Creating Ensemble of Realistic Synthetic Distribution Networks

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***Abstract*—The abstract goes here.**

***Index Terms*—IEEE, IEEEtran, journal, LATEX, paper, template.**

I. Introduction

NETWORKED infrastructures are vital components of present day society and is instrumental for its proper functioning [1]. A common structural attribute of such networked infrastructures is that they are located close to human settlements since their principal function is to provide *life-line* for the society. Examples of some of these networked infrastructures include power system network, communication network, transportation network etc. Furthermore, the inter-dependencies among these networked infrastructures is well known.

The US government has termed these networks as critical infrastructures since a failure in one network can lead to cascade events in several dependent networks and eventually cause an immense impact on the nation's economy. In recent times, there has been an increased interest in studying the resilience and reliability of such networks during extreme events. The complex non-linear correlations between different networks has drawn attention of many researchers [2]–[4]. However, a fundamental problem to perform such analysis is the lack of data pertaining to such networked infrastructures. Additionally, emerging technologies and policies are being proposed to make these networks resilient to failures. For example, in case of power distribution system, the role of distributed energy resources (DERs) is being widely acknowledged as an efficient means to alleviate the network resiliency. The focus of power system research has moved towards efficient deployment and control of DERs in the distribution system [5]–[9]. However, there remains a lack of realistic test case scenarios to implement the proposed methodologies for their validation. Most of the algorithms and technologies are validated against standard IEEE test feeders or small sized distribution network. This unavailability is primarily due to the sensitive nature of the data as well as its enormous volume.

To this end, this paper proposes a methodology to synthetically generate distribution networks which connect substations to individual residential consumers through medium voltage (MV) and low-voltage (LV) networks. Thereafter, an ensemble of such synthetic networks can be produced by fine adjustments of design parameters.

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II. Creation of Secondary Network

The goal of this step is to create the secondary distribution network connecting local transformers along a given road network link $l \in \mathcal{L}_R$ to the set of $\mathcal{V}_H$ residences mapped to it. However, the local transformer locations are not known beforehand. Therefore, we assume probable transformer locations along the link based on standard engineering practice [10] and represent this set as $\mathcal{V}_T$. We need to connect the residence nodes in a forest of *starlike* trees with each tree rooted at a transformer node.

We further consider a Delaunay triangulation [11] of the residence and probable transformer nodes to obtain a possible set of edges $\mathcal{E}_D$ from which the secondary network edges are to be selected. Hence, we construct a connected network $\mathcal{G}_D := (\mathcal{V}, \mathcal{E}_D)$ with $\mathcal{V} = \mathcal{V}_T \bigcup \mathcal{V}_H$. Any edge $e \in \mathcal{E}_D$ is defined by incident nodes (i,j) where $i,j \in \mathcal{V}$. The aim of this step can be formalized through the following problem statement.

Problem II.1 (Secondary network creation problem)**.** *Given a connected network $\mathcal{G}_D(\mathcal{V}, \mathcal{E}_D)$ where $\mathcal{V} = \mathcal{V}_T \bigcup \mathcal{V}_H$, find $\mathcal{E}_D^\star \subseteq \mathcal{E}_D$ such that the induced subgraph network $\mathcal{G}_D^\star(\mathcal{V}^\star, \mathcal{E}_D^\star)$ is a forest of star-like trees with each tree rooted at some $t \in \mathcal{V}_T^\star$ where $\mathcal{V}^\star = \mathcal{V}_T^\star \bigcup \mathcal{V}_H$ and $\mathcal{V}_T^\star \subseteq \mathcal{V}_T$.*

Edge variables We introduce binary variables $\{x_e\}_{e\in\mathcal{E}_D} \in \{0,1\}^{|\mathcal{E}|_D}$. Variable $x_e = 1$ indicates that the edge is present in the optimal topology and vice versa. Each edge $e = (i,j)$ is assigned a flow variable f_e directed from the source node i to destination node j. The binary variable and flows can be stacked in $|\mathcal{E}_D|$-length vectors $\mathbf{x}$ and $\mathbf{f}$ respectively.

Node variables The average hourly load demand at the i^{th} residence node is denoted by p_i and is strictly positive. We stack these average hourly load demands at all residence nodes in a $|\mathcal{V}_H|$ length vector $\mathbf{p}$.

Degree constraint. Statistical surveys on distribution networks in [12] show that residences along the secondary network are mostly connected in series with at most 2 neighbors. This is ensured by (1) which limits degree of residence nodes to 2.

$$\sum_{e:(h,j)} x_e \leq 2, \quad \forall h \in \mathcal{V}_H \tag{1}$$

Power flow constraints. For the connected graph $\mathcal{G}_D(\mathcal{V}, \mathcal{E}_D)$, we define the $|\mathcal{E}_D| \times |\mathcal{V}|$ branch-bus incidence matrix $\mathbf{A}_{\mathcal{G}_D}$ with the entries as

$$\mathsf{A}_{\mathcal{G}_D}(e,k) := \begin{cases} 1, & k = i \\ -1, & k = j \\ 0, & \text{otherwise} \end{cases} \qquad \forall e = (i,j) \in \mathcal{E} \tag{2}$$

Since the order of rows and columns in $\mathbf{A}_{\mathcal{G}_D}$ is arbitrary, we can partition the columns as $\mathbf{A}_{\mathcal{G}_D} = \begin{bmatrix}\mathbf{A_T} & \mathbf{A_H}\end{bmatrix}$, without loss of generality, where the partitions are the columns corresponding to transformer and residence nodes respectively. We define $\mathbf{A_H}$ as the reduced branch-bus incidence matrix.

By ignoring network losses, the *linearized distribution flow* LDF model gives the relation between power consumption at the nodes and power flow along edges as (3a). Note that the optimal network is obtained from $\mathcal{G}_D$ after removing the edges for which $x_e = 0$. Therefore, (3b) enforces flows $f_e = 0$ for non-existing edges. The value of $\overline{f}$ in (3b) can be chosen to be the power flow capacities in the edges.

$$\mathbf{A_H}^T \mathbf{f} = \mathbf{p} \tag{3a}$$

$$-\overline{f}\mathbf{x} \leq \mathbf{f} \leq \overline{f}\mathbf{x} \tag{3b}$$

Ensuring radial topology In our case, this condition is satisfied by the node power flow condition in (3a) if all the residential nodes consume power which is a reasonable assumption. This is an extension of the proposition in [5] which considers renewable generation at the nodes. However, contrary to the previous work, the following proposition is a special case where every non-root node has only positive load demand. Using the power balance constraints to ensure radial topology reduces the number of constraints when dealing with large sized networks.

Proposition II.1. *A graph $\mathcal{G}_D(\mathcal{V}, \mathcal{E}_D)$, with $\mathcal{V} = \mathcal{V}_T \bigcup \mathcal{V}_H$, with reduced branch-bus incidence matrix $\mathbf{A_H}$ (corresponding to columns of $\mathcal{V}_H$) and node power demand vector $\mathbf{p} \in \mathbb{R}^{\mathcal{V}_H}$, with strictly positive entries, is connected if and only if there exists a vector $\mathbf{f} \in \mathbb{R}^{|\mathcal{E}_D|}$, such that (3a) is satisfied.*

Once the connectivity of the induced subgraph $\mathcal{G}_D^\star$ is ensured the radiality requirement can be enforced from a known graph theory property: *a forest with n nodes and m components has $n-m$ edges*. In our case, $|\mathcal{V}_H| + |\mathcal{V}_T^\star|$ nodes need to be covered and there would be $|\mathcal{V}_T^\star|$ components which leads us to the following constraint.

$$\sum_{e\in\mathcal{E}_D} x_e = n_h \tag{4}$$

Generating optimal network topology An edge $e : (i,j) \in \mathcal{E}_D$ for all $i,j \in \mathcal{V}$ is assigned a weight $w(i,j)$

$$w(i,j) = \begin{cases} \infty, & \text{if } i,j \in \mathcal{V}_T \\ \mathsf{dist}(i,j) + \lambda \mathsf{C}(u,v), & \text{otherwise} \end{cases} \tag{5}$$

where $\mathsf{dist} : \mathcal{V} \times \mathcal{V} \to \mathbb{R}$ denotes the geodesic distance between the nodes i,j. The function C denotes if the edge crosses the road link and is defined as

$$\mathsf{C}(u,v) = \begin{cases} 0, & \text{if } u,v \text{ are on same side of link} \\ 2, & \text{if } u,v \text{ are on opposite side of link} \\ 1, & \text{if } u \in \mathsf{V_R} \text{ or } v \in \mathsf{V_R} \end{cases}$$

λ is a weight factor to penalize multiple crossing of edges over the road links. It also penalizes multiple edges emerging from the root node. The weights are stacked in $|\mathcal{E}|$-length vector $\mathbf{w}$. Thereafter, a forest with trees rooted at the probable transformer nodes and spanning all the nodes of G is considered as the secondary network. This is obtained after solving the optimization problem.

$$\begin{aligned} &\min_{\mathbf{x}} \ \mathbf{w}^T \mathbf{x} \\ &\text{s.to. } (1), (3), (4) \end{aligned} \tag{6}$$

III. Creation of Primary Network

In this section, the goal is to create the primary distribution network which connects the substations to the local transformers. For this purpose, we aggregate the load at residences to the local transformer locations which have been obtained as an output from the preceding step. The objective of the current step is to connect all these local transformer locations to the substation.

We assume that the primary distribution network almost follows the road network and therefore the latter may be used as a proxy network for the former. Hence, the goal of the current step is to select edges along the road network graph such that all local transformer locations along road links are covered. The road network nodes which define the road network graph can be considered to be dummy points with no load, while the transformer locations have aggregated residential load demands. The road network nodes are only required to be covered in order to connect the local transformer points. In other words, the road network nodes cannot be leaf nodes in the created primary network.

Finally, a substation can have multiple feeder lines connecting different local group of residences. This is usual in a rural geographic region where such local groups are distinctly observable. Therefore, the primary network may be created as a forest of trees with the roots connected to the substation though high voltage feeder lines. The trees are rooted at some road network node and covers all the local transformers. The formal problem statement is provided here.

Problem III.1 (Primary network creation problem)**.** *Given a connected network $\mathcal{G}_R(\mathcal{V},\mathcal{E}_R)$ where $\mathcal{V}=\mathcal{V}_R\bigcup\mathcal{V}_T$, find $\mathcal{E}_R^\star\subseteq\mathcal{E}_R$ such that the induced subgraph network $\mathcal{G}_R^\star(\mathcal{V}^\star,\mathcal{E}_R^\star)$ is a forest of trees with each tree rooted at some $r\in\mathcal{V}_R^\star$ where $\mathcal{V}^\star=\mathcal{V}_R^\star\bigcup\mathcal{V}_T$ and $\mathcal{V}_R^\star\subseteq\mathcal{V}_R$.*

Node variables. The node set $\mathcal{V}=\mathcal{V}_R\bigcup\mathcal{V}_T$ comprises of road and transformer nodes respectively. The residential average hourly demands are aggregated at the transformer locations and stacked in a $|\mathcal{V}_T|$-length vector $\mathbf{p}$. Let v_i represent the voltage at the node i. The node voltages can be stacked in $|\mathcal{V}_R|+|\mathcal{V}_T|$ length vectors $\mathbf{v}$.

We assign binary variables $\{y_r,z_r\}_{r\in\mathcal{V}_R}\in\{0,1\}$. Variable $y_r=1$ indicates that road network node r is part of the primary network and vice versa. Variable $z_r=0$ indicates that road network node r is a root node (connected to substation through a high voltage feeder line) in the primary network and $z_r=1$ indicates otherwise. The binary variables are stacked in $|\mathcal{V}_R|$-length vectors $\mathbf{y}$ and $\mathbf{z}$ respectively.

Edge variables In order to identify which edges comprise of the primary network, each edge e is assigned a binary variable $\{x_e\}_{e\in\mathcal{E}_R}\in\{0,1\}^{|\mathcal{E}_R|}$. Variable $x_e=1$ indicates that the edge is part of the optimal primary network and vice versa. The binary variable for all edges and edge power flows can be stacked in a $|\mathcal{E}_R|$-length vectors $\mathbf{x}$ and $\mathbf{f}$ respectively.

Connectivity constraint Let $e:(r,j)\in\mathcal{E}_R$ denote an edge which is incident on the road node $r\in\mathcal{V}_R$. $\sum_{e:(r,j)}x_e$ is the degree of node r in graph $\mathcal{G}_R$.

$$\sum_{e:(r,j)}x_e\leq|\mathcal{E}_R|y_r,\qquad\forall r\in\mathcal{V}_R\tag{7a}$$

$$\sum_{e:(r,j)}x_e\geq y_r,\qquad\forall r\in\mathcal{V}_R\tag{7b}$$

$$1-z_r\leq y_r,\qquad\forall r\in\mathcal{V}_R\tag{7c}$$

$$\sum_{e:(r,j)}x_e\geq 2(y_r+z_r-1),\qquad\forall r\in\mathcal{V}_R\tag{7d}$$

If r is not included ($y_r=0$) in the primary network, (7a) and (7b) ensures that there are no incident edges on r since $\sum_{e:(r,j)}x_e=0$. Further, (7c) ensures $z_r=1$ which implies that an unselected road node is treated as a non-root node.
If r is included in primary network and is not a root node ($y_r=1,z_r=1$), it has to be a transfer node with a minimum degree of 2 which is ensured by (7d) through $\sum_{e:(r,j)}x_e\geq 2$. The binary variables y_r,z_r can be stacked in n_r-length vectors $\mathbf{y}$ and $\mathbf{z}$ respectively.
If r is included in primary network and is a root node ($y_r=1,z_r=0$), (7a), (7b) and (7d) ensures that the degree of the road node is positive.

Ensuring radiality constraint. From graph theory, we know that a forest with n nodes and m components has $n-m$ edges. In our case, the total number of nodes is $|\mathcal{V}_T|+\sum_{r\in\mathcal{V}_R}y_r$, while the number of components is the number of root nodes, i.e, $\sum_{r\in\mathcal{V}_R}(1-z_r)$. Therefore, the radiality constraint is given below

$$\sum_{e\in\mathcal{E}_R}x_e=|\mathcal{V}_T|+\sum_{r\in\mathcal{V}_R}y_r-\sum_{r\in\mathcal{V}_R}(1-z_r)\tag{8}$$

However, this is not a sufficient condition for radiality since it does not avoid formation of disconnected components with cycles. Therefore, we need to enforce constraints to avoid formation of cycles in individual trees.

Power balance and flow constraints We can define the branch-bus incidence matrix $\mathbf{A}_{\mathcal{G}_\mathbf{R}}\in\mathbb{R}^{|\mathcal{V}|\times|\mathcal{E}_R|}$. Since the order of rows and columns in $\mathbf{A}_{\mathcal{G}_\mathbf{R}}$ is arbitrary, we can partition the rows without loss of generality as $\mathbf{A}_{\mathcal{G}_\mathbf{R}}=\begin{bmatrix}\mathbf{A_T} & \mathbf{A_R}\end{bmatrix}$. Here, the partitions $\mathbf{A_T}$ and $\mathbf{A_R}$ are obtained by stacking the columns of $\mathbf{A}_{\mathcal{G}_\mathbf{R}}$ corresponding to transformer and road nodes respectively.

$$\mathbf{A_T}^T\mathbf{f}=\mathbf{p}\tag{9a}$$

$$-\overline{s}(1-\mathbf{z})\leq\mathbf{A_R}^T\mathbf{f}\leq\overline{s}(1-\mathbf{z})\tag{9b}$$

$$-\overline{f}\mathbf{x}\leq\mathbf{f}\leq\overline{f}\mathbf{x}\tag{9c}$$

(9a) ensures that a path exists between each transformer node and a root node. If a road node is not a root node (with $z_r=1$), (9b) enforces that the consumption at the node is 0. If a road node is a root node (with $z_r=0$), the power consumption/injection at the node is limited by the feeder capacity limits $[-\overline{s},\overline{s}]$. (9c) ensures that if an edge $e\in\mathcal{E}_s$ is selected ($x_e=1$), the power flow is limited by the line capacity limits $[-\overline{f},\overline{f}]$.

Voltage constraints. The created primary network should also satisfy the operational voltage constraints. The long HV lines from the substation to the root nodes in the optimal primary distribution network end in voltage regulators which ensure that the root nodes have a voltage of 1pu. This is ensured by (10a). (10b) limits the voltage at all nodes within the acceptable limits of $[\underline{v}, \overline{v}]$.

$$(1 - v_r) \leq z_r \ \forall r \in \mathcal{V}_{s_R} \tag{10a}$$

$$\underline{v}\mathbf{1} \leq \mathbf{v} \leq \overline{v}\mathbf{1} \tag{10b}$$

$$-(1 - x_e)M \leq v_i - v_j - r_e f_e \leq (1 - x_e)M, \ \forall e \in \mathcal{E}_s \tag{10c}$$

If r_e denotes the resistance of the line $e : (i, j) \in \mathcal{E}_R$, the Linearized Distribution Flow (LDF) model relates the squared voltage magnitude to power flows linearly as $v_i^2 - v_j^2 = 2r_e f_e$ where f_e is the entry from the vector $\mathbf{f}$ corresponding to edge e. The squared voltage can be approximated as $v_i^2 \approx 2v_i - 1$ which leads to the relation $v_i - v_j = r_e P_e$ (see [13]). Notice that (10c) enforces this constraint on only those edges $e \in \mathcal{E}_s$ for which $x_e = 1$. Here, a tight bound of M can be selected as $M = \overline{v} - \underline{v}$.

Generating optimal primary network Each edge $e = (u, v) \in \mathcal{E}_s$ is assigned a weight $w_e = w(u, v) = \mathsf{dist}(u, v)$ which is the geodesic distance between the nodes. Additionally, for every road node $r \in \mathcal{V}_{s_R}$, we compute its geodesic distance from the substation s and is denoted by d_r. The optimal primary network topology is obtained by solving the optimization problem.

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y},\mathbf{z}} \ & \sum_{e \in \mathcal{E}_s} x_e w_e + \sum_{e \in \mathcal{V}_{s_R}} (1 - z_r) d_r \\ \text{s.to. } & (7), (8), (9), (10) \end{aligned} \tag{11}$$

IV. Ensemble of Synthetic Networks

TABLE I: List of parameters to create ensembles of networks

Notation	Parameter	Network
r	line parameters	Secondary Network
λ	penalty factor	
$\overline{f}$	maximum flow limit in secondary network	
$\overline{s}$	maximum feeder rating	Primary Network
N_{feeder}	maximum number of feeders	

We look into the following aspect of the cumulative hop distribution.

$$F_{\theta,\phi}(k) = \frac{|\{i | i \in \mathcal{V}^\star, d(i, \mathsf{root}) \leq k\}|}{|\{i | i \in \mathcal{V}^\star\}|} \tag{12}$$

where $|\{\cdot\}|$ denotes the cardinality of the set $\{\cdot\}$

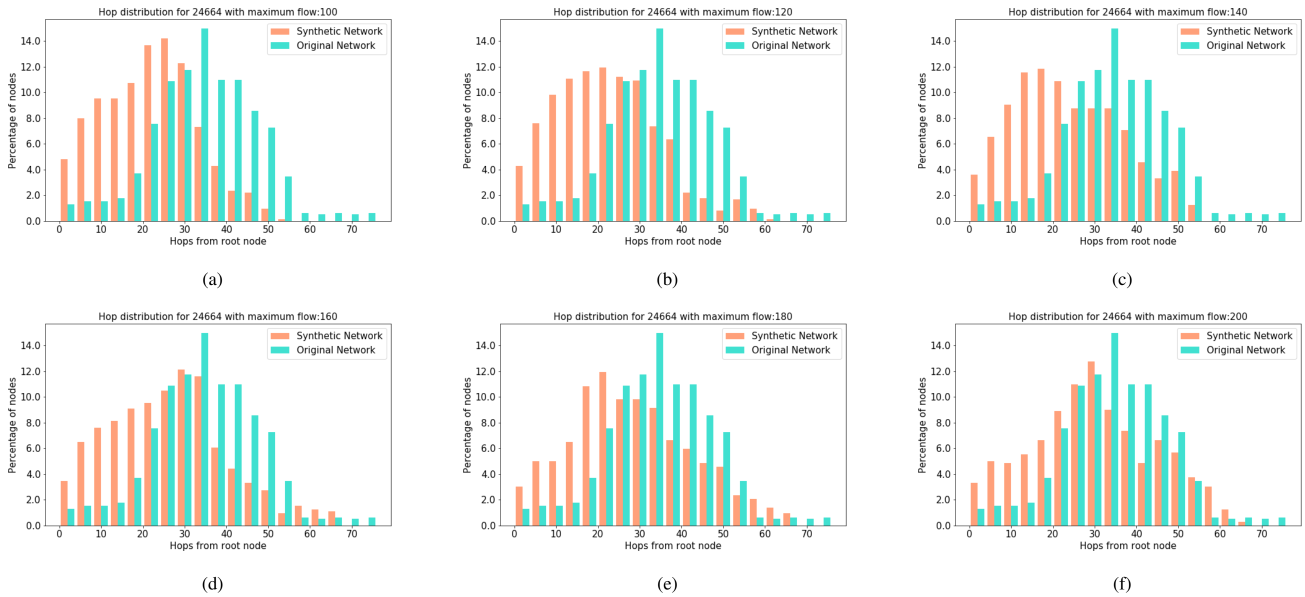

Fig. 1: Hop distribution of ensemble of synthetic networks generated with different line flow limits and with a fixed upper limit of feeder lines from a substation. The maximum rating of distribution lines is exceeded from 100 pu to 200 pu in steps of 20 pu (with base rating as 1kVA). As the allowable rating is increased, more number of nodes tend to be placed away from the root node. In other words, increasing the maximum allowable flow limit generates primary networks with longer chains.

V. Conclusion

The conclusion goes here.

Appendix A

Statement. A graph $\mathcal{G}_D(\mathcal{V},\mathcal{E}_D)$, with $\mathcal{V}=\mathcal{V}_T\bigcup\mathcal{V}_H$, with reduced branch-bus incidence matrix $\mathbf{A_H}$ (corresponding to columns of $\mathcal{V}_H$) and node power demand vector $\mathbf{p}\in\mathbb{R}^{\mathcal{V}_H}$, with strictly positive entries, is connected if and only if there exists a vector $\mathbf{f}\in\mathbb{R}^{|\mathcal{E}_D|}$, such that (3a) is satisfied.

Proof. The proof follows along similar lines to proof of Proposition 1 in [5]. For the sake of completeness, this has been put in this paper. Proving by contradiction, suppose $\mathcal{G}_D(\mathcal{V},\mathcal{E}_D)$ is not connected and there exists $\mathbf{f}\in\mathbb{R}^{|\mathcal{E}|_D}$ satisfying the proposed equality. If $\mathcal{G}_D(\mathcal{V},\mathcal{E}_D)$ is not connected, then there exists a connected component $\mathcal{G}_\mathcal{S}(\mathcal{V}_\mathcal{S},\mathcal{E}_\mathcal{S})$ which is a maximal connected subgraph with $\mathcal{V}_\mathcal{S}\subset\mathcal{V}$ and $\mathcal{V}_T\bigcap\mathcal{V}_\mathcal{S}=\emptyset$. Let $\mathbf{A}_\mathcal{S}$ denote the bus incidence matrix of $\tilde{\mathcal{G}}_\mathcal{S}$. By definition, it holds that $\mathbf{1}^T\mathbf{A}_\mathcal{S}=\mathbf{0}$.

Since graph $\mathcal{G}_\mathcal{S}(\mathcal{V}_\mathcal{S},\mathcal{E}_\mathcal{S})$ is a maximal connected subgraph of $\mathcal{G}_D$, there exists no edge (i,j) with $i\in\mathcal{V}_\mathcal{S}$ and $j\in\mathcal{V}_{\overline{\mathcal{S}}}$, where $\mathcal{V}_{\overline{\mathcal{S}}}=\mathcal{V}\setminus\mathcal{V}_\mathcal{S}$. Since the order of rows and columns of $\mathbf{A_H}$ are arbitrary, we can partition without loss of generality as

$$\mathbf{A_H}=\begin{bmatrix}\mathbf{A}_{\overline{\mathcal{S}}} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_\mathcal{S}\end{bmatrix}$$

We can partition vectors $\mathbf{f}$ and $\mathbf{p}$ conformably to $\mathbf{A_H}$ to get the following equality

$$\begin{bmatrix}\mathbf{A}_{\overline{\mathcal{S}}} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_\mathcal{S}\end{bmatrix}\begin{bmatrix}\mathbf{f}_{\overline{\mathcal{S}}}\\ \mathbf{f}_\mathcal{S}\end{bmatrix}=\begin{bmatrix}\mathbf{p}_{\overline{\mathcal{S}}}\\ \mathbf{p}_\mathcal{S}\end{bmatrix}$$

where $\mathbf{p}_\mathcal{S},\mathbf{p}_{\overline{\mathcal{S}}}$ are respectively the vectorized power demands at nodes $i\in\mathcal{V}_\mathcal{S}$ and $i\in\mathcal{V}_{\overline{\mathcal{S}}}$. From the second block, it implies that

$$\mathbf{A}_\mathcal{S}\mathbf{f}_\mathcal{S}=\mathbf{p}_\mathcal{S}\Rightarrow\mathbf{1}^T\mathbf{p}_\mathcal{S}=\mathbf{1}^T\mathbf{A}_\mathcal{S}\mathbf{f}_\mathcal{S}=\mathbf{0} \tag{13}$$

Since all entries of $\mathbf{p}$ are strictly positive from the initial assumption, we have $\mathbf{1}^T\mathbf{p}_\mathcal{S}\neq 0$ which contradicts (13) and completes the proof. □

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