Syntax Reference Sheet for First-order Logic

This document gives our official rules for syntactically well-formed formulas (*wff*'s) in first-order logic. For the rules for syntactic derivation, see the Natural Deduction Rules handout.

Let a formula be any finitely long string of symbols in our language. Let t, t' etc. stand for terms (constant or variable symbols) in FOL, x for any variable symbol, P and Q stand for any well-formed formulas, R stand for any predicate symbol, and $*$ stand for any binary connective.[1] Then the class of well-formed formulas is recursively defined as follows:

1. The atomic formulas are well-formed formulas. The atmoic formulas are of the form:

    a) $t = t'$

    b) $R(t, t'...)$, where the number of term symbols match the arity of the predicate symbol

2. Formulas of the form $(P * Q)$ are well-formed formulas

3. Formulas of the form $\neg P$ are well-formed

4. $\forall x P$ is a wff and $\exists x P$ is a wff

5. Nothing else is a well-formed formula

Observations:

Non-sentences (open formulas) can count as wff's. We need them to so that we can feed them back into the definition to make well-formed formulas that are sentences, by putting a quantifier in front.

The variable bound by the quantifier (in rule 4) doesn't have to be in P.

There are no sentence letters in our language.

These rules are supposed to make sure that it's always clear what the scope of the quantifiers is and what the arguments of the predicates are. They are also supposed to make sure that terms never go in slots meant for sentences and vice versa.

Notice that there are a lot of brackets cluttering up our well-formed formulas. For convinience, in informal exposition, we will often leave out brackets when it is obvious and unambiguous how to add them to make a wff. In practice, this usually means leaving out the brackets that enclose the whole rest of the formula. But officially those brackets are always present.[2]

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[1] Note well that these symbols are ones we are introducing to talk about FOL, *not* symbols in FOL itself.

[2] For the sticklers among you: we agree that in the language we use to talk about FOL (a mix of English and symbolic language) we'll use the abbreviated bracketless formulas as names for the full formulas, with all their brackets, in FOL itself.