Bootcamp : Maths

Baptiste Rouger

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1 Session 1

1.1 Calculus

Calculus : study of changes.

2 types of calculus : differential and integral calculus.

Differential : distance $\rightarrow$ speed
Integration : speed $\rightarrow$ distance

1.1.1 Derivative

Definition
The derivative is the slope of the tangent line on a point.
Definition : The derivative of f at x_0 is the slope of the line tangent to the graph of f at x_0.

$$\lim_{\Delta x \to 0}; \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x_0)$$

Some derivatives $f(x) = \frac{1}{x}(x > 0)$
$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x} = -\frac{1}{x_0^2 + x_0 \Delta x}$

$$f'(x_0) = \lim_{\Delta x \to 0}; -\frac{1}{x_0^2 + x_0 \Delta x} = -\frac{1}{x_0^2}$$

$f(x) = x^n$

$$f'(x) = \lim_{\Delta x \to 0}; \frac{(x + \Delta x)^n - x^n}{\Delta x} = \lim_{\Delta x \to 0}; \frac{(\binom{n}{0} x^n \Delta x^0 + \binom{n}{1} x^n \Delta x^1 + ...) - x^n}{\Delta x} = nx^{n-1}$$

$(f+g)'(x) = f'(x) + g'(x)$

Continuity
Notations : $f'(x) = \frac{dy}{dx} = \frac{df}{dx} = Df = \frac{d}{dx} f =$ y'

Example : 400 ft , $y = 400 - 16t^2$
$400 - 16t^2 = 0$
$t = 5$
On average, -80 ft per second. But we want to know the instantaneous speed when the pumpkin hits the ground :
$f'(x) = -32t$

$f'(5) = -160 ft.s^{-1}$

$f(x)$ is continuous at x_0 if

$$lim_{x\to x_0}; f(x) = f(x_0) \Leftrightarrow \lim_{x\to x_0}; f(x) - f(x_0) = 0$$

Theorem : If f is differentiable at x_0 then f is continuous at x_0

$$\lim_{x\to x_0}; f(x) - f(x_0) = \lim_{x\to x_0}; \frac{f(x)-f(x_0)}{x-x_0}(x-x_0) = f'(x_0).0 = 0$$

Derivatives of composed functions

$$\begin{aligned}(u+v)'(x) &= \lim_{\Delta x\to 0}; \frac{(u+v)(x+\Delta x)-(u+v)(x)}{\Delta x}\\ &= \lim_{\Delta x\to 0}; \frac{u(x+\Delta x)+v(x+\Delta x)-u(x)-v(x)}{\Delta x}\\ &= u'(x)+v'(x)\end{aligned}$$

$$\begin{aligned}(uv)' &= u'v+v'u\\ (\frac{u}{v})' &= \frac{u'v-v'u}{v^2}\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0}; \frac{\sin x}{x} &= 1\\ \lim_{x\to 0}; \frac{\cos x-1}{x} &= 0\end{aligned}$$

$$\begin{aligned}f'(x) &= \frac{d}{dx}\sin x\\ &= \lim_{x+\Delta x}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}\\ &= \lim_{x+\Delta x}\frac{\sin x\cos\Delta x+\sin\Delta x\cos x-\sin x}{\Delta x}\\ &= \lim_{x+\Delta x}\frac{\sin(\cos\Delta x-1)}{\Delta x}+\lim_{x+\Delta x}\frac{\cos x\sin\Delta x}{\Delta x}\\ &= 0+\cos x\end{aligned}$$

Chain rules $\quad y = f(g(t))$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dx}$$

$y = f(x) = \sin x$
$x = g(t) = t^2$
$y = sin(t^2)$

$$\frac{d}{dt}(\sin t^2) = (\frac{dy}{dx})(\frac{dx}{dt}) = (\cos x)(2t) = \cos(t^2)2t$$

$f(g(x))' = g'(x)f'(g(x))$

Implicit differntiation $y = f(x)$
$x^2 + y^2 = 1$

$\frac{d}{dx}(x^a) = ax^{a-1}$
$a = m/n; y = x^{m/n}; y^n = x^m$
$ny^{n-1}.\frac{dy}{dx} = mx^{m-1}$
$\frac{dy}{dx} = \frac{m}{n}\frac{x^{m-1}}{y^{n-1}} = \frac{m}{n}\frac{x^{m-1}}{x^{m-1^{n-1}}}$
$= \frac{m}{n}\frac{x^{m-1}}{x^{\frac{m(n-1)}{n}}}$
$\frac{m}{n}x^{\frac{m}{n}-1} = ax^{a-1}$

$$\begin{aligned}
\frac{d}{dx}(x^a) &= ax^{a-1} \\
a = m/n; y = x^{m/n}; y^n &= x^m \\
ny^{n-1}.\frac{dy}{dx} &= mx^{m-1} \\
\frac{dy}{dx} &= \frac{m}{n}\frac{x^{m-1}}{y^{n-1}} \\
&= \frac{m}{n}\frac{x^{m-1}}{x^{m-1^{n-1}}} \\
&= \frac{m}{n}\frac{x^{m-1}}{x^{\frac{m(n-1)}{n}}} \\
\frac{m}{n}x^{\frac{m}{n}-1} &= ax^{a-1}
\end{aligned}$$

1.2 Differential equations

1.3 Linear Algebra