Bootcamp : Physics

Baptiste Rouger

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carles.blanch-mercader@curie.fr

Contents

Part I
Brownian motion

Diffusion give rise to *material transport*. It gives a quantitative understanding of many phenomenon (permeability...).

1 Historical aspects :

1834 : Clapeyron proposed a phenomenological equation for the ideal gas : $PV = nRT$, $PV = N.K_B.T$ (N : number of particules, K_B Boltzman constant $1.38 \times 10^{-23} \frac{kg.m}{K.s^2}$)

1828 : Brown : studied pollen that was moving

1905 : Einstein : $\langle x^2 \rangle = 2DT$ (D : diffusion coefficient)

1908 : Perrin : confirmed the Einstein guess

2 Random-walk

Simplest model that describes Brownian motion. A particle has the same probabiblity to move to the right and to the left. After two jumps, there is one way to 2 ($\frac{1}{4}$) and -2 ($\frac{1}{4}$), and 2 ways to 0 ($\frac{1}{2}$).

$$\langle a \rangle = \frac{1}{m} \sum_{i=1}^{m} a_i = \sum_{i=1}^{m} p_i a_i$$
$$\langle a^2 \rangle = \frac{1}{m} \sum_{i=1}^{m} a_i^2$$

$\langle a \rangle$ the average position, with m the number of realisations, i the i-th realisation and a_i the i-th realisation. In our example : $\langle a \rangle = \frac{1}{4}(2 + -2 + 0 + 0) = 0$ and $\langle a^2 \rangle = \frac{1}{4}(2^2 + -2^2 + 0^2 + 0^2) = 2$

In the 3 jumps case :

- 1 way to 3
- 1 way to -3
- 3 ways to 1
- 3 ways to -1

$\langle a \rangle = \frac{1}{8}(3 + -3 + 3 \times 1 + 3 \times -1) = 0$ and $\langle a^2 \rangle = \frac{1}{8}(3^2 + -3^2 + (3 \times 1)^2 + (3 \times -1)^2) = 3$

$$N - jumps = \langle a \rangle = 0 \text{ and } \langle a^2 \rangle = N$$

If N is large enough, distribution of position is a binomial distribution.

If we say that the boxes have a l length, then $x = a \times l$, t_0 is the time delay between successive jumps. So $\frac{t}{t_0} = N$

$$\langle x \rangle = \langle a \rangle l = 0$$

$$\begin{aligned} \langle x^2 \rangle &= \langle a^2 \rangle l^2 \\ &= l^2 N \\ &= t \frac{l^2}{t_0} \\ &= 2t \frac{l^2}{2t_0} \end{aligned}$$
$$So\ D = \frac{l^2}{2t_0}$$

Example :

- $D = 100\mu m^2/s$
- $l_{bacteria} = 1\mu m$
- $l_{cell} = 20\mu m$
- $l_{arm} = 1m$

$$t_{bacteria} = \frac{l^2}{2D} = \frac{(1\mu m)^2}{2(100\mu m^2/s)} = 5ms$$
$$t_{cell} = \frac{l^2}{2D} = \frac{(20\mu m)^2}{2(100\mu m^2/s)} = 2s$$
$$t_{arm} = \frac{l^2}{2D} = \frac{(1m)^2}{2(100\mu m^2/s)} = 300years$$

Q : How long does it take to tRNA to find a ribosome ?

- $N_{tRNA} = 200000$ molecules / bacteria
- the length of the bacteria is around $1\mu m$ long
- The volume of the bacteria : $V = 1\mu m^3$
- $C_{tRNA} = \frac{200000}{1}$
- $d = \sqrt[3]{\frac{1}{C_{tRNA}}} \approx 0.016\mu m$
- So $D = 100\mu m/s^2 \rightarrow t = \frac{1^2}{2D} \approx 1\mu s$
- But ribosomes synthetize at 40 nucleotides per seconds, the the synthesis is about of 2 seconds for a 80 mucleotides tRNA.

So the ribosomes binds quickly to the tRNA, and takes longer to synthesize.

What is the concentration ? $C = \frac{Number\ of\ particles}{Volume} = 200000 molec/\mu m$. Although, the concentration is not uniform is the bacteria.
What is flux (j) ? Flux the number of particules that pass through an area A per unit of time. $j \propto 2 - 1 = 1$

$$\frac{\#\ of\ particles}{\text{time step} \times \text{area of the plane}}$$

Example :

- $N_{H_2O} = 10^{10} molecules$
- $T = 3000sec$
- $A = 6\mu m^2$

$$j = \frac{10^{10}}{3 \times 10^3 \times 6} = 10^6 \frac{molecules}{s.\mu m^2}$$

Fick's Law

$$j = -D\frac{\partial c}{\partial x}$$

Darcy's Law

$$v = -d\frac{\partial P}{\partial x} \text{d : mobility coeef}$$

Ohm's Law

$$\overrightarrow{j_E} = \sigma\overrightarrow{E} = -\sigma\frac{\partial V}{\partial x} \quad \text{sigma : electric conductivity, V : electric potential}$$

Fourier's Law

$$j_T = -K\frac{\partial T}{\partial x} \quad \text{T : temperature, K : thermal conductivity}$$

Random walk :

$$j = \frac{particles(\rightarrow) - particles(\leftarrow)}{t_0 A}$$
$$particles(\rightarrow) = N(x) \times \frac{1}{2} = C(x) V_{Box} \frac{1}{2} = C(x) A l \frac{1}{2}$$
$$particles(\leftarrow) = N(x+l) \frac{1}{2} = C(x+l) A l \frac{1}{2}$$

$$\begin{aligned} j &= \frac{C(x)\frac{l}{2} - C(x+l)\frac{l}{2}}{t_0} \\ &= \frac{l^2}{2t_0} \left(\frac{C(x) - C(x+l)}{l} \right) \\ &= -D \frac{\partial C}{\partial x} \end{aligned}$$

Law of mass conservation : $\#part@_{t+t_0} = \#part@_t + \#part_{entering} - \#part_{leaving}$
$\#part_{entering} = j(x) A t_0$, $part_{leaving} = j(x+l) A t_0$
$\#part@_{t+t_0} = C(x, t+t_0) l A$

$$\begin{aligned} (...) &= \left(\frac{C(x, t+t_0) - C(x,t)}{t_0} \right) l A t_0 \\ &= \frac{\partial C}{\partial t} l A T_0 \\ \frac{\partial C}{\partial t} &= -\frac{\partial j}{\partial x} \end{aligned}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

$$\frac{\partial C}{\partial t} = 0, \frac{\partial^2 C}{\partial x^2} = 0, \frac{\partial C}{\partial t} = 0$$

If there is a concentration gradient :

$$C = -Kx \text{with K the slope of the conentration gradient}$$

This is a non-equilibrium state

$$\begin{aligned} \frac{\partial C}{\partial x} &= -K \\ \frac{\partial C}{\partial t} &= 0 \\ \frac{\partial^2 C}{\partial x^2} &= 0 \end{aligned}$$

Example 3 : There is a film of water, and a drop of ink on it. We just take a plan to consider the diffusion. d is the size of the drop.

$$C(x,t) = \frac{N}{\sqrt{4\pi D t}} e^{\frac{x^2}{4Dt}}$$

Example 4 : There are centain molecules inside a cell. There is much more molecules outside the cell. There is a flux a the interface of the membrane.

$$\begin{aligned} j_M &= -P_m \Delta C \text{ with : } \Delta C = C_{ext} - C_{int} \\ R &= 10\mu m; P_m = 20\mu m/s \\ \frac{\partial C_i}{\partial t} &= \frac{j_m A}{V} = \frac{P_m A}{V}(C_e - C_i) \\ \frac{\partial C_i}{\partial t} &= -\frac{P_m A}{V} C_i + \frac{P_m A}{V} C_e \end{aligned}$$

$$\tau = \frac{V}{P_m A} = \frac{L^3}{\frac{L}{T}L^2} = T$$
$$= \frac{\frac{4}{3}\pi R^3}{P_m 4\pi R^2} = \frac{R}{3P_m} = \frac{1}{6}sec$$

$$\frac{\partial C_i}{\partial t} = -\frac{1}{t}(C_i - C_e)$$
$$\to C_i(t) = C_e - C_e e^{-t/\tau}$$

3 Bias Random walk

This is the case where the probabiblity of going left is not the same than to go right. $K_+ \neq K_-$

$$F = \gamma v \to v = \frac{F}{\gamma} \text{ avec } \gamma \text{ le coefficient de friction}$$

$$j_{New} = vC$$
$$j = -D\frac{\partial C}{\partial c} + vc$$

$\Delta x = v\Delta t$ and $N = C\Delta x A = cA\Delta t v$

$$j = \frac{\#_{part}}{time.area} = \frac{CvA\Delta t}{\Delta t A}$$
$$= Cv = c\frac{F}{\gamma}$$

$$j_{tot} = -D\frac{\partial C}{\partial x} + Cv$$

Equilibrium situation :

$$0 = -D\frac{\partial C}{\partial x} + C\frac{F}{\gamma}$$

With the force $F = cst$

$$\frac{\partial C}{\partial x} = \frac{CF}{\gamma}; \frac{\partial C}{\partial x} = \frac{CF}{\gamma D}$$
$$\frac{\partial C}{\partial x} = \frac{C}{\lambda} \text{ as } \lambda = \frac{\gamma D}{F}$$
$$\frac{\partial C}{\partial x} = \frac{C}{\lambda}; \frac{\partial C}{C} = \frac{\partial x}{\lambda}$$

$$\int \frac{\partial C}{C} = \ln C - \ln C_0 = \ln\frac{C}{C_0}$$
$$\int \frac{\partial x}{\lambda} = \frac{x}{\lambda}$$

$$\text{So : } \ln\frac{C}{C_0} = \frac{x}{\lambda}$$
$$C = C_0 e^{x/\lambda}$$

Same case as above, but with $F = -Kx$

$$j_{Tot} = 0 = -D\frac{\partial C}{\partial x} + C\frac{F}{\gamma} = -D\frac{\partial C}{\partial x} - \frac{CKx}{\gamma} = C$$

Solution :

$$D\frac{\partial C}{\partial x} = -\frac{KCx}{\gamma}; \frac{\partial C}{\partial x} = -\frac{KCx}{\gamma D}$$

$$\frac{dC}{dx} = \frac{-KCx}{\gamma D}; \frac{dC}{C} = -\frac{KCx}{\gamma D}dx;$$

$$\int \frac{dC}{C} = \ln C - \ln C_0$$

$$\int \frac{-Kx}{\gamma D}dx = -\frac{K}{\gamma D}\frac{x^2}{2}$$

$$\ln\frac{C}{C_0} = -\frac{Kx^2}{2\gamma D} \Rightarrow$$

$$C = C_0 e^{\frac{-Kx}{2\gamma D}}$$

We expect that : $\sigma^2 = \frac{\gamma D}{K}$