Question 1: Successor Function [50 points]

Grading Scheme

1. (30 points) Compiles and passes Example 1 and 2. 15 points each.
2. (20 points) Additional test cases. 2 points each.

Example 1:

```
$java successor 3 2 1 3 2 0
0 2 0
3 0 0
3 2 1
2 2 1
3 1 1
```

Example 2:

```
$java successor 11 5 2 6 3 1
0 3 1
6 0 1
6 3 0
11 3 1
6 5 1
6 3 2
9 0 1
7 3 0
4 5 1
6 4 0
5 3 2
6 2 2
```

Test Case 1:

```
$java successor 9 9 9 6 3 8
0 3 8
6 0 8
6 3 0
9 3 8
6 9 8
6 3 9
9 0 8
9 3 5
0 9 8
6 9 2
5 3 9
6 2 9
```

Test Case 2:

```
$java successor 2147483647 2147483647 0 2000000000 2000000000 0
0 2000000000 0
2000000000 0 0
2147483647 2000000000 0
2000000000 2147483647 0
2147483647 1852516353 0
1852516353 2147483647 0
```

Test Case 3:

```
$java successor 6 6 2 3 3 1
0 3 1
6 3 1
6 0 1
0 6 1
3 3 0
3 3 2
2 3 2
4 3 0
3 0 1
3 6 1
3 2 2
3 4 0
```

Test Case 4:

```
$java successor 4 6 5 1 3 2
0 3 2
4 3 2
4 0 2
0 4 2
1 3 0
1 3 5
3 3 0
0 3 3
1 0 2
1 6 2
1 0 5
1 5 0
```

Test Case 5:

```
$java successor 4 5 6 3 2 1
0 2 1
4 2 1
4 1 1
```

```
0 5 1
3 2 0
3 2 6
4 2 0
0 2 4
3 0 1
3 5 1
3 0 3
3 3 0
```

Test Case 6:

```
$java successor 2 1 1 1 0 0
0 0 0
2 0 0
0 1 0
1 0 1
0 0 1
1 1 0
```

Test Case 7:

```
$java successor 123 789 654 100 200 300
0 200 300
123 200 300
123 177 300
0 300 300
100 200 0
100 200 654
123 200 277
0 200 400
100 0 300
100 789 300
100 0 500
100 500 0
```

Test Case 8:

```
$java successor 321 321 345 0 221 300
321 221 300
221 0 300
0 221 0
0 221 345
300 221 0
0 0 300
0 321 300
0 321 200
0 176 345
```

Test Case 9:

```
$java successor 1 1 1 0 0 0
1 0 0
0 0 1
0 1 0
```

Test Case 10:

```
$java successor 1 1 1 1 1 0
0 1 0
1 1 1
0 1 1
1 0 0
1 0 1
```

Question 2: State Space [50 points]

An (m, n, k)-puzzle is a sliding puzzle with m columns, n rows, and k empty squares, where $1 \leq k < mn$. As usual, one move is to move a single tile to an adjacent (up, down, left, right) empty square, if available.

1. (10 points) How many tiles are there in a (m, n, k)-puzzle in general? For example, there are 8 tiles in a $(3, 3, 1)$-puzzle.
   (Solution.)
   There are $m \times n$ total spaces in the puzzle, In which k are empty. So we need $m \times n - k$ total tiles to fill up the puzzle.

   (Grading Scheme.)
   General Case:

   (a) Getting to $m * n$: 5 points
   (b) Getting to $m * n - k$: 10 points

2. (20 points) Let each tile have a distinct name. How many distinct states are there in the state space? Show your derivation.
   (Solution.)

   (a) Suppose k = 0, We have $m \times n$ distinct spaces and $m \times n$ distinct tiles. The number of states is the number of permutaions the tiles can have. The Answer is $(m \times n)!$
   (b) Suppose k > 1, then we can treat the spaces as indistinguishable tiles. We divide previous answer by the permutaion of k distince tiles. The answer is

   $$\frac{(m \times n)!}{k!}$$

   (**Note.** $\binom{n \times m}{k} \times (n \times m - k)!$ is also a correct answer.)

   (Grading Scheme.)
   General Case:

   (a) Getting to $m * n$: 10 points
   (b) Getting to $(m * n)!$: 15 points
   (c) Getting to $\frac{(m*n)!}{k!}$: 20 points

   Additional deductions:

   (a) No derivation: -2 points

3. (20 points) Draw a graph that corresponds to the state space of a $(2, 2, 1)$-puzzle, and **briefly describe your graph**. This is not an art project: You may represent the states in ways easy for you to type, and you do not necessarily need drawing programs – you may even use plaintext. You can also hand draw the graph, but please clearly show the nodes and edges.

   (Grading Scheme.)

   (a) Explaining the graph (5 points)

   (b) Nodes are drawn correctly (10 points)

      i. Only 4 nodes: 4 points
      ii. Only 12 reachable nodes: 6 points
      iii. All 24 nodes: 10 points

      Additional deductions:

      i. Missing node: -0.5

   (c) Edges are drawn correctly (5 points)

      i. Only edges for reachable states: 3 points
      ii. All 24 edges (4 edges if only has 4 nodes): 5 points

      Additional deductions:

      i. Missing edge: -0.5 points

      Common cases:

      i. 4 nodes with all edges and explanation: 14 points
      ii. 12 nodes with all edges and explanation: 16 points
      iii. 24 nodes with 12 edges and explanation: 18 points

(Solution.)

In the (2,2,1) puzzle, we have 24 distinct states. Observe that one can never exchange the position of two tiles. There must be some unreachable states. Label the three tiles as (a,b,c). The empty space can be between (a,b), (b,c), or (a,c). Rotating by 90 degrees gives us 12 states when the tiles are (a,b,c) in clockwise order. The other 12 states can be reached when the tiles are (a,c,b) in clockwise order.

Now we draw the states. Since each state only has 2 successors, each connected componect must be a loop. Draw as follows: