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Multi-objective planning of electrical distribution systems using dynamic programming

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ABSTRACT

This paper presents a novel dynamic programming approach for multi-objective planning of electrical distribution systems. In this planning, the optimal feeder routes and branch conductor sizes of a distribution system are determined by simultaneous optimization of cost and reliability. The multiple planning objectives are minimization of: (i) installation and operational cost, and (ii) interruption cost. The first objective function consists of the installation cost of new feeder branches and substations, maintenance cost of the existing and new feeder branches, and the cost of energy losses. The second objective function measures the reliability of the distribution network in terms of the associated interruption costs for all the branches, which includes the cost of non-delivered energy, cost of repair, and the customer damage cost due to interruptions. A dynamic programming based planning algorithm for optimization of the feeder routes and branch conductor sizes is proposed. A set of Pareto solutions is obtained using a weighted aggregation of the two objectives with different weight settings. The proposed approach is evaluated on 21-, 54-, and 100-node distribution systems. The simulation test results are analyzed with various case studies and are compared with those of two existing planning approaches based on multi-objective evolutionary algorithm.

1. Introduction

Electric power distribution system planning is an important technology for power utilities in the deregulated power market [1,2]. A typical distribution system planning is broadly categorized either as a *static* or an *expansion* planning [1]. The static planning is a one-step planning of a new network, whereas an expansion planning is adopted to plan a network taking the load growth at the existing nodes and/or inclusion of additional load nodes. An expansion planning can be of single stage for single horizon year or multi-stage, i.e., stage-by-stage expansion. A proper planning of a distribution system not only saves expenditure for the utilities but also helps to meet customer satisfaction, which is very important in the competitive power market. A lot of computer-based distribution systems planning approaches are reported during the past three decades. State-of-the-art reviews of the reported works can be found in [3,4]. The distribution system planning is essentially an optimization process to obtain a number of planning/design variables such as: (i) size and location of distribution substation, (ii) number of feeders and their routes, and (iii) branch conductor sizes. The planning objectives include minimization of the installation cost of new facilities (substations/feeders/branches), cost of capacity addition of existing facilities, maintenance cost of the feeders and network power loss, and maximization of the network reliability. This optimization is also subject to some constraints, such as substation/feeder capacity limit, node voltage deviation limit, and network radiality.

In the early works [3–5], the planning model is formulated with one objective, i.e., minimization of installation cost and the cost of energy losses. The network reliability, an important aspect in the competitive power market, is also considered as another objective [6–29]. The network reliability is maximized by optimizing different reliability objective functions, such as total (cost of) non-delivered energy [6–13,24–28], customer outage cost [14], customer interruption cost [15,16], and contingency-load-loss index [29]. Two approaches have been used for optimizing the cost and reliability. In the first approach [6–16], both objectives are aggregated to obtain a single solution, while the second approach [17–29] takes the conflicting natures of the cost and reliability into account by simultaneous optimization of the two objectives to obtain a set of non-dominated solutions, called Pareto solutions [19,20] and a decision maker or the planning engineer selects one solution for implementation.

The main challenge in this planning is to devise a solution strategy as the objective functions are typically nonlinear, non-convex,

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non-differentiable with discrete and continuous decision variables. The difficulty increases with higher dimensions that depend on the number of nodes in the network. The reported solution strategies fall into two categories: (i) deterministic algorithms and (ii) heuristics-based algorithms. The deterministic algorithms are based on mathematical optimization technique. They can always produce same output for a given input. The heuristics-based algorithm can produce an acceptable solution to a problem in many practical scenarios, but there is no formal proof of its optimality. The deterministic algorithms that have been used for this problem are: nonlinear mixed integer programming [7], dynamic programming [6,8], nonlinear programming [9,10], Benders' decomposition [17,18], etc. Most of the heuristics-based algortithms applied to this problem are based on the evolutionary algorithms (EAs), such as genetic algorithm (GA) [12–16,19–24], tabu search (TS) [25,26], artificial immune system (AIS) [27], particle swarm optimization (PSO) [28,29], and honey bee mating optimization [30].

The evolutionary computation techniques are used as solution strategies in most of the Pareto-based multi-objective planning approaches [19–30] due to their multi-point search capability, which helps to obtain a set of non-dominated solutions in a single run. However, the major drawback of a multi-objective evolutionary algorithm (MOEA) is that the convergence is not always guaranteed. On the contrary, the deterministic algorithms are well known for their good convergence characteristics. Till date, few works [17,18] have reported the use of deterministic algorithms for simultaneous optimization of multiple objectives to obtain a set of non-dominated solutions. In [17], a two-step approach based on linear programming for optimization of continuous variables followed by integer programming for optimization of integer variables has been used. The formulation approximates the quadratic cost function due to energy losses as a linear function and solves it by linear programming. In [18], the mixed-integer programming (MIP) is used with commercial MIP-solver GAMS. The branch conductor size optimization has not been considered in both the approaches in view of both cost and reliability objectives as in some MOEA-based works [20,28]. There is another powerful deterministic algorithm, i.e., the dynamic programming, which can deal with this type of objective functions efficiently. Although, it is used in [6,8], none of the two approaches deals with simultaneous optimization of the objective functions. Moreover, it is reported in [8] that the computation time of GAMS is reasonably higher than that of dynamic programming for the same distribution system planning problem.

Motivated by all these issues, an attempt is made to investigate the use of dynamic programming for simultaneous optimization of the objective functions in a distribution system planning problem. The two objective functions of the proposed multi-objective planning model are formulated as: (i) *total installation and operational cost* and (ii) *total interruption cost*. The total installation and operational cost is the sum of the total installation costs of new facilities (substations/feeders/branches) and incremental capacity addition of the existing facilities, annual maintenance cost, and the discounted present value of the cost of energy losses. The second objective is the minimization of the total cost associated with the interruptions in all the branches and it includes three components, i.e., cost of non-delivered energy, cost of fault repair/maintenance, and customer damage cost due to interruptions. The first two components are utilities' cost due to faults and the last component is the customer cost due to interruptions. The last component, a measure of customer dissatisfaction, is very important in competitive markets. The solution strategy proposed in this work is based on dynamic programming for optimization of the feeder routes and branch conductor sizes. As the cost and reliability conflict with each other, a set of Pareto solutions is obtained using weighted aggregation of the objectives with different weight settings. Each weight combination yields one solution. The proposed approach is validated on three systems, i.e., 21-, 54-, and 100-node distribution systems, and on both static as well as expansion planning problems. The results are analyzed, with different case studies, and compared with the results of two MOEA-based planning approaches [20,28].

The key contributions of this paper are:

- A multi-objective planning algorithm using dynamic programming is proposed to determine the optimal feeder routes and branch conductor sizes with simultaneous optimization of cost and reliability.
- This planning algorithm is applicable for both static and expansion planning of distribution systems. It can also be used for the planning of both single and multi-feeder networks.
- An empirical simulation study is carried out to show the advantages of the dynamic programming based conductor size optimization over the conductor size selection. A qualitative and quantitative performance comparison between the proposed approach and two other previously reported MOEA-based approaches is provided to bring out the relative merits and demerits.

The organization of the paper is as follows. The modeling of distribution systems is briefly discussed in Section 2. The multi-objective planning model for electrical distribution systems and the proposed multi-objective dynamic programming approach are given in Sections 3 and 4, respectively. The simulation results are presented in Section 5. Section 6 concludes the paper. A list of symbols used in this paper is provided in the Appendix A.

2. Distribution system modeling

A typical distribution system consists of various components, such as substation, feeder, and load. This section provides the modeling of each component of a distribution network in the context of proposed planning approach. It is to be noted that this work deals with planning of primary distribution systems, which act as a liaison between transmission system and secondary distribution systems.

2.1. Substation modeling

A substation is the source of a distribution network. Sometimes, a distribution network is fed from two different substations. However, in this planning approach, this case is not considered. A substation consists of primary distribution transformers, switchgears, and several switching and protective equipments. In addition, a substation may be equipped with voltage regulators and shunt capacitors banks. In distribution system planning, a substation modeling basically includes the determination of the optimal size and site of a substation [1,2]. Since the location of a substation involves several social and political issues, most of the planning approaches do not include this as an optimization variable. Thus, in this planning approach, the optimal size of the substation located at a specified site is determined in static planning. For expansion planning, the capacity addition required to meet the additional load demand is determined. This is done by optimizing the installation cost of the substation and/or the cost of capacity addition of the existing substation. Total cost of a substation installation and/or capacity addition typically includes the cost of installation and/or capacity addition of substation switchgear, protective and metering arrangements, switching arrangements and installation cost of transformer and its accessories. The substation voltage level is generally specified by the system planner and it is used as the reference voltage in the planning.

2.2. Feeder modeling

A feeder brings power from substation to load points/nodes in distribution networks. Single or multiple radial feeders are used in this planning approach. Basically, the feeder routes and branch conductor sizes are optimized by minimizing the installation and/or capacity addition cost along with the cost of energy loss. To determine the cost of energy loss, the power loss at each feeder branch is determined by performing load flow. The forward/backward sweep load flow technique is used. The impedance of a feeder branch is computed by the specified resistance and reactance of the conductors used in the branch construction. The optimal branch conductor for a feeder branch is determined from a set of different types and sizes of branch conductors. The network reliability is modeled by considering the interruption/fault in each feeder branch, taken one at a time. The fault in a feeder branch is simulated by its failure rate and repair duration which are obtained from conductor specifications.

2.3. Load modeling

The load at a node of any primary distribution network is the total load demand of the secondary distribution transformer connected at the node. It is basically the total load of the secondary distribution network fed from the node. In this planning approach, balanced three-phase constant power load model is used [2]. In general, the load demand data at each node is obtained using load forecasting method and the data are directly used in the planning stage.

3. Multi-objective planning model for electrical distribution systems

The objectives of this planning approach are to optimize the cost and reliability simultaneously so as to obtain an economical and yet reliable network. In this work, the cost is optimized by minimizing the *total installation and operational cost*, considered as objective function-1. The reliability is optimized by minimizing the total cost associated with network failure, named as the *total interruption cost* (i.e., objective function-2). The objective function-1 consists of the costs due to installation of new facilities (substations/feeders/branches), incremental capacity addition of existing facilities, annual maintenance, and discounted present value of the cost of the energy losses. The objective function-2 consists of three components, i.e., cost of non-delivered energy, cost of fault repair/maintenance, and the customer damage cost due to interruptions; the first two components are utility's cost and the last component is the customer cost, due to interruptions. The first two parts of the reliability objective are formulated according to formulation given in [20]. The utility components are the expenditures due to fault repair/maintenance (function of system average failure rate) and the cost of non-delivered energy/revenue loss (function of system average outage duration). The customer component is basically a measure of customer dissatisfaction due to faults. It is a function of the *composite customer damage function* (CCDF) [31] and the average outage duration. The CCDF typically includes the costs due to loss of production for the manufacturing industries, loss of sale for the commercial customers and additional expenditure due to alternative supply for the domestic customers. It is expressed as a sum of the weighted damage costs (in \$/MW/h) due to different types of customers; the weights may vary depending on the percentage of different types of customers in an area and their relative importance. The mathematical expressions for the objective functions are given in:

$$\begin{aligned} C_{IO} = & \sum_{(i,j)\in A_{br}} \left\{ \left(C^{I_b}_{i,j} l_{i,j} \right) + \left(C^{M_b}_{i,j} l_{i,j} \right) t_a + C^V P^l_{i,j} t_a \vartheta D_F \right\} y_{i,j} \\ & + \sum_{(i,j)\in E_{br}} \left\{ \left(C^{I_b}_{i,j} l_{i,j} \right) + \left(C^{M_b}_{i,j} l_{i,j} \right) t_a + C^V P^l_{i,j} t_a \vartheta D_F \right\} \\ & + \sum_{k=1}^{N_s} C^{I_s}_k + \sum_{k=1}^{N_{se}} C^{IC}_k y_k \end{aligned} \quad (1)$$

$$\begin{aligned} C_{Fa} = & \sum_{(i,j)\in A_{br}} \left\{ C^{Ot}_{i,j} + C^{NDE}_{i,j} d_{i,j} + C^{CCDC}_{i,j} d_{i,j} \right\} \lambda_{i,j} P_{i,j} t_a l_{i,j} y_{i,j} \\ & + \sum_{(i,j)\in E_{br}} \left\{ C^{Ot}_{i,j} + C^{NDE}_{i,j} d_{i,j} + C^{CCDC}_{i,j} d_{i,j} \right\} \lambda_{i,j} P_{i,j} t_a l_{i,j} \end{aligned} \quad (2)$$

where the power loss in any branch is a quadratic function as follows,

$$P^l_{i,j} = 3 I^2_{i,j} r_{i,j} \quad (3)$$

The additional branches and substations are chosen from a set of pre-defined conductor and substation sizes, respectively. The optimization is carried out under the following constraints: (i) power demand and supply balance, (ii) limits on power flows in substation and feeder branches according to the respective capacities, (iii) upper and lower limits for the node voltages, and (iv) network radiality. A feeder branch capacity is decided based on the thermal limit of its conductor. This optimization problem is essentially a bi-objective minimization problem to find the optimal feeder routing along with the optimum branch conductor sizes. Two different categories of branch conductors are used in the planning. The first category of conductors has lower failure rate. Hence, they are more reliable and costlier. The second category of conductors has higher failure rate. Hence, they are less reliable and cheaper. Hence, if all or most of the branches of a network are built with the second type of conductor, the network becomes costlier yet reliable and vice versa. Apart from the branch conductor selection, the network topology/structure has significant influence on the reliability of the network. The network topologies with more lateral branches are more reliable. However, they are costlier because the circuit length increases with more lateral branches.

3.1. Pareto-dominance principle [20]

The Pareto-dominance principle states that for an m-objective optimization (say, minimization) problem, a solution x is said to dominate another solution y if

$$\forall i, f_i(x) \leqslant f_i(y), \text{ and } \exists j, \quad \text{such that } f_j(x) < f_j(y) \quad (4)$$

where $f_i|_{i=1,\cdots,m}$ are the objective functions. The solutions which are not dominated by each other are called the non-dominated solutions. A set of non-dominated solutions constitutes a Pareto front. The simultaneous optimization of multiple objective functions can be done in various ways for example, weighted aggregation method, Pareto-based method, lexicographic ordering, non-Pareto-based method, etc. as can be obtained in [29]. The approach followed in the paper is the weighted aggregation method, in which all objectives are aggregated with weights. The objective functions are firstly normalized and then aggregated with different combinations of the weights so as to obtain the whole Pareto-set.

4. Multi-objective dynamic programming for planning of distribution systems

The two key ingredients of the dynamic programming are optimal substructure and overlapping sub-problems [32,33]. Firstly, the dynamic programming decomposes a multi-stage decision problem into several overlapping sub-problems [32] as shown in Fig. 1. Then, it solves all the sub-problems recursively, in a bottom-up fashion starting with an independent sub-problem, say sub-problem-1 in Fig. 1. A problem exhibits optimal substructure if an optimal solution consists of all the optimal solutions of its sub-problems.

As noted earlier, both the objective functions for the problem at hand are functions of branch power flows. The power flow in a branch depends on the load in all the downstream nodes. The optimization of a branch/feeder route can be carried out after optimization of all its downstream branches. Thus, the distribution system planning problem can be decomposed into several overlapping sub-problems. In this work, the use of dynamic programming for optimization of network topology and branch conductor sizes has been investigated via two implementation methods: (i) non-iterative two-step method, and (ii) iterative two-step method. In this section, the two support subroutines, i.e., network topology optimization and branch conductor size optimization are described followed by the constraint handling techniques and the main algorithm.

4.1. Step #1: Network topology optimization

The optimization of network topology starts with geographical division of the network service area into a number of stages (say, M) as shown in Fig. 2a; M is a user-defined parameter. For static planning, the first stage is assigned to contain the substation only whereas, for expansion planning, the existing network is kept in the first stage. The example shown in Fig. 2a is for a typical static planning problem. The feeder routes are optimized starting from stage-M proceeding towards stage-1 in a backward sequence. The reason to go for this backward sequence optimization is that the power flow in any branch depends on all of its following nodes' load demand as shown in Fig. 2b (L_i = load demand at node i). The power flows in the terminal branches are completely independent of the power flows in any other branch. The optimization starts with any furthest node of the last stage as the reference node (Fig. 2c); thus this reference node becomes a leaf node. Then, all other nodes in this stage are connected one by one so as to minimize a weighted objective function constructed using the normalized installation and operational cost (C_{IO}^{norm}) and the normalized interruption cost (C_{Fa}^{norm}), as given in:

$$C_T = w_1 C_{IO}^{norm} + w_2 C_{Fa}^{norm}, \quad \text{such that } w_1 + w_2 = 1 \tag{5}$$

During this optimization, the binary variable (y) for all optimal branches/feeder routes is one and it is zero for all other routes. For example, if a branch between nodes i–j is found to be optimal the value of y for this branch is 1, i.e., $y_{i,j} = 1$. In this way the dynamic programming handles the integer variables of this planning. The continuous variable, i.e., power flow in each branch, is obtained by performing load flow. The predefined weight combinations determine the relative importance of the respective objectives.

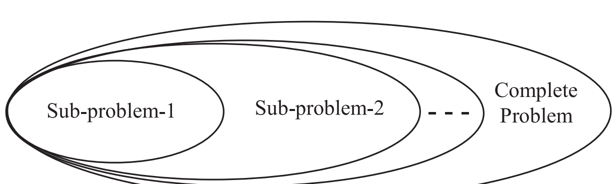

Fig. 1. Decomposition of a problem into several overlapping sub-problems in dynamic programming.

The optimization of all subsequent stages is performed sequentially as shown in Fig. 2c. The optimization of one stage yields a sub-network. The sub-network obtained after optimization of one stage along with the sub-networks of all the previous stages forms a partial network (PN) which becomes an input to the next stage. The partial network evolves from stage to stage as shown in Fig. 2d and, after the first stage optimization; it becomes the final (complete) network topology.

All nodes are connected one by one. Whenever a node is added to the network, it becomes a part of the partial network and it is never considered to be added again with the network. Thus, there is no chance of any violation of the radiality constraint. This is an important advantage of the proposed approach over the approach used in [8] as there is no need to impose additional penalty due to violation of the radiality constraint thereby saving some computational efforts.

The social and environmental obstacles in the network service area can be easily considered in the proposed algorithm defining a (binary) node connectivity matrix (B). If there is no obstacle to building a feeder route between nodes p and q, then $B(p,q) = 1$; else $B(p,q) = 0$. The subroutine for building a sub-network j is shown in Fig. 3, which is basically used in each stage of the optimization of network topology. It is done using two arrays, i.e., $\{\alpha\}$, $\{\beta\}$ consisting of the nodes of previous and current stages, respectively. At the very beginning, i.e., at stage M, $\{\alpha\}$ is initialized with any one of the furthest leaf nodes (the chosen reference node) and $\{\beta\}$ is initialized with rest of the nodes of stage-M. The network building processes for both static and expansion planning are same for all the stages except for stage-1. In stage-1, $\{\beta\}$ consists of the substation node only and all the nodes of the existing network in static and expansion planning, respectively. In stage-1, the best feeder route between the substation and the partial network (built so far) is determined for static planning and the best feeder route between the existing and the partial networks is determined in expansion planning.

The building process of a partial network is illustrated with an example in Fig. 4. The partial network, as shown in Fig. 4a, consists of nodes 1–4 and nodes 5–7 are to be connected with this network. For addition of one node in the partial network, the elements of the two arrays are: $\{\alpha\} = \{1,2,3,4\}$ and $\{\beta\} = \{5,6,7\}$. To add a node to the partial network, the best possible feeder route is to be determined from the all possible feeder routes as indicated by broken lines. If any obstacle exists in any route (for example, say, between nodes 7–2), the corresponding element of the B matrix becomes zero {i.e., $B(7,2) = 0$ for route 7–2} and it is discarded. Out of all possible routes, the best route is selected based on the objective function C_T. For example, if the best route is found as 5–3, it is connected with the network (Fig. 2b) and the arrays are updated accordingly (i.e., $\{\alpha\}$ becomes $\{1,2,3,4,5\}$ and $\{\beta\}$ becomes $\{6,7\}$). During this route selection process, the node, which is to be connected with the network, is considered to have a fictitious source for performing load flow. The idea is to obtain the best route which can carry the load demands of the partial network. The location of this fictitious source changes after addition of each node and it becomes the original source (substation node) only after optimization of stage-1. As the location this fictitious source changes from node to node, the amount of power flow through any route also changes. For example, the amount of power flow through route 5–3 is different if the fictitious source is located at 6 compared to the fictitious source location at node 7, as shown in Fig. 4c and d. The amount of power flow through all branches in a stage can be obtained after the partial network topology optimization of this stage.

This approach can also be extended for obtaining multi-feeder networks. In those cases, the division of the service area can be done in different ways as shown in Fig. 5 for double and triple

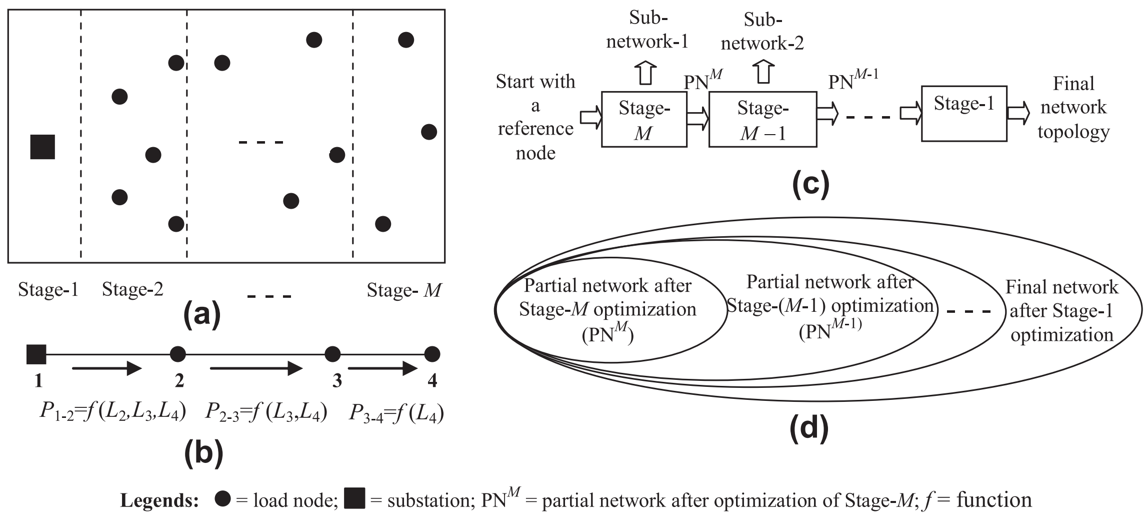

Fig. 2. Multi-stage decision process in distribution network topology optimization: (a) division of the service area of a distribution system into a number of stages; (b) an example showing the dependency of branch flow; (c) multi-stage decision process to get network topology using dynamic programming; (d) stage by stage evolution of the network.

feeder networks. For multi-feeder networks, the optimization starting from stage-M to stage-1 is performed for all the feeders in a similar manner as described above. The element of stage-1 for each feeder is substation for static planning (and the existing network for expansion planning).

4.2. Step #2: Branch conductor size optimization

Branch conductor size optimization is performed only after obtaining the complete/partial network topology. The conductor size for each branch is optimized from the η_C number of available conductor sizes. Each conductor size has different capacity, failure rate and repair duration. Before the branch conductor optimization, a load flow is performed to obtain minimum conductor size required to satisfy branch current capacity constraint for each branch. These sizes are stored into an array min_cond_size {min_cond_size(l, m, k) = minimum conductor size required to satisfy branch current capacity constraint for the kth branch of the mth stage of the lth feeder}. The conductor size used for any branch is kept above the required minimum size. The branch conductor sizes are optimized sequentially starting from any terminal branch of the last stage, i.e., stage-M, and moving towards the substation stage by stage. In any stage, the conductor size optimization of all the terminal branches is done first (in any order) before going for other non-terminal branches. A typical single-feeder network with seven branches is shown in Fig. 6 to illustrate the process. Total number of stages in this example is 3. Hence, the branch conductor optimization can be started either from *Br*-7 or *Br*-6 of stage-3. For this stage, the sequence of optimization can be *Br*-7 → *Br*-6 → *Br*-5. The reason for this sequential optimization is the dependency of branch power flow as explained earlier. For example, the power flow in *Br*-5 depends on the branch flow in *Br-6* while the power flows in *Br*-7 and *Br*-6 do not depend on any other branch's flow. After stage-3, this optimization moves toward stage-2 and so on. During any intermediate stage, (say, stage-2) this optimization again starts with any terminal branch and continues till conductor size for all branches are optimized. During this optimization step, the branch conductor sizes are optimized by minimizing the weighted objective function given in Eq. (5). The same procedure is followed for all the feeders in a multi-feeder network. The subroutine for branch conductor size optimization is given in Fig. 7.

4.3. Constraint handling techniques

The constraints in this planning optimization are taken care of as follows:

- Power demand and supply balance is met by the load flow.
- The node voltage limit violation information is incorporated in terms of a penalty factor in the objective function given in Eq. (5). The penalty factor, computed as the product of the absolute value of the maximum node voltage deviation from a specified nominal value and a very high integer number, is added to the value of C_T. This penalty factor is used in the optimization of both network topology and conductor sizes.
- The branch current capacity constraint is maintained by performing load flow to obtain minimum conductor size required to satisfy branch current capacity constraint for each branch. These sizes are stored into an array. The conductor size used for any branch is always kept above the required minimum size. If the substation power demand exceeds its capacity, a decision on incremental capacity addition for the substation is taken.
- The radiality constraint is never violated as the nodes are connected one by one. If a node is added to the network, it is deleted from the array $\{\beta\}$ and added to the array $\{\alpha\}$. Hence, there is no chance of connecting any node twice with the network.

4.4. Complete algorithm

The optimization of network topology and branch conductor sizes is performed using dynamic programming with two types of implementation schemes: (i) non-iterative two-step method, and (ii) iterative two-step method. In the non-iterative method, the branch conductor size optimization is done after network topology optimization. In the iterative approach, both the optimization steps are carried out iteratively in each planning stage. The complete flow charts for both the methods are given in Fig. 8.

5. Simulation results

The proposed multi-objective distribution system planning algorithm using dynamic programming is evaluated via computer simulation studies using MATLAB 7.0 on two typical distribution

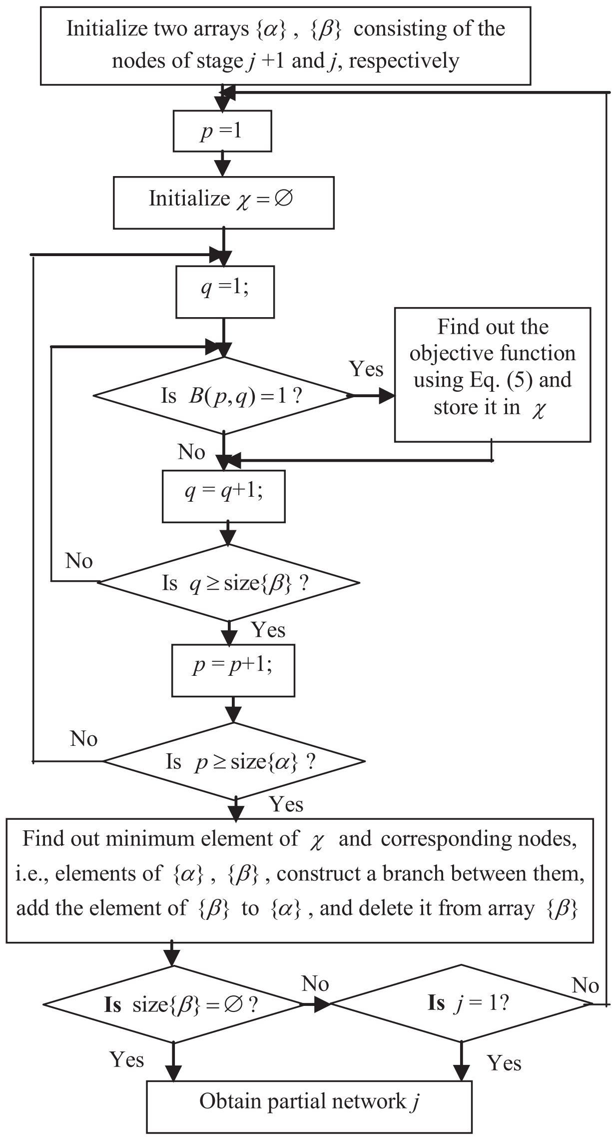

Fig. 3. Subroutine for building partial network j after optimization of Stage j (indices p and q represent an element of arrays $\{\alpha\}$ and $\{\beta\}$, respectively).

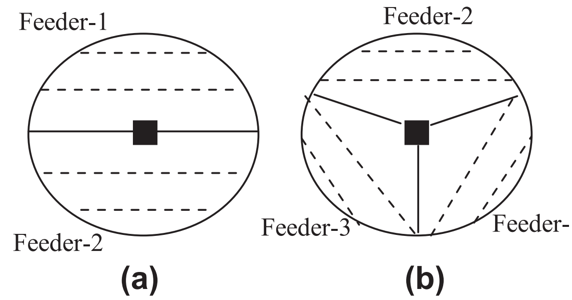

Fig. 5. Division of network service area to obtain multi-feeder networks (a) double feeder, (b) Triple feeder.

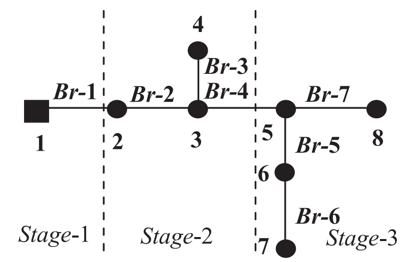

Fig. 6. A typical single-feeder network.

system planning problems. The first type is an expansion planning problem for a 21-node distribution system [20,34] in which there are four existing branches and the remaining 16 nodes are to be connected via multi-objective optimization. The second type is a static planning problem, i.e., planning of a completely new network. Two different distribution systems are taken for the static planning, i.e., 54-node [35] and 100-node systems [20,34]. The customer damage cost is assumed to be 0.67 \$/MW/min of failure [28,31]. The detailed derivation of this cost function can be obtained from [31]. For simplicity, all the nodes are assumed to have same customer damage cost. In reality, this may vary from node to node depending upon the percentage of different customers. Some typical features of these systems are:

- All the three chosen systems have one substation (at a suitably chosen location).
- Nine different conductor sizes are used in the optimization of the 21-node and 100-node systems. Two conductor sizes are used for the 54-node system.

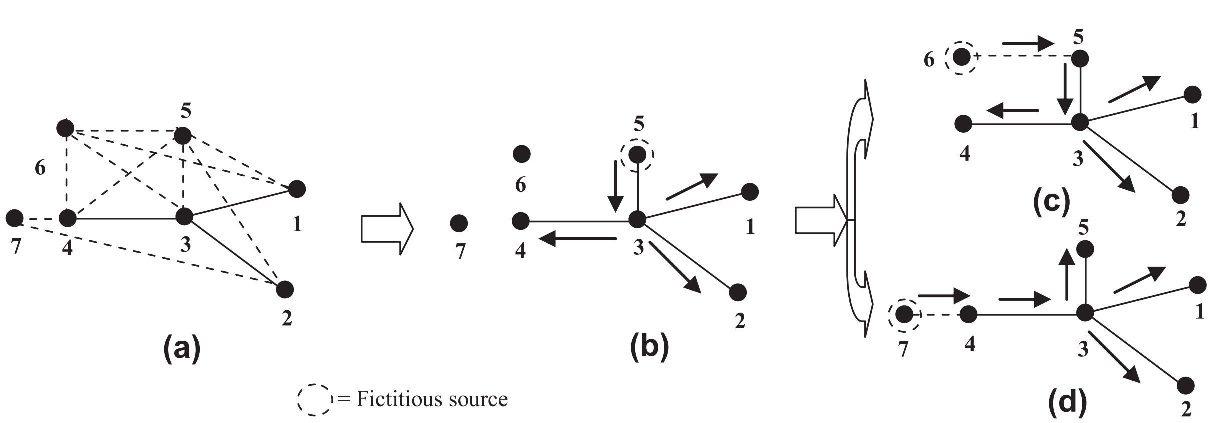

Fig. 4. An example to show the building process of partial network: (a) a partial network shown in solid lines along with several possible feeder routes shown with broken lines, (b) the best possible feeder route to add one node with fictitious source at 5, (c and d) two possible connections to add one node further with fictitious sources at 6 and 7, respectively.

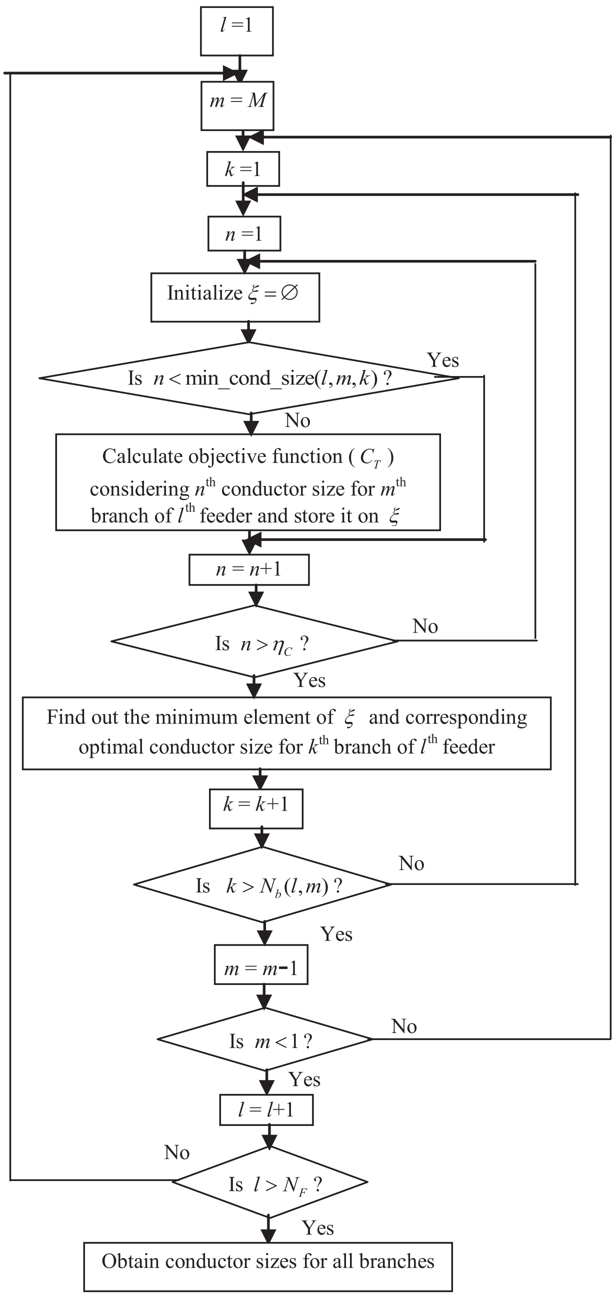

Fig. 7. Subroutine for branch conductor size optimization.

- Minimum and maximum node voltage limits are taken as 0.92 and 1.08 p.u., respectively. Substation voltage is assumed to be 1.05 p.u.
- Base voltages for 21-node, 54-node, 100-node systems are 13.8 kV, 12 kV, and 34.8 kV, respectively.

5.1. Expansion planning

For the expansion planning problem of the 21-node distribution system, 50 different combinations of weights, uniformly varying between 0 and 1, are considered. The number of stages (M) is taken as 5. The set of non-dominated solutions constituting a Pareto-front, obtained with the proposed multi-objective dynamic programming, is shown in Fig. 9. From this set of non-dominated network set, the decision maker can choose one for implementation considering the relative emphasis of each objective. It is instructive to examine how the two objective function values on the Pareto-front (of the networks obtained) change with different weights assigned to each objective. These variations with respect to weight w_1 are shown in Fig. 10. It indicates that one objective increases with the weight while other decreases. The reason behind this is: an increase in w_1 increases the relative importance of objective function-1 in the weighted objective function of Eq. (5) and thus decreases the relative importance of objective function-2. Hence, the objective function-1 reaches its best at $w_1 = 1$. This also illustrates the conflicting nature of the two objective functions. It gives an idea to the system planner on the required investment for certain network reliability.

Two sample networks, i.e., the most economical and most reliable networks as indicated in the Pareto-front, are shown in Fig. 11. The existing branches are shown with bold lines and the node numbers and conductor sizes are shown by bold and italic numerals, respectively. The substation is at node 1. These solutions illustrate that the most reliable network (Fig. 11a) consists of branches with conductor size 9 that has lower failure rate but higher installation cost and it has more lateral branches that reduces power flow in some branches. The most economical network (Fig. 11b) is possibly the shortest path network having conductor sizes (between 1 and 7) with lower installation cost and higher failure rate.

In Fig. 12, the way partial networks evolve to the final optimal network by dynamic programming is shown for better visualization of the network topology optimization for the weight combination (0.5, 0.5). The division of the stages is shown with dotted lines and the existing network between nodes (1–5) is also shown.

5.1.1. Advantages of conductor size optimization over conductor size selection

The conductor size optimization is very important in the distribution system planning. Most of the previous works are based on conductor size selection. The minimum conductor size required to satisfy branch current capacity constraint is selected for each branch. The conductor size optimization is carried out only in few works [20,27,28]. An investigation is carried out to study the influence of the conductor size optimization on the Pareto-fronts. The results are shown in Fig. 13. A much better Pareto-front is obtained with the planning incorporating conductor size optimization. It is to be noted that the computational time (Processor: Intel Pentium D CPU, 3 GHz, 1 GB RAM) for the planning with conductor size optimization (1.895 min) is not much higher than that of planning with conductor size selection, i.e., 1.69 min.

5.1.2. Performance comparison of non-iterative and iterative two-step methods

The performances of the non-iterative and iterative two-step methods are compared for the 21-node distribution system considering 50 different combinations of weights. The respective Pareto-fronts are shown in Fig. 14. It is observed that better Pareto-front is obtained with the iterative two-step approach. The computational time required for the iterative and the non-iterative two-step approaches are 9.15 min and 1.895 min, respectively. Thus, the iterative method is computationally very much expensive compared to the non-iterative method. It is expected as the conductor size optimization is carried out in each planning stage. The computational time with the iterative approach for higher node systems could be extremely high. Hence, all subsequent studies on higher node systems are carried out with the non-iterative two-step method.

5.2. Static planning

The proposed algorithm is also applied for static planning of the 54-node and 100-node systems. The node locations and kVA

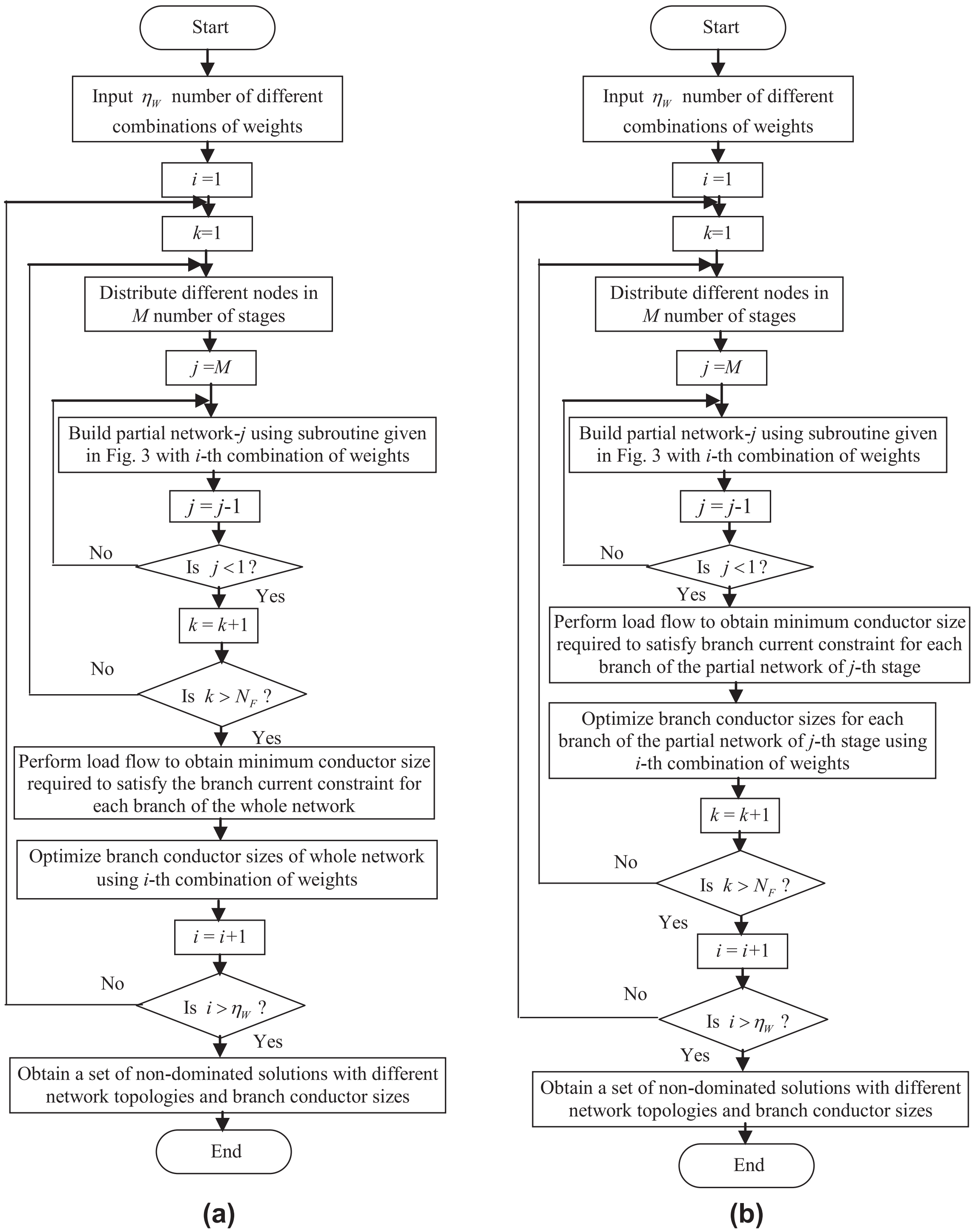

Fig. 8. Overall flow chart of multi-objective dynamic programming for multi-feeder electrical distribution system planning: (a) non-iterative two-step method and (b) iterative two-step method.

demands for the 54-node system are taken from [35]. In [35], the reliability is not considered in the planning. To incorporate reliability aspect into the planning of this network, two types of conductors, as given in Table 1, are assumed. The data for the 100-node system are given in [34]. In this planning, 50 and 25 different combinations of weights are used for the 54-node and 100-node system, respectively. The numbers of stages are taken as 5 and 10 for the 54-node and 100-node system, respectively. The Pareto-fronts for the 54- and 100-node systems are shown in Figs. 15 and 16, respectively. The objective of this simulation study is to show the performance of the proposed algorithm on higher dimensional systems, i.e., higher node systems, and also to validate the algorithm for the static planning. The computational times for the 54-node and 100-node systems are 60.15 min and 398.79 min, respectively. It is to be noted that less number of weight combinations are intentionally taken for the 100-node system as higher computational time is required to execute a single weight combination. This illustrates that the dynamic

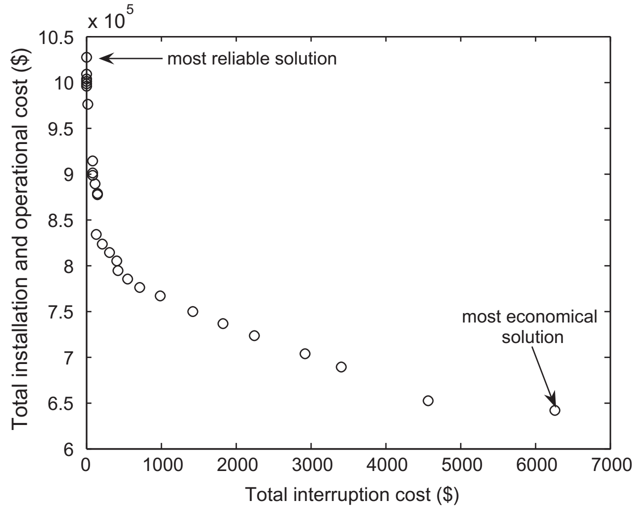

Fig. 9. Pareto-front for the expansion planning of 21-node distribution system.

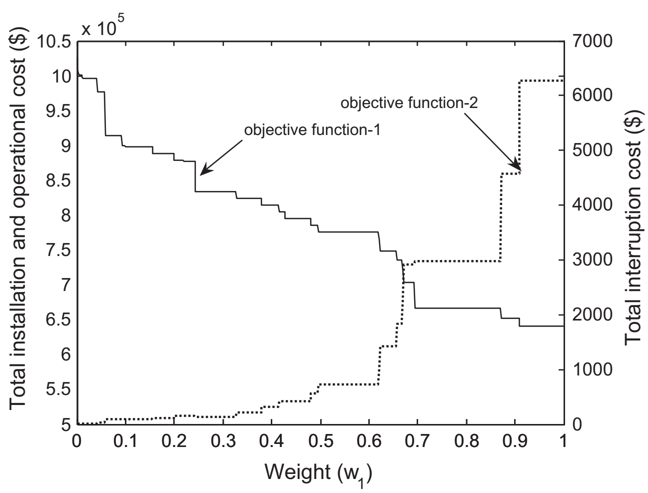

Fig. 10. Dependency of objective functions with weight.

programming requires high computational time for the planning of higher node systems.

5.2.1. Multi-feeder network planning

In the planning case studies presented above, the number of feeders is assumed to be 1. Since the distribution systems generally consist of a large number of nodes, the system planner generally thinks of planning a multi-feeder network. This can be achieved using the proposed approach as discussed earlier (Fig. 5). The location of the substation is approximately chosen in the middle of the service area and division of the service area is carried out as shown in Fig. 5. The feeder routes are optimized sequentially, i.e., starting with feeder-1 followed by feeder-2, 3, and so on, as given in the flowchart of Fig. 8. The proposed approach is applied to obtain two-feeder networks for the 54-node system. The Pareto-front obtained with two-feeder networks is compared with that of a single feeder network in Fig. 17. It shows that the upper part of the Pareto-front is better in case of double feeder network while the lower half is better in case of single feeder network. It is obvious as the single feeder networks are economical but less reliable. On the contrary, the double feeder networks are more reliable and hence, less economical. The power loss and node voltages are significantly improved with double feeder networks compared to those of single feeder networks as shown in Fig. 18.

5.2.2. Sensitivity test (with different number of stages)

There is a user-defined parameter, i.e., number of stages (M). A sensitivity test is required to assess the performance of the proposed approach with respect to this parameter. The Pareto-fronts obtained, for the 54-node system, with two values of M, i.e., 2 and 5, are shown in Fig. 19. The results illustrate that the performance of the proposed approach is better with less number of stages. The reason is that, for smaller values of M, there are more number of nodes per stage. Therefore, more numbers of combinations are to be evaluated for getting a sub-network from each stage and this also increases the computational time. This points to the fact that the global best can only be obtained from all possible combinations of branches in one go and that can be obtained with $M = 1$. The computational time with (for the processor specification given earlier) $M = 2$ is 237.8742 min. which is much higher than that of the execution time with $M = 5$, i.e., 61.3942 min. The computational time is also system dependent and it generally increases with number of nodes in the system. In general, a trade-off between the accuracy in terms of obtaining better results and computational time should be done. It is left to the system planner to decide about the acceptability of a network design.

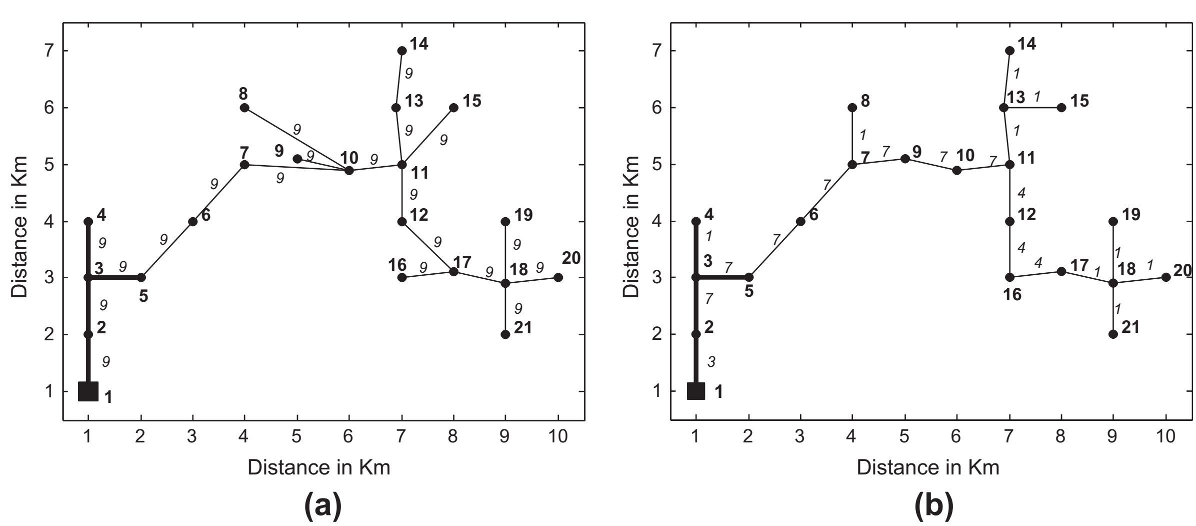

Fig. 11. Two sample networks from the Pareto-front: (a) most reliable network and (b) most economical network.

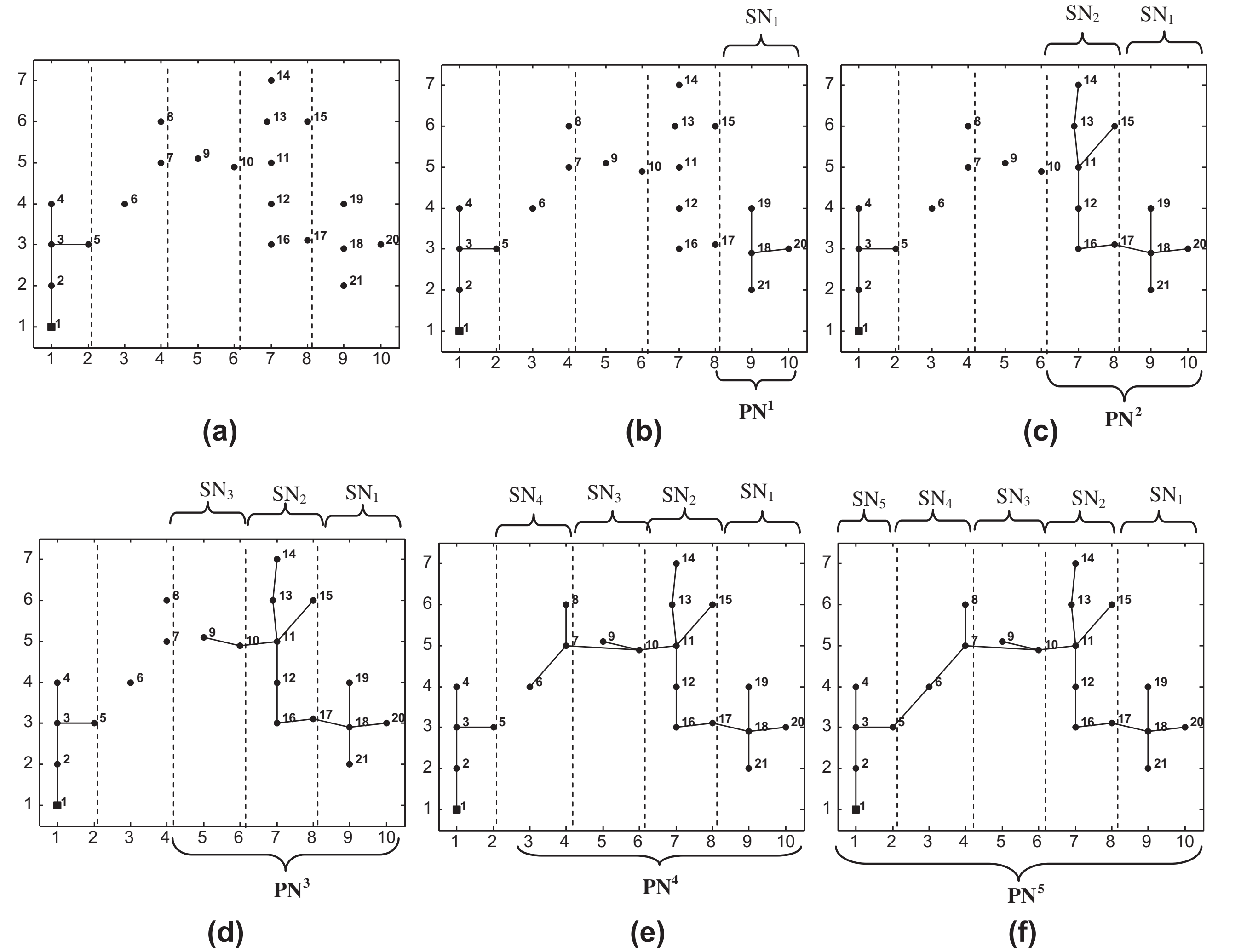

Fig. 12. Stage by stage evolution of the network for 21-node system using weights (0.5, 0.5) and number of stages = 5: (a) before optimization, and partial networks after optimization of (b) stage-5, (c) stage-4, (d) stage-3, (e) stage-2, and (f) stage-1 (final network).

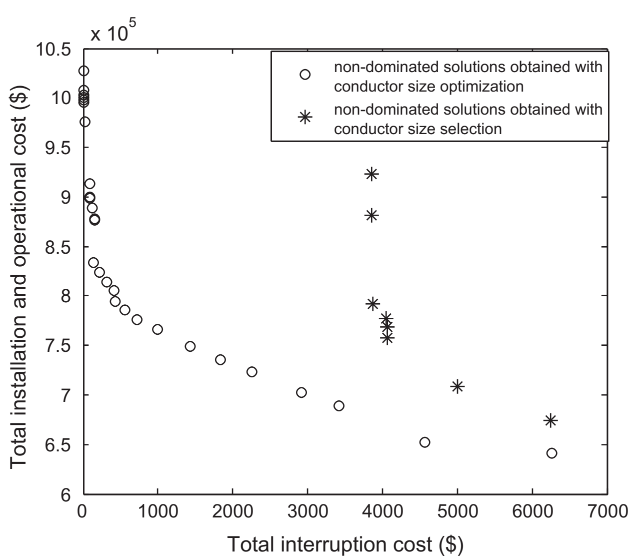

Fig. 13. Pareto-fronts obtained with conductor size selection and conductor size optimization.

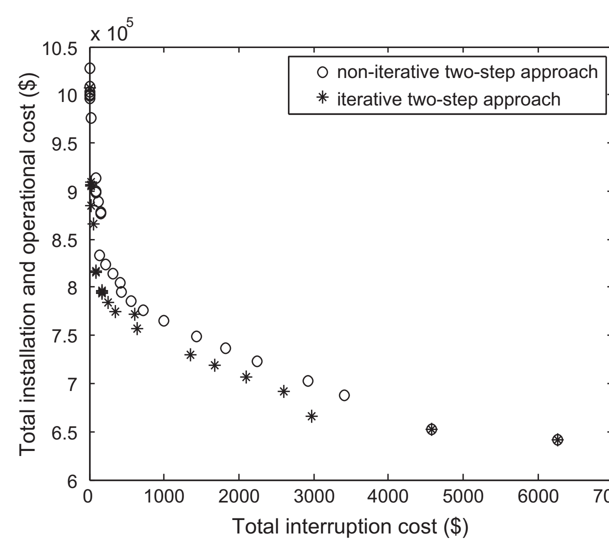

Fig. 14. Pareto-fronts obtained with non-iterative two-step method and iterative two-step method.

Table 1
Specification of conductor sizes for optimization of 54-node system.

Conductor type	Current capacity (A)	Resistance (Ω)	Reactance (Ω)	Failure rate (fault/km/year)	Failure duration (h/fault/year)	Installation cost ($)
1	150	0.5762	0.5184	0.096	10.75	10000
2	230	0.4724	0.2875	0.064	8.95	15000

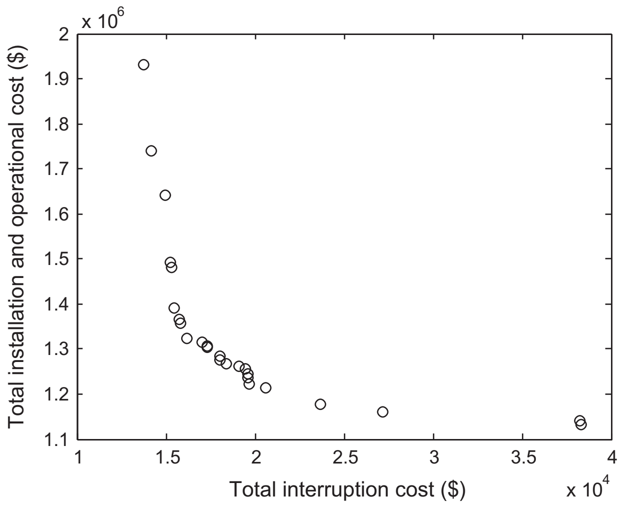

Fig. 15. Pareto front for the static planning for 54-node distribution system.

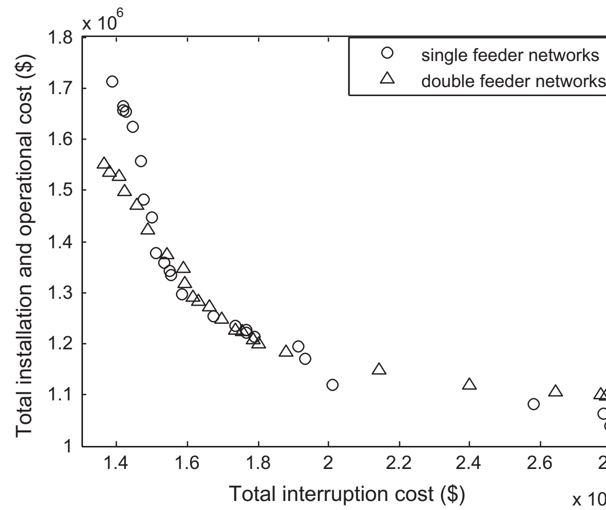

Fig. 17. Comparison of the Pareto-fronts between single and double feeder networks.

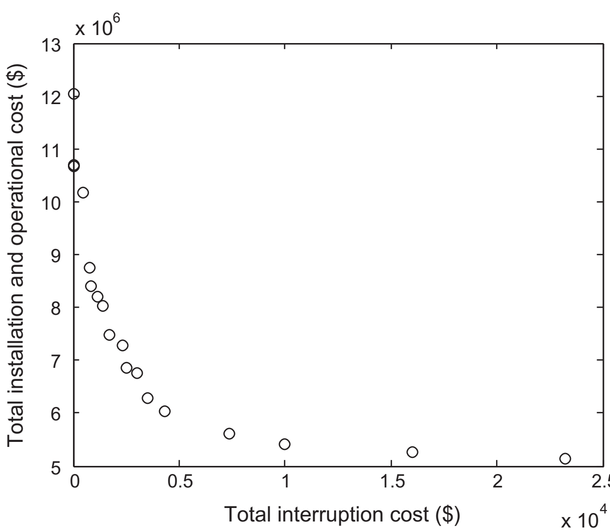

Fig. 16. Pareto front for the static planning for 100-node distribution system.

5.2.3. Performance comparison with MOEA

In this section, the performance of the proposed multi-objective dynamic programming (MODP)-based planning algorithm is compared with two MOEA-based planning algorithms, published in the literature. The first one is the multi-objective GA (MOGA)-based planning algorithm [20]. The second one is the multi-objective PSO (MOPSO)-based planning algorithm [28]. The reason for choosing these two specific algorithms is that the branch conductor optimization is carried out along with network topology optimization in both the approaches. Moreover, the test systems used in the optimization in both the works are same as that of the present work. As the objective function formulation of the present work is different from [20], the MODP is applied to optimize the objective functions formulated in [20] for expansion planning of the 21-node distribution system. The qualitative and quantitative comparisons are summarized as follows.

- A quantitative performance comparison of the most reliable and the most economical solutions obtained from the proposed dynamic programming and the results reported in [20] are shown in Table 2. The results clearly illustrate the better performance with MODP. Moreover, the MODP is simpler to implement compared to the MOGA because there is no need for so many heuristic crossover and mutation operators as used in the MOGA.
- The Pareto-fronts for the 21-node and 100-node systems obtained with the proposed MODP and MOPSO [28] are shown in Figs. 20 and 21, respectively. In MODP, 50 different combinations of weights for the 21-node system and 5 different combinations of weights for the 100-node system are used. The number of stage is taken as 5 for both systems. The results show that the lower half of the Pareto-front for the 21-node system is better with the MODP than that of the MOPSO. The computational times to obtain the Pareto-front for 21-node system are almost the same for both the approaches (1.895 min for the proposed MODP and 1.765 min for the MOPSO). The Pareto-fronts for the 100-node system are mostly similar. A better spread of solutions can be obtained with the MODP. However, the computational time with the proposed MODP is very high, i.e., 370 min. to execute 5 different combinations of weights to obtain the Pareto-front. On the contrary, the whole Pareto-front is obtained within 50 min. with the MOPSO.

In conclusion, it can be said that a better result can be obtained with the MODP compared to the MOEAs. But, it suffers from the curse of dimensionality for higher node systems.

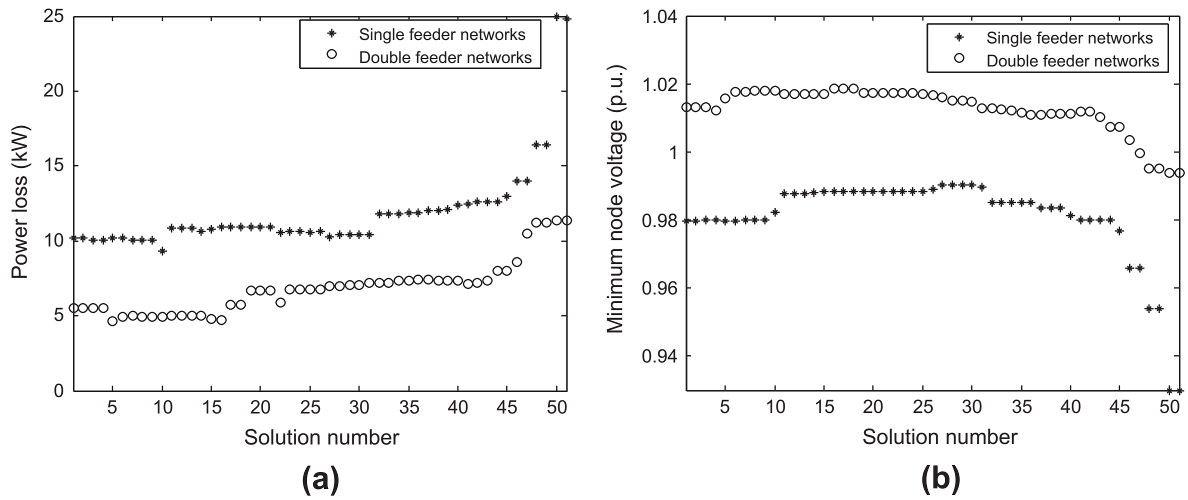

Fig. 18. Comparison between single and double feeder networks in terms of (a) power loss and (b) minimum node voltage.

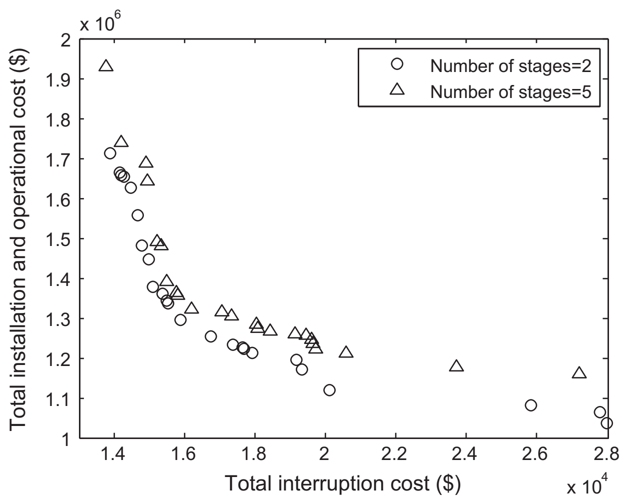

Fig. 19. Pareto-fronts for the 54-node system for two different number of stages (i.e., $M = 2$ and $M = 5$).

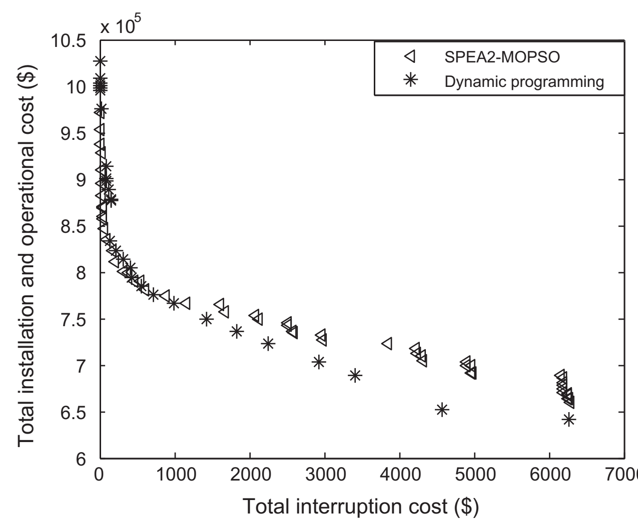

Fig. 20. Pareto-fronts for 21-node distribution system obtained with the MODP and MOPSO.

Table 2
Comparison of results between MOGA-based planning [20] and MODP-based planning.

Solutions	Objective functions (MOGA [20]) ($)		Objective functions ($) (MODP)	
	First	Second	First	Second
Most economical	6.7×10^5	843.79	6.5×10^5	639.6468
Most reliable	17.07×10^5	7.71	14.75×10^5	5.9175

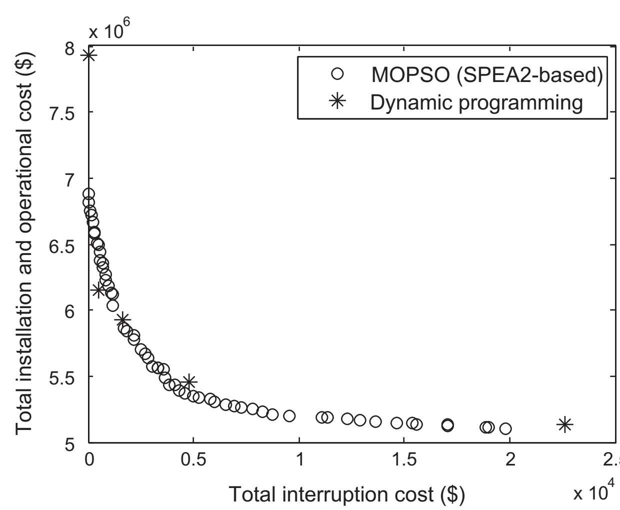

Fig. 21. Pareto-fronts for 100-node distribution system obtained with the MODP and MOPSO.

6. Conclusion

In this work, a multi-objective dynamic programming approach for electrical distribution system planning has been proposed. The two objective functions are formulated as: (i) *total installation and operational cost* and (ii) *total interruption cost*. The first objective represents the cost of a network. The second objective is a measure of network reliability. Both non-iterative and iterative two-step dynamic programming methods are proposed for optimization of the feeder routes and branch conductor sizes. In both the methods, a set of Pareto solutions is determined using weighted aggregation of the objectives with different settings of weights. Obviously, the performance of the iterative approach is found to be better, but it is very much computationally expensive. The advantage of the conductor size optimization over the conductor size selection is shown with an empirical study. The proposed approach is validated on both static and expansion planning problems using three different test systems, i.e., 21-, 54-, and 100-node distribution systems. It is also applicable for the planning of both single and multi-feeder networks. Finally, the performance comparisons with two MOEA-based planning algorithms illustrate that better performance can be obtained with the proposed approach. However, it has a limitation that it suffers from the curse of dimensionality. However, the proposed algorithm can be further extended so as to incorporate the sectionalizing switches, tie-lines, capacitor banks and the distributed generation into the planning. The incorporation of the uncertainty associated with the load demand into the planning model can be a future scope of research. The proposed approach can also be modified as a multi-stage formulation problem for expansion planning. This needs further investigations.

Appendix A

List of symbols

C_{IO}	Total installation and operational cost ($)
C_{Fa}	Total interruption cost ($)
C^{I_b}	Branch installation cost per unit length ($/km)
C^R	Conductor replacement cost per unit length ($/km)
C^{M_b}	Annual branch maintenance cost ($/km/year)
C^V	Variable cost i.e., cost of energy losses ($)
$l_j(P_j)$	Length (power flow) of branch i–j
$P^l_{i,j}$	Power loss in branch i–j
$I_{i,j}(r_{i,j})$	Current (resistance) in branch i–j
t_a	Total planning time (year)
ϑ	Load loss factor
y	Binary decision variable for capacity addition
C^{I_s}	Substation installation cost ($)
C^{IC}	Substation incremental capacity addition cost ($)
C^{Ot}	Utility outage cost per unit length ($/MW/fault)
C^{NDE}	Cost of non-delivered energy ($/MW/h)
C^{CCDC}	Composite customer damage cost ($/MW/h)
λ	Branch average failure rate
d	Branch average failure duration per failure
D_F	Discount factor $\left(D_F = \frac{1}{(1+u)^t};\ u = \text{discount rate}\right)$
N_b	Number of additional branches to be installed
E_{br}	A set of existing branches in the network
A_{br}	Total number of allowable branches for feeder routes
N_s	Number of additional substations to be installed
N_{se}	Number of existing substations
C_T	Weighted objective function
C^{norm}_{IO}	Normalized total installation and operational cost
C^{norm}_{Fa}	Normalized total interruption cost
w_1, w_2	Weights assigned to objective functions 1, 2
M	Total number of stages
PN^M	Partial network after optimization of stage-M
$\{\alpha\}, \{\beta\}$	A set of the nodes of previous and current stages
η_C	Number of available conductor sizes
N_F	Number of feeders
η_w	Total number of different weight combinations
$N_b(l, m)$	Number of branches in the mth stage of lth feeder

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