Lecture 3: Robust Cognitive Models

Jeff Rouder

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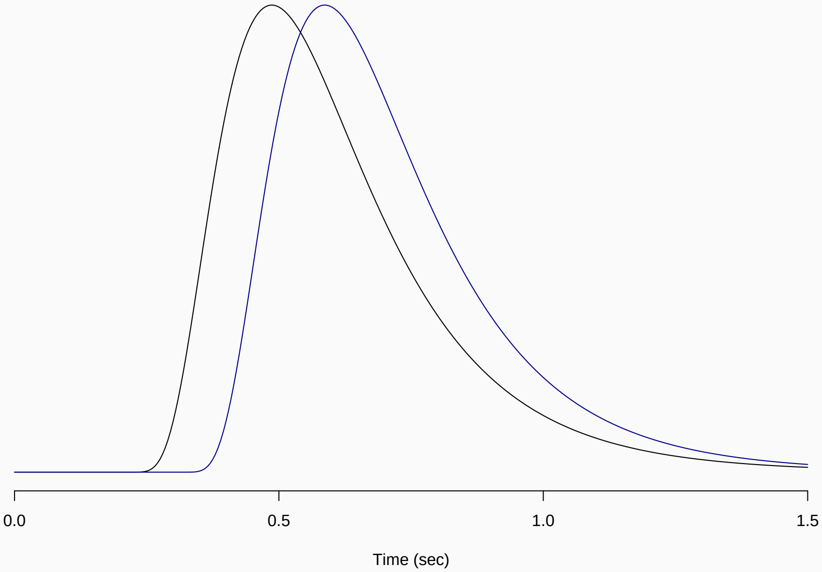
Some RT modeling

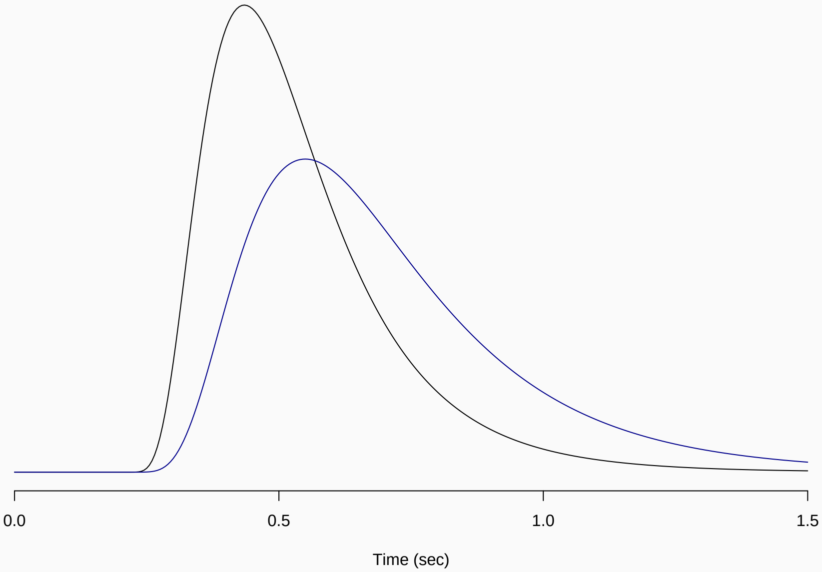
Distance From 5

- 2, 3 4 and 6, 7, 8
- Is there an analog representation where 2 is further from 5 than 4 is?
- Rouder et al. (2005). Locus of RT effects
- Shift, Scale, and Shape

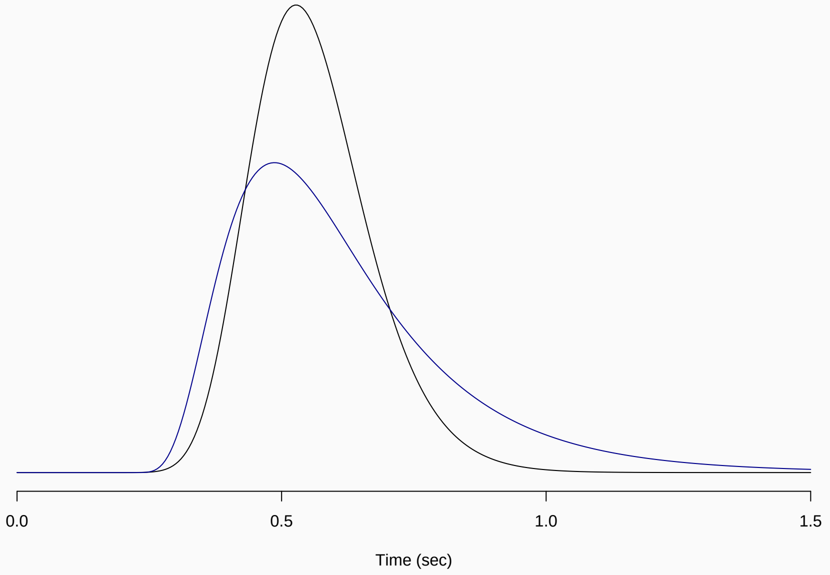
Difference in Shift



Difference in Scale



Difference in Shape



Your Turn/ My Turn Too

- Load Up https://raw.githubusercontent.com/PerceptionCognitionLab/data0/master/lexDec-dist5/ld5.all
- Columns at https://github.com/PerceptionCognitionLab/data0/blob/master/lexDec-dist5/ld5.txt
- cleaning code can be sourced at https://raw.githubusercontent.com/rouderj/robust21/main/ld5clean.R

Your Turn? My turn?

```
link1 = "https://raw.githubusercontent.com/rouderj/"
link2 = "robust21/main/ld5clean.R"
link=paste(link1,link2,sep="")
source(url(link))
dat=ld5MakeClean()
```

Your Turn

- Analyze the mean RT.
- What is the effect of digit on mean RT?
- Analog Representation?

Your Turn

- How strong is this effect?
- Test is easy, can you show the strength graphically?

Distributional Model

For one person:

$$Y_k \sim \text{Lognormal}(\psi, \theta, \beta)$$

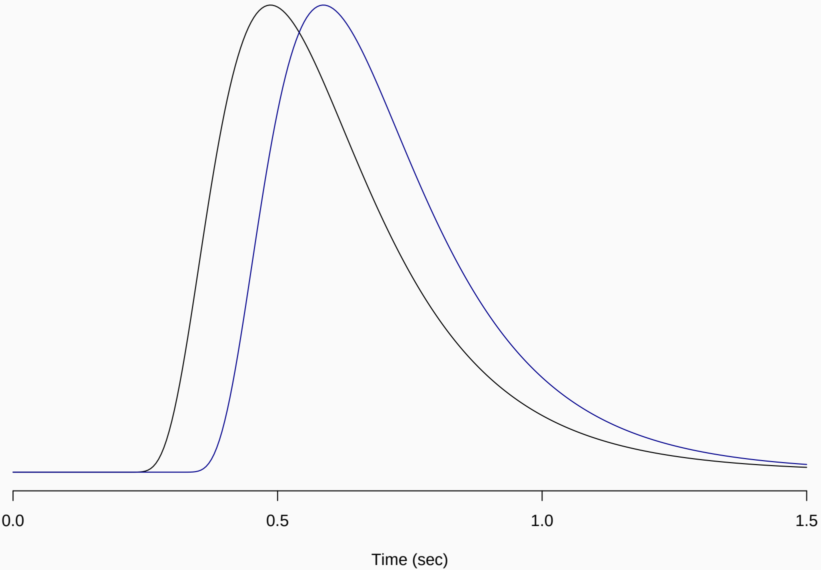
$$f_{Y_k}(t; \psi, \theta, \beta) = \frac{1}{(t-\psi)\sqrt{2\pi\beta}} \exp\left(-\frac{(\log(t-\psi)-\theta)^2}{2\beta}\right)$$

Alt:

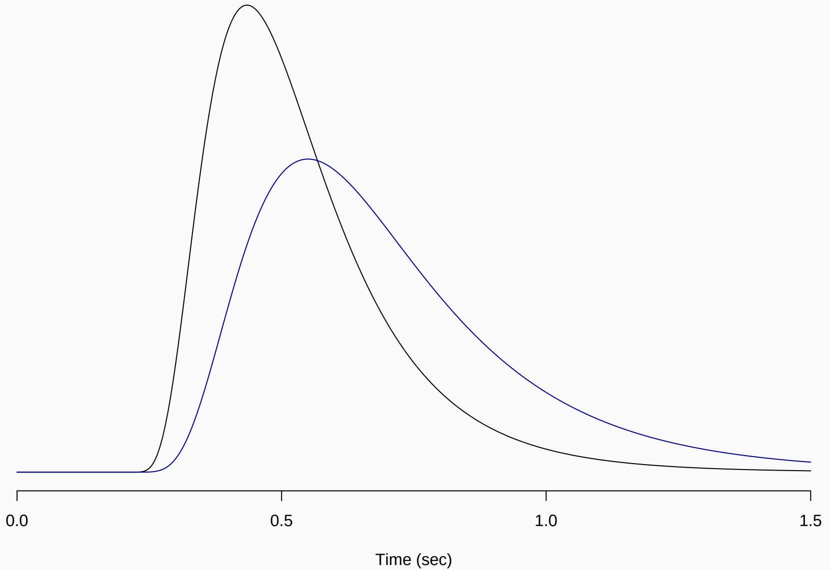
$$\log(Y_k - \psi) \sim \text{N}(\theta, \beta)$$

- shift: ψ (sec)
- scale:exp(θ)
- shape: β

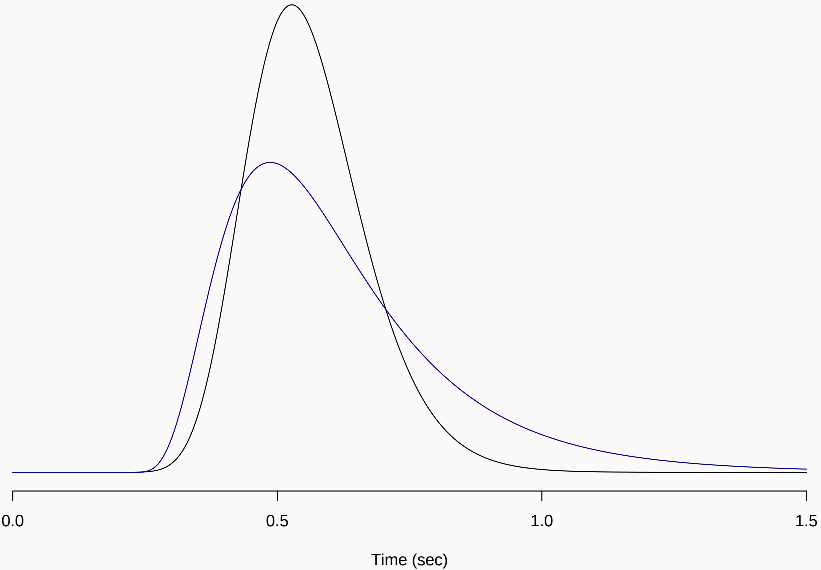
Shift .2 vs. .3



Scale .3 vs .45



Shape, .2 vs. .5



Likelihood

- Let ψ, θ, β be the density evaluated at some point y for parameters (ψ, θ, β).
- If there is one observation, y_1, $L(\psi, \theta, \beta; y_1) = f(y_1; \psi, \theta, \beta)$.
- If there are two observations, $L(\psi, \theta, \beta; y_1, y_2) = f(y_1; \psi, \theta, \beta) \times f(y_2; \psi, \theta, \beta)$
- For $\boldsymbol{y} = (y_1, \ldots, y_n)$, $L(\psi, \theta, \beta; \boldsymbol{y}) = \prod_{i=1}^{n} f(y_i; \psi, \theta, \beta))$
- For $\boldsymbol{y} = (y_1, \ldots, y_n)$, $l(\psi, \theta, \beta; \boldsymbol{y}) = \log L(\psi, \theta, \beta; \boldsymbol{y}) = \sum_{i=1}^{n} \log f(y_i; \psi, \theta, \beta))$

Likelihood

```
#par[1]=shift
#par[2]=scale
#par[3]=shape

logLike=function(par,y) #maximize
  sum(dlnorm(y-par[1],par[2],par[3],log=T))

nll=function(par,y) #minimize
  -sum(dlnorm(y-par[1],par[2],par[3],log=T))
```

Your Turn

- Let's use maximum likelihood to estimate the three parameters.
- Grab the first person, digit 2 condition as data. See if you can use above function to find out some good estimates of ψ, θ, β. HINT: you formalize the search using `optim()`

My Turn

```
## $par
## [1]  0.3765913 -2.2160193  0.4462156
##
## $value
## [1] -91.43569
##
## $counts
## function gradient
##      142       NA
##
## $convergence
## [1] 0
##
## $message
## NULL
```

General Model over People and Conditions

Most General:

- ith subject, $i = 1, \ldots, I$
- jth condition, $j = 1, \ldots, J$
- kth replicate, $k = 1, \ldots, K_{ij}$

$$Y_{ijk} \sim \text{Lognormal}(\psi_{ij}, \theta_{ij}, \beta_{ij})$$

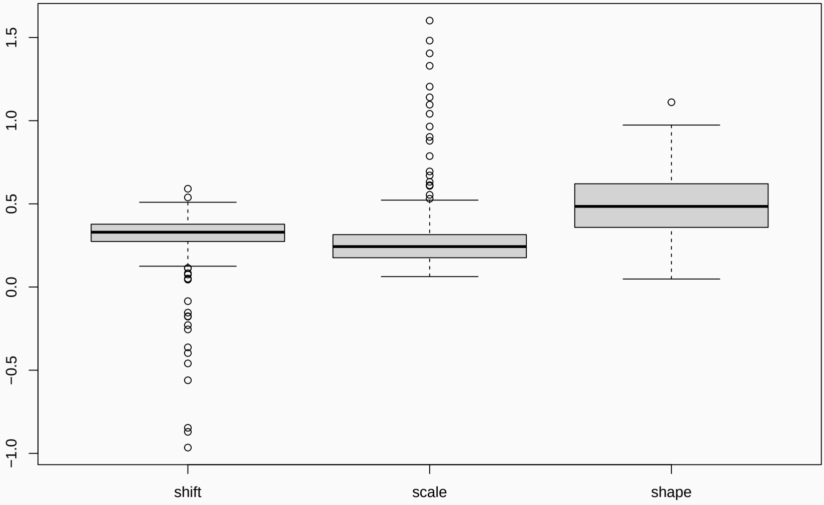
Your turn

- Estimate all the parametersby looping through each person and condition. It should not take too long.
- Examine the pattern of estimates. What can you learn?

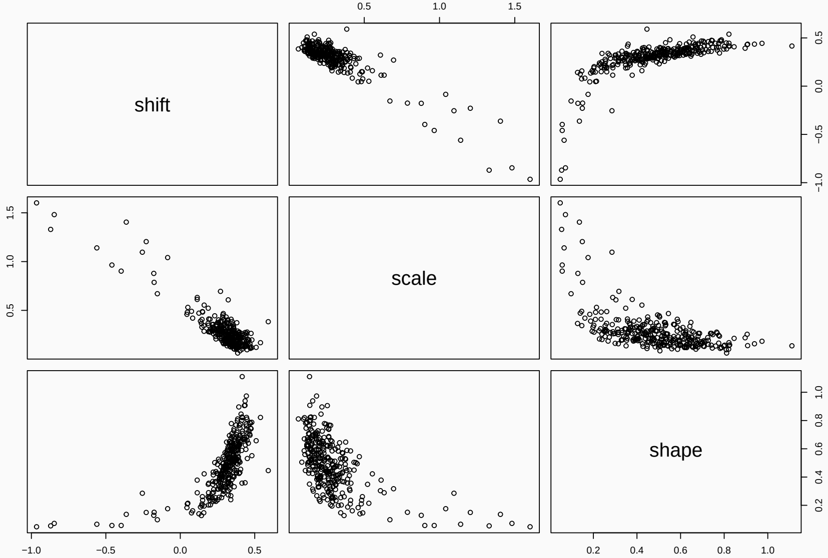
My Turn

My Turn

My Turn

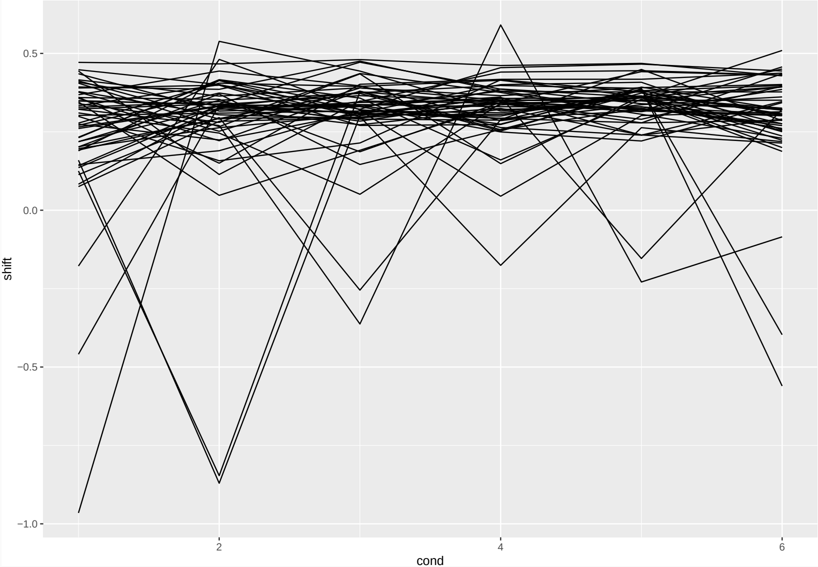


My Turn



Your Turn

My Turn



Shape Differences

- So flexible
- Need a theory (say diffusion model)
- For our exploration, how about one shape
- Benefit:
  - statistical?
  - interpretation, differences in scale become meaningful

How about one shape?

$$Y_{ijk} \sim \text{Lognormal}(\psi_{ij}, \theta_{ij}, \beta)$$

Homework: Your Turn

- How might you estimate this model?

Homework: Your Turn

- What kind of plots could you make to help decide if the effect was primarily in shift or scale?

Homework: Your Turn

- What would be a hierarchical extension that you might pursue?