Bootcamp : Physics

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Contents

Part I
Brownian motion

Diffusion give rise to *material transport*. It gives a quantitative understanding of many phenomenon (permeability...).

1 Historical aspects :

1834 : Clapeyron proposed a phenomenological equation for the ideal gas : $PV = nRT$, $PV = N.K_B.T$ (N : number of particules, K_B Boltzman constant $1.38 \times 10^{-23} \frac{kg.m}{K.s^2}$)

1828 : Brown : studied pollen that was moving

1905 : Einstein : $\langle x^2 \rangle = 2DT$ (D : diffusion coefficient)

1908 : Perrin : confirmed the Einstein guess

2 Random-walk

Simplest model that describes Brownian motion. A particle has the same probabiblity to move to the right and to the left. After two jumps, there is one way to 2 ($\frac{1}{4}$) and -2 ($\frac{1}{4}$), and 2 ways to 0 ($\frac{1}{2}$).

$$\langle a \rangle = \frac{1}{m} \sum_{i=1}^{m} a_i = \sum_{i=1}^{m} p_i a_i$$

$$\langle a^2 \rangle = \frac{1}{m} \sum_{i=1}^{m} a_i^2$$

$\langle a \rangle$ the average position, with m the number of realisations, i the i-th realisation and a_i the i-th realisation. In our example : $\langle a \rangle = \frac{1}{4}(2 + -2 + 0 + 0) = 0$ and $\langle a^2 \rangle = \frac{1}{4}(2^2 + -2^2 + 0^2 + 0^2) = 2$

In the 3 jumps case :

- 1 way to 3
- 1 way to -3
- 3 ways to 1
- 3 ways to -1

$\langle a \rangle = \frac{1}{8}(3 + -3 + 3 \times 1 + 3 \times -1) = 0$ and $\langle a^2 \rangle = \frac{1}{8}(3^2 + -3^2 + (3 \times 1)^2 + (3 \times -1)^2) = 3$

$$N - jumps = \langle a \rangle = 0 \text{ and } \langle a^2 \rangle = N$$

If N is large enough, distribution of position is a binomial distribution.

If we say that the boxes have a l length, then $x = a \times l$, t_0 is the time delay between successive jumps. So $\frac{t}{t_0} = N$

$$\langle x \rangle = \langle a \rangle l = 0$$

$$\begin{aligned} \langle x^2 \rangle &= \langle a^2 \rangle l^2 \\ &= l^2 N \\ &= t \frac{l^2}{t_0} \\ &= 2t \frac{l^2}{2t_0} \\ So\ D &= \frac{l^2}{2t_0} \end{aligned}$$

Example :

- $D = 100\mu m^2/s$
- $l_{bacteria} = 1\mu m$
- $l_{cell} = 20\mu m$
- $l_{arm} = 1m$

$$t_{bacteria} = \frac{l^2}{2D} = \frac{(1\mu m)^2}{2(100\mu m^2/s)} = 5ms$$

$$t_{cell} = \frac{l^2}{2D} = \frac{(20\mu m)^2}{2(100\mu m^2/s)} = 2s$$

$$t_{arm} = \frac{l^2}{2D} = \frac{(1m)^2}{2(100\mu m^2/s)} = 300years$$