

# Bootcamp : Physics

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## Part I

# Brownian motion

Diffusion give rise to *material transport*. It gives a quantitative understanding of many phenomenon (permeability...).

## 1 Historical aspects :

**1834** : Clapeyron proposed a phenomenological equation for the ideal gas :  $PV = nRT$ ,  
 $PV = N.K_B.T$  (N : number of particles,  $K_B$  Boltzman constant  $1.38 \times 10^{-23} \frac{kg.m}{K.s^2}$ )

**1828** : Brown : studied pollen that was moving

**1905** : Einstein :  $\langle x^2 \rangle = 2DT$  (D : diffusion coefficient)

**1908** : Perrin : confirmed the Einstein guess

## 2 Random-walk

Simplest model that describes Brownian motion. A particle has the same probability to move to the right and to the left. After two jumps, there is one way to 2 ( $\frac{1}{4}$ ) and -2 ( $\frac{1}{4}$ ), and 2 ways to 0 ( $\frac{1}{2}$ ).

$$\langle a \rangle = \frac{1}{m} \sum_{i=1}^m a_i = \sum_{i=1}^m p_i a_i$$
$$\langle a^2 \rangle = \frac{1}{m} \sum_{i=1}^m a_i^2$$

$\langle a \rangle$  the average position, with m the number of realisations, i the i-th realisation and  $a_i$  the i-th realisation.  
In our example :  $\langle a \rangle = \frac{1}{4}(2 + -2 + 0 + 0) = 0$  and  $\langle a^2 \rangle = \frac{1}{4}(2^2 + -2^2 + 0^2 + 0^2) = 2$

In the 3 jumps case :

- 1 way to 3
- 1 way to -3
- 3 ways to 1
- 3 ways to -1

$$\langle a \rangle = \frac{1}{8}(3 + -3 + 3 \times 1 + 3 \times -1) = 0 \text{ and } \langle a^2 \rangle = \frac{1}{8}(3^2 + -3^2 + (3 \times 1)^2 + (3 \times -1)^2) = 3$$

$$N - jumps = \langle a \rangle = 0 \text{ and } \langle a^2 \rangle = N$$

If N is large enough, distribution of position is a binomial distribution.

If we say that the boxes have a  $l$  length, then  $x = a \times l$ ,  $t_0$  is the time delay between successive jumps. So  $\frac{t}{t_0} = N$

$$\langle x \rangle = \langle a \rangle l = 0$$

$$\langle x^2 \rangle = \langle a^2 \rangle l^2$$

$$= l^2 N$$

$$= l^2 \frac{t}{t_0}$$

$$= 2t \frac{l^2}{2t_0}$$

$$\text{So } D = \frac{l^2}{2t_0}$$

**Example :**

- $D = 100\mu m^2/s$
- $l_{bacteria} = 1\mu m$
- $l_{cell} = 20\mu m$
- $l_{arm} = 1m$

$$t_{bacteria} = \frac{l^2}{2D} = \frac{(1\mu m)^2}{2(100\mu m^2/s)} = 5ms$$

$$t_{cell} = \frac{l^2}{2D} = \frac{(20\mu m)^2}{2(100\mu m^2/s)} = 2s$$

$$t_{arm} = \frac{l^2}{2D} = \frac{(1m)^2}{2(100\mu m^2/s)} = 300years$$

$Q$  : How long does it take to tRNA to find a ribosome ?

- $N_{tRNA} = 200000$  molecules / bacteria
- the length of the bacteria is around  $1\mu m$  long
- The volume of the bacteria :  $V = 1\mu m^3$
- $C_{tRNA} = \frac{200000}{1}$
- $d = \sqrt[3]{\frac{1}{C_{tRNA}}} \approx 0.016\mu m$
- So  $D = 100\mu m/s^2 \rightarrow t = \frac{1^2}{2D} \approx 1\mu s$
- But ribosomes synthetize at 40 nucleotides per seconds, the the synthesis is about of 2 seconds for a 80 nucleotides tRNA.

So the ribosomes binds quickly to the tRNA, and takes longer to synthesize.

What is the concentration ?  $C = \frac{\text{Number of particles}}{\text{Volume}} = 200000molec/\mu m$ . Although, the concentration is not uniform in the bacteria.

What is flux ( $j$ ) ? Flux the number of particles that pass through an area  $A$  per unit of time.  $j \propto 2 - 1 = 1$

$$\frac{\# \text{ of particles}}{\text{time step} \times \text{area of the plane}}$$

Example :

- $N_{H_2O} = 10^{10}$  molecules
- $T = 3000sec$
- $A = 6\mu m^2$

$$j = \frac{10^{10}}{3 \times 10^3 \times 6} = 10^6 \frac{\text{molecules}}{s \cdot \mu m^2}$$

Fick's Law

$$j = -D \frac{\partial c}{\partial x}$$

Darcy's Law

$$v = -d \frac{\partial P}{\partial x} \text{ : mobility coef}$$

Ohm's Law

$$\vec{j}_E = \sigma \vec{E} = -\sigma \frac{\partial V}{\partial x} \quad \text{sigma : electric conductivity, V : electric potential}$$

Fourier's Law

$$j_T = -K \frac{\partial T}{\partial x} \quad T : \text{temperature, K : thermal conductivity}$$

Random walk :

$$j = \frac{\text{particles}(\rightarrow) - \text{particles}(\leftarrow)}{t_0 A}$$

$$\text{particles}(\rightarrow) = N(x) \times \frac{1}{2} = C(x) V_{Box} \frac{1}{2} = C(x) A l \frac{1}{2}$$

$$\text{particles}(\leftarrow) = N(x+l) \frac{1}{2} = C(x+l) A l \frac{1}{2}$$

$$j = \frac{C(x) \frac{l}{2} - C(x+l) \frac{l}{2}}{t_0}$$

$$= \frac{l^2}{2t_0} \left( \frac{C(x) - C(x+l)}{l} \right)$$

$$= -D \frac{\partial C}{\partial x}$$

Law of mass conservation :  $\#part@_{t+t_0} = \#part@_t + \#part_{entering} - \#part_{leaving}$   
 $\#part_{entering} = j(x) A t_0$ ,  $\#part_{leaving} = j(x+l) A t_0$   
 $\#part@_{t+t_0} = C(x, t+t_0) l A$

$$(\dots) = \left( \frac{C(x, t+t_0) - C(x, t)}{t_0} \right) l A t_0$$

$$= \frac{\partial C}{\partial t} l A t_0$$

$$\frac{\partial C}{\partial t} = - \frac{\partial j}{\partial x}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

$$\frac{\partial C}{\partial t} = 0, \frac{\partial^2 C}{\partial x^2} = 0, \frac{\partial C}{\partial x} = 0$$

If there is a concentration gradient :

$$C = -Kx \text{ with } K \text{ the slope of the concentration gradient}$$

This is a non-equilibrium state

$$\frac{\partial C}{\partial x} = -K$$

$$\frac{\partial C}{\partial t} = 0$$

$$\frac{\partial^2 C}{\partial x^2} = 0$$

*Example 3* : There is a film of water, and a drop of ink on it. We just take a plan to consider the diffusion.  
 $d$  is the size of the drop.