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Abstract—The abstract goes here.

Index Terms—synthetic distribution networks, MILP formulation, radiality, heuristic algorithms.

I. INTRODUCTION

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mds

August 26, 2015

A. Subsection Heading Here

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II. PRELIMINARIES

A. Distribution system

The distribution network consists of overhead power lines, underground cables, pole top transformers and various control and metering equipment that bring electrical power from high voltage (HV)(greater than 33kV) transmission system to the end residential consumers requiring a low voltage (LV) level (208-480V). This is normally done in a two-step procedure: first the high transmission level voltage is stepped down to medium voltage (MV) level at distribution substations which forms an interface between the transmission and distribution networks; thereafter, the voltage is further stepped down to LV level near the consumer locations at the local distribution transformers. These are either pole-top transformers for overhead distribution lines or pad-mounted for underground cables.

The MV level network is responsible for distributing power from distribution transformers to pole-top/pad-mounted transformers and comprises of the *primary distribution* system. The main feeder from the distribution substation forms the backbone of this network with several laterals from it serving the local distribution transformers. In most cases, the primary distribution feeder and connected laterals follow the road network from distribution substations to reach the local transformers near residences. After the voltage is stepped down to LV level at local distribution transformers, the power is distributed to individual residential homes through the *secondary distribution* network.

Distribution systems (primary and secondary) are usually configured in a *radial* structure where power flow is unidirectional from source to consumers. Therefore, for primary distribution the flow is directed from the distribution substation transformer through main feeder and laterals to the individual local distribution transformers. Similarly, the secondary network carries power from pole-top/pad-mounted transformers to the residential consumers through secondary feeders. Such radial structure ensures protection coordination among reclosures, breakers and downstream fuses in the distribution system feeders [1]. However, unidirectional power flow no longer holds true when distribution system is operated with distributed energy resources (DERs) which inject power at the consumer ends [2].

B. Available datasets

In this work, we try to generate the distribution network for a given area (county/town/city) from different open source

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publicly available information. These data pertain to following sources.

- Transportation network data published by NAVTEQ [3].
- Geographical location of high voltage (HV) and extra-high voltage (EHV) substations from data sets published by U.S. Energy Information Administration (EIA) [4].
- Residential electric power demand information developed in the models by [5].

The NAVTEQ transportation network data is available in the form of an edge-list or a list of links. Each link e has an associated integer level l_e from the set $\{1, 2, 3, 4, 5\}$ that describes the link type (e.g., a level-1 ($l_e = 1$) link could correspond to an Interstate road segment, while a level-5 ($l_e = 5$) link could correspond to a residential driveway). All links with level $l_e \leq 2$ are removed from the dataset. These links are dropped since components of the distribution network (e.g., homes) are typically not located along these links. Furthermore, connected components in the road network graph of considerable size (number of nodes greater than 10) are only considered. This is done to remove small connected components along which a radial primary distribution feeder network cannot exist.

The synthetic population data and the substation information is available for the entire United States and stored in a database together with the NAVTEQ data. The synthetic distribution network is generated for a particular area which may represent a county, state etc. For this purpose, the required data set is selected from the master database based on the zip code information. The available data set is therefore listed as below.

- The road network represented in the form of a graph $R = (V_R, L_R)$, where V_R and L_R are respectively the sets of nodes and links of the network. Each node in the graph has an associated spatial embedding in form of longitude and latitude.
- The set of substations $S = \{s_1, s_2, \dots, s_M\}$, where the area consists of M substations and their respective geographical location data.
- The set of residential buildings $H = \{h_1, h_2, \dots, h_N\}$, where the area consists of N home locations. Each building is associated with longitude-latitude information and an hourly load demand profile.

C. Proposed Approach

The goal of this work is to generate separately the primary and secondary distribution networks to connect the distribution substations to all residential building locations. Therefore, the problem of synthesizing realistic distribution networks is considered to be a two-step bottom-up procedure: the first step constructs the secondary distribution network connecting the residential buildings to pole-top/pad-mounted transformers and the second step involves connecting these local transformers to distribution substations through feeders and laterals. As mentioned earlier in Section II-A, the road network can be used as a proxy for the primary distribution network. Therefore, the locations of local pole-top transformers are considered to be internal points on the road network links.

To summarize the overall procedure, the tasks would be the following:

- 1) Evaluate a mapping between sets of residential buildings and road network links such that each residence is mapped to the nearest road link. Additionally, identify probable locations of local distribution transformers along these road network links.
- 2) Connect the local distribution transformers to mapped residences in a radial configuration to resemble a typical secondary distribution network.
- 3) Identify subset of road network links which connects distribution substation transformers to the local pole-top/pad-mounted transformers in the form of a radial feeder network.

Each of these tasks are discussed in details in the following sections.

III. NODE IDENTIFICATION FOR SECONDARY NETWORK

This section details the proposed methodology to identify subsets of residential buildings and sets of point on the road network which would be used in the successive steps to generate the secondary distribution network. To this end, the residential buildings are required to be mapped to the nearest road network link. The points along road network would serve as local distribution transformers delivering power to the residential buildings mapped to the link.

Algorithm 1 is used to compute the nearest road network link to a given point with associated spatial embedding. The point is represented as a 2-dimensional vector $\mathbf{p} \in \mathbb{R}^2$ with the longitude and latitude as its entries. Similarly, the road network link is denoted as a line segment vector \mathbf{l} connecting terminal point vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$ and related by $\mathbf{l} = \mathbf{u} - \mathbf{v}$.

For this algorithm, a bounding box of suitable size is evaluated for each road network link. This is done such that any point in the bounding box is within a radius r from any internal point of the road network link $\mathbf{l} = \mathbf{u} - \mathbf{v}$. The bounding box is given as

$$\mathbf{B}_l = \{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_1\|_2 \leq r, \forall \mathbf{x}_1 = \theta \mathbf{u} + (1 - \theta) \mathbf{v}, \theta = [0, 1]\} \quad (1)$$

Similarly, bounding box is considered for the point of interest and is given by

$$\mathbf{B}_p = \{\mathbf{x} \mid \|\mathbf{x} - \mathbf{p}\|_2 \leq r\} \quad (2)$$

The intersections between the bounding box of the point and those for the links are stored and indexed in a *quad-tree* data structure [6]. This information is retrieved to identify the few links which are comparably nearer to the point than the others.

In order to create the mapping between road network links and residential buildings, the nearest link to each home is evaluated using Algorithm 1. Let the generated mapping be denoted by $f : H \rightarrow L_R$ and subsequently we can evaluate the inverse mapping $f^{-1} : L_R \rightarrow H$ which gives the set of residences mapped to each road network link..

As mentioned previously in Section II-C, internal points along the road links are assumed to be probable locations of local distribution transformer. Therefore, an additional

Algorithm 1 Find the nearest link in L_R to a given point \mathbf{p} .

Require: Radius for bounding boxes r , a mapping $\text{dist} : \mathbb{R}^2 \times L_R \rightarrow \mathbb{R}^+$ such that $\text{dist}(\mathbf{p}, \mathbf{l})$ is the shortest distance between point $\mathbf{p} \in \mathbb{R}^2$ and link segment $\mathbf{l} = \mathbf{u} - \mathbf{v}$ for $\mathbf{l} \in L$ and $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$.

- 1: **for** each link $\mathbf{l} \in L$ **do**
- 2: evaluate bounding box $\mathbf{B}_\mathbf{l}$ for each link \mathbf{l} using Eq. 1.
- 3: Evaluate bounding box $\mathbf{B}_\mathbf{p}$ for point \mathbf{p} using Eq. 2.
- 4: Find the bounding boxes $\mathbf{B}_{\mathbf{l}_1}, \mathbf{B}_{\mathbf{l}_2}, \dots, \mathbf{B}_{\mathbf{l}_m}$ corresponding to the links $\mathbf{l}_1, \mathbf{l}_2, \dots, \mathbf{l}_m$ which intersect with $\mathbf{B}_\mathbf{p}$.
- 5: Find the link $\mathbf{l}^* \in L'$, where $L' = \{\mathbf{l}_1, \mathbf{l}_2, \dots, \mathbf{l}_m\} \subseteq L_R$ such that $\mathbf{l}^* = \arg \min_{\mathbf{l} \in L} \text{dist}(\mathbf{p}, \mathbf{l})$.

objective of this step is to identify the points along the road network link where local distribution transformers may be placed. One option may be to place the transformers at the terminals of road network links. Another option may be to interpolate multiple points along the link which are a definite length apart from each other. In this paper, we have undertaken the second option.

Algorithm 2 Identify nodes for secondary network generation.

Require: Road network graph $R = (V_R, L_R)$, set of residential buildings H , minimum distance between transformers d .

- 1: **for** each building $h \in H$ **do**
- 2: find mapping $f : H \rightarrow L_R$ using Algorithm 1 to generate the nearest link $e \in L_R$.
- 3: **for** each link $l = (u, v) \in L_R$ **do**
- 4: find the inverse mapping $f^{-1} : L_R \rightarrow H$ which generates the set of buildings H_l associated with l .
- 5: Interpolate m points $T_l = \{t_1, t_2, \dots, t_m\}$ on link l between road nodes u and v which are d distance apart from each other.

IV. CREATION OF SECONDARY NETWORK

This section details the generation of secondary distribution networks connecting set of residential buildings H_l mapped to a road network link $l \in L_R$ with set of local distribution transformer nodes T_l located along l . Without loss of generality, we only consider non-trivial road links which are mapped to at least one residential building, i.e., $f^{-1}(l) = H_l \neq \emptyset$. The generated network should be a *forest* of disconnected trees with each tree rooted at one of the transformer nodes $t \in T_l$. The forest should cover all residential buildings in such a way that the overall length of distribution lines is minimized and intersections/overlaps of primary and secondary networks are avoided.

A. Graph generation, node and edge variables

In the proposed methodology, for a given non-trivial road network link $l \in L_R$, a fully connected undirected graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ is constructed with node set \mathcal{V} and edge set \mathcal{E} that are incident on \mathcal{V} . The node set comprises of all the residences mapped to the link l and the set of transformers along l and denoted by $\mathcal{V} = H_l \cup T_l$. Any edge $e \in \mathcal{E}$ is defined by incident

nodes (i, j) where $i, j \in \mathcal{V}$. Since \mathcal{G} is fully connected, the edge set \mathcal{E} consists of all pairs of entries (i, j) for $i, j \in \mathcal{V}$. The aim of this step is to select an optimal subgraph $\tilde{\mathcal{G}} := (\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ which is a forest of disconnected trees.

In this paper, we consider each tree in the generated forest to be a single phase secondary distribution network rooted at a local distribution transformer. Therefore, the single phase distribution network with n_h residences and n_t transformers can be modeled using $\mathcal{G}(\mathcal{V}, \mathcal{E})$ where the node set \mathcal{V} corresponds to n_h load buses H_l and n_t feeder/transformer buses T_l and edge set \mathcal{E} comprises of overhead lines or underground cables. In order to identify which edges are required to be connected in the optimal topology, we introduce binary variables $\{x_e\}_{e \in \mathcal{E}} \in \{0, 1\}^{|\mathcal{E}|}$. Variable $x_e = 1$ indicates that the edge is present in the optimal topology and vice versa. Each edge $e = (i, j)$ is assigned a direction from the source node i to destination node j .

B. Node constraints

Each node $i \in \mathcal{V}$ either consumes power (if it is a residential building) or it injects power into the network (if it is a pole-top/pad-mounted transformer). Since each residence is associated with an hourly load demand profile, the average hourly load can be computed and be denoted by p_i . Let v_i represent the voltage at the node i . The nodal voltages and power injections at all nodes can be stacked in $n_t + n_h$ length vectors \mathbf{v} and \mathbf{p} respectively with the transformer nodes first followed by the residential nodes. The transformer voltages are assumed to be known and fixed at v_o . The constraints on node voltages are listed as

$$\underline{v}\mathbf{1} \leq \mathbf{v} \leq \bar{v}\mathbf{1} \quad (3)$$

C. Ensuring radial topology

Given the fully connected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, we define a subgraph $\tilde{\mathcal{G}}(\mathcal{V}, \tilde{\mathcal{E}})$ with $\tilde{\mathcal{E}} = \mathcal{E} \setminus \{e : e \in \mathcal{E}, x_e = 0\}$. The subgraph $\tilde{\mathcal{G}}$ denotes the optimal secondary distribution network. The edge-node relationship in graph \mathcal{G} is denoted in the $(n_h + n_t) \times |\mathcal{E}|$ branch-bus incidence matrix \mathbf{A} with the entries as

$$\mathbf{A}_{e,k} := \begin{cases} +1, & k = i \\ -1, & k = j \\ 0, & \text{otherwise} \end{cases} \quad \forall e = (i, j) \in \mathcal{E} \quad (4)$$

We separate the rows of \mathbf{A} into rows corresponding to residences and those corresponding to transformers as $\mathbf{A} = \begin{bmatrix} \mathbf{A}_T \\ \mathbf{A}_H \end{bmatrix}$. Let \mathbf{A}_H denote the reduced branch-bus incidence matrix for the graph \mathcal{G} . Similarly let $\tilde{\mathbf{A}} \in \mathbb{R}^{(n_h + n_t) \times |\mathcal{E}|}$ denote the branch-bus incidence matrix for the subgraph $\tilde{\mathcal{G}}$ and $\tilde{\mathbf{A}}_H$ be the corresponding reduced branch-bus incidence matrix. From the proposition in [7] a graph $\tilde{\mathcal{G}}(\mathcal{V}, \tilde{\mathcal{E}})$ with reduced branch-bus incidence matrix $\tilde{\mathbf{A}}_H$ is connected if and only if there exists a vector $\mathbf{f} \in \mathbb{R}^{|\tilde{\mathcal{E}}|}$, such that

$$\tilde{\mathbf{A}}_H \mathbf{f} = \mathbf{1} \quad (5)$$

To provide an intuition, the vector \mathbf{f} can be assumed to be flows along the edges of the subgraph $\tilde{\mathcal{G}}$ resulting in unit injection at each residential home node. For this flow to be feasible, there must be withdrawal of n_h units from all the transformer nodes and a path must exist for all injections in home nodes to reach one of the transformer nodes. Therefore, this constraint ensures that all home nodes are connected to at least one transformer node. The flow vector \mathbf{f} does not relate to the actual line flows in the network.

It is to be noted that $\tilde{\mathbf{A}}_H$ is obtained from \mathbf{A}_H after removing the edges for which $x_e = 0$. Therefore, the constraint in Eq. 5 can be expressed with matrix \mathbf{A}_H by forcing flows f to be zero for non-existing edges. This is ensured by the following set of constraints.

$$\begin{aligned} \mathbf{A}_H \mathbf{f} &= 1 \\ -Mx_e &\leq f_e \leq Mx_e, \quad \forall e \in \mathcal{E} \end{aligned} \quad (6)$$

Once the connectivity of the subgraph $\tilde{\mathcal{G}}$ is ensured the radiality requirement can be enforced by the following constraint

$$\sum_{e \in \mathcal{E}} x_e = n_h \quad (7)$$

D. Edge constraints

The edge constraints can be added as a heuristic method to avoid solving the power flow problem for the distribution network. In this work, two such constraints are considered: maximum allowable number of homes to serve in one leg for each transformer node and maximum degree of interconnection for home nodes.

In order to implement the first constraint, a slight alteration of Eq. 6 is required. It is mentioned earlier that f_e does not relate to the actual flows and rather it indicates the number of downstream home nodes. Therefore, the constraint is ensured if M in Eq. 6 is considered to be the maximum allowable number of homes to be served by each transformer node in one leg.

To consider the degree constraint for residence nodes, the unoriented branch-bus incidence matrix \mathbf{I} is considered with entries

$$I_{e,k} := \begin{cases} 1, & k = i \\ 1, & k = j \\ 0, & \text{otherwise} \end{cases} \quad \forall e = (i, j) \in \mathcal{E} \quad (8)$$

The degree constraint is ensured by the following

$$\mathbf{I} \mathbf{x} \leq \mathbf{k} \quad (9)$$

where vector \mathbf{k} stacks the maximum allowable degrees for each node.

E. Power flow constraints

The *linearized distribution flow* (LDF) model has been extensively used for several grid optimization tasks in order to relate power injections and flows to voltages. By ignoring network losses, the LDF model gives the relation between power injections at the home nodes and power flows along

edges of optimal graph $\tilde{\mathcal{G}}$ with reduced branch-bus incidence matrix $\tilde{\mathbf{A}}_H$ as

$$\mathbf{p} = \tilde{\mathbf{A}}_H \mathbf{h} \quad (10)$$

where \mathbf{h} is the vector of power flows along all edges of the graph. Similar to the connectivity constraint, this constraint can be related to the reduced branch-bus incidence matrix of the fully connected graph \mathcal{G} as

$$\begin{aligned} \mathbf{A}_H \mathbf{h} &= 1 \\ -Mx_e &\leq h_e \leq Mx_e, \quad \forall e \in \mathcal{E} \end{aligned} \quad (11)$$

However, unlike the previous case, h relates to the actual flows in the network. Therefore, the value of M in Eq. 11 can be chosen to be the power flow limits in the edges.

If r_e denotes the resistance of the line $e : (i, j) \in \mathcal{E}$, the LDF model relates the squared voltage magnitude to power flows linearly as $v_i^2 - v_j^2 = 2r_e P_e$ where P_e is the entry from the vector \mathbf{h} corresponding to edge e . The squared voltage can be approximated as $v_i^2 \approx 2v_i - 1$ which leads to the relation $v_i - v_j = r_e P_e$. Notice that this constraint is only activated for those edges $e \in \mathcal{E}$ for which $x_e = 1$. Therefore, we can enforce this constraint as

$$-M(1 - x_e) \leq v_i - v_j - r_e P_e \leq M(1 - x_e) \quad (12)$$

F. Generating optimal network topology

For a non-trivial road network link $l \in \mathcal{L}_R$, with at least one residence mapped to it (i.e., $f^{-1}(l) = \mathcal{H}_l \neq \emptyset$), we assign a label $L_H(h, l)$ to each home $h \in \mathcal{H}_l$ to indicate the side of the road link (first or second) on which it is located.

$$L_H(h, l) = \begin{cases} 1, & \text{if } h \text{ is on first side of road link } l \\ -1, & \text{if } h \text{ is on second side of road link } l \end{cases} \quad (13)$$

We assign a label $L(v)$ to each node $v \in \mathcal{V}$ as

$$L(v) = \begin{cases} L_H(v, l), & \text{if } v \in f^{-1}(l) \\ 0, & \text{if } v \in \mathcal{T}_e \end{cases} \quad (14)$$

The edges $(u, v) \in \mathcal{E}$ for all $u, v \in \mathcal{V}$ are assigned weights $w(u, v)$

$$w(u, v) = \begin{cases} \infty, & \text{if } u, v \in \mathcal{T}_e \\ \text{dist}(u, v) + \lambda C(u, v), & \text{otherwise} \end{cases} \quad (15)$$

where $\text{dist} : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}$ denotes the Euclidean distance between the residential locations u, v . The function C denotes if the edge crosses the nearest road link and is defined as

$$\begin{aligned} C(u, v) &= |L(u) - L(v)| \\ &= \begin{cases} 0, & \text{if } u, v \text{ are on same side of road link} \\ 2, & \text{if } u, v \text{ are on opposite side of road link} \\ 1, & \text{if } u \in \mathcal{V}_R \text{ or } v \in \mathcal{V}_R \end{cases} \end{aligned} \quad (16)$$

λ is a weight factor to penalize multiple crossing of edges over the road links. It also penalizes multiple edges emerging from the root node. Thereafter, a forest with trees rooted at the probable transformer nodes and spanning all the nodes of \mathcal{G} is considered as the secondary network.

V. CONCLUSION

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APPENDIX A

PROOF OF THE FIRST ZONKLAR EQUATION

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APPENDIX B

Appendix two text goes here.

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