Bootcamp : Physics

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Contents

Part I
Brownian motion

Diffusion give rise to *material transport*. It gives a quantitative understanding of many phenomenon (permeability...).

1 Historical aspects :

1834 : Clapeyron proposed a phenomenological equation for the ideal gas : $PV = nRT$,
$PV = N.K_B.T$ (N : number of particules, K_B Boltzman constant $1.38 \times 10^{-23} \frac{kg.m}{K.s^2}$)

1828 : Brown : studied pollen that was moving

1905 : Einstein : $\langle x^2 \rangle = 2DT$ (D : diffusion coefficient)

1908 : Perrin : confirmed the Einstein guess

2 Random-walk

Simplest model that describes Brownian motion. A particle has the same probabiblity to move to the right and to the left. After two jumps, there is one way to 2 ($\frac{1}{4}$) and -2 ($\frac{1}{4}$), and 2 ways to 0 ($\frac{1}{2}$).

$$\langle a \rangle = \frac{1}{m} \sum_{i=1}^{m} a_i = \sum_{i=1}^{m} p_i a_i$$

$$\langle a^2 \rangle = \frac{1}{m} \sum_{i=1}^{m} a_i^2$$

$\langle a \rangle$ the average position, with m the number of realisations, i the i-th realisation and a_i the i-th realisation.
In our example : $\langle a \rangle = \frac{1}{4}(2 + -2 + 0 + 0) = 0$ and $\langle a^2 \rangle = \frac{1}{4}(2^2 + -2^2 + 0^2 + 0^2) = 2$

In the 3 jumps case :

- 1 way to 3
- 1 way to -3
- 3 ways to 1
- 3 ways to -1

$\langle a \rangle = \frac{1}{8}(3 + -3 + 3 \times 1 + 3 \times -1) = 0$ and $\langle a^2 \rangle = \frac{1}{8}(3^2 + -3^2 + (3 \times 1)^2 + (3 \times -1)^2) = 3$

$$N - jumps = \langle a \rangle = 0 \text{ and } \langle a^2 \rangle = N$$

If N is large enough, distribution of position is a binomial distribution.

If we say that the boxes have a l length, then $x = a \times l$, t_0 is the time delay between successive jumps. So $\frac{t}{t_0} = N$

$$\langle x \rangle = \langle a \rangle l = 0$$

$$\begin{aligned} \langle x^2 \rangle &= \langle a^2 \rangle l^2 \\ &= l^2 N \\ &= t \frac{l^2}{t_0} \\ &= 2t \frac{l^2}{2t_0} \end{aligned}$$

$$So\ D = \frac{l^2}{2t_0}$$

Example :

- $D = 100\mu m^2/s$
- $l_{bacteria} = 1\mu m$
- $l_{cell} = 20\mu m$
- $l_{arm} = 1m$

$$t_{bacteria} = \frac{l^2}{2D} = \frac{(1\mu m)^2}{2(100\mu m^2/s)} = 5ms$$
$$t_{cell} = \frac{l^2}{2D} = \frac{(20\mu m)^2}{2(100\mu m^2/s)} = 2s$$
$$t_{arm} = \frac{l^2}{2D} = \frac{(1m)^2}{2(100\mu m^2/s)} = 300 years$$

Q : How long does it take to tRNA to find a ribosome ?

- $N_{tRNA} = 200000$ molecules / bacteria
- the length of the bacteria is around $1\mu m$ long
- The volume of the bacteria : $V = 1\mu m^3$
- $C_{tRNA} = \frac{200000}{1}$
- $d = \sqrt[3]{\frac{1}{C_{tRNA}}} \approx 0.016\mu m$
- So $D = 100\mu m/s^2 \rightarrow t = \frac{1^2}{2D} \approx 1\mu s$
- But ribosomes synthetize at 40 nucleotides per seconds, the the synthesis is about of 2 seconds for a 80 mucleotides tRNA.

So the ribosomes binds quickly to the tRNA, and takes longer to synthesize.

What is the concentration ? $C = \frac{Number\ of\ particles}{Volume} = 200000 molec/\mu m$. Although, the concentration is not uniform is the bacteria.
What is flux (j) ? Flux the number of particules that pass through an area A per unit of time. $j \propto 2 - 1 = 1$

$$\frac{\#\ of\ particles}{\text{time step} \times \text{area of the plane}}$$

Example :

- $N_{H_2O} = 10^{10} molecules$
- $T = 3000 sec$
- $A = 6\mu m^2$

$$j = \frac{10^{10}}{3 \times 10^3 \times 6} = 10^6 \frac{molecules}{s.\mu m^2}$$

Fick's Law

$$j = -D\frac{\partial c}{\partial x}$$

Darcy's Law

$$v = -d\frac{\partial P}{\partial x} \text{d : mobility coeef}$$

Ohm's Law

$$\overrightarrow{j_E} = \sigma \overrightarrow{E} = -\sigma\frac{\partial V}{\partial x} \quad \text{sigma : electric conductivity, V : electric potential}$$

Fourier's Law

$$j_T = -K\frac{\partial T}{\partial x} \quad \text{T : temperature, K : thermal conductivity}$$

Random walk :

$$j = \frac{particles(\rightarrow) - particles(\leftarrow)}{t_0 A}$$
$$particles(\rightarrow) = N(x) \times \frac{1}{2} = C(x)V_{Box}\frac{1}{2} = C(x)Al\frac{1}{2}$$
$$particles(\leftarrow) = N(x+l)\frac{1}{2} = C(x+l)Al\frac{1}{2}$$

$$\begin{aligned} j &= \frac{C(x)\frac{l}{2} - C(x+l)\frac{l}{2}}{t_0} \\ &= \frac{l^2}{2t_0}\left(\frac{C(x) - C(x+l)}{l}\right) \\ &= -D\frac{\partial C}{\partial x} \end{aligned}$$

Law of mass conservation : $\#part@_{t+t_0} = \#part@_t + \#part_{entering} - \#part_{leaving}$
$\#part_{entering} = j(x)At_0$, $part_{leaving} = j(x+l)At_0$
$\#part@_{t+t_0} = C(x, t+t_0)lA$

$$\begin{aligned} (...) &= \left(\frac{C(x,t+t_0) - C(x,t)}{t_0}\right) lAt_0 \\ &= \frac{\partial C}{\partial t} lAT_0 \\ \frac{\partial C}{\partial t} &= -\frac{\partial j}{\partial x} \end{aligned}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

$$\frac{\partial C}{\partial t} = 0, \frac{\partial^2 C}{\partial x^2} = 0, \frac{\partial C}{\partial t} = 0$$

If there is a concentration gradient :

$$C = -Kx \text{with K the slope of the conentration gradient}$$

This is a non-equilibrium state

$$\begin{aligned} \frac{\partial C}{\partial x} &= -K \\ \frac{\partial C}{\partial t} &= 0 \\ \frac{\partial^2 C}{\partial x^2} &= 0 \end{aligned}$$

Example 3 : There is a film of water, and a drop of ink on it. We just take a plan to consider the diffusion. d is the size of the drop.

$$C(x,t) = \frac{N}{\sqrt{4\pi Dt}} e^{\frac{x^2}{4Dt}}$$

Example 4 : There are centain molecules inside a cell. There is much more molecules outside the cell. There is a flux a the interface of the membrane.

$$\begin{aligned} j_M &= -P_m \Delta C \text{ with : } \Delta C = C_{ext} - C_{int} \\ R &= 10\mu m; P_m = 20\mu m/s \\ \frac{\partial C_i}{\partial t} &= \frac{j_m A}{V} = \frac{P_m A}{V}(C_e - C_i) \\ \frac{\partial C_i}{\partial t} &= -\frac{P_m A}{V}C_i + \frac{P_m A}{V}C_e \end{aligned}$$

$$
\begin{aligned}
\tau &= \frac{V}{P_m A} = \frac{L^3}{\frac{L}{T} L^2} = T \\
&= \frac{\frac{4}{3}\pi R^3}{P_m 4\pi R^2} = \frac{R}{3P_m} = \frac{1}{6} sec
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial C_i}{\partial t} &= -\frac{1}{t}(C_i - C_e) \\
\rightarrow C_i(t) &= C_e - C_e e^{-t/\tau}
\end{aligned}
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