MATHS ASSIGNMENT 6

Date

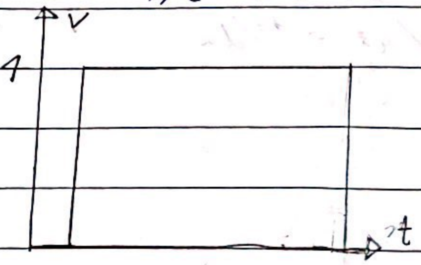
SAAD RIAZUDDIN MAHMOOD (22803)

EXERCISE SET 2.1

Q1) (a) velocity = gradient

$v = (30-0)/(10-2.5) = 4$

(b)



Q2) AT $t=4$

$v =$ gradient

$v \approx (70-0)/(8-2)$

$v \approx 11.67$

AT $t=8$

$v =$ gradient

$v \approx (140-0)/(10-4)$

$v \approx 23.33$

Q3) (a) 0

(b) $t = 0, 2, 4, 8$

(c) maximum → $t=1$

minimum → $t=3$

(d) $v =$ gradient

$v = (20-0)/(1.5-4.5)$

$v = -6.67$

Q4) (a) decreasing

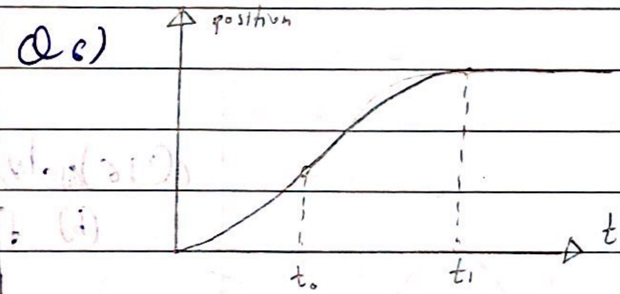
(b) increasing

(c) increasing

(d) decreasing

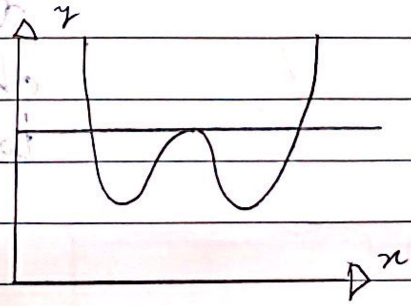
Q5) It is a straight line with its gradient equal to the velocity.

Q6)



Till $t=t_0$, the velocity increases, so does the slope since the gradient of the position-time graph is equal to gradient. From t_0 to t_1, the velocity decreases so the slope decreases. At t_1, the velocity drops to 0 so the slope also becomes 0.

Q7)



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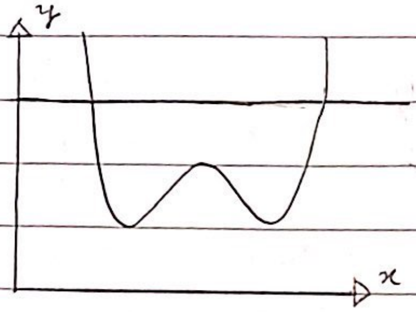
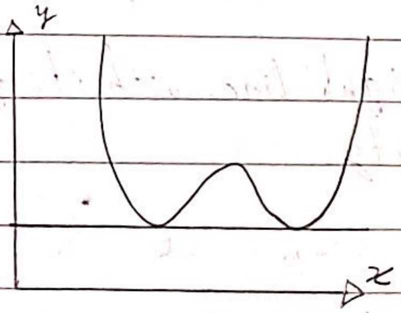
Q8)



Q9)



Q10)

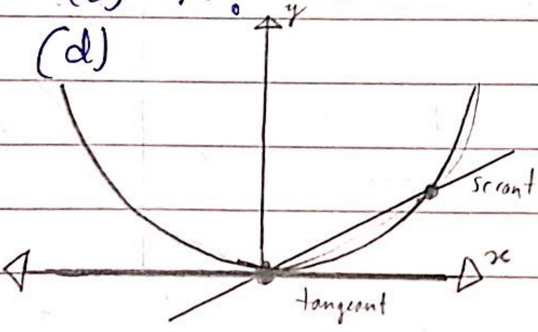
Q11) (a) $dy/dx = (2(1)^2 - 2(0)^2)/(1-0)$

$dy/dx = 2$

(b) $4(0) = 0$

(c) $4x_0 = dy/dx$

(d)



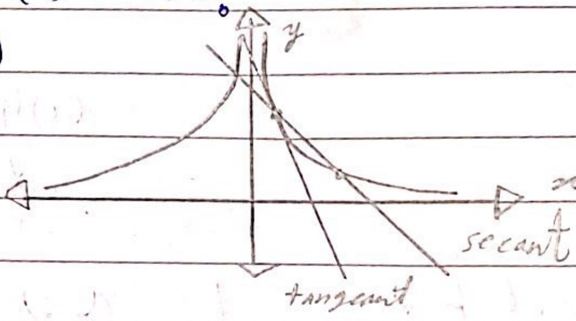
Q14) (a) $dy/dx = ((1/2^2)-(1/1^2))/(2-1)$

$dy/dx = -0.5$

(b) $-2/(1^3) = -2$

(c) $-2/x_0^3 = dy/dx$

(d)



Q16) (a) $dy/dx = 2x_0 + 3$

(b) $f'(2) = 7$

Q17) $dy/dx = 1 + x^{1/2} = 1 + \frac{1}{2\sqrt{x}}$

$f'(1) = 3/2$

Q19) True if we let $h = x - 1$.

Q20) False, secant line intersects twice while tangeant line intersects once.

Q21) False, it represents the change in position with respect to time.

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Q25) (a) At $t=0$

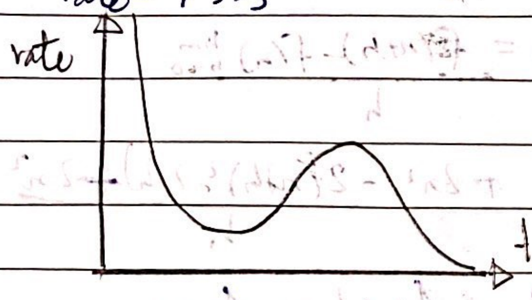
(b) gradient = growth rate

$= (200-75)/(17.50) = 7.14$

(c) 14 years

rate $= (200-0)/(17.5-2.5)$

rate $= 13.3$



Q28 (a) $v = (4.5(12)^2 - 4(0)^2)/(12-6)$

$v = 54$

(b) $f'(x) = 9t$

$f'(6) = 54$

$v = 54$

Q29) (a) $v = (6(4)^4 - 6(2)^4)/(4-2)$

$v = 720$

(b) $s'(t) = 24t^3$

$s'(2) = 24(2)^3 = 192$

$v = 192$

Q30) this is because to find the gradient of a point on a curve we need a tangent and to find the gradient of this tangent but we only have one point. the other point is taken to be arbitrary which lies on the graph. the ~~line~~ line joining these two points is the secant line. The tangent at point P is defined to as the line that is formed when this ~~arb~~ arbitrary point comes very close to Point P.

Q31) the instantaneous velocity at $t=1$ is equal to the limit as $h \to 0$ of the average velocity during the interval between $t=1$ and $t=1+h$.

EXERCISE SET 2.3

Q1) (a) $f'(1) = (5.5-2)/(2-0.5) = 2.33$

$f'(3) = 0$

$f'(5) = (3+1)/(6-5) = -2$

$f'(6) = (3-0)/(-2+4) = 1$

Q2) $f'(4) < f'(0) < f'(2) < 0 < f'(3)$

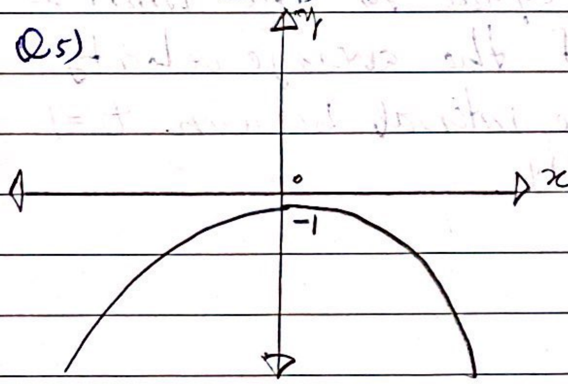
Q3) (a) $f'(a)$ will be the slope of the line

(b) $f'(2) = 3$

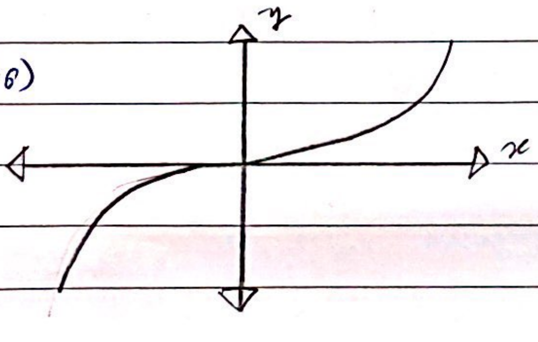
(c) $f'(2) = 3$

Q4) $f'(1) = \dfrac{-1-2}{-1-1} = \dfrac{3}{2}$

Q5)



Q6)



Q7) $y = mx + c$

$y = 5x + c$

$-1 = 5(3) + c$

$c = -16$

$y = 5x - 16$

Q9) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{2(x+h)^2 - 2x^2}{h} = \dfrac{2(x^2+h^2+2xh) - 2x^2}{h}$

$= \dfrac{h^2 + 4xh}{h} = 4x + h = 4x$

Q10) $f'(x) = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \dfrac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$

$= \dfrac{2x+2h+1-2x-1}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} = \dfrac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$

$= \dfrac{2}{2\sqrt{2x+1}} = \dfrac{1}{\sqrt{2x+1}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$

Q15) $f'(x) = \dfrac{f(x+h) - f(x)}{h}$

$f'(x) = \dfrac{1/(x+h) - 1/x}{h}$

$f'(x) = \dfrac{x - x - h}{hx(x+h)} = \dfrac{-h}{hx^2 + h^2x}$

$$= \frac{-1}{x^2+hx} = -1/x^2$$

Q20) $$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \frac{1/\sqrt{x+h-1} - 1/\sqrt{x-1}}{h}$$

$$f'(x) = \frac{\sqrt{x-1} - \sqrt{x+h-1}}{h\sqrt{(x-1)(x+h-1)}}$$

$$f'(x) = \frac{x-1-x-h+1}{h\sqrt{(x-1)(x+h-1)}(\sqrt{x-1}+\sqrt{x+h-1})}$$

$$f'(x) = \frac{-h}{h((x-1)\sqrt{x+h-1} + (x+h-1)\sqrt{x-1})}$$

$$f'(h) = \frac{-1}{(x-1)\sqrt{x-1} + (x-1)\sqrt{x-1}}$$

$$f'(h) = \frac{-1}{(2x-2)\sqrt{x-1}}$$

$$f'(h) = \frac{-1}{2(x-1)^{3/2}}$$

Q21) $$f'(t) = \frac{f(t+h) - f(t)}{h}$$

$$f'(t) = \frac{4(t+h)^2 - 4t^2}{h}$$

$$f'(t) = \frac{4t^2 + 4h^2 + 8th - 4t^2}{h}$$

$$f'(t) = 4h + 8t$$

$$f'(t) = 8t$$

Q22) $$V'(r) = \frac{V(r+h) - V(r)}{h}$$

$$V'(r) = \frac{4/3\,\pi(r+h)^3 - 4/3\,\pi r^3}{h}$$

$$V'(r) = \frac{4/3\,\pi(r^2+h^2+2rh)(r+h) - 4/3\,\pi r^3}{h}$$

$$V'(r) = \frac{4/3\,\pi(r^3+rh^2+2r^2h+hr^2+h^3+2rh^2) - 4/3\,\pi r^3}{h}$$

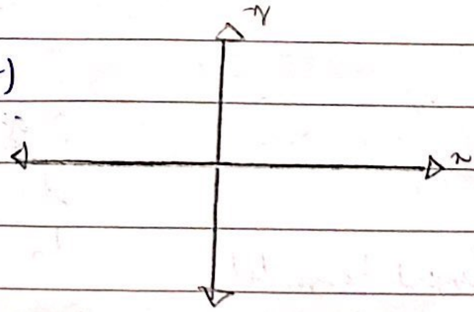
$$V'(r) = \frac{\pi(4r^3 - 4r^3 + 8r^2h + 4hr^2 + 4h^3 + 2rh^2)}{3h}$$

$$V'(r) = \frac{\pi(8r^2 + 4r^2)}{3} = \frac{12}{3}\pi r^2$$

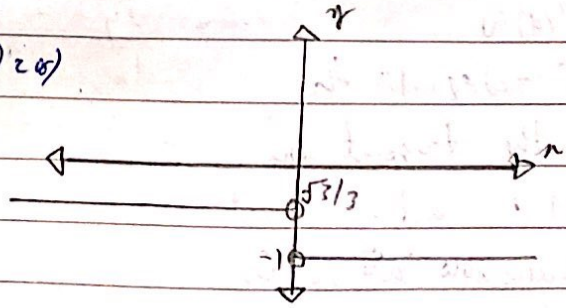
$$V'(r) = 4\pi r^2$$

Q23) (a) D (b) F (c) B
(d) C (e) A (f) E

Q25)



Q26)



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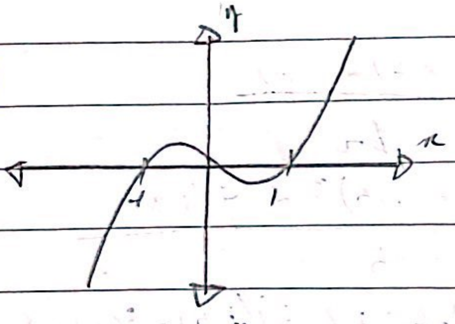
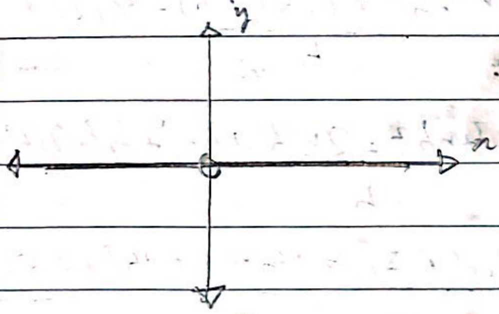
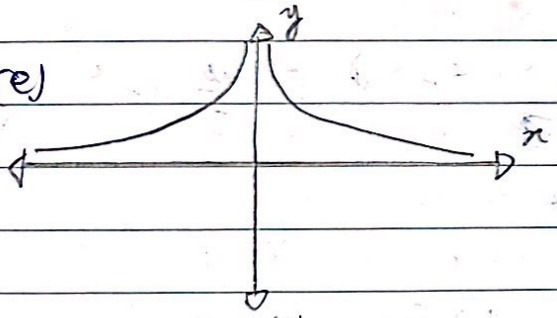
(c)



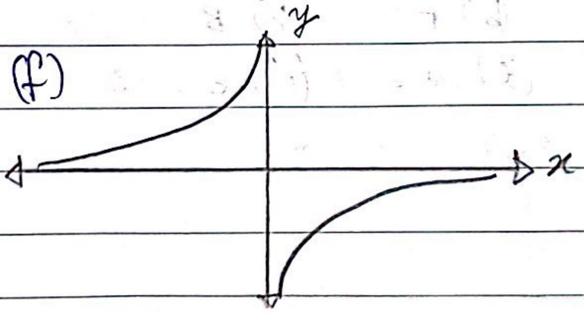
(d)



(e)



(f)



Q27) False, if tangent horizontal then $f'(a)=0$

Q28) True, $f'(-2)$ equals the slope of the tangent line.

Q29) False, $|x|$ is continuous but not differentiable at $x=0$.

Q30) True: if a function is differentiable at a point, then has to be continuous too

Q31) (a) $f(x)=\sqrt{x}$, $a=1$

(b) $f(x)=x^2$, $a=3$

Q32) (a) $f(x)=\cos x$, $a=\pi$

(b) $f(x)=x^7$, $a=1$

Q33) $\frac{dy}{dx} = \frac{(1-(x+h)^2)-(1-x)^2}{h} = \frac{(1-x^2-h^2-2xh)-(1-x)^2}{h}$

$= \frac{\cancel{1}-\cancel{x^2}-h^2-2xh-\cancel{1}+\cancel{x^2}+2x}{h}$

$= -2x-h = -2(1)-0 = -2$

Q34) $\frac{dy}{dx} = \frac{\frac{x+2+h}{x+h}-\frac{x+2}{x}}{h}$

$\frac{dy}{dx} = \frac{x(x+2+h)-(x+2)(x+h)}{hx(x+h)}$

$\frac{dy}{dx} = \frac{-2}{x(x+h)} = \frac{-2}{x^2}$

$\frac{dy}{dx} = \frac{-1}{2}$

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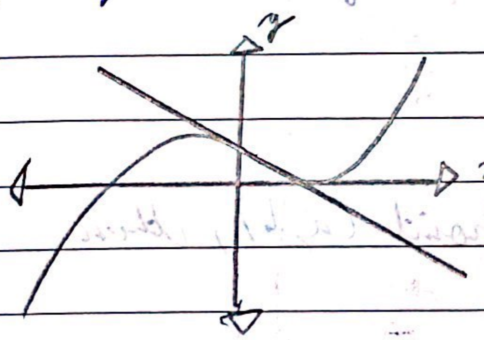
Q35) $y = -2x + 1$



Q39) $\frac{f(3) - f(1)}{3-1} = \frac{2.2 - 2.12}{2}$

$= 0.04$

$\frac{f(2) - f(1)}{2-1} = \frac{2.34 - 2.12}{1}$

$= 0.22$

$\frac{f(2) - f(0)}{2-0} = \frac{2.34 - 0.58}{2}$

$= 0.88$

(b) tangent slope ≈ 0.8

$\frac{f(2) - f(0)}{2-0}$ is best estimate

$\frac{f(3) - f(1)}{3-1}$ is worst estimate

Q40)(a) $f'(0.5) = \frac{f(1) - f(0)}{1-0}$

$= \frac{2.12 - 0.58}{1} = 1.54$

(b) $f'(2.5) = \frac{f(3) - f(2)}{3-2} = \frac{2.2 - 2.34}{1} = -0.14$

Q41)(a) dollars per feet ($/ft)

(b) $f'(x) \approx$ prize of extra foot

(c) If each extra foot added, increase the total prize, then $f'(x)$ is +ve.

(d) $f'(300) \approx \frac{f(301) - f(300)}{301-300} = f(301) \approx f(300) + 1000$

$\approx f(300) = 1600$

So extra money for foot is $1000

Q42)(a) $\frac{\text{gallon}^2}{\text{dollar}}$

(b) Increasing this amount of paint would be sold for one more dollar for each gallon.

(c) negative ∵ increase in the amount of paint leads to decrease in the amount sold

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Q55) A function f is said to be differentiable if limit at every point exists such that.

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

exists. If f is differentiable at each point (a, b), then f is differentiable everywhere

Q56) If f is differentiable at x_0, f is continuous at x_0.