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Decentralized Online Portfolio Selection with Transaction Costs

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Abstract
Online portfolio selection provides sequential decision-making on asset allocation without estimating the distribution function of asset return in advance, which is suitable for complex financial market. However, transaction costs generated during the portfolio update process have a significant impact on the return of investment strategy. In addition, some existing online portfolio models are prone to centralized investment. Thus, this paper firstly proposes a decision-making framework aiming to balance transaction costs and investment returns by using L1 norm (i.e., the Manhattan Distance) as the regularization term of the objective function. Second, a framework of decentralized online portfolio with transaction costs (DTC) is provided by taking the entropy of the portfolio as a constraint. Third, using two approaches of predicting asset return, two strategies of DTC1 and DTC2 are proposed correspondingly. Lastly, the performance of DTC1 and DTC2 is verified by using historical data from financial market. The results show that DTC1 and DTC2 can effectively deal with reasonable transaction costs and improve related strategies in the case of non-zero transaction costs.

Keywords Online portfolio selection · Decentralized investment · Entropy constraint · Transaction costs · Asset return prediction

1 Introduction

In 1952, Markowitz (1952) proposed the mean-variance model, which pioneered the modern investment portfolio. Many scholars have carried out research on this basis and achieved rich research results. The mean-variance model usually assumes that the returns of risky assets follow a certain statistical distribution. Under this assumption, the mean and variance of returns are calculated to measure the expected return and risk respectively. In this way, investors can allocate different investment weights to achieve

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excess returns or avoid investment risks. In contrast, online portfolio selection does not pre-estimate the distribution functions (Cover, 1991), but more often uses artificial intelligence techniques to predict future returns of risky assets and design optimal investment strategies.

The backtest on historical trading data shows that the reversal online portfolio strategies based on reversal effect perform well in return (Li et al., 2012, 2015; Lai et al., 2020; Huang et al., 2016, 2018; Cai & Ye, 2019). This type of strategy is proposed based on the "mean reversion" effect of the stock market, which holds that stock prices tend to average out in the long run. Although reversal strategies perform well on many datasets, most of them do not consider transaction costs in the decision-making process. As a result, they often perform poorly in the test of non-zero transaction costs. Later, Guo et al. (2021) improved the price prediction method of Online Moving Average Reversion (OLMAR) strategy (Li et al., 2015) and proposed the Adaptive Online Net Profit Maximization (AOLNPM) strategy considering transaction costs. Although transaction costs are taken into account in the portfolio model construction of AOLNPM strategy, it is still a return-maximizing model in essence, and this point is the same as OLMAR strategy. Therefore, both the AOLNPM strategy and the OLMAR strategy tend to concentrate on a small number of outstanding performing assets when making portfolio investments, which can still bring greater risks. As a matter of fact, the investment weight adjustment of the portfolio in each period is large. Once the tracking error occurs, its competitive performance will be greatly reduced.

In order to overcome these shortcomings, this paper firstly proposes a framework for dealing with transaction costs by adding the L1 norm (i.e., the Manhattan Distance) of the investment weight adjustment to the objective function as a regularization term. Secondly, to reduce or disperse the risk of investment portfolio, we learn from the idea of entropy and take the entropy of the portfolio as a constraint in the model construction. Based on this, we design a decentralized online portfolio framework with transaction costs (DTC). Thirdly, we use the methods of moving average and adaptive moving average to predict the stock price, and propose DTC1 and DTC2 strategies respectively by combining the predicted values with the DTC framework. Finally, we use the trading data of the stock market to conduct numerical analysis, and verify the effectiveness and feasibility of our DTC1 and DTC2 strategies, which shows that the strategies have excellent performance compared with other existing related strategies.

The main structure of the rest of this paper is as follows. Section 2 introduces the existing related studies. Section 3 uses L1 norm and entropy constraint to propose a diversified online portfolio framework considering transaction costs, and then designs two online portfolio strategies, that is, DTC1 and DTC2. Section 4 carries out extensive numerical experiments. Section 5 concludes this paper.

2 Related Works

In order to clearly review the existing online portfolio selection studies, we would like to summarize them into five types. The first type is often adopted as the baseline, which is called "Benchmark". The most classic benchmark is the Market strategy, which requires investors to evenly invest their funds in each asset at the beginning of the investment and not to adjust their positions thereafter (Li & Hoi, 2014). Another benchmark is called the "Best stock", which is to choose the best asset to invest at the end of the entire investment period (Li & Hoi, 2014). Moreover, Constant Rebalanced Portfolios (CRP) are adjusted each period to keep the asset position unchanged (Cover, 1991). The Best Constant Rebalanced Portfolio (BCRP) is the optimal CRP strategy to solve the maximum cumulative return problem.

The second type is called momentum strategy, which believes that assets that have risen in price today will still do well in the future, so investors should increase the investment weight in these assets. The Universal Portfolio (UP) strategy (Cover, 1991) allocates funds to many CRP strategies and gives greater weight to those with better historical performance. The Exponential Gradient (EG) strategy (Helmbold et al., 1998) tracks the stocks with the best performance in the previous period, and introduces relative entropy to maintain the information in the previous period. Most existing momentum strategies are improved on the basis of UP and EG strategies. For example, Zhang et al. (2022) updates the learning rate of the EG strategy by maximizing the recent cumulative return using the price data in a window. Both Yang et al. (2020) and Zhang et al. (2023) adopted multiple Exponential Gradient (EG(η)) strategies with different values of parameter η as experts, and they used the Weak Aggregating Algorithm to aggregate the recommendations of finite and infinite experts, respectively, to determine the next portfolio. Therefore, UP and EG are often compared as classic momentum strategies.

The third type is the pattern matching strategy, which searches for similar samples from historical data in a specific time window to optimize the current portfolio. Lai et al. (2018) proposed the Kernel based Trend Pattern Tracking strategy, which uses the kernel function to measure the similarity between the current asset allocation and the predicted price relatives. Khedmati and Azin (2020) used four clustering algorithms to match the historical time window similar to the recent time window, and designed an optimal function considering transaction costs to adapt to more complex markets.

The fourth type is called reversal strategy, which assumes that the price of a stock that does well in the current period will decline in the next period. It is believed that the stock price will tend to average out in the long run. The research direction of reversal strategy can be divided into two categories. One is to construct the weight transfer function and then design the online portfolio strategy. The other needs to make stock price forecast first, and then construct the update function of investment weight. Both Anti-correlation (AC) strategy (Borodin et al., 2004) and Confidence Weight Mean Reversion (CWMR) strategy (Li et al., 2013) build weight transfer models through stock price correlation analysis. Thereafter, Chu et al. (2019) conducted research on the basis of AC strategy and incorporated Kalman filter and wavelet analysis into the model. Passive Aggressive Mean Reversion (PAMR) (Li et al., 2012) and OLMAR (Li et al., 2015) strategies need to predict stock prices first. Then they use a famous online learning algorithm, namely passive aggressive learning algorithm, to achieve mean reversion. Lai et al. (2020) automatically assigned separate weights to different assets according to the performance of each asset, which improves the performance of OLMAR strategy. Guan and An (2019) combined the linear function and L2 norm into the OLMAR model to establish a locally adaptive online portfolio system. Huang et al. (2016) used the median to estimate the price relatives of future stocks and proposed the Robust Median Reversion (RMR) strategy. Later, Huang et al. (2018) used combination forecasting methods to improve the accuracy of stock price forecasting. In addition, to enhance temporal heterogeneity, Cai and Ye (2019) used Gaussian weighting function to weight historical stock price data in the moving window. The numerical analysis of these strategies shows that they can obtain huge excess returns.

The fifth type is to take transaction costs into account. In the process of multi-period investment portfolio, the investment proportion will be constantly adjusted, which will result in a series of transaction costs. This will be an important factor affecting investment decisions. Therefore, many scholars have incorporated transaction costs into online portfolios for further study. Kozat and Singer (2011) gave the Semiconstant Rebalanced Portfolios (SRP) strategy with transaction costs, and proved that the wealth realized by this strategy is asymptotic to the wealth of the best SRP strategy. For the reversal strategy, some scholars have carried out extended research combined with transaction costs. Most of these studies constructed the objective function by maximizing expected returns and adding transaction costs as adjustment terms (Li et al., 2018; Guo et al., 2021). The numerical experiment results show that this can indeed reduce the sensitivity of the reversal strategy to transaction costs.

However, the real reason for the huge transaction costs of reversal strategy is that this kind of strategy tends to track the best performing stocks, which will lead to a large adjustment of the portfolio weight in each period. This kind of strategy tends to underperform when the best stocks being tracked go wrong. Diversification can help to solve this problem, which can help investors avoid the risk of overheating of a single stock and stabilize the return of the investment portfolio. Therefore, simply considering transaction costs optimization cannot effectively solve this problem. What we need to do further is to design decentralized investment strategy. Philippatos and Wilson (1972) first introduced entropy into the investment decision-making process. They used entropy to replace the uncertainty contained in the portfolio measured by variance. Therefore, by adding L1 regularization term and entropy constraint into the online portfolio selection, we propose a DTC framework in this paper. On this basis, we design DTC1 and DTC2 strategies. The feasibility and effectiveness of the strategy are verified by numerical analysis using historical data of different stock markets.

3 The DTC Strategy

3.1 Problem Setting

Suppose an investor invests in a security market with m stocks for n trading periods. Let the closing price vector of m stocks in period t be $\mathbf{p}_t = [p_t^1, p_t^2, \dots, p_t^m]^\top$, $t = 1, 2, \dots, n$, where p_t^i, $i = 1, 2, \dots, m$, represents the closing price of stock i in period t, and $\top$ represents transpose in linear algebra. In a trading period, the growth or decline of price is expressed by price relative, then the price relative vector of m stocks in period t is $\mathbf{x}_t = [x_t^1, x_t^2, \ldots, x_t^m]^\top$, where $x_t^i = \frac{p_t^i}{p_{t-1}^i}$. Obviously, $x_t^i \in \mathbb{R}_+^m$.

The portfolio vector is denoted as $\mathbf{b}_t = [b_t^1, b_t^2, \ldots, b_t^m]^\top$, where b_t^i represents the investment proportion of stock i in period t. At the beginning of period t, the investor obtains a new portfolio $\mathbf{b}_t$ for the coming trading period according to the portfolio strategy, and updates the portfolio $\tilde{\mathbf{b}}_{t-1}$ to $\mathbf{b}_t$ at the end of period $t-1$, where $\tilde{b}_{t-1}^i = \frac{b_{t-1}^i x_{t-1}^i}{\mathbf{b}_{t-1}^\top \mathbf{x}_{t-1}}$. Assuming that investors are self-financed and short selling is not allowed, there is $b_t^i \ge 0$ and $\sum_{i=1}^m b_t^i = 1$. The daily return in period t is $\mathbf{b}_t^\top \mathbf{x}_t$. At the end of the entire trading period, the final cumulative wealth is denoted as S_n. In this paper, the initial wealth is set as $S_0 = 1$, so that S_n is calculated as

$$S_n = S_0 \prod_{t=1}^n \left(\mathbf{b}_t^\top \mathbf{x}_t - c \| \mathbf{b}_t - \tilde{\mathbf{b}}_{t-1} \|_1 \right), \tag{1}$$

where c is the transaction cost rate.

Online portfolio selection is a dynamic decision-making process. The goal is to find a sequence of portfolio vectors that maximizes cumulative wealth and thus achieves asset preservation and appreciation. Many online portfolio models usually imply two basic assumptions: sufficient liquidity in the market and zero shock costs.

3.2 Strategy Framework

Transaction costs are one of the main concerns for investors. Ignoring transaction costs will lead to an ineffective portfolio. Transaction costs mainly come from the adjustment of the weights in the portfolio. The size of portfolio weight adjustment at the beginning of period t can be expressed as

$$Q_t = \sum_{i=1}^m |b_t^i - \tilde{b}_{t-1}^i|. \tag{2}$$

Most existing online portfolios aim to maximize expected returns. We now add the L1 norm of the portfolio weight adjustment as a regularization term into the objective function, which can better balance expected returns and transaction costs, and is more suitable for actual investment activities. Therefore, the online portfolio strategy after considering transaction costs can be transformed into the following optimization framework for solving b_t^i:

$$\begin{aligned} &\max \sum_{i=1}^m b_t^i \hat{x}_t^i - \lambda \sum_{i=1}^m |b_t^i - \tilde{b}_{t-1}^i| \\ &s.t. \begin{cases} \sum_{i=1}^m b_t^i = 1, \\ 0 \le b_t^i \le 1, i = 1, 2, \ldots, m. \end{cases} \end{aligned} \tag{3}$$

In the above optimization framework (3), $\hat{x}_t^i$ is the predicted value of the price relative of stock i in period t; λ is the trade-off parameter and the larger λ is, the more the strategy pays attention to transaction costs.

We found that most reversal portfolio strategies tend to invest all of their money in a single or a few stocks, resulting in a strategy that is more risky in reality. In this paper, after incorporating transaction costs into the optimization problem, we further take decentralized investment into account in the model, which can help to reduce the risk of the strategy. In traditional investment portfolios, the entropy E_n of the investment portfolio is often used to measure the degree of decentralization, which is expressed as:

$$E_n(\mathbf{b}_t) = -\sum\nolimits_{i=1}^{m} b_t^i \ln b_t^i. \tag{4}$$

It is clear that E_n achieves the maximum value when the investment is uniform while E_n achieves the minimum value when the investor invests all of his money in one stock. Then in this study, in addition to optimizing the transaction costs, the portfolio entropy constraint is further incorporated into the optimization framework to increase the decentralization of the investment proportion, so as to reduce the risk. Therefore, our proposed online portfolio optimization framework considering both transaction costs and decentralization constraints is as follows.

$$\begin{aligned} &\max \sum\nolimits_{i=1}^{m} b_t^i \hat{x}_t^i - \lambda \sum\nolimits_{i=1}^{m} |b_t^i - \tilde{b}_{t-1}^i| \\ &s.t. \begin{cases} \sum_{i=1}^{m} b_t^i = 1, \\ 0 \le b_t^i \le 1, i = 1, 2, \dots, m, \\ E_n(\mathbf{b}_t) \ge \xi, \end{cases} \end{aligned} \tag{5}$$

where ξ is the threshold parameter, and the larger ξ is, the more decentralized the weights are in the portfolio. The optimization framework (5) is a nonlinear optimization problem, and an analytical solution cannot be obtained. Therefore, in the numerical example, the *fmincon* package of MATLAB will be used for numerical solutions.

3.3 Price Prediction

3.3.1 Exponential Smoothing Average

The DTC strategy decision-making process requires to predict the stock price relatives. Li et al. (2015) used the exponential smoothing average to predict the stock prices. Assuming that the stock price in the next period will reverse to the moving average of the stock price, the prediction formula of the stock closing price in the next period is:

$$\hat{p}_t^i = \alpha p_{t-1}^i + (1-\alpha)\hat{p}_{t-1}^i = \alpha p_{t-1}^i + \alpha(1-\alpha)p_{t-2}^i + \cdots + (1-\alpha)^{t-2} p_1^i, \tag{6}$$

where $0 \le \alpha \le 1$ is the decay factor, p_{t-1}^i is the closing price of stock i in period $t-1$, and $\hat{p}_t^i$ is the predicted closing price of stock i in period t. Thus, the predicted price relative of stock i in period t is $\hat{x}_t^i = \frac{\hat{p}_t^i}{p_{t-1}^i}$. Exponential smoothing average method not only uses all historical data, but also assigns a greater weight to recent observation data by adjusting the α, so that the predicted value can quickly reflect the actual changes in the market. It is a commonly used forecasting method. In addition, this paper adopts the same exponential smoothing average method as the OLMAR strategy (Li et al., 2015) to predict the price, which is more convenient to directly compare the performance differences in the two strategies, and can better demonstrate the superiority and effectiveness of the optimization model proposed in this paper. Therefore, this paper proposes the DTC1 strategy that applies the predicted value of stock price based on the above exponential smoothing average to optimization framework (5).

3.3.2 Adaptive Exponential Smoothing Average

The exponential smoothing average method described above is widely used to predict stock prices in online portfolio strategies, but it still has some limitations, that is, all risky assets share the same decay factor α, which is set to be constant over all investment periods. Due to the complexity and non-stationarity of the actual market environment, it may be better to adjust α as the market corresponding to different asset price forecasts changes. In order to improve the accuracy of stock prediction, Guo et al. (2021) designed an adaptive exponential smoothing average method to predict stock prices based on formula (6). The key to this method is that the decay factor α changes with the investment period and the stock. The idea of this method is as follows. Firstly, based on formula (6), the predicted value of the stock price relative can be obtained as

$$\hat{x}_t^i = \alpha_t^i + (1-\alpha_t^i)\frac{\hat{x}_{t-1}^i}{x_{t-1}^i}, \tag{7}$$

where α_t^i is the decay factor of stock i in period t. Thus, the prediction error of stock price relative is

$$x_t^i - \hat{x}_t^i = \left(x_t^i - \frac{\hat{x}_{t-1}^i}{x_{t-1}^i}\right) - \left(1 - \frac{\hat{x}_{t-1}^i}{x_{t-1}^i}\right)\alpha_t^i. \tag{8}$$

When x_t^i is known at the end of period t, the decay factor α_t^i of the next investment period needs to be considered in the following four cases:

Case 1. $x_t^i > \hat{x}_t^i$ and $x_{t-1}^i > \hat{x}_{t-1}^i$;
Case 2. $x_t^i > \hat{x}_t^i$ and $x_{t-1}^i \le \hat{x}_{t-1}^i$;
Case 3. $x_t^i \le \hat{x}_t^i$ and $x_{t-1}^i > \hat{x}_{t-1}^i$;
Case 4. $x_t^i \le \hat{x}_t^i$ and $x_{t-1}^i \le \hat{x}_{t-1}^i$.

According to (8), let

$$A = \left(x_t^i - \hat{x}_t^i\right). \tag{9}$$

$$B = \left(x_t^i - \frac{\hat{x}_{t-1}^i}{x_{t-1}^i} \right). \tag{10}$$

$$C = \left(1 - \frac{\hat{x}_{t-1}^i}{x_{t-1}^i} \right). \tag{11}$$

For Case 1, note that the prediction error in (8) is positive, i.e. $A > 0$, and B is a known fixed value that can be obtained in period $t + 1$, and $C > 0$. In this case, in order to reduce the prediction error, A should be made to approach 0, so the prediction error can be reduced by increasing the decay factor α_{t+1}^i of the next period. Then, the calculation formula of α_{t+1}^i is:

$$\alpha_{t+1}^i = \alpha_t^i + \gamma, \tag{12}$$

where $\gamma > 0$ is the step size of the adjustment.

For Case 2, $A > 0$, B is a known fixed value, and $C < 0$. In this case, in order for A to approach 0, we need to decrease the decay factor α_{t+1}^i, whose calculation formula is:

$$\alpha_{t+1}^i = \alpha_t^i - \gamma. \tag{13}$$

Similarly, for Case 3, the calculation formula of α_{t+1}^i is equation (13) while for Case 4, equation (12) is used to update α_{t+1}^i.

Then, this paper proposes the DTC2 strategy that applies the predicted value of stock price based on the above adaptive exponential smoothing average to optimization framework (5).

4 Numerical Experiments

In this section, we carry out extensive numerical experiments to analyze the performance of the proposed strategies DTC1 and DTC2 using indicators such as cumulative wealth and risk-adjusted returns. Meanwhile, we select relevant strategies to compare the results with respect to different indicators to demonstrate the effectiveness of our proposed strategies. We also conduct the parameter sensitivity analysis to test the robustness of the strategies DTC1 and DTC2.

4.1 Data and Parameter Setting

The datasets used in this section are SH50, CSI100, and SWYJ from the Chinese stock market, DJIA30 and NDX100 from the US stock market, and HSI66 from the Hong Kong Stock Exchange. The datasets collected from Chinese stock markets, among which SH50 dataset is from Shanghai Stock Exchange 50 index, CSI100 dataset is from China Securities 100 index, and the assets in SWYJ dataset are the industry index, which is from Shenwan Primary industry index. The stock data in DJIA30 dataset is from the Dow Jones Industrial Index, and the stock data in NDX100 dataset is from the Nasdaq 100 index. Table 1 provides detailed information of all the datasets used in this paper.

In the experiment, for DTC1 strategy, we set decay factor $\alpha = 0.5$ according to reference Li et al. (2015). For DTC2 strategy, according to reference Guo et al. (2021), the initial value of decay factor α_1^i is set to 0.5, and the step size γ is set to 10^{-5}. The trade-off parameter λ is set to 0.05, and threshold parameter ξ is set to 1. The sensitivity analysis of λ and ξ will be further carried out later.

4.2 Performance Metrics

In the experiment of this section, the following five indicators are used to measure the performance of each strategy (Li et al., 2012; Zhang et al., 2022).

- Cumulative Wealth: the total return achieved by a portfolio strategy over the entire trading period, is a standard indicator.
- Sharpe Ratio: reflects the excess return per unit of risk, and the larger the indicator, the better.
- Information Ratio: describes the risk-adjusted return from the perspective of active management, and the larger the indicator, the better.
- Maximum Drawdown: an important risk indicator that describes the maximum loss a portfolio could face, and the smaller the indicator, the better.
- Calmar Ratio: reflects the relationship between annualized percentage returns and the maximum drawdown, and the larger the indicator, the better.

4.3 Comparison Strategies

To analyze the performance of DTC1 and DTC2 strategies, this paper compares them with benchmark strategies and state of the art online strategies. The comparison strategies and corresponding parameter settings are described as follows:

- Market: Buy and hold strategy with equal investment weights.
- Best: Buy and hold the best stock in the market, which is a strategy of hindsight.
- BCRP: Best constant rebalanced portfolio strategy with hindsight.

Table 1 Summary of benchmark datasets for real markets

Dataset	Region	#Stocks	Time span	#Periods
SH50	CN	29	2020.3.1–2022.3.1	486
CSI100	CN	47	2020.3.1–2022.3.1	486
SWYJ	CN	28	2017.8.1–2021.8.1	974
DJIA30	US	28	2006.3.1–2022.3.1	4028
NDX100	US	61	2006.3.1–2022.3.1	4028
HSI66	HKG	42	2017.8.1–2022.3.1	1130

- UP: Universal Portfolio strategy is proposed by Cover (Cover, 1991) and the parameters are set as minimum coordinate $\delta_0 = 0.004$, spacing of grid $\delta = 0.005$, number of samples $m = 10$, and number of steps in random walk $S = 5$ according to the study of efficient implementation of the Universal Portfolio algorithm proposed by Kalai and Vempala (2002).
- EG: Exponential gradient strategy (Helmbold et al., 1998), the parameter of lerning rate is set as $\eta = 0.05$.
- CAEGc: Continuous Aggregating Exponential Gradient strategy with transaction costs (Zhang et al., 2023), the parameter of discrete expert index set is set as $\tilde{E} = \left\{\eta_{\min}, \eta_{\min} + 0.01, \ldots, \eta_{\max}\right\}$, where $\eta_{\min} = 0.01$ and $\eta_{\max} = 0.2$.
- AC: Anti-correlation strategy (Borodin et al., 2004), the parameter of window size is set as $W = 30$.
- PAMR: Passive Aggressive Mean Reversion strategy (Li et al., 2012), the sensitivity parameter is set as $\epsilon = 0.5$.
- RMR: Robust Median Reversion strategy (Huang et al., 2016), the sensitivity parameter is set as $\epsilon = 5$ and the window length parameter is set as $w = 5$.
- OLMAR: Online Moving Average Reversion strategy (Li et al., 2015), the reversion threshold parameter is set as $\epsilon = 10$ and the decaying factor parameter is set as $\alpha = 0.5$

4.4 Cumulative Wealth

Table 2 shows the cumulative wealth achieved by our proposed strategies DTC1, DTC2, and other ten related comparison strategies on each dataset under three different transaction costs. In this paper, we study the cumulative wealth of each strategy using the transaction cost rates $c = 0\%$, $c = 0.25\%$ and $c = 0.5\%$.

From Table 2, several conclusions can be drawn. Firstly, as transaction cost rates continue to increase, it can be seen that the cumulative wealth of different types of strategies changes differently. Secondly, the reversal strategies (AC, PAMR, OLMAR, and RMR) can achieve good cumulative wealth at zero transaction costs in most cases, but their cumulative wealth decreases significantly when considering transaction costs. For example, when the transaction cost rate reaches 0.25%, the cumulative wealth of OLMAR strategy on all datasets decreases very quickly, and when the transaction cost rate reaches 0.5%, the cumulative wealth of OLMAR strategy on all datasets approaches zero. The most likely reason is that these reversal strategies tend to invest money in a single stock, which is significantly affected by transaction costs. Thirdly, although the momentum strategies (UP and EG) may achieve less cumulative wealth at zero transaction costs than some reversal strategies, they are much less affected by transaction costs as transaction costs increase. Moreover, our proposed strategies (DTC1 and DTC2) not only achieve high cumulative wealth, but also perform robustly at different transaction cost rates and are less affected by transaction costs. It indicates that it is necessary to study the online portfolio selection strategy with non-zero transaction costs. It can be seen that the DTC framework proposed in this paper can synthesize the advantages of

Table 2 Cumulative wealth of each strategy on different datasets

Strategies	c	SH50	CSI100	SWYJ	DJIA30	NDX100	HSI66
	0%	4.58	5.16	2.62	77.23	100.70	12.60
Best	0.25%	4.58	5.16	2.61	77.13	100.57	12.57
	0.5%	4.56	5.14	2.60	76.84	100.19	12.54
	0%	4.58	5.16	2.62	150.79	554.94	12.60
BCRP	0.25%	4.58	5.16	2.61	144.47	512.67	12.57
	0.5%	4.56	5.14	2.60	127.03	404.19	12.54
	0%	1.35	1.60	**1.13**	8.62	21.59	1.76
Market	0.25%	1.35	1.59	**1.13**	8.61	21.56	1.76
	0.5%	1.35	1.59	1.12	8.58	21.48	1.75
	0%	1.30	1.67	1.08	9.92	37.26	1.58
UP	0.25%	1.28	1.65	1.05	8.62	30.90	1.43
	0.5%	1.22	1.53	0.95	5.71	18.25	1.31
	0%	1.31	1.67	1.08	9.93	37.22	1.58
EG	0.25%	1.30	1.65	1.07	9.50	35.14	1.53
	0.5%	1.26	1.60	1.05	8.33	29.59	1.47
	0%	1.31	1.67	1.08	10.12	37.91	1.58
CAEGc	0.25%	1.29	1.64	1.07	9.24	33.68	1.52
	0.50%	1.27	1.60	1.06	8.45	29.93	1.47
	0%	1.36	**3.25**	0.48	60.53	**9136.94**	1.17
AC	0.25%	1.10	**2.53**	0.34	11.32	1299.99	0.40
	0.5%	0.57	1.19	0.12	0.07	3.70	0.14
	0%	**1.64**	1.12	0.58	2.67	41.99	1.57
PAMR	0.25%	0.62	0.44	0.07	0.00	0.01	0.02
	0.5%	0.03	0.03	0.00	0.00	0.00	0.00
	0%	1.34	0.70	0.53	5.70	14.68	1.78
RMR	0.25%	0.61	0.32	0.11	0.01	0.03	0.05
	0.5%	0.06	0.03	0.00	0.00	0.00	0.00
	0%	1.24	0.71	0.55	4.38	53.08	1.54
OLMAR	0.25%	0.49	0.28	0.09	0.00	0.03	0.02
	0.5%	0.03	0.02	0.00	0.00	0.00	0.00
	0%	1.43	1.74	**1.13**	81.03	1923.35	1.87
DTC1	0.25%	**1.43**	1.74	**1.13**	75.49	**1412.51**	1.85
	0.5%	**1.42**	**1.73**	**1.13**	70.32	**1036.43**	1.82
	0%	1.42	1.74	**1.13**	**96.07**	1296.82	**1.92**
DTC2	0.25%	1.42	1.74	**1.13**	**89.74**	933.85	**1.91**
	0.5%	**1.42**	**1.73**	**1.13**	**83.82**	671.84	**1.90**

The top results under three transaction cost rates on each dataset (excluding the Best and BCRP strategies, which are in hindsight) are highlighted in **bold**

reversal strategy and momentum strategy. In the case of non-zero transaction costs, both DTC1 and DTC2 strategies can achieve good competitive performance.

There are two very likely reasons for the excellent performance of DTC1 and DTC2 strategies. On the one hand, transaction costs are taken into account in the decision-making process, aiming to maximize the net profit of the portfolio each period, which is closer to the actual investment process. On the other hand, the decentralization constraint is added to avoid concentrated investment in a single stock, reducing the risk. Therefore, from the perspective of cumulative wealth, the optimization algorithm obtained from (5) that considers both transaction costs and decentralization constraints can make good use of the advantages of reversal effect. In addition, DTC1 and DTC2 strategies can overcome the shortcomings of existing online portfolio strategies that do not perform well when considering transaction costs.

It is worth noting that the DTC2 strategy is actually an improvement of the DTC1 strategy. However, in the numerical tests in this section, the performance of the DTC2 strategy is not guaranteed to be completely better than that of the DTC1 strategy. The reason may be that DTC2 strategy does not set the step size γ dynamically in the decision-making process. For the parameter γ, Guo et al. (2021) looks for appropriate parameter values for different datasets. For simplicity, this paper adopts only a fixed parameter γ to verify the strong applicability of the proposed strategy DTC2. Thus, DTC2 performs slightly worse than DTC1 for some datasets.

4.5 Risk-Adjusted Return

The maximum drawdown is used to measure the risk of a strategy. The larger the value, the higher the risk of the strategy. Sharpe ratio, information ratio and Calmar ratio are the indicators to measure the risk-adjusted return of a strategy. The larger the value, the stronger the competitive performance of the strategy. The calculation for the above four indicators is based on reference Li et al. (2012). In this paper, we also set the risk-free return to 0.04. Tables 3, 4, 5 and 6 provide the experimental results of different strategies in terms of these four indicators. We divide each table into two parts, where Best and BCRP strategies are often referred to as hindsight strategies, and they are difficult to implement in actual investment activities. Other strategies are online strategies. In online portfolio research, when the strategy performance is only worse than Best and BCRP, we can still consider it has strong competitive performance.

To assess the risk, we calculate the maximum drawdown of each strategy, and the results are listed in Table 3. The results show that the Best and BCRP strategies have higher maximum drawdown on most datasets, which indicates that the Best and BCRP strategies can achieve excellent cumulative wealth, but both face greater risk.

The DTC1 and DTC2 strategies proposed in this paper are close to the smallest maximum drawdown risk on all datasets and have relatively robust performance, especially compared to the reversal strategies (i.e., AC, PAMR, RMR, and OLMAR). In addition, the DTC1 and DTC2 strategies have the smallest maximum drawdown on the SWYJ and NDX100 datasets, which demonstrates that DTC1 and DTC2 face lower risks than other strategies.

Table 3 Maximum drawdown on cumulative wealth

Maximum drawdown	SH50	CSI100	SWYJ	DJIA30	NDX100	HSI66
Best	0.4344	0.4732	0.3579	0.6087	0.9454	0.4094
BCRP	0.4344	0.4729	0.3579	0.6122	0.6788	0.4094
Market	0.1858	0.2200	**0.3706**	0.4793	0.5757	**0.2676**
UP	0.1595	**0.1884**	0.3736	**0.4521**	0.5288	0.2761
EG	0.1593	0.1903	0.3738	0.4528	0.5316	0.2755
CAEGc	**0.1588**	0.1902	0.3738	0.4529	0.5255	0.2754
AC	0.3680	0.4076	0.6723	0.7877	0.7842	0.6084
PAMR	0.4279	0.4234	0.5717	0.7694	0.7453	0.4610
RMR	0.2770	0.4377	0.6131	0.6894	0.6373	0.2774
OLMAR	0.4029	0.4545	0.5966	0.7460	0.7308	0.5392
DTC1	0.1694	0.1941	**0.3706**	0.5039	0.4836	0.2761
DTC2	0.1697	0.1950	**0.3706**	0.4960	**0.4575**	0.2702

The smallest results of the maximum drawdown on each dataset (excluding the Best and BCRP strategies, which are in hindsight) are highlighted in bold

Table 4 Sharpe ratios

Sharpe ratio	SH50	CSI100	SWYJ	DJIA30	NDX100	HSI66
Best	2.1150	1.8898	0.7981	0.8416	0.1461	1.7172
BCRP	2.1150	1.8913	0.7981	0.6841	0.3852	1.7172
Market	0.5772	0.8959	−0.0438	0.5143	0.4121	0.5371
UP	0.5304	1.0776	−0.1000	0.5506	0.4492	0.4393
EG	0.5414	1.0673	−0.0952	0.5540	0.4499	0.4389
CAEGc	0.5427	1.0680	−0.0962	**0.5587**	0.4536	0.4360
AC	0.2583	**1.2611**	−0.4621	0.2924	**0.4834**	−0.0015
PAMR	0.5767	0.0440	−0.6033	0.0409	0.3117	0.2062
RMR	0.2755	−0.4319	−0.6655	0.1944	0.2072	0.2514
OLMAR	0.1714	−0.3956	−0.6310	0.1426	0.2357	0.1575
DTC1	**0.7566**	1.1851	−0.0420	0.3635	0.3405	0.6714
DTC2	0.7565	1.1849	−0.0419	0.3729	0.3025	**0.6991**

The top results of Sharpe ratio on each dataset (excluding the Best and BCRP strategies, which are in hindsight) are highlighted in bold

To further evaluate risk-adjusted returns, Tables 4, 5 and 6 show the Sharpe ratio, Calmar ratio and information ratio for each strategy without transaction costs. The results show that in most cases, DTC1 and DTC2 strategies are consistently at the forefront of these three indicators and are the two prominent strategies among online portfolio strategies. For example, compared with the OLMAR strategy in the reversal strategies, the Sharpe ratio, Calmar ratio and information ratio of DTC1 and DTC2 strategies on all datasets are higher than those of OLMAR strategy, showing obvious advantages. It is noted that the price forecasting method of the DTC framework is similar to that of OLMAR strategy, but the biggest difference lies

Table 5 Calmar ratios

Calmar ratio	SH50	CSI100	SWYJ	DJIA30	NDX100	HSI66
Best	2.6254	2.6891	0.7595	0.5129	0.3534	2.1594
BCRP	2.6254	2.6910	0.7595	0.6015	0.7135	2.1594
Market	0.8819	1.1973	0.0832	0.3006	0.3677	0.5676
UP	0.8897	1.5668	0.0521	0.3391	0.4807	0.4384
EG	0.9067	1.5417	0.0547	0.3406	0.4772	0.4401
CAEGc	0.9115	1.5433	0.0542	0.3436	0.4854	0.4384
AC	0.4572	**1.9691**	−0.2486	0.3711	0.9797	0.0642
PAMR	0.6519	0.1372	−0.2235	0.0822	0.3530	0.2605
RMR	0.5685	−0.3784	−0.2405	0.1667	0.2869	0.2734
OLMAR	0.2871	−0.3514	−0.2343	0.1297	0.3856	0.2101
DTC1	1.1211	1.6448	**0.0843**	0.6612	**1.1704**	0.6163
DTC2	**1.1365**	1.6985	0.0825	**0.6985**	1.0356	**0.6538**

The top results of Calmar ratio on each dataset (excluding the Best and BCRP strategies, which are in hindsight) are highlighted in bold

Table 6 Information ratios

Information ratio	SH50	CSI100	SWYJ	DJIA30	NDX100	HSI66
Best	0.1114	0.0861	0.0731	0.0464	0.0220	0.0858
BCRP	0.1114	0.0861	0.0731	0.0394	0.0296	0.0858
Market	–	–	–	–	–	–
UP	−0.0262	0.0371	−0.0362	0.0099	0.0239	−0.0265
EG	−0.0245	0.0366	−0.0347	0.0113	0.0245	−0.0273
CAEGc	−0.0245	0.0363	−0.0352	0.0129	0.0250	−0.0275
AC	0.0192	**0.0773**	−0.0328	**0.0285**	**0.0397**	0.0096
PAMR	0.0296	−0.0257	−0.0552	0.0021	0.0212	0.0060
RMR	0.0110	−0.0548	−0.0607	0.0058	0.0098	−0.0274
OLMAR	0.0050	−0.0471	−0.0569	0.0033	0.0189	0.0090
DTC1	**0.0355**	0.0594	**0.0119**	0.0239	0.0277	0.0103
DTC2	0.0323	0.0589	−0.0026	0.0241	0.0311	**0.0285**

The top results of information ratio on each dataset (excluding the Best and BCRP strategies, which are in hindsight) are highlighted in bold

in the design of the portfolio optimization model, which plays an important role in the effectiveness of the strategies proposed in this paper.

Comparing the DTC2 strategy with another reversal strategy, the AC strategy, it can be found that the DTC2 strategy has lower risk on all datasets.

In terms of the Sharpe ratio, Calmar ratio, and information ratio, AC strategy are slightly higher than those of DTC2 strategy on some datasets, however, DTC2 strategy are significantly higher than those of AC strategy on other datasets.

For example, for the Sharpe ratio, AC is 6.04% higher than that of DTC2 on the CSI100 dataset, while DTC2 is 100.22% higher than that of AC on the HSI66 dataset. In addition, compared with the CAEG strategy in the momentum strategies, the Calmar ratio and information ratio of DTC1 and DTC2 strategies on all datasets are higher than those of CAEG strategy, while the Sharpe ratio is slightly lower than that of CAEG strategy on only two datasets. All these indicate that the DTC framework has strong competitive performance in terms of risk-adjusted returns.

Combined with the above analysis, it can be seen that DTC1 and DTC2 strategies can achieve better cumulative wealth than other strategies and still have strong competitiveness in risk-adjusted returns in most cases, which indicates that the DTC framework proposed in this paper can not only effectively reduce risk, but also maintain a high return performance. These results show that our proposed DTC framework can achieve a good balance between benefits and risks, and have better competitive performance.

4.6 Transaction Costs

Transaction costs have a significant impact on the cumulative wealth of an investment strategy. In order to further study the relationship between transaction cost rate and cumulative wealth, Fig. 1 shows the cumulative wealth comparison between DTC1 and DTC2 strategies and other reversal strategies (AC, PAMR, RMR, and OLMAR) under different transaction cost rates.

Figure 1 shows that, firstly, with the increase of transaction costs, the cumulative wealth of AC, PAMR, RMR, and OLMAR strategies decreases significantly. Especially on NDX100 dataset, the cumulative wealth of PAMR, OLMAR and RMR strategies declines exponentially. Secondly, in the case of zero transaction costs, the cumulative wealth obtained by DTC1 and DTC2 strategies on some datasets is less than that obtained by some other reversal strategies, however, DTC1 and DTC2 strategies outperform the reversal strategies on all datasets when the transaction cost rate is greater than 0.3%. Finally, it is obvious that as transaction costs increase, the cumulative wealth of the reversal strategies (AC, PAMR, RMR, and OLMAR) is less than 1 or even reduces to close to 0 in most datasets, but there is little fluctuation in the cumulative wealth of DTC1 and DTC2 strategies. In conclusion, DTC1 and DTC2 strategies are less affected by transaction costs and superior to other reversal strategies in the case of non-zero transaction costs.

4.7 Parameter Sensitivity

We then evaluate the sensitivity of parameters λ and ξ in model (5), where λ is the trade-off parameter and ξ is the threshold parameter. Figures 2 and 3 are heat maps that show the parameter sensitivity analysis on cumulative wealth with trade-off parameter λ changing from 0.01 to 1 and threshold parameter ξ changing from 0 to 2, on DTC1 and DTC2 strategies, respectively. Each point on the graph corresponds to the cumulative wealth obtained by DTC1 and DTC2 strategies under different

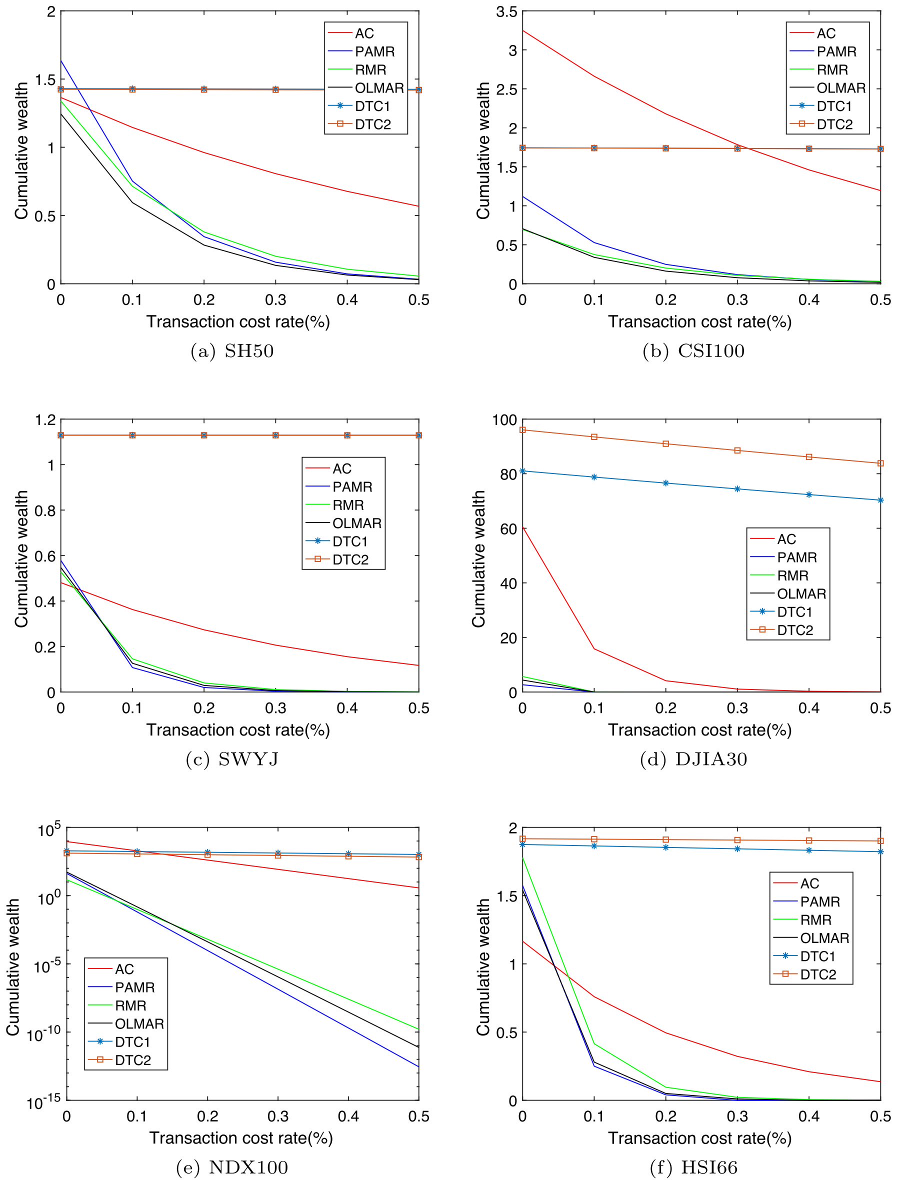

Fig. 1 Cumulative wealth of different strategies under different transaction cost rates

parameter combinations. The different regions in each graph we look at have different colors that represent different cumulative wealth achieved by our proposed strategy DTC1 or DTC2 under different combinations of λ and ξ. The color bar on the right of each subfigure shows the scale of cumulative wealth, and the darker the color, the more wealth.

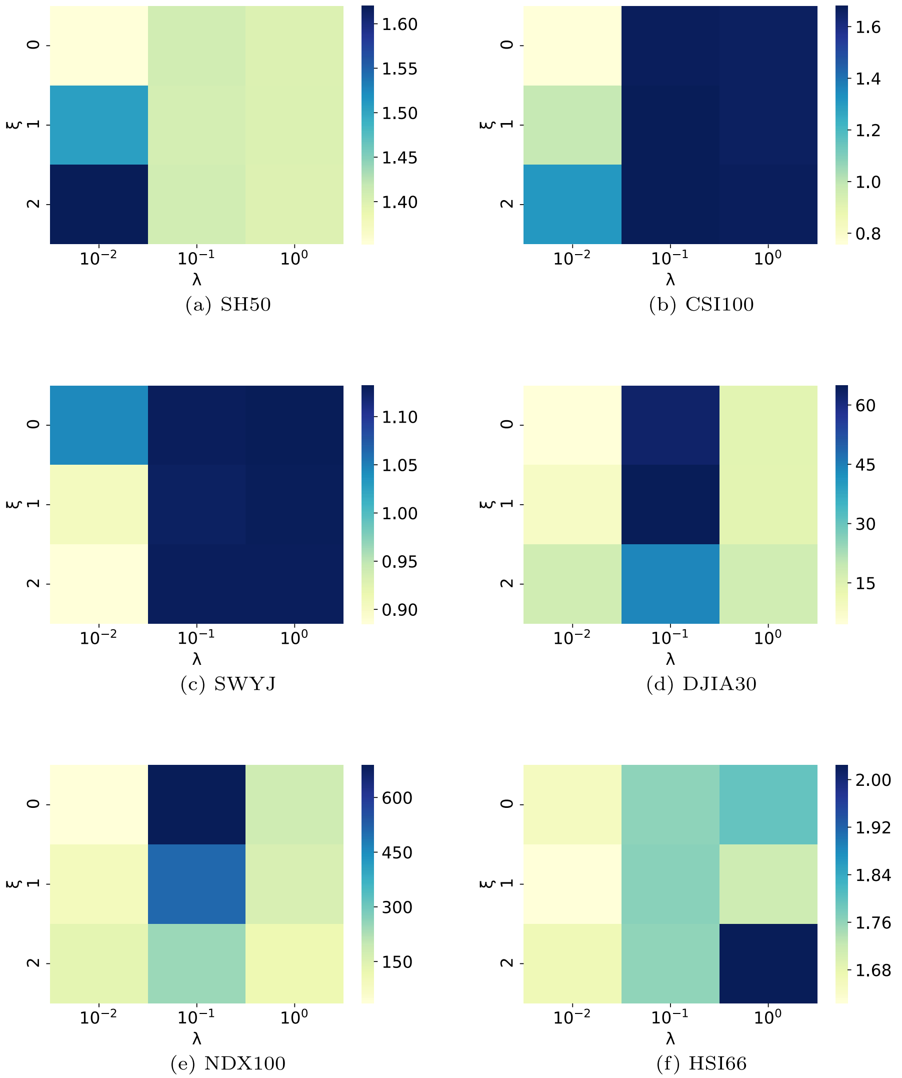

Fig. 2 Parameter sensitivity of DTC1 with respect to λ and ξ at zero transaction costs

First of all, as can be seen from Fig. 2, the dark regions on each dataset are not completely consistent, indicating that it is difficult to ensure the optimal performance of the strategy under different market environments through fixed parameter values. However, from the range displayed in the color bar, it can be found that the performance of DTC1 strategy in the Chinese market (SH50, CSI100, SWYJ, and HSI66) is very little affected by the parameters. In the US market (DJIA30 and NDX100), when the parameter combination λ is about 0.1 and ξ is 0 to 1, the DTC1 strategy still maintains stable performance.

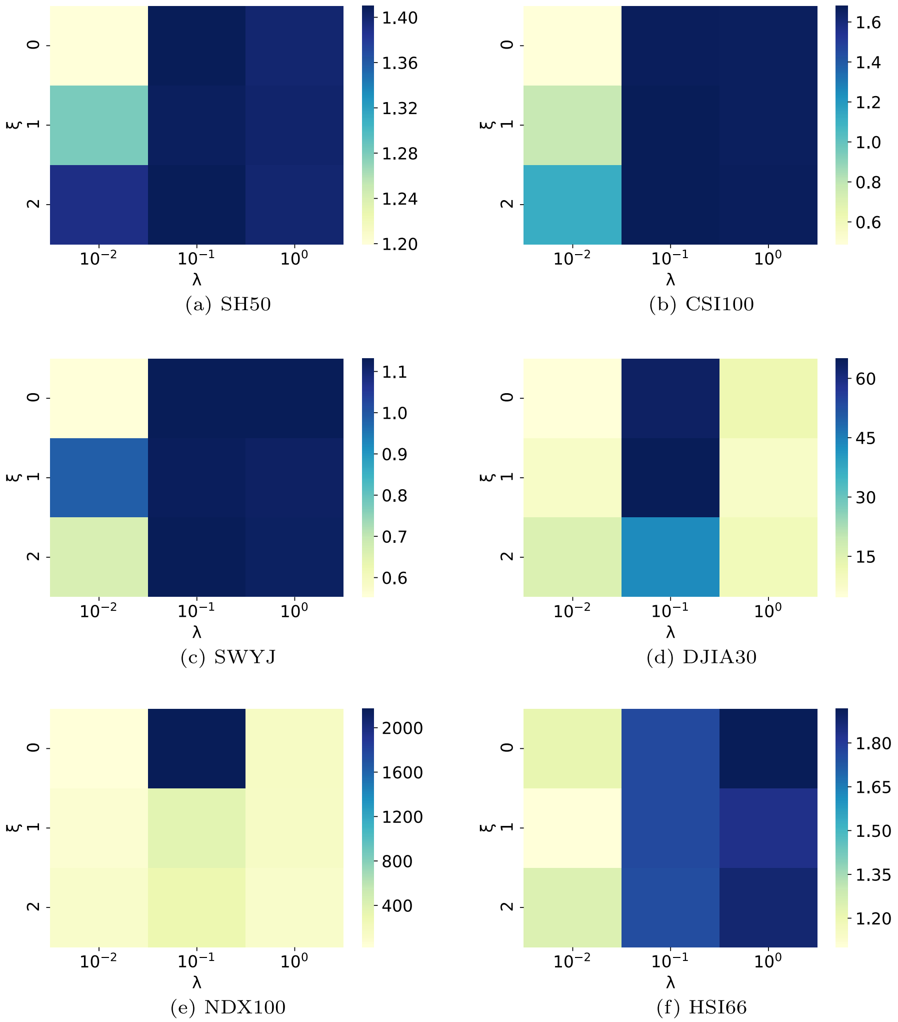

Fig. 3 Parameter sensitivity of DTC2 with respect to λ and ξ at zero transaction costs

Besides, Fig. 3 shows that the influence of the parameter combination of λ and ξ on the DTC2 strategy is similar to that on the DTC1 strategy. However, due to the different methods of DTC1 and DTC2 to predict stock prices, there are still some differences in the effects of different parameters on the returns of the two strategies.

By comparing Figs. 2 and 3, it can be found that on NDX100 dataset, DTC1 and DTC2 both perform best when λ is about 0.1 and ξ is about 0, but their optimal performance differs greatly, and DTC2 is obviously better than DTC1.

In addition, it can be seen from Table 2 that DTC1 is superior to DTC2 on NDX100 dataset. It further indicates that the adaptive exponential smoothing average forecasting method may have greater potential space for application.

In the experiments in this section, we simply set λ to 0.05 and ξ to 1, which are satisfactory but obviously not optimal results. It demonstrates that DTC1 and DTC2 strategies are less affected by parameter values and are not very sensitive to parameter selection, so they can select parameters within a certain range and maintain good performance.

5 Conclusion

This paper studies the online portfolio selection with transaction costs. First of all, a framework for dealing with linear transaction costs is proposed, which makes a trade-off between maximizing expected returns and minimizing transaction costs. In addition, considering that the existing reversal online portfolios are prone to centralized investment, we take the entropy of portfolio as the constraint in the model, and design a decentralized online portfolio strategy (DTC) with transaction costs. Furthermore, based on two price prediction methods of exponential smoothing average and adaptive exponential smoothing average, we propose DTC1 and DTC2 strategies, respectively. The backtest results on multiple datasets show that the DTC1 and DTC2 strategies can effectively deal with reasonable transaction costs and improve the existing strategies in the case of non-zero transaction costs. Compared with the existing momentum strategies, the DTC framework has little difference in risk but better return performance. And compared with the existing reversal strategies, the DTC framework has less risk and competitive risk-adjusted returns, especially when the transaction cost rate is high, and the performance of the DTC framework is more stable. Nevertheless, there are still some further improvements in this paper, such as the relatively simple method of predicting prices. In the future, we consider adopting machine learning or deep learning methods to improve the accuracy of forecasts, further refine our investment strategy, and inform decision-making ideas for many investors.

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Declarations

Conflict of interest The authors have no conflict of interest to declare that are relevant to the content of this article.

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