The one-way, random-effects ANOVA test proceeds as follows: The F statistic is a ratio of two different estimates of σ^2, one based on between-participant variation and the other based on within-participant variation. Under the null hypothesis, these estimates should be close; under the alternative, the between-participant variation will tend to be larger than the within-participant variation.

Derivation of the between-participant variation is as follows: Note that $Y_{ijk} \sim \mathrm{N}(\mu_{ij}, \sigma^2)$, and, consequently $\bar{Y}_{ij} \sim \mathrm{N}(\mu_{ij}, \sigma^2/K_{ij})$, where K_{ij} is the number of responses for the ith person in the jth condition. The effect, $d_i = \bar{Y}_{i2} - \bar{Y}_{i1}$ is distributed as

$$d_i \sim \mathrm{N}\left(\mu_{i2} - \mu_{i1}, \frac{\sigma^2}{K_{i1}} + \frac{\sigma^2}{K_{i2}}\right).$$

Let K_i^* denote the effective sample size for the ith effect:

$$K_i^* = \frac{K_{i1}K_{i2}}{K_{i1} + K_{i2}}.$$

Then,

$$d_i \sim \mathrm{N}\left(\mu_{i2} - \mu_{i1}, \frac{\sigma^2}{K_i^*}\right).$$

Let $d_i^* = d_i\sqrt{K_i^*}$. Then, under the null that $\mu_{i2} - \mu_{i1}$ is constant, the following expression is a between-participant estimator of σ^2:

$$s_1^2 = \frac{\sum_i (d_i^* - \bar{d}^*)^2}{(I-1)},$$

where $\bar{d}^*$ is the mean of all d_i^*. The degrees-of-freedom associated with this estimator is $I - 1$.

The expression for within-subject variation follows from the usual considerations:

$$s_2^2 = \frac{\sum_{ijk}(Y_{ijk} - \bar{Y}_{ij})^2}{\sum_{ij}(K_{ij} - 1)} = \frac{\sum_{ijk}(Y_{ijk} - \bar{Y}_{ij})^2}{N - IJ},$$

where $N = \sum_{ij} K_{ij}$ is the total number of observations. The degrees-of-freedom associated with this estimator is $N - IJ$.

With these expressions, the F statistic is

$$F = \frac{s_1^2}{s_2^2},$$

which under the null is distributed as an F distribution with $\nu_1 = I - 1$ and $\nu_2 = N - IJ$ degrees-of-freedom.