Symbolic computing with SymPy

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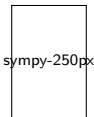
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Materials will be available at `http://github.com/rouckas/sympy-slides/`

What is SymPy

Goal
Provide a symbolic manipulation library in Python.

"SymPy is an open source Python library for symbolic mathematics. It aims to become a full-featured computer algebra system (CAS) while keeping the code as simple as possible in order to be comprehensible and easily extensible. SymPy is written entirely in Python and has only one dependence (mpmath)."

Past and present

History

- ▶ Ondřej Čertík started the project in 2005.
- ▶ Development took off in 2007 when SymPy first participated in Google Summer of Code. We have participated in every Google Summer of Code since.
- ▶ In 2011, Aaron Meurer (who also joined from Google Summer of Code) took over as lead developer.

Current Status

- ▶ Over 750 contributors.
- ▶ Current code base has over 500,000 lines of code and documentation.

Why SymPy?

- ▶ Standalone
- ▶ Full featured
- ▶ BSD licensed
- ▶ Embraces Python
- ▶ Usable as a library

Why SymPy for data scientists?
It is up to you

Research Articles citing SymPy

- ▶ Google Scholar list
- ▶ Zotero group
- ▶ Sympy paper

Projects Using SymPy

- ▶ **Sage**: A CAS, which includes many open source mathematical libraries.
- ▶ **SageMathCloud**: A web-based cloud computing and course management platform for computational mathematics.
- ▶ **Mathpix**: An iOS App, that detects handwritten math as input, and uses SymPy to evaluate it and generate the relevant steps to solve the problem.
- ▶ **PyDy**: Multibody Dynamics with Python.
- ▶ **IKFast**: IKFast is a robot kinematics compiler provided by OpenRAVE. It analytically solves robot inverse kinematics equations and generates optimized C++.
- ▶ **Octave Symbolic**: Symbolic package adding symbolic calculation features to GNU Octave.
- ▶ **galgebra**: Geometric algebra (previously `sympy.galgebra`).

Projects Using SymPy

- ▶ **SymPy.jl**: Provides a Julia interface to SymPy using PyCall.
- ▶ **Mathics**: A free, general-purpose online CAS featuring Mathematica compatible syntax and functions.
- ▶ **SfePy**: Simple finite elements in Python.
- ▶ **Quameon**: Quantum Monte Carlo in Python.
- ▶ **Lcapy**: Experimental Python package for teaching linear circuit analysis.
- ▶ **Quantum Programming in Python**: Quantum 1D Simple Harmonic Oscillator and Quantum Mapping Gate.
- ▶ **LaTeX Expression project**: Easy LaTeX typesetting of algebraic expressions in symbolic form with automatic substitution and result computation.
- ▶ **Symbolic statistical modeling**: Adding statistical operations to complex physical models.

Features

- **Core Capabilities**
  - Basic arithmetic: Support for operators such as +, -, *, /, ** (power)
  - Simplification
  - Expansion
  - Functions: trigonometric, hyperbolic, exponential, roots, logarithms, absolute value, spherical harmonics, factorials and gamma functions, zeta functions, polynomials, special functions, . . .
  - Substitution
  - Numbers: arbitrary precision integers, rationals, and floats
  - Noncommutative symbols
  - Pattern matching

- **Polynomials**
  - Basic arithmetic: division, gcd, . . .
  - Factorization
  - Square-free decomposition
  - Gröbner bases
  - Partial fraction decomposition
  - Resultants

- **Calculus**
  - Limits: $\lim_{x \to 0} x \log(x) = 0$
  - Differentiation
  - Integration: It uses extended Risch-Norman heuristic
  - Taylor (Laurent) series

- **Solving equations**
  - Polynomial equations
  - Algebraic equations
  - Differential equations
  - Difference equations
  - Systems of equations

- **Combinatorics**
  - Permutations
  - Combinations
  - Partitions
  - Subsets
  - Permutation Groups: Polyhedral, Rubik, Symmetric, . . .
  - Prufer and Gray Codes

Features

- **Discrete math**
  - Binomial coefficients
  - Summations
  - Products
  - Number theory: generating prime numbers, primality testing, integer factorization, . . .
  - Logic expressions
- **Matrices**
  - Basic arithmetic
  - Eigenvalues/eigenvectors
  - Determinants
  - Inversion
  - Solving
  - Abstract expressions
- **Geometric Algebra**
- **Geometry**
  - points, lines, rays, segments, ellipses, circles, polygons, . . .
  - Intersection
  - Tangency
  - Similarity

- **Plotting**
  - Coordinate modes
  - Plotting Geometric Entities
  - 2D and 3D
  - Interactive interface
  - Colors
- **Physics**
  - Units
  - Mechanics
  - Quantum
  - Gaussian Optics
  - Pauli Algebra
- **Statistics**
  - Normal distributions
  - Uniform distributions
  - Probability
- **Printing**
  - Pretty printing: ASCII/Unicode pretty printing, LaTeX
  - Code generation: C, Fortran, Python

Let's try it

Conclusion

Recommendations

- ▶ Try SymPy yourself
- ▶ If you have a problem, consult the documentation `http://docs.sympy.org` and contact the community via mailing list or Stack Overflow
- ▶ If you find an issue, report it
- ▶ SymPy developers will help you to fix it
- ▶ Get involved in SymPy development. It is easy also thanks to `https://github.com/sympy/sympy/wiki/Development-workflow`

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