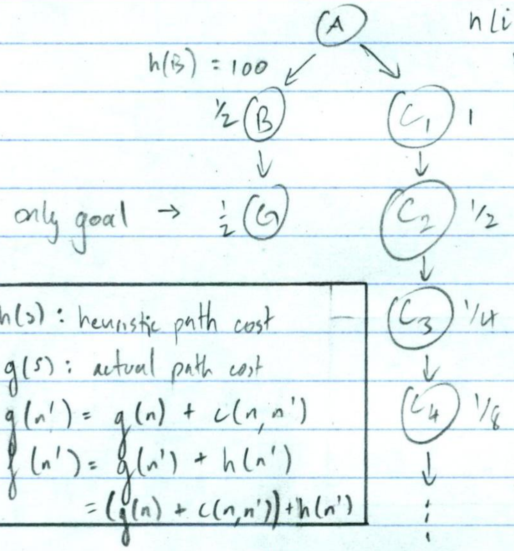
CS 540 - HW4

Roumen Guha

1.



$h(i) = 0$
$\forall i,\ i \neq B$

$h(s)$: heuristic path cost
$g(s)$: actual path cost
$g(n') = g(n) + c(n, n')$
$f(n') = g(n') + h(n')$
$\quad = (g(n) + c(n,n')) + h(n')$

(a) <u>Def</u>: a heuristic function is admissible if it never overestimates the true cost of reaching the state.

$h(i) \leq 1/2, \ \forall i$ //

This should say h(B) <= 1/2 (sorry!)

(b) The A^* algorithm

(i) OPEN ← s
(ii) if (OPEN.isEmpty())
  return failure;
(iii) n ← OPEN (minimum f(n))
  CLOSED ← n
(iv) if (n.isGoal())
  return backPointers(n, s);
(v) n_i' = successors(n); $\forall i$
  · if (OPEN.contains(n_i') || !CLOSED.contains(n_i'))
    OPEN ← n_i'
  · else if ($g(n_2') < g(n_1')$)
    - travel along n_2', redirect back pointers accordingly
    - OPEN ← n_2'
  · else
    do nothing
(vi) Go to (ii)

<u>FIRST ITERATION</u> $\boxed{\left(S^{P}_{f,g,h}\right)}$

(i) OPEN = $\{A^{\phi}_{0,0,0}\}$
(iii) S ← min(OPEN)
  CLOSED = $\{A^{\phi}_{0,0,0}\}$, OPEN = { }
(v) successors(A) = $\{B^{A}_{\frac{201}{2}, \frac{1}{2}, 100},\ C^{A}_{1,1,0}\}$

$$\boxed{\begin{array}{l} \text{OPEN} = \{C^{A}_{1,1,0},\ B^{A}_{\frac{201}{2},\frac{1}{2},100}\} \\ \text{CLOSED} = \{A^{\phi}_{0,0,0}\} \end{array}}$$

<u>SECOND ITERATION</u>

(iii) n ← min(OPEN)
  CLOSED = $\{A^{\phi}_{0,0,0},\ C^{A}_{1,1,0}\}$
(v) successors(C_1) = $\{C^{C_1}_{2\ \frac{3}{2},\frac{3}{2},0}\}$

$$\boxed{\begin{array}{l} \text{OPEN} = \{C^{C_1}_{2\ \frac{3}{2},\frac{3}{2},0},\ B^{A}_{\frac{201}{2},\frac{1}{2},100}\} \\ \text{CLOSED} = \{A^{\phi}_{0,0,0},\ C^{A}_{1\ 1,1,0}\} \end{array}}$$

<u>THIRD ITERATION</u>

(iii) n ← min(OPEN)
  CLOSED = $\{A^{\phi}_{0,0,0},\ C^{A}_{1\ 1,1,0},\ C^{C_1}_{2\ \frac{3}{2},\frac{3}{2},0}\}$
(v) successors(C_2) = $\{C^{C_2}_{3\ \frac{7}{4},\frac{7}{4},0}\}$

$$\boxed{\begin{array}{l} \text{OPEN} = \{C^{C_2}_{3\ \frac{7}{4},\frac{7}{4},0},\ B^{A}_{\frac{201}{2},\frac{1}{2},100}\} \\ \text{CLOSED} = \{A^{\phi}_{0,0,0},\ C^{A}_{1\ 1,1,0},\ C^{C_1}_{2\ \frac{3}{2},\frac{3}{2},0}\}. \end{array}}$$

FOURTH ITERATION

(iii) $n \leftarrow \min(\text{OPEN})$

$\text{CLOSED} = \{ A^{\phi}_{0,0,0},\ C_1{}^{A}_{1,1,0},\ C_2{}^{C_1}_{\frac{3}{2},\frac{3}{2},0},\ C_3{}^{C_2}_{\frac{7}{4},\frac{7}{4},0} \}$

(v) successors$(C_3) = \{ C_4{}^{C_3}_{\frac{15}{8},\frac{15}{8},0} \}$

$\text{OPEN} = \{ C_4{}^{C_3}_{\frac{15}{8},\frac{15}{8},0},\ B^{A}_{\frac{201}{2},\frac{1}{2},100} \}$

$\text{CLOSED} = \{ A^{\phi}_{0,0,0},\ C_1{}^{A}_{1,1,0},\ C_2{}^{C_1}_{\frac{3}{2},\frac{3}{2},0},\ C_3{}^{C_2}_{\frac{7}{4},\frac{7}{4},0} \}$

FIFTH ITERATION

(iii) $n \leftarrow \min(\text{OPEN})$

$\text{CLOSED} = \{ A^{\phi}_{0,0,0},\ C_1{}^{A}_{1,1,0},\ C_2{}^{C_1}_{\frac{3}{2},\frac{3}{2},0},\ C_3{}^{C_2}_{\frac{7}{4},\frac{7}{4},0},\ C_4{}^{C_3}_{\frac{15}{8},\frac{15}{8},0} \}$.

(v) successors$(C_4) = \{ C_5{}^{C_4}_{\frac{31}{16},\frac{31}{16},0} \}$

$\text{OPEN} = \{ C_5{}^{C_4}_{\frac{31}{16},\frac{31}{16},0},\ B^{A}_{\frac{201}{2},\frac{1}{2},100} \}$

$\text{CLOSED} = \{ A^{\phi}_{0,0,0},\ C_1{}^{A}_{1,1,0},\ C_2{}^{C_1}_{\frac{3}{2},\frac{3}{2},0},\ C_3{}^{C_2}_{\frac{7}{4},\frac{7}{4},0},\ C_4{}^{C_3}_{\frac{15}{8},\frac{15}{8},0} \}$

(c) $\{f(C_i)\} = 1, \ 3/2, \ 7/4, \ 15/8, \ 31/16, \ \ldots$

$$\lim_{i \to \infty} f(C_i) = \sum_{i=0}^{\infty} \left(\tfrac{1}{2}\right)^i \rightarrow 1$$
$$= 1 + \cancel{\sum_{i=1}^{\infty} \left(\tfrac{1}{2}\right)^i}$$
$$= 2 \ //$$

(d) Because, since the limit of $f(C_i) = 2$, $f(C_i)$ will approach 2 as $i \to \infty$ but will never truly be 2. Our value of $f(B) = 100.5$, which is much greater than 2, so we will never pop it out of our OPEN priority queue, since C_i is infinitely long.

(e) $h(B) < 2$. Since if the limit of $f(C_i)$ eventually reaches some value close to 2, it will eventually surpass some $h(B) < 2$, and we will expand into B.

This should actually be f(B) < 2, therefore h(B) < 1.5 (sorry!)

(f) Sufficient condition, since it was not necessary that $h(B) \leq 1/2$ to reach the goal state.

2. $\text{successors}(x) = \{ y : y \in [x-10, x+10] \}$

$$f(x) = \max \{ 4 - |x|,\ 2 - |x-6|,\ 2 - |x+6| \}$$

$$p = \exp\left\{ - \frac{|f(y) - f(x)|}{T} \right\}, \qquad T = 2(0.9)^i$$

SA algorithm (want to maximize $f()$)

```
current = initialState(problem);
for i = 1 to ∞
   T = schedule(i);
   if (T == 0)
       return current;
   next = selectRandomSuccessorState(current);
   deltaE = f(next) - f(current);
   if (deltaE > 0)
       current = next;
   else
       p = exp(deltaE/T);
       if (rand() ≤ p)
           current = next;
end
```

Iteration i	Random Successor y	Random Number z	Current point x	The Temperature $T = 2(0.9)^i$	P(Moving to $y \mid y, T$) $p = \exp(\text{deltaE}/T)$	deltaE $f(y) - f(x)$
1	3	0.102	2	1.800	($z <$) 0.574.	-1 (< 0)
2	1	0.223	3	1.620	1	2
3	1	0.504	1	1.458	1	0
4	4	0.493	1	1.312	0.102 ($< z$)	-3 (< 0)
5	2	0.312	1	1.181	($z <$) 0.4288	-1 (< 0)
6	3	0.508	2	1.063	0.390 ($< z$)	-1 (< 0)
7	4	0.982	2	0.957	0.124 ($< z$)	-2 (< 0)
8	3	0.887	2	0.861	0.313 ($< z$)	-1 (< 0)

↑ if $> z$ accept random successor

↑ if $< z$ reject random successor

↑ if < 0 accept with some prob. p.

3. (a) For n trees, if each state is a possible permutation of n trees, there are $n!$ states.

(b) Every state has $(n-1)$ neighbors, so neighborhood cover $= \dfrac{n-1}{n!}$

(c) Let $n = 119{,}113$ trees
$\therefore n! \approx 3 \times 10^{552{,}885}$

(d) $$f(t_i) = d(0, t_1) + \sum_{i=1}^{n-1} d(t_i, t_{i+1}) + d(t_n, 0)$$

$\therefore$ $(n+1)$ distances travelled. If 10 km per distance, worst-case total distance =
$10\text{ km} \cdot (119{,}113 + 1) = 1{,}191{,}140\text{ km} = 740{,}140$ miles
≈ 3 LD.

Average Earth-Moon distance = 385,000 km.

(e) $10\text{m} \cdot (119{,}113 + 1) = 1{,}191{,}140\text{ m} = 740.14$ miles
$= 1{,}191$ km best-case total distance

(f) Even assuming best-case of 740 miles, at 25 miles/hour with no deviations in speed, it would take $740/25 = 29.6$ hours to complete the route, and so cannot be completed in a day.