Lecture 2: Robust Cognitive Models

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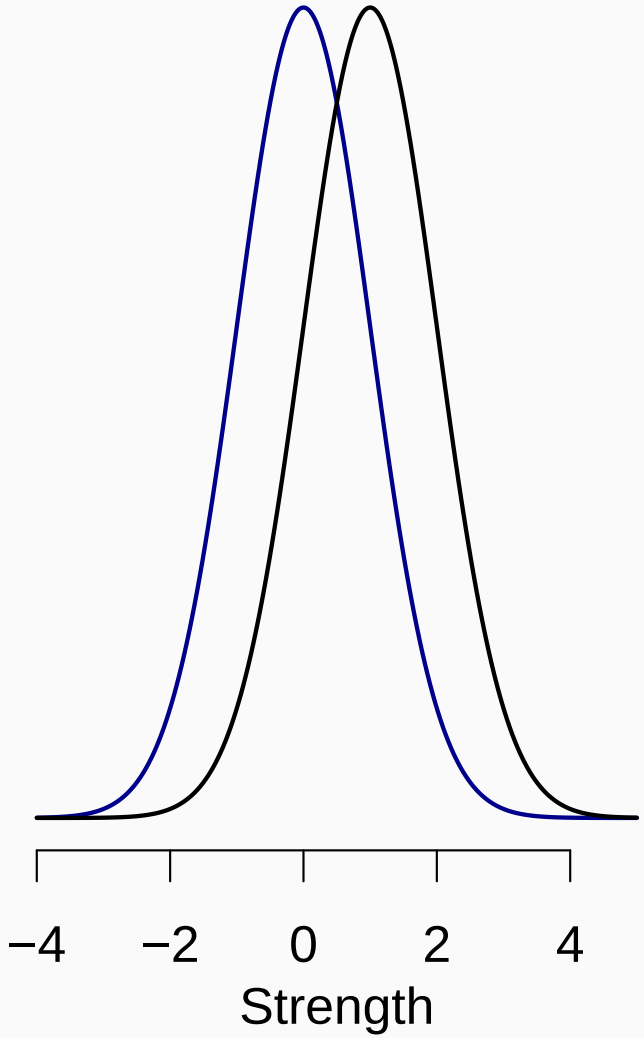
Two Signal-Dection Models

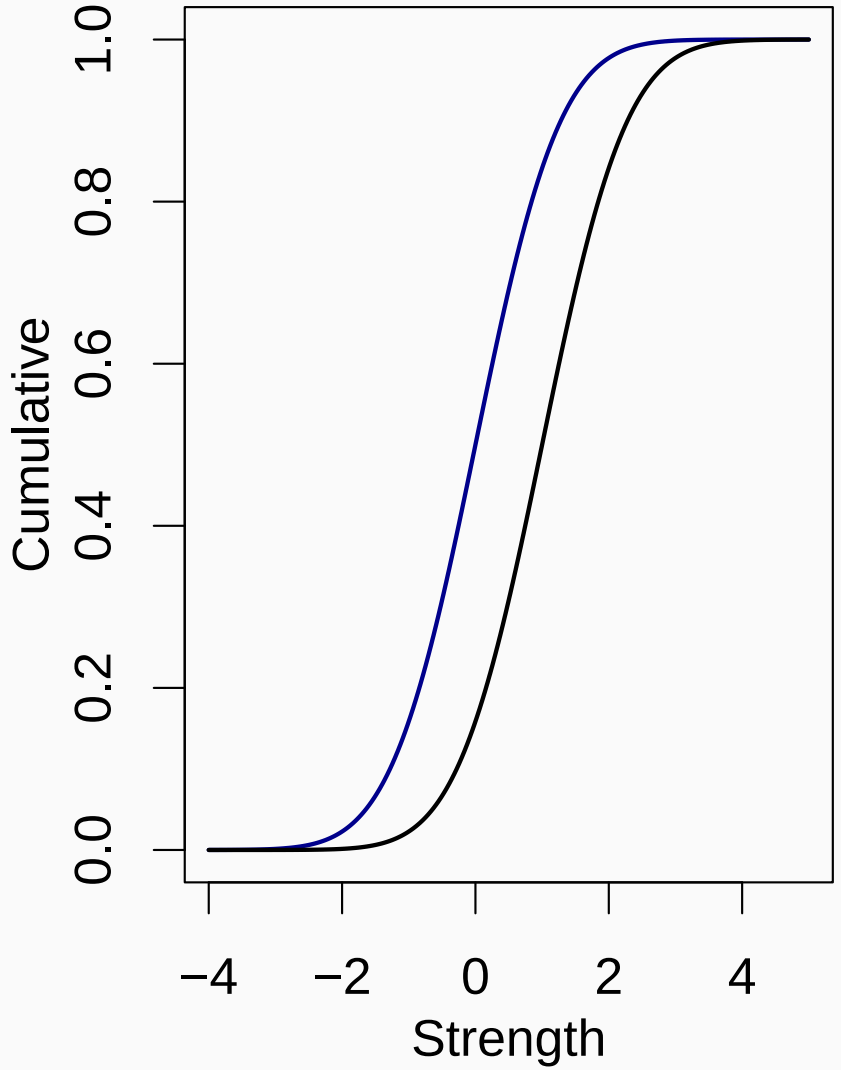
- EVSD (equal variance normal signal detection)
- UVSD (unequal variance normal signal detection)

Model And Boundary Conditions

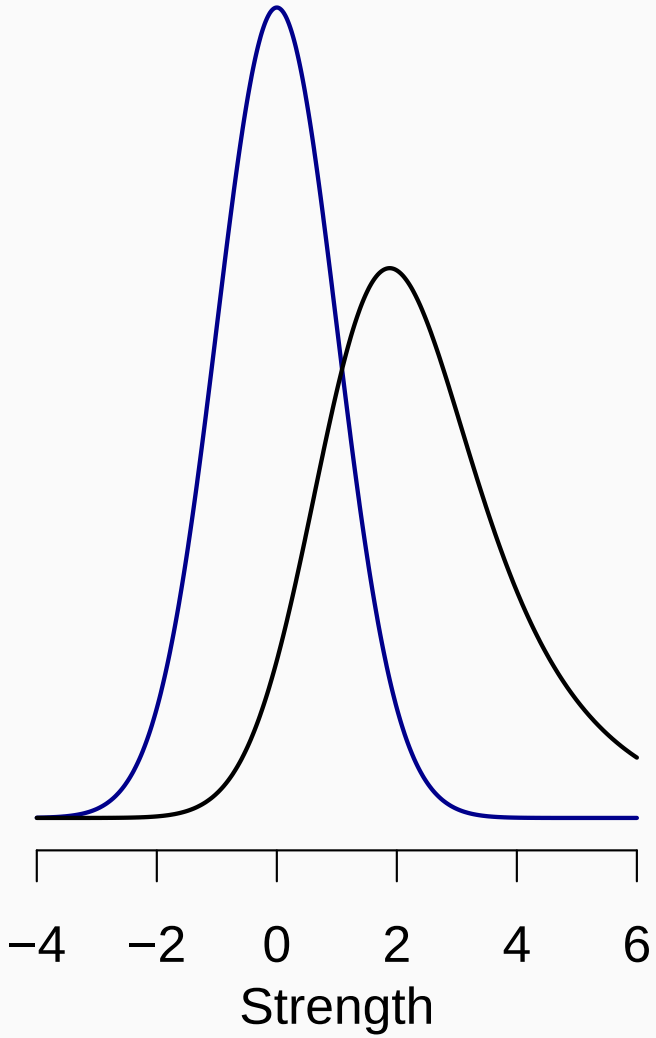
- I think it is helpful to be aware of certain cases or boundary conditions where the predictions and nature of the model become most clear.
- Boundary Condition:
  - Can studying an item reduce its mnemonic strength?
  - Does that make sense?
  - Be wary of models that specify any reduction in strength due to study.
  - Old items at least as strong as new items.

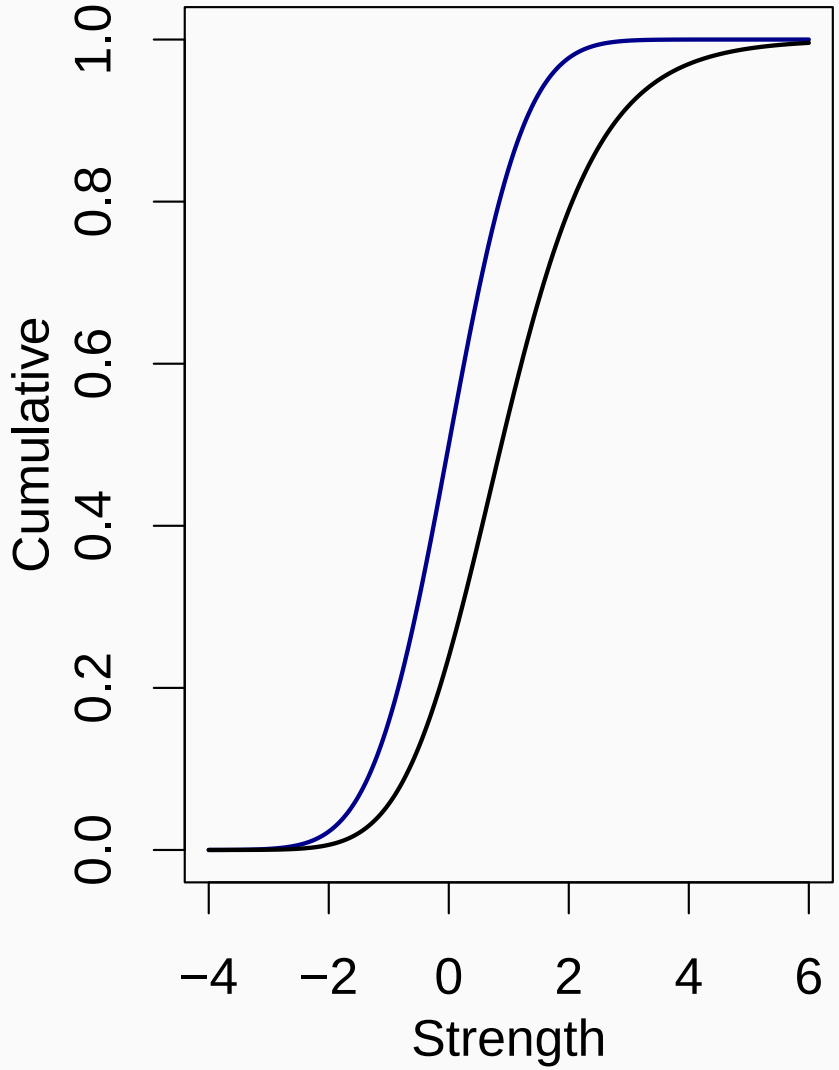
EVSD Meets Boundary so long as $\mu \geq 0$.



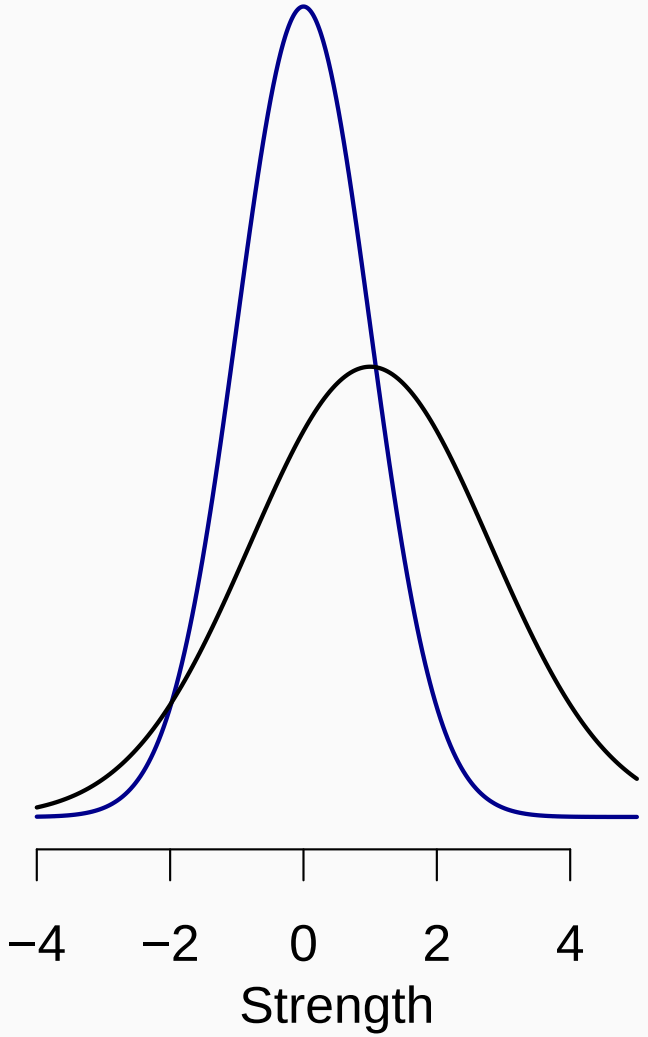


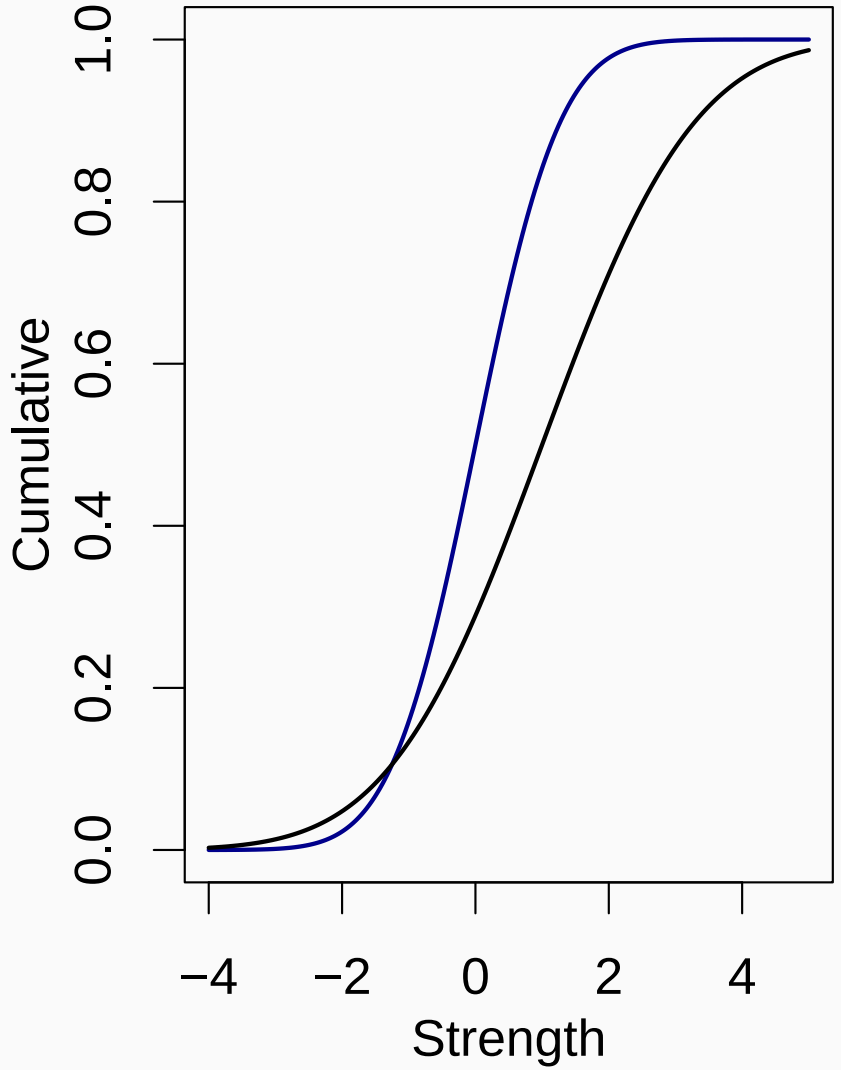
Always Stronger, Expontential Additon Study



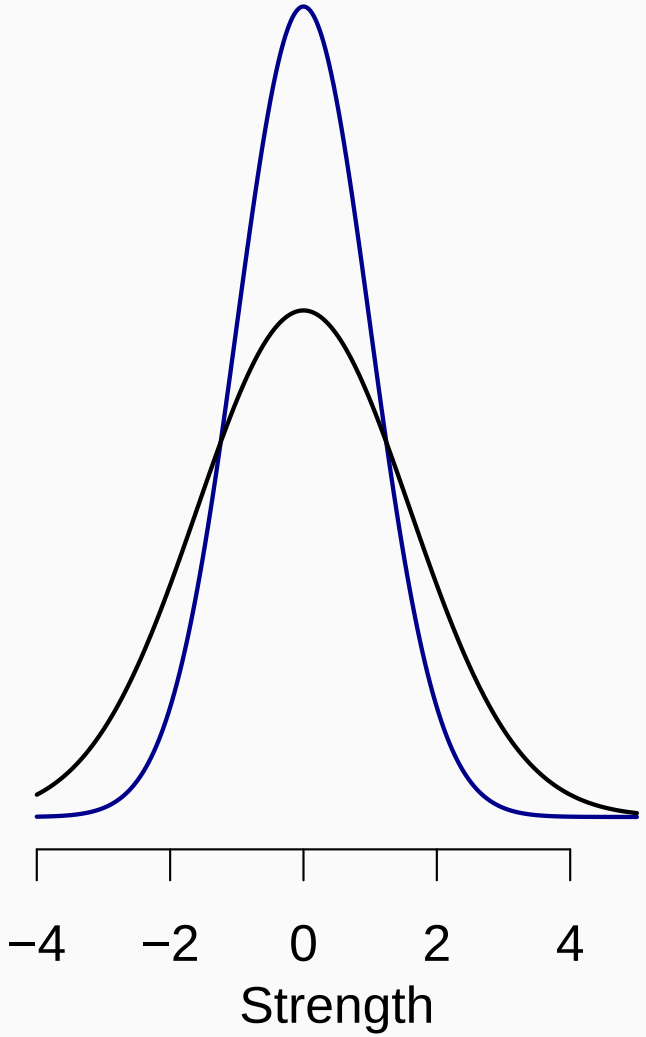


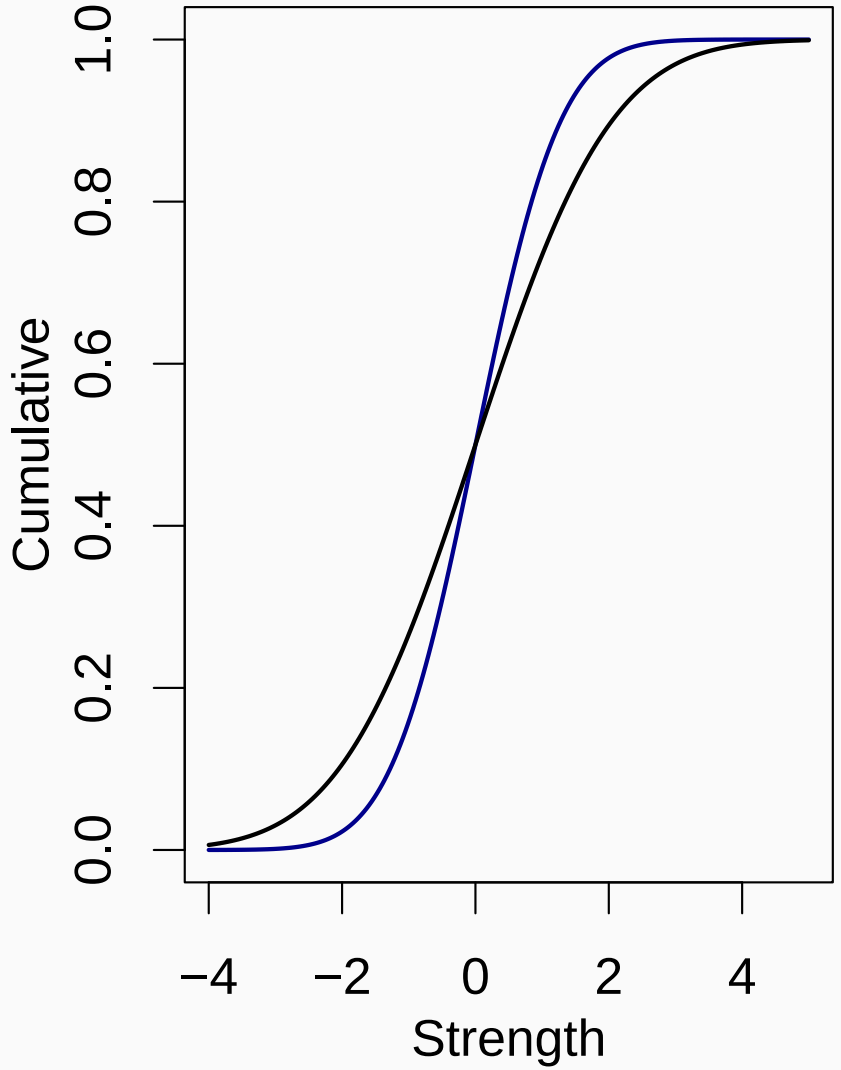
Not Always Stronger, UVSD





Worst Case





Boundary Condition

Stochastic dominance

- Relationship holds across the distribution
- Unambiguously bigger
- No crossings in cumulatives
- ROCs cant cross
- A good model should not force a violation of stochastic dominance
- UVSD does just that

Experiment With Varying Strength Conditions

- Study some words for 1 sec, 2 sec, 5 sec

Your Turn

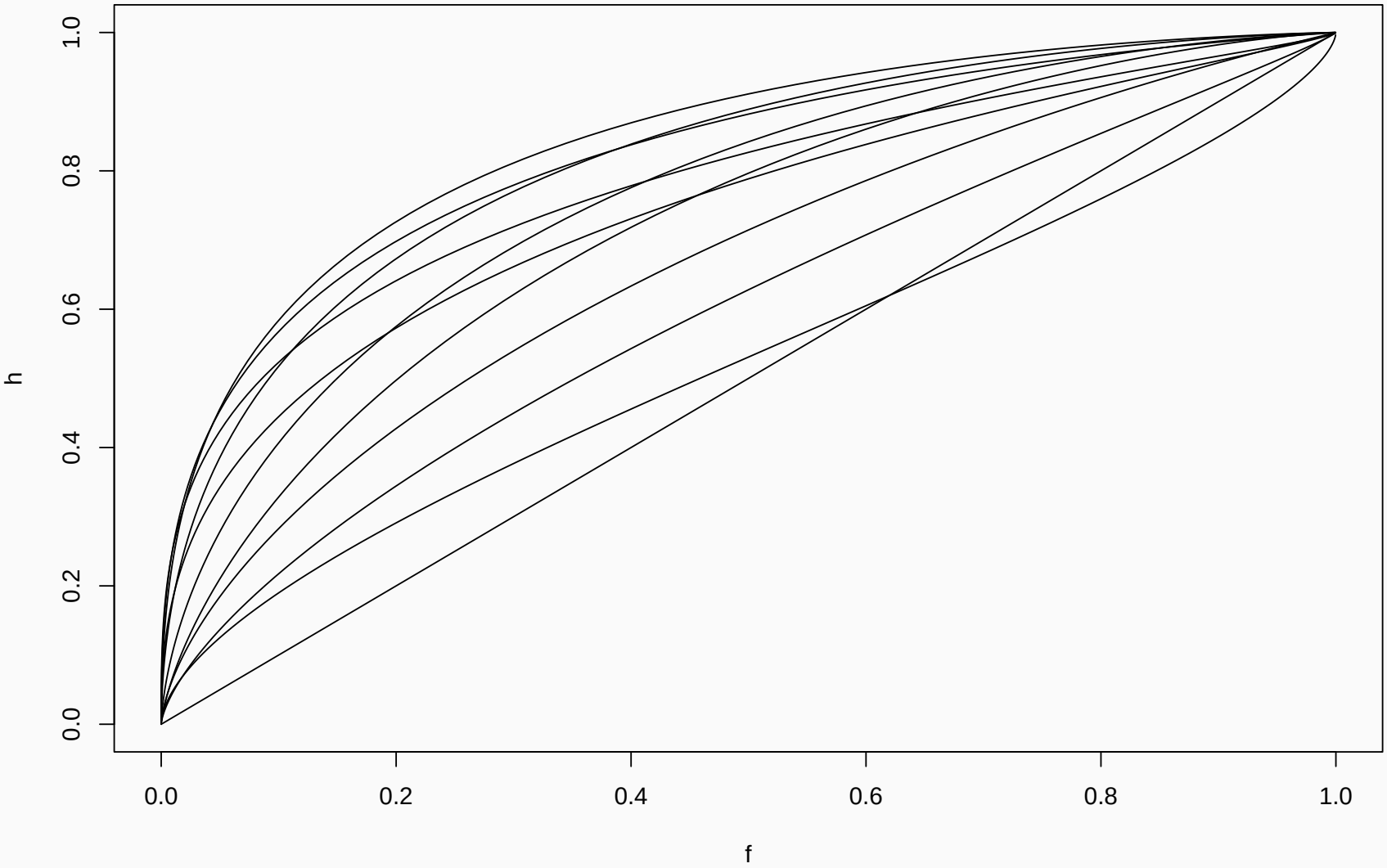
Formally expand UVSD to include a few strength conditions.

- Certainly μ is affected by strength?
- How about σ^2?
  - constant
  - free
  - tied to μ?

Your Turn

- Program up your multi-condition UVSD model in R
- Draw the resulting ROCs? Do they cross?
  - Always?
  - Sometimes?
  - Never?

My Turn



Your Turn

- Can you envision a gamma signal detection model?
- What might it look like?
- Does it make sense?

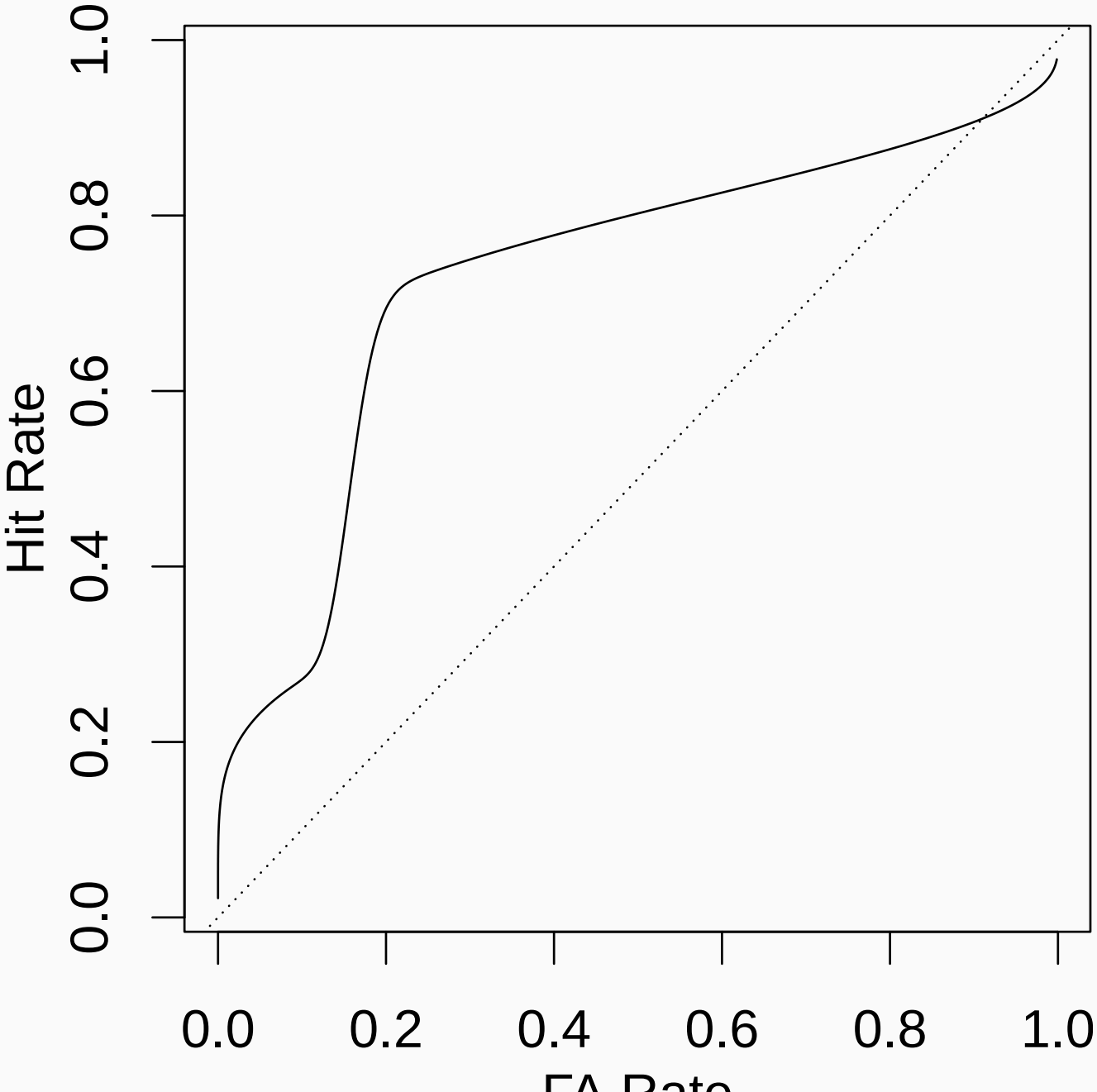
What is the core of signal detection?

- Latent strength is unidimensional
- Responses are governed by distributions relative to criteria

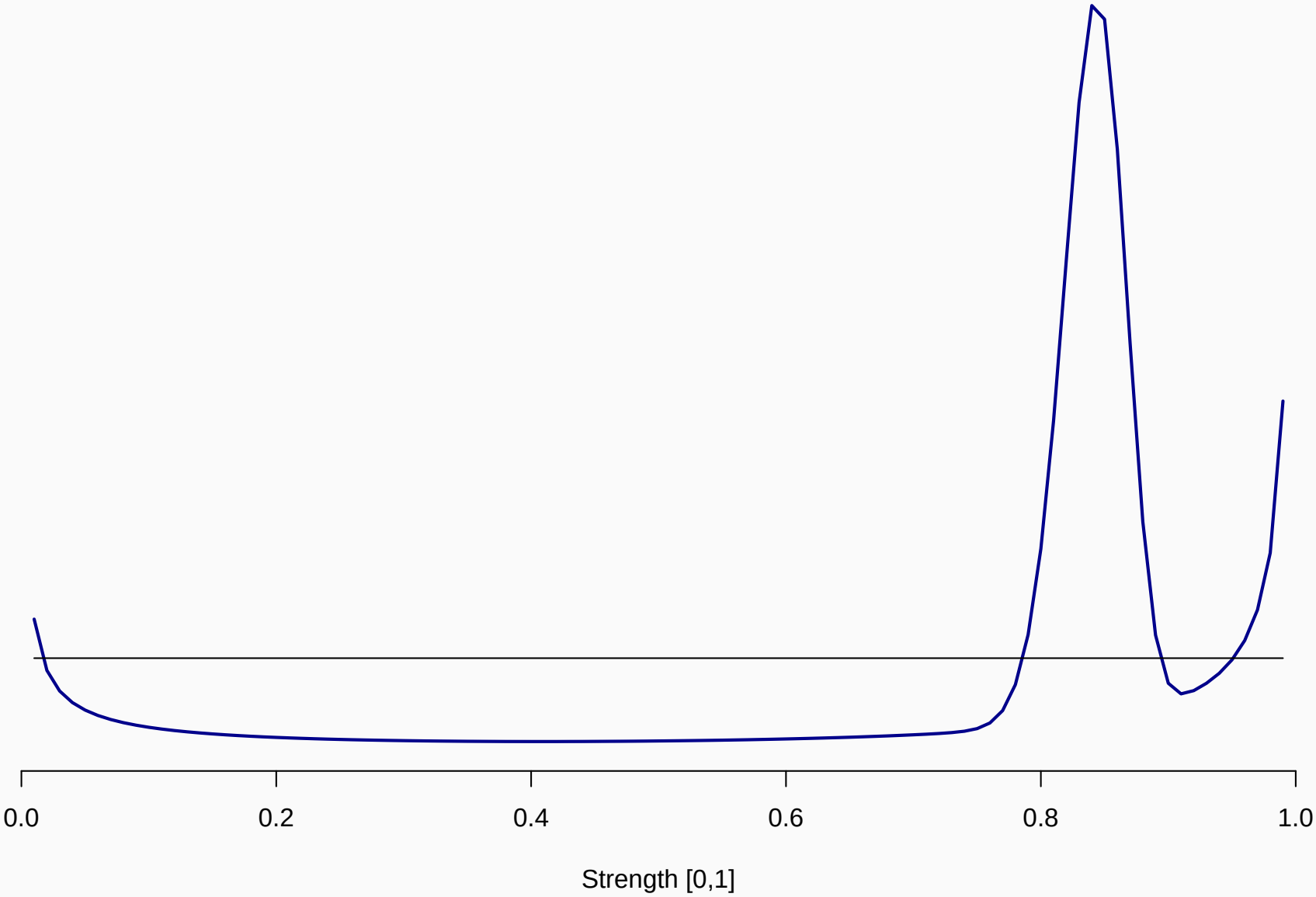
This core always hold

- It is unfalsifiable.
- Easiest proof is by construction.
  - X for new items; Y for old items
  - curve is $h(f)$.
  - set $X \sim \mathrm{U}(0,1)$ (for new items)
  - to calculate Y, $1 - F_Y(y) = h(y), \quad y \in [0,1]$
  - also, $f_Y(1-y) = dh(y)/dy$

Example, Is this whacky ROC signal-detection representable?



It is:



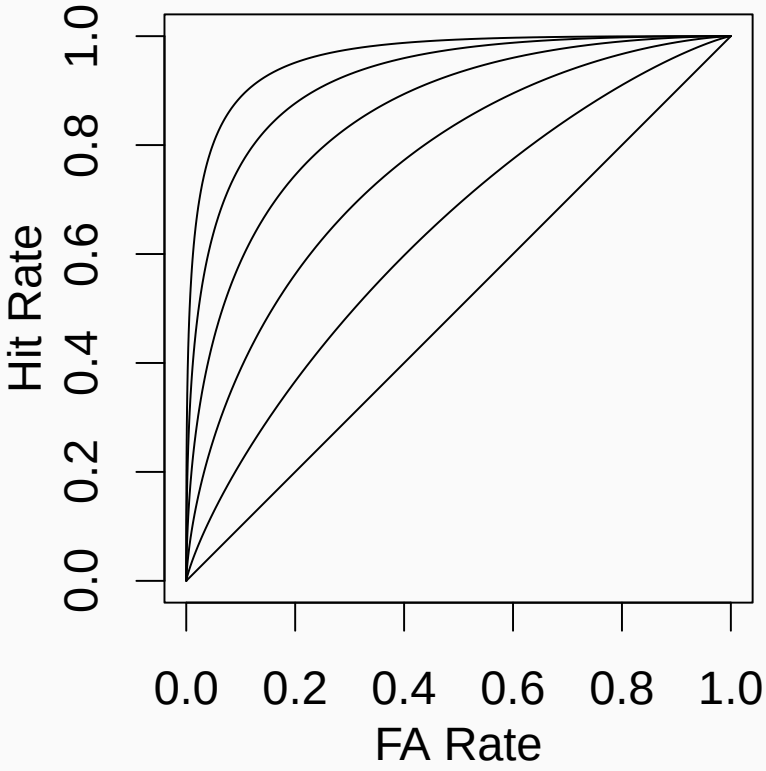
Signal Detection is Not Robust

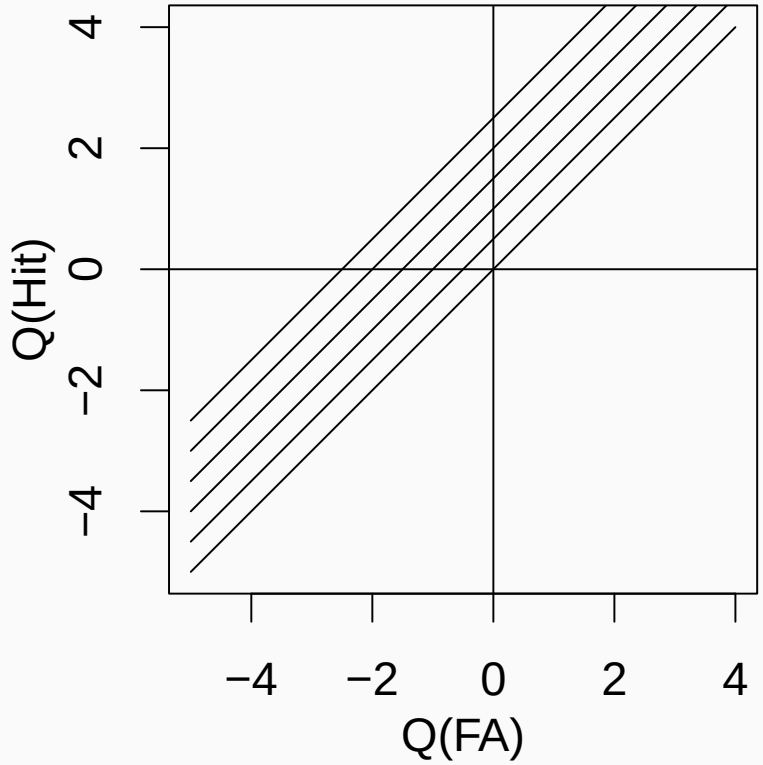
- Either you buy the normal as part of the process (arbitrary)
- Or you lose falsifiability
- UVSD is just a hack to fit data:
  - must buy unequal variance normal
  - must live with indominance
  - example of injudicious modeling.

Are There Robust Signal Detection Models?

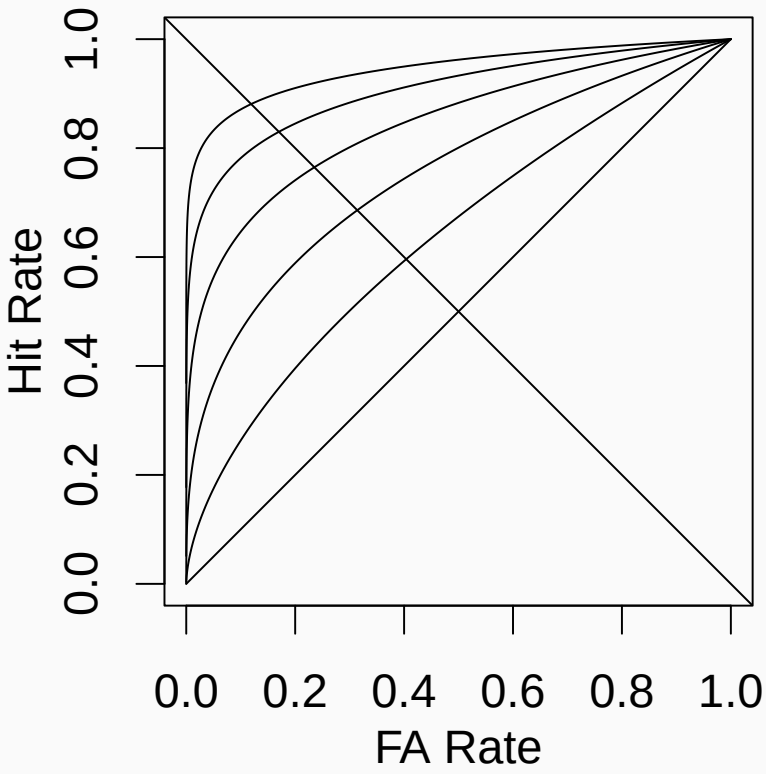
- Strength affects one part of the distribution.
- Scalar representability?

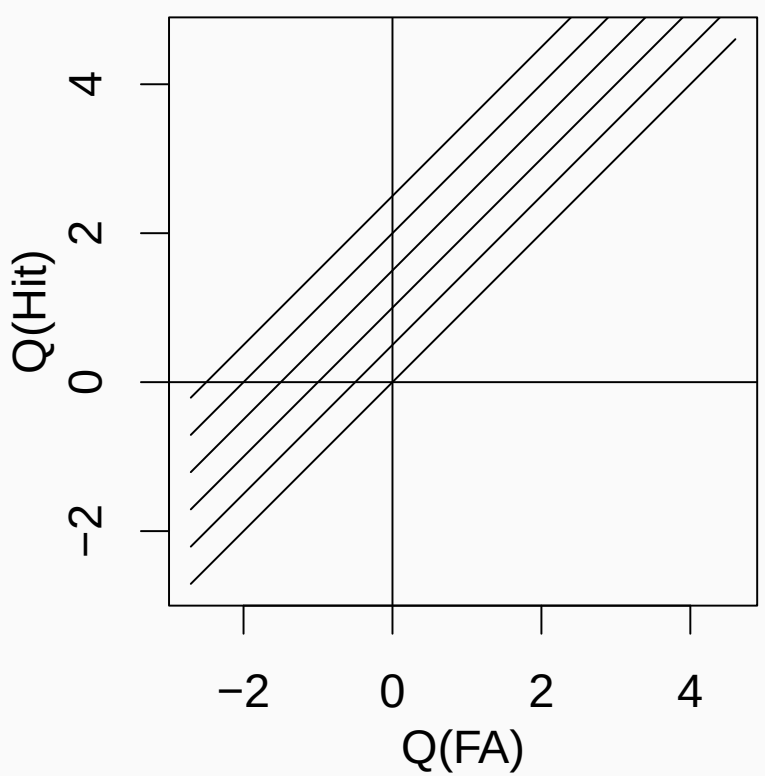
EVSD is Scalar Representable



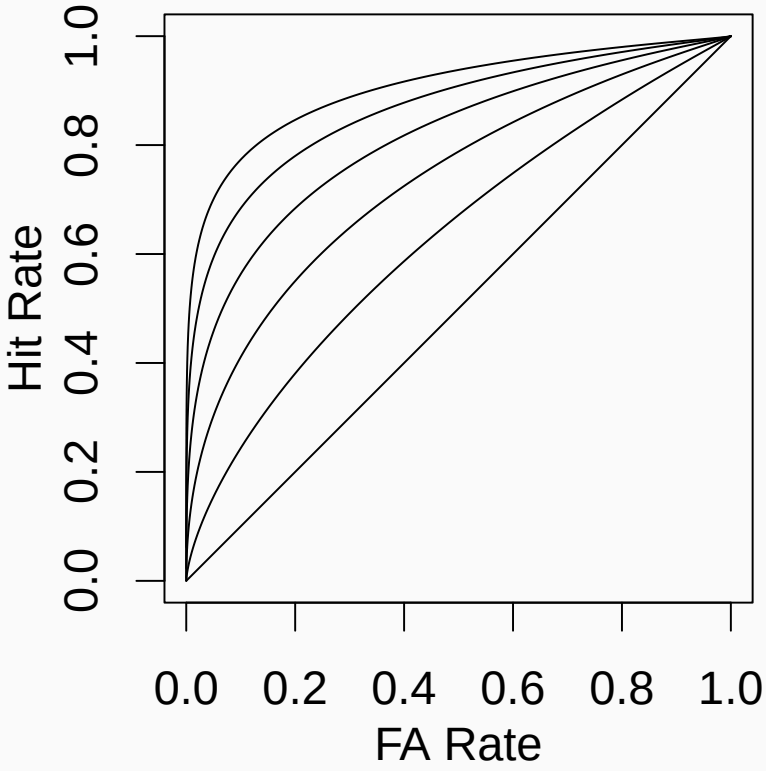


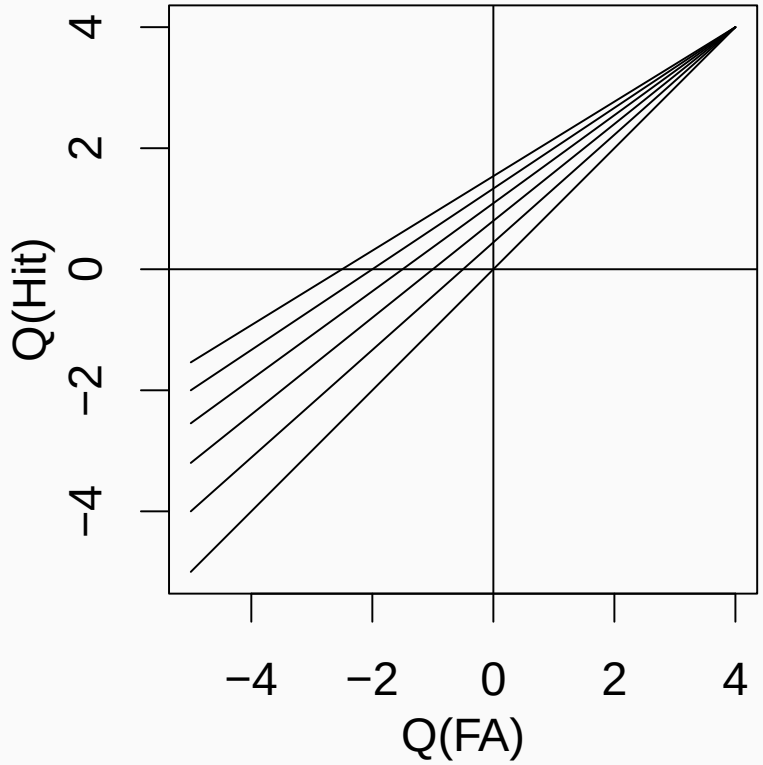
Gamma is Scalar Representable





UVSD is Not Scalar Representable





A robust signal detection model admits a function Q that linearizes ROCs with constant slope so that there is one difference—intercept, which is strength.

Extension To Item Response Theory

I ppl each respond to J dichotomous items. Let $Y_{ij} = 0, 1$ for incorrect and correct responses, respectively.

$$Y_{ij} \sim \text{Bernoulli}(\pi_{ij})$$

Item Ordering

If item A is harder than item B, then it is harder than item B for all people.

Rasch Model

$$Y_{ij} \sim \text{Bernoulli}(\pi_{ij})$$

$$\pi_{ij} = \frac{1}{1 + \exp[-(\gamma_i - \alpha_j)]}$$

- γ_i: ability of the ith person
- α_j: difficulty of the jth item

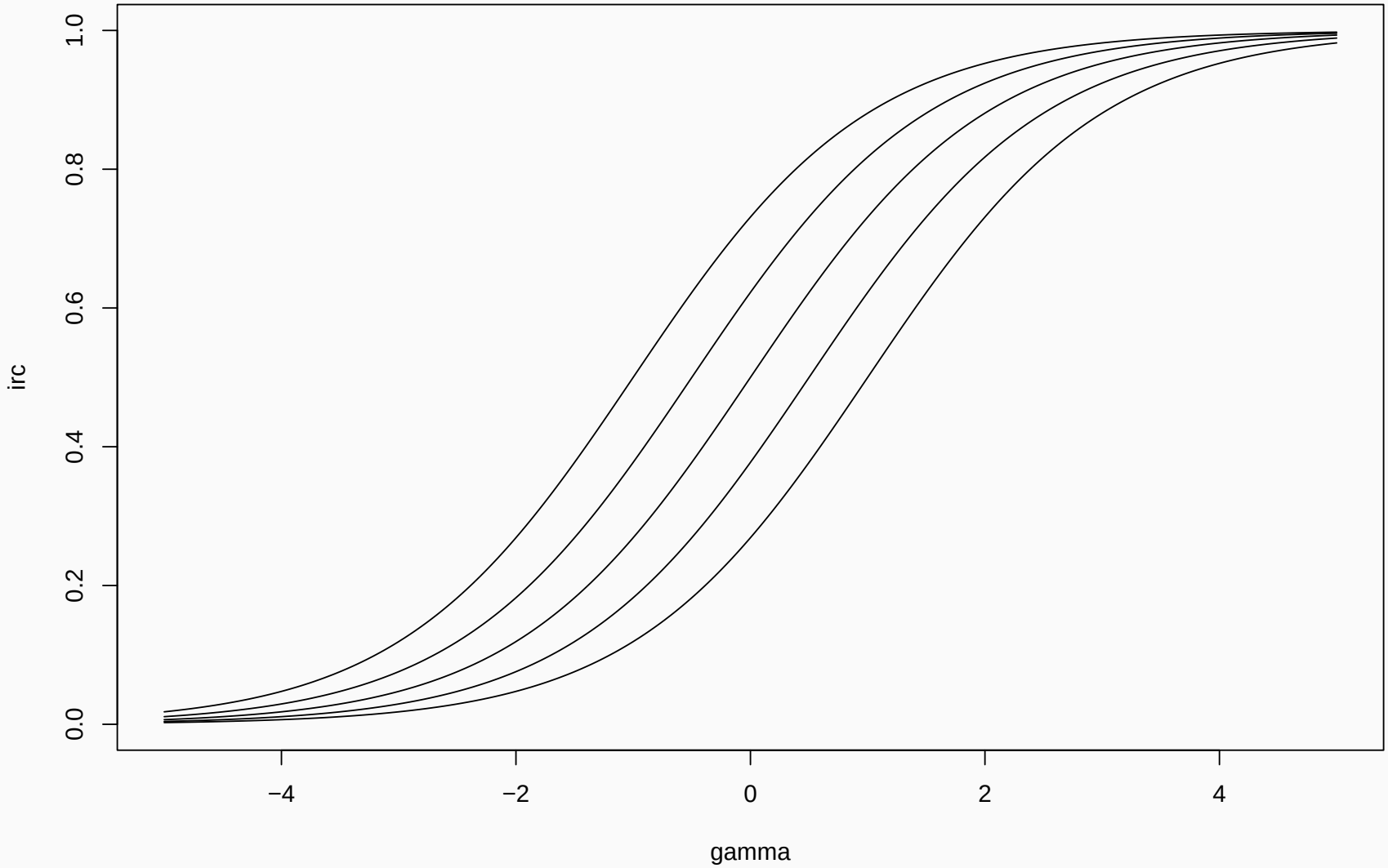
Rasch Model

- Is the Rasch model identifiable as is?
- What is needed?

Your Turn

- Does the Rasch model exhibit item ordering?
- Hint: Draw an item response curve which is a function of γ_i. It should increase as people get better.
- Now compare a few items

My Turn



2PL

Much like in signal detection, researchers thought that the Rasch model was insufficient to capture real world response curves.

Your Turn

Draw some response curves for 2PL

Our Turn

- 2PL is the leading IRT base model
- Changes in discriminability force violations of item ordering
- When might 2PL be appropriate?
- When might it not?

Your Turn

Can you think of general model we could use estimation to explore whether items order?

- two parameters for items (shift, scale)
- both can vary and there is the possibility of crossing
- both can vary and there is the possibility of not crossing