Introduction to Probability

Permutations & Combinations

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- Permutations
- Combinations

What is Probability

Probability by definition, is the frequency at which some event happens out of greater number of outcomes.

Thus, to find probabilities, we have to be able to count in complex ways.

Introduction: Permutations

Definition:
The number of different ways that a certain number of objects can be **arranged in order** from a larger number of objects.

Example: If there are ‘n’ objects, how many different ways we can make Ordered list of size ‘r’

$$P(n,r) = \frac{n!}{(n-r)!}$$

Introduction: Combinations

Definition:
The number of different ways that a certain number of objects AS A GROUP can be **selected** from a larger number of objects.

In brief, *The number of ways that 'r' objects can be selected from a larger number of 'n' objects.* ***Order does not matter***

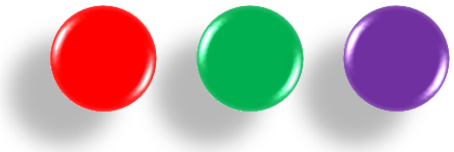
Often said as '***n* choose *r***'

Example: If there are 'n' objects, how many different ways we can select Groups or sets of size 'r':

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

Visualizing the difference

Permutation



In how many ways we can arrange the three colored balls ?

1st Position : 3 ways

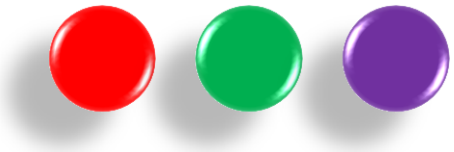
2nd Position: 2 ways

3rd Position: 1 way

Total Ways = 3 x 2 x 1 = 6 ways

Ordered List

Combination



In how many ways we select the three colored balls ?

One Way.

The order does not matter here.

Un-Ordered group or **set**

Example

A horse race includes 10 horses. You are given two betting options:

a) You can choose the top three horses regardless of finishing order

b) You can choose the top three horses in exact order of finish.

How many outcome are there for each outcome ?

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Solution:

a) Top 3 in any order:

C(10,3) = 10! / (10-3)! x 3! = 8x9x10 / 3x2 = **120 possible outcomes**

a) Top 3 in exact order:

P(10,3) = 10! / (10-3)! = 8x9x10 = **720 possible outcomes**

Example

During most of the season, a MLB team has 25 active players. Ignoring the playing position on the field, a manager must select 9 Players for each game.

a) How many **unique groups or sets** of 9 players out of the 25 players could be potential line ups ? (Think a bucket of 9 players)

b) How many line ups (ordered list) could the manager make using 9 players out of the 25 ? (Hence the batting order)

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Solutions:

a) $C(25,9) = 2042975$

b) $P(25,9) = 7.41 \times 10^{11}$

Example

There are 30 stocks in the Dow Jones Industrial Average. Over the next 6 months you decide to invest $1000 in a different stock each month.

a) How many unique stock groups of 6 stocks are possible ?
b) How many investment lists could be made ? The order of investment to be made over time ?

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Solutions:
a) C(30, 6) = 593775
b) P(30, 6) = 427518000

Example

A state lottery selects 6 numbers out of 52. The numbers are selected as a group. There are no repeated numbers and the order of the numbers selected does not matter.

a) How many possible lottery winners does this lottery have ?

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a) How many possible lottery winners does this lottery have ?

Solution:
C(52, 6) = 20358520

Example

In a move to collect more money from its citizens without calling it a tax, a state comes up with an insane lottery idea. The state will hold this lottery every 10 years and people can buy virtual tickets over that time period.

The catch of the lottery is:
Not only you have to select the correct 6 numbers, you also have to correctly choose the ORDER in which they exit the lotto machine. You do that, you win.

Question is, how many possible winners does this lottery have ?

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Solution:
P(52,6) = 14658134400