Cairo University
Faculty of Computers & Information
Information Technology Department

Signals and Systems

Sheet No. 5

Problem No.1 :-

For the continuous-time periodic signal $x(t) = 2 + \cos(\frac{2\pi}{3}t) + 4\sin(\frac{5\pi}{3}t)$, determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Problem No.2 :-

Use the Fourier series analysis equation (3.39) to calculate the coefficients a_k for the continuous-time periodic signal $x(t) = \begin{cases} 1.5; 0 \le t \le 1 \\ -1.5; 1 \le t \le 2 \end{cases}$ with fundamental frequency

$$\omega_0 = \pi$$

Problem No.3 :-

Each of the two sequences $x_1[n]$ and $x_2[n]$ has a period N = 4 , and the corresponding Fourier series coefficients are specified as $x_1[n] \longleftrightarrow a_k$, $x_2[n] \longleftrightarrow b_k$ where $a_0 = a_3 = \frac{1}{2}a_2 = 1$ and $b_0 = b_1 = b_2 = b_3 = 1$, using the multiplication property , determine the Fourier series coefficients c_k for the signal $g[n] = x_1[n]x_2[n]$.

Problem No.4 :-

Determine the Fourier series representations for the following signals :

(a) $x(t) = e^{-t}$ where -1 < t < 1 and it is periodic with period 2 .

(b) $x(t) = \begin{cases} \sin \pi t; 0 \le t \le 2 \\ 0; 2 < t \le 4 \end{cases}$ with period 4 .

(c) each x(t) in figure 5.1

Problem No.5 :-

Let $x(t) = \begin{cases} t; 0 \le t \le 1 \\ 2 - t; 1 \le t \le 2 \end{cases}$ be periodic signal with fundamental period T = 2 and Fourier coefficients a_k ,

(a) Determine the value of a_0

(b) Determine the Fourier series representation of dx(t)/dt .

(c) Use the result of part(b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of x(t) .

Problem No.6 :-

In each of the following , we specify the Fourier series coefficients of a signal that is periodic with period 8 , Determine the signal x[n] in each case :

(a) $a_k = \cos\left(\frac{k\pi}{4}\right) + \sin\left(\frac{3k\pi}{4}\right)$ (b) $a_k = \begin{cases} \sin\left(\frac{k\pi}{4}\right), 0 \le k \le 6 \\ 0; k = 7 \end{cases}$

Problem No.7 :-

Let $x[n] = \begin{cases} 1; 0 \le n \le 7 \\ 0; 8 \le n \le 9 \end{cases}$ be a periodic signal with fundamental period N = 10 and Fourier series coefficients a_k . Also let g[n] = x[n] - x[n-1] .

(a) Show that g[n] has a fundamental period of 10 .

(b) Determine the Fourier series coefficients of g[n] .

(c) Using the Fourier series coefficients of g[n] and the First-Difference property , determine a_k for $k \neq 0$.

Problem No.8 :-

Let x(t) be a periodic signal with fundamental period T and Fourier series coefficients a_k , Derive the Fourier series coefficients of each of the following signals in terms of a_k :

(a) $x(t-t_0) + x(t+t_0)$ (b) Ev{x(t)}

Problem No.9 :-

Suppose we are given the following information about a continuous-time periodic signal with period 3 and Fourier coefficients a_k :

1.$a_k = a_{k+2}$

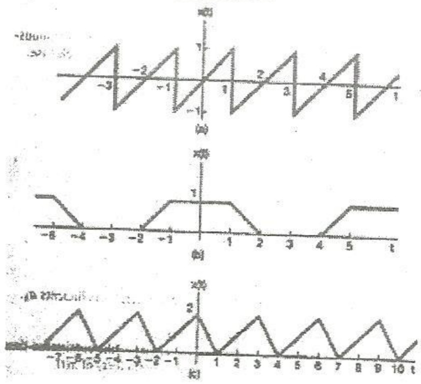
2.$a_k = a_{-k}$

3. $\int_{-0.5}^{0.5} x(t)dt = 1$

4. $\int_{0.5}^{1.5} x(t)dt = 2$

Determine x(t) .

Fig 5.1



Good Luck