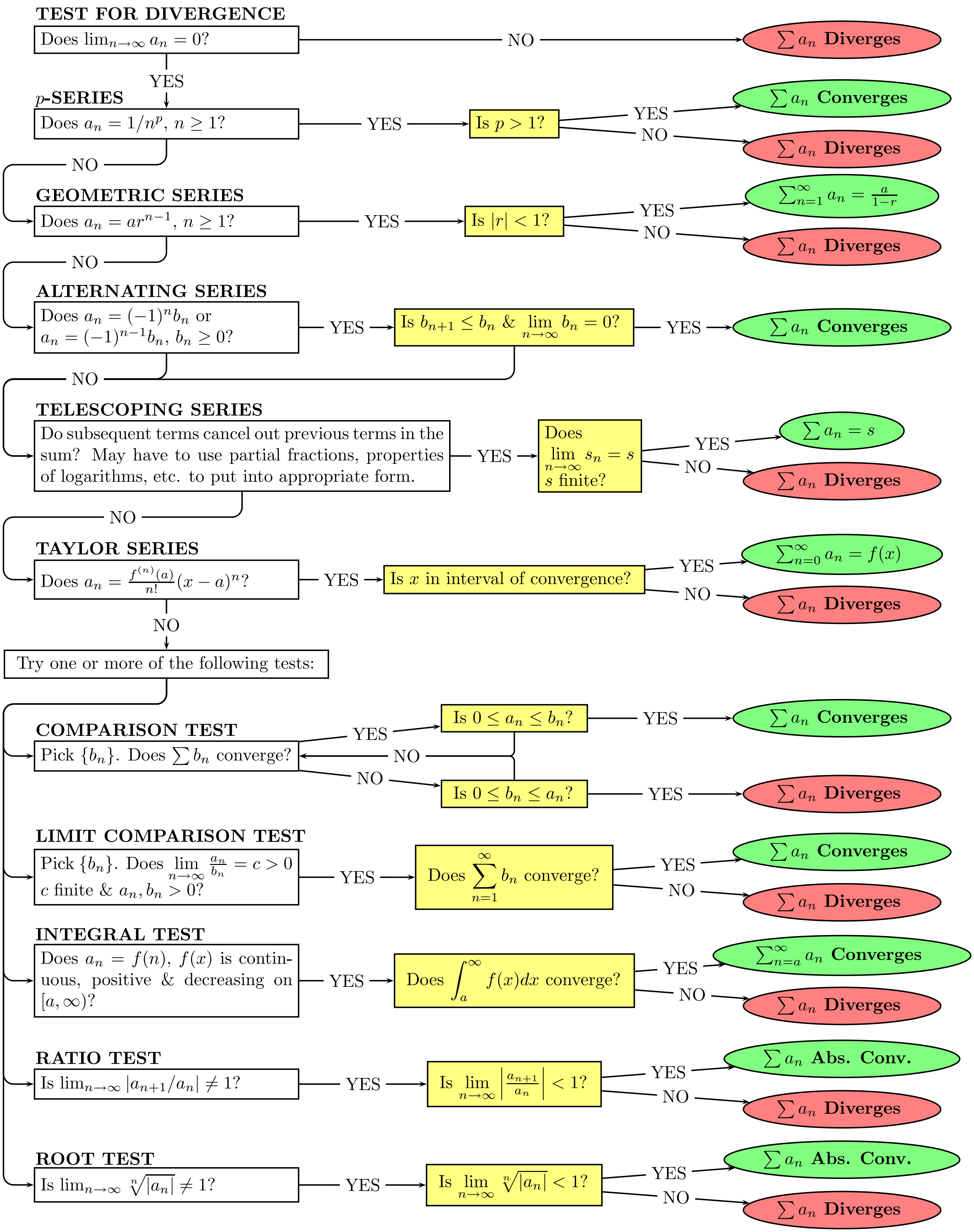
Series Convergence/Divergence Flow Chart



1. Convergence and Divergence Tests for Series

Test	When to Use	Conclusions
Divergence Test	for any series $\sum_{n=0}^{\infty} a_n$	Diverges if $\lim_{n\to\infty} \lvert a_n\rvert \neq 0$.
Integral Test	$\sum_{n=0}^{\infty} a_n$ with $a_n \geq 0$ and a_n decreasing	$\int_1^{\infty} f(x)dx$ and $\sum_{n=0}^{\infty} a_n$ both converge/diverge where $f(n) = a_n$.
Comparison Test	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ if $0 \leq a_n \leq b_n$	$\sum_{n=0}^{\infty} b_n$ converges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ diverges $\Longrightarrow \sum_{n=0}^{\infty} b_n$ diverges.
Limiting Comparison Test	$\sum_{n=0}^{\infty} a_n, (a_n > 0)$. Choose $\sum_{n=0}^{\infty} b_n$, $(b_n > 0)$ if $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ with $0 < L < \infty$	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge/diverge
	if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$	$\sum_{n=0}^{\infty} b_n$ converges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ converges.
	if $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$	$\sum_{n=0}^{\infty} b_n$ diverges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ diverges.
Convergent test for alternating Series	$\sum_{n=0}^{\infty} (-1)^n a_n \quad (a_n > 0)$	converges if $\lim_{n\to\infty} a_n = 0$ and a_n is decreasing
Absolute Convergence	for any series $\sum_{n=0}^{\infty} a_n$	If $\sum_{n=0}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=0}^{\infty} a_n$ converges, (definition of absolutely convergent series.)
Conditional Convergence	for any series $\sum_{n=0}^{\infty} a_n$	if $\sum_{n=0}^{\infty} \lvert a_n\rvert$ diverges but $\sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ conditionally converges
Ratio Test: Root Test:	For any series $\sum_{n=0}^{\infty} a_n$, Calculate $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$ Calculate $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$	there are 3 cases: if $L < 1$, then $\sum_{n=0}^{\infty} \lvert a_n\rvert$ converges ; if $L > 1$, then $\sum_{n=0}^{\infty} \lvert a_n\rvert$ diverges; if $L = 1$, no conclusion can be made.

2. Important Series to Remember

Series	How do they look	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$ and diverges if $\lvert r\rvert \geq 1$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ and diverges if $p \leq 1$

Binomial Series

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$
$$= a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \cdots + nab^{n-1} + b^n$$