CS 540 - HW8 - Roumen Guha 11/15/18

1. Propositional Logic. Proof by Resolution.

Given $p \Rightarrow (q \Rightarrow r)$, (1)

prove $(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$ (2)

Convert (1) to Conjunctive Normal Form (CNF):

$$
\begin{aligned}
& p \Rightarrow (q \Rightarrow r) \\
= \; & \neg p \vee (q \Rightarrow r) && \text{implication elimination} \\
= \; & \neg p \vee (\neg q \vee r) && \text{implication elimination.} \\
= \; & (\neg p \vee \neg q) \vee (\neg p \vee r) && \text{distributivity of } \vee \text{ over } \wedge \\
= \; & \neg p \vee \neg q \vee r && (3)
\end{aligned}
$$

Convert (2) to CNF:

$$
\begin{aligned}
& (p \Rightarrow q) \Rightarrow (p \Rightarrow r) \\
= \; & (\neg p \vee q) \Rightarrow (\neg p \vee r) && \text{implication elimination} \\
= \; & \neg(\neg p \vee q) \vee (\neg p \vee r) && \text{implication elimination} \\
= \; & (p \wedge \neg q) \vee (\neg p \vee r). && \text{de Morgan.} \quad (4)
\end{aligned}
$$

Negation of (4)

$$
\begin{aligned}
= \; & \neg((p \wedge \neg q) \vee (\neg p \vee r)) \\
= \; & \neg(p \wedge \neg q) \wedge \neg(\neg p \vee r) && \text{de Morgan} \\
= \; & (\neg p \vee q) \wedge (p \wedge \neg r) && \text{de Morgan} \\
= \; & \cancel{(p \wedge \neg r \wedge \neg p)} \vee (p \wedge \neg r \wedge q) && \text{distributivity of } \wedge \text{ over } \vee \\
= \; & (p \wedge \neg r \wedge q) \\
= \; & \neg(\neg p \vee \neg q \vee r) && \text{de Morgan} \quad (5)
\end{aligned}
$$

Resolving (3) and (5), we get empty. Therefore:

$$p \Rightarrow (q \Rightarrow r) \models (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$$

2. Translation to First Order Logic (F.O.L.)

(a) Question : What jumps higher than a building?
Answer : Everything, buildings don't jump.

(i) Everything that can jump jumps higher than a building.

(ii) Variables : x and y $\in \{$ everything in the world $\}$.

(iii) CanJump(x) : true if x is able to jump
IsBuilding(x) : true if x is a building
JumpsHigherThan(x, y) : true if x jumps higher than y

(iv) $\forall x \forall y (x \neq y)$ CanJump$(x) \wedge$ IsBuilding$(y) \Rightarrow$ JumpsHigherThan(x, y)

(b) (i) There must be exactly one honest politician at the party.

(ii) Variables : $x, y \in \{$ politicians at the party $\}$.

(iii) IsHonest(x) : true if x is honest

(iv) $\forall x \forall y (x \neq y)$ IsHonest$(x) \Rightarrow \neg$IsHonest$(y))$

3. Hierarchical Clustering

(a)

	Madison	Seattle	Boston	Vancouver	Winnepeg	Montreal
Madison	0	1617	931	1654	597	800
Seattle	1617	0	2486	121	1153	2283
Boston	931	2486	0	2501	1344	250
Vancouver	1654	121	2501	0	1159	2291
Winepeg	597	1153	1344	1159	0	1132
Montreal	800	2283	250	2291	1132	0

call c_0 = Madison
c_1 = Seattle
c_2 = Boston
c_3 = Vancouver
c_4 = Winnepeg
c_5 = Montreal

$d(\{c_0, c_4\}, \{c_1, c_3\}) = \max\{1617, 1654, 1153, 1159\} = 1654$

$d(\{c_0, c_4\}, \{c_2, c_5\}) = \max\{597, 800, 1344, 1132\} = 1344$

$d(\{c_1, c_3\}, \{c_2, c_5\}) = \max\{2486, 2283, 2501, 2291\} = 2486$

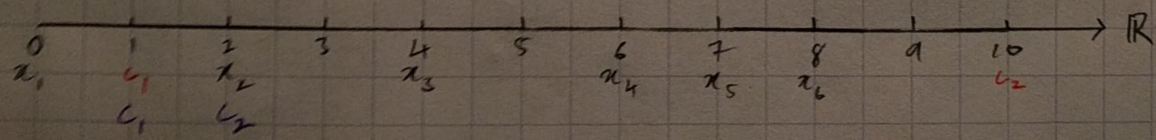
(b)

Iteration	New Cluster	Distance (complete-linkage)	Clusters
0	—	—	$\{c_0\}, \{c_1\}, \{c_2\}, \{c_3\}, \{c_4\}, \{c_5\}$
1	$\{c_1, c_3\}$	121	$\{c_0\}, \{c_1, c_3\}, \{c_2\}, \{c_4\}, \{c_5\}$
2	$\{c_2, c_5\}$	250	$\{c_0\}, \{c_1, c_3\}, \{c_2, c_5\}, \{c_4\}$
3	$\{c_0, c_4\}$	597	$\{c_0, c_4\}, \{c_1, c_3\}, \{c_2, c_5\}$
4	$\{\{c_0, c_4\}, \{c_2, c_5\}\}$	1,344	$\{c_0, c_2, c_4, c_5\}, \{c_1, c_3\}$

(c)

Iteration	New Cluster	Distance (complete-linkage)	Clusters
0	—	—	$\{c_0\}, \{c_1\}, \{c_2\}, \{c_3\}, \{c_4\}, \{c_5\}$.
1	$\{c_0, c_2\}$	931	$\{c_0, c_2\}, \{c_1\}, \{c_3\}, \{c_4\}, \{c_5\}$
2	$\{c_4, c_5\}$	1,132	$\{c_0, c_2\}, \{c_1\}, \{c_3\}, \{c_4, c_5\}$
3	$\{\{c_3\}, \{c_4, c_5\}\}$	2,291	$\{c_0, c_2\}, \{c_1\}, \{c_3, c_4, c_5\}$
4	$\{\{c_0, c_2\}, \{c_1\}\}$	2,486	$\{c_0, c_1, c_2\}, \{c_3, c_4, c_5\}$

4. K-Means Clustering.



(a)

Iteration	Centers (c_1, c_2)	Clusters	New Centers (c_1^{new}, c_2^{new})	Energy (Distortion)
0	—	$\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}$	—	—
1	(1, 10)	$\{x_1, x_2, x_3\}, \{x_4, x_5, x_6\}$	(2, 7)	$(2)^2 + 0 + (2)^2 + (1)^2 + 0 + (1)^2 = 10$
2	(2, 7)	$\{x_1, x_2, x_3\}, \{x_4, x_5, x_6\}$	(2, 7)	10

(b)

Iteration	Centers (c_1, c_2)	Clusters	New Clusters (c_1^{new}, c_2^{new})	Energy (Distortion)
0	—	$\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}$	—	—
1	$(1, 2)$	$\{x_1\}, \{x_2, x_3, x_4, x_5, x_6\}$	$(0, 5.4)$	$0 + (3.4)^2 + (1.4)^2 + (0.6)^2 + (1.6)^2 + (2.6)^2 = 23.2$
2	$(0, 5.4)$	$\{x_1, x_2\}, \{x_3, x_4, x_5, x_6\}$	$(1, 6.25)$	$1^2 + 1^2 + 2.25^2 + 0.25^2 + 0.75^2 + 1.75^2 = 10.75$
3	$(1, 6.25)$	$\{x_1, x_2\}, \{x_3, x_4, x_5, x_6\}$	$(1, 6.25)$	10.75

(c) the scheme used in part (a) is better, since it resulted in a lower distortion.