Introduction to Probability

Set Theory & Venn Diagram

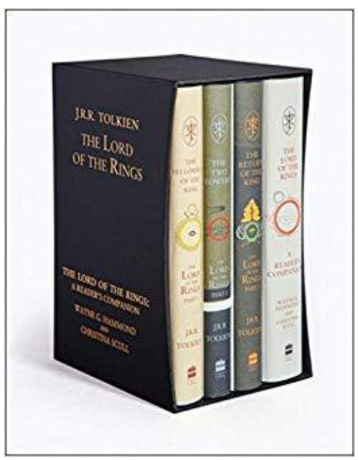
Set Theory & Venn Diagram

- Sets and Elements
- Equivalent Sets
- Subsets
- Proper Subsets
- Empty Sets
- Set Notation
- Set Builder Notation
- Set Conclusion
- Intersection of Sets
- Union of Sets
- Disjoint Set
- Universal Set
- Compliment of a Set
- Venn Diagram Region Map

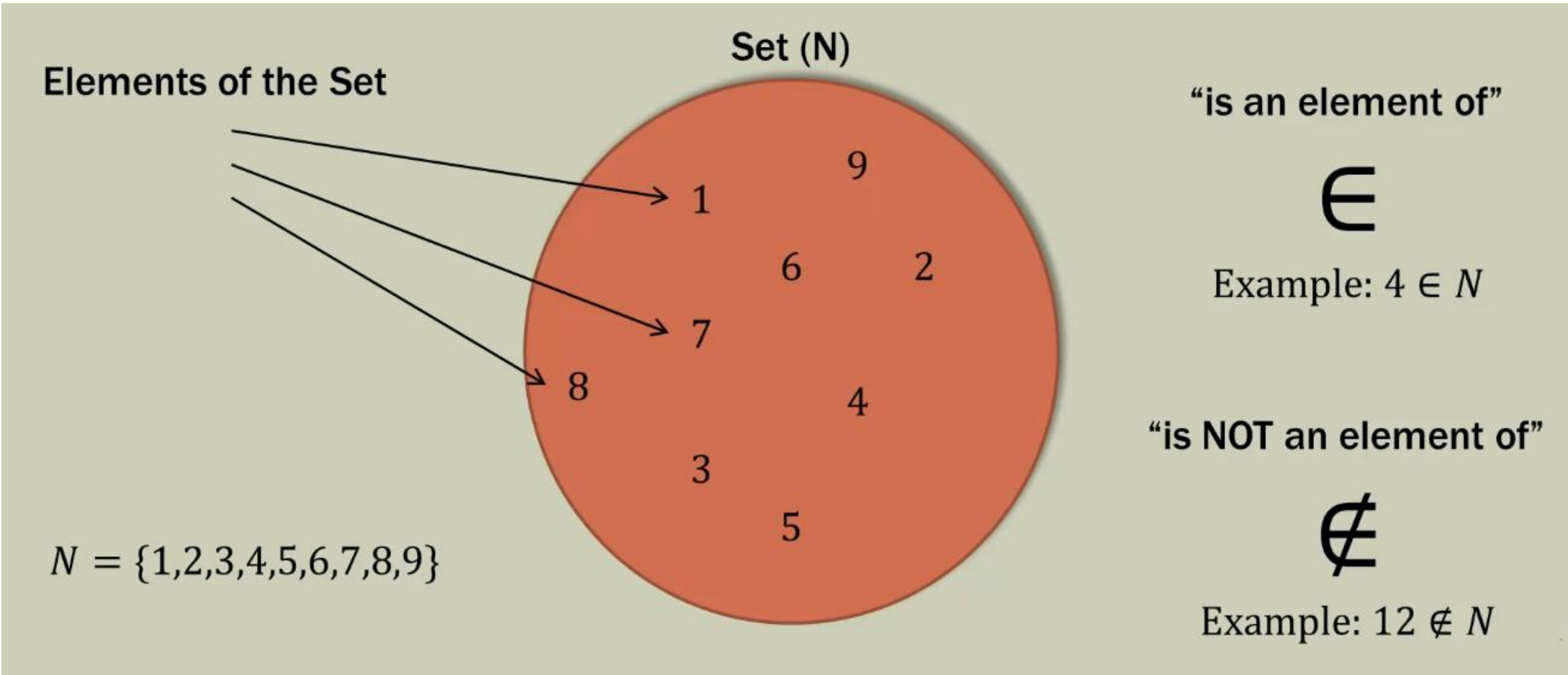
What is a Set ?

In Layman terms:
Set is a collection of things which are grouped together based on some criteria that makes them similar.

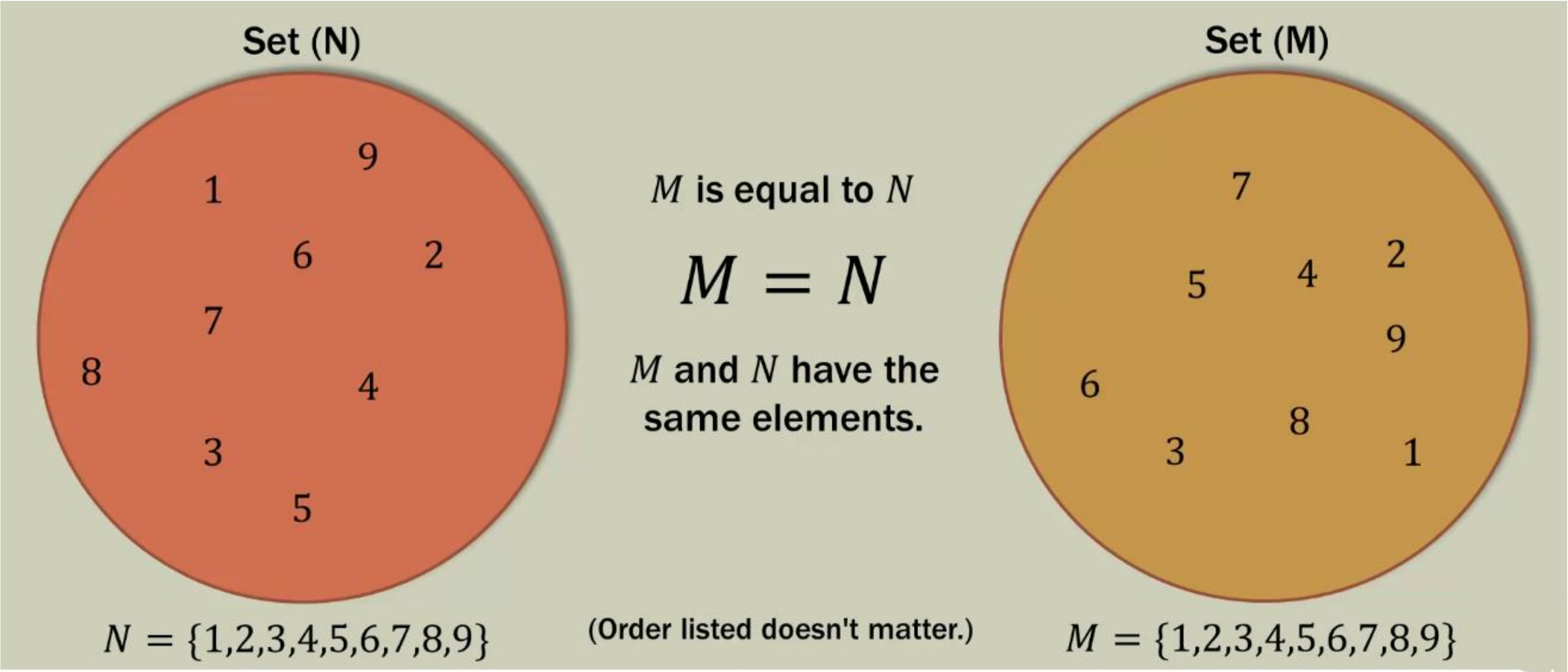
The easiest way to think about a “SET” are things in everyday life:



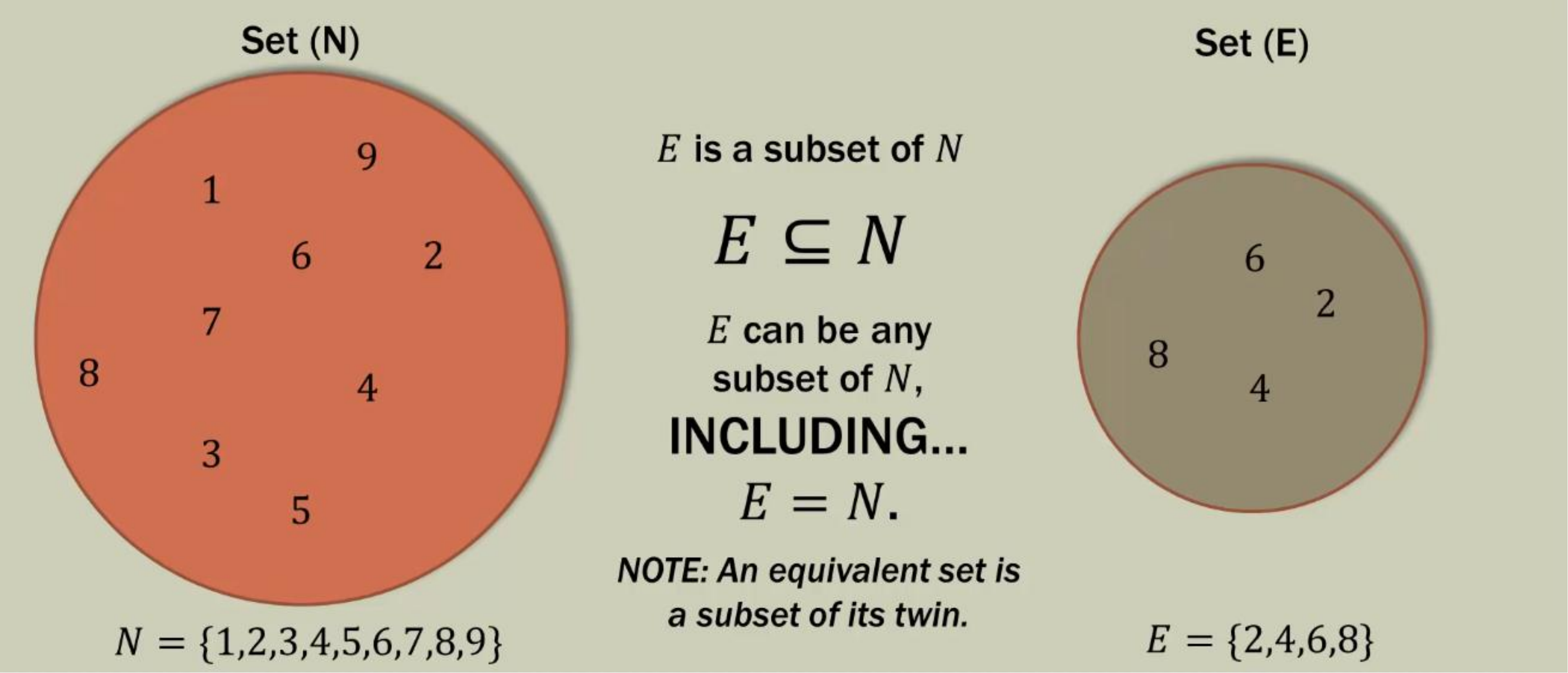
Sets and Elements



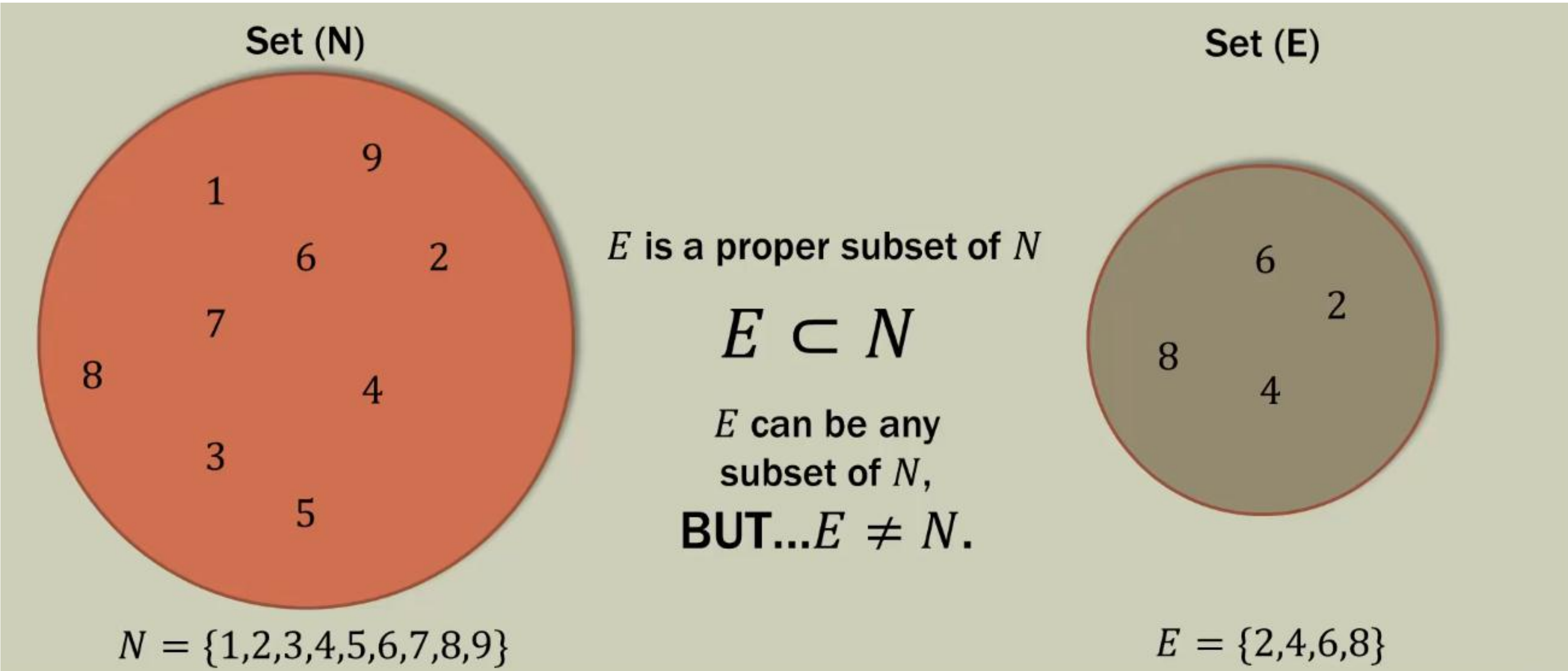
Equivalent Sets



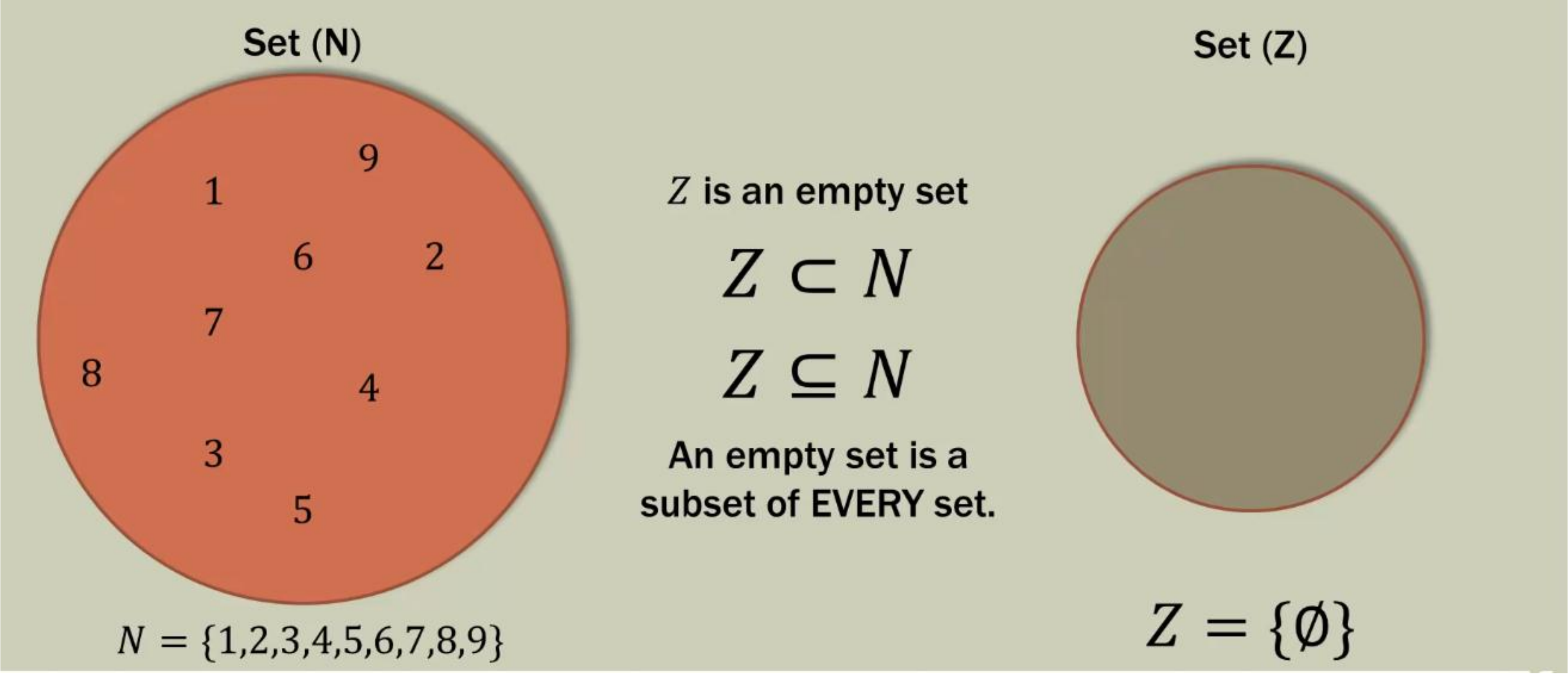
Subsets


Set (N)
9
1
6
2
7
8
4
3
5
$N = \{1,2,3,4,5,6,7,8,9\}$
E is a subset of N
$E \subseteq N$
E can be any subset of N,
INCLUDING...
$E = N$.
NOTE: An equivalent set is a subset of its twin.
Set (E)
6
2
8
4
$E = \{2,4,6,8\}$

Proper Subsets



Empty Set



Set (N)
1
9
6
2
7
8
4
3
5
$N = \{1,2,3,4,5,6,7,8,9\}$
Z is an empty set
$Z \subset N$
$Z \subseteq N$
An empty set is a
subset of EVERY set.
Set (Z)
$Z = \{\emptyset\}$

Subsets Vs. Proper Subsets

$$S = \{1,2,3\}$$

Set	Subset	Proper Subset
{Ø} or {}	✓	✓
{0}	✓	✓
{1}	✓	✓
{2}	✓	✓
{3}	✓	✓
{0,1}	✓	✓
{0,2}	✓	✓
{1,2}	✓	✓
{0,1,2}	✓	✗

Set Notation

We often use set notation to represent a list of outcomes

<u>Example: Single Coin Flip</u>

$S = \{ H,T \}$

<u>Example: Single Die Roll</u>

$S = \{ 1,2,3,4,5,6 \}$

<u>Example: Rain in a day</u>

$S = \{ \textit{Rain, No-Rain, Rain} \}$

<u>Example: Flip **Two Indistinguishable Coins**</u>

$S = \{ (H,H), (H,T), (T,T) \}$

<u>Example: Flip Two Distinguishable Coins</u>

$S = \{ (H1,H2), (H1,T2), (H2,T1), (T1,T2) \}$

Set Builder Notation

Set Builder Notation is a way of writing out what we want in our set

C = { n | n is an odd integer less than 10 }
C = { n | n is an odd integer < 10 }
Set Notation: C = {1,3,5,7,9}

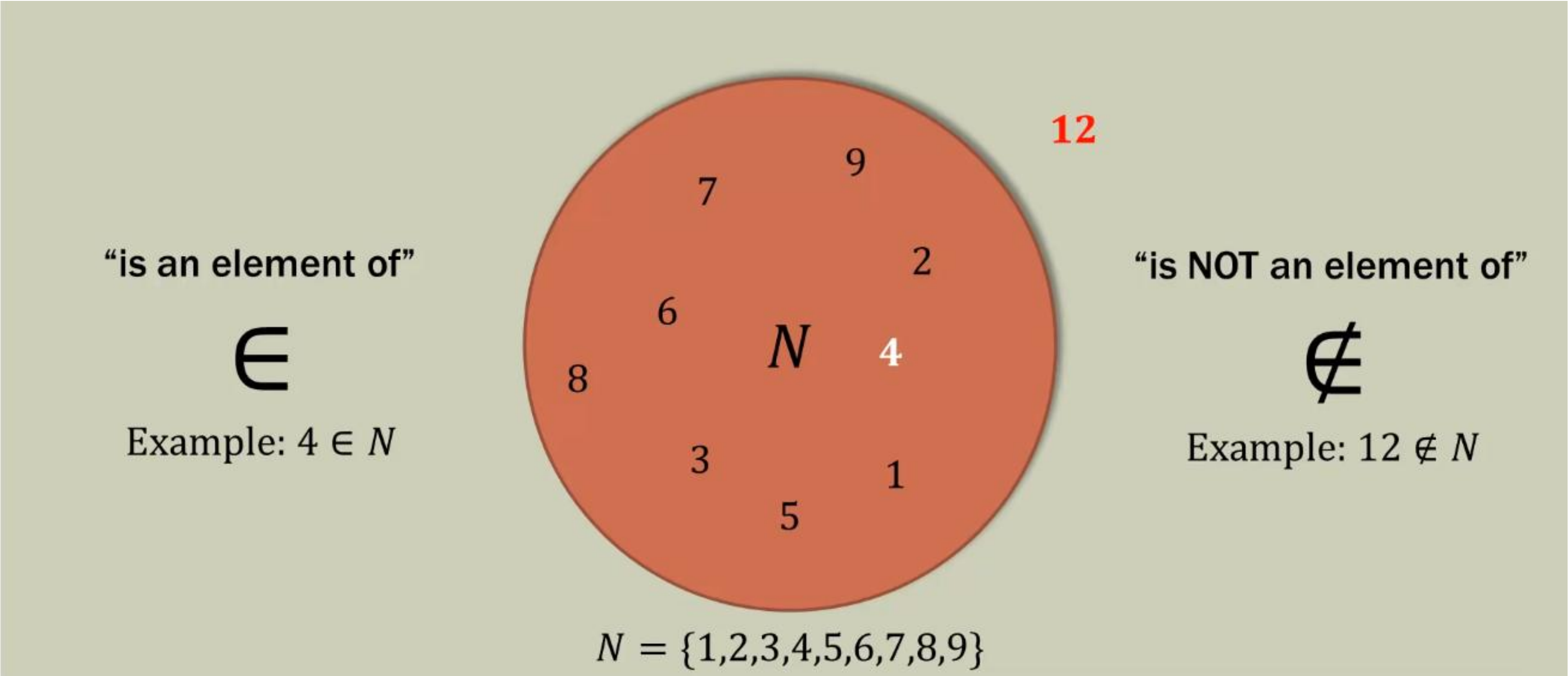
D = { n | n is one of the seven days }
Set Notation: P = {Mon, Tue, Wed, Thu, Fri, Sat, Sun}

Set Conclusions

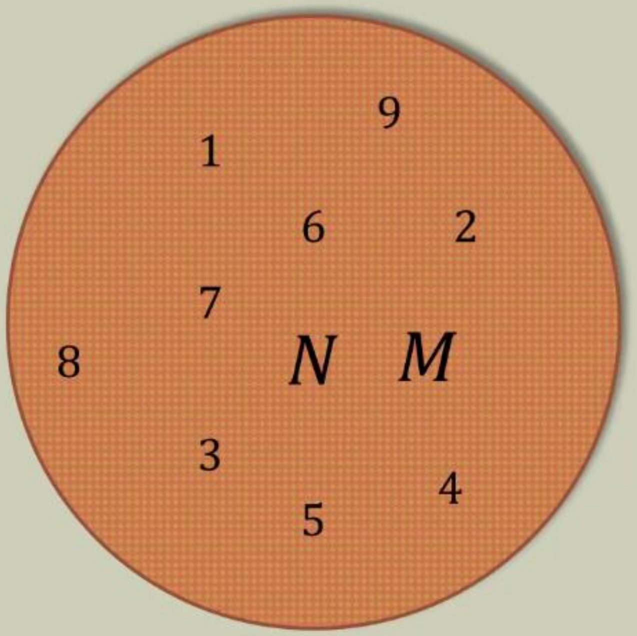
- Set is a simple way of categorizing things based on some common criterion.
- We do this all the time. Usually we group things together e.g. clustering clothes together, gadgets together etc.
- <u>Real-life examples:</u>
  Amazon offers "similar items" based on the set concept.
  Google offers "similar apps" in its app store.

Set Operations

Set and Elements



Equivalent Sets

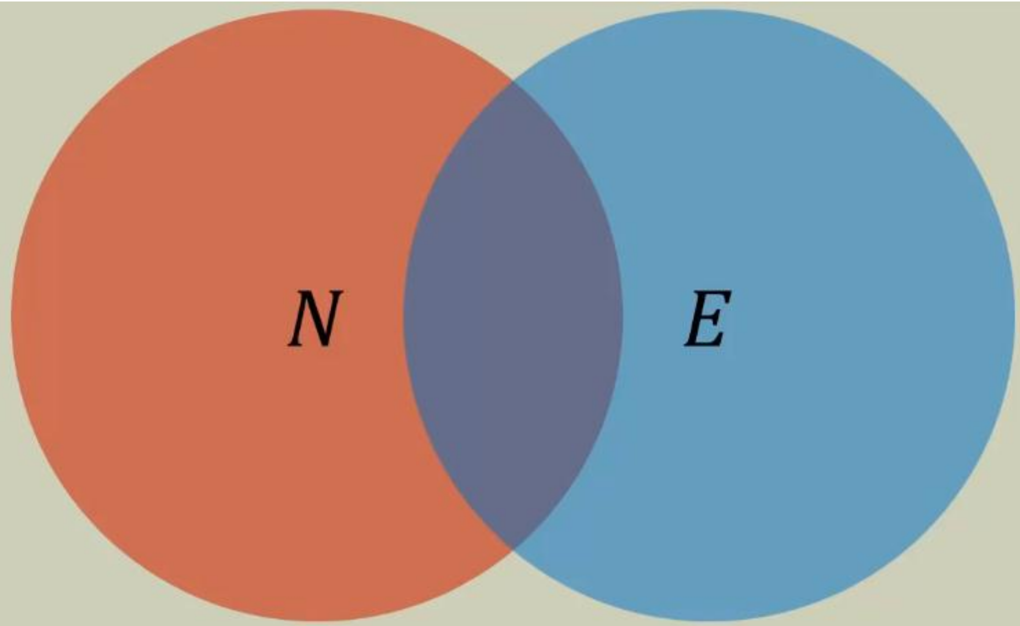


$N = \{1,2,3,4,5,6,7,8,9\}$ $M = \{1,2,3,4,5,6,7,8,9\}$

$$M = N$$
$$N = M$$

$$N \subseteq M$$
$$M \subseteq N$$

Set Operations: Intersection



Neither set is inside the area of the other. Some elements of N will be outside of E and some elements of E will be outside of N.

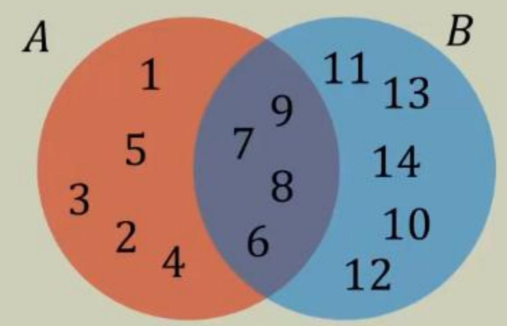
Set Operations: Intersection

$$A = \{1,2,3,4,5,6,7,8,9\}$$

$$B = \{6,7,8,9,10,11,12,13,14\}$$

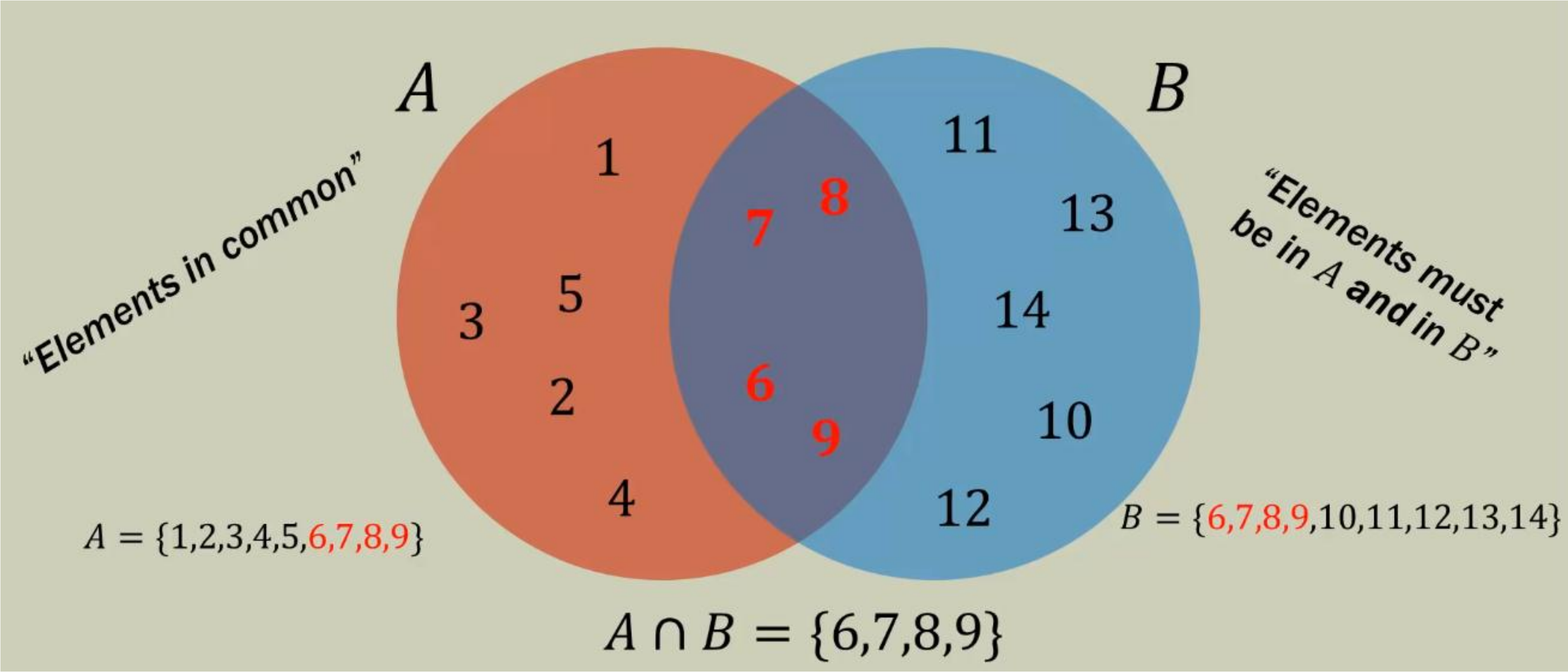
$$A = \{1,2,3,4,5,6,7,8,9\}$$

$$B = \{6,7,8,9,10,11,12,13,14\}$$

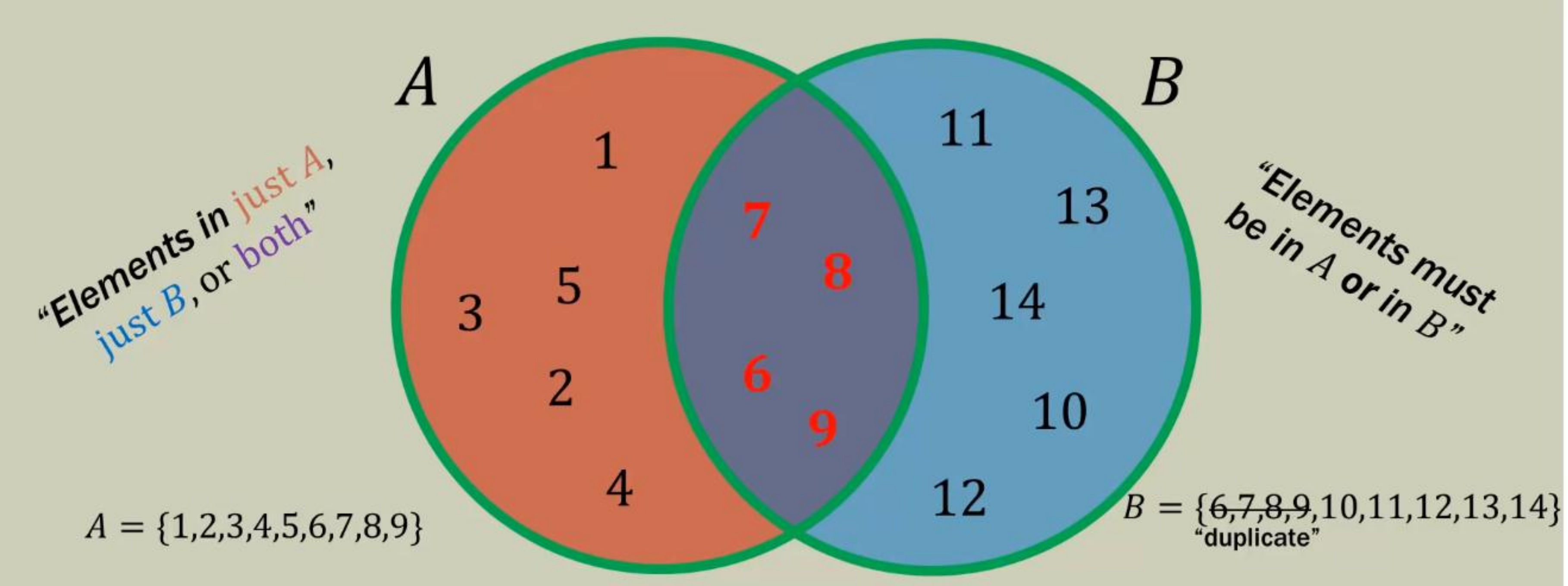


What elements do these sets have in common?
What elements do they share?
Where do these two sets overlap?

Set Operations: Intersection

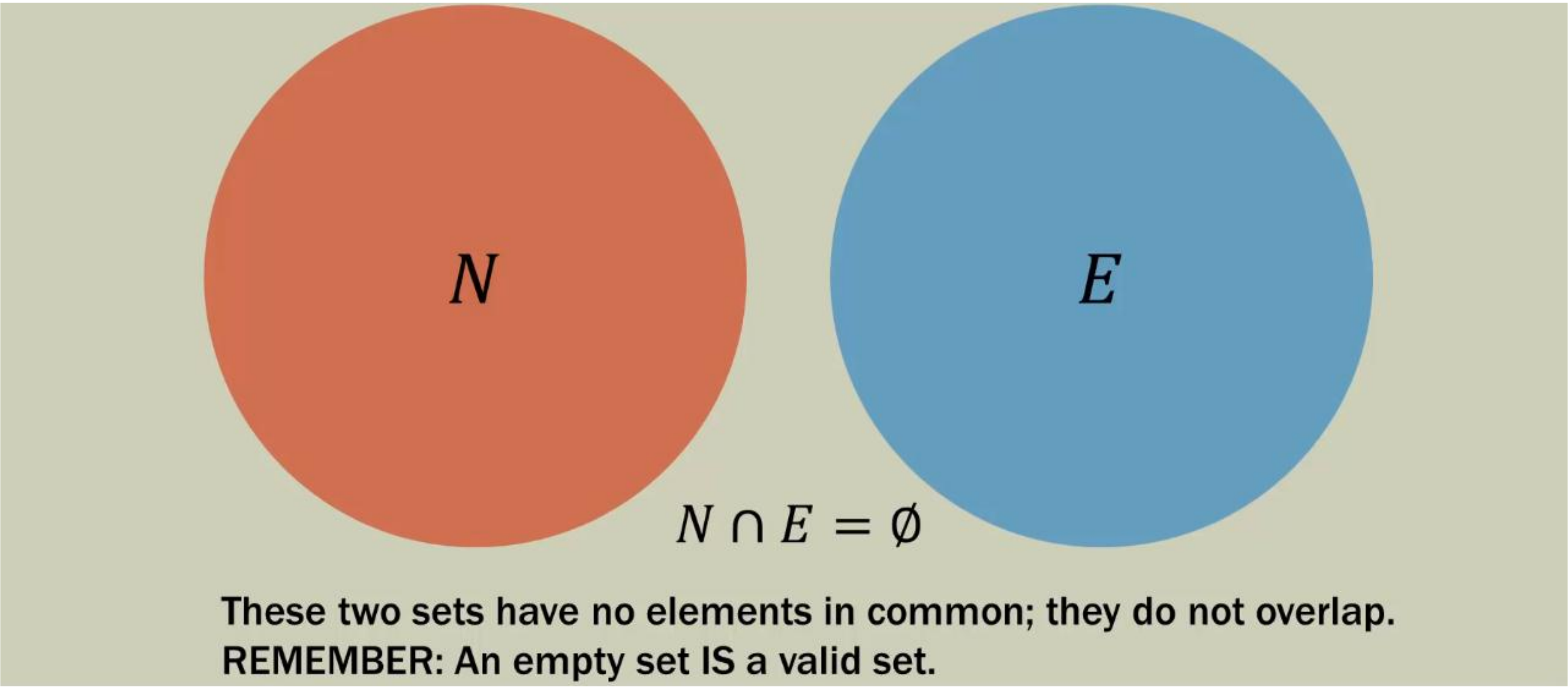


Set Operations: Union



$$A \cup B = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$$

Disjoint Set

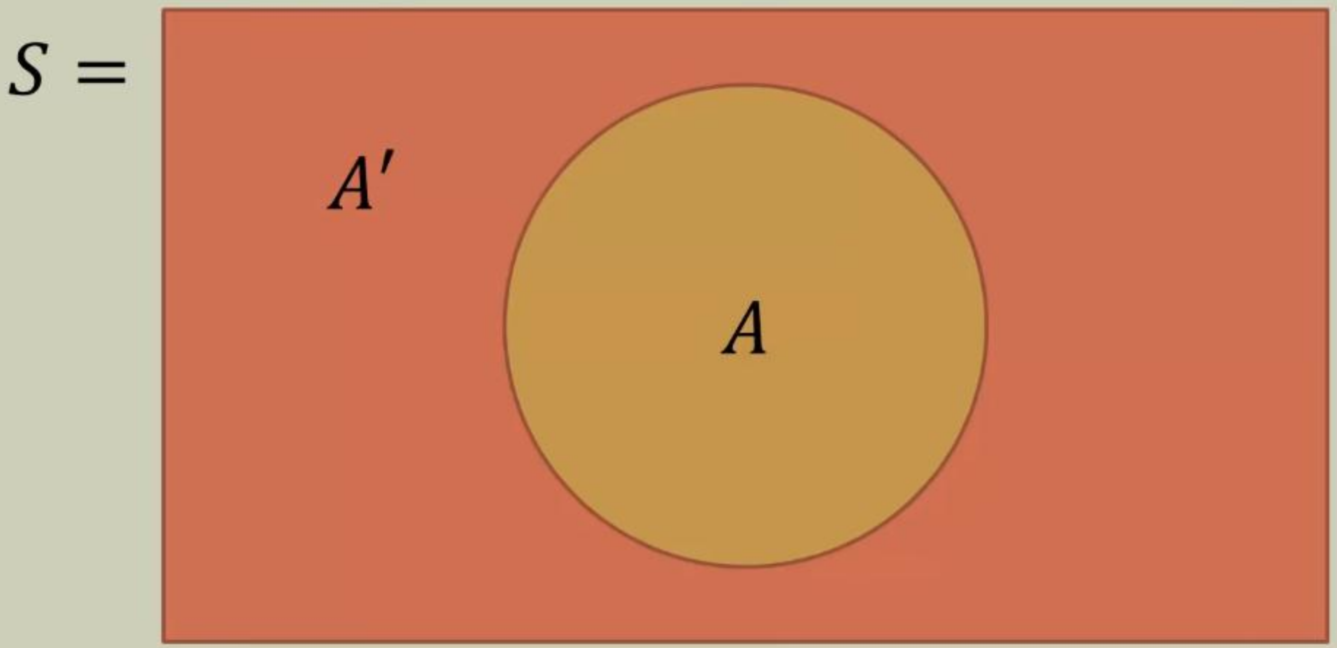


Universal Set

$S =$

The universal set contains all elements we are interested in; depends on problem.

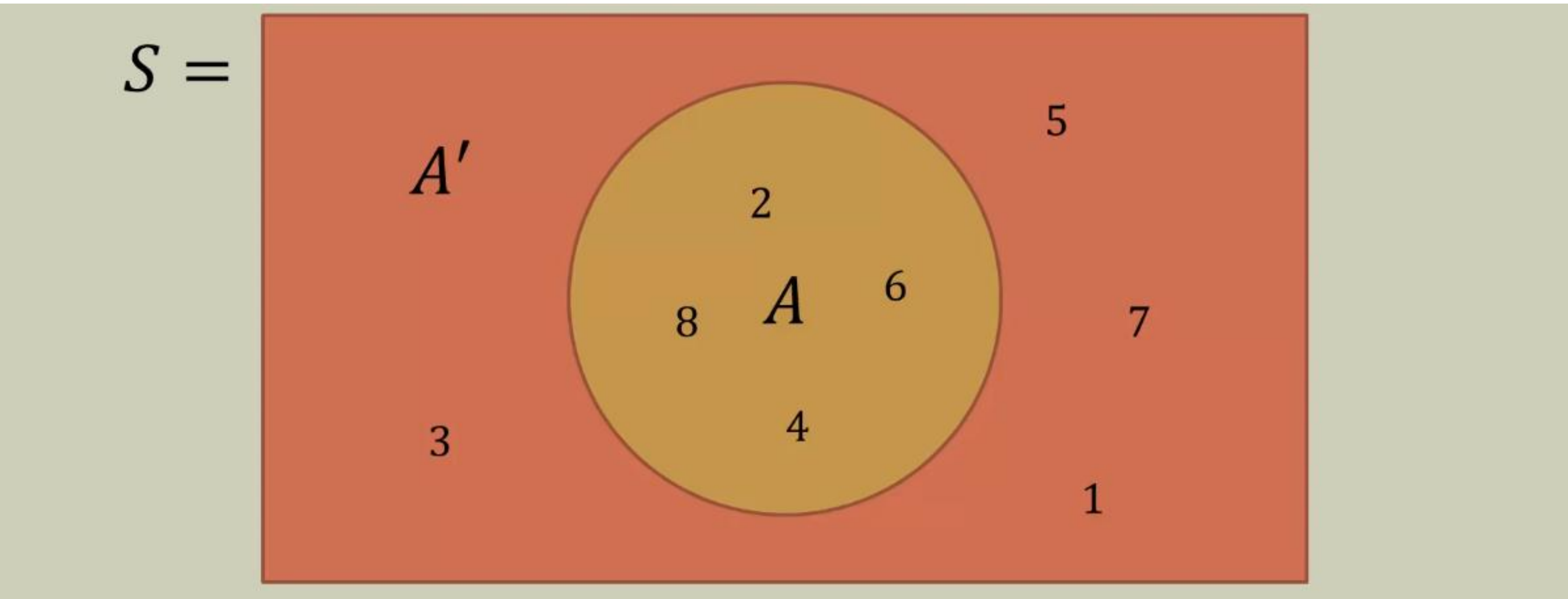
Compliment of a Set



"Not A"

Of everything in S, some are also in A; but usually not everything. If an element is in S but not in A then it is in the complement set A'.

Compliment of a Set

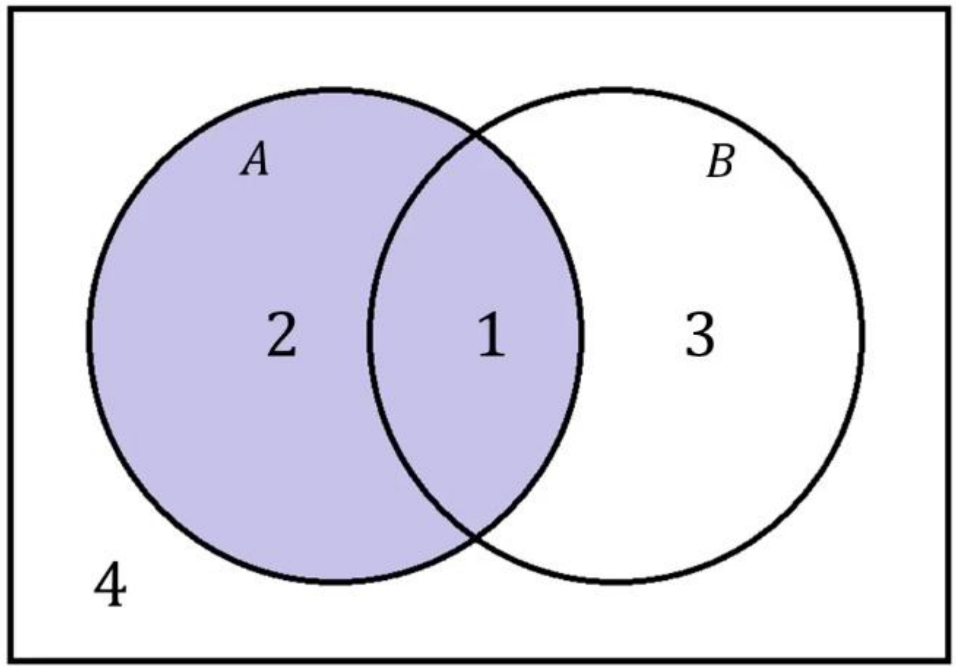


$S = \{1,2,3,4,5,6,7,8]$ $\quad A = \{2,4,6,8\}$ $\quad A' = ?$ $\quad A' = \{1,3,5,7\}$

Venn Diagram Region Map

Venn Diagram Region "Map"

$U_r =$

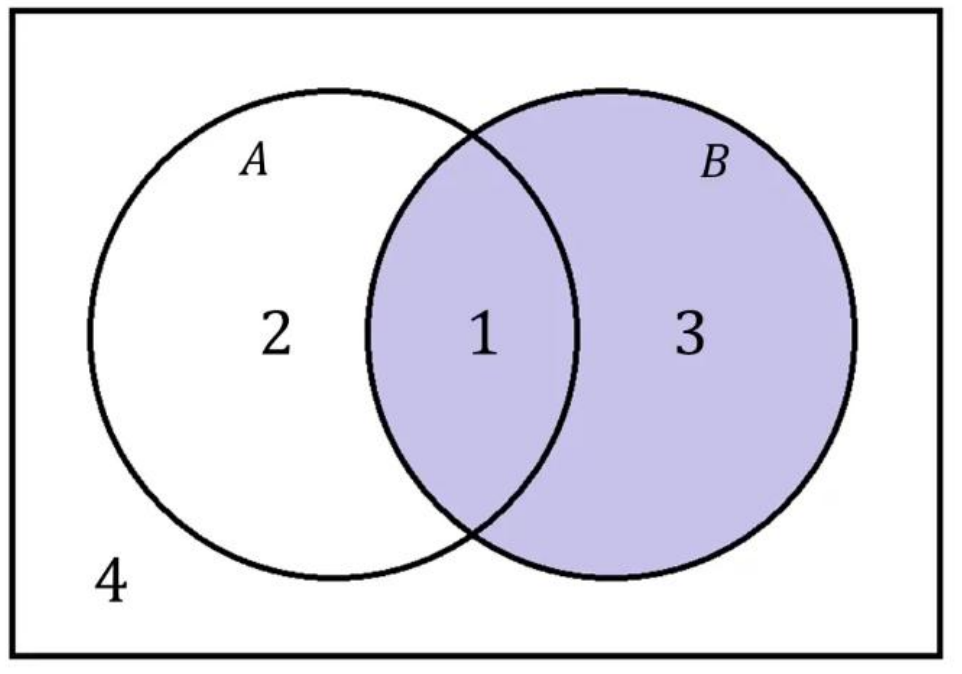


Which region(s) make up A?

2,1

Venn Diagram Region "Map"

$U_r =$

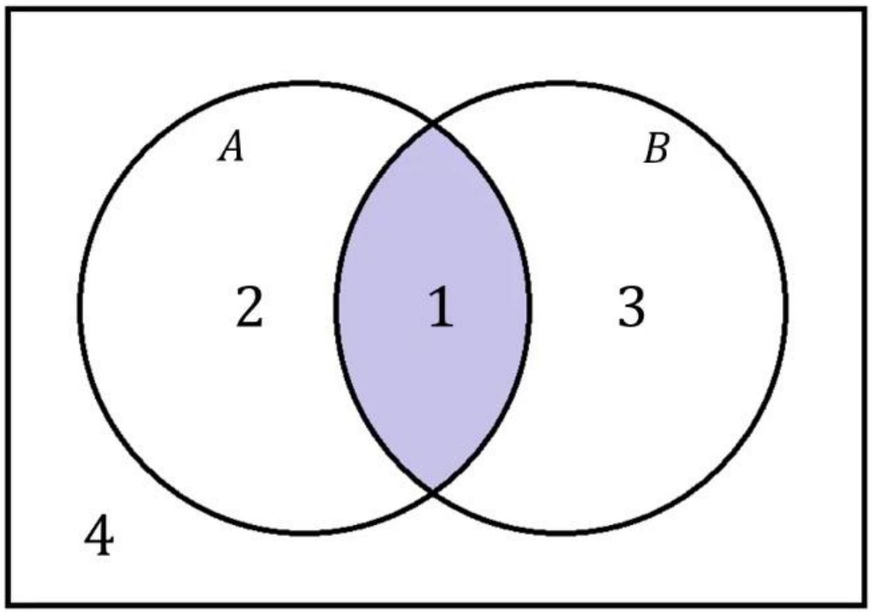


Which region(s) make up B?

3,1

Venn Diagram Region "Map"

$U_r =$

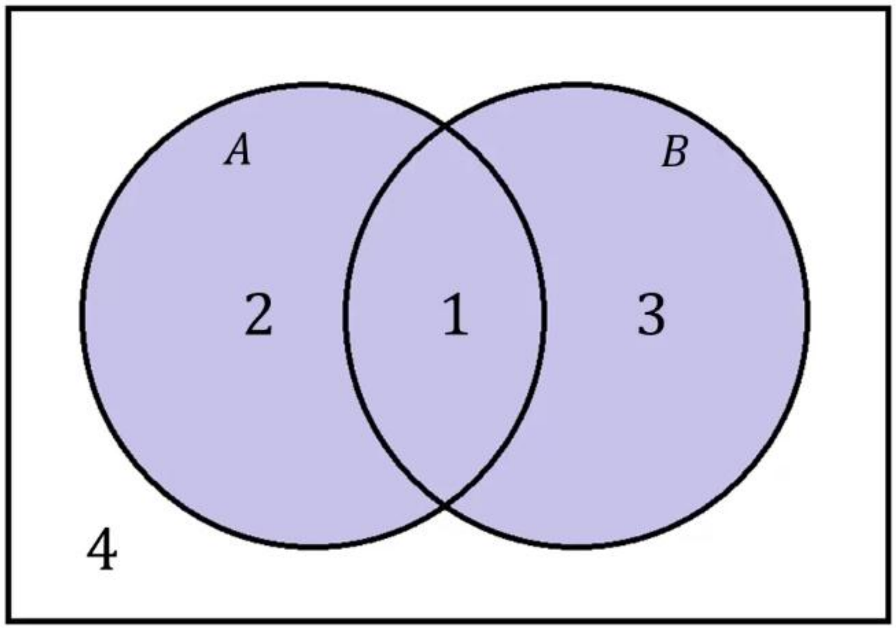


Which region(s) make up $A \cap B$?

1

Venn Diagram Region "Map"

$U_r =$



Which region(s) make up $A \cup B$?

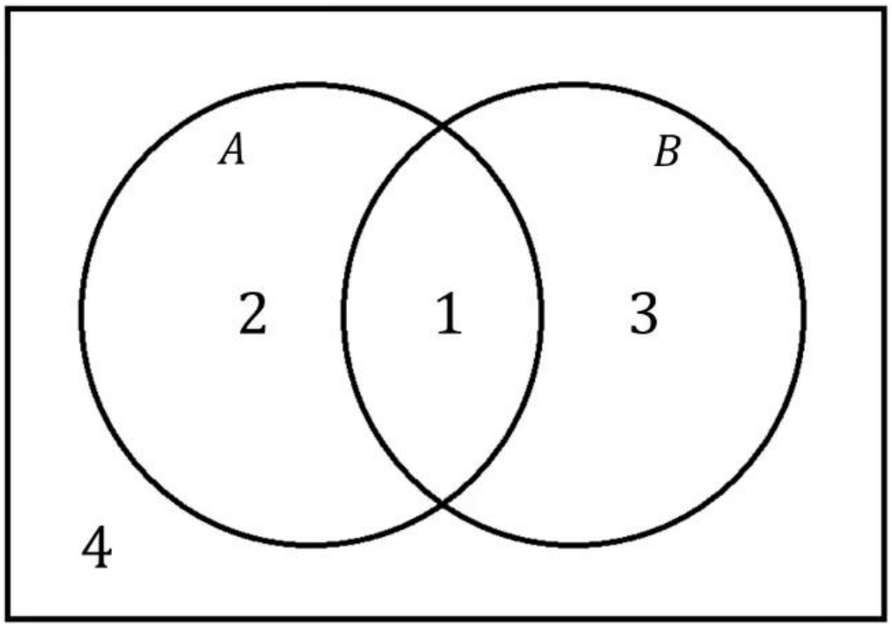
1,2,3

Exercise

Venn Diagram Region "Map"

Question

$U =$



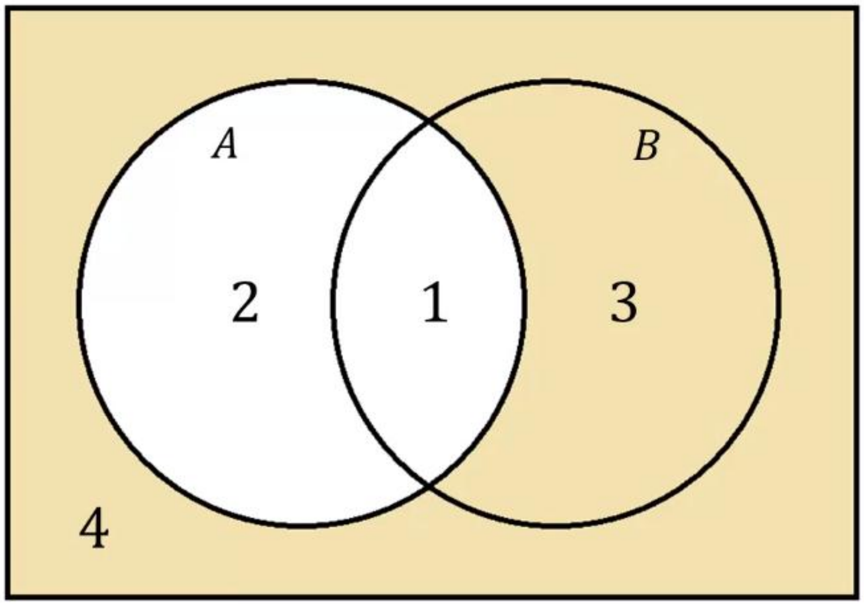
What region(s) are in the universal set U?

1,2,3,4

Which regions make up A'?

Venn Diagram Region "Map"

Solution:



What region(s) are in the universal set U?

1,2,3,4

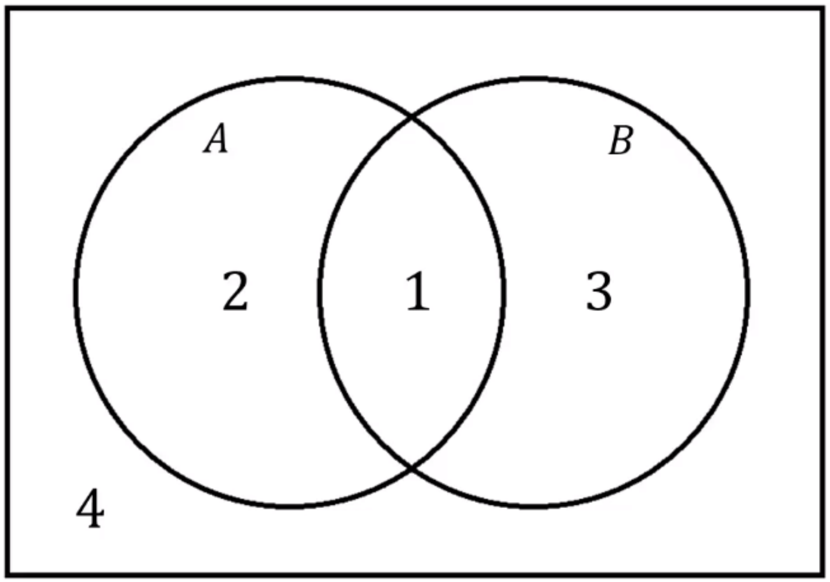
Which regions make up A'?

3,4

Venn Diagram Region "Map"

Question:

$U_r =$



What region(s) are in the universal set U?

1,2,3,4

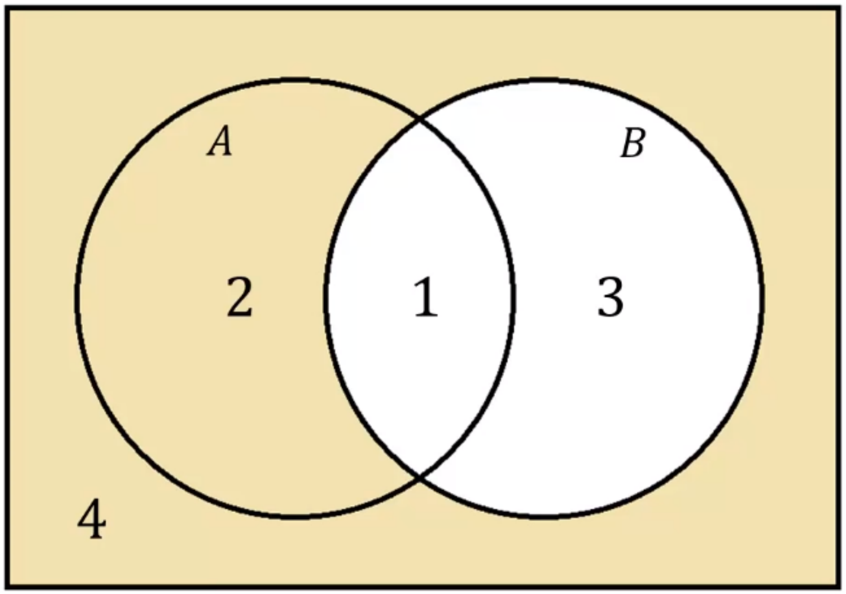
Which regions make up B'?

If B is made up of regions 1 & 3, then NOT B has to be everything else...

Venn Diagram Region "Map"

Solution:

$U_r =$



What region(s) are in the universal set U?

1,2,3,4

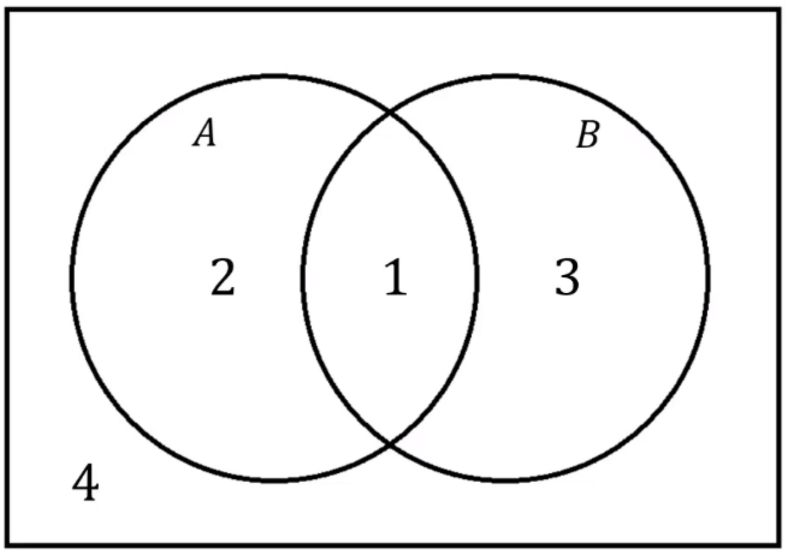
Which regions make up B'?

2,4

Venn Diagram Region "Map"

Question:

$U_r =$



What region(s) are in the universal set U?

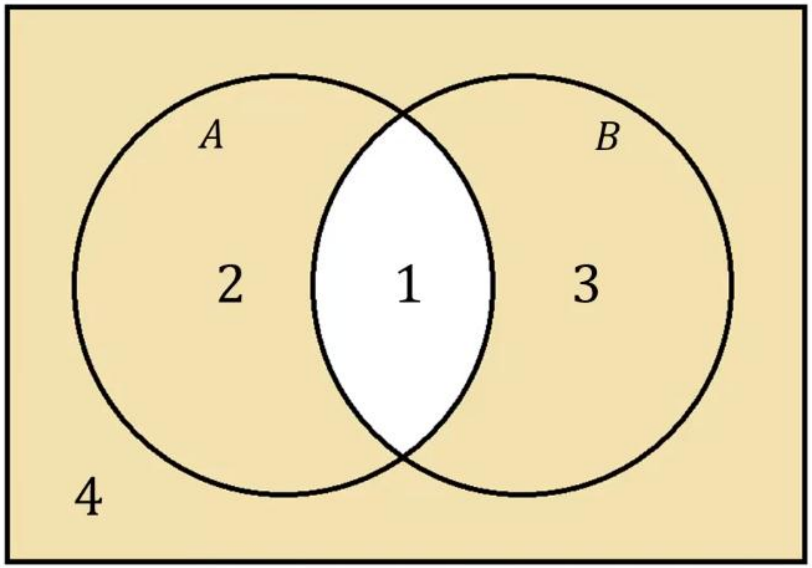
1,2,3,4

Which regions make up $(\mathrm{A} \cap \mathrm{B})'$?

Venn Diagram Region "Map"

Solution:

$U_r =$



What region(s) are in the universal set U?

1,2,3,4

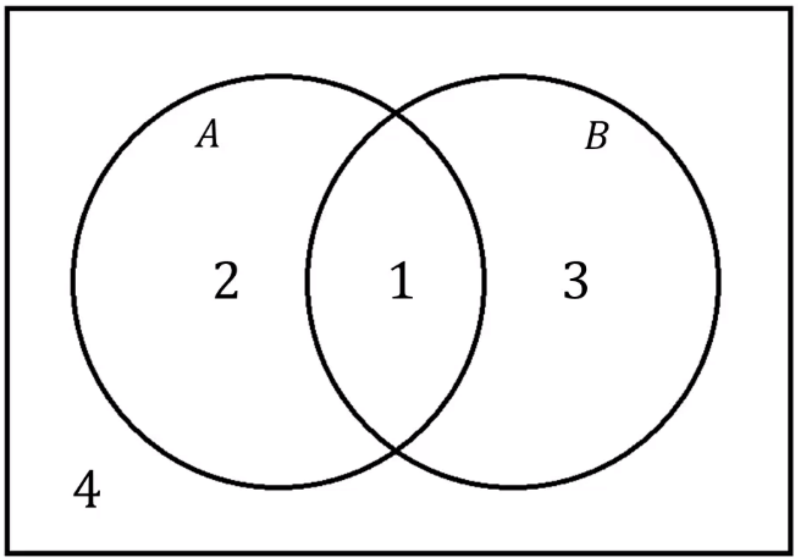
Which regions make up $(A \cap B)'$?

If $A \cap B$ is region 1, then everything that is NOT that must be regions 2,3,4

Venn Diagram Region "Map"

Question:

$U_r =$



What region(s) are in the universal set U?

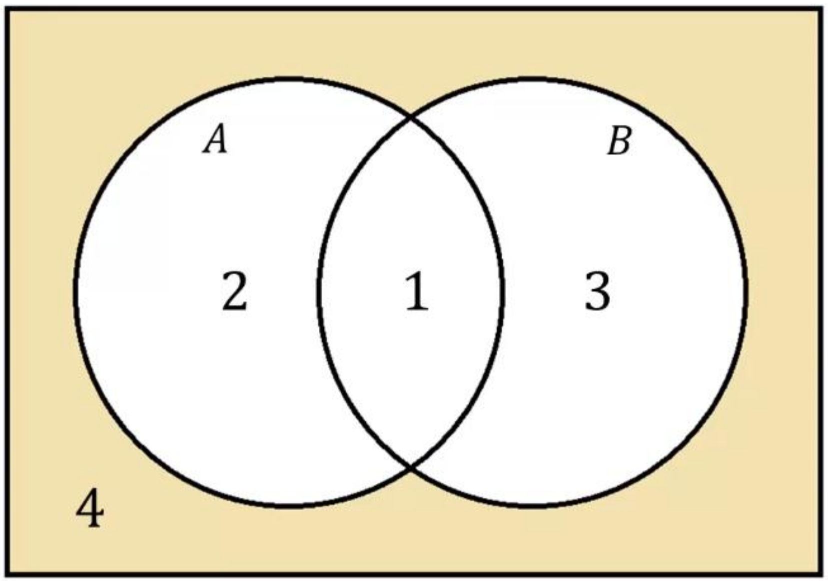
1,2,3,4

Which regions make up $(A \cup B)'$?

Venn Diagram Region "Map"

Solution:

$U_r =$



What region(s) are in the universal set U?

1,2,3,4

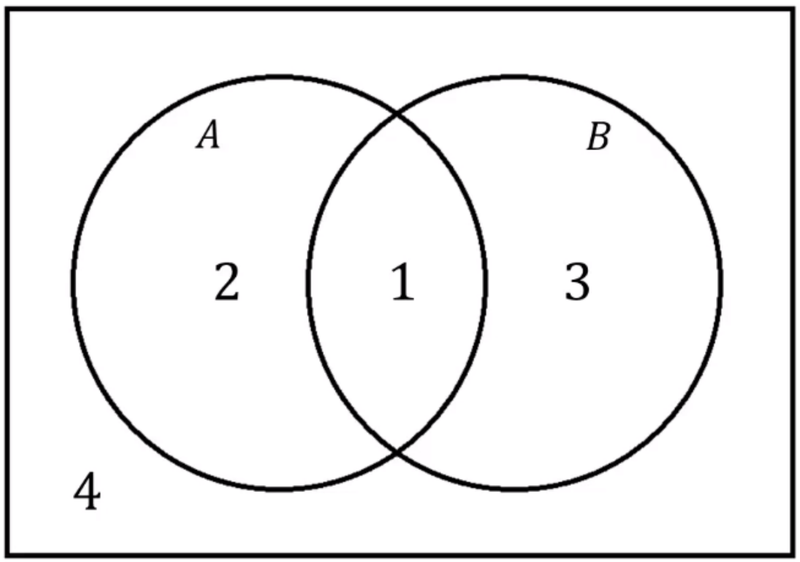
Which regions make up $(A \cup B)'$?

If $A \cup B$ is region 1,2,3 then everything that is NOT that must be region 4

Venn Diagram Region "Map"

Question:

$U_r =$



What region(s) are in the universal set U?

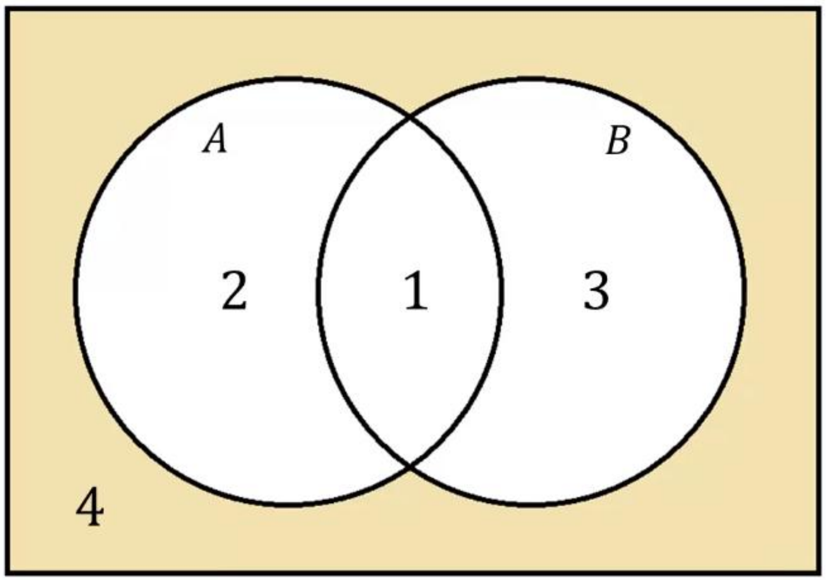
1,2,3,4

Which regions make up $(A' \cap B')$?

Venn Diagram Region "Map"

Solution:

$U_r =$

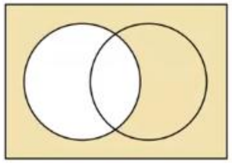


What region(s) are in the universal set U?

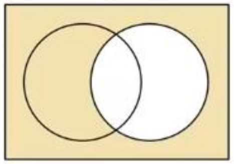
1,2,3,4

Which regions make up $(A' \cap B')$?

$A' = \{3,4\}$



$B' = \{2,4\}$



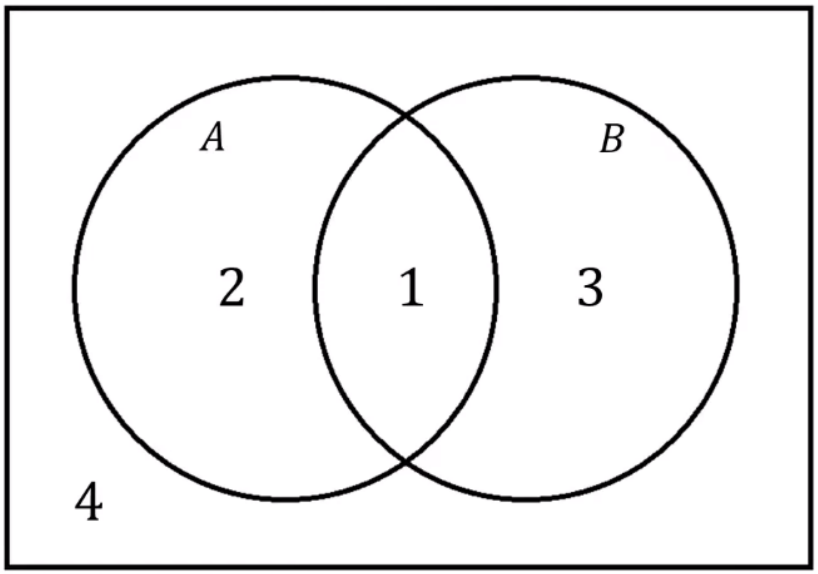
What region(s) do they share?

4

Venn Diagram Region "Map"

Question:

$U_r =$



What region(s) are in the universal set U?

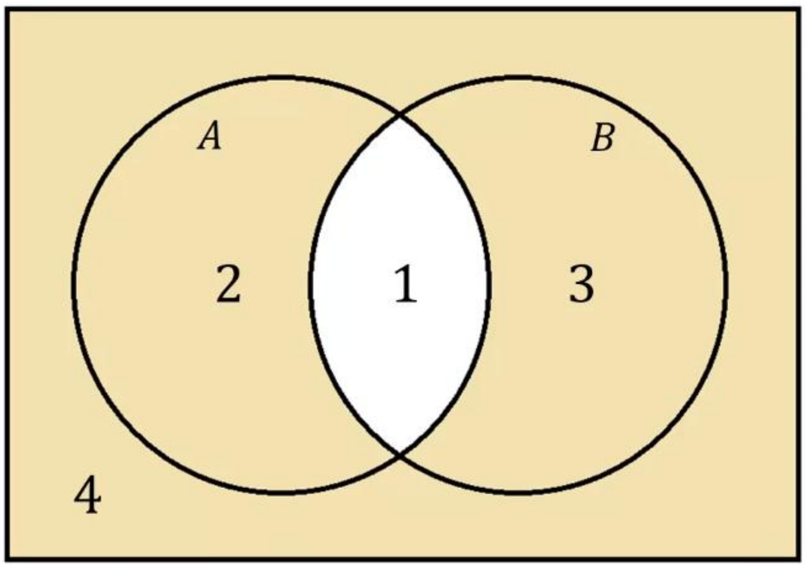
1,2,3,4

Which regions make up $(A' \cup B')$?

Venn Diagram Region "Map"

Solution:

$U_r =$

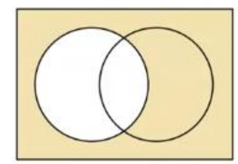


What region(s) are in the universal set U?

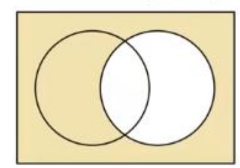
1,2,3,4

Which regions make up $(A' \cup B')$?

$A' = \{3,4\}$



$B' = \{2,4\}$



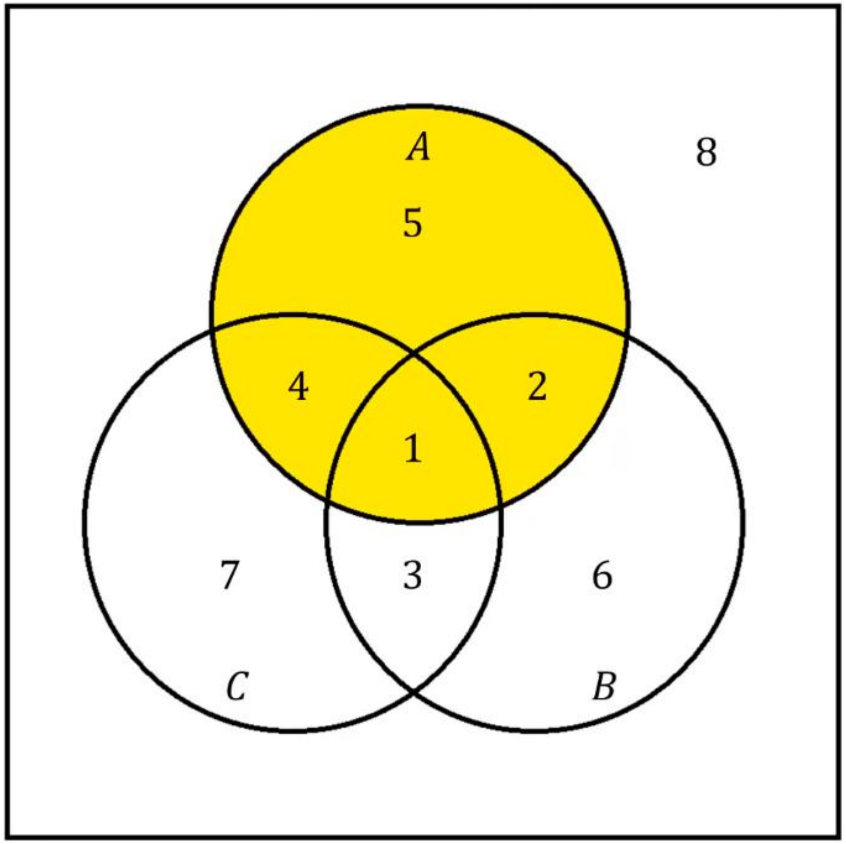
What region(s) are in either or both?

2,3,4

Venn Diagram Region "Map"

Example:

$U_r =$

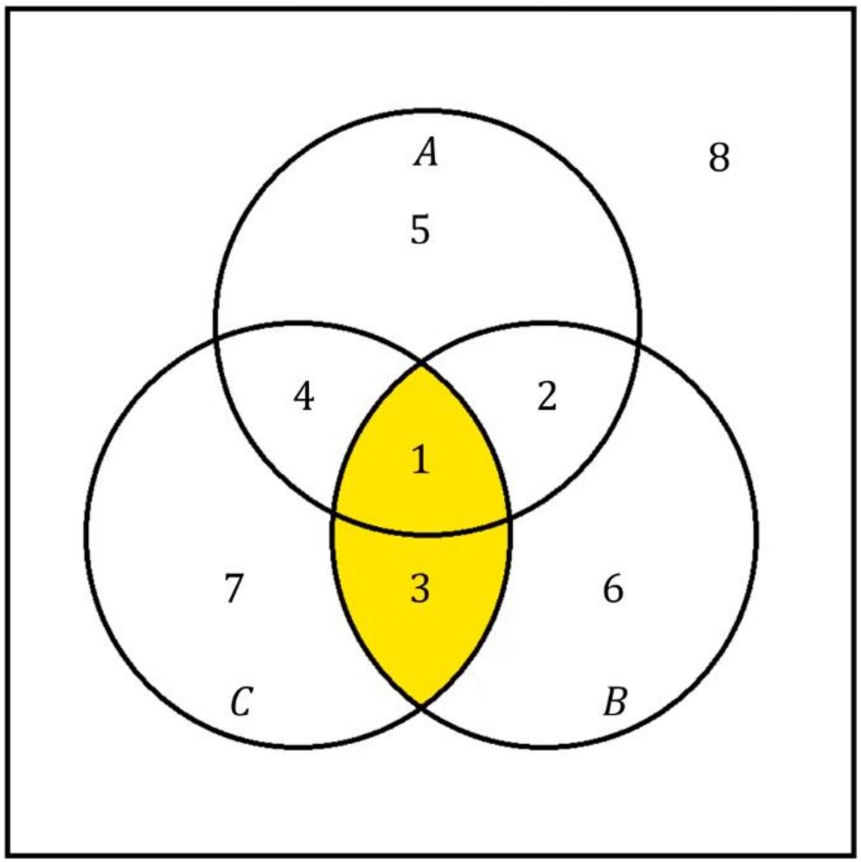


$A = \{1,2,4,5\}$

Venn Diagram Region "Map"

Example:

$U_r =$

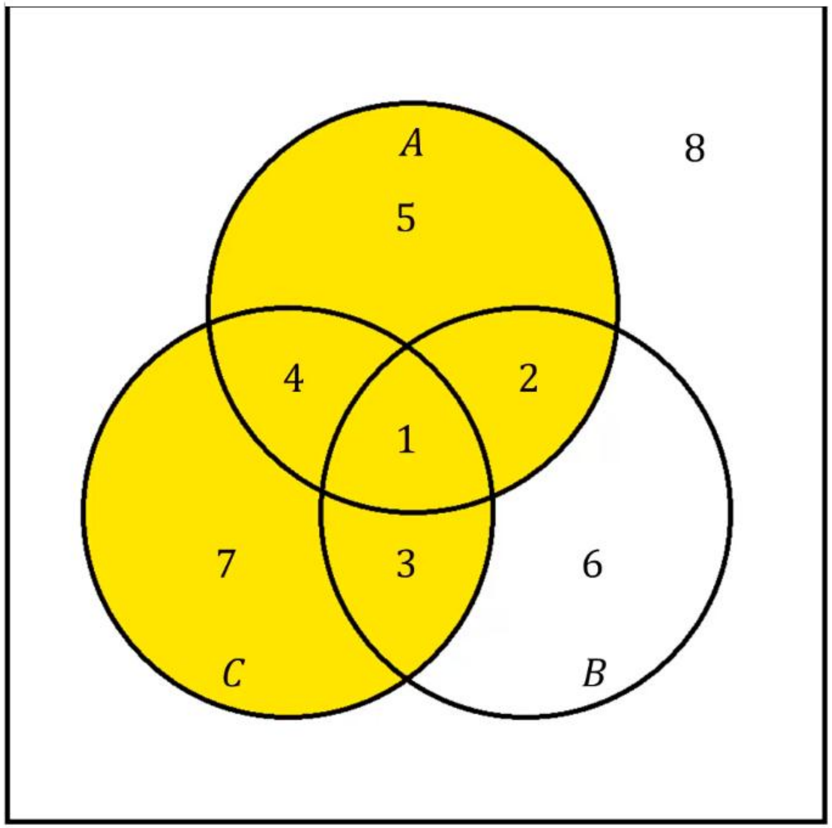


$B \cap C = \{1,3\}$

Venn Diagram Region "Map"

Example:

$U_r =$

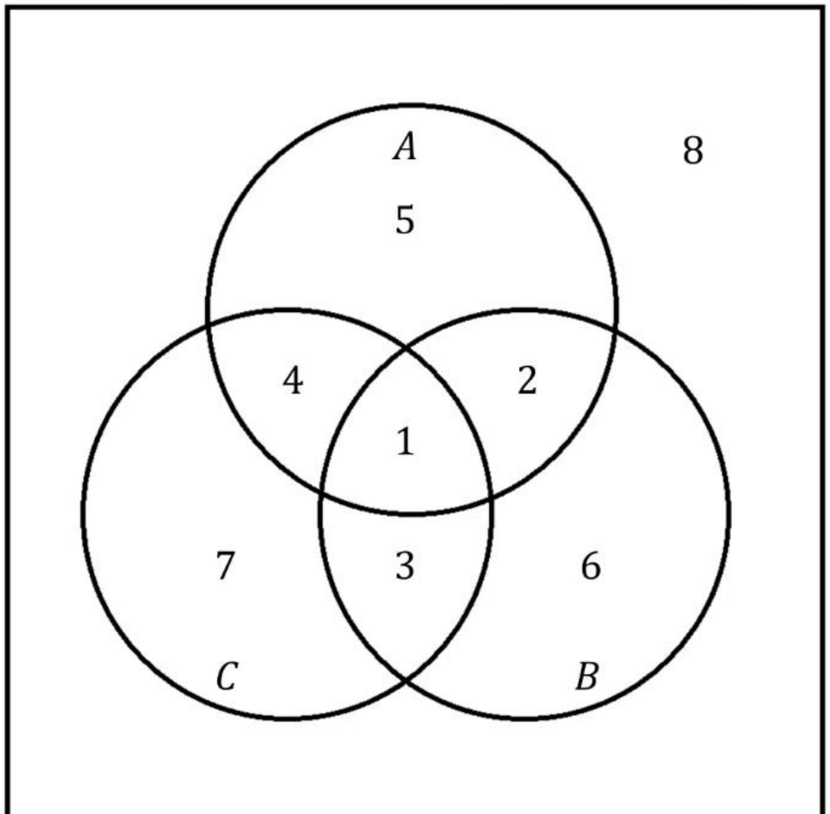


$A \cup C = \{1,2,4,5,3,7\}$

Venn Diagram Region "Map"

Question:

$U_r =$

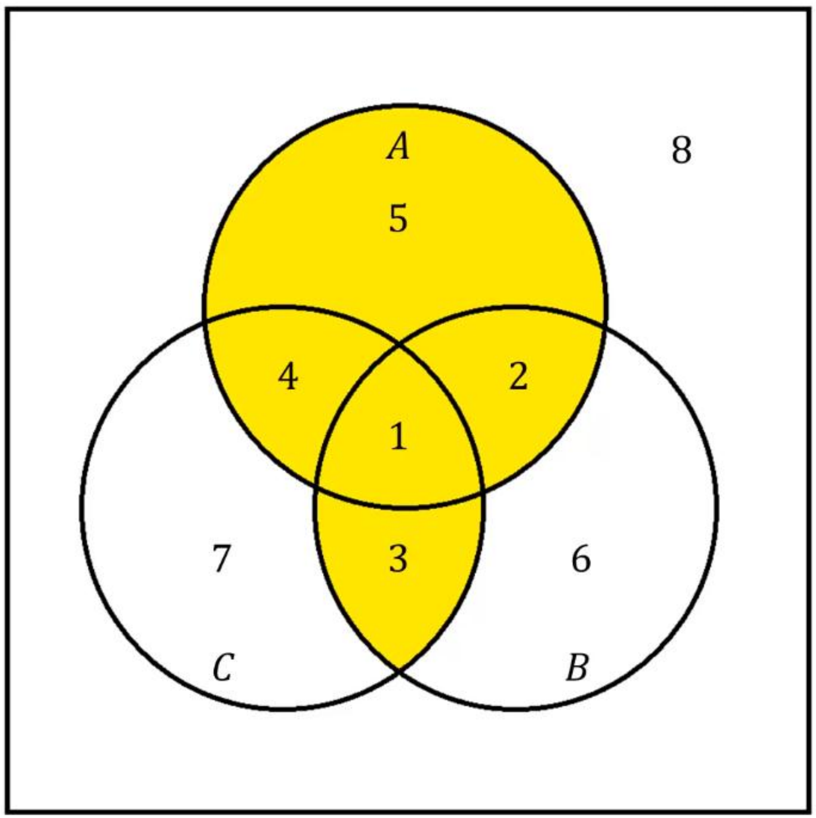


Which region(s) comprise $(A \cup (B \cap C))$?

Venn Diagram Region "Map"

Solution:

$U_r =$



Which region(s) comprise $(A \cup (B \cap C))$?

Step 1: Do the inner () first. So what is $B \cap C$?

1,3

Step 2: What regions make up A?

1,2,4,5

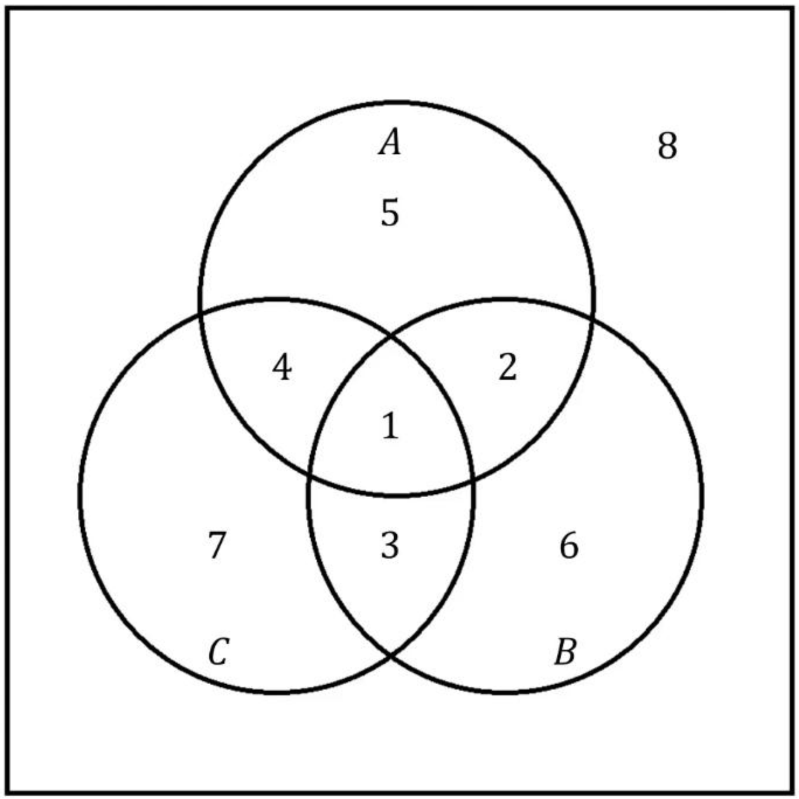
Step 3: Complete the union.

1,2,3,4,5

Venn Diagram Region "Map"

Question:

$U_r =$

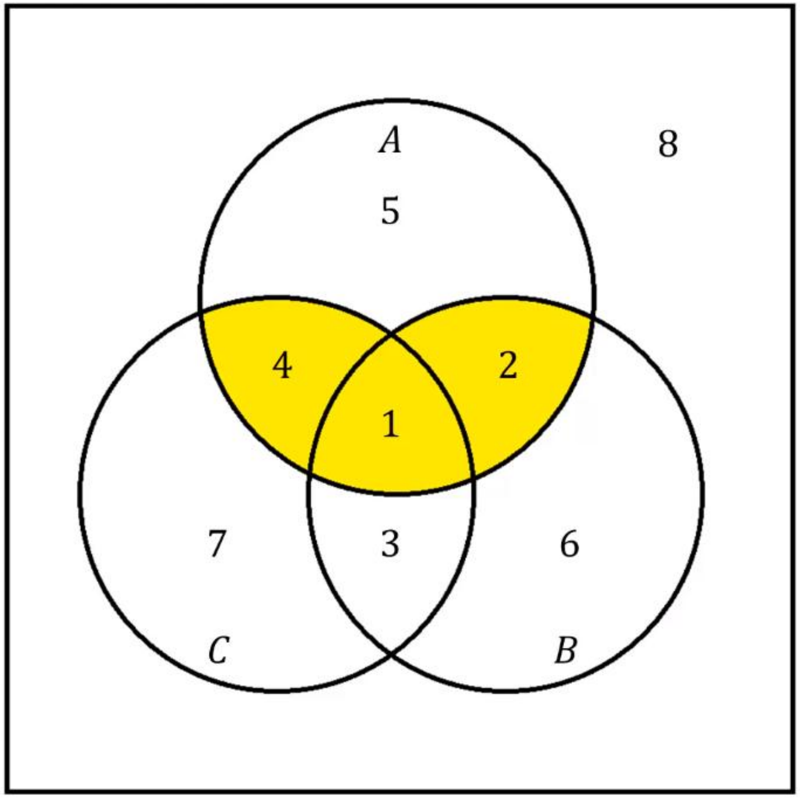


Which region(s) comprise $(A \cap (B \cup C))$?

Venn Diagram Region "Map"

Solutions:

$U_r =$



Which region(s) comprise $(A \cap (B \cup C))$?

Step 1: Do the inner () first. So what is $B \cup C$?

1,2,6,3,7,4

Step 2: What regions make up A?

1,2,4,5

Step 3: Find common regions.

1,2,4

Venn Diagram Region - Summary

- A Venn diagram can be broken down into several discrete regions.
- Each set, subset, intersection, union etc. can then be written as a list of regions.
- From those list you can then figure out intersections, unions, etc., whatever problem is asking you to find.
- Once you have your list based answer, you can then go back and shade in the appropriate regions in the Venn diagram.