Operations Research Letters 63 (2025) 107360

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

Factor based forecasts in universal portfolios via Dirichlet weights

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ARTICLE INFO

Keywords:
Universal portfolio
Online portfolio selection
Dirichlet portfolio
Robo advisor
Factor models
Asset allocation

ABSTRACT

We revisit the online portfolio allocation problem that seeks to maximize the long term portfolio growth rate. We propose a new technique that modifies the concentration parameter of the Dirichlet distribution to incorporate cross-sectional return forecasts into the universal portfolio. We analytically establish that under certain conditions, the wealth generated by the factor Dirichlet portfolio dominates that generated by its uniform Dirichlet counterpart. We corroborate our analytical results with empirical studies on equity markets.

1. Introduction

Portfolio selection is a well-studied problem across finance, economics, operations research and computer science. While there are many lenses to viewing this problem, two major approaches have been considered in the past: offline single or finite multi-period optimization and growth-optimal online portfolio selection.

In offline single or finite multi-period optimization, the focus is on how one can come up with a set of weights for assets that satisfy certain global criteria for a given period of investment. These criteria may include return maximization, risk minimization, portfolio diversification, minimizing worst-case drawdowns, maximizing Sharpe or Sortino ratios, to name a few. Individual assets may have constraints on how much we can invest in them. There may be restrictions or allowances for taking short positions. The problem may be subject to various conditions of information availability – for example, the expected returns for the periods evaluated and the covariance matrix for the assets that are eligible may or may not be available.

In growth-optimal online portfolio selection, the focus is on online algorithms for arriving at the best average growth rate for the portfolio over an infinite horizon. As before, various sources of information could be present here, including but not limited to the financial information or covariance matrix for the assets and in some cases return estimates.

The first line of research is more suited to problems with a finite investment horizon, while the second fits the case where the horizon is infinite. This work focuses on the second approach to maximize the growth rate of the portfolio in the infinite horizon and its applications to long-term investment management problems.

The crux of the online portfolio selection problem lies in being able to tweak the allocation as and when new information is available to achieve certain well-defined investment goals over a sufficiently long time period. Recent reductions in trading costs in global markets and the emergence of longer market hours have brought increased attention to this problem, as these features make online portfolio changes less expensive and more convenient. This problem has ubiquitous appeal: To a market maker who wants to manage her inventory through the trading day and to a pension fund manager who plans asset allocation over many years.

The classic solution to the problem of maximization of the growth rate of the online portfolio was given by Cover's Universal Portfolio [5], in which it was shown that a performance-weighted combination of assets can beat the best constant rebalanced portfolio in the long run. Cover named this a 'Universal Portfolio' as it needed no look-ahead information, and it was theoretically guaranteed to do better than a baseline portfolio under all conditions. In continuation of the original work, Cover and Ordentlich [7] showed general results for the performance of the online portfolio in the presence of extra information. However, none of these solutions explore any structural information models for the market. For example, assets may be highly correlated, or only a few common factors could drive them. Can the online allocation methods be modified to use this information? These additional questions arise from the nature of equity and commodity markets, as these markets are often driven by sentiment, news, and common factors that describe market evolution in a sparse fashion. Specifically, we address the question of how this type of information can be effectively leveraged to our benefit.

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https://doi.org/10.1016/j.orl.2025.107360
Received 11 October 2024; Received in revised form 22 May 2025; Accepted 22 August 2025

Available online 3 September 2025

Cover and Ordentlich [7] proposed to use a uniform Dirichlet distribution to generate portfolio weights to compute the universal portfolio. Augmenting that approach, we use the concentration parameter of the Dirichlet distribution to harness information biases that reflect the underlying structure of the market. This structure could be a factor model or any other alpha forecast model. We show that using this information source to alter the concentration parameter of the Dirichlet distribution leads to another class of universal portfolios that have enhanced properties in terms of their long-term expected growth rate. We also analytically show the relative performance gain from this method.

We validate our approach using a few factors that are known to influence equity market returns. We use data spanning multiple years across two major indices to show that the portfolios that utilize Dirichlet factor weights show considerable out-performance over the plain universal portfolio in terms of the growth rate. In summary, our contributions are as follows:

- We propose a generic method of online asset allocation using the properties of the Dirichlet distribution to incorporate information from a factor model that drives asset returns.
- We show that such a portfolio theoretically dominates the universal portfolio in the long run, as described in the context of Cover [7].
- We illustrate the practical application of this method by testing it on two global equity portfolios, namely the constituents of the NASDAQ-100 and FTSE-100.

1.1. Literature review

Cover [5] shows that one can come up with an adapted strategy for online portfolio selection that asymptotically outperforms the best stock in the portfolio. One portfolio that achieves this is the performance-weighted average of all the possible portfolios in the m-1 simplex, where performance is defined as the cumulative return to date for the m assets. In other words, Cover provides a formulation to calculate the universal portfolio, which is equivalent to having an infinite number of portfolio managers on the m-1 simplex, and calculating a performance-weighted average portfolio as the universal portfolio.

Cover's [5] central result is that the exponential growth rate of this 'non-anticipating' fully adapted portfolio with no look-ahead bias asymptotically approaches the growth rate of the best constant weight re-balanced portfolio (best CRP) with the benefit of hindsight. The update in each step tilts towards the portfolios that have performed well so far and continuously tracks a weighted average portfolio. Note that as Cover [5] restricts the benchmark portfolio to constant proportional weight re-balanced portfolios; it is worth noting that although Cover's choice of a benchmark is reasonable within the context of what is presented, without this restriction of constant weight re-balancing, a portfolio with only the advantage of hindsight can achieve significantly higher returns as shown in Singer [16]. It is observed that while Cover's universal portfolio eventually surpasses the best stock in the long run, it may require significant time to establish itself as the front-runner. Modifications to Cover's portfolio that demonstrate faster empirical convergence have been proposed, including the work by Sullivan and Edelman [15], who appropriately name these 'adaptive portfolios'.

In subsequent work done as an extension to Cover [5], Cover and Ordentlich [7] find a tight error bound on the maximum deviation between the best constant re-balanced portfolio in hindsight and the one they arrive at using side information without the benefit of hindsight.

In related work, Helmbold et al. [10] show that exponential multiplicative updates yield almost the same performance as the universal portfolio and have lesser memory requirements. The authors use a utility function that trades off a performance measure with a distance measure. The performance is based on $\boldsymbol{x_t}$, the current relative price vector while the distance measure is chosen to be the relative entropy between the new weight vector and the current one. In various empirical tests, the algorithm achieves better out-of-sample performance than Cover's [5] portfolio and some other benchmarks.

Kozat and Singer [13] provide formulations for semi constant re-balanced portfolios where re-balancing does not happen every period. Kalai and Vempala [12] propose an alternate sampling method to simulate the universal portfolios – instead of a uniform sampling, they suggest a distribution that favors portfolios that perform well. The authors provably improve the exponential complexity of computing the universal portfolio. As a sharp contrast to the universal portfolio type idea where the best-performing assets are assumed to perform well in the future, some methods take a contrarian approach. Borodin et al. [4] show that a forecast step that uses mean-reversion can be used to get a desirable portfolio. There is also a line of work that explores meta-algorithms (MAs). Das and Bannerjee [8] use daily S&P500 data for the past 21 years to show that MAs outperform existing portfolio selection algorithms by several orders of magnitude. As a good summary of available work, Li et al. [14] have written a comprehensive review paper that describes many of the major techniques in this area.

In this work, we are the first to demonstrate a generic technique to incorporate asset-level cross-sectional forecasts into the universal portfolio scheme by leveraging the properties of the Dirichlet distribution. We show that under certain conditions, this new portfolio will asymptotically dominate Cover's uniform Dirichlet portfolio [7]. This lets us construct an online portfolio that combines Cover's insight with well-known factor effects in markets. More generally, our technique can be used in any market where the investor has a cross-sectional forecasting model for the asset universe of interest. Our work is closely related to [12] as instead of using the portfolio performance, we skew the distribution to weigh factors that are known to drive equity returns.

2. Constant re-balanced portfolios in factor-driven markets

We begin with a description of our setup and notations. The portfolio has m assets. A new price vector is revealed for each period. A long-only, fully invested portfolio is defined as a set of non-negative allocations (no shorting) to each asset such that the weights sum to one. The sequence of allocations that a trader comes up with as time goes by is called a strategy. In a real market scenario, the trader will have only information up to time k, including the k^{th} period revelation to decide the allocation $\boldsymbol{b_k}$. A constant re-balanced strategy is a strategy that uses a constant value of allocation $\boldsymbol{b_k} = \boldsymbol{b}$ through all the periods. For example, if the trader has chosen $\boldsymbol{b}$ as $(1/m, 1/m ... 1/m)$, she will start with equal wealth invested in each asset and, as time goes by, maintain the same investment in each asset.

2.1. Growth rate of multi-factor portfolios

In this section we derive the relationship between the growth of a constant re-balanced portfolio and the growth of the underlying factors when the portfolio constituents are assumed to follow a log-factor model. A stock market as modeled in Cover [6] is represented as a vector of price-relatives that are assumed to follow a distribution $F(x)$. Define X_i as the one-period price-relative for asset i. We set $X_i = p_i^{end}/p_i^{start}$ where p_i's are measured at the start and end of the time-period (single period model). The wealth generated by engaging $\boldsymbol{b}$ over the single period price-relatives is $S = \boldsymbol{b^t} X$. As the investment model involves re-investing of the capital every period, we are interested in the cumulative product of these single period gains and thereby characterize the expected logarithm of this gain [6] as:

$$W(b, F) = \int \log b^t x \, dF(x) = E\left[\log b^t X\right] \tag{1}$$

The optimal $\boldsymbol{b}^*$ that maximizes (1) also maximizes the doubling rate of the portfolio as shown in [6]. We now consider a k-factor log returns factor model of the market where known factors drive the growth of the market. Using this setup, we derive the relationship between the growth of a constant re-balanced portfolio as described above and the factor growth rates. For literature on factor-based models in general see [9] and [11] particularly for momentum factor-related effects. The factor model for asset j we use is:

$$R_{ji} = (r_{1i})^{\beta_j^1} (r_{2i})^{\beta_j^2} ..(r_{Ki})^{\beta_j^K} \quad j \in (1...m)$$

where r_{ki} are returns attributed to factor k and β_j^k denotes the factor exposure to the j^{th} asset, both in the time period i. The average growth rate of this portfolio is given by:

$$GR_{avg} = \frac{1}{n} E \sum_{i=1}^{n} \log(\sum_{j=1}^{m} b_j R_j)$$

$$GR_{avg} = \frac{1}{n} E \sum_{i=1}^{n} \log(\sum_{j=1}^{m} b_j r_{1i}^{\beta_j^1} r_{2i}^{\beta_j^2} ..r_{Ki}^{\beta_j^K})$$

By the AM-GM inequality and by the property that log is a monotone function:

$$GR_{avg} \geq \frac{1}{n} E \sum_{i=1}^{n} \log(m \prod_{j=1}^{m} (b_j r_{1i}^{\beta_j^1} r_{2i}^{\beta_j^2} ..r_{Ki}^{\beta_j^K})^{\frac{1}{m}})$$

Simplifying the RHS:

$$= \log m + \frac{1}{mn} E \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \log(b_j r_{1i}^{\beta_j^1} r_{2i}^{\beta_j^2} ..r_{Ki}^{\beta_j^K}) \right]$$

$$= \log m + \frac{1}{mn} E \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \left(\log b_j + \beta_j^1 \log r_{1i} + .. + \beta_j^K \log r_{Ki} \right) \right]$$

$$= \log m + \frac{1}{m} \sum_{j=1}^{m} \log b_j + \frac{1}{m} \sum_{j=1}^{m} \beta_j^1 E \left[\frac{1}{n} \sum_{i=1}^{n} \log(r_{1i}) \right]$$

$$+ .. + \frac{1}{m} \sum_{j=1}^{m} \beta_j^K E \left[\frac{1}{n} \sum_{i=1}^{n} \log(r_{Ki}) \right]$$

$$= \log m + \frac{1}{m} \sum_{j=1}^{m} \log b_j + \frac{1}{m} \sum_{j=1}^{m} \beta_j^1 G_1 + .. + \frac{1}{m} \sum_{j=1}^{m} \beta_j^K G_K$$

Finally we have:

$$GR_{avg} \geq \beta_1^{AM} G_1 + .. + \beta_K^{AM} G_K - \log \frac{1}{m\, b^{GM}}$$

where AM stands for arithmetic mean and GM stands for geometric mean and G_k is the buy and hold returns from factor k. The main insight from the result is that the growth rate of a portfolio that re-balances regularly can do better than the underlying factor growth rates, depending on the average β exposure of the assets underlying the portfolio and how diversified the portfolio is. The last term denotes a portfolio concentration cost term that vanishes for a $1/m$ diversified portfolio.

2.2. *Universality of Dirichlet factor portfolios under a single factor model*

Next, we present a modification of Cover's [5] universal portfolio selection process to incorporate economic information. We focus on the alpha parameter of the Dirichlet sampling on the unit simplex. Recall that alpha is a concentration parameter of the Dirichlet distribution, in the sense that an alpha that is uniform across assets leads to portfolios with no prior knowledge. On the other hand, an alpha where one component is large compared to the others would tend to bias the portfolio towards that asset. We propose using Dirichlet($\alpha = \beta$) distributions, where α is the Dirichlet concentration parameter and β are factor loadings of assets to create universal portfolios that perform better than the benchmark universal portfolio that Cover [5] suggests. To show that the portfolio is universal, recall that Cover and Ordentlich [7] in result (24) show that the total wealth generated by the Dirichlet $(1, 1, 1...1)$ distribution is:

$$W_{\mathbf{1}} = E_{F(x),b} \left[\int_{b \in D_{\mathbf{1}}} S_n(b, X^n)\, d\mu(b) \right]$$

where

$$S_n(b, X^n) = \prod_{i=1}^{n} b^t X_i$$

The method that we propose involves the same process, and samples repeatedly from a Dirichlet distribution ($\alpha = \beta$) that is driven by factor exposures that each asset has to the underlying factor. In that case, the wealth process is given by:

$$W_{\beta} = E_{F(x),b} \left[\int_{b \in D_{\beta}} S_n(b, X^n)\, d\mu(b) \right]$$

Keeping in mind that Cover and Ordentlich [7] show that the Dirichlet $(1, 1..., 1)$ process leads to a universal portfolio, we are interested in analyzing the properties of the ratio of these wealth processes. A ratio where the denominator is the naive Dirichlet wealth with no information and the numerator is the new informed Dirichlet wealth process with knowledge of the factor βs describes the relative performance of these portfolios:

$$R_n = \frac{E_{F(x),b} \left[\int_{b \in D_{\beta}} S_n(b, X^n)\, d\mu(b) \right]}{E_{F(x),b} \left[\int_{b \in D_{\mathbf{1}}} S_n(b, X^n)\, d\mu(b) \right]}$$

$$R_n = \frac{\int_{b \in D_{\beta}} E_{F(x),b} \left[S_n(b, X^n) \right]\, d\mu(b)}{\int_{b \in D_{\mathbf{1}}} E_{F(x),b} \left[S_n(b, X^n) \right]\, d\mu(b)}$$

$$R_n = \frac{\int_{b \in D_{\beta}} E_{F(x),b} \left[\prod_{i=1}^{n} b^t X_i \right]\, d\mu(b)}{\int_{b \in D_{\mathbf{1}}} E_{F(x),b} \left[\prod_{i=1}^{n} b^t X_i \right]\, d\mu(b)}$$

We proceed to show that this ratio converges to a number larger than one, which would show that the wealth gains from the Dirichlet($\beta_1, \beta_2, \beta_3 ... \beta_m$) in expectation dominate the gains from the Dirichlet(1,1,1...1) portfolio. The one-factor model between the asset returns and market returns is as follows in a single period:

$$\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_m \end{bmatrix} = \begin{bmatrix} \beta_1\, R_{mkt} \\ \beta_2\, R_{mkt} \\ . \\ . \\ \beta_m\, R_{mkt} \end{bmatrix} + \epsilon$$

where ϵ is mean zero gaussian noise vector. In this case only positive factor loadings are permitted. This leads to the ratio being evaluated as:

$$R_n = \frac{\int_{b \in D_{\beta}} E_{F(x),b} \left[\prod_{i=1}^{n} R_{mkt}^i \prod_{i=1}^{n} b_i^t \beta \right]\, d\mu(b)}{\int_{b \in D_{\mathbf{1}}} E_{F(x),b} \left[\prod_{i=1}^{n} R_{mkt}^i \prod_{i=1}^{n} b_i^t \beta \right]\, d\mu(b)}$$

$$R_n = \frac{\int_{b \in D_{\beta}} E_{F(x),b} \left[\prod_{i=1}^{n} R_{mkt}^i \right] \prod_{i=1}^{n} E_{F(x),b} \left[b_i^t \right] \beta\, d\mu(b)}{\int_{b \in D_{\mathbf{1}}} E_{F(x),b} \left[\prod_{i=1}^{n} R_{mkt}^i \right] \prod_{i=1}^{n} E_{F(x),b} \left[b_i^t \right] \beta\, d\mu(b)}$$

$$R_n = \frac{\int_{\Delta} E_{F(x),b^{\beta}} \left[\prod_{i=1}^{n} R_{mkt}^i \right] \prod_{i=1}^{n} E_{F(x),b^{\beta}} \left[b_i^{\beta} \right]^t \beta\, d\mu(b)}{\int_{\Delta} E_{F(x),b^1} \left[\prod_{i=1}^{n} R_{mkt}^i \right] \prod_{i=1}^{n} E_{F(x),b^1} \left[b_i^1 \right]^t \beta\, d\mu(b)}$$

$$R_n = \frac{\int_{\Delta} \left[m^n (\sum_{j=1}^{m} \beta_j^2)^n \right]\, d\mu(b)}{\int_{\Delta} \left[(\sum_{j=1}^{m} \beta_j)^{2n} \right]\, d\mu(b)}$$

where we use the expectation of the Dirichlet distribution to be the $\boldsymbol{\beta} / \sum \beta_i$ vector. We see this is a ratio of two integrals over the unit (nonnegative) simplex and the numerator dominates the denominator:

$$R_n = \left[\frac{m(\sum_{j=1}^{m} \beta_j^2)}{(\sum_{j=1}^{m} \beta_j)^2} \right]^n \geq 1 \tag{2}$$

By Cauchy-Schwartz R_n is greater than 1. We have shown that the new factor tilted portfolio is also universal.

2.3. Multi-factor portfolios

In the case of multi-factor portfolios, we present a proof by construction. We show that for any arbitrary factor, it is possible to construct a new portfolio from the original assets, such that in this new portfolio, we can dominate the uniform Dirichlet portfolio along that chosen factor. We consider the multi-factor model:

$$\begin{bmatrix} X_{1i} \\ X_{2i} \\ . \\ . \\ X_{mi} \end{bmatrix} = \begin{bmatrix} \beta_1^1 \, f_{1i} \\ \beta_1^2 \, f_{1i} \\ . \\ . \\ \beta_1^m \, f_{1i} \end{bmatrix} + \begin{bmatrix} \beta_2^1 \, f_{2i} \\ \beta_2^2 \, f_{2i} \\ . \\ . \\ \beta_2^m \, f_{2i} \end{bmatrix} + \dots + \begin{bmatrix} \beta_K^1 \, f_{Ki} \\ \beta_K^2 \, f_{Ki} \\ . \\ . \\ \beta_K^m \, f_{Ki} \end{bmatrix} + \epsilon$$

Let $\mathcal{A} = S_1, S_2, S_3, \dots, S_m$ be the current asset universe of m stocks. We wish to make a transformation of this asset space. We make two assumptions to this end:

- The factors $f_1, f_2 \dots f_K$ are tradable in the market.
- The factors can be shorted to create a factor neutral portfolio.

We show a transformation that lets us construct a portfolio that is exposed to only one factor at a time. Consider a new asset space $\mathcal{A}*$ as:

$$\mathcal{A}* = \begin{bmatrix} S_1 - \beta_1^1 f_1 - . - \beta_{k-1}^1 f_{k-1} - \beta_{k+1}^1 f_{k+1} - . - \beta_K^1 f_K \\ S_2 - \beta_1^2 f_1 - . - \beta_{k-1}^2 f_{k-1} - \beta_{k+1}^2 f_{k+1} - . - \beta_K^2 f_K \\ . \\ . \\ S_m - \beta_1^m f_1 - . - \beta_{k-1}^m f_{k-1} - \beta_{k+1}^m f_{k+1} - . - \beta_K^m f_K \end{bmatrix}$$

Where the new asset space takes short positions on all factors except factor k and is also tradable in the market as each of its linear components is tradable. By construction, the new assets in $\mathcal{A}*$ follow the single factor model:

$$\begin{bmatrix} X_1^* \\ X_2^* \\ . \\ . \\ X_m^* \end{bmatrix} = f_{ki} \, \boldsymbol{\beta_k} + \epsilon$$

Consequently the results from Section 2.2 hold and we can set $\beta = \beta_k$ to dominate the uniform Dirichlet portfolio in this new asset space. Hence we have shown that in the case of multi-factor markets, for any given factor, it is possible to construct a new asset universe where the Dirichlet factor portfolio is guaranteed to dominate the uniform Dirichlet portfolio asymptotically.

3. Empirical results: Dirichlet factor portfolios

We highlight the practical application of our results by showing three use cases across two separate equity market settings, the NASDAQ 100 and the FTSE 100. For both asset universes, we show that it is possible to construct Dirichlet portfolios with factors that are already known in the literature to impact the cross-section of returns.

One relevant factor is company size, as existing literature suggests that smaller firms tend to outperform their larger counterparts [9]. To approximate size in a straightforward manner, we use the traded price at any given time as an indicative measure. While this can be critiqued, it serves as a simple calculation keeping our information set within the pricing domain. Note that to get to a set of weights that can be used for the Dirichlet sampling, a few cross-sectional normalizations are also needed. As a starting example, we use the transformations shown in Algorithm 1 to obtain an alpha value for our Dirichlet simulation at every step.

Table 1
Factor based Dirichlet alpha.

	Factor Name	Dirichlet Alpha
1	Size	$\left[\frac{\frac{1}{p}}{min(\frac{1}{p})}) \right]$
2	Momentum	$\max \left\{ e^{w(Z(r),3)}, 1 \right\}$
3	Short-Term Reversion	$\max \left\{ e^{w(Z(-r[-9:]),3)}, 1 \right\}$

Algorithm 1 Calculating portfolio weights for SizeFactor based Dirichlet portfolio.

```
while t ≠ T do
    α_jt ← 1/p_jt
    α_jt ← α_jt / min(α_jt)
Require: Simulate N-Managers Dirichlet portfolios from Dirichlet(α = α_t)
Require: Calculate Universal Portfolios from factor-biased managers and up-
  date the wealth process
    w_jt ← ∫(Sbdb) / ∫(Sdb)
    t ← t + 1
end while
```

The result of this algorithm is a portfolio that shows favorable growth characteristics. We show the results for a few such factor portfolios in Fig. 1A for NASDAQ 100 constituents from June 2005 and April 2025. Results are repeated for FTSE 100 constituents in Fig. 1B.

3.1. NASDAQ 100 constituents

In this example we used a selection of 65 assets (listed cash equity) from the US stock market that are a subset of the NASDAQ 100 index constituents. Tickers that were part of the index as of April 25th, 2025 and had availability of daily price time-series data for at least 5,000 trading days were chosen as the asset universe. The tickers were retrieved from the NASDAQ web page [3]. on the same date, 102 tickers were filtered using the above criteria to yield 65 tickers with at least the minimum threshold of price history. Daily price data was downloaded from EOD HistoricalData's (a commercial service) end of day API [1].

Table 1 shows the computational functional form behind each of the factors shown in the charts. Given we have access only to price data for this study, we limit the experiment to price-based factors. For example, the momentum factor is calculated as a function of the cumulative return of the asset (r). In this case, we cross-sectionally z-score (Z) the cumulative return (r) to date, winsorize (w) the z-score between -3 and 3, exponentiate it, and then floor the result to ensure a minimum value of 1. As a contrast to long-term momentum, the short-term reversion factor uses a 9 day lookback to calculate an alpha factor that benefits from short term weaknesses in the recent returns. These back-of-the-envelope functions are meant to be easy to calculate and at the same time intend to mimic the factor target. They also serve the purpose of adhering to the allowed values for a Dirichlet distribution α parameter input. It is worth noting that these transformations are merely ways to mimic the β exposure and do not violate the linear factor models presented in Section 2.

The results are shown in Fig. 1A. Note that this back-test has an inherent look-ahead bias in that it uses assets that survive from June 2005 and April 2025. Hence the asset universe comprises of higher quality assets than the NASDAQ at the start of the period. Since we are only comparing the relative performance of the naive uniform-Dirichlet portfolio to the factor-based Dirichlet portfolios, this chart serves its purpose as a relative comparison and is not indicative of absolute performance. The back-test shows out-performance for all factor Dirichlet portfolios in terms of the growth rate of the portfolio.

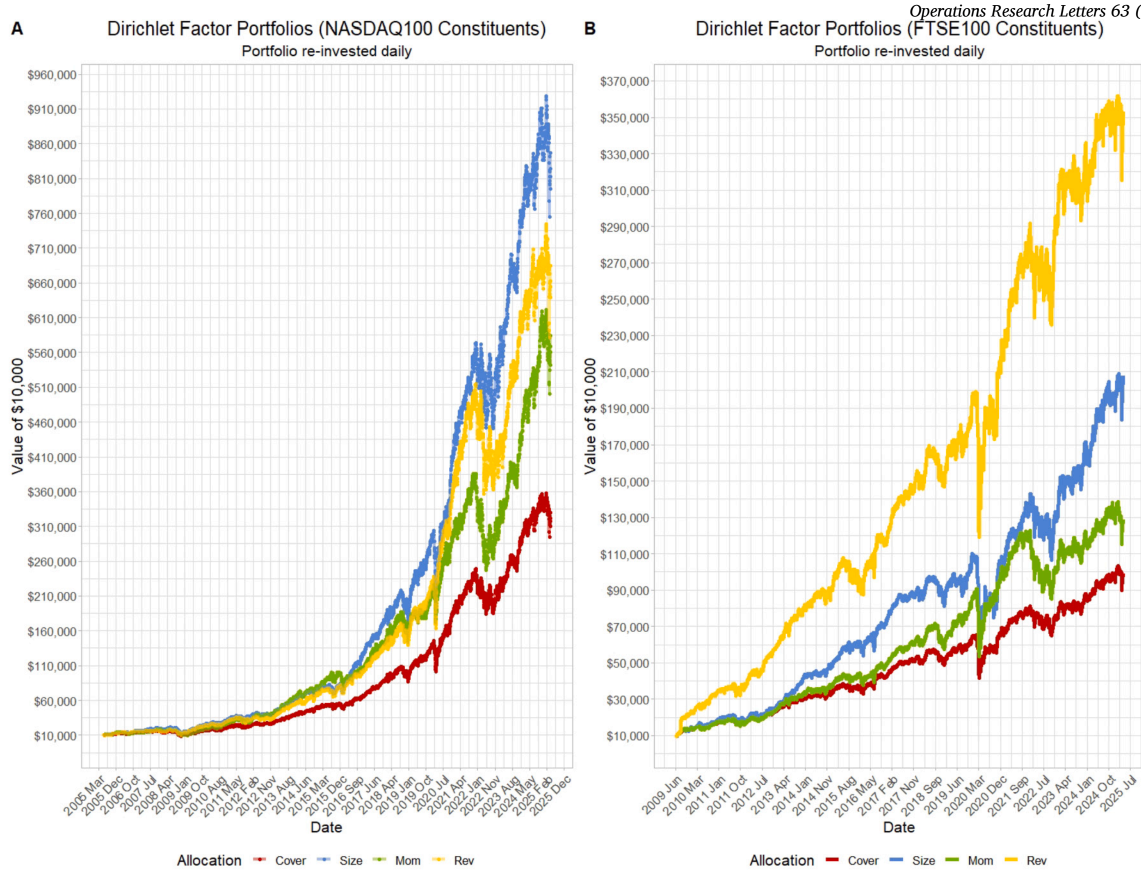

Fig. 1. Backtest results for NASDAQ100 (Chart A) and FTSE100 (Chart B) constituents with various Dirichlet factors. Mom and Rev stand for the Momentum and Short-term Reversion portfolio respectively.

3.2. FTSE 100 constituents

Now we use a selection of 80 cash equity assets from the stocks listed on the London Stock Exchange (LSE) and are part of the FTSE 100. The criteria for selection were the same as in the previous example with NASDAQ 100 stocks. Tickers that were part of the FTSE index as of April 25 2025 and had availability of daily price time-series data for 4,000 trading days as of that date were chosen as the asset universe. In the FTSE dataset, the size effects are quite large. To control for this effect and not let it influence our results too much, we remove assets with a minimum price of less than $1 through the period considered. Only four such assets was removed from the dataset in this case. The tickers were retrieved from the LSE web page [2]. Trading conditions and factors are the same as described in the section on NASDAQ constituents. Similar outsize performance is observed for factor Dirichlet portfolios, with both the momentum and short-term reversion based Dirichlet portfolios dominating the Cover allocation (Fig. 1B).

4. Conclusion

In summary, we have introduced a generative process designed to construct robust, diversified universal portfolios that achieve superior performance compared to the naive uniform Dirichlet universal portfolio allocation. We have analytically shown that the portfolios are universal and that they dominate portfolios that arise out of uniform Dirichlet sampling. Applications of this technique have been demonstrated on real market data from the U.S. and European equities markets. We believe that this technique can be used by small and large investment managers and robo-advisors for long-term investment gain, and by algorithmic traders to set target portfolios for short-term incremental alpha generation. Further work could involve online learning of the parameter for Dirichlet inputs based on the recent performance of factors in the market, and being able to blend them in composite portfolios. Another line of work could include blending price and non-price factors to create mixture portfolios that outperform.

CRediT authorship contribution statement

Purushottam Parthasarathy: Writing – review & editing, Visualization, Supervision, Methodology, Formal analysis, Conceptualization, Writing – original draft, Validation, Software, Investigation, Data curation. **Avinash Bhardwaj:** Validation, Resources, Writing – review & editing, Supervision, Funding acquisition. **Manjesh K. Hanawal:** Writing – review & editing, Supervision, Project administration, Validation, Resources.

References

[1] EOD Historical Data, https://eodhistoricaldata.com. (Accessed 25 April 2025).
[2] FTSE100, https://www.londonstockexchange.com/indices/ftse-100/constituents/table. (Accessed 25 April 2025).
[3] NASDAQ100, https://www.nasdaq.com/market-activity/quotes/nasdaq-ndx-index. (Accessed 25 April 2025).
[4] Allan Borodin, Ran El-Yaniv, Vincent Gogan, Can we learn to beat the best stock, Adv. Neural Inf. Process. Syst. 16 (2003).
[5] Thomas M. Cover, Universal portfolios, Math. Finance 1 (1) (1991) 1–29.
[6] Thomas M. Cover, Elements of Information Theory, John Wiley & Sons, 1999.
[7] Thomas M. Cover, Erik Ordentlich, Universal portfolios with side information, IEEE Trans. Inf. Theory 42 (2) (1996) 348–363.
[8] Puja Das, Arindam Banerjee, Meta optimization and its application to portfolio selection, in: Proceedings of the 17th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2011, pp. 1163–1171.
[9] Eugene F. Fama, Kenneth R. French, Common risk factors in the returns on stocks and bonds, J. Financ. Econ. 33 (1) (1993) 3–56.
[10] David P. Helmbold, Robert E. Schapire, Yoram Singer, Manfred K. Warmuth, On-line portfolio selection using multiplicative updates, Math. Finance 8 (4) (1998) 325–347.

[11] Jegadeesh Narasimhan, Sheridan Titman, Returns to buying winners and selling losers: implications for stock market efficiency, J. Finance 48 (1) (1993) 65–91.

[12] Adam Tauman Kalai, Santosh Vempala, Efficient algorithms for universal portfolios, J. Mach. Learn. Res. (2002) 423–440.

[13] Suleyman S. Kozat, Andrew C. Singer, Universal semiconstant rebalanced portfolios, Math. Finance 21 (2) (2011) 293–311.

[14] Bin Li, Steven C.H. Hoi, Online portfolio selection: a survey, ACM Comput. Surv. 46 (3) (2014) 1–36.

[15] Patrick O'Sullivan, David Edelman, Adaptive universal portfolios, Eur. J. Finance 21 (4) (2015) 337–351.

[16] Yoram Singer, Switching portfolios, Int. J. Neural Syst. 8 (04) (1997) 445–455.