A Constructive Heuristic Algorithm for Distribution System Planning

Marina Lavorato, *Student Member, IEEE*, Marcos J. Rider, *Member, IEEE*, Ariovaldo V. Garcia, *Member, IEEE*, and Rubén Romero, *Senior Member, IEEE*

***Abstract*—A constructive heuristic algorithm (CHA) to solve distribution system planning (DSP) problem is presented. The DSP is a very complex mixed binary nonlinear programming problem. A CHA is aimed at obtaining an excellent quality solution for the DSP problem. However, a local improvement phase and a branching technique were implemented in the CHA to improve its solution. In each step of the CHA, a sensitivity index is used to add a circuit or a substation to the distribution system. This sensitivity index is obtained by solving the DSP problem considering the numbers of circuits and substations to be added as continuous variables (relaxed problem). The relaxed problem is a large and complex nonlinear programming and was solved through an efficient nonlinear optimization solver. Results of two tests systems and one real distribution system are presented in this paper in order to show the ability of the proposed algorithm.**

***Index Terms*—AMPL, constructive heuristic algorithm, distribution system planning, KNITRO, mixed binary nonlinear programming, power systems optimization.**

NOTATION

The notation used throughout this paper is reproduced below for quick reference.

Sets:

Ω_l	Sets of branches (existing and proposed).
Ω_b	Sets of nodes.
Ω_{b_s}	Sets of bus substation nodes (existing and proposed, $\Omega_{b_s} \subset \Omega_b$).
Ω_{b_i}	Sets of connected nodes in the node $i(\Omega_{b_i} \subset \Omega_b)$.
Ω_a	Sets of circuit type.
Ω_f	Set of power flow directions ($\Omega_f=\{ij/i \in \Omega_b$ and $j \in \Omega_{b_i}\}$).

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M. Lavorato, M. J. Rider, and A. V. Garcia are with the Department of Electrical Energy Systems, University of Campinas, Campinas, SP, Brazil (e-mail: lavorato@dsee.fee.unicamp.br; mjrider@dsee.fee.unicamp.br; ari@dsee.fee.unicamp.br).

R. Romero is with the Faculdade de Engenharia de Ilha Solteira, UNESP—Universidade Estadual Paulista, Departamento de Engenharia Elétrica, Ilha Solteira, SP, Brazil (e-mail: ruben@dee.feis.unesp.br).

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Constants:

κ_l	Capital recovery rate of circuit constructions.
κ_s	Capital recovery rate of substation reinforcement or construction.
$c_{ij,a}$	Construction cost of branch ij of type a (US$/km).
c_{f_i}	Substation fixed cost at node i (US$).
α	Number of hours in one year (8760 h).
τ_l	Interest rate of the cost of power losses.
τ_s	Interest rate of substation operation cost.
ϕ_l	Loss factor of circuits.
ϕ_s	Loss factor of substations.
c_l	Cost per unit of energy lost (US$/kWh).
c_{v_i}	Substation operation cost at node i (US$/kVAh^2).
ΔV	Voltage thresholds.
$n^0_{ij,a}$	Existent circuit in branch ij of type a.
l_{ij}	Circuit length of branch ij.
$\bar{S}^0_i$	Maximum apparent power limit of existent substation at node i.
$\bar{S}_i$	Maximum apparent power limit of substation reinforcement or construction at node i.
V^{nom}	Nominal voltage magnitude.
$\bar{S}_{ij,a}$	Maximum apparent power limit of branch ij of type a.
n_b	Number of nodes ($n_b=\|\Omega_b\|$).
n_{b_s}	Number of bus substation nodes ($n_{b_s}=\|\Omega_{b_s}\|$).
P_{D_i}	Active power demand at node i.
Q_{D_i}	Reactive power demand at node i.
$g_{ij,a}$	Conductance of branch ij of type a.
$b_{ij,a}$	Susceptance of branch ij of type a.

Functions:

G_{ij}	Nodal conductance matrix elements.
B_{ij}	Nodal susceptance matrix elements.
P_i	Active power calculated at node i.
Q_i	Reactive power calculated at node i.

$P_{ij,a}$ Active power flow of type a that leaves node i toward node j.

$Q_{ij,a}$ Reactive power flow of type a that leaves node i toward node j.

$S_{ij,a}$ Apparent power flow of type a that leaves node i toward node j ($S_{ij,a}=\sqrt{P_{ij,a}^2+Q_{ij,a}^2}$).

$\hat{n}_{ij,a}$ Total circuit number of branch ij of type a.

v Total investment and operation cost

Variables:

$n_{ij,a}$ Circuit number that can be added on branch ij of type a.

m_i Substation number that can be added on node i.

V_i Voltage magnitude at node i.

θ_{ij} Difference of phase angle between nodes i and j.

P_{S_i} Active power provided by substation at node i.

Q_{S_i} Reactive power provided by substation at node i.

$n_{ij,a}^{+}$ Added circuit number in branch ij of type a during the CHA iterative process.

m_i^{+} Substation number added on node i during the CHA iterative process.

n^{+R} Vector of ranked added circuits.

I. Introduction

THE main objective of the distribution system planning (DSP) problem is to provide a reliable and cost effective service to consumers while ensuring that voltages and power quality are within standard ranges. Several objective functions, including new equipment installation cost, equipment utilization rate, reliability of the target distribution system and loss minimization should be evaluated considering an increase of system loads and newly installed loads for the planning horizon. New optimization models, new techniques aimed to find optimal or even good solutions for the DSP problem are still needed, considering the location and size of substations and circuits, the construction of new circuits as well as new substations or, alternatively, the reinforcement of the existing ones, in order to allow a viable system operation in a pre-defined horizon [1].

According to the planning horizon the DSP problem can be classified into short-range (one to four years) or long-range (five to 20 years) problem [2]. According to the model, one can have a static problem, in which only one stage (planning horizon) is considered, or a dynamic problem, or a multistage problem, in which several planning horizons are considered in the same problem [3]. The latter problem is not addressed in the present paper. Some DSP research also include the primary-secondary distribution planning [4] and the DSP problem with distributed generation [3] and [5].

In this paper the long-range DSP problem is modeled as a mixed integer (binary) nonlinear programming (MBNLP) problem. The binary nature of the decision variables represents the construction (or not) of new circuits or reconductoring of existing circuits, considering different types of conductor, and represents also the construction (new) or reinforcement (existing) of substations. The objective is to minimize the total investment cost (fixed and variable costs) subject to operation constraints [6]. A special constraint of the problem is the system radial operation and also the impossibility of a load be fed by more than one substation.

In the literature there are several approaches proposed to solve the DSP problem. Among them are discrete optimizations like branch-exchange [7] and branch-and-bound [4] algorithms. In such approaches the search space size of the problem leads to a big computational effort. Techniques based in meta-heuristics like genetic algorithms [8], [9], simulated annealing [10], [11], tabu search [12], ant colony system [13], and evolutive algorithms [14], [15] are also present in the literature. The majority of the proposed methods solve a load flow problem to calculate the operation point of the system and to verify the viability of each investment proposal. Heuristic algorithms are also proposed to guarantee the radial system configuration. Meta-heuristics are techniques that can find good solutions for the DSP problem [9], [13], [16]–[18].

The constructive heuristic algorithm (CHA) has also been proposed to solve the DSP problem [19], [20]. The CHA is robust, easy to be applied and it normally converges to a local solution with a finite iterations number. In [19] and [20] the DSP problem is modeled as a mixed integer quadratic programming problem and the objective is to minimize the cost of construction of new circuits and substations and also the cost of the active power loss in the circuits. A linearized model is employed to represent the operation constraints which affect the quality and precision of the results. The CHAs were also successfully used to solve other power system optimization problems like the reconfiguration of distribution systems [21], the expansion planning of power system transmission [22] and the capacitor allocation in power distribution systems [23].

In this work a CHA is proposed to solve the DSP problem modeled as an MBNLP problem. In each CHA iteration, a nonlinear programming problem (NLP) is solved to obtain a sensitivity index that is used to add a circuit or a substation to the distribution system. The NLP is obtained through relaxation of the binary nature of decision variables which are considered as continuous (but restricted) variables. The objective of the NLP problem is to minimize the operation and construction costs of the distribution system (construction of new circuits and/or substation plus the costs associated with the circuits active power losses) in a time range previously defined; and the constraints are the demand attended, the voltage levels between limits, the capacity of both circuits and substations and the radial configuration of the system. A nonlinear optimization commercial solver was used to solve the NLP. A branching technique is implemented in the CHA to avoid system mesh operations and also to avoid that loads be fed by different substations. It is known that, when the CHA is applied to large and complex problems, the solution obtained usually is not optimal. In this work, the final solution of the CHA is improved through a proposed simple local improvement technique. Results with two tests systems and one real distribution system are shown in the test section.

II. Distribution System Expansion Planning Model

The DSP problem is modeled in this paper as follows:

$$\begin{aligned}\min f &= \kappa_l \sum_{(ij)\in\Omega_l}\sum_{a\in\Omega_a}(c_{ij,a}n_{ij,a}l_{ij}) + \kappa_s \sum_{i\in\Omega_{b_s}}(c_{f_i}m_i)\\ &+ \delta_l \sum_{(ij)\in\Omega_l}\sum_{a\in\Omega_a}(g_{ij,a}\widehat{n}_{ij,a}(V_i^2+V_j^2-2V_iV_j\cos\theta_{ij}))\\ &+ \delta_s \sum_{i\in\Omega_{b_s}}(c_{v_i}(P_{S_i}^2+Q_{S_i}^2)) \end{aligned} \quad (1)$$

s.t.

$$P_i - P_{S_i} + P_{D_i} = 0 \quad \forall\, i\in\Omega_b \quad (2)$$

$$Q_i - Q_{S_i} + Q_{D_i} = 0 \quad \forall\, i\in\Omega_b \quad (3)$$

$$1-\frac{\Delta V}{100} \le \frac{V_i}{V^{\mathrm{nom}}} \le 1+\frac{\Delta V}{100} \quad \forall\, i\in\Omega_b \quad (4)$$

$$P_{S_i}^2+Q_{S_i}^2 \le \left(\bar{S}_i^0 + m_i\overline{S}_i\right)^2 \quad \forall\, i\in\Omega_{b_s} \quad (5)$$

$$P_{ij,a}^2+Q_{ij,a}^2 \le (\widehat{n}_{ij,a}\bar{S}_{ij,a})^2 \quad \forall\,(ij)\in\Omega_f,\ \forall\, a\in\Omega_a \quad (6)$$

$$\sum_{a\in\Omega_a}\widehat{n}_{ij,a} \le 1 \quad \forall\,(ij)\in\Omega_l \quad (7)$$

$$n_{ij,a}\in[0,1] \quad \forall\,(ij)\in\Omega_l, \quad \forall\, a\in\Omega_a \quad (8)$$

$$m_i\in[0,1] \quad \forall\, i\in\Omega_{b_s} \quad (9)$$

$$\sum_{(ij)\in\Omega_l}\sum_{a\in\Omega_a}\hat{n}_{ij,a} = n_b - n_{b_s} \quad (10)$$

$$\widehat{n}_{ij,a} = n_{ij,a}^0 + n_{ij,a} \quad \forall\,(ij)\in\Omega_l, \quad \forall\, a\in\Omega_a \quad (11)$$

where $\delta_l{=}\alpha\tau_l\phi_l c_l$ and $\delta_s = \alpha\tau_s\phi_s$. The objective function (1) is the total investment and operation cost based on [6]. The first part represents the investment cost (construction of circuits and construction/reinforcement of substations), the second and third part represent the annual cost of power losses and substation operation, respectively. Equations (2) and (3) represent the conventional equations of load balance and the elements of P_i and Q_i are given by (12) and (13), respectively:

$$P_i = V_i\sum_{j\in\Omega_b}V_j[G_{ij}(\hat{n}_{ij,a})\cos\theta_{ij} + B_{ij}(\hat{n}_{ij,a})\sin\theta_{ij}] \quad (12)$$

$$Q_i = V_i\sum_{j\in\Omega_b}V_j[G_{ij}(\hat{n}_{ij,a})\sin\theta_{ij} - B_{ij}(\hat{n}_{ij,a})\cos\theta_{ij}]. \quad (13)$$

Equation (4) represents the constraints of voltage magnitude of nodes and (5) represents the maximum capacity of substation i. Note that in (5), both the reinforcement of existing substation ($\bar{S}_i^0 \neq 0$) and the construction of new substation ($\bar{S}_i^0 = 0$) are modeled. The elements of active and reactive power flow in branch ij of type a of (6) are given by

$$P_{ij,a} = \hat{n}_{ij,a}[V_i^2 g_{ij,a} - V_iV_j(g_{ij,a}\cos\theta_{ij} + b_{ij,a}\sin\theta_{ij})]$$

$$Q_{ij,a} = \hat{n}_{ij,a}[-V_i^2 b_{ij,a} - V_iV_j(g_{ij,a}\sin\theta_{ij} - b_{ij,a}\cos\theta_{ij})].$$

The binary investment variables $n_{ij,a}$ and m_i are the decision variables and a feasible operation solution for the distribution system depends on its value. The remaining variables represent the operating state of a feasible solution. For a feasible investment proposal, defined through specified values of $n_{ij,a}$ and m_i, several feasible operation states are possible. Equation (7) assures that duplication of circuits (existing and proposed) is not allowed. Equations (8) and (9) represent the binary nature of circuits and substations that can be added to the distribution system, respectively. The element (circuit or substation) is constructed if the corresponding value is equal to one and is not constructed if it is equal to zero.

In the literature, (10) is considered as a sufficient condition to generate radial connected solutions [3], [5], [6]. However, from the graph theory, it is known that this equation is a necessary condition but not a sufficient one. A subgraph T is a tree if the subgraph meets both conditions as follows: 1) the subgraph has (n_b-1) arcs and 2) it is connected. Equation (10) guarantees the first condition and the second condition is guaranteed by (2) and (3) (load balance), provided that there is power demand at each node, for the optimization technique is required to generate a feasible solution connecting all system nodes. This means that the final solution for the problem is a connected system and has radial topology. For the DSP, it is a valid assumption that all buses have non zero loads, and this assumption is considered in this work.

III. Constructive Heuristic Algorithm

The DSP problem as formulated in (1)–(11) is an MBNLP problem. It is a complex combinatorial problem that can lead to a combinatorial explosion in the number of alternatives that has to be tested. Considering the number of circuits and substations ($n_{ij,a}$ and m_i) as continuous variables, the problem becomes an NLP problem still difficult to solve. Although the solution of this relaxed DSP problem may not be an alternative for planning (fractional number of circuits and substations), it serves to build a useful sensitivity index used in the CHA.

A CHA may be viewed as an iterative process in which a good solution of a complex problem is built step by step. The CHA is robust and has good convergence characteristics. In the case of the DSP problem, in each step, a substation or a single circuit is added to the distribution system based on the sensitivity indexes shown in (14) and (15). The iterative process ends when a feasible solution is found (generally this is a good quality solution). The previously mentioned sensitivity indexes Substation Sensitivity Index (*SSI*) and Circuit Sensitivity Index (*CSI*) are based on the maximum MVA power flow in circuits and on the MVA power of substation provided, respectively, obtained in the solution of the NLP problem. The sensitivity indexes are a function of the operational characteristics of the distribution system only:

$$SSI = \max_{i\in\Omega_{b_s}}\{\sqrt{P_i^2+Q_i^2}, \quad \forall\, m_i \neq 0\} \quad (14)$$

$$CSI = \max_{(ij)\in\Omega_f, a\in\Omega_a}\{S_{ij,a}, \quad \forall\, n_{ij,a}\neq 0\}. \quad (15)$$

The NLP problem used to find the sensitivity indexes is obtained from (1)–(11), considering the number of new circuits and substations (continuous but limited between 0 and 1) as continuous variables and adding two new parameters to the constraint sets (5), (8), (9), and (11) as shown in (16)–(19):

$$P_{S_i}^2+Q_{S_i}^2 \le (\bar{S}_i^0 + m_i^+\bar{S}_i + m_i\bar{S}_i)^2 \qquad \forall\, i\in\Omega_{b_s} \quad (16)$$

$$0 \le n_{ij,a} \le 1 - n_{ij,a}^{+} \qquad \forall\,(ij) \in \Omega_l,\ \forall\, a \in \Omega_a \tag{17}$$
$$0 \le m_i \le 1 - m_i^{+} \qquad \forall\, i \in \Omega_{b_s} \tag{18}$$
$$\hat{n}_{ij,a} = n_{ij,a}^{0} + n_{ij,a}^{+} + n_{ij,a} \quad \forall\,(ij) \in \Omega_l,\ \forall\, a \in \Omega_a. \tag{19}$$

At the end of the CHA iterative process, the total investment and operation cost, given by (20), is obtained by using the solution of the last NLP problem:

$$\begin{aligned} v = {} & \kappa_l \sum_{(ij)\in\Omega_l} \sum_{a\in\Omega_a} (c_{ij,a} n_{ij,a}^{+} l_{ij}) + \kappa_s \sum_{i\in\Omega_{b_s}} (c_{f_i} m_i^{+}) \\ & + \delta_l \sum_{(ij)\in\Omega_l} \sum_{a\in\Omega_a} (g_{ij,a}(n_{ij,a}^{0} + n_{ij,a}^{+}) \\ & \times (V_i^2 + V_j^2 - 2V_iV_j\cos\theta_{ij})) \\ & + \delta_s \sum_{i\in\Omega_{b_s}} (c_{v_i}(P_{S_i}^2 + Q_{S_i}^2)). \end{aligned} \tag{20}$$

The substation and circuit feasible indexes (21) and (22) are used to define the stop criterion of the CHA:

$$SFI = \sum_{i\in\Omega_{b_s}} (m_i) \tag{21}$$
$$CFI = \sum_{(ij)\in\Omega_l} \sum_{a\in\Omega_a} (n_{ij,a}). \tag{22}$$

The CHA is very simple and is presented in the flowchart given in Fig. 1. It must be pointed out that at the initialization (current topology) it is assumed that $m_i^{+}=0, \quad \forall\, i \in \Omega_{b_s}$ and $n_{ij,a}^{+}=0, \quad \forall\,(ij) \in \Omega_l, \quad \forall\, a \in \Omega_a$ and then the NLP problem (1)–(4), (6), (7), (10), and (16)–(19) is solved. Firstly, all needed substations were added in the CHA and the circuits were considered in the sequence. The circuits must be added to the system in such a way that a tree that starts from an existing substation bus is constructed. The dilemma between reinforcing an existent substation or adding a new one, and also adding a new circuit or reconductoring of existing circuits depends on the NLP problem. The optimum operation point of the system is also obtained through the NLP problem. To improve the convergence and to reduce the CPU time to solve NLP problems, it is recommended that the previous NLP solution be used as initial point.

Since the binary nature of the investment variables was relaxed, the constraint (10) may be satisfied with fractional number of circuits. As from this stage, at each step of the CHA, one circuit is added to the actual topology, one investment variable is fixed (binary value) and the number of routes to build the system becomes smaller. This means that as a variable of investment is fixed, less fractional variables appear in the solution of the NLP problem and, as a consequence, only at the end of the iterative process a radial topology for the system is found.

Fig. 2 shows two cases that should be avoided at each CHA iteration when adding a circuit to the current topology: 1) generation of a mesh and 2) connection of one load to different substations. In case 1) the index *CSI* may suggest that circuit $l{-}m$ be added. But, as previously mentioned, due to the relaxation of the binary nature of the investment variables, this circuit may create a mesh, which is not allowed; to avoid this, the CHA must

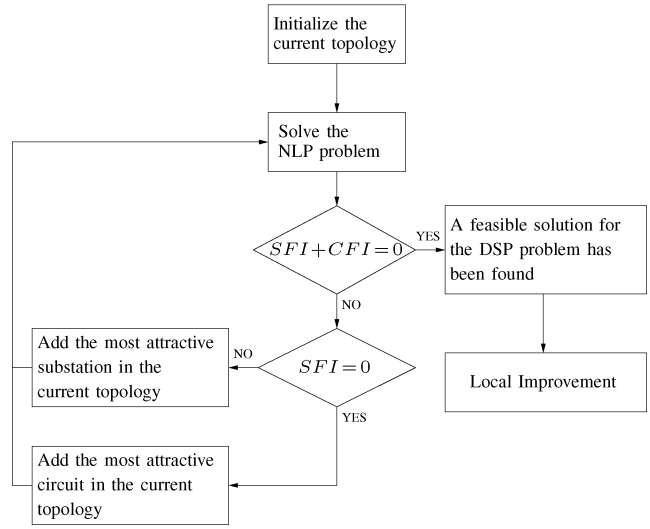

Fig. 1. Constructive heuristic algorithm flowchart.

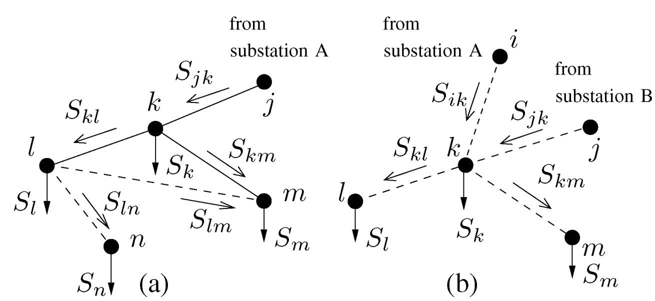

Fig. 2. Illustrative examples.

verify the radial condition of the current topology including the $l{-}m$ circuit. In case 2), node k is connected to two substations, which is not allowed either. In this case the CHA creates two different problems (branching), which are separately analyzed. In one problem, circuit $i{-}k$ is added to the current topology and circuit $j{-}k$ is removed. In the other problem, circuit $j{-}k$ is added and circuit $i{-}j$ is removed. By doing this, two problems are solved independently and the final solution is the one with the lower cost.

The Local Improvement Phase (LIP) flowchart is shown in Fig. 3, where ACN is the total number of added circuits by the CHA. In the first step added circuits are ranked in a descending order by using investment and operation costs as criteria. In the sequence each of these circuits are removed in that order and the NLP problem solved. If the proposed circuit is the removed, it is maintained in the configuration. Otherwise the new circuit is included. v^n is the new total planning cost and it is compared with the best known value. The circuits in the final topology represent the solution of the CHA (distribution system expansion plan).

IV. Tests and Results

The CHA proposed to solve the DSP problem was written in AMPL (a mathematical programming language) [24] and the solution of the NLP was solved by using the nonlinear programming solver KNITRO 5.2 (Nonlinear Interior-point Trust Region Optimizer) [25] (called with default options). The numer-

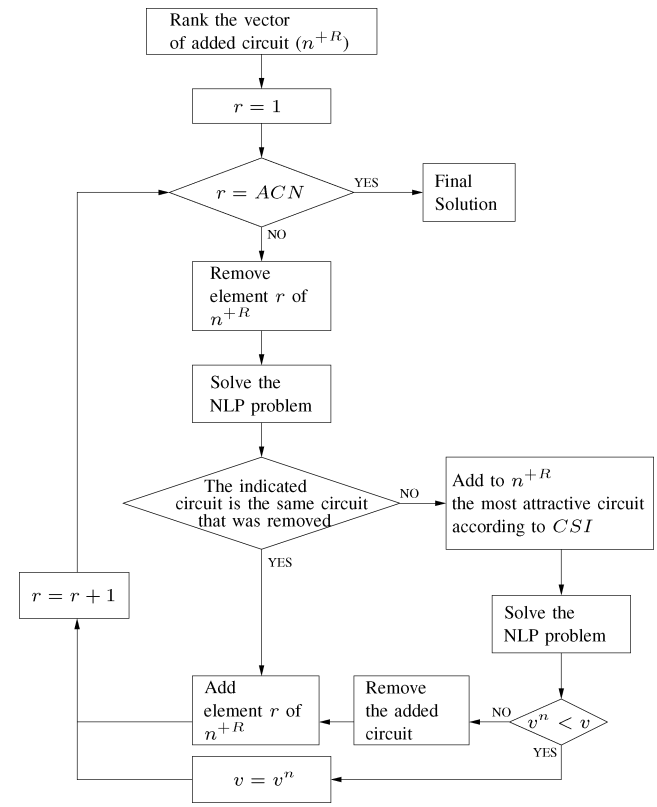

Fig. 3. Local improvement phase flowchart.

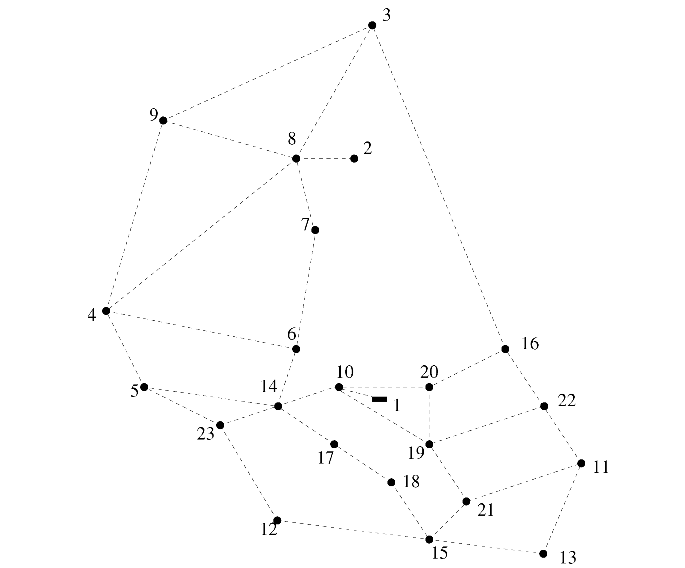

Fig. 4. The 23-node system—proposed feasible routes.

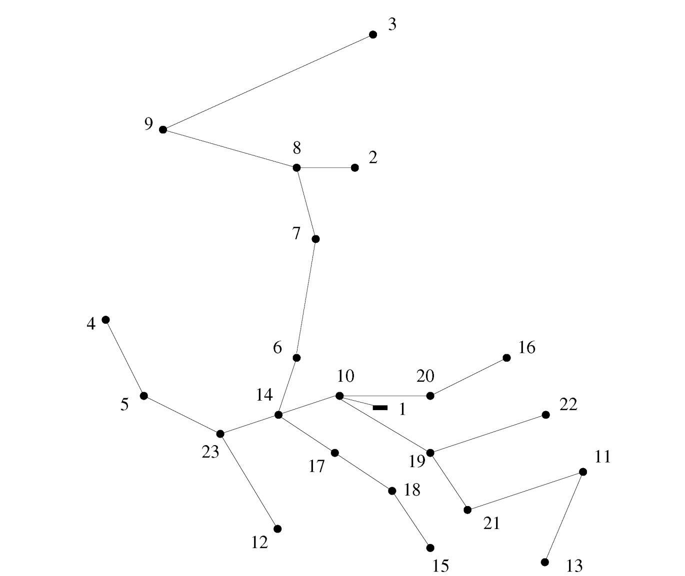

Fig. 5. Distribution system expansion plan for Test 1.

ical results have been obtained using a PC Intel® Core™2 Duo 6700, 2 GB RAM. Two test systems and a real system have been used to show the performance and robustness of the CHA for the DSP problem.

A. The 23-Node Distribution System

The 23-node distribution system, whose data are available in [10], [13], is a 34.5-kV distribution system, supplied by a 10-MVA substation, which feeds an oil production area with 21 load nodes. The proposed feasible routes are displayed in Fig. 4. All aluminum conductors $1/0$ and $4/0$ are used with parameters given in [26], the cost per kilometer for circuits is 10 kUS$/km and 20 kUS$/km, respectively. Two kinds of tests were performed: *Test 1*—a planning considering one existing substation; and *Test 2*—a planning considering two substations, an existing one and a candidate substation. For this test the voltage thresholds is 3%, the average power factor is equal to 0.9, the cost of energy losses is 0.05 US$/kWh, the loss factor equals 0.35, the interest rate is 0.1 and the planning period extends to 20 years.

The NLP problem that is solved at each iteration of the CHA, for both tests, has the following characteristics: 117 variables (24 unconstrained and 93 constrained), one equality linear constraint, 46 equality nonlinear constraints, 35 linear inequality constraints, and 141 inequality nonlinear constraints.

1) Test 1—Planning of the Distribution System: This test has the objective to show a comparison between results obtained with the proposed method and results presented in the literature by using meta-heuristics. Table I shows the sequence in which the circuits were added to the system by the CHA, the circuit costs, the active power losses costs and the total investment cost. The circuits added in the local improvement phase are also shown. Due to the radial configuration constraint it is possible to preview the total number of iterations that is equal to $2(n_b - 1) + 1$. In this test the CHA solves a total of 45 NLP problems and the solution is depicted in Fig. 5.

Table II shows a comparison between the results presented in [10] and [13] and the results obtained by the proposed method. The topology (circuit cost) obtained is the same. The authors guess that the difference in the operation cost may be due to differences in the way the substation voltage limits are considered. In the present non linear model the substation voltage is a variable, and it can assume values in an interval ($0.97\ \mathrm{pu} \leq V \leq 1.03\ \mathrm{pu}$ in this test). Such detail of the model is not available in [10] and [13]. It should be also noted that the initial solution obtained by the CHA, without the local improvement phase, is a

TABLE I
ITERATIVE PROCESS OF THE CHA FOR TEST 1

Iteration CHA	Added circuits	Circuit cost (US$)	Losses cost (US$)	Total cost (US$)
0	–	0	37851	37851
1	$n^{+}_{01-10} = 1$	2021	37701	39722
2	$n^{+}_{10-14} = 1$	6318	37750	44068
3	$n^{+}_{06-14} = 1$	14495	36848	51343
4	$n^{+}_{10-19} = 1$	20444	36848	57292
5	$n^{+}_{06-07} = 1$	28625	31666	60291
6	$n^{+}_{07-08} = 1$	35491	31667	67158
7	$n^{+}_{14-23} = 1$	40351	31667	72018
8	$n^{+}_{19-21} = 1$	45901	31666	77567
9	$n^{+}_{19-22} = 1$	51728	31667	83395
10	$n^{+}_{03-08} = 1$	78807	25912	104719
11	$n^{+}_{10-20} = 1$	85780	24117	109896
12	$n^{+}_{05-23} = 1$	92189	24066	116255
13	$n^{+}_{14-17} = 1$	96671	24069	120739
14	$n^{+}_{15-21} = 1$	102740	24065	126805
15	$n^{+}_{11-21} = 1$	109134	24067	133200
16	$n^{+}_{03-09} = 1$	127336	22407	149742
17	$n^{+}_{04-05} = 1$	136738	20987	157725
18	$n^{+}_{12-23} = 1$	143523	19946	163470
19	$n^{+}_{16-20} = 1$	148542	19947	168488
20	$n^{+}_{11-13} = 1$	153594	19946	173541
21	$n^{+}_{17-18} = 1$	158006	19946	177952
22	$n^{+}_{02-08} = 1$	158762	19947	178709
Iteration LI	Removed/added circuits	Circuit cost (US$)	Losses cost (US$)	Total cost (US$)
1	$n^{+}_{03-08} = 0$			
	$n^{+}_{08-09} = 1$	152250	19921	172171
2	$n^{+}_{15-21} = 0$			
	$n^{+}_{15-18} = 1$	151892	20227	172119

TABLE II
PRESENT WORTH COSTS FOR TEST 1 (US$)

Solutions	Circuit cost	Losses cost	Total cost
Final [13]	151892	21021	172913
Final [10]	151892	21007	172899
Before LIP	158762	19947	178709
Final	151892	20227	172119

good quality solution for the proposed problem. The total CPU time is equal to 28.98 s.

2) Test 2—Planning of the Distribution System and Substations: In this test, the maximum capacity of substation at node 1 is set to 4 MVA and, in node 2, there is a candidate substation with a maximum capacity of 4 MVA as well, with a construction cost of 1000 kUS$ and the operation cost of this substation is 0.1 US$/kVAh^2. The solution obtained by CHA is shown in Fig. 6. Table III shows a summary of the results obtained with the proposed method.

Fig. 7 shows the step sequence of CHA to find the best solution. Note that two branchings were made in the iterative process. The best solution is Solution 1. In this test, 114 NLP problems were solved (considering the three found solutions). The total CPU time is equal to 111.86 s.

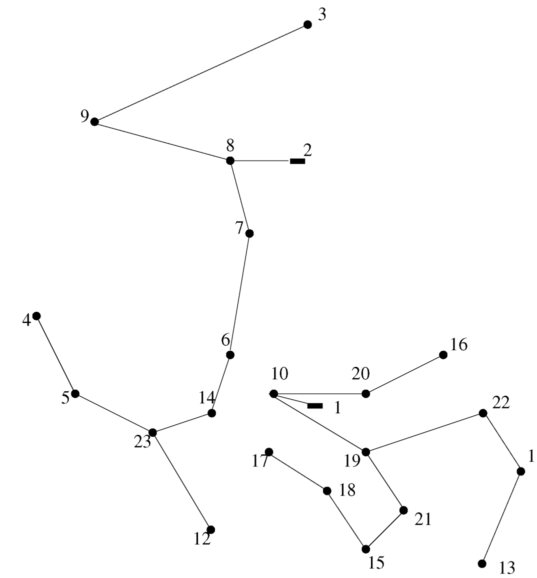

Fig. 6. Distribution system expansion plan for Test 2.

TABLE III
SUMMARY OF RESULTS FOR TEST 2 (US$)

Solutions	Circuit cost	Losses cost	Substation cost	Operation cost	Total cost
Before LIP	155694	14687	1000000	6493490	7663871
Final	149712	14259	1000000	6492761	7656732

B. The 54-Node Distribution System

The 54-node distribution system data and the proposed feasible routes are available in [8]. It is a 13.5-kV distribution system, supplied by a 107.8-MVA, which feeds 50 load nodes. This test aims at the planning of the distribution system considering four substations, two existing substations with possible expansion and two candidate substations. Two kinds of conductors were used, with a maximum current 90-A and 110-A, respectively. For this test the voltage thresholds is 5%, the average power factor is equal to 0.92, the cost of energy losses is 0.1 US$/kWh, the loss factor equals 0.35, the interest rate is 0.1, the operation cost of these substations is 0.1 US$/kVAh^2 and the planning period extends to 20 years.

The NLP problem that is solved at each iteration of the CHA has the following characteristics: 201 variables (143 bounded and 58 frees), one equality linear constraint, 108 equality nonlinear constraints, 44 linear inequality constraints, and 214 inequality nonlinear constraints. In this test 151 NLP problems were solved. The total CPU time is equal to 183.98 s for two branchings.

The solution obtained by CHA is shown in Fig. 8. Table IV shows the result obtained with the proposed method. The local improvement phase does not improve the solution found by the CHA. Note that the methodology has constructed four radial systems, and in order to attend the loads, two new substations

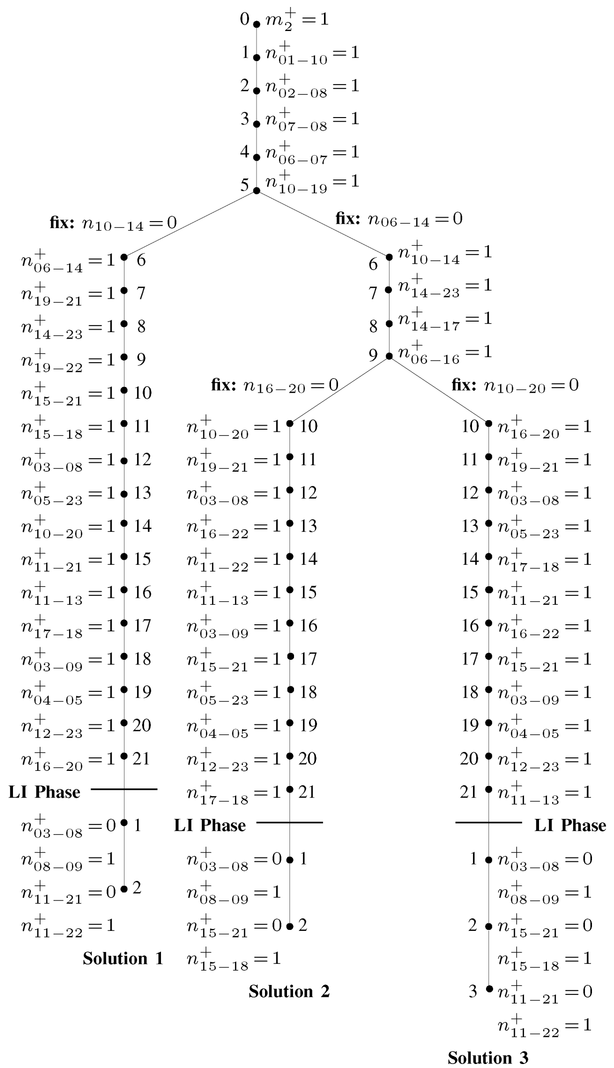

Fig. 7. Tree of the CHA for Test 2

TABLE IV
SUMMARY OF RESULTS FOR A 54-NODE DISTRIBUTION SYSTEM (US$)

Solutions	Circuit cost	Losses cost	Substation cost	Operation cost	Total cost
Final	39580	2672	540000	2933183	3515435

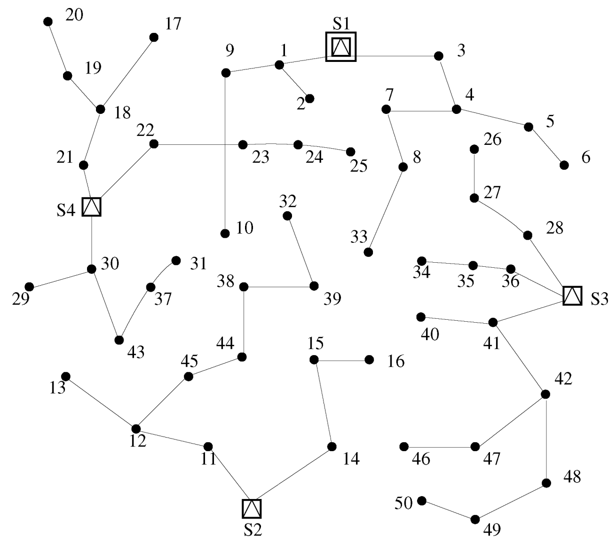

Fig. 8. Distribution system expansion plan for a 54-node distribution system.

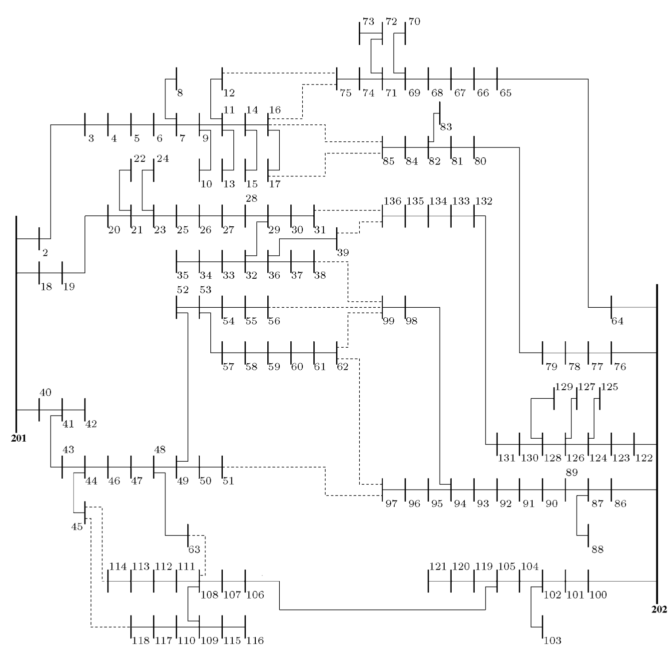

Fig. 9. The 136-node real distribution system.

were needed ($S3$ and $S4$) and the expansion of only substation $S1$.

C. The 136-Node Real Distribution System

The 136-node real distribution system is shown in Fig. 9. This is a 13.8-kV distribution system, supplied by two substations, 201 and 202, with 15 MVA and 10 MVA, respectively, and an increase of the loads fed by substation 202 is provided. These two substations feed 134 load nodes. However, in the current scenario, substation 202 is operating at its maximum capacity, and has no physical resources to increase this capacity. In that case it is necessary to plan the loads change from one substation to another, and in order to do this, circuits must be added, as well as which circuits must be removed (in fact opened) to respect the radial operation constraint. The dotted lines in Fig. 9 represent the circuits that may be added. To solve this problem it is necessary to consider the system fed by substation 202 as a system to be constructed. However there is no cost to add existing circuits (cost is equal to zero).

For this test the maximum allowable voltage drop is 7% and over-voltage up to 5%, the average power factor is equal to 0.92, the cost of energy losses is 0.1 US$/Wh, loss factor equals 0.35, interest rate is 0.1, the operation cost of these substation is 0.1 US$/kVAh^2.

The NLP problem that is solved at each iteration of the CHA has the following characteristics: 363 variables (224 bounded and 139 free), one equality linear constraint, 274 equality nonlinear constraints, and 300 inequality nonlinear constraints. In this test 459 NLP problems were solved. The total CPU time is equal to 345.16 s for two branchings.

TABLE V
SUMMARY OF RESULTS FOR 136-NODE REAL DISTRIBUTION SYSTEM (US$)

Solutions	Circuit cost	Losses cost	Operation cost	Total cost
Final	5360	6919	5461279	5473558

A summary of the results obtained is shown in Table V. Fifteen loads were transferred to substation 201 and the new system topology has seven new circuits, $n^+_{16-85} = 1, n^+_{31-136} = 1, n^+_{38-99} = 1, n^+_{51-97} = 1, n^+_{63-108} = 1, n^+_{45-114} = 1$ and $n^+_{45-118} = 1$, and seven circuits were opened, $n^+_{84-85} = 0, n^+_{93-94}=0, n^+_{96-97}=0, n^+_{106-107} = 0, n^+_{108-109} = 0, n^+_{108-111} = 0$ and $n^+_{134-135} = 0$. It can be observed that the CHA suggests the transfer of loads and the construction of new circuits considering the power loss of all distribution system and not only the cost of the new circuits candidates to be added. In this test, the local improvement phase does not improve the solution found by the CHA.

V. CONCLUSION

A constructive heuristic algorithm aimed to solve the power system distribution expansion planning was presented. The CHA has some advantages like robustness, and quickly presents viable investment proposals.

A nonlinear programming problem, in which the costs of system operation and construction of circuits and substations are minimized, subject to constraints as to attend the demand, the voltage magnitude between limits, the capacity of circuits and substations respected and also the radial configuration of the system, was solved by using a robust commercial software.

In the proposed method, a branching technique to avoid infeasible operation cases and also a local improvement technique to evaluate the CHA solution are included. Results obtained show the capability of the method to find an expansion plan to distribution systems, and the topology obtained in some tests were identical to those presented in the literature. Results also show the capacity of the method in solving problems in which construction of new substations and the transfer load to other substation or feeder are possible.

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REFERENCES

[1] T. Gönen, *Electric Power Distribution Systems Engineering*. New York: McGraw-Hill, 1986.

[2] H. Lee Willis, *Power Distribution Planning Reference Book*. New York: Marcel Dekker, 1997.

[3] S. Haffner, L. F. A. Pereira, L. A. Pereira, and L. S. Barreto, "Multistage model for distribution expansion planning with distributed generation-part I: Problem formulation," *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 915–923, Apr. 2008.

[4] P. C. Paiva, H. M. Khodr, J. A. D. Domímgues-Navarro, J. M. Yusta, and A. J. Urdaneta, "Integral planning of primary-secondary distribution systems using mixed integer linear programming," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1134–1143, May 2005.

[5] S. Haffner, L. F. A. Pereira, L. A. Pereira, and L. S. Barreto, "Multistage model for distribution expansion planning with distributed generation-part II: Numerical results," *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 924–929, Apr. 2008.

[6] J. L. Bernal-Agustín, "Aplicación de Algoritmos Genéticos al Diseño Optimo de Sistemas de Distrubuición de Energía Eléctrica," Ph.D. dissertation, Univ. Zaragoza, Zaragoza, Spain, 1998.

[7] E. Míguez, J. Cidrás, E. Díaz-Dorado, and J. L. García-Dornelas, "An improved branch-exchange algorithm for large-scale distribution network planning," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 931–936, Nov. 2002.

[8] V. Miranda, J. V. Ranito, and L. M. Proença, "Genetic algorithm in optimal multistage distribution network planning," *IEEE Trans. Power Syst.*, vol. 9, no. 4, pp. 1927–1933, Nov. 1994.

[9] I. J. Ramirez-Rosado and J. L. Bernal-Agustín, "Genetic algorithm applied to the design of large power distribution systems," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 696–703, May 1998.

[10] J. M. Nahman and D. M. Peric, "Optimal planning of radial distribution networks by simulated annealing technique," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 790–795, May 2008.

[11] V. Parada, J. A. Ferland, M. Arias, and K. Daniels, "Optimization of electric distribution feeders using simulated annealing," *IEEE Trans. Power Del.*, vol. 19, no. 3, pp. 1135–1141, Jul. 2004.

[12] A. Baykasoglu, S. Owen, and N. Gindy, "Solution of goal programming models using a basic taboo search algorithm," *J. Oper. Res. Soc., Nottingham*, vol. 50, no. 9, pp. 960–973, 1999.

[13] J. F. Gómez, H. M. Khodr, P. M. Oliveira, L. Ocque, J. M. Yusta, R. Villasana, and A. J. Urdaneta, "Ant colony system algorithm for the planning of primary distribution circuits," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 996–1004, May 2004.

[14] F. Mendoza, J. L. Bernal Agustín, and J. A. Domínguez-Navarro, "NSGA and SPEA applied to multiobjective design of power distribution systems," *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp. 1938–1945, Nov. 2006.

[15] E. Díaz-Dorado, J. Cidrás, and E. Míguez, "Application of evolutionary algorithms for the planning of urban distribution networks of medium voltage," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 879–884, Aug. 2002.

[16] S. K. Khator and L. C. Leung, "Power distribution planning: A review of model and issues," *IEEE Trans. Power Syst.*, vol. 12, no. 3, pp. 1151–1159, Aug. 1997.

[17] S. Najafi, S. H. Hosseinian, M. Abedi, A. Vahidnia, and S. Abachezadeh, "A framework for optimal planning in large distribution networks," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 1019–1028, May 2009.

[18] G. Y. Yang, Z. Y. Dong, and K. P. Wong, "A modified differential evolution algorithm with fitness sharing for power system planning," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 514–522, May 2008.

[19] M. Ponnavaikko and P. Rao, "Distribution system planning through a quadratic mixed integer programming approach," *IEEE Trans. Power Del.*, vol. 2, no. 4, pp. 1157–1163, Oct. 1987.

[20] S. Bhowmik, S. K. Goswami, and P. K. Bhattacherjee, "Distribution system planning through combined heuristic and quadratic programing approach," *Elect. Mach. Power Syst.*, vol. 28, no. 1, pp. 87–103, Jan. 2000.

[21] T. E. McDermott, I. Drezga, and R. P. Broadwater, "A heuristic nonlinear constructive method for distribution system reconfiguration," *IEEE Trans. Power Syst.*, vol. 14, no. 2, pp. 478–483, May 1999.

[22] M. J. Rider, A. V. Garcia, and R. Romero, "Power system transmission network expansion planning using AC model," *IET, Gen., Transm., Distrib.*, vol. 1, no. 5, pp. 731–742, Sep. 2007.

[23] I. C. Silva Junior, S. Carneiro Junior, E. J. Oliveira, J. S. Costa, J. L. R. pereira, and P. A. N. Garcia, "A heuristic constructive algorithm for capacitor placement on distribution system," *IEEE Trans. Power Syst.*, vol. 23, no. 4, pp. 1619–1626, Nov. 2008.

[24] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd ed. Pacific Grove, CA: Brooks/Cole-Thomson Learning, 2003.

[25] R. H. Byrd, J. Nocedal, and R. A. Waltz, "KNITRO: An integrated package for nonlinear optimization," in *Large-Scale Nonlinear Optimization*. New York: Springer-Verlag, 2006, pp. 35–59.

[26] L. L. Grigsby, *Electric Power Engineering Handbook*. Boca Raton, FL: CRC/IEEE Press, 2001.

Marina Lavorato (S'07) received the B.Sc. and M.Sc. degrees in electrical engineering from the Federal University of Juiz de Fora, Juiz de Fora, Brazil, in 2002 and 2004, respectively. Currently, she is pursuing the Ph.D. degree in electrical engineering at the University of Campinas, Campinas, Brazil.

Her areas of research are the development of methodologies for the optimization and planning and control of electrical power systems.

Marcos J. Rider (S'97–M'06) received the B.Sc. (Hons.) and P.E. degrees from the National University of Engineering, Lima, Perú, in 1999 and 2000, respectively, the M.Sc. degree from the Federal University of Maranhão, Maranhão, Brazil in 2002, and the Ph.D. degree from the University of Campinas, Campinas, Brazil, in 2006, all in electrical engineering.

His areas of research are the development of methodologies for the optimization, planning and control of electrical power systems, and applications of artificial intelligence in power systems.

Ariovaldo V. Garcia (M'89) received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from the University of Campinas (UNICAMP), Campinas, Brazil, in 1974, 1977, and 1981, respectively.

He is a Professor in the Electrical Energy Systems Department at UNICAMP. He has served as a consultant for a number of organizations. His general research interests are in the areas of planning and control of electrical power systems.

Rubén Romero (M'93–SM'08) received the B.Sc. and PE. degrees from the National University of Engineering, Lima, Peru, in 1978 and 1984, respectively, and the M.Sc. and Ph.D. degrees from the University of Campinas, Campinas, Brazil, in 1990 and 1993, respectively.

Currently he is a Professor of Electrical Engineering at the São Paulo State University (FEIS-UNESP), Ilha Solteira, Brazil. His general research interests are in the areas of planning of electrical power systems.