VX	Velocity in the X direction
$V_y$	VY	Velocity in the Y direction
$V_z$	VZ	Velocity in the Z direction
$V_t$	VT	Velocity along the trajectory
$R_a$	RA	Distance from center of earth to re-entry body

Symbols (Cont.)

<u>Symbol</u>	<u>Computer Symbol</u>	
H	ALT	Distance from surface of earth to re-entry body
R	R	Radius of earth - Constant (20,902,890 ft.)
$\bar{r}$	-	Geocentric location vector
$g_o$	GO	Acceleration of gravity at sea level - constant (32.174 ft/sec <sup>2</sup> )
g	GRAV	Acceleration of gravity at altitude
U	U	Earth's rotation in radians per second - constant ( $2\pi/24 \times 3600$ )
$\sigma$	SIGMA	Angle of a spherical triangle - see Figure 3
u	AMU	$(g_o R^2)^{\frac{1}{2}}$
$(\frac{w}{C_D A})$	BALCO	Ballistic coefficient - Estimated from shape and dimensions
$\rho$	DEN	Air density from tape for IBM 1410 program - computed internally in IBM 1620 program

PHYSICAL CONSTANTS

$$R = 20,902,890 \text{ feet}$$

$$g_o = 32.174 \text{ ft/sec}^2$$

$$U = 2\pi/24 \times 3600 \text{ radians/sec.}$$

$$u^2 = g_o R^2$$

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## INTRODUCTION

This computer program may be used to study the free flight and re-entry portions of a ballistic trajectory. The information calculated and printed by the computer includes time of flight, location in both X, Y, and Z from earth's center and latitude, longitude and altitude from earth's surface, acceleration, velocity, flight path angle, bearing angle and range in nautical miles.

The model chosen for use in this computer program was taken from References 1 and 2. Point mass and a spherical earth are assumed. The basic acceleration equations are developed in Reference 1. Reference 2 discusses a similar computer program which uses the same acceleration equations and many of the other relationships used in this computer program.

A portion of the computer program discussed in Reference 3 was used to compute density in the IBM 1620 Computer program. This portion is an integral part of that program.

### Library Functions Used:

SQRT  
SIN  
COS  
EXP  
ATAN

## ANALYSIS

### Initial Conditions

The computer program will accept initial conditions at burn out or at any time thereafter until impact. The items of information needed are punched on two IBM cards which are placed at the end of the deck.

<u>Items</u>	<u>Comments</u>
$\delta$	Positive when missile is climbing.
$\beta$	Clockwise horizontal angle from north
$\theta$	Initial $\theta_0 = 0$ when flying eastward and $\theta_0 = \pi/2$ when flying toward the west
$\phi$	Positive in northern hemisphere
H	Altitude above earth's surface
R	20,902,890 ft. (earth's radius)
$V_t$	Velocity along the trajectory
$g_0$	32.174 ft/sec <sup>2</sup>
U	$2\pi/24 \times 3600$ radians per second
$(\frac{w}{C_D A})$	Ballistic coefficient

All angles are put into the computer in radians. The computer program is designed to accept and compute English units. All angles are printed in degrees. (Program SP-9-E)

The computer program may be modified to accept, compute and print metric units by using metric units for H, R,  $V_t$ ,  $g_0$  and by using conversion

factors for the altitude and density in the density subroutine in the IBM 1620 computer program. A deck in metric units has been prepared. (SP-9-M)

#### Development of Equations

In order to provide the proper initial conditions, the following set of co-ordinate transformations were used as an integral part of the computer program. (See Figure 1):

$$X = R_a \cos \phi \cos \theta \quad (1)$$

$$Y = R_a \cos \phi \sin \theta \quad (2)$$

$$Z = R_a \sin \phi \quad (3)$$

By definition:

$$H = R_a - R \quad (4)$$

It follows that:

$$R_a = (X^2 + Y^2 + Z^2)^{\frac{1}{2}} \quad (5)$$

In order to convert the input velocity along the trajectory to components in the X, Y, and Z directions, the angles  $\delta$ ,  $\beta$ ,  $\phi$ , and  $\theta$  are used. See Figure 1.  $\delta$  is the angle the vehicle makes with the local horizontal.

$$V_t \sin \delta = \text{a velocity component along } R_a \quad (6)$$

$$V_t \sin \delta \sin \phi = \text{a velocity component along the } Z \text{ axis} \quad (7)$$

$$V_t \sin \delta \cos \phi = \text{a velocity component along } R_a \text{ projected into the plane of the Equator} \quad (8)$$

$$V_t \sin \delta \cos \phi \sin \theta = \text{a velocity component along the } Y \text{ axis} \quad (9)$$

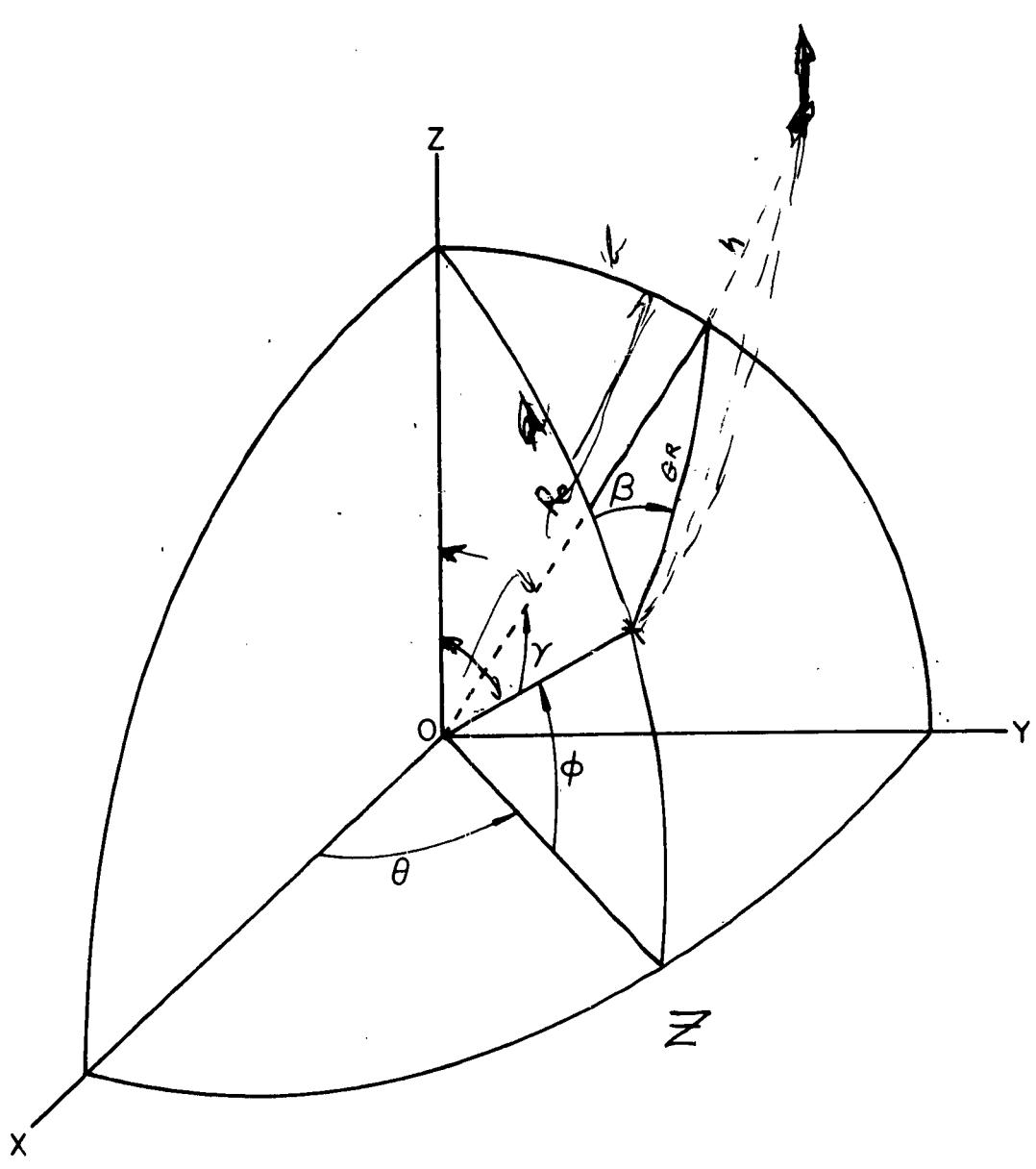


FIGURE 1

$V_t \sin \delta \cos \phi \cos \theta$  = a velocity component along the X axis (10)

$V_t \cos \delta$  = a velocity component parallel to the earth's surface (11)

$V_t \cos \delta \sin \beta$  = a velocity component along a parallel of latitude (12)

$V_t \cos \delta \sin \beta \sin \theta$  = a velocity component along the X axis (13)

$V_t \cos \delta \sin \beta \cos \theta$  = a velocity component along the Y axis (14)

$V_t \cos \delta \cos \beta$  = a velocity component along a meridian of longitude (15)

$V_t \cos \delta \cos \beta \sin \phi$  = a velocity component along  $R_a$  projected into the plane of the Equator (16)

$V_t \cos \delta \cos \beta \sin \phi \sin \theta$  = a velocity component along the Y axis (17)

$V_t \cos \delta \cos \beta \sin \phi \cos \theta$  = a velocity component along the X axis (18)

$V_t \cos \delta \cos \beta \cos \phi$  = a velocity component along the Z axis (19)

The following three equations are integral parts of the computer program. The signs (+ or -) of the sin and cos functions of  $\delta$ ,  $\beta$ ,  $\phi$ , and  $\theta$  were analyzed as the angles changed. The proper sign was given to the components used.

Combining (10), (13), and (18):

$$V_x = V_t (\sin \delta \cos \phi \cos \theta - \cos \delta \sin \beta \sin \theta - \cos \delta \cos \beta \sin \phi \cos \theta) \quad (20)$$

Combining (9), (14), and (17):

$$V_y = V_t (\sin \delta \cos \phi \sin \theta + \cos \delta \sin \beta \cos \theta - \cos \delta \cos \beta \sin \phi \sin \theta) \quad (21)$$

Combining (7) and (19):

$$V_z = V_t (\sin \delta \sin \phi + \cos \delta \cos \beta \cos \phi) \quad (22)$$

The component velocities can be used to find the velocity along the trajectory.

$$V_t = (V_x^2 + V_y^2 + V_z^2)^{\frac{1}{2}} \quad (23)$$

If one takes into account the air resistance, the variation of the gravitational and centrifugal field and earth's rotation, the equation of motion for the ballistic vehicle becomes:

$$m \ddot{\bar{r}} = m \bar{g} + \bar{W} + \bar{k}_c + \bar{k}_n \quad (24)$$

where

$m$  = mass of vehicle

$\bar{r}$  = radius vector from center of earth to vehicle

$\bar{g}$  = acceleration due to gravity

$\bar{W}$  = air resistance

$\bar{k}_c$  = Coriolis force

$\bar{k}_n$  = centrifugal force

Examining equation (24) term by term, one obtains:

$$m \bar{g} = - \frac{m u^2}{R_a^2} \frac{\bar{r}}{R_a} \quad (25)$$

where

$$u^2 = g_o R^2$$

R = radius of earth

R<sub>a</sub> = distance from earth's center to vehicle

$$\bar{w} = - \bar{v}_c v^{n-1} \quad (26)$$

where c and n are factors which depend on air density, the shape of the vehicle and the velocity;

$$\bar{k}_c = 2m (\bar{v} \times \bar{U}) \quad (27)$$

where

$\bar{U}$  = angular velocity of the rotation of earth in radians per second

and

$$\bar{k}_n = m U^2 r \cos \phi = m U^2 (X^2 + Y^2)^{\frac{1}{2}} \quad (28)$$

In this analysis, the air resistance was assumed to be proportional to the square of velocity so that equation (26) becomes

$$\bar{w} = \frac{\bar{v} \rho g V m}{2 \left( \frac{\bar{w}}{C_D A} \right)} \quad (29)$$

Writing equations (25) through (29) in component form, we obtain:

$$A_x = - \frac{u^2 X}{R_a^3} - \frac{\rho V t g V_x}{2 \left( \frac{\bar{w}}{C_D A} \right)} + 2 V_y U + U^2 X \quad (30)$$

$$A_y = - \frac{u^2 Y}{R_a^3} - \frac{\rho V t g V_y}{2 \left( \frac{\bar{w}}{C_D A} \right)} - 2 V_x U + U^2 Y \quad (31)$$

$$A_z = -\frac{u^2 Z}{R_a^3} - \frac{\rho V_t g V_z}{2 \left( \frac{w}{C_D A} \right)} \quad (32)$$

Integrating these equations in computer form, we obtain

$$V_{x_n} = V_{x(n-1)} + A_x \Delta t \quad (33)$$

$$V_{y_n} = V_{y(n-1)} + A_y \Delta t \quad (34)$$

$$V_{z_n} = V_{z(n-1)} + A_z \Delta t \quad (35)$$

$$X_n = X_{n-1} + V_x \Delta t \quad (36)$$

$$Y_n = Y_{n-1} + V_y \Delta t \quad (37)$$

$$Z_n = Z_{n-1} + V_z \Delta t \quad (38)$$

From Figure 1, the relationships pertaining to the angles are:

$$\phi = \arcsin \frac{Z}{R_a} \quad (39)$$

$$\theta = \arctan \frac{Y}{X} \quad (40)$$

$$\delta = \arcsin \frac{\Delta R_a / \Delta t}{V_t} \text{ where } \Delta R_a / \Delta t = \text{average rate of change in altitude} \quad (41)$$

In spherical trigonometry, the law of cosines states:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad (42)$$

The law of sines states:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad (43)$$

where angles A, B, and C represent the angles of the spherical triangle on the earth's surface and a, b, and c represent the sides of the triangle and the angles subtended by them at earth's center. See Figure 2.

Now refer to Figure 3.

$$\cos \gamma = \cos \text{Co}\phi_o \cos \text{Co}\phi + \sin \text{Co}\phi_o \sin \text{Co}\phi \cos \Delta\theta \quad (44)$$

where

$$\Delta\theta = \theta - \theta_o \text{ and}$$

$$\text{Co}\phi_o = 90^\circ - \phi_o, \text{ then } \cos \text{Co}\phi_o = \sin \phi_o \text{ and } \sin \text{Co}\phi_o = \cos \phi_o$$

$$\text{Co}\phi = 90^\circ - \phi, \text{ then } \cos \text{Co}\phi = \sin \phi \text{ and } \sin \text{Co}\phi = \cos \phi$$

Equation (44) then becomes

$$\cos \gamma = \sin \phi_o \sin \phi + \cos \phi_o \cos \phi \cos \Delta\theta \quad (45)$$

$$\gamma = \arccos(\sin \phi_o \sin \phi + \cos \phi_o \cos \phi \cos \Delta\theta) \quad (46)$$

Then range =  $\gamma R$ ,  $\gamma$  is in radians. Earth's radius,  $R = 3440.239$  nautical miles.

$$\text{Range} = \gamma (3440.239) \quad (47)$$

Refer again to Figure 3.

$$\frac{\sin \sigma}{\sin \text{Co}\phi_o} = \frac{\sin \Delta\theta}{\sin \gamma} \quad \text{but } \sin \text{Co}\phi_o = \cos \phi_o \quad (48)$$

$$\sin \sigma = \frac{\cos \phi_o \sin \Delta\theta}{\sin \gamma} \quad (49)$$

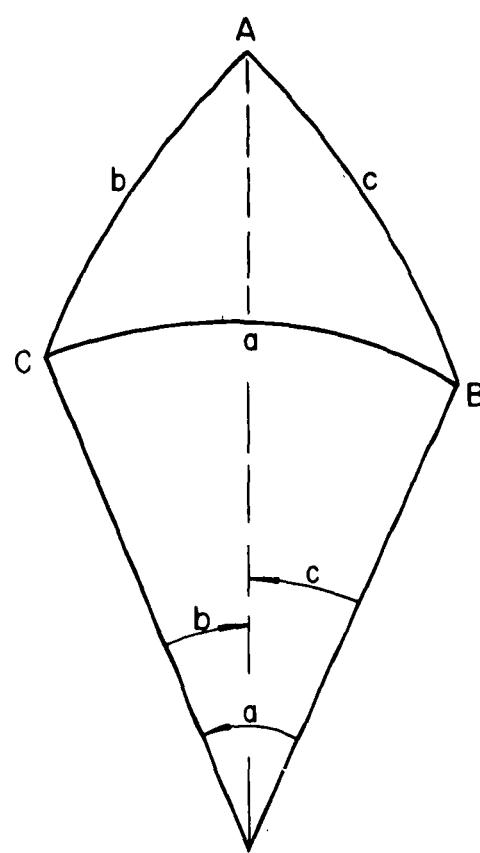


FIGURE 2

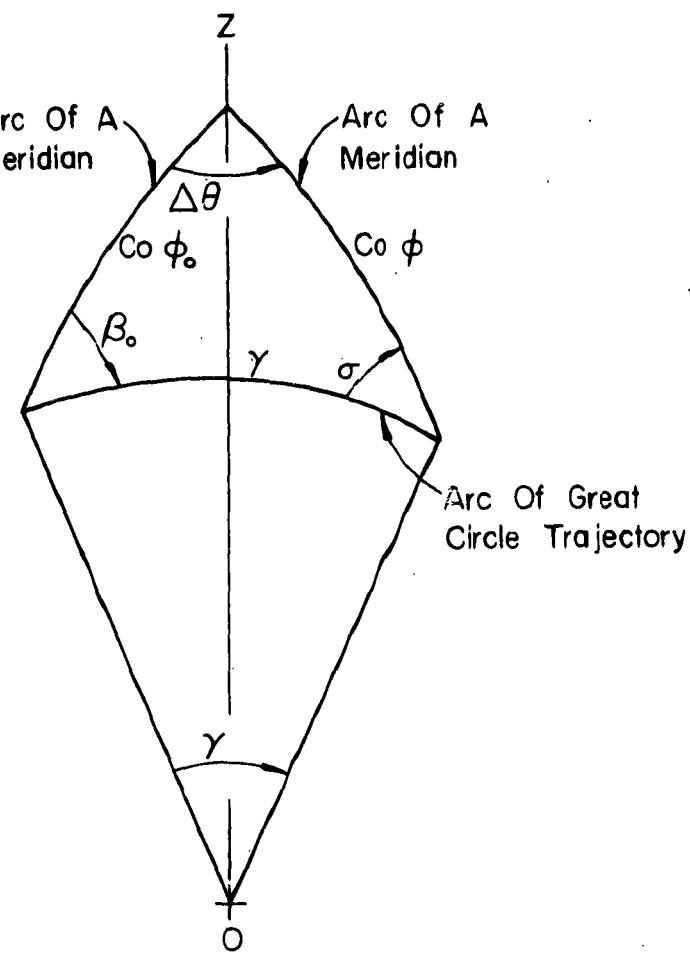


FIGURE 3

$$\sigma = \arcsin\left(\frac{\cos \phi_0 \sin \Delta\theta}{\sin \gamma}\right) \quad (50)$$

Sigma ( $\sigma$ ) is used to compute bearing ( $\beta$ ). In order to minimize the error caused by assuming that the trajectory follows an exact great circle (it would follow an exact great circle if the earth were not rotating), a smaller spherical triangle is used in the computer program.

Let symbols with subscript 1 stand for the next to last values computed. The symbols without subscripts stand for the newest values computed. Let  $\gamma_3$  equal the range increment angle.

Equation (46) now becomes:

$$\gamma_3 = \arccos\left[\sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos(\theta - \theta_1)\right] \quad (51)$$

Equation (50) becomes

$$\sigma = \arcsin\left(\frac{\cos \phi_1 \sin(\theta - \theta_1)}{\sin \gamma_3}\right) \quad (52)$$

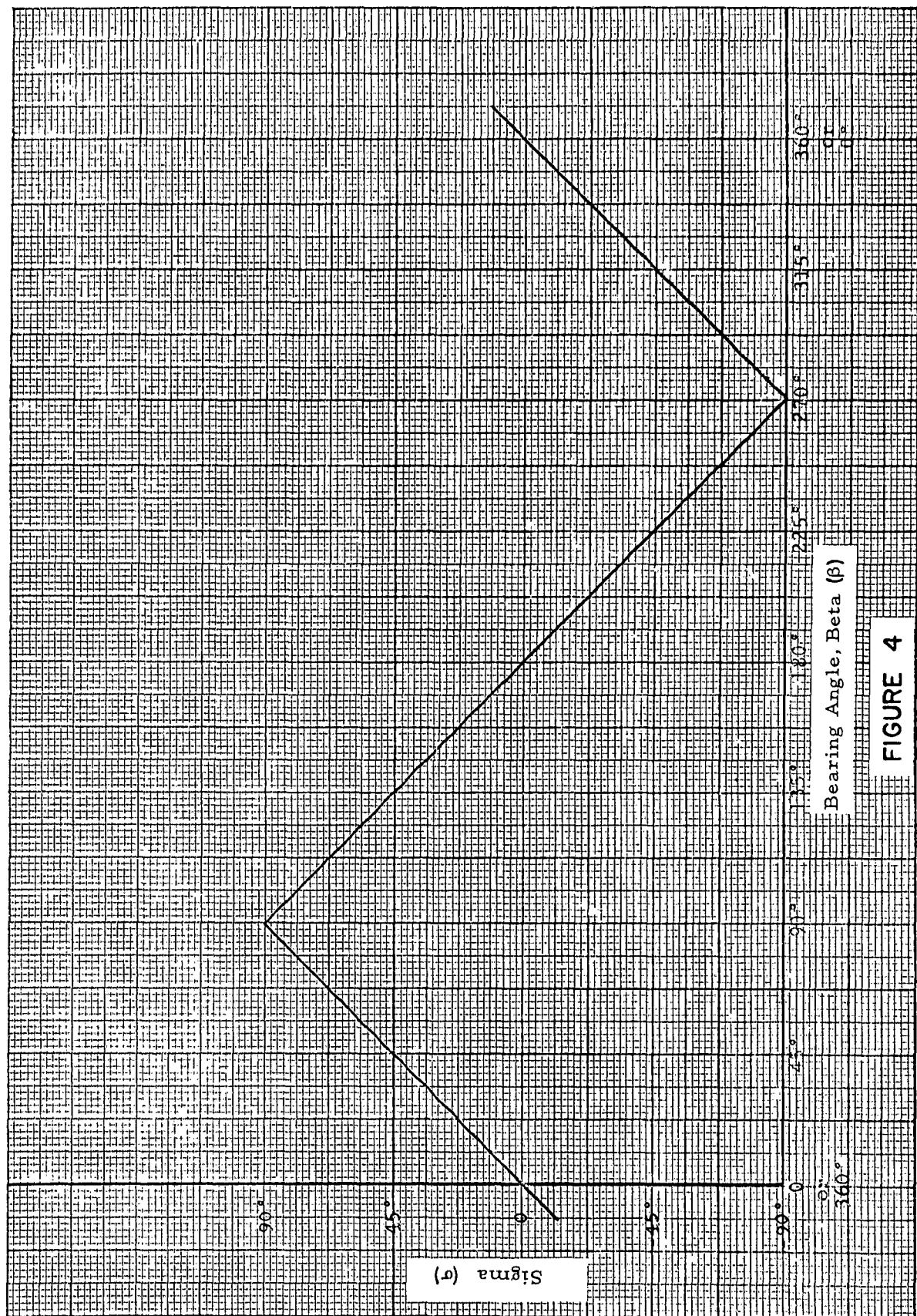
The computer uses the tangent function to determine the value of sigma ( $\sigma$ ). The value determined will lie between  $-90^\circ$  and  $+90^\circ$ . A graph showing sigma plotted against beta is shown in Figure 4.

From Figure 4 we can see that if beta is between  $0$  and  $90^\circ$

$$\beta = \sigma \quad (53)$$

If beta is between  $90^\circ$  and  $270^\circ$ ,

$$\beta = \pi - \sigma \quad (54)$$



**FIGURE 4**

If beta is greater than  $270^\circ$ ,

$$\beta = 2\pi + \sigma \quad (55)$$

The five equations (51) through (55) are integral parts of the computer program.

In order to compute  $A_t$ , which is the acceleration along the trajectory, let  $V_t$  = the last velocity computed along the trajectory,

$V_{t_1}$  = the next to last velocity computed along the trajectory

$\Delta t$  = the time increment being used the computer  
then

$$A_t = \frac{V_t - V_{t_1}}{\Delta t} \quad (56)$$

## DISCUSSION

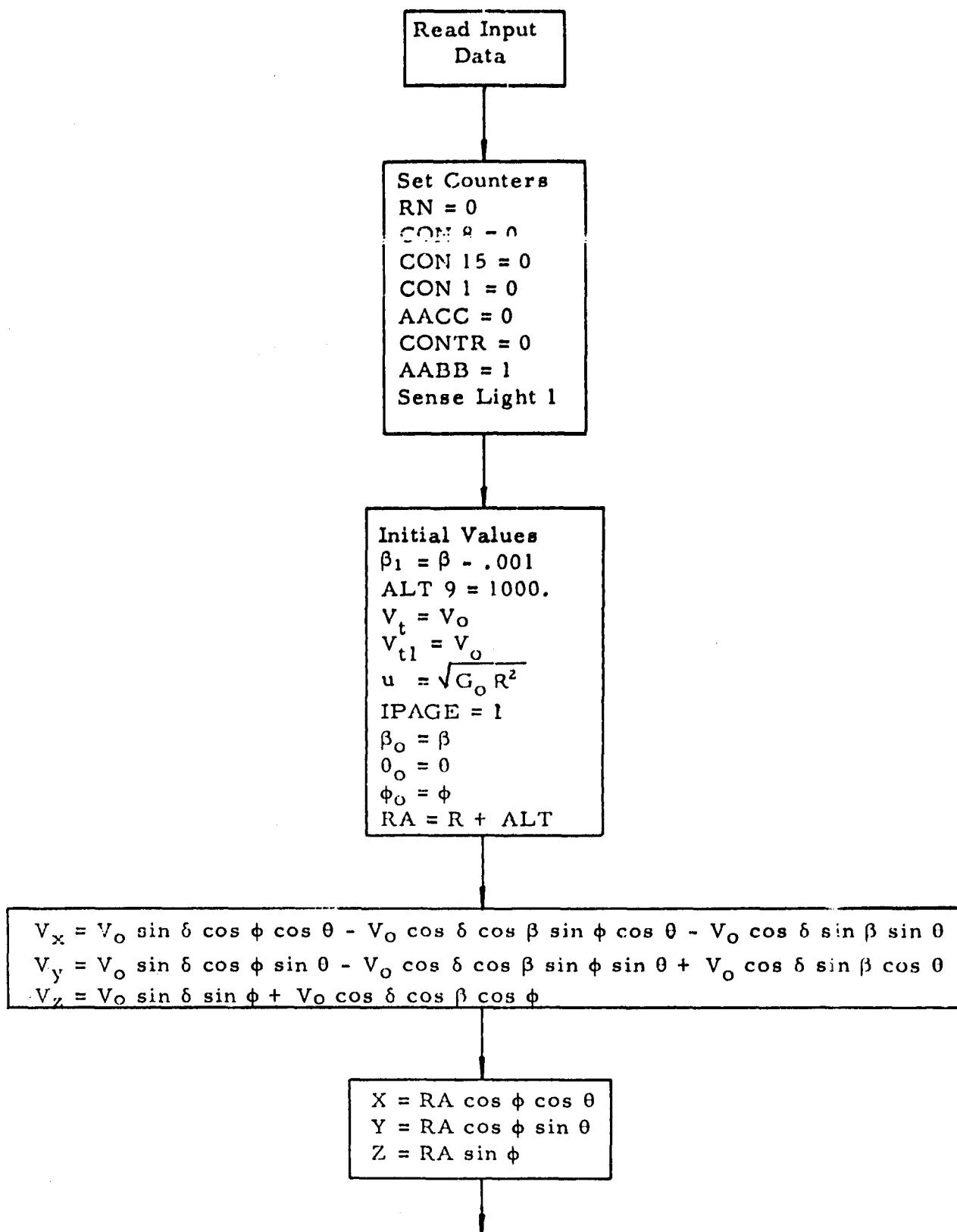
The computer program assumes a spherical earth and a point-mass.

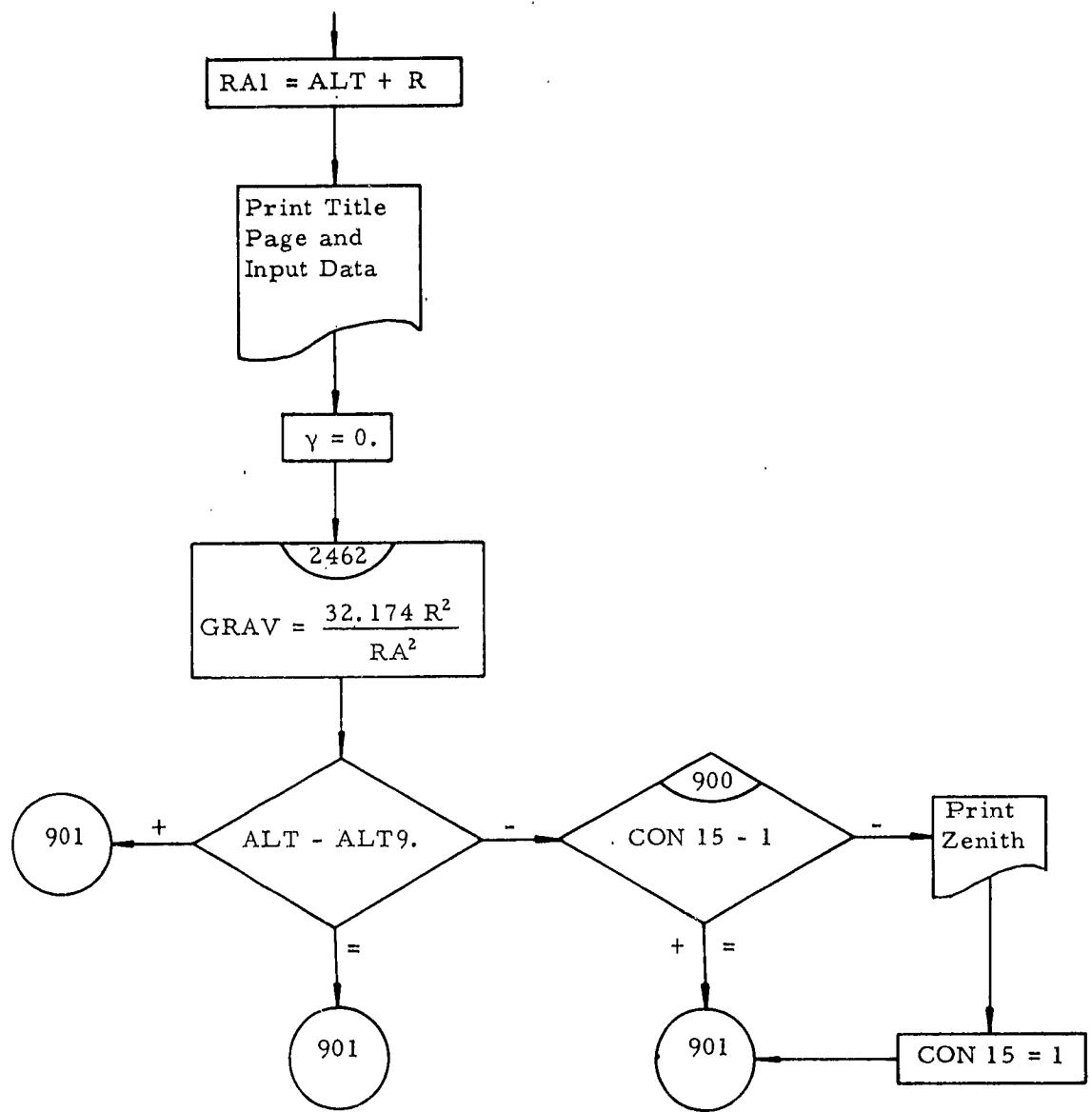
Other conditions are those which actually exist at earth's surface and at altitude from the surface.

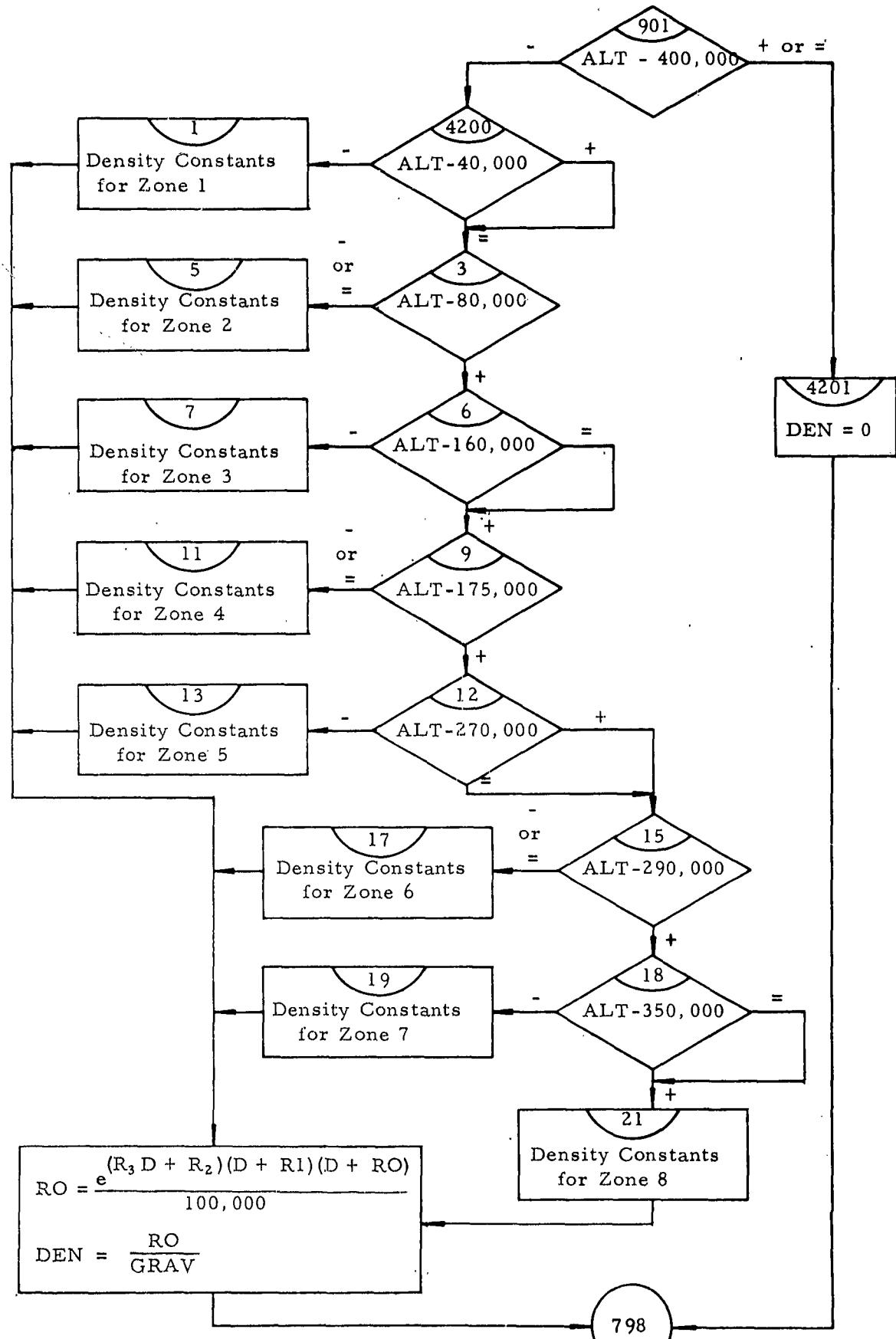
The time increment used by the computer is one second during the portion of the trajectory where conditions change slowly. During the re-entry portion of the trajectory when the altitude of the re-entry body is below 150,000 feet, the time increment is decreased to 0.1 second to increase accuracy. In order to give desired accuracy, this program was written in double precision. The program uses about 39,950 digits of memory. It is believed that this program produces highly accurate results.

For some of the angle relationships, it is assumed that the trajectory follows an exact great circle. The errors caused by this assumption are very small because of the short time increments used and the methods chosen to compute these relationships. These relationships are computer outputs and are not used to drive the computer.

**FLOW CHART FOR FREE FLIGHT AND RE-ENTRY TRAJECTORY**





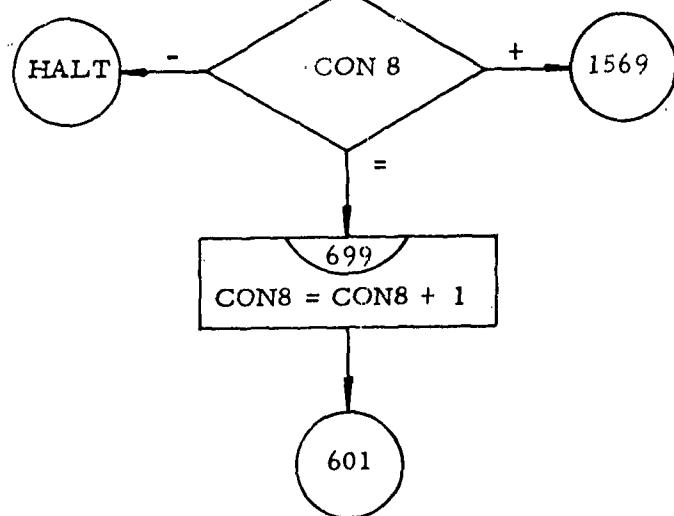


798

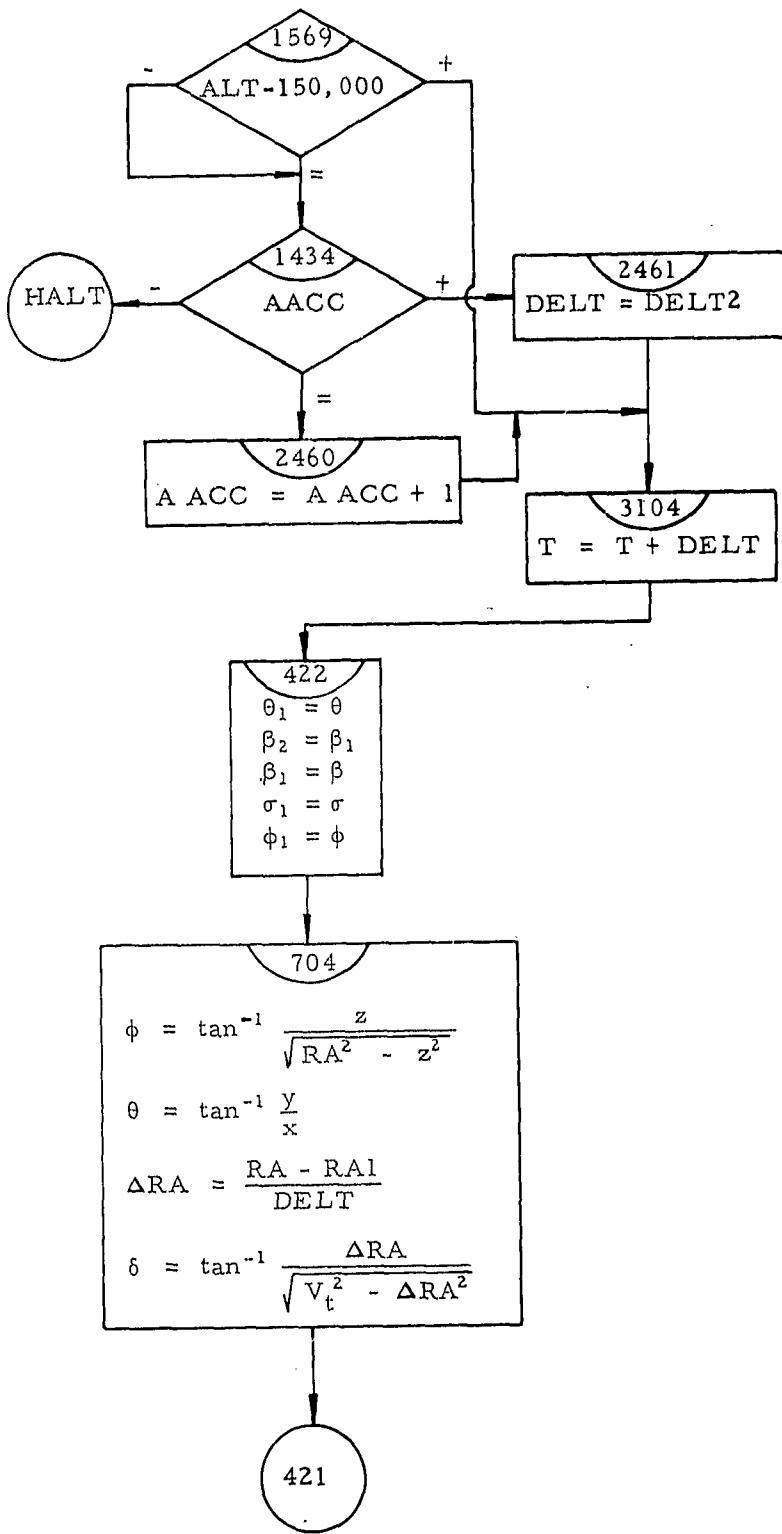
$$A_x = 2V_y U + U^2 X - \frac{u^2 X}{RA^3} - \frac{\rho g V_x V_t}{2 \text{BALCO}}$$

$$A_y = U^2 X - 2V_x U - \frac{u^2 Y}{RA^3} - \frac{\rho g V_y V_t}{2 \text{BALCO}}$$

$$A_z = -\frac{u^2 Z}{RA^3} - \frac{\rho g V_z V_t}{2 \text{BALCO}}$$



(601 on page 23)

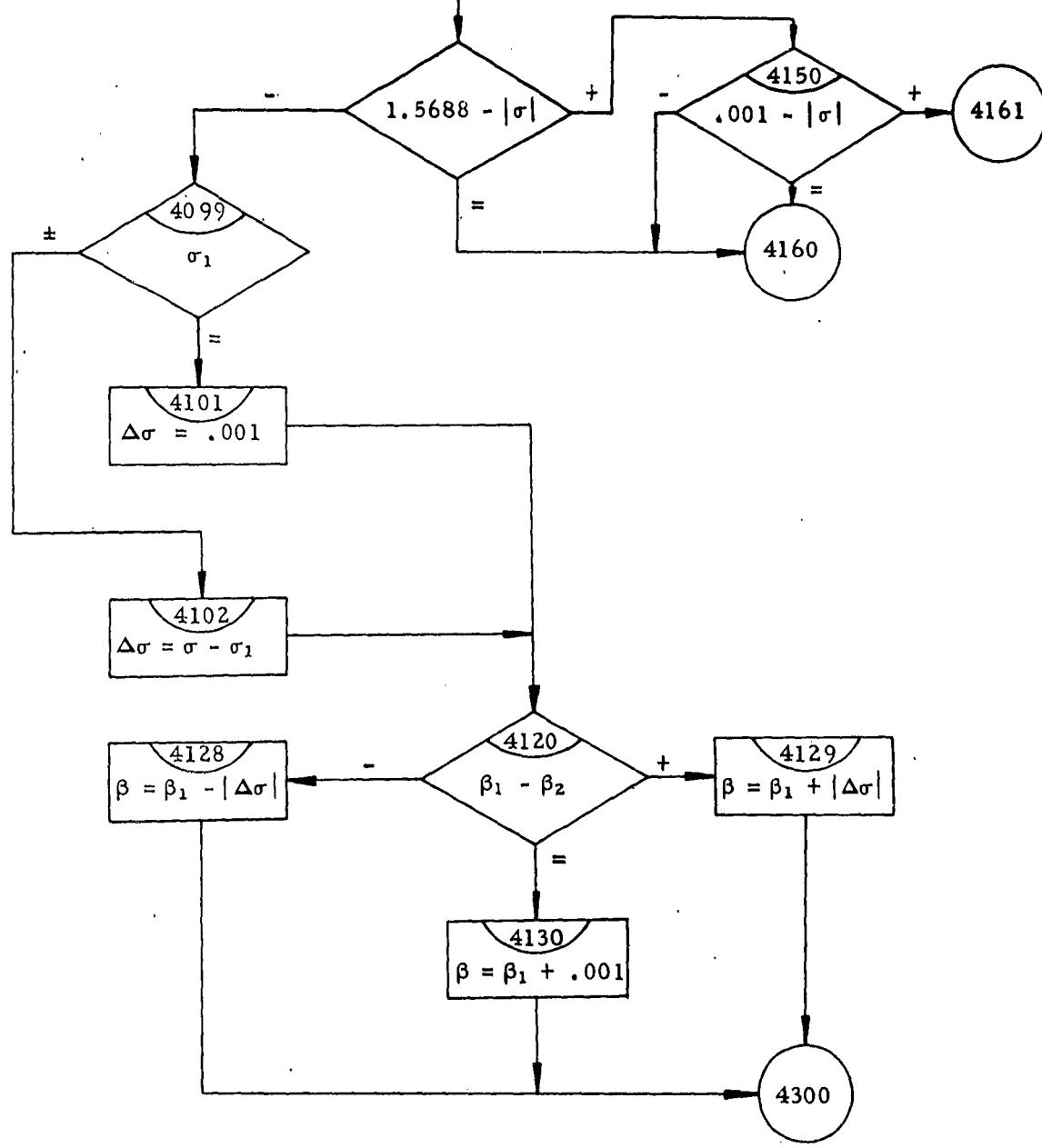


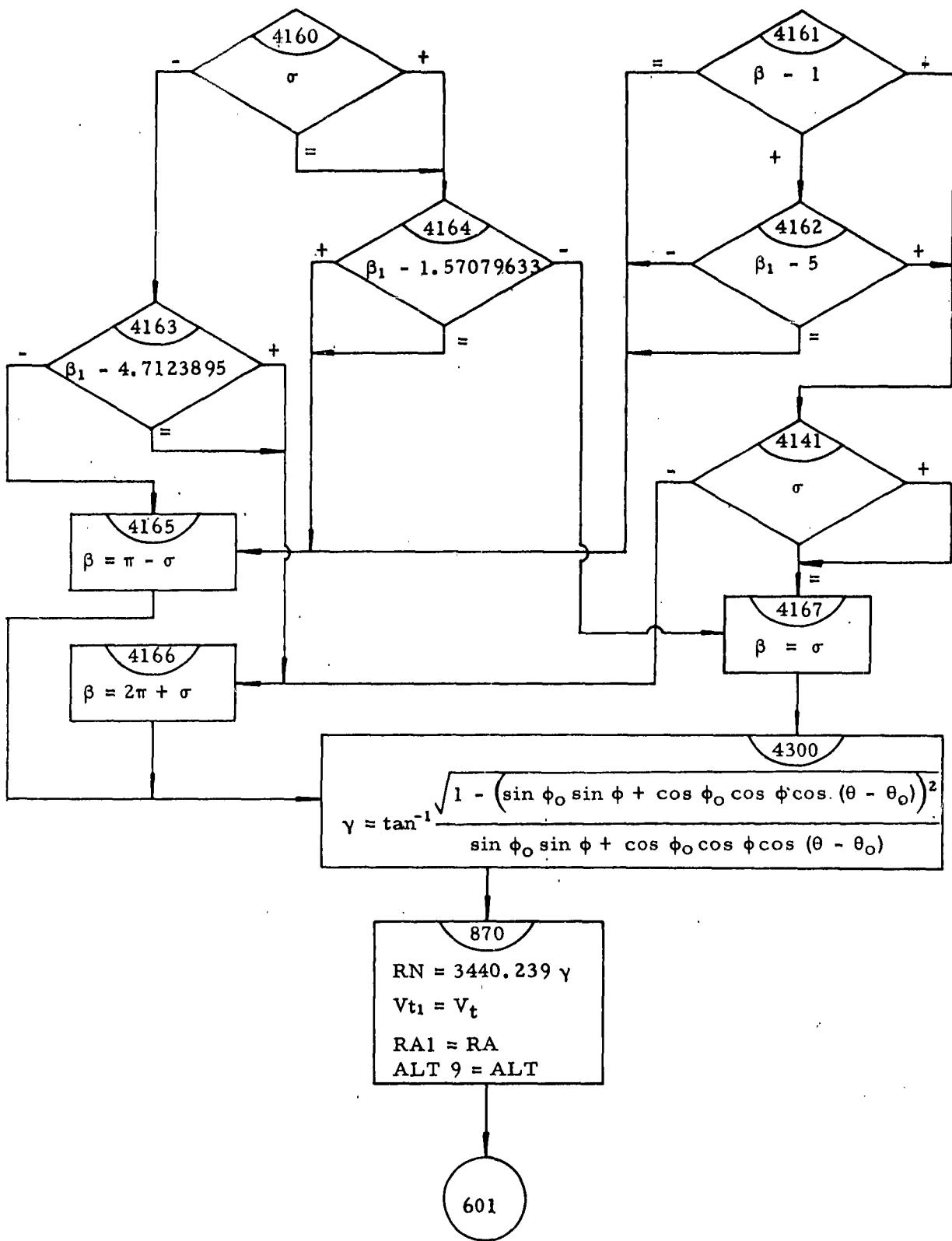
421

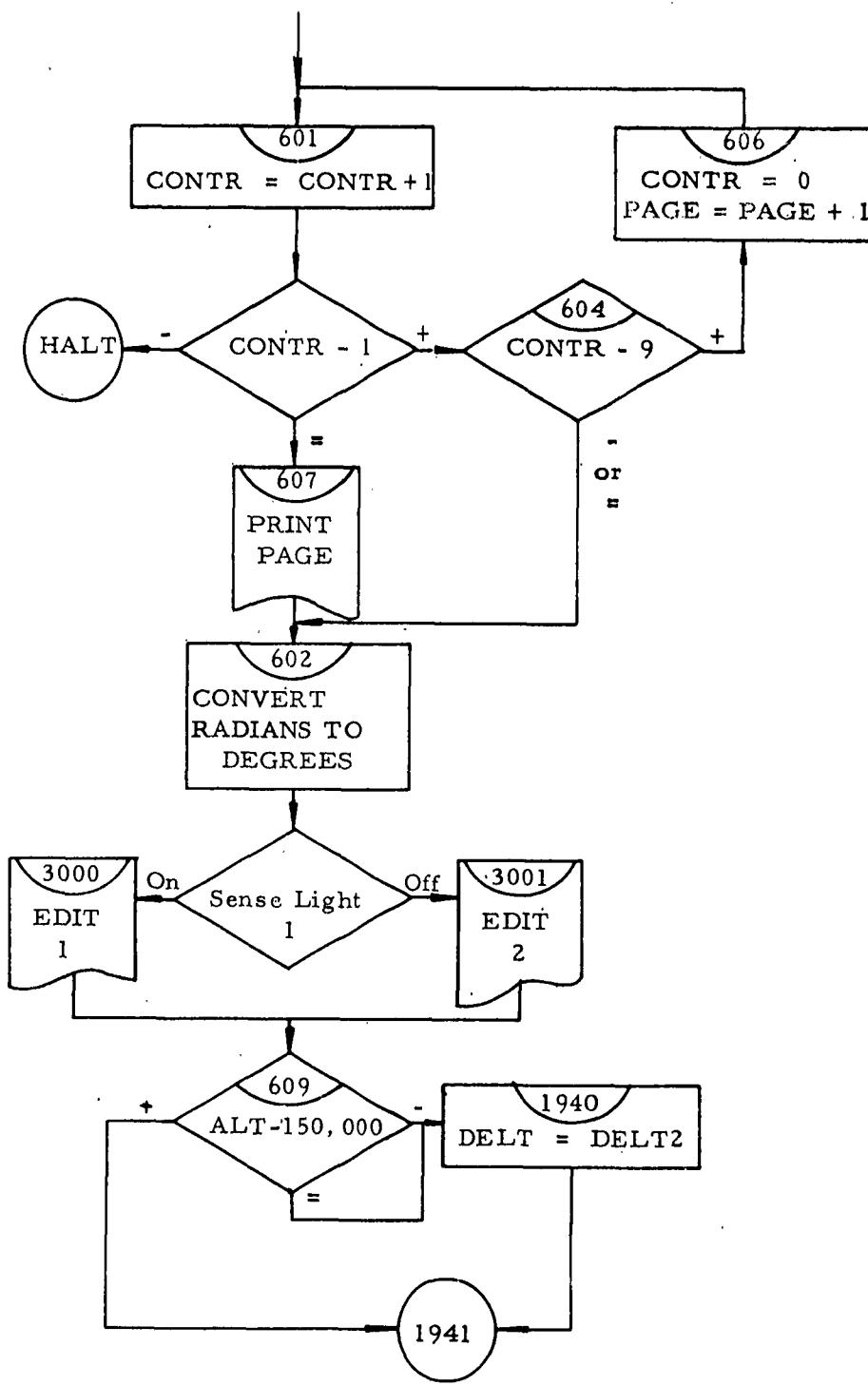
$$\Delta\theta = \theta - \theta_1$$

$$\gamma_3 = \tan^{-1} \frac{\sqrt{1 - (\sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos \Delta\theta)^2}}{\sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos \Delta\theta}$$

$$\sigma = \tan^{-1} \frac{\sin \Delta\theta \cos \phi_1}{\sqrt{\sin^2 \gamma_3 - (\sin \Delta\theta \cos \phi_1)^2}}$$







1941

$$V_x = V_x + A_x * \text{DELT}$$

$$V_y = V_y + A_y * \text{DELT}$$

$$V_z = V_z + A_z * \text{DELT}$$

$$X = X + V_x * \text{DELT}$$

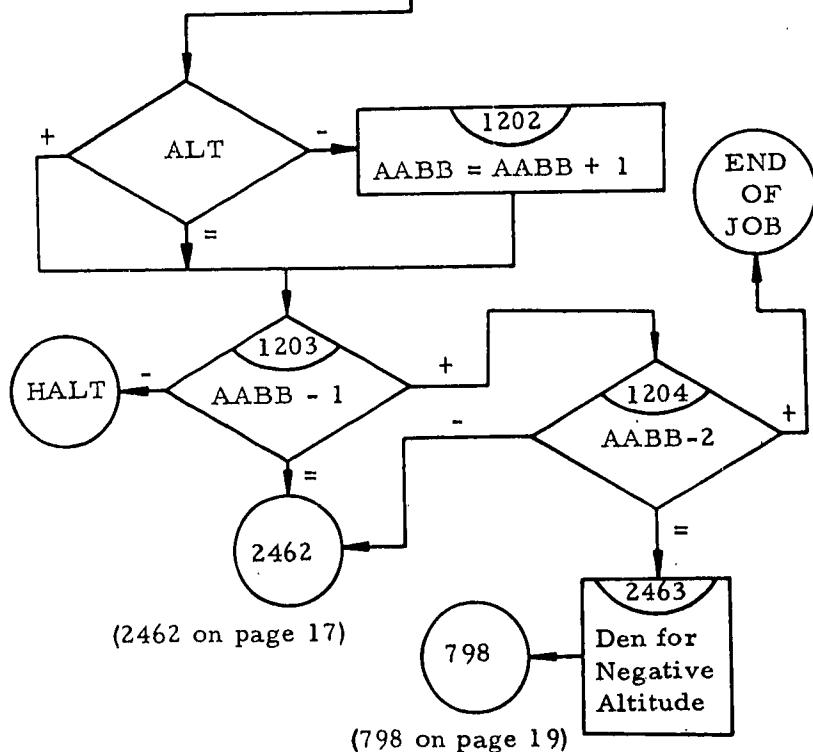
$$Y = Y + V_y * \text{DELT}$$

$$Z = Z + V_z * \text{DELT}$$

$$V_t = (V_x^2 + V_y^2 + V_z^2)^{\frac{1}{2}}$$

$$A_t = \frac{V_t - V_{t1}}{\text{DELT}}$$

$$RA = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\text{ALT} = RA - R$$


FORTRAN STATEMENTS

English Units

C FREE FLIGHT AND RE-ENTRY  
C PROGRAMMED BY W.B.WARREN  
C FOR RESEARCH STAFF  
C 10 NOVEMBER 1962  
C PROGRAM NO. SP 9 ENGLISH UNITS.

C  
C PHYSICAL CONSTANTS  
C GO # GRAVITATIONAL ACCELERATION AT SEA LEVEL - 32.174 FT/SEC\*\*2  
C U # EARTH ROTATION -  $\frac{2\pi}{24*3600}$  RADIANS/SEC  
C R # RADIUS OF EARTH - 20,902,890 FEET  
C BALCO # BALLISTIC COEFFICIENT - CONSTANT  
C MU\*\*2 #  $GO*R^{**2} / 32.174 * 20,902,890^{**2}$   
C  
C OTHER INPUT DATA  
C DEL #ANGLE BETWEEN HORIZONTAL AND MISSILE AXIS--POSITIVE UP  
C BETA #AZIMUTH ANGLE CLOCKWISE FROM NORTH TO MISSILE AXIS  
C THETA #LONGITUDE OF MISSILE  
C PHI #LATITUDE OF MISSILE  
C H #HEIGHT OF MISSILE ABOVE EARTH SURFACE  
C RHO #ATMOSPHERIC DENSITY --TABLE  
C RA # MISSILE ALTITUDE FROM CENTER OF EARTH  
C VO # MISSILE BURNOUT VELOCITY #VT AT T#0  
READ 100,DEL,BETA,THETA,PHI,ALT  
READ100,R,VO,GO,U,BALCO  
READ 100,T,DELT,DELT2  
CON8 # 0.  
SENSE LIGHT 1  
CON15 # 0.  
CON1 # 0.  
AACC # 0.  
CONTR#0.  
AARB # 1.  
BETA1# BETA - .001  
ALT9 #1000.  
RN#G.  
VT # VO  
VT1 #VO  
AMU #  $\sqrt{GO*R^{**2}}$   
IPAGE # 1  
BETA0 # BETA  
THETO # THETA  
PHIC # PHI  
RA # ALT&R  
OVX # $VO * \sin\theta * \cos\phi * \cos\beta * \cos\alpha - VO * \cos\theta * \cos\phi * \cos\beta * \sin\alpha$   
1F% $\phi * \cos\theta * \cos\beta - VO * \cos\theta * \sin\phi * \sin\beta$   
OVY # $VO * \sin\theta * \cos\phi * \sin\beta * \cos\alpha - VO * \cos\theta * \cos\phi * \sin\beta * \sin\alpha$

```
1F%PHI#*SINF%THETA#&VO*COSF%DEL#*SINF%BETA#*COSF%THETA#
OVZ #VO*SINF%DEL#*SINF%PHI#&VO*COSF%DEL#*COSF%BETA#*COSF%PHI#
X #RA*COSF%PHI#*COSF%THETA#
Y #RA*COSF%PHI#*SINF%THETA#
Z # RA*SINF%PHI#
RA1 # ALT & R
PRINT 4999,IPAGE
PRINT 5000
PRINT 5001
PRINT 5027
PRINT 5002
PRINT 5003
PRINT 5004
PRINT 5010
PRINT 5011
PRINT 5012
PRINT 5013
PRINT 5014
PRINT 5015,DEL,BETA,THETA,PHI
PRINT 5016,ALT,BALCO,RA,VO
GAMMA # 0.
2462 GRAV # %32.174*R**2# /RA**2
IF %ALT - ALT9#900,901,901
900 IF %CON15 - 1.#3840,901,901
3840 PRINT 5050
5050 FORMAT %1HK,35X,6HZENITH#
CON15 # 1.
901 IF %ALT-400000.#4200,4201,4201
4200 IF %ALT - 40000.#1,2,3
1 D # ALT/10000.
R3# -0.37728875E-02
R2# 0.52352523E-02
R1# -0.31047929E00
R0# 0.89444960E01
GO TO 50
2 CONTINUE
3 IF %ALT-80000.#4,5,6
4 CONTINUE
5 D # %ALT-40000.#/10000.
R3# 0.23298604E-05
R2# 0.21620268E-03
R1# -0.47974207E00
R0# 0.75402419E01
GO TO 50
6 IF %ALT - 160000.#7,8,9
7 D # %ALT-80000.#/10000.
R3# 0.26359811E-04
R2# 0.80943560E-02
R1# -0.51509227E00
R0# 0.56272847E01
GO TO 50
8 CONTINUE
9 IF %ALT-175000.#10,11,12
```

```

10 CONTINUE
11 D # %ALT-160000.□/10000.
   R3# -0.57066660E-04
   R2# 0.30080CC0E-03
   R1# -0.36384173E600
   R0# 0.20423758E601
   GO TO 50
12 IF %ALT-270000.□13,14,15
13 D # %ALT-175000.□/10000.
   R3# -0.84014648E-03
   R2# -0.16797257E-02
   R1# -0.32931657E600
   R0# 0.15021266E601
   GO TO 50
14 CONTINUE
15 IF %ALT-290000.□16,17,18
16 CONTINUE
17 D # %ALT-270000.□/10000.
   R3# 0.
   R2# 0.36480C09E-03
   R1# -0.6137624CE600
   R0# -0.25094864E601
   GO TO 50
18 IF %ALT-350000.□19,20,21
19 D # %ALT-290000.□/10000.
   R3# 0.17070042E-02
   R2# -0.16579218E-02
   R1# -0.63898718E600
   R0# -0.37292327E601
   GO TO 50
20 CONTINUE
21 D # %ALT-350000.□/10000.
   R3# -0.38550824E-02
   R2# 0.65901945E-01
   R1# -0.68705559E600
   R0# -0.72589563E601
50 R0#%EXPF%%R3*D&R2□*D&R1□*D&R0□□/100000.
   DEN # R0/GRAV
I   7980AX#2.*VY*U&U**2*X-%AMU**2*X□/RA**3□-%DEN*GRAV *VX*VT□/%2.*BALCO
I   1□
I   OAY#U**2*Y-2.*VX*U-%AMU**2*Y□/RA**3□-%DEN*GRAV *VY*VT□/%2.*BALCO
I   1□
I   AZ#%-1.□*%%AMU**2*Z□/RA**3□&%DEN*GRAV *VZ*VT□/%2.*BALCO□□
I   IF %CON8□4000,699,1569
T   699 CON8 # CON8 & 1.
I   GO TO 601
4201 DEN # 0.
I   GO TO 798
1569 IF %ALT -150000.□1434,1434,3104
1434 IF %AACC□4000,2460,2461
2460 AACC # AACC & 1.
I   GO TO 3104
2461 DELT #DELT2

```

```

3104 T # T & DELT
422 THET1 # THETA
    BETA2 #BETA1
    BETA1 # BETA
    SIGMA1#SIGMA
    PHI1 # PHI
704 PHI # ATANF%Z/%SQRTE%RA**2-Z**2000
    THETA #ATANF%Y/X0
    DELRA # %RA - RA1#DELT
    DEL # ATANF%%DELRA#/%SQRTE%VT**2-DELRA**2000
421 DTHET#THETA -THET1
    GAMMA3# ATANF%%SQRTE%1.-%SINF%PHI1#*SINF%PHI0#&%COSF%PHI1#*COSF%P
    1H1#*COSF%DTHET#0#**200/%SINF%PHI1#*SINF%PHI0#&COSF%PHI1#*COSF%PHI0#
    2COSF%DTHET#0#
    SIGMA # ATANF%%SINF%DTHET#*COSF%PHI1#/%SQRTE%$SINF%GAMMA3#**2-%S
    1INF%OTHE#*COSF%PHI1#**200
    IF %1.5688 - %ABSF%SIGMA#4099,4160,4150
4099 IF %SIGMA1#4102,4101,4102
4101 DELSIG # .001
    GO TO 4120
4102 DELSIG # 'SIGMA -SIGMA1
4120 IF%BETA1-BETA2#4128,4130,4129
4128 BETA# BETA1-%ABSF%DELSIG#0#
    GO TO 4300
4130 BETA# BETA1&.001
    GO TO 4300
4129 BETA# BETA1 &%ABSF%DELSIG#0#
    GO TO 4300
4150 IF %.C01-%ABSF%SIGMA#4160,4160,4161
4161 IF %BETA - 1.0#4141,4165,4162
4141 IF %SIGMA#4166,4167,4167
4162 IF %BETA1 - 5.0#4165,4165,4141
4160 IF %SIGMA# 4163,4164,4164
4163 IF %BETA1-4.7123895#4165,4166,4166
4165 BETA #3.14159265-SIGMA
    GO TO 4300
4166 BETA #6.28318530 & SIGMA
    GO TO 4300
4164 IF %BETA1 - 1.57079632675#4167,4165,4165
4167 BETA # SIGMA
4300 GAMMA # ATANF%%SQRTE%1.-%SINF%PHI0#*SINF%PHI0#&%COSF%PHI0#*COSF%P
    1H1#*COSF%THETA-THET0#0#**200/%SINF%PHI0#*SINF%PHI0#&COSF%PHI0#*COSF%
    2%PHI0#*COSF%THETA-THET0#0#
870 RN #GAMMA * 3440.239
    VT1#VT
    RA1# RA
    ALT9 #ALT
601 CONTR #CONTR & 1.
    IF %CONTR -1.0#4000,607,604
604 IF %CONTR-9.0#602,602,606
606 CONTR #0.
    IPAGE #IPAGE & 1
    GO TO 601

```

```

607 PRINT 5017,IPAGE
602 AA1 #DEL* 57.2958
AA2 #BETA*57.2958
AA3 #THETA*57.2958
AA4 #PHI *57.2958
PRINT 5005,T,X,Y,Z
AA6 # SIGMA * 57.2958
AA7 # GAMMA3 * 57.2958
PRINT 5006,ALT,VX,VY,VZ
PRINT 5007,VT,AX,AY,AZ
PRINT 5008,AA2,RN,DEN,GRAV
IF %SENSE LIGHT 1 3000,3001
3000 PRINT 5028,AA4,AA3,AA1
GO TO 609
3001 PRINT 5025,AA4,AA3,AA1,AT
609 IF %ALT - 150000.0 1940,1940,1941
1940 DELT #DELT2
1941 VX # VX & AX*DELT
VY # VY & AY * DELT
VZ # VZ & AZ * DELT
X # X & VX * DELT
Y # Y & VY * DELT
Z # Z & VZ * DELT
VT # SQRTF%VX**2 & VY**2 & VZ**2
AT #%VT- VT1/DELT
RA # SQRTF%X**2 & Y**2 & Z**2
ALT #RA - R
IF %ALT 1202,1203,1203
1202 AABB # AABB & 1.
1203 IF %AABB-1.0 4000,2462,1204
1204 IF %AABB-2.0 2462,2463,8000
2463 DEN #.C02376&%ALT/%-5000.00*%.0027448-.0023769
GO TO 798
5028 FORMAT %1H ,2X,5HPHI ,E15.8,2X,5HTHETA,E15.8,2X,5HDELTA,E15.8
100 FORMAT %5E15.8
5010 FORMAT %1HK,25X,42HPhysical Constants -same for all missiles-
5011 FORMAT %1H ,25X,57HGravitational Acceleration at sea level 32.174
1 FT/SEC**2
5012 FORMAT %1H ,30X,44HEarth Rotation 2PI/24 X 3600 Radians/Second
5013 FORMAT %1H ,30X,32HRadius of Earth 20,902,890 FEET
5014 FORMAT %1HK,5X,12HOther Inputs
5015 FORMAT %1H ,5HDEL ,E15.8,2X,5HBETA ,E15.8,2X,5HTHETA,E15.8,2X
1,5HPHI ,E15.8
5016 FORMAT %1H ,5HALT ,E15.8,2X,5HBALCO,E15.8,2X,5HRA ,E15.8,2X
1,5HVO ,E15.8
5017 FORMAT %1H1,79X,5HPAGE ,I4
8000 PRINT 8001
8001 FORMAT %1H1,//////////20X,32H*****END OF JOB*****//1H1
4999 FORMAT %1H1,32X,31HBrown Engineering Company, Inc.,4X,5HPAGE ,I4
5000 FORMAT %1H ,30X,35HFREE FLIGHT AND RE-ENTRY TRAJECTORY
5001 FORMAT %1H ,35X,24HPROGRAMMED BY W.B.WARREN
5027 FORMAT %1H ,30X,35HTHEORETICAL DERIVATION - C.F.OSTNER
5002 FORMAT %1H ,38X,18H RESEARCH STAFF

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CARD TO PRINT 80/80

5003 FORMAT %1H ,39X,16H10 NOVEMBER 1962□  
5004 FORMAT %1HK,39X,31HPROGRAM NO. SP 9 ENGLISH UNITS.□  
50050FORMAT %1HK,2X,5HTIME ,E15.8,2X,5HX ,E15.8,2X,5HY ,E15.8,2X,  
15HZ ,E15.8□  
5006 FORMAT %1H ,2X,5HALT ,E15.8,2X,5HVX ,E15.8,2X,5HVY ,E15.8,2X,  
15HVZ ,E15.8□  
5007 FORMAT %1H ,2X,5HVT ,E15.8,2X,5HAX ,E15.8,2X,5HAY ,E15.8,2X,  
15HAZ ,E15.8□  
5008 FORMAT %1H ,2X,5HBETA ,E15.8,2X,5HRANGE,E15.8,2X,5HDEN ,E15.8,2X,  
15HGRAV ,E15.8□  
5025 FORMAT %1H ,2X,5HPHI ,E15.8,2X,5HTHETA,E15.8,2X,5HDELTA,E15.8,2X,  
15HACCEL,E15.8□  
4000 STOP  
END

FORTRAN STATEMENTS

Metric Units

C FREE FLIGHT AND RE-ENTRY  
C PROGRAMMED BY W.B.WARREN  
C FOR RESEARCH STAFF  
C 10 NOVEMBER 1962

C  
C PROGRAM NO. SP 9 METRIC UNITS.

C PHYSICAL CONSTANTS  
C GO#GRAVITATIONAL ACCELERATION AT SEA LEVEL - 9.80665 METERS/SEC\*\*2  
C U # EARTH ROTATION - %2\*PI/%24\*3600 RADIANS/SEC  
C R # RADIUS OF EARTH - 6,371,230 METERS  
C BALCO # BALLISTIC COEFFICIENT - CONSTANT.  
C MU\*\*2 # GO\*R\*\*2 # 9.80665 \* 6,371,230\*\*2

C OTHER INPUT DATA  
C DEL #ANGLE BETWEEN HORIZONTAL AND MISSILE AXIS--POSITIVE UP  
C BETA #AZIMUTH ANGLE CLOCKWISE FROM NORTH TO MISSILE AXIS  
C THETA #LONGITUDE OF MISSILE  
C PHI #LATITUDE OF MISSILE  
C ALT #HEIGHT OF MISSILE ABOVE EARTH SURFACE  
C RA # MISSILE ALTITUDE FROM CENTER OF EARTH  
C VO # MISSILE BURNOUT VELOCITY #VT AT T#0  
READ 100,DEL,BETA,THETA,PHI,ALT  
READ 100,R,VO,GO,U,BALCO.  
READ 100,T,DELT,DELT2

CON8 # 0.

SENSE LIGHT 1

CON15 # 0.

CON1 # 0.

AACC # 0.

CONTR#0.

AABB # 1.

BETA1# BETA - .001

ALT9 #1000.

RNC#C.

VT # VO

VT1 #VO

AMU # SQRTF%GO\*R\*\*2

IPAGE # 1

BETAO # BETA

THETO # THETA

PHIC # PHI

RA # ALT&R

AK # 16.0184

OVX #VO\*SINF%DEL#\*COSF%PHI#\*COSF%THETA#-VO\*COSF%DEL#\*COSF%BETA#\*SIN  
1F%PHI#\*COSF%THETA#-VO\*COSF%DEL#\*SINF%BETA#\*SINF%THETA#

OVY #VO\*SINF%DEL#\*COSF%PHI#\*SINF%THETA#-VO\*COSF%DEL#\*COSF%BETA#\*SIN  
1F%PHI#\*SINF%THETA#&VO\*COSF%DEL#\*SINF%BETA#\*COSF%THETA#

OVZ #VO\*SINF%DEL#\*SINF%PHI#&VO\*COSF%DEL#\*COSF%BETA#\*COSF%PHI#  
X #RA\*COSF%PHI#\*COSF%THETA#

Y #RA\*COSF%PHI#\*SINF%THETA#

Z # RA\*SINF%PHI<sup>o</sup>  
RA1 # ALT & R  
PRINT 4999,IPAGE  
PRINT 5000  
PRINT 5001  
PRINT 5027  
PRINT 5002  
PRINT 5003  
PRINT 5004  
PRINT 5010  
PRINT 5011  
PRINT 5012  
PRINT 5013  
PRINT 5014  
PRINT 5015,DEL,BETA,THETA,PHI  
PRINT 5016,ALT,BALCO,RA,VO  
GAMMA # 0.  
2462 GRAV # %GO\*R\*\*2<sup>o</sup>/RA\*\*2  
IF %ALT - ALT900,901,901  
900 IF %CON15 - 1.03840,901,901  
3840 PRINT 5050  
5050 FORMAT %1HK,35X,6HZENITH<sup>o</sup>  
CON15 # 1.  
901 ALT # ALT \* 3.280833  
IF %ALT-400000.04200,4201,4201  
4200 IF %ALT - 40000.01,2,3  
1 D # ALT/10000.  
R3# -0.37728875E-02  
R2# 0.52352523E-02  
R1# -0.31047929E&00  
R0# 0.89444960E&01  
GO TO 50  
2 CONTINUE  
3 IF %ALT-80000.04,5,6  
4 CONTINUE  
5 D # %ALT-40000.0/10000.  
R3# 0.23298604E-05  
R2# 0.21620268E-03  
R1# -0.47974207E&00  
R0# 0.75402419E&01  
GO TO 50  
6 IF %ALT - 160000.07,8,9  
7 D # %ALT-80000.0/10000.  
R3# 0.26359811E-04  
R2# 0.80943560E-02  
R1# -0.51509227E&00  
R0# 0.56272847E&01  
GO TO 50  
8 CONTINUE  
9 IF %ALT-175000.010,11,12  
10 CONTINUE  
11 D # %ALT-160000.0/10000.  
R3# -0.57066660E-04

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R2# 0.30080000E-03
R1#-0.36384173E&00
R0# 0.20423758E&01
GO TO 50
12 IF %ALT-270000.□13,14,15
13 D # %ALT-175000.□/10000.
R3# -0.84014648E-03
R2# -0.16797257E-02
R1# -0.32931657E&00
R0# 0.15021266E&01
GO TO 50
14 CONTINUE
15 IF %ALT-290000.□16,17,18
16 CONTINUE
17 D # %ALT-270000.□/10000.
R3# 0.
R2# 0.36480009E-03
R1#-0.61376240E&00
R0#-0.25094864E&01
GO TO 50
18 IF %ALT-350000.□19,20,21
19 D # %ALT-290000.□/10000.
R3# 0.17070042E-02
R2# -0.16579218E-02
R1# -0.63898718E&00
R0# -0.37292327E&01
GO TO 50
20 CONTINUE
21 D # %ALT-350000.□/10000.
R3# -0.38550824E-02
R2# 0.65901945E-01
R1# -0.68705559E&00
R0# -0.72589563E&01
50 R0#%EXPF%%R3*D&R2□*D&R1□*D&R0□□/100000.
DEN #%R0*AK*RA**2□/R**2
ALT # ALT / 3.280833
7980AX#2.*VY*U&U**2*X-%AMU**2*X□/RA**3□-%DEN*GRAV *VX*VT□/%2.*BALCO
1□□
0AY#U**2*Y-2.*VX*U-%AMU**2*Y□/RA**3□-%DEN*GRAV *VY*VT□/%2.*BALCO
1□□
AZ#%-1.□*%%AMU**2*Z□/RA**3□&%DEN*GRAV *VZ*VT□/%2.*BALCO□□□
IF %CON8□4000,699,1569
699 CON8 # CON8 & 1.
GO TO 601
4201 DEN # 0.
ALT # ALT / 3.280833
GO TO 798
1569 IF %ALT - 45720.096□1434,1434,3104
1434 IF %AACC□4000,2460,2461
2460 AACC # AACC & 1.
GO TO 3104
2461 DELT #DELT2
3104 T # T & DELT

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```

422 THET1 # THETA
    BETA2 #BETA1
    BETA1 # BETA
    SIGMA1#SIGMA
    PHI1 # PHI
704 PHI # ATANF%Z/%SQRTE%RA**2-Z**2000
    THETA #ATANF%Y/X0
    DELRA # %RA - RA1#DELT
    DEL # ATANF%%DELRA#/%SQRTE%VT**2-DELRA**2000
421 DTHET#THETA - THET1
    GAMMA3# ATANF%%SQRTE%1.-%%SINF%PHI1#*SINF%PHI0#&%COSF%PHI1#*COSF%P
    1HI#*COSF%DTHET#000**200/%SINF%PHI0#*SINF%PHI#&COSF%PHI1#*COSF%PHI#
    2COSF%DTHET#000
    SIGMA # ATANF%%SINF%DTHET#*COSF%PHI1#/%SQRTE%%SINF%GAMMA3#00**2-%S
    1INF%DTHET#*COSF%PHI1#00**200
    IF %1.5688 - %ABSF%SIGMA#0004099,4160,4150
4099 IF %SIGMA1#4102,4101,4102
4101 DELSIG # .001
    GO TO 4120
4102 DELSIG # SIGMA -SIGMA1
4120 IF%BETA1-BETA2#4128,4130,4129
4128 BETA# BETA1-%ABSF%DELSIG#0
    GO TO 4300
4130 BETA# BETA1&.001
    GO TO 4300
4129 BETA# BETA1 &%ABSF%DELSIG#0
    GO TO 4300
4150 IF %.001-%ABSF%SIGMA#0004160,4160,4161
4161 IF %BETA - 1.0#4141,4165,4162
4141 IF %SIGMA#4166,4167,4167
4162 IF %BETA1 - 5.0#4165,4165,4141
4160 IF %SIGMA# 4163,4164,4164
4163 IF %BETA1-4.7123895#4165,4166,4166
4165 BETA #3.14159265-SIGMA
    GO TO 4300
4166 BETA #6.28318530 & SIGMA
    GO TO 4300
4164 IF %BETA1 - 1.57079632675#4167,4165,4165
4167 BETA # SIGMA
4300 GAMMA # ATANF%%SQRTE%1.-%%SINF%PHI0#*SINF%PHI#&%COSF%PHI0#*COSF%P
    1HI#*COSF%THETA-THET0#00**200/%SINF%PHI0#*SINF%PHI#&COSF%PHI0#*COSF%
    2%PHI#*COSF%THETA-THET0#000
870 RN #GAMMA * 3440.239
    VT1#VT
    RA1# RA
    ALT9 #ALT
601 CONTR #CONTR & 1.
    IF %CONTR -1.0#4000,607,604
604 IF %CONTR-9.0#602,602,606
606 CONTR #0.
    IPAGE #IPAGE & 1
    GO TO 601
607 PRINT 5017,IPAGE

```

```

602 AA1 #DEL * 57.2958
AA2 #BETA*57.2958
AA3 #THETA*57.2958
AA4 #PHI *57.2958
PRINT 5005,T,X,Y,Z
AA6 # SIGMA * 57.2958
AA7 # GAMMA3 * 57.2958
PRINT 5006,ALT,VX,VY,VZ
PRINT 5007,VT,AX,AY,AZ
PRINT 5008,AA2,RN,DEN,GRAV
IF %SENSE LIGHT 10 3000,3001
3000 PRINT 5028,AA4,AA3,AA1
GO TO 609
3001 PRINT 5025,AA4,AA3,AA1,AT
609 IF %ALT - 45720.0960 1940,1940,1941
1940 DELT #DELT2
1941 VX # VX & AX*DELT
    VY # VY & AY * DELT
    VZ # VZ & AZ * DELT
    X # X & VX * DELT
    Y # Y & VY * DELT
    Z # Z & VZ * DELT
    VT # SQRTF%VX**2 & VY**2 & VZ**2
    AT #VT-VT1/DELT
    RA # SQRTF%X**2 & Y**2 & Z**2
    ALT #RA - R
    IF %ALT 1202,1203,1203
1202 AABB # AABB & 1.
1203 IF %AABB-1.04000,2462,1204
1204 IF %AABB-2.02462,2463,8000
2463 DEN #.002376&%ALT/-5000.00*%.0027448-.0023769
GO TO 798
5028 FORMAT %1H ,2X,5HPHI ,E15.8,2X,5HTHETA,E15.8,2X,5HDELTA,E15.8
100 FORMAT %5E15.8
5010 FORMAT %1HK,25X,42HPhysical Constants - SAME FOR ALL MISSILES-
5011 FORMAT %1H ,25X,62HGravitational Acceleration at Sea Level 9.8066
15 Meters/Sec**2
5012 FORMAT %1H ,30X,44HEarth Rotation 2PI/24 X 3600 Radians/Second
5013 FORMAT %1H ,30X,33HRADIUS OF EARTH 6,371,230 METERS
5014 FORMAT %1HK,5X,12HOTHER INPUTS
5015 FORMAT %1H ,5HDEL ,E15.8,2X,5HBETA ,E15.8,2X,5HTHETA,E15.8,2X
1,5HPHI ,E15.8
5016 FORMAT %1H ,5HALT ,E15.8,2X,5HBALCO,E15.8,2X,5HRA ,E15.8,2X
1,5HV0 ,E15.8
5017 FORMAT %1H1,79X,5HPAGE ,I4
8000 PRINT 8C01
8001 FORMAT %1H1,/////////20X,32H*****END OF JOB*****//1H1
4999 FORMAT %1H1,32X,31HBrown Engineering Company, Inc.,4X,5HPAGE ,I4
5000 FORMAT %1H ,30X,35HFree Flight and Re-Entry Trajectory
5C01 FORMAT %1H ,35X,24HProgrammed by W.B.WARREN
5027 FORMAT %1H ,30X,35HTheoretical Derivation - C.F.OSTNER
5002 FORMAT %1H ,38X,18H Research Staff
5003 FORMAT %1H ,39X,16H10 NOVEMBER 1962

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CARD TO PRINT 80/80

5004 FORMAT %1HK,39X,31HPROGRAM NO. SP 9 METRIC UNITS.  
50050FORMAT %1HK,2X,5HTIME ,E15.8,2X,5HX ,E15.8,2X,5HY ,E15.8,2X,  
15HZ ,E15.8□  
5006 FORMAT %1H ,2X,5SHALT ,E15.8,2X,5HVX ,E15.8,2X,5HVY ,E15.8,2X,  
15HVZ ,E15.8□  
5007 FORMAT %1H ,2X,5HVT ,E15.8,2X,5HAX ,E15.8,2X,5HAY ,E15.8,2X,  
15HAZ ,E15.8□  
5008 FORMAT %1H ,2X,5HBETA ,E15.8,2X,5HRANGE,E15.8,2X,5HDEN ,E15.8,2X,  
15HGRAV ,E15.8□  
5025 FORMAT %1H ,2X,5HPHI ,E15.8,2X,5HTHETA,E15.8,2X,5HDELTA,E15.8,2X,  
15HACCEL,E15.8□  
4000 STOP  
END

## REFERENCES

1. Kooy, Uytenbogaart, Ballistics of the Future, N. V. Detechnische Uitgeverij H. Stam.
2. Pope, Bennie E., "Body Dynamics During Re-Entry", Brown Engineering Company, Inc. Technical Note R-5A, (Unclassified Version), August 28, 1962.
3. Minshew, H. M., "A FORTRAN Program to Calculate Atmospheric Properties", Brown Engineering Company, Inc. Technical Note R-27, October, 1962.