Discrete Mathematics

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1 Proposition

Definition: A statement that can is either true or false

Examples of proposition:

- 3+2=4
- 1+1=2
- Elephants can fly

Examples of statements that are not proposition:

- Bring me a glass of water
- Is the window open?
- This statement is false

1.1 Problem with human languages

Languages like English and Bengali are okay for normal conversations but they are to precise enough for mathematics. Lets see an example:

“Take a lalmohon or kalojam”
Does that mean I can take both? Or, just one?

Confusing right? That is why we have to learn some symbols and definitions to ensure that there is no ambiguity.

1.2 Propositional Variable

Definition: A variable that represents a proposition

Example:
P = Elephants can fly
P is false

Conventionally, we use letters: p,q,r,s to represent propositional variables

2 Compound Proposition

Definition: New proposition formed from existing proposition(s) using logical operator(s)

George Boole first discussed about compound proposition in his book, “The laws of Thought”

2.1 Not

Negation of $P = \neg P$

P	$\neg P$
T	F
F	T

Example:
If P= Birds can fly
then, $\neg P$= Birds can't fly

2.2 Conjunction/And

Conjunction of P and Q = $P \wedge Q$

$P \wedge Q$ is true, only if both P and Q are true

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example:
If P= Roses are red and Q= Violets are blue
then, $P \wedge Q$= Roses are red and violets are blue

2.3 Disjunction/Or

Disjunction of P and Q = $P \vee Q$

$P \vee Q$ is true, if at least one of P and Q is true and false otherwise

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:
If P= Roses are red
then, $P \vee \neg P$= Roses are red or roses are not red

2.4 Exclusive OR

Exclusive OR of P and $Q = P \oplus Q$

$P \oplus Q$ is true, if only one of P and Q is true and false otherwise

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Example:
If P= Roses are red
then, $P \lor \neg P$= Either roses are red or roses are not red

2.5 Implication

P imples $Q = P \implies Q$

$P \implies Q$ is false, if P is true and Q is false and true otherwise

P	Q	$P \oplus Q$
T	T	T
T	F	F
F	T	T
F	F	T

Think of implication as a contract.

Example:
If P= I get elected and Q= I will plant trees
then, $P \implies Q$= If I get elected, then I will plan trees

Terminologies:

- The *if-part* is called hypothesis/antecident/premise
- The *then-part* is called consequence/conclusion

2.6 If and only if/IFF

P IFF Q = $P \Leftrightarrow Q$

$P \Leftrightarrow Q$ means P and Q are logically equivalent

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Example:
If P is $x^2 - 4 \geq 0$ and Q is $|x| \geq 2$
then, $P \Leftrightarrow Q$= For some values of both P and Q are true and for others, both are false