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Knowing power grids and understanding complexity science

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Abstract: Complex networks theory has been well established as a useful framework for studying and analysing structure, dynamics and evolution of many complex systems. Infrastructural and man-made systems like power grids, gas and water networks and the internet, have been also included in this network framework, albeit sometimes ignoring the huge historical body of knowledge surrounding them. Although there seems to exist clear evidence that both complexity approach in general, and complex networks in particular, can be useful, it is necessary and profitable to put forward some of the limits that this scheme is facing when dealing with not so complex but rather complicated systems like the power grid. In this introductory paper, we offer a critical revision of the usefulness of the complexity and complex networks' approach in this later case, highlighting both its strengths and weaknesses. At the same time we emphasise the disconnection between the so called complex and the more traditional engineering communities as one of the major drawbacks in the advent of a true body of understanding, more than simply knowing the subtleties of this kind of complex systems.

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1 Engineered systems and complexity

Complexity can be considered a relatively new and basically interdisciplinary field that deals with the so called complex systems. It is worth to stress that although experimental observations are possible, these are usually not considered a part of the field of complex systems itself, which is primarily devoted to theoretical developments (Newman, 2011). Although there is no general agreement on a strict definition of a 'complex system', these are usually composed of many interacting units often called 'agents' (which can be consumers, species, vehicles, neurons, etc.) capable of many different types of interactions (*Nature*, 2008). This definition is usually completed with that of emergent behaviour, so that the collective behaviour of those parts together is more than the sum of their individual behaviours. These emergent phenomena are usually related with non-linear positive and negative feedback loops among composing parts (which means that small changes in the initial conditions can have a big effect at a global level) and the appearance of self-organising processes, which tends to drive them out of any perennial static equilibrium (i.e., classical thermodynamic principles do not apply). Classic and so-believed examples of complex systems include ecosystems, insect colonies, human societies, financial markets and infrastructural systems like the internet, road networks, gas pipe work or the power grid.

In the particular later context of power networks, complexity appears as hotchpotch where many different concepts from complexity science have been applied and analysed. Efforts involved in trying to characterise the power grid as a complex system have revolved essentially around the following issues [see Mei et al. (2011) for a complete review and references therein].

1.1   Criticality

As said before, self-organising processes are one usual outcome of complex systems. In any self-organising process, a pattern appears that can be clearly recognised and, far from subjective opinions, statistically quantified. Among the many statistical signatures of self-organisation, those involved in the characterisation of phase-transitions and critical states have become the most remarkable, like scaling relations, power-law divergences of some quantities, universality, fractality, etc. Notably, the appearance of algebraic probability distributions, especially power laws, in terms of event or object sizes, is often thought to be the signature of hierarchy, robustness, criticality and universal underlying mechanisms (Bak, 1996). In the case of power transmission networks, time series of usual measures of blackout size like energy not served, power loss or number of customers affected, have been shown to be algebraically distributed in many countries. This apparent ubiquitous evidence have led to believe and try to demonstrate that power systems tend to self-organise near a critical point and that there may be some universality ruling the inner depths of these systems [see Dobson et al. (2007) for a complete review and references therein].

1.2   Complex networks

In the last years, complex networks theory has appeared as a new framework to study complex systems (Newman, 2010). A network is a simplified representation of a system. It reduces it to a graph, an abstract entity that captures essentially its structure. Although some amount of specificity is lost, this is an advantage when questions that arise from difficult hypothesis can be answered with limited computational power and simplifying assumptions. Complex networks' theory applied to infrastructures in general, and power grid systems in particular, has produced a huge amount of literature basically centred around the three following issues:

- *Structure:* In order to understand the structure of the interactions between the components of a network, much effort has been done in defining metrics to characterise the topology of real networks [see Pagani and Aiello (2013) for a complete review and references therein]. Some of them, such as the degree (i.e., number of edges connecting an element to others), degree probability distribution, clustering coefficient (i.e., number of connected triples) and average path length (i.e., number of edges needed to travel from one element of the graph to another) have been used thoroughly in order to identify unifying principles and common statistical properties which, in turn, has allowed the development of models of networks (Boccaletti et al., 2006). A complete characterisation of a network's topology is motivated by the expectancy that modelling the structure of a complex network would lead to a better understanding of its evolutionary mechanisms and to a better comprehension of its dynamical and functional behaviour (Solé et al., 2003). In the case of power grids, some of the questions addressed with the analysis of the structure of their topology have tried to answer which are the measures that best characterise it and whether they are useful in order to understand and classify these systems. If topology and dynamics are deeply interwoven, another important question is whether we can find evidences of 'weak' or conversely 'robust' topologies and how different topologies stand the impact of different kinds of

damages (Rosato et al., 2007; Solé et al., 2008). This last aspect is essential for a critical infrastructure as the power grid.

- *Dynamics:* In the case of complex networks, dynamics refers to the case in which the processes involved in the redistribution of flows are taken into account. It involves the modelling of flows of the physical quantities of interest over the network and in power networks, it has been basically used to model blackouts and the robustness of the network to withstand malfunctions and attacks (Albert et al., 2000; Motter and Lai, 2002). When it comes to modelling the dynamics, the situation is far more complicated since the components of a network may have different dynamical behaviours and flows are often a highly variable quantity, both in space and time. The usual shortcut to overcome these difficulties has been the assumption of a characteristic load of an element as a measure of its capacity and the association of this load to a topological measure defined on it. The most used topological measure has been one standard measure of node centrality (Boccaletti et al., 2006): the so called betweenness centrality of that element (i.e., how many shortest paths are crossing that particular node or edge). The robustness of the network has been analysed by means of cascading processes, where an element of the system, characterised by a finite capacity (i.e., the maximum load that this element can handle and directly correlated to its topological value of betweenness) is eliminated. This fact changes the shortest paths between nodes and, consequently, the distribution of loads, creating overloads on some other nodes that, similarly, are eliminated from the network. This leads to a new redistribution of loads and the new overloaded nodes are removed. The redistribution process continues until when, at a certain time, all the remaining nodes have values under or equal to its capacity. Somehow related to this analysis, there has been lately remarkable research in *network synchronisation*, where networks of oscillators with particular distributions of frequencies are interpreted as models of frequency synchronisation dynamics among generators and loads (Lozano et al., 2012).

- *Evolution:* In analysing topology and dynamics of any networked system, we usually can only see the present outcome of a huge evolving process taking place in multiple spatial and temporal scales. Such process is driven by multiple and, usually, unknown forces. As these unknown forces shape and, at the same time, are being shaped by the evolving structure and dynamics of the network, its growth can be an intricate process. Here dynamics impels new topological forms that, at the same time, modify the flows of information between its constituents. Therefore, in order to fully comprehend a network we must deal with its evolution, as well as with its structure and topology (Dorogovtsev and Mendes, 2001). Unlike social networks for example, where topologies and dynamics can be easily retrieved from past accumulated data (Kumar et al., 2006; Leskovec et al., 2007), information regarding past evolutionary stages in infrastructural systems is difficult to obtain and usually not available at all. This drawback complicates the processes of corroborating assumptions imposed by models or validating the appearance of other intermediate evolutionary processes. With very few exceptions, like in Buzna and collaborators (Buzna et al., 2009), very little is known about the evolution of infrastructural networks. Its study is an essential component of the complex networks research agenda in order to shed light on some fundamental questions. As networked systems grow by adding elements and, at the same time, by coupling their dynamics to those

already present, the level of interaction between elements varies and this process modifies the fluxes of information and energy flows. Infrastructure networks in general (and power grids in particular) belong to the engineering field, where objectives and constraints are clear a priori. However, most technological networks have been continuously going through changes, spanning and crossing urban and natural systems from their early stages, adapting and being adapted by human societies, landscapes, territories and other constraints. This adaptive process has the power to modify the initial objective functions.

2   Complex or complicated?

There are huge conceptual and practical problems when engineering and complexity sciences are tried to be connected. The truth is that finding most of the characteristics of complex systems in man-made ones like the power grid is a difficult task. Concepts like emergence, self-organisation and adaptation are far from the engineering purposes and difficult, if not impossible, to experience in the man-made realm. Engineering is about assembling pieces that work in specific ways: that is, designing (complicated or not so) systems. These systems are robust in terms of redundancies but not adaptation: once a transmission line is constructed, it remains there for years to come and the system must deal with its fixed topology, and associated impact in the system's dynamics, whether it is beneficial for the whole functioning scheme or not in the long term. In the power grid case, self-organisation does not exist: synchronisation and phase angles must be maintained constantly under schedule. And so, it cannot lead to any kind of emergent behaviour or pose the system to any critical state (if we consider blackouts not as an emergent phenomenon but as an expected malfunction output and so, understandable from the individual behaviours of the system's composing elements).

Complexity and network theory have offered a new and fresh approach to an old system, that of the power grid. However, and paradoxically, it seems its usefulness has been relatively undermined by its oversimplifying assumptions. From a general conceptual point of view, there is one main drawback associated to the oversimplification of any real power network. This is that for power grids we can:

a   obtain a closed form mathematical model of the many equations that assess the physical system

b   manage to solve it in the time it is needed (being it offline or even online).

Thus, the question about why do we need simplified (or approximated) complex networks models if we can get better answers with fully precise models seems an obvious one. The approach offered by complex networks theory, with purely topological analysis (or even extended ones to take into account minimal electrical information), has been useful in order to detect critical components and evaluate topological robustness. The assessment of these results has been done by means of correlating topological and extended degrees (Bompard et al., 2009) with load shedding in real systems (over several

time spans) or even with statistical failure data of real blackouts and failures (Solé et al., 2008). However, results have been not conclusive and their applicability could not be generalised. Graph theory and applied statistical physics alone cannot account for a full characterisation of an electricity grid: there is a need for the deeper interplay between structural and electrical aspects.

As far as the dynamics is concerned, it is important here to stress the meaning of 'dynamical behaviour' (or simply dynamics) when dealing with complex networks, in contrast with its meaning in the power generation, operation and control field. While complex networks dynamics is related to the flow of information, energy or matter through the networked system and the different temporal values that characterise the resulting feature vector, power systems dynamics is related to frequency, synchronisation, swings and transient stability performance. If the word dynamics is used as in the complex networks acceptation, power grids seem to be an optimal candidate for the kind of dynamic analysis presented above, since cascading failures have been usually the prologue to huge blackouts (Nedic et al., 2006). But at the same time, power system operation, flow and generation turns out to be one of the most mathematically complicated problems encountered in engineering nowadays. All the variables and processes involved in such calculations demonstrate that power grids do not concentrate flows depending on topological centrality measures (like betweenness) nor the pool of sinks and sources are necessarily constrained by shortest paths (Wood and Wollenberg, 1996). Therefore, most of the initial complex networks dynamic models encountered in the literature have only a qualitative role as explanatory theory. Although they provide some indications on the actions that can be performed in order to decrease undesired effects such as congestions, avalanches of node breakdown and cascading failures, these are not realistic neither sufficiently accurate to explain the power grid dynamics. If the word dynamics is used as in the engineering acceptation, it does not mean a change in the configuration or topology (and not even the power flows over the lines). Dynamics is referred to the voltage and angular behaviour over time which concerns on stability. It does not mean a change in the configuration, but on the voltage and angular behaviour over time. Therefore, blackouts are due to voltage and angular instability and cannot be explained by any of the previous approaches.

It seems reasonable to ask ourselves whether the power grid is only complicated or there are other aspects that can turn it into a complex system. The answer is that maybe the physical network is not complex, but when other layers of agents are present, an increased number of non-linear interactions and unexpected phenomena can easily arise. These new agents, different from solely transformers, substations or lines, can be multiple heterogeneous decision makers at national and transnational levels, like transmission system operators (each one running and managing a subsystem of a global interconnected transmission system), policy makers, regulators or market participants. Topology (and the complex networks approach) appears then as just one more of the many dimensions of performance that influence global risk and security. In addition to purely electrical dynamics, the specific operative status of the grid is then a multivariable and multileveled problem, with possibilities for emergent phenomena and self-organising processes to develop.

3   From knowledge to understanding

There exists an important distinction between knowledge and understanding. While the former is usually constituted by sets of data, facts, observations or behaviours, the later appears when we appreciate that two or more very different-seeming phenomena can be treated as similar in some way (Lander, 2010). Complexity science has allowed the creation of understanding out of validated bits of knowledge and there seems to be no doubt about the usefulness of some of its concepts for understanding and optimising engineered systems (Ottino, 2004). In the particular case of power grids, the challenges are basically two:

1   to make useful models and theories that come from simplified schemes

2   to detect how far we can go in elucidating new metrics for addressing dynamical properties of the system's components, being them transformers, lines or users.

Some of the engineering areas where complexity concepts applied to power systems are useful and with a high potential of developing sound scientific frameworks in the short and mid-terms would be the following:

- *Smart grids*: Complex networks approach has been extremely useful in modelling and classifying topologies of real networks (Albert and Barabási, 2002). Some topologies are shown to be more efficient than others when information, resources or energy transportation are taken into account as fluxes (Pagani and Aiello, 2011). On the other hand, some biologically inspired mathematical models have captured the basic dynamics of network adaptability and have produced solutions with properties comparable to or better than those of real-world infrastructure networks (Tero et al., 2010). Such models may provide a useful starting point to improve routing protocols and topology control for infrastructural networks. This could be applied, for example, as a tool to support the modelling of low and medium voltage smart grids or distribution networks designed from scratch (Pagani and Aiello, 2012).

- *Agents:* It has been previously stated that, in the particular field of power systems, complexity appears when users of many kind interact with the physical system. When interacting and thoughtful agents appear, dependencies among the elements become increasingly important, stochastic processes arise and we are finally dealing with adaptive systems, usually modelled by computational units termed agents (Miller and Page, 2007; Railsback and Grimm, 2012). Agent (or individual)-based modelling has been traditionally linked to complexity and computer sciences and allows to modelling unique and autonomous entities that usually interact with each other and their environment locally. Agent-based modelling lets us address problems that concern adaptive behaviour and emergence phenomena, very much connected to prosumers' (i.e., small producers and consumers of energy at the same time) activities and dynamic behaviour of the power grid.

- *Interdependent networks:* Power systems are the underlying support of most infrastructural systems. Malfunctions and failures in the former dramatically affect the rest of interconnected networks. When interdependent networks are taken in consideration, complexity and complex networks approaches could be once again useful. There is a need for new robustness measures that capture not only purely topological aspects, but also the functionality of the system as a whole. And these

approaches are useful in capturing fundamental differences between real loads and commonly used load estimators, meta-vulnerabilities (Kurant and Thiran, 2006), or developing analytical models to study cascading failures in interdependent networks (Newman et al., 2005; Buldyrev et al., 2010; Brummitt et al., 2012).

Above all these issues, there is a pressing requirement of communication among scholars and academic traditions. Results from complex approximations have to be checked with real power networks' operators and facilities. A need of interaction and collaboration between complexity and complex networks communities, on the one hand, and electrical engineers, on the other, is compulsory. In order to fully develop the former issues, where complexity science can clearly help engineering science and conversely, this communicative gap between these two communities of knowledge must be overcome. This is the only way to finally become communities of understanding.

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