

SERIES CONVERGENCE/DIVERGENCE FLOW CHART

TEST FOR DIVERGENCE

Does $\lim_{n \rightarrow \infty} a_n = 0$? NO $\rightarrow \sum a_n$ Diverges

YES

p-SERIES

Does $a_n = 1/n^p, n \geq 1$? YES \rightarrow Is $p > 1$? YES $\rightarrow \sum a_n$ Converges
NO $\rightarrow \sum a_n$ Diverges

NO

GEOMETRIC SERIES

Does $a_n = ar^{n-1}, n \geq 1$? YES \rightarrow Is $|r| < 1$? YES $\rightarrow \sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$
NO $\rightarrow \sum a_n$ Diverges

NO

ALTERNATING SERIES

Does $a_n = (-1)^n b_n$ or $a_n = (-1)^{n-1} b_n, b_n \geq 0$? YES \rightarrow Is $b_{n+1} \leq b_n$ & $\lim_{n \rightarrow \infty} b_n = 0$? YES $\rightarrow \sum a_n$ Converges

NO

TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form. YES \rightarrow Does $\lim_{n \rightarrow \infty} s_n = s$ YES $\rightarrow \sum a_n = s$
NO $\rightarrow \sum a_n$ Diverges

NO

TAYLOR SERIES

Does $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$? YES \rightarrow Is x in interval of convergence? YES $\rightarrow \sum_{n=0}^{\infty} a_n = f(x)$
NO $\rightarrow \sum a_n$ Diverges

NO

Try one or more of the following tests:

COMPARISON TEST

Pick $\{b_n\}$. Does $\sum b_n$ converge? YES \rightarrow Is $0 \leq a_n \leq b_n$? YES $\rightarrow \sum a_n$ Converges
NO \rightarrow Is $0 \leq b_n \leq a_n$? YES $\rightarrow \sum a_n$ Diverges

LIMIT COMPARISON TEST

Pick $\{b_n\}$. Does $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$ c finite & $a_n, b_n > 0$? YES \rightarrow Does $\sum_{n=1}^{\infty} b_n$ converge? YES $\rightarrow \sum a_n$ Converges
NO $\rightarrow \sum a_n$ Diverges

INTEGRAL TEST

Does $a_n = f(n)$, $f(x)$ is continuous, positive & decreasing on $[a, \infty)$? YES \rightarrow Does $\int_a^{\infty} f(x) dx$ converge? YES $\rightarrow \sum_{n=a}^{\infty} a_n$ Converges
NO $\rightarrow \sum a_n$ Diverges

RATIO TEST

Is $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$? YES \rightarrow Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$? YES $\rightarrow \sum a_n$ Abs. Conv.
NO $\rightarrow \sum a_n$ Diverges

ROOT TEST

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$? YES \rightarrow Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$? YES $\rightarrow \sum a_n$ Abs. Conv.
NO $\rightarrow \sum a_n$ Diverges

1. Convergence and Divergence Tests for Series

Test	When to Use	Conclusions
Divergence Test	for any series $\sum_{n=0}^{\infty} a_n$	Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$.
Integral Test	$\sum_{n=0}^{\infty} a_n$ with $a_n \geq 0$ and a_n decreasing	$\int_1^{\infty} f(x)dx$ and $\sum_{n=0}^{\infty} a_n$ both converge/diverge where $f(n) = a_n$.
Comparison Test	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ if $0 \leq a_n \leq b_n$	$\sum_{n=0}^{\infty} b_n$ converges $\implies \sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ diverges $\implies \sum_{n=0}^{\infty} b_n$ diverges.
Limiting Comparison Test	$\sum_{n=0}^{\infty} a_n, (a_n > 0)$. Choose $\sum_{n=0}^{\infty} b_n, (b_n > 0)$ if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ with $0 < L < \infty$	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge/diverge
	if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$	$\sum_{n=0}^{\infty} b_n$ converges $\implies \sum_{n=0}^{\infty} a_n$ converges.
	if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$	$\sum_{n=0}^{\infty} b_n$ diverges $\implies \sum_{n=0}^{\infty} a_n$ diverges.
Convergent test for alternating Series	$\sum_{n=0}^{\infty} (-1)^n a_n \quad (a_n > 0)$	converges if $\lim_{n \rightarrow \infty} a_n = 0$ and a_n is decreasing
Absolute Convergence	for any series $\sum_{n=0}^{\infty} a_n$	If $\sum_{n=0}^{\infty} a_n $ converges, then $\sum_{n=0}^{\infty} a_n$ converges, (definition of absolutely convergent series.)
Conditional Convergence	for any series $\sum_{n=0}^{\infty} a_n$	if $\sum_{n=0}^{\infty} a_n $ diverges but $\sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ conditionally converges
Ratio Test: Root Test:	For any series $\sum_{n=0}^{\infty} a_n$, Calculate $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = L$ Calculate $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = L$	there are 3 cases: if $L < 1$, then $\sum_{n=0}^{\infty} a_n $ converges ; if $L > 1$, then $\sum_{n=0}^{\infty} a_n $ diverges; if $L = 1$, no conclusion can be made.

2. Important Series to Remember

Series	How do they look	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	Converges to $\frac{a}{1-r}$ if $ r < 1$ and diverges if $ r \geq 1$
p -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ and diverges if $p \leq 1$

Binomial Series

$$(a+b)^n = \sum_{i=0}^n \binom{n}{i} a^{n-i} b^i$$

$$= a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \dots + nab^{n-1} + b^n$$