Demystifying Hierarhcical Models

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Hierarchical Models Seem Magical

$$Y_{ij} \sim \mathsf{N}(\mu_i, \sigma^2)$$
$$\mu_i \sim \mathsf{N}(\nu, \delta)$$

is the same as:

$$Y_{ij} \sim \mathsf{N}(\nu + \mu_i, \sigma^2)$$
$$\mu_i \sim \mathsf{N}(0, \delta)$$

More standard:

$$Y_{ij} \sim \mathsf{N}(\mu + \mu_i, \sigma^2)$$
$$\alpha_i \sim \mathsf{N}(0, \delta)$$

If there are two people, there are three parameters, and that seems like magic!

What Happens With 2 People and 3 Parameters?

```
source('jags/normJ.R')
```

```
## Loading required package: R2jags

## Loading required package: rjags

## Loading required package: coda

## Linked to JAGS 4.3.1

## Loaded modules: basemod,bugs

##
## Attaching package: 'R2jags'

## The following object is masked from 'package:coda':
##
##     traceplot
```

What Happens With 2 People and 3 Parameters?

```
sub=rep(1:2,each=100)
tMu=c(8,12)
y=rnorm(200,tMu[sub],5)
dat=data.frame(sub,y)
prior=list("a"=10,"b"=10^2,'r1'=10^2,'r2'=10^2)
out=runM2a(dat,prior)
```

```
## module glm loaded

## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 200
##    Unobserved stochastic nodes: 5
##    Total graph size: 418
```

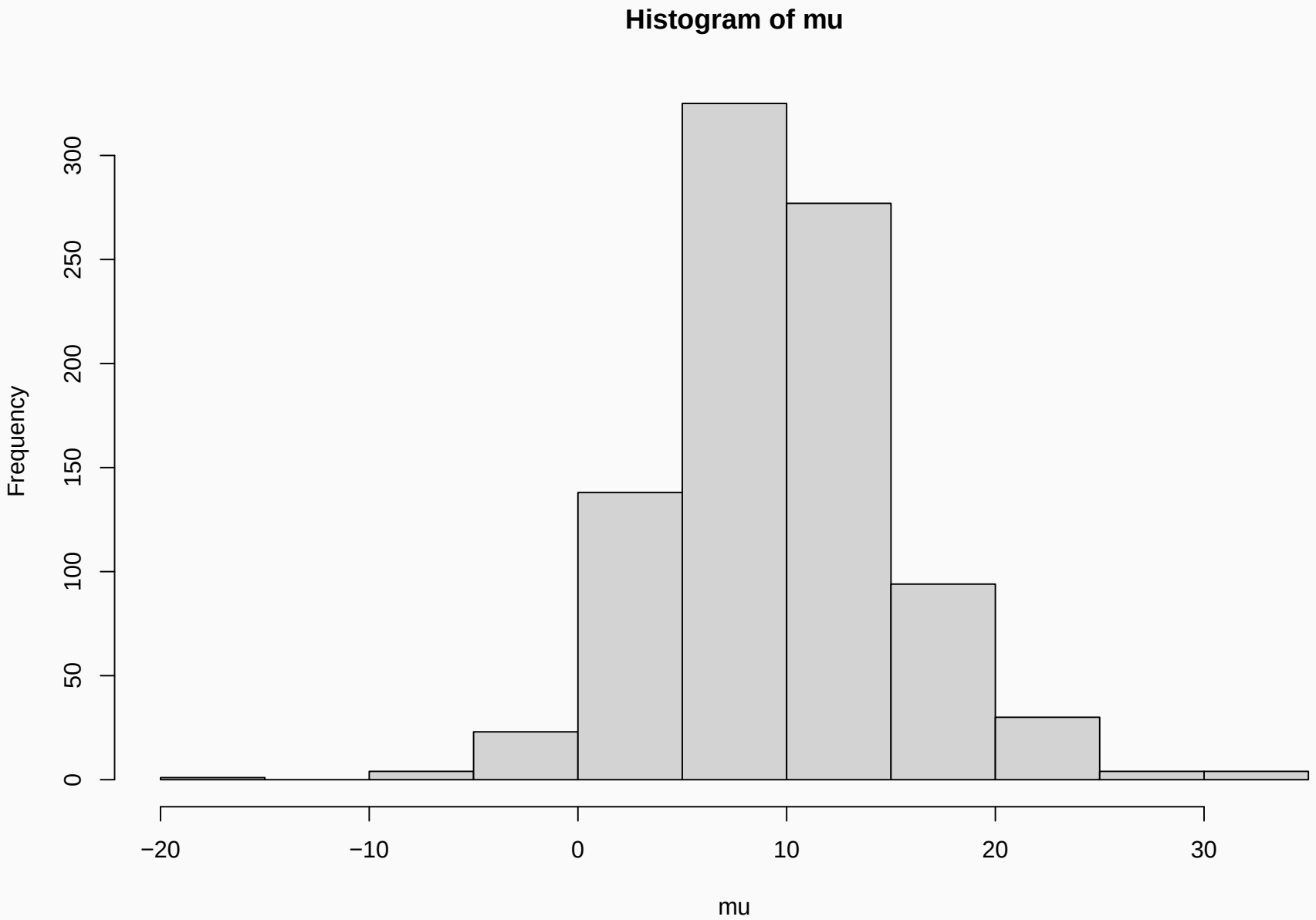
How Well Should We Be Able To Localize Parameter?

SEM $= \sigma/\sqrt{n} = 5/10 = .5$

We should know all parameter ± 1.

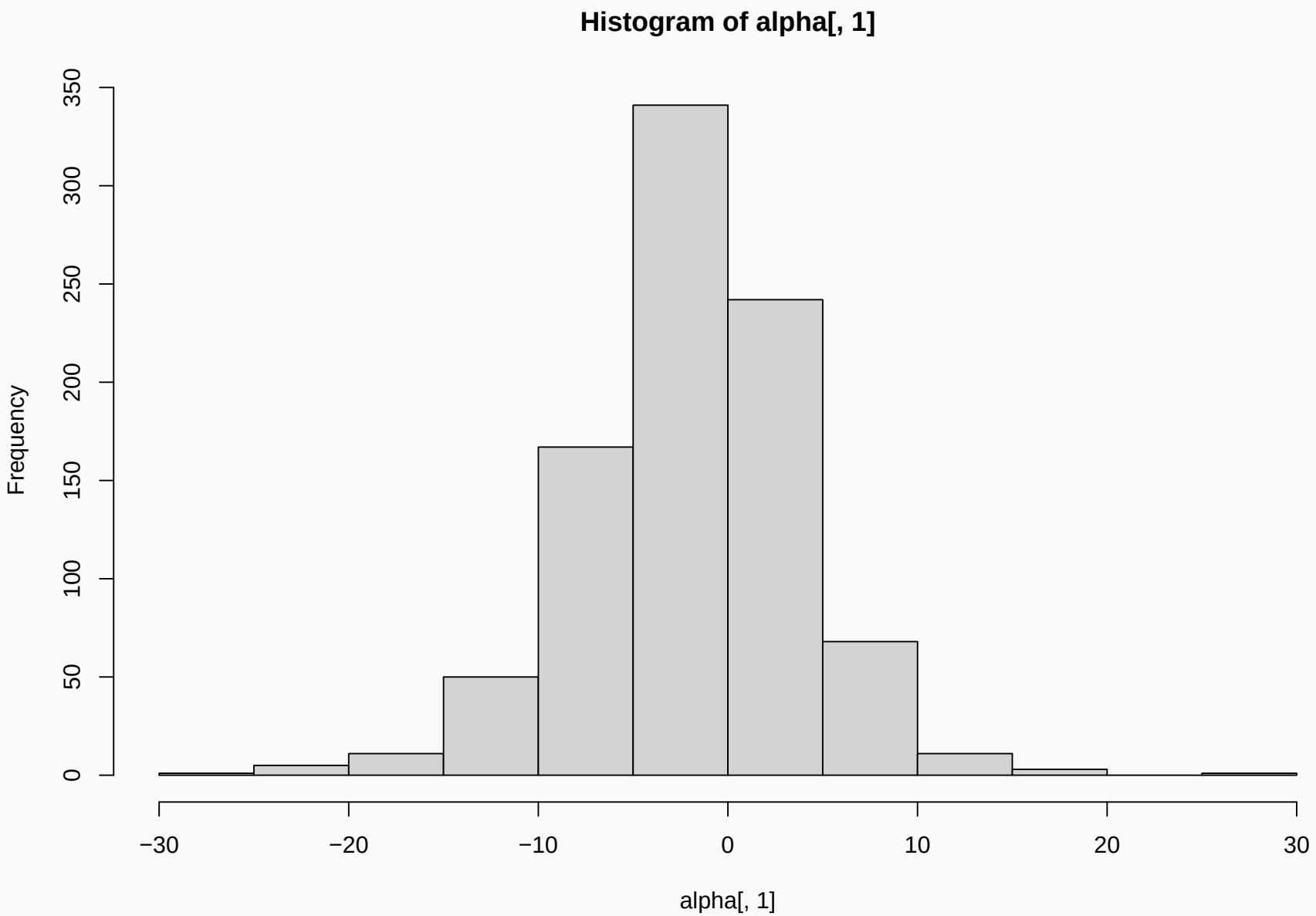
Yet, Can't Localize Parameters Well

```
hist(mu)
```



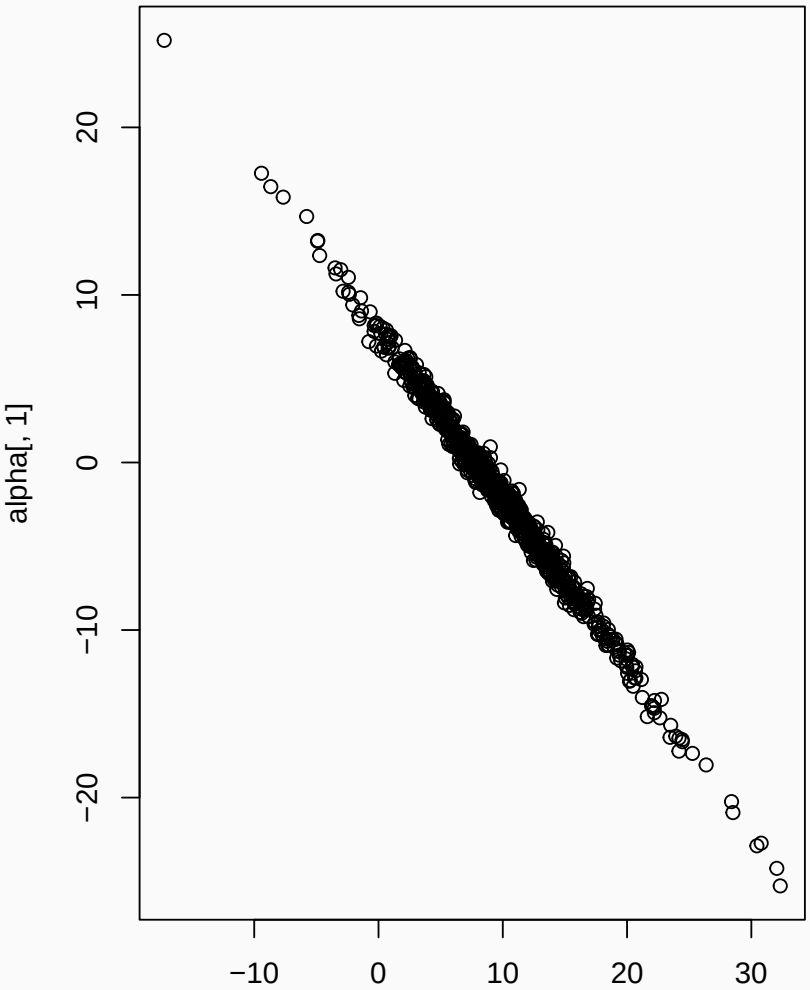
Can't Localize Parameters Well

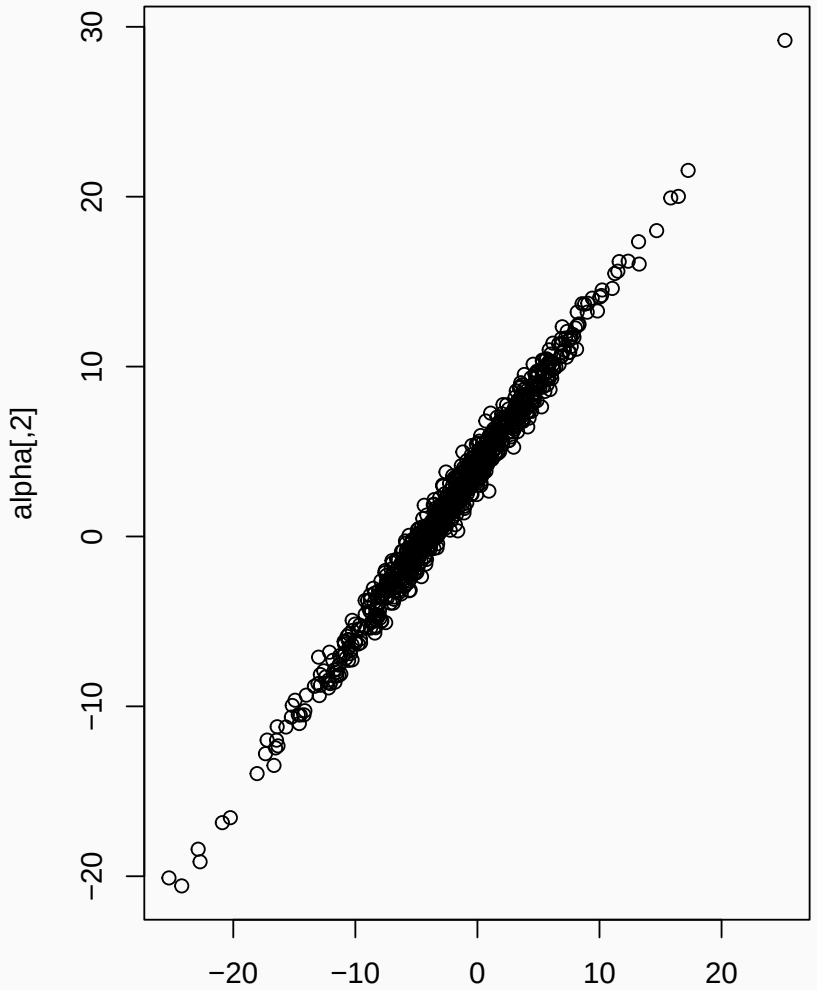
```
hist(alpha[,1])
```



Correlated Parameters ...

```
par(mfrow=c(1,2))
plot(mu,alpha[,1])
plot(alpha)
```





Morale of the Story

There is no magic here. Usually, it matters little how we parameterized the hierarchical models because the cost of one parameter is not great if there are a lot of people.