Dated: Assignment

a $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$$a \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + c \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + b + c = 0$

$2a + b - c = 0$

$3a - b + 0 = 0$

$3a = b$

$a = b/3$

$\cancel{2b/3 + b - c = 0}$

$\cancel{\frac{2b+3b}{3} = c}$

$3a = b$

$2a + b - c = 0$

$3a + 3a - c = 0$

$6a = c$

$a + b + c = 0$

$a + 3a + 6a = 0$

$10a = 0$

$\boxed{a = 0}$

$3a = b$

$3(0) = b \Rightarrow \boxed{b = 0}$

$6a = c$

$6(0) = c$

$\boxed{c = 0}$

Linearly independent

Dated:

b) $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}3\\-1\\2\end{bmatrix}$

$$a\begin{bmatrix}1\\0\\1\end{bmatrix} + b\begin{bmatrix}1\\-1\\0\end{bmatrix} + c\begin{bmatrix}3\\-1\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$a + b + 3c = 0$

$0 - b - c = 0$

$a + 0 + 2c = 0$

$a = -2c$

$-b = -c$

$a + b + 3c = 0$

$-2c - c + 3c = 0$

$-3c + 3c = 0$

$0 = 0$

$a = -2c$

$a = -2(0)$

$a = 0$

$b = -c$

$b = -(0)$

$b = 0$

$$2\begin{bmatrix}1\\0\\1\end{bmatrix} + 1\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}3\\-1\\2\end{bmatrix}$$

$2 + 1 = 3$

$0 - 1 = -1$

$2 + 0 = 2$

$2V_1 + V_2 = V_3$

$$2\begin{bmatrix}1\\0\\1\end{bmatrix} + \begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}3\\-1\\2\end{bmatrix}$$

since $V_1 + V_2 = V_3$, making $v_3$ the set linearly dependent

Dated:

b

$$\begin{bmatrix}1\\0\\0\\-1\end{bmatrix} \begin{bmatrix}1\\0\\2\\0\end{bmatrix} \begin{bmatrix}2\\0\\4\\0\end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c}1&1&2&0\\0&0&0&0\\0&2&4&0\\-1&0&0&0\end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c}1&1&2&0\\0&2&4&0\\0&0&0&0\\-1&0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&1&2&0\\0&2&4&0\\0&0&0&0\\0&1&2&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&1&2&0\\0&1&2&0\\0&0&0&0\\0&1&2&0\end{array}\right]$$

$R_1 + R_4$ $\qquad \frac{1}{2}R_2$ $\qquad -R_2 + R_4$

$$\Rightarrow \left[\begin{array}{ccc|c}1&1&2&0\\0&1&2&0\\0&0&0&0\\0&0&0&0\end{array}\right]$$

$x_3 = \alpha$

$x_2 + 2(\alpha) = 0$

$x_2 = -2\alpha$

$x_1 + x_2 + 2x_3 = 0$

$x_1 + -2\alpha + 2(\alpha) = 0$

$x_1 = 0$

free variables ∴ the ~~m~~ set will be linearly dependent with a non-trivial solution

c

$$\begin{bmatrix}1\\0\\0\\-1\end{bmatrix} \begin{bmatrix}1\\0\\2\\0\end{bmatrix} \begin{bmatrix}1\\0\\4\\0\end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c}1&1&1&0\\0&0&0&0\\0&2&4&0\\-1&0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&1&1&0\\0&2&4&0\\0&0&0&0\\-1&0&0&0\end{array}\right]$$

$R_2 \leftrightarrow R_3$ $\qquad \frac{1}{2}R_2$

$$\Rightarrow \left[\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\0&0&0&0\\-1&0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\0&0&0&0\\0&1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\0&1&1&0\\0&0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\0&0&1&0\\0&0&0&0\end{array}\right]$$

$R_1 + R_4$ $\qquad R_3 \leftrightarrow R_4$ $\qquad R_2 - R_3$

∴ Trivial Solution

Linearly Independent.

$x_3 = 0$

$x_2 + 2x_3 = 0$

$x_2 + 2(0) = 0$

$x_2 = 0$

$x_1 + x_2 + x_3 = 0$

$x_1 + 0 + 0 = 0$

$x_1 = 0$

Dated:

4a

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 0$$
$$2x_1 + x_2 + x_4 = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \underset{R_2 - 2R_1 \to R_2}{\Rightarrow} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 0 \end{array}\right] \underset{-R_2 \to R_2}{\Rightarrow}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \end{array}\right] \underset{R_1 - R_2 \to R_1}{\Rightarrow} \left[\begin{array}{cccc|c} \underline{1} & 0 & -1 & 1 & 0 \\ 0 & \underline{1} & 2 & -1 & 0 \end{array}\right]$$

$$x_2 + 2x_3 - x_4 = 0$$
$$\Rightarrow x_2 = -2x_3 + x_4$$

$$x_1 - x_3 + x_4 = 0$$
$$\Rightarrow x_1 = x_3 - x_4$$

$$N(A) \Rightarrow x_3 \to \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad ; \quad x_4 \to \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Dated:

d ~~R~~ Vectors set in Part b is linearly dependent because it has ~~no~~ a non-trivial solution as the solution set has free variables ($x_3$) ~~resulting~~

Part C has a trivial solution as the solution set ~~it~~ ~~trivial~~ (has a solution with zeroes in the form of $x_1, x_2$ & $x_3$) therefore making it linearly independent with a unique solution.

Part C has a trivial solution set (with zeroes in the form of $x_1, x_2$ & $x_3$) therefore making it linearly independent with a unique solution.

Dated:

39 $\begin{bmatrix}1\\2\\1\end{bmatrix}$ $\begin{bmatrix}1\\1\\-1\end{bmatrix}$ $\begin{bmatrix}0\\1\\2\end{bmatrix}$ $\begin{bmatrix}3\\4\\-1\end{bmatrix}$

$$\left[\begin{array}{cccc|c}1&1&0&3&0\\2&1&1&4&0\\1&-1&2&-1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&0&3&0\\0&1&-1&2&0\\1&-1&2&-1&0\end{array}\right]$$

$-2R_1+R_2 \to R_2$ $\qquad$ $-R_1+R_3 \to R_3$

$$\Rightarrow \left[\begin{array}{cccc|c}1&1&0&3&0\\0&-1&1&-2&0\\0&-2&2&-4&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&0&3&0\\0&1&-1&2&0\\0&-2&2&-4&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&0&3&0\\0&1&-1&2&0\\0&0&0&0&0\end{array}\right]$$

$-R_2 \to R_2$ $\qquad$ $2R_2+R_3$ $\qquad$ $-R_2+R_1 \to R_1$

$$\Rightarrow \left[\begin{array}{cccc|c}1&0&1&1&0\\0&1&-1&2&0\\0&0&0&0&0\end{array}\right]$$

$x_3 = \alpha$

$x_2 - 1 + 2\alpha = 0$

$x_2 = -2\alpha + 1$

$x_1 - 2\alpha + 1 + \alpha = 0$

$x_1 = 2\alpha - 1 - \alpha$

$\Rightarrow \alpha - 1$

$x_1$ ; $x_2$ ; $x_3$

$\alpha - 1$ $\quad$ $-2\alpha+1$ $\quad$ $\alpha$

b dimension = 2

c $\begin{bmatrix}1\\1\\-1\end{bmatrix} + \begin{bmatrix}0\\1\\2\end{bmatrix} \Rightarrow \begin{bmatrix}1\\2\\1\end{bmatrix}$ $\quad$ : $\quad$ $\begin{bmatrix}1\\2\\1\end{bmatrix} - \begin{bmatrix}1\\1\\-1\end{bmatrix} \Rightarrow \begin{bmatrix}0\\1\\2\end{bmatrix}$

$\therefore \begin{bmatrix}1\\1\\-1\end{bmatrix}$ ; $\begin{bmatrix}0\\1\\2\end{bmatrix}$ $\to$ first basis $\quad$ : $\quad$ $\therefore \begin{bmatrix}1\\2\\1\end{bmatrix}$ ; $\begin{bmatrix}1\\1\\-1\end{bmatrix}$ $\to$ Second basis

Dated:

5

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\\1\end{bmatrix} \qquad \text{assuming } V_4 \Rightarrow \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&-1&1&1&0\\0&0&-1&1&0\\0&1&1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&-1&1&1&0\\0&1&1&1&0\\0&0&-1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&-1&1&1&0\\0&0&2&2&0\\0&0&-1&1&0\end{array}\right]$$

$R_3 \leftrightarrow R_4$ $\qquad$ $R_2+R_3 \to R_3$ $\qquad$ $R_1+R_2 \to R_2$

$$\Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\1&0&2&2&0\\0&0&2&2&0\\0&0&-1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\1&0&2&2&0\\-1&0&0&0&0\\0&0&-1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\1&0&2&2&0\\0&1&1&1&0\\0&0&-1&1&0\end{array}\right]$$

$R_3-R_2 \to R_3$ $\qquad$ $R_1+R_3 \to R_3$ $\qquad$ $R_3+R_4 \to R_1$

$$\Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\1&0&2&2&0\\0&1&0&2&0\\0&0&-1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&-1&1&1&0\\0&1&0&2&0\\0&0&-1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&-1&1&1&0\\0&0&1&3&0\\0&0&-1&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&-1&1&1&0\\0&0&1&3&0\\0&0&0&4&0\end{array}\right]$$

$-R_1+R_2 \to R_2$ $\qquad$ $R_2+R_3 \to R_3$ $\qquad$ $R_3+R_4 \to R_4$ $\qquad$ $-R_2 \to R_2$

$$\Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&1&-1&-1&0\\0&0&1&3&0\\0&0&0&4&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&1&0&2&0\\0&0&1&3&0\\0&0&0&4&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&1&1&1&0\\0&1&0&2&0\\0&0&1&3&0\\0&0&0&1&0\end{array}\right] \Rightarrow \left[\begin{array}{cccc|c}1&0&1&1&0\\0&1&0&2&0\\0&0&1&3&0\\0&0&0&1&0\end{array}\right]$$

$R_2+R_3 \to R_2$ $\qquad$ $\frac{1}{4}R_4 \to R_4$ $\qquad$ $R_1-R_2 \to R_1$ $\qquad$ $R_1-R_2 \to R_1$

$$\Rightarrow \left[\begin{array}{cccc|c}1&0&0&-4&0\\0&1&0&2&0\\0&0&1&3&0\\0&0&0&1&0\end{array}\right] \Rightarrow \begin{array}{cccc}x_1 ; & x_2 ; & x_3 ; & x_4\\-4 & 2 & 3 & 1\end{array}$$

$\Rightarrow$ no free variable. ~~All~~ 4th vectors ~~are~~ is unique $\therefore$ the matrix is linearly independent

Dated:

b) Dimension = 2

c) Matrix A:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Matrix B:

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$N(B) \Rightarrow$

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 \end{array}\right]$$

$-R_2 \to R_2$ $\qquad$ $R_2 + R_1 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & -2 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$R_2 + R_1 \to R_2$ $\qquad$ $-R_2 \to R_2$

$x_1 - x_3 + x_4 = 0$

$x_1 = x_3 - x_4$

$x_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad$ $x_4 \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$N(B) \Rightarrow x_3 \to \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \leftarrow x_4$

$x_2 + 2x_3 - x_4 = 0$

$x_2 \Rightarrow x_4 - 2x_3$

$N(A) = x_3 \; ; \; x_4$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

*Dated:*

6. $v_1 = 3i - 2j + k \qquad v_2 = i + 3j - k$

a) $\vec{n} = \begin{vmatrix} i & -j & k \\ 3 & -2 & 1 \\ 1 & 3 & -1 \end{vmatrix} = i\begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix}$

$\vec{n} = \; = -2i + 4j + 11k$

$\begin{vmatrix} i & -j & k \\ 1 & 3 & -1 \\ -2 & 4 & 11 \end{vmatrix} = i\begin{vmatrix} 3 & -1 \\ 4 & 11 \end{vmatrix} - j\begin{vmatrix} 1 & -1 \\ -2 & 11 \end{vmatrix} + k\begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix}$

a vector that lies on the plane. $\leftarrow \; = 35i - 9j - 2k$

b) normal vetor to a plane does not lie on the plane

so: $\vec{n} = -2i + 7j + 11k$

c) $\vec{w} \cdot \vec{n} = (2)(-2) + (-1)(4) + (3)(11)$

$= -4 - 7 + 33$

$= 25 \neq 0$

$\vec{w}$ does not lie on the plane.

Dated:

a) det = Ad - bc

$= a(0) - b(a+b)$

$\Rightarrow 0 - ba - b^2$

$\Rightarrow -ba - b^2 \Rightarrow -b(a-b)$

$\therefore$ this is a subspace of $M_{2\times2}$

b) $\begin{bmatrix} a & b \\ a+b & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a+b & 0 \\ a & b \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\Rightarrow \begin{bmatrix} a+b & 0 \\ a & b \end{bmatrix} \Rightarrow \begin{bmatrix} b & a-b \\ a & b \end{bmatrix} \Rightarrow \begin{bmatrix} b & -b \\ 0 & -b-a \end{bmatrix}$

$R_1 - R_2$ $\quad$ $R_2 - a \rightarrow R_2$ $\quad$ $R_2 \rightarrow \frac{1}{b-a} R_2$

$\begin{bmatrix} b & -b \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{1}{b} R_1 \rightarrow R_1$ $\quad$ $R_2 + R_1 \rightarrow R_1$

Dimension = 2

c) Basis

$\begin{bmatrix} a & b \\ a+b & 0 \end{bmatrix}$

# Homework 3

Dated:

~~8.~~

8. a) this set cannot be linearly independent because there are four vectors and only three variables. even if three of them were linearly independent, at least one of them is redundant, and can therefore be expressed as a linear combination of the others.

b)

9. $3x^2+2x+3$, $3x+4$, $5x^2+x-1$, $-2x^2+1$.

$$3x+4 = a(3x^2+2x+3) + b(5x^2+x-1) + c(-2x^2+1)$$

$$0x^2+3x+4 = 3ax^2+2ax+3a + 5bx^2 + bx - 5b - 2cx^2 + c$$

$0 = 3a+5b-2c$ $\quad$ $3 = 2a+b$ $\quad$ $4 = 3a - b + c$

Dated:

$0 = 3a + 5b - 2c$

$\underline{8 = 6a - 2b + 2c}$

$8 = 9a + 3b$ . $3 = 2a + b$ .

$\hookrightarrow b = 3 - 2a$

$8 = 9a + 3(3 - 2a)$

$8 = 9a + 9 - 6a$

$8 = 3a + 9$

$\boxed{a = \frac{-1}{3}}$

$b = 3 - 2a$

$= 3 - 2(\frac{-1}{3})$

$\boxed{b = \frac{11}{3}}$

$4 = 3a - b + c$

$c = 4 - 3a + b$

$= 4 - 3(\frac{-1}{3}) + \frac{11}{3}$

$= 4 + 1 + \frac{11}{3}$

$\boxed{c = \frac{26}{3}}$

$3x + 4 = -\frac{1}{3}(3x^2 + 2x + 3) + \frac{11}{3}(5x^2 + x - 1) + \frac{26}{3}(-2x^2 + 1)$

Dated:

10. a) the vector $\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$ is the only one whose $x_2$ and $x_4$ values are non-zero. this means that the $x_2$ and $x_4$ values of any linear combination of these vectors ~~must be~~ will be dependent on one another. there is no way for $x_2 = 1$ and $x_4 = 2$, for example. so the vectors cannot span $\mathbb{R}^4$

b) the vector $\begin{bmatrix}2\\0\\2\\0\end{bmatrix}$ is a multiple of the vector $\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$ so it is redundant. excluding that, there are three vectors and four variables so at least two of them will be dependant on one another. therefore the vectors cannot span $\mathbb{R}^4$

11. Section 3.3.

7. $x_1$ $x_2$ $x_3$ → all independent.

$y_1 = x_2 - x_1$
$y_2 = x_3 - x_2$
$y_3 = x_3 - x_1$

~~$y_3 = (x_3 - x_2) - (x_2 - x_1)$~~

$$y_3 = x_3 - x_1$$
$$= (x_3 - x_2) - (x_2 - x_1)$$
$$y_3 = y_2 - y_1$$

since $y_3$ can be written as a linear combination of $y_2$ and $y_1$, $y_1$, $y_2$, and $y_3$ are not linearly independent.

Dated:

15. if a set of vectors $\{v_1, v_2, \dots, v_n\}$ is linearly independent, then $c_1v_1 + c_2v_2 + \dots c_nv_n = 0$ has only one solution, such that $c_1 = c_2 = \dots = c_n = 0$

~~since~~ any subset of ~~set~~ $\{v_1 \dots v_n\}$ can only have solutions that are a subset of $c_1 = c_2 = \dots c_n = 0$. since the only value in that set is 0, ~~that is~~ there can be no other solution for the subset. therefore the subset will also be linearly independent.

19. since $v \in V$, there exist scalar ~~values~~ values such that
$v = c_1v_1 + c_2v_2 + \dots + c_nv_n$.
since $v$ can be represented as a linear combination of the set, $v, v_1, v_2, \dots, v_n$ must be linearly dependent.

20. assume that $v_2 \dots v_n$ span $V$.
since $v_1 \in V$, there ~~exists~~ exists a linear combination such that
$v_1 = c_2v_2 + \dots c_nv_n$
since $v_1$ can be represented as a linear combination of the others, $v_1, v_2, \dots, v_n$ cannot be linearly independent.
this is a contradiction, so our assumption ~~[illegible]~~ must be incorrect.