Dated:

Assignment 4

1. a)

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

inverse of:

1 : 1
2 : 3
3 : 2
4 : 4

b) i)

$$5x + 23 \equiv 6 \pmod{47}$$
$$5x \equiv -17 \pmod{47}$$
$$5x \equiv 30 \pmod{47}$$
$$x \equiv 6 \pmod{47}$$

ii)

$$9x + 80 \equiv 2 \pmod{81}$$
$$9x \equiv -78 \pmod{81}$$
$$9x \equiv 3 \pmod{81}$$
$$3x \equiv 1 \pmod{81}$$

since 81 is a multiple of 3, no solution of x exists

Dated:

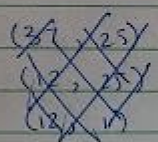
iii) $30x + 3y \equiv 0 \pmod{37}$ $\qquad$ $y \equiv 4 + 13x \pmod{37}$

$30x + 3(4 + 13x) \equiv 0 \pmod{37}$

$30x + 12 + 2x \equiv 0 \pmod{37}$

$32x \equiv -12 \pmod{37}$

$32x \equiv 25 \pmod{37}$



$37 = 1(25) + 12$

$25 = 2(12) + 1$

$1 = 25 - 2(12)$

$1 = 3(25) - 2(37) \pmod{37}$

$1 \equiv 3(25) \pmod{37}$

$32x \equiv \overset{3\times}{25} \pmod{37}$

$3\times$

$96x \equiv 1 \pmod{37}$

$22x \equiv 1 \pmod{37}$

$37 = 1(22) + 15$

$22 = 1(15) + 7$

$15 = 2(7) + 1$

$1 = 15 - 2(7)$

$1 = 3(15) - 2(22)$

$1 = 3(37) - 5(22)$

$1 = -5(22) \pmod{37}$

$1 = 32(22) \pmod{37}$

$22x \equiv 1 \pmod{37}$

$x = 32$

Dated:

$$y \equiv 4 + 13(32) \pmod{37}$$
$$y \equiv 13 \pmod{37}$$

c) (5688, 2010)

(1668, 2010)

(1668, 342)

(300, 342)

(300, 42)

(6, 42)

(6, 0) → 6 = gcd.

d) $52j + 15k = 3$

~~$52j + 15k = 1$~~

$52 = 3(15) + 7$

$15 = 2(7) + 1$

$1 = 15 - 2(7)$

$1 = 7(15) - 2(52)$

$52(-2) + 15(7) = 1$

$52(-6) + 15(21) = 3$

$j = -6$

$k = 21$

a) $31 = 1(21) + 10$

$21 = 2(10) + 1$

$1 = 21 - 2(10)$

$1 = 3(21) - 2(31)$

b) $31a + 21b = 1010$

$31(-2) + 21(3) = 1$

$31(-2020) + 21(3030) = 1010$ $\qquad \frac{3030}{31} \approx 97$

$31(-2020 + (21 \times 97)) + 21(3030 - (31 \times 97)) = 1010$

$31(17) + 21(23) = 1010$

17 31-cent coins + 23 21-cent coins.

c) ~~there's no other possible solution because we would need to add multiples of 21 to~~

there's no other possible solution where the amounts of coins aren't negative

3. $k = 1$ 3 - prime

$k = 2$ 7 - prime

$k = 3$ 31 - prime

$k = 4$ 211 - prime

$k = 5$ 2311 - prime

$k = 6$ 30031 - not prime.

we aren't always guaranted a prime number, however, the proof is still valid because, if not prime, then ~~the product must~~ it must be a product of primes. however that ~~is not possible~~ should not be possible because it isn't divisible by any number on the list.
therefore the contradiction holds.