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DISTRIBUTION SYSTEM PLANNING THROUGH A QUADRATIC MIXED INTEGER PROGRAMMING APPROACH

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Abstract - This paper presents a new approach for the optimal sizing and siting of substations and network routing problem. The solution approach proposed is a nonlinear programming approach. The problem has been formulated as a Quadratic Mixed Integer programming (QMIP) problem in terms of the fixed costs of the substations and lines and the present worth of the energy loss costs of the line segments. The solution to this QMIP problem is obtained in two stages. In the first stage the quadratic programming problem is solved following the procedure developed by Wolfe using simplex method and treating all the variables as continuous variables. In the second stage, a procedure has been suggested to integerize the values of the integer variables. The proposed method is validated using a numerical example.

INTRODUCTION

The optimal distribution system planning problem is becoming increasingly important in the context of the increasing cost of construction, materials and equipment and energy conservation. This problem was studied by the authors in their earlier works [1, 2] using generalized approaches. In the generalized approaches, the studies are based on the assumption that the system conditions including load density are the same throughout the area, and that the area is virgin with no electrical network existing in the area to start with. Thus, the outcome of these studies results in that the optimal size obtained for all the substations is the same and substations are located at uniform intervals in the area with all the feeders being identical in configuration and loading - an ideal optimal solution for an area with uniform system conditions. This approach is more appropriate for rural distribution systems because of the uncertainties associated with the future loads and load locations, and the flexibility in selecting the feasible sites for substations and feeder routings. This approach thus serves only as a tool for formulating general guidelines for system expansion. But, in the case of urban distribution systems and subtransmission systems, the uncertainties are reduced through master developmental plans. The load locations, demand levels and the feasible sites for substations are known fairly in advance. The system conditions are rarely uniform. For such areas the generalized approaches are not suitable. It, thus, becomes necessary to adopt specific problem solution procedures for determining the optimal sizes and locations of the substations and the actual routings of the feeders to serve the actual demands of the individual demand locations.

Mathematical programming techniques, such as integer programming (IP), mixed integer programming (MIP), transportation models, transhipment methods and branch and bound techniques were extensively used for solving the problems of this nature. The problem of network synthesis and timing of capital investment for creating new facilities to meet future demands was solved by Boardman and Hogg [3] and Adams and Laughton [4] using dynamic programming methods. Masud [5] employed linear and integer programming to optimize the substation size and timing of substation expansion or new construction. Adams and Laughton [6] used mixed integer programming approach to the planning of electrical power networks. Their method was based on fixed-cost transportation-type models and they represented the nonlinear network loss cost by a piecewise linear cost function. Crawford [7] adopted a minimum path algorithm and the Ford and Fulkerson's transportation algorithm [8] for locating and sizing distribution substations, and deriving their optimal service areas. A branch and bound capacitated transhipment method was used by Hindi and Brameller [9, 10] for determining the optimal layout of a radial low voltage distribution network. This method also considered the network loss cost as a linearized function. Wall, Thompson and Northcote-Green [11] used a fast upper bounded transhipment code for planning the optimal radial distribution networks. Thompson and Wall [12] later proposed a branch and bound model for the optimal choice of substation locations. Their models again are based on linear cost approximation of the nonlinear network loss costs. Gonen and Foote [13] presented a total model for optimizing the number, size and location of substations, network routing, conductor sizing, reconductoring and load transfer simultaneously using mixed integer programming. The network loss cost was considered in this model by piecewise linearization. Sun et al [14] formulated the optimal distribution substation and primary feeder planning problem as a fixed charge transhipment network problem and suggested a solution methodology which employs the branch-and-bound algorithm and includes explicit modelling of fixed charge and variable cost components for improved accuracy. Kaplan and Braunstein [15] employed a graphoanalytical method for optimal substation siting.

It can be seen from the above review of the existing work in this direction that the majority of them are based on linear models and hence the nonlinear cost functions of the substations and feeder segments are linearized.

In this investigation, the above problem is formulated as a Quadratic Mixed Integer Programming (QMIP) problem. The solution to the problem is obtained in two stages. In the first stage, the quadratic programming problem is solved following the procedure developed by Wolfe [16] using simplex method [17, 18] and treating all the variables as continuous variables. The non-integer values, thus obtained for such of those variables which should have been integers are integerized in the second stage. A numerical example is

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solved using the proposed method and the results are presented.

PROBLEM DEFINITION

The main objective of this study is to optimize the number, size and location of substations, and the number, loading and routing of the feeders in an area for a given demand level at the load locations, subject to certain constraints.

Feeder Loss Cost

It is known from the fundamentals that the peak power loss in a feeder segment between the nodes i and j can be obtained as a function of the power flow (in kVA) in the segment as

$$P_{L_{ij}} = c'_{\ell_{ij}}\, p_{ij}^2 \tag{1}$$

where

$$c'_{\ell_{ij}} = \frac{0.001\, r}{(kV)^2}\, \ell_{ij} \tag{2}$$

Using (1) the present worth of the annual energy loss costs of the feeder segment during its life can be obtained as

$$E_p = c_{\ell_{ij}}\, p_{ij}^2 \tag{3}$$

where

$$c_{\ell_{ij}} = \frac{8.76\, r}{(kV)^2}\, \ell_{ij} \sum_{k=1}^{NLF} \frac{c_{e_k}(LLF_k)}{(1+u)^k} \tag{4}$$

Feeder Segment Cost

The cost of a new feeder segment between any two nodes i and j is represented by its fixed cost component, $c_{f_{ij}}$ $(i = 1,\ldots, NT)$, $(j = 1,\ldots, ND)$. For an existing feeder segment the fixed cost can be taken as zero.

Substation Cost

The fixed cost of a new feasible substation includes the cost of equipment, material and construction as well as the present worth of the constant transformation loss costs and the cost of infeed circuits to the new substations. For an existing substation, the fixed cost can be assumed to be zero.

Feeder Bay Cost

The cost of an existing feeder bay at a substation is taken as zero in the model. The possible feeder connections to a substation define the feasible number of new feeder bays at the substation. The cost of a new feeder bay includes the cost of a circuit breaker, measuring equipment, the materials and construction.

Cost Function

The cost function can be defined as

F = Substation fixed cost of all the feasible substation locations including the cost of infeed circuits
+ the cost of constant transformation losses at all the feasible substations
+ feeder bay costs
+ feeder segment costs
+ feeder loss costs

Statement of the Problem

The problem can thus be stated as

$$F = \sum_{i=NES+1}^{NS} c_{s_i}\delta_i + \sum_{i=NES+1}^{NS} c_{t\ell c}\,\delta_i + \sum_{i=1}^{NS}\sum_{j=1}^{ND} c_b\,\delta_{ij} + \sum_{i=1}^{NT}\sum_{j=1}^{ND} c_{f_{ij}}\,\delta_{ij} + \sum_{i=1}^{NT}\sum_{j=1}^{ND} c_{\ell_{ij}}\, p_{ij}^2 \tag{5}$$

Subject to

$$\delta_i = 1, \quad (i = 1,\ldots, NES) \tag{6}$$

$$\delta_i \in [\,0,1\,], \ (i = NES+1, \ldots, NS) \tag{7}$$

$$\delta_{ij} \in [\,0,1\,], \ (i = 1,\ldots, NT), \ (j = 1,\ldots, ND) \tag{8}$$

$$\sum_{j=1}^{NT} \delta_{ij} \geq 1, \quad (i = 1,\ldots, ND) \tag{9}$$

$$\sum_{j=1}^{ND} \delta_{ij} \leq NF_{max}\,\delta_i, \ (i = 1,\ldots, NS) \tag{10}$$

$$\sum_{\substack{i=1\\ i\neq j}}^{NT} (p_{ij} - p_{ji}) = D_j, \quad (j = 1,\ldots, ND) \tag{11}$$

$$\sum_{j=1}^{ND} p_{ij} \leq S_i\,\delta_i, \ (i = 1,\ldots, NS) \tag{12}$$

$$p_{ij} \leq U_{ij}\,\delta_{ij}, \ (i = 1,\ldots, NT), \ (j = 1,\ldots, ND) \tag{13}$$

$$p_{ij} \geq 0 \quad (i = 1,\ldots, NT), \ (j = 1,\ldots, ND) \tag{14}$$

SOLUTION METHOD

The problem stated in (5) - (14) is a quadratic programming problem with mixed integer variables subject to linear inequality constraints. The quadratic programming problem, without integer constraints on the integer variables can be solved following the procedure developed by Wolfe [16] using simplex method. This requires modification of the 'zero-one' integer constraints stated in (7) and (8) as

$$\delta_i \leq 1.0 \quad (i = NES+1, \ldots, NS) \tag{15}$$

$$\delta_i \geq 0.0 \quad (i = NES+1, \ldots, NS) \tag{16}$$

$$\delta_{ij} \leq 1.0 \quad (i = 1,\ldots, NT), \ (j = 1,\ldots, ND) \tag{17}$$

$$\delta_{ij} \geq 0.0 \quad (i = 1,\ldots, NT), \ (j = 1,\ldots, ND) \tag{18}$$

After solving the quadratic programming problem using Wolfe's method, the non-integer values of the integer variables obtained in the solution process, can be integerized subsequently following the procedure suggested later in this section.

Quadratic Programming (QP) Approach

The problem defined in (5),(6),(9)-(18) can be stated under the framework of Quadratic Programming as

$$\text{Minimize} \quad f(\underline{X}) = \underline{C}^T \underline{X} + \frac{1}{2} \underline{X}^T Q \underline{X} \tag{19}$$

$$\text{Subject to} \quad A \underline{X} \leq \underline{B} \tag{20}$$

$$\underline{X} \geq \underline{0} \tag{21}$$

where

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \underline{C} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & & & \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{bmatrix}, \text{ and } A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & & \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix} \tag{22}$$

In (19), the term $\frac{1}{2} \underline{X}^T Q \underline{X}$ represents the quadratic part of the objective function (5). Q is required to be either positive definite or positive semidefinite. It may be noted here that if $Q = 0$, the problem reduces to an LP problem.

Wolfe's theorem 2 states that if $\underline{X} \geq 0$, $A\underline{X} = \underline{B}$, and there exists a $\underline{\theta} \geq 0$ ($\underline{\theta}$ is an n by 1 vector) and λ (λ is an m by 1 vector) such that

$$\underline{\theta}^T \underline{X} = \underline{0} \tag{23}$$

and

$$Q\underline{X} - \underline{\theta} + A^T \underline{\lambda} + \underline{C} = \underline{0} \tag{24}$$

then $\underline{X}$ is the solution of the following problem.

$$\text{Min} \left[\underline{C}^T \underline{X} + \frac{1}{2} \underline{X}^T Q \underline{X} : \underline{X} \geq \underline{0},\ A\underline{X} = \underline{B} \right] \tag{25}$$

The above theorem is stated for problems with equality constraints. In the case of inequality constraints of the form (20), or of the form

$$A' \underline{X} \geq \underline{B}' \tag{26}$$

the inequality constraints are reduced to equality constraints, respectively, as

$$A \underline{X} + \underline{Y} = \underline{B} \tag{27}$$

$$A'\underline{X} - \underline{Y}' = \underline{B}' \tag{28}$$

where $\underline{Y}$ and $\underline{Y}'$ are slack variables.

Using Lagrange multiplier technique [17] and Wolfe's approach, the necessary conditions can be written as

$$c_j - \theta_j + \sum_{i=1}^{n} x_i q_{ij} + \sum_{i=1}^{m} \lambda_i a_{ij} = 0, \quad (j=1,..,n) \tag{29}$$

$$A_i \underline{X} + y_i = b_i, \quad (i = 1,\dots, m) \tag{30}$$

$$x_j \geq 0, \quad (j = 1,\dots, n) \tag{31}$$

$$y_i \geq 0, \quad (i = 1,\dots, m) \tag{32}$$

$$\lambda_i \geq 0, \quad (i = 1,\dots, m) \tag{33}$$

$$\theta_j \geq 0, \quad (j = 1,\dots, n) \tag{34}$$

$$\lambda_i y_i = 0, \quad (i = 1,\dots, m) \tag{35}$$

$$\theta_j x_j = 0 \quad (j = 1,\dots, n) \tag{36}$$

The solution of the above system of equations can be obtained by following the recursive procedure suggested by Wolfe which is given below :

Given a basis and basic solution satisfying (30) to (36)

$$c_j - \theta_j + \sum_{i=1}^{n} x_i q_{ij} + \sum_{i=1}^{n} \lambda_i a_{ij} + z_j = 0, (j=1,..,n) \tag{37}$$

and $\sum_{k=1}^{n} z_k > 0$, make one change of basis in the simplex procedure for minimizing the linear form

$$W = \sum_{k=1}^{n} z_k \tag{38}$$

under the side conditons that for $k=1,\dots,n$, if x_k is in the basis do not admit θ_k and if θ_k is in the basis, do not admit x_k to satisfy the nonlinear constraint (36): similarly, to satisfy (35), if y_k is in the basis do not admit λ_k and if λ_k is in the basis do not admit y_k.

The recursive step is repeated until the equation (38) vanishes, yielding $z = 0$. The x-part of the terminal basic solution is a solution to the quadratic programming problem.

It can be seen from the above that with the exception of (35) and (36), the necessary conditions are linear functions of the variables x_j, y_i, λ_i and θ_j. Thus the solution of the original quadratic programming problem can be obtained by finding a non-negative solution to the set of $m + n$ linear equations given by (29) and (30), which is also made to satisfy the $m + n$ equations stated in (35) and (36).

If Q is either a positive definite or positive semi-definite matrix, $f(\underline{X})$ will be a convex function and, therefore, any local minimum of the problem will be the global minimum. Further, the solution of (29), (30), (35) and (36) must be unique, as the number of equations in the necessary conditions stated in (29) to (36) is equal to the number of variables i.e. $2(n+m)$. Hence, the feasible solution that satisfies (29) to (36), if it exists, must give the optimum solution of the quadratic programming problem directly.

In the case of problems having inequality constraints of the form (28) or equality constraints, surplus variables are introduced which are forced to zero in the final solution. In such cases, the initial simplex tableau is converted into a canonical form to obtain the intitial basic solution for providing the necessary starting point to the simplex procedure.

Integerization Procedure for the QMIP Problem

In the problem under consideration, integer variables are defined to represent the status of the potential substation locations and the feasible feeder branch elements in the final solution. Assignment of a value of 'one' to these integer variables during the optimization process is an indication of their selection in the final optimal solution. On the other hand assignment of 'zero' indicates their rejection. Hence, the values obtained for these variables in the final solution should be either zero or one. But, the Wolfe's quadratic programming solution procedure suggested for solving this problem does not ensure zero/one integer values to these variables. The values obtained through the QP solution procedure for the integer variables in the final solution are mostly non-integer fractions. Thus, there is a need for integerizing the optimal fractional values of the variables which should take integer values.

A two stage procedure is suggested for this purpose. In the first stage, the QP solution obtained as above, is modified by integerizing the fractional values of the substation variables δ_i ($i = N \in S+1, \ldots, NS$), to the nearest integer variables (i.e. either zero or one). In the second stage the QP problem is redefined retaining only the selected substation sites and the associated feeder branch elements. At this stage appropriate constraints are introduced to force the values of the integer variables (greater than or equal to 0.5) associated with the substation locations to one in the final solution. The revised QP problem is solved again using the procedure suggested in the previous section. In the solution thus obtained, the values of the integer variables associated with the feasible feeder branch elements may be non-integer fractions. Rounding off these fractions again to their nearest integer values (zero or one) gives the desired optimal radial configuration of the distribution feeders.

In the mixed integer programming problem discussed here, fractional values assigned to the integer variables and the corresponding minimum objective function do not constitute a feasible solution. Hence a rounding-off procedure is introduced in the solution algorithm discussed above to obtain a feasible and an optimal solution in both the stages. It is to be noted that the function f(x) in (19) is a strictly convex function and the rounding-off the fractional values to either zero or one (depending on whether this value is less than 0.5 or greater than or equal to 0.5) basically increases the minimum value of the objective function on either side of the optimal value obtained. Further, during the solution procedure, the fractional values (real variables) assigned to the integer variables depend upon their coefficients. Higher the value of these coefficient, lower are the values assigned to these variables to minimize the objective function. Thus, the obvious choice of these integer variables is to round them off to one if their values are greater than or equal to 0.5 or make them zero if they are less than 0.5. The increase in the minimum value of objective function was observed to be the least with the above rounding-off procedure. Thus rounding-off the fraction greater than or equal to 0.5 to one and rejecting the rest leads to the optimal radial configuration of the distribution feeders in the area.

NUMERICAL EXAMPLE

Consider the example shown in Fig. 1. It is a problem of selecting optimal 33 kV feeder routings and the optimal locations for 132/33 kV substations to feed the demands at the 33/11 kV substations in the area. The general data, load data, the feasible potential sites for locating the grid substations and the feasible routes for the 33 kV feeders are given in the Tables 1 to 4.

Table 1

General Data

1. Load factor	: 0.40
2. Loss load factor	: 0.21
3. Cost of energy	: Rs. 0.3
4. Discount rate	: 0.10
5. Power factor	: 0.80
6. System life	: 25 years
7. Conductor size	: Raccoon (ACSR)
8. Fixed cost of the substation	: Rs. 3 million
9. Feeder bay cost	: Rs. 0.1 million
10. Feeder cost per km	: Rs. 0.4 million

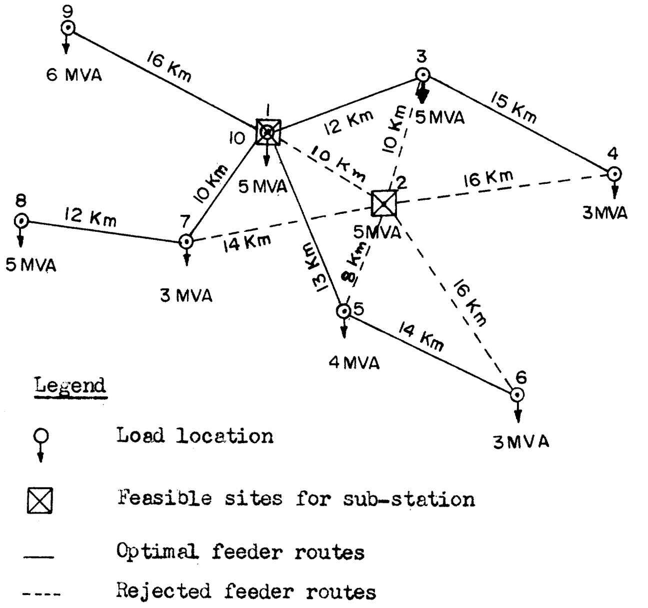

Fig. 1 A Sample System

Table 2

Load Data

S.No.	Node No.	Demand (MVA)	S.No.	Node No.	Demand (MVA)
1	3	5	5	7	3
2	4	3	6	8	5
3	5	4	7	9	6
4	6	3	8	10(1)	5

Table 3

Substation Data

S.No.	Feasible location number	Max. feasible capacity (MVA)	Cost in million Rs.	Cost of feeder bay in million Rs.	Integer variable name
1	1	50	3.1	0.1	x_1
2	2	50	3.1	0.1	x_2

Defining the variables as in Tables 3 and 4, and using the data listed therein, the problem can be stated as

$$\begin{aligned}\min_{\underline{X}} F = {} & 3.1\,x_1 + 3.1\,x_2 + 0.58\,x_3 + 0.62\,x_4 + 0.50\,x_5 \\ & + 0.10\,x_6 + 0.50\,x_7 + 0.50\,x_8 + 0.42\,x_9 + 0.74\,x_{10} \\ & + 0.74\,x_{11} + 0.66\,x_{12} + 0.48\,x_{13} + 0.64\,x_{14} + 0.60\,x_{15} \\ & + 0.56\,x_{16} + 0.74\,x_{17} + 0.0205\,x_{18}^2 + 0.0222\,x_{19}^2 \\ & + 0.0171\,x_{20}^2 + 0.0171\,x_{22}^2 + 0.0171\,x_{23}^2 + 0.0137\,x_{24}^2 \\ & + 0.0274\,x_{25}^2 + 0.0274\,x_{26}^2 + 0.0240\,x_{27}^2 + 0.0205\,x_{28}^2 \\ & + 0.0274\,x_{29}^2 + 0.0257\,x_{30}^2 + 0.0240\,x_{31}^2 + 0.0274\,x_{32}^2\end{aligned}$$

Subject to

$x_1 \le 1,\; x_2 \le 1,\; x_3 \le 1,\; x_4 \le 1,\; x_5 \le 1,\; x_6 \le 1,$

$x_7 \le 1,\ x_8 \le 1,\ x_9 \le 1,\ x_{10} \le 1,\ x_{11} \le 1,\ x_{12} \le 1,$

$x_{13} = 1,\ x_{14} \le 1,\ x_{15} \le 1,\ x_{16} \le 1,\ x_{17} \le 1$

$x_3 + x_4 + x_5 + x_6 + x_{17} \le 4\, x_1$;

$x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} \le 4\, x_2$; $x_6 + x_7 + x_{14} \ge 1$;

$x_3 + x_8 + x_{15} \ge 1$; $x_{15} + x_{11} \ge 1$;

$x_4 + x_9 + x_{16} \ge 1$; $x_{10} + x_{16} \ge 1$;

$x_5 + x_{12} + x_{13} \ge 1$; $x_{17} + x_{14} \ge 1$; $x_{28} = 5$;

$x_{29} + x_{32} = 6$; $x_{27} + x_{20} - x_{28} = 3$; $x_{21} + x_{22} - x_{29} = 5$;

$x_{18} + x_{23} - x_{30} = 5$; $x_{30} + x_{26} = 3$; $x_{19} + x_{24} - x_{31} = 4$;

$x_{25} + x_{31} = 3$; $x_{18} + x_{19} + x_{20} + x_{21} + x_{32} \le 50\, x_1$;

$x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \le 50\, x_2$; $x_{18} \le 12\, x_3$;

$x_{19} \le 12\, x_4$; $x_{20} \le 12\, x_5$; $x_{22} \le 12\, x_7$; $x_{23} \le 12\, x_8$;

$x_{24} \le 12\, x_9$; $x_{27} \le 12\, x_{12}$

$$x_1, x_2, \ldots, x_{32} \ge 0 \qquad (40)$$

After defining the cost function (39) and obtaining the constraints (40), the matrices A, B, C and Q are formed which are fed to the QMIP software package, developed for this purpose. The QMIP software package builds the Wolfe's constraints as defined by (37) and the simplex tableau is developed. The problem is then solved using the solution algorithm discussed in the previous section

The results obtained for the example system are given in Table 5. The optimal network configuration, thus obtained is shown in full lines in Fig. 1.

Table 5

Optimal Solution of the QMIP Problem of Fig. 1

1. The optimal value of the cost function : Rs.12.6204 million
2. The optimal substation location ($x_1 = 1$) : 1
3. Optimal network links

Integer variable	From node	To node	Power flow (MVA)
$x_3 = 1$	1	3	$x_{18} = 8$
$x_4 = 1$	1	5	$x_{19} = 7$
$x_5 = 1$	1	7	$x_{20} = 8$
$x_6 = 1$	1	10	$x_{21} = 11$
$x_{13} = 1$	7	8	$x_{28} = 5$
$x_{14} = 1$	10	9	$x_{29} = 6$
$x_{15} = 1$	3	4	$x_{30} = 3$
$x_{16} = 1$	5	6	$x_{31} = 3$

4. The optimal number of feeders at the substation 1 : 4

Table 4

Feasible Feeder Branch Data

S.No.	From Node	To Node	Length (km)	Cost in million Rs.	Loss cost coefficient (million Rs./MVA^2)	Maximum Feeder capacity(MVA)	Integer variable name	Flow variable name
1	1	3	12	0.48	0.0205	12	x_3	x_{18}
2	1	5	13	0.52	0.0222	12	x_4	x_{19}
3	1	7	10	0.40	0.0171	12	x_5	x_{20}
4	1	10	0	0.00	0	-	x_6	x_{21}
5	2	10	10	0.40	0.0171	12	x_7	x_{22}
6	2	3	10	0.40	0.0171	12	x_8	x_{23}
7	2	5	8	0.32	0.0137	12	x_9	x_{24}
8	2	6	16	0.64	0.0274	12	x_{10}	x_{25}
9	2	4	16	0.64	0.0274	12	x_{11}	x_{26}
10	2	7	14	0.56	0.0240	12	x_{12}	x_{27}
11	7	8	12	0.48	0.0205	12	x_{13}	x_{28}
12	10	9	16	0.64	0.0274	12	x_{14}	x_{29}
13	3	4	15	0.60	0.0257	12	x_{15}	x_{30}
14	5	6	14	0.56	0.0240	12	x_{16}	x_{31}
15	1	9	16	0.64	0.0274	12	x_{17}	x_{32}

DISCUSSION

The example studied is a problem with 8 demand locations, two feasible potential sites for constructing substations and 15 feasible branch elements for network connections. The number of constraints posed for this problem is 43. The 32 variable problem with 43 constraints is very effectively solved through the proposed QMIP solution approach. The only limitation, as can be seen from the solution approach, is the increase in the number of variables during the solution process when solved using the QMIP approach. For the present problem the number of variables increases to 182 during the solution process, though it is a 32 variable problem. However, with the advent of large and fast computing systems, it is no more a limitation for solving large distribution system problems.

While defining the flow constraints in (11) two variables p_{ij} and p_{ji} in a branch between the nodes i and j, indicating the power flows from the nodes i to j and j to i, respectively, are assumed. However, while constructing the flow constraints for a particular problem, it is not necessary to assume both the variables for a particular branch when its definite direction of flow is known. This will help in reducing the number of variables and hence the size of the problem. In the present example, it can be seen from Table 4 that only one variable for every branch has been assumed, since the possibility of power flowing in the other direction, in this case either does not arise or is obviously uneconomical. But, in large systems with more substation locations it is advisable to assume both the variables for such of those branch elements for which it is difficult to decide a-priori the economical direction of flow.

CONCLUSIONS

In this paper, the problem of optimal sizing and siting of the substations and network routing is studied. The model incorporates the actual nonlinear loss cost function of the feeders in its quadratic form. The solution to the QMIP problem has been obtained in two stages, using in the first stage, the solution procedure suggested by Wolfe for solving the quadratic programming problem and applying, in the second stage, an integerization procedure suggested in this paper for obtaining the optimal integer values of the integer-valued variables. The optimization procedure suggested is very effective in obtaining the optimal network solutions without the need for linearizing the nonlinear loss cost functions.

NOTATIONS

p_{ij}	power flow in the feeder branch between the nodes i and j in kVA
r	per-phase resistance of the feeder in ohms/km
kV	circuit voltage in kilo volts
ℓ_i	length of the i^{th} feeder branch in km
c_{e_k}	cost of energy at the k^{th} year in Rs./kWh
LLF_k	loss load factor during the k^{th} year
u	annual discount rate in p.u.
NT	number of total nodes
NLF	feeder life in years
NS	number of substation nodes including the existing ones
NES	number of existing substation nodes
ND	number of demand nodes
c_{s_i}	fixed cost of the i^{th} substation including the cost of infeed circuit required to feed the i^{th} substation in Rupees
$c_{t\ell c}$	cost of the constant transformation loss in Rupees
c_b	feeder bay cost per bay in Rupees
$c_{f_{ij}}$	cost of the feeder segment between the nodes i and j in Rupees. $c_{f_{ii}} = 0, \ (i = 1,\dots, NT)$
$c_{\ell_{ij}}$	loss cost coefficient given by (4) $c_{\ell_{ii}} = 0, \ (i = 1,\dots, NT)$
D_j	load demand at node j, in kVA
S_i	maximum feasible capacity at substation i, in kVA
U_{ij}	upper bound of branch flow from node i to j, given by the thermal capacity of the conductor used, in kVA
δ_i	binary integer variable which denotes the decision to select site i for a new substation (= 1 denotes selection and 0 indicates rejection)
δ_{ij}	binary integer variable which denotes the decision to select a branch between the nodes i and j for network connection (= 1 denotes selection and 0 denotes rejection)
λ	Lagrange multiplier
NF_{max}	maximum permissible number of feeder bays at a substation
A_i	i^{th} row of the A matrix.

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