ANOVA

Analysis Of Variance

Analysis of Variance

- Introduction
- Types of ANOVA

Introduction

Why ANOVA ?

- Using various tests for Hypothesis, we have been comparing two populations.
  - Independent Samples t-test (random)
  - Matched sample t-test (paired)

- However, this limit us to the comparison of two populations only.

- If you wish to compare the means of more than two populations each containing several levels or subgroups we use ANOVA

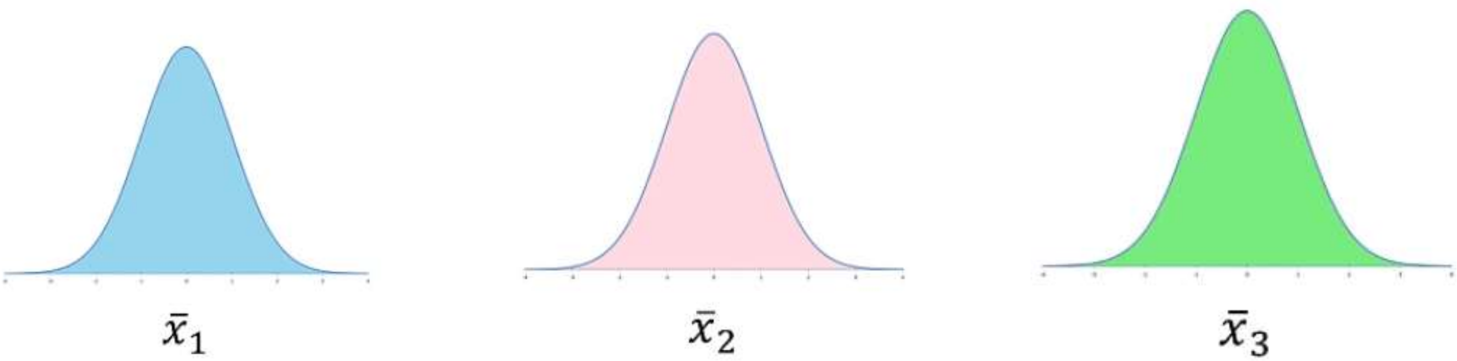
- **AN**alysis **O**f **VA**riance

Concept of ANOVA

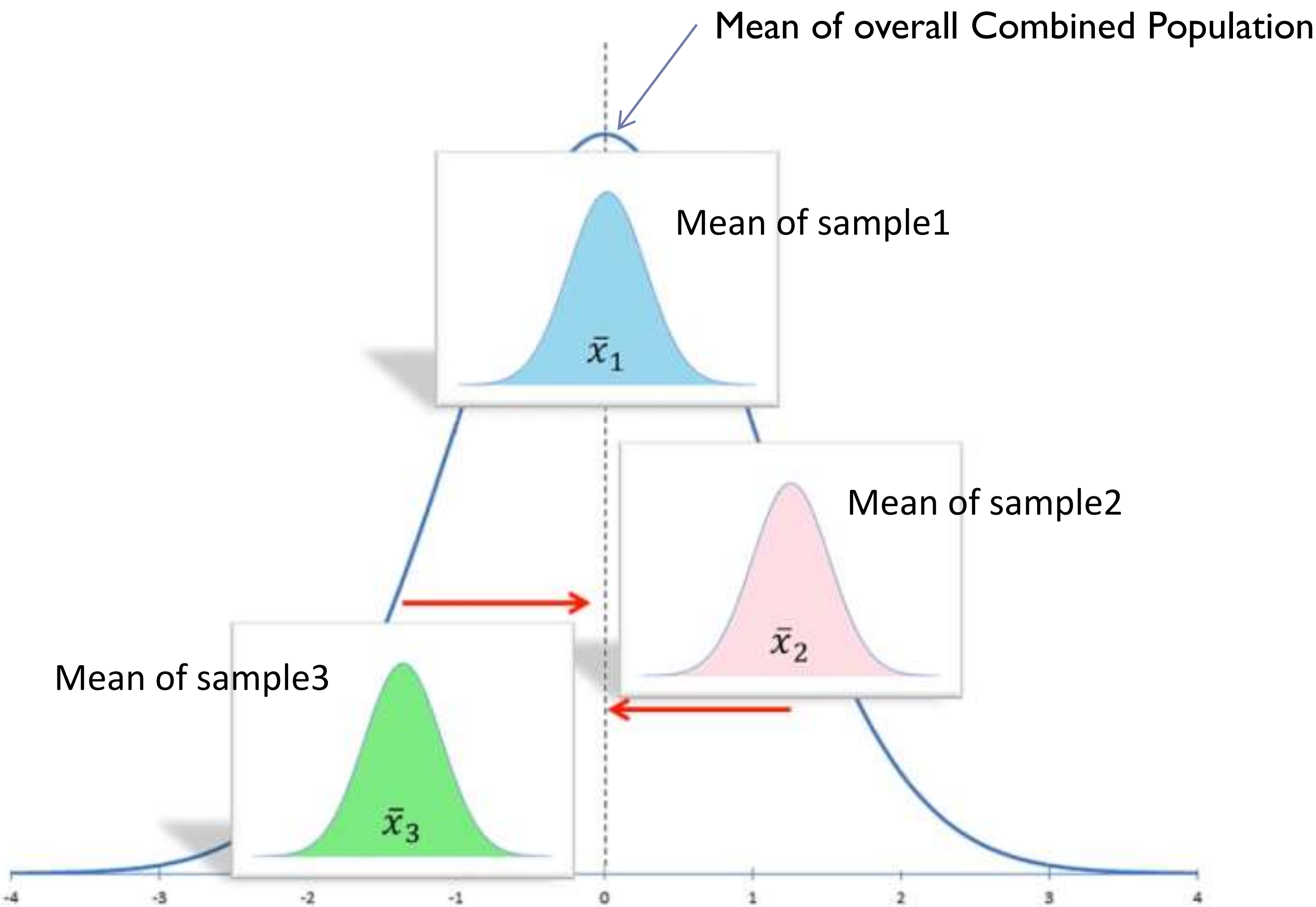
ANOVA is used when we wish to compare more than two populations/sample.

Suppose we want to compare THREE sample means to see if a difference exists among them or not.
Basically, What we are asking is:

- Do all of these means comes from a common population ?
- Or they come from different/unique populations ?

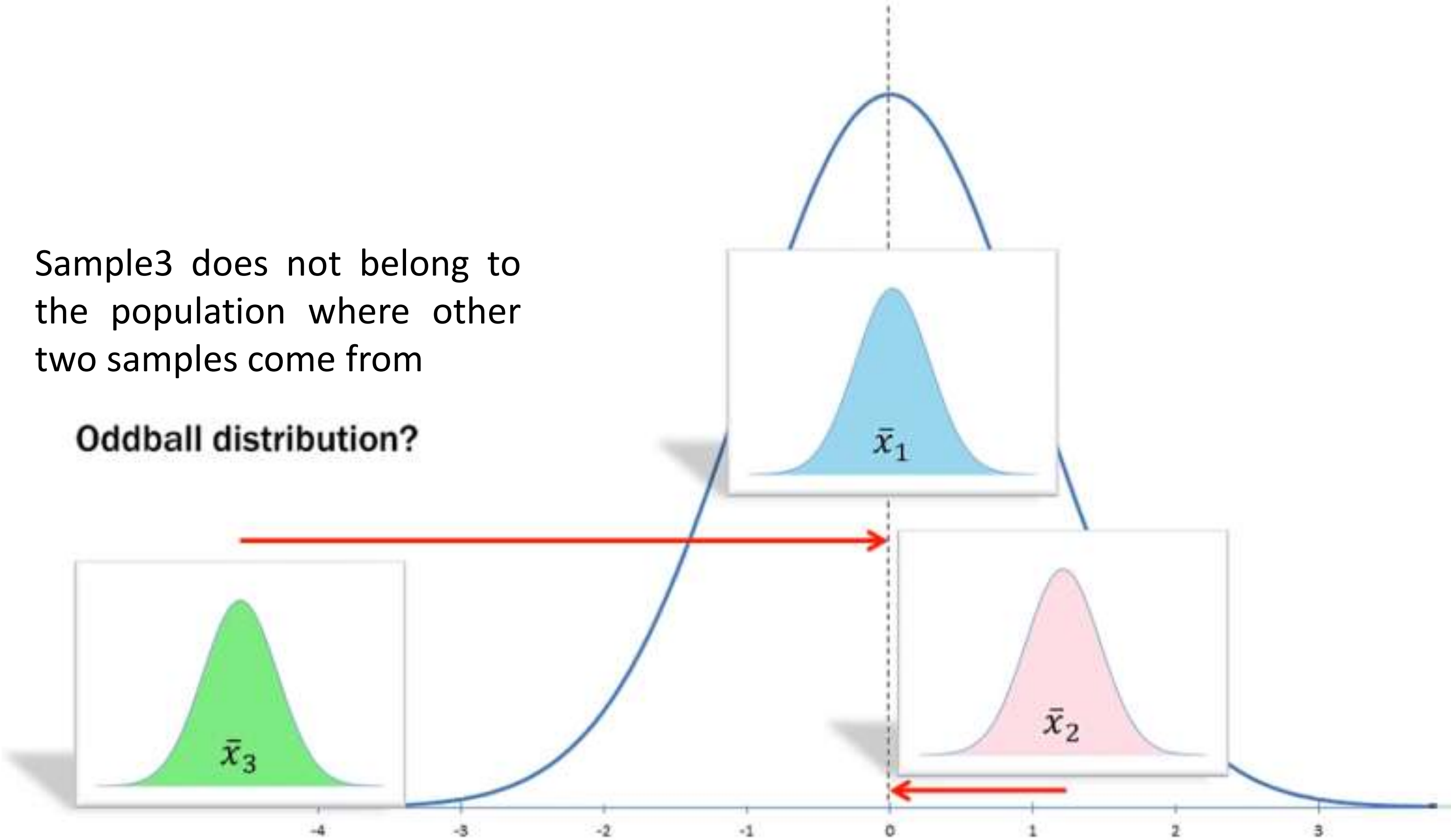


Concept of ANOVA



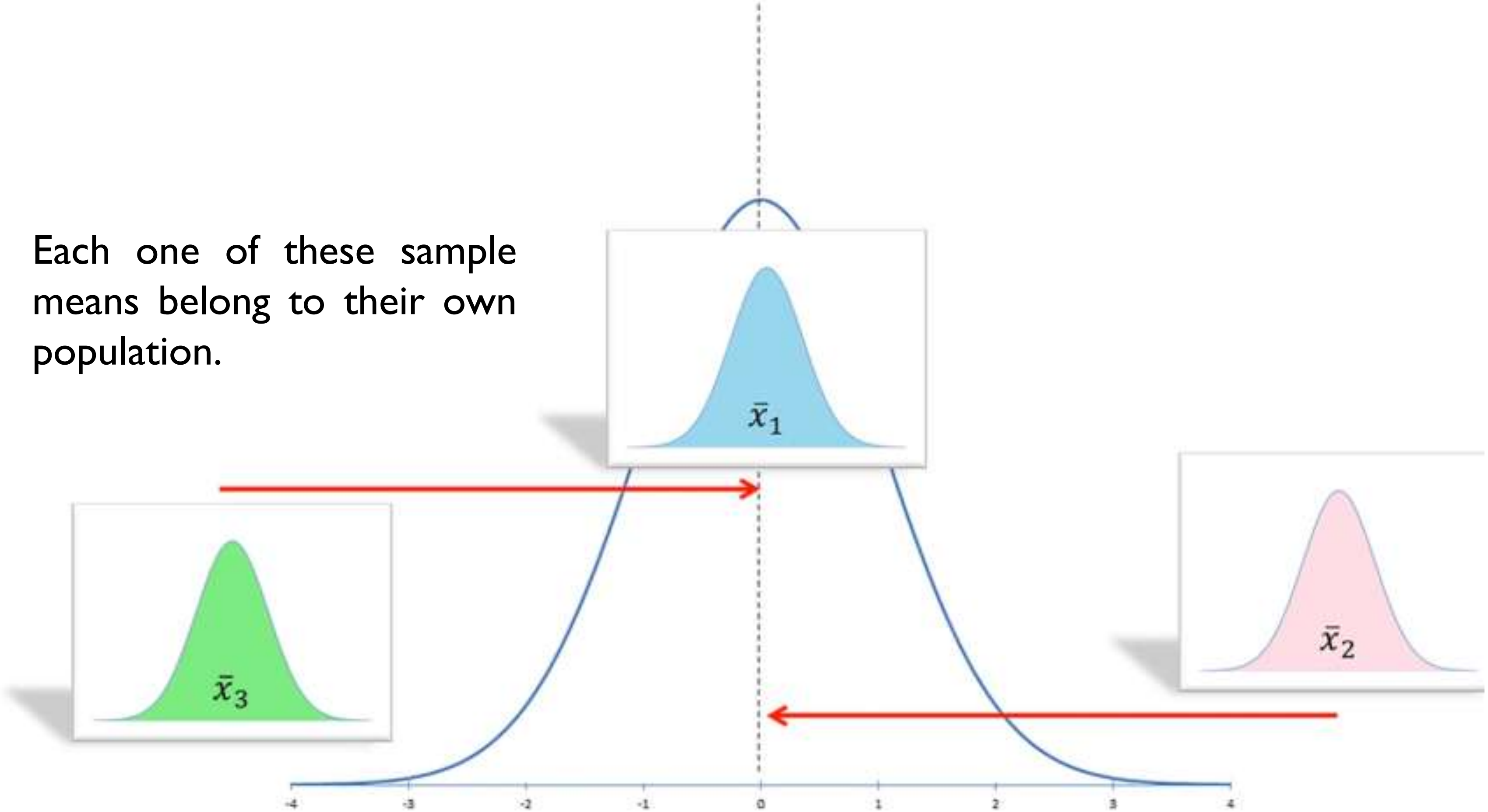
Concept of ANOVA

Sample3 does not belong to the population where other two samples come from



Concept of ANOVA

Each one of these sample means belong to their own population.

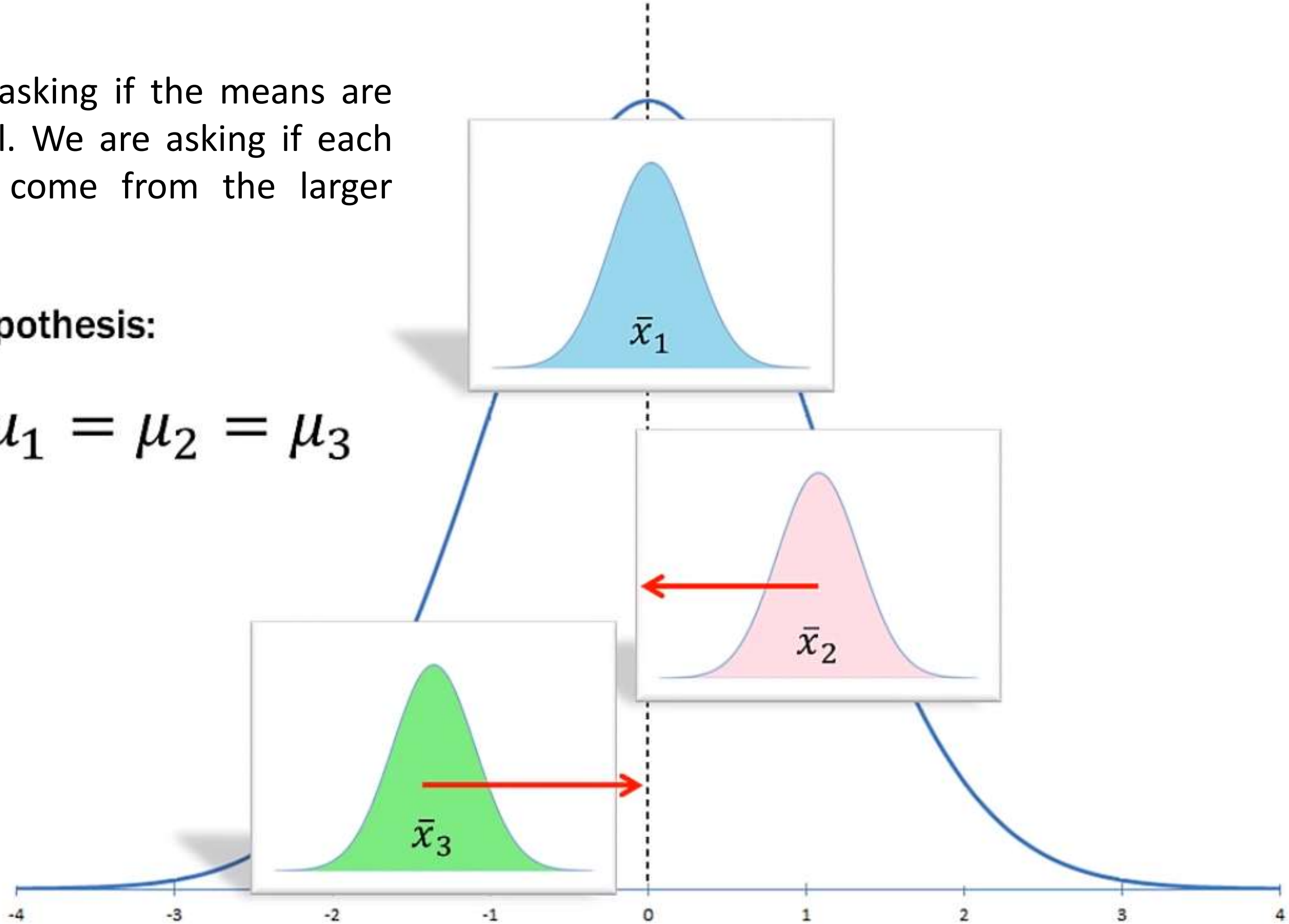


Concept of ANOVA

We are not asking if the means are exactly equal. We are asking if each mean likely come from the larger population

Null hypothesis:

$$H_0: \mu_1 = \mu_2 = \mu_3$$



Concept of ANOVA

Why are we using ANOVA and why not multiple t-tests like we did in case of single sample and two sample ?

It is simple because the error rate compounds if we use t-test for all pairs. Let's see how ?

We have three samples:

$\bar{x}_1$

$\bar{x}_2$

$\bar{x}_3$

The Pairs for t-test will be: $H_0: \bar{x}_1 = \bar{x}_2; \alpha = .05$ $H_0: \bar{x}_1 = \bar{x}_3; \alpha = .05$ $H_0: \bar{x}_2 = \bar{x}_3; \alpha = .05$

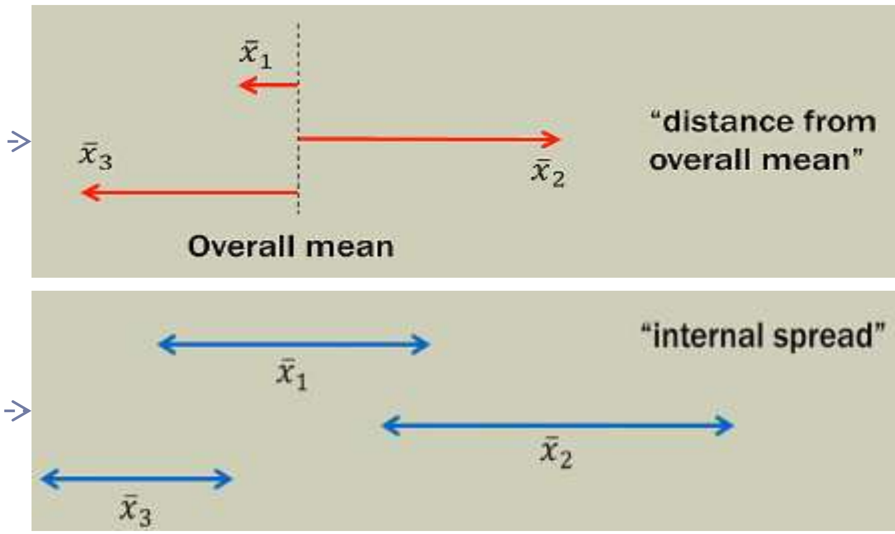
Pairwise Comparison means : **Three t-tests ALL with** α = 0.05
Alpha is Type I error rate (at 95% Confidence Interval)

The error compounds with each test: (0.95)*(0.95)*(0.95) = 0.857
α = 1 – 0.857 = 0.143

Concept of ANOVA

ANOVA is a variability ratio (F Ratio):

$$\frac{\mathit{Variability}\ \text{AMONG/BETWEEN}\ \mathit{the\ means}}{\mathit{Variability}\ \text{AROUND/WITHIN}\ \mathit{the\ distribution}} = \frac{\mathit{Variance\ Between}}{\mathit{Variance\ Within}}$$



$$Total\ Variance = Variance\ Between + Variance\ Within$$

Partitioning – Separating total variance into its component parts

If the 'Variability BETWEEN the means' is greater, numerator will be relatively larger Hence ratio will be much greater than 1.
i.e. It means, the samples most likely do not come from the common population; **REJECT NULL HYPOTHESIS.**

Concept of ANOVA

$$\frac{Variance\ Between}{Variance\ Within} = \frac{Variance\ Among}{Variance\ Around}$$

$$\frac{LARGE}{small} = Reject\ H_0$$

At least one mean is an outlier

$$\frac{similar}{similar} = Fail\ to\ Reject\ H_0$$

Means are fairly close to overall mean and distributions overlap a bit

$$\frac{small}{LARGE} = Fail\ to\ Reject\ H_0$$

Means are very close to overall mean and the distributions melt together.

Types of ANOVA

Types of ANOVA

Types of ANOVA

- One Way ANOVA *(One Factor ANOVA)*
- Two way ANOVA without Replication *(Two Factor ANOVA)*
- Two way ANOVA with Replication *(Two Factor ANOVA)*

Why ANOVA ?

- Using various tests for Hypothesis, we have been comparing two populations.
  - Independent Samples t-test (random)
  - Matched sample t-test (paired)

- However, this limit us to the comparison of two populations only.

- If you wish to compare the means of more than two populations each containing several levels or subgroups we use ANOVA

- **AN**alysis **O**f **VA**riance

Example – One way ANOVA

Twenty One students at University of Madrid in Spain were selected for a test on few common topics combined.
7 first year students, 7 second year students, 7 third year students were randomly selected.

Students undertook assessment having maximum score of 100.
We are interested in whether or not a difference exists somewhere between the three different year levels ?

Example – One way ANOVA

Single Factor
'year of student'

Columns / Groups

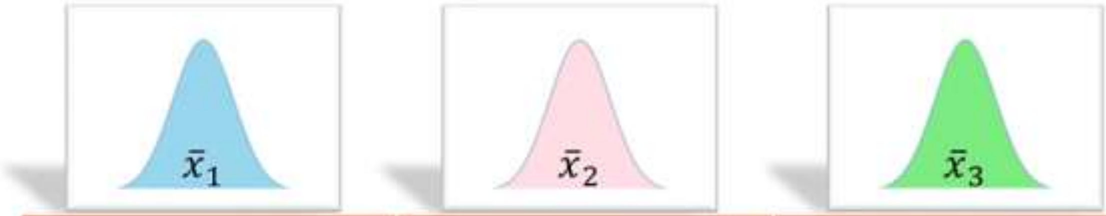
Random sample within each group.

Year 1 Scores	Year 2 Scores	Year 3 Scores
82	71	64
93	62	73
61	85	87
74	94	91
69	78	56
70	66	78
53	71	87

Also known as the "Completely Randomized Design"

Example – One way ANOVA

Step 1: Calculate Mean of each column
Step 2: Calculate Overall Mean



Year 1 Scores	Year 2 Scores	Year 3 Scores
82	71	64
93	62	73
61	85	87
74	94	91
69	78	56
70	66	78
53	71	87
$\bar{x}_1 = 71.71$	$\bar{x}_2 = 75.29$	$\bar{x}_3 = 76.57$

Overall Mean:

The mean of all 21 scores taken together.

$$\bar{\bar{x}} = 74.52$$

Example – One way ANOVA

Step 3: Calculate Sum of Squares (SST, SSC, SSE)

$$SS = \sum (x - \mu)^2$$

SST = SSC + SSE

Where

SST = Sum of square Totals or Total Sum of Squares, which is

Sum of square of (Each item in all samples – Overall Mean)

SSC = Sum of Square of Columns, which is

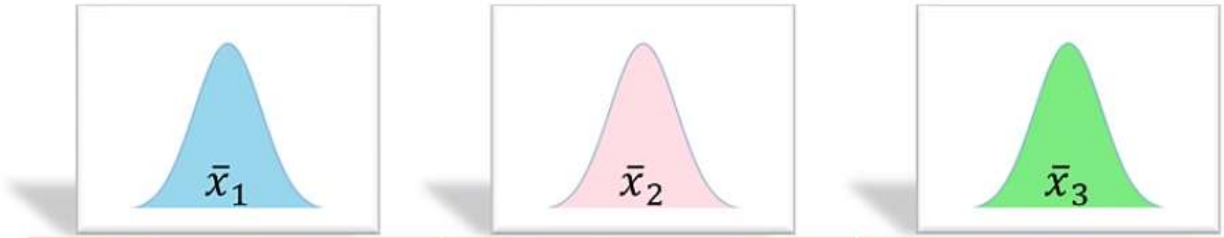
Sum of square of (Each Group Mean – Overall Mean)

SSE = Sum of Square or Sum of Square of Errors, which is

Sum of square if (Each item in a group – Mean of that group)

Example – One way ANOVA

Step 3: Calculate Sum of Squares (SST, SSC, SSE)



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$\bar{x}_1 = 71.71$	$\bar{x}_2 = 75.29$	$\bar{x}_3 = 76.57$

SST

(total / overall) sum of squares

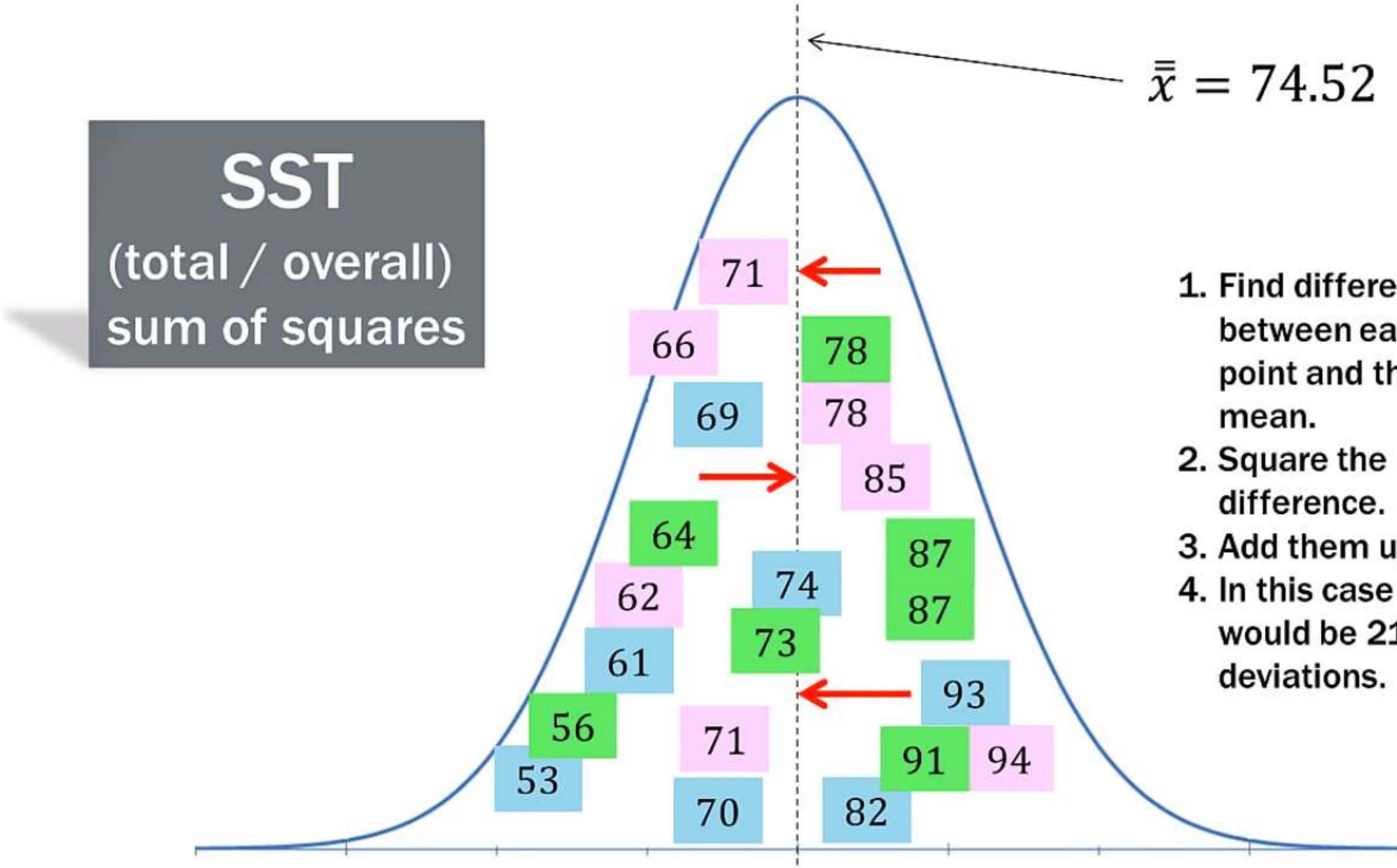
1. Find difference between each data point and the overall mean.
2. Square the difference.
3. Add them up

$$\bar{\bar{x}} = 74.52$$

SST = 2901.238

Example – One way ANOVA

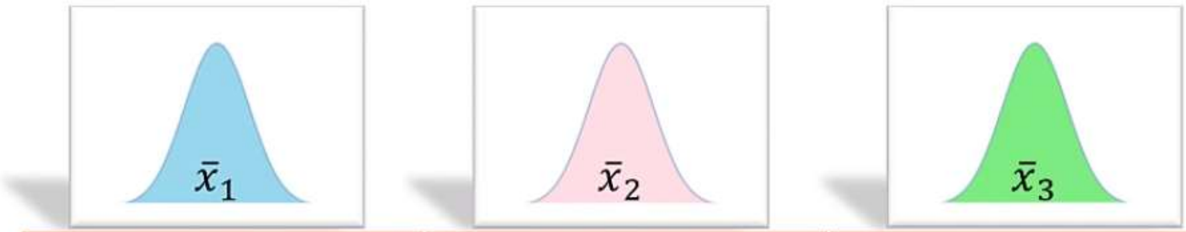
Step 3: Calculate Sum of Squares (SST, SSC, SSE)



1. Find difference between each data point and the overall mean.
2. Square the difference.
3. Add them up.
4. In this case there would be 21 squared deviations.

Example – One way ANOVA

Step 3: Calculate Sum of Squares (SST, SSC, SSE)



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70	66	78
53	71	87
$\bar{x}_1 = 71.71$	$\bar{x}_2 = 75.29$	$\bar{x}_3 = 76.57$

SSC
(column/ between) sum of squares

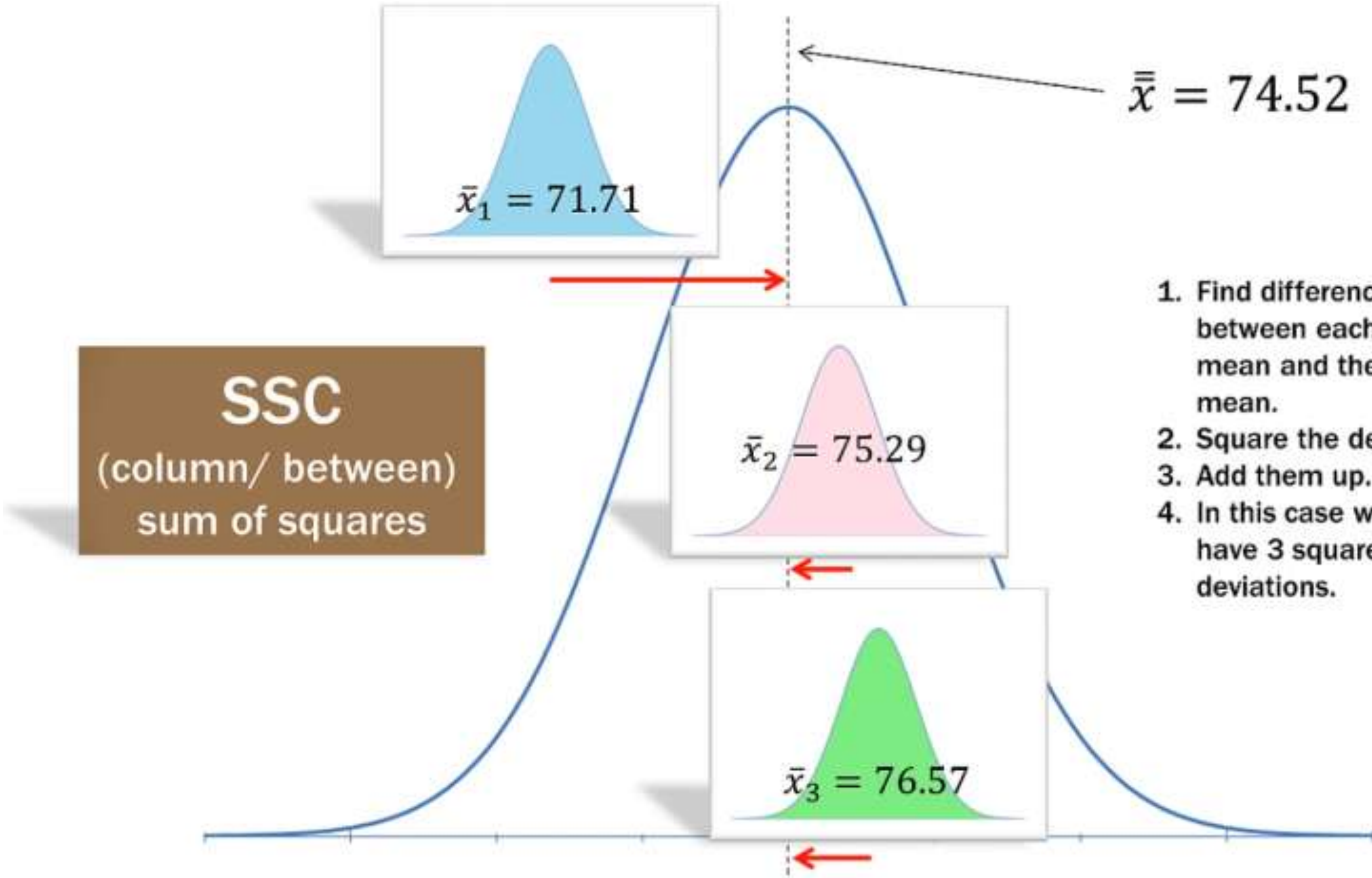
SSC = 88.66

$$\bar{\bar{x}} = 74.52$$

1. Find difference between each group mean and the overall mean.
2. Square the deviations.
3. Add them up.
4. In this case we would have 3 squared deviation

Example – One way ANOVA

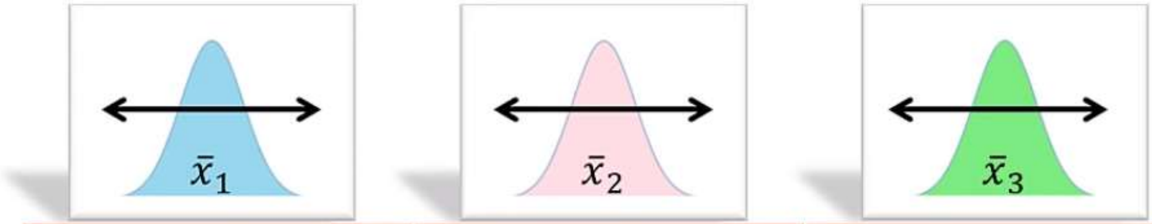
Step 3: Calculate Sum of Squares (SST, SSC, SSE)



1. Find difference between each sample mean and the overall mean.
2. Square the deviations.
3. Add them up.
4. In this case we would have 3 squared deviations.

Example – One way ANOVA

Step 3: Calculate Sum of Squares (SST, SSC, SSE)



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53	71	87
$\bar{x}_1 = 71.71$	$\bar{x}_2 = 75.29$	$\bar{x}_3 = 76.57$

SSE
(within / error)
sum of squares

SSE = 2812.571

$\cancel{\bar{\bar{x}} = 74.52}$

1. Find difference between each data point and its **column** mean.
2. Square each deviation.
3. Add them up the squared deviations.
4. In this case we would have 21 squared deviations

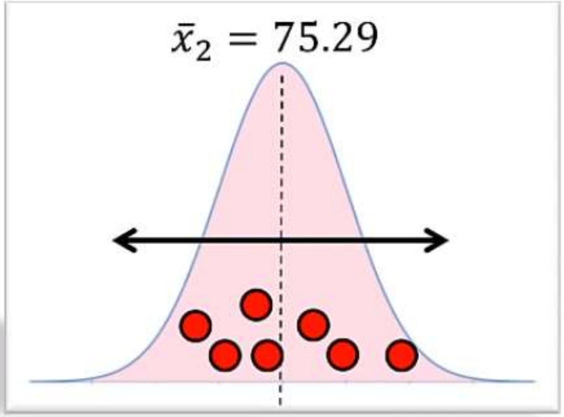
Example – One way ANOVA

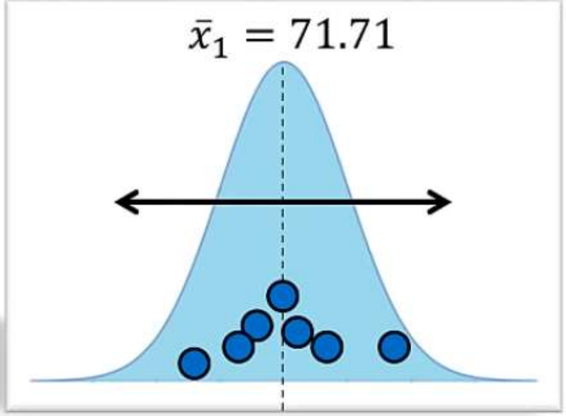
Step 3: Calculate Sum of Squares (SST, SSC, SSE)

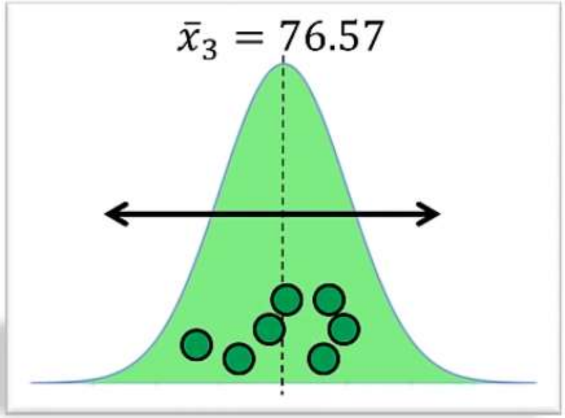
SSE

(within / error) sum of squares

1. Find difference between each data point and its **column** mean.
2. Square each deviation.
3. Add them up the squared deviations.
4. In this case we would have **21** squared deviations.





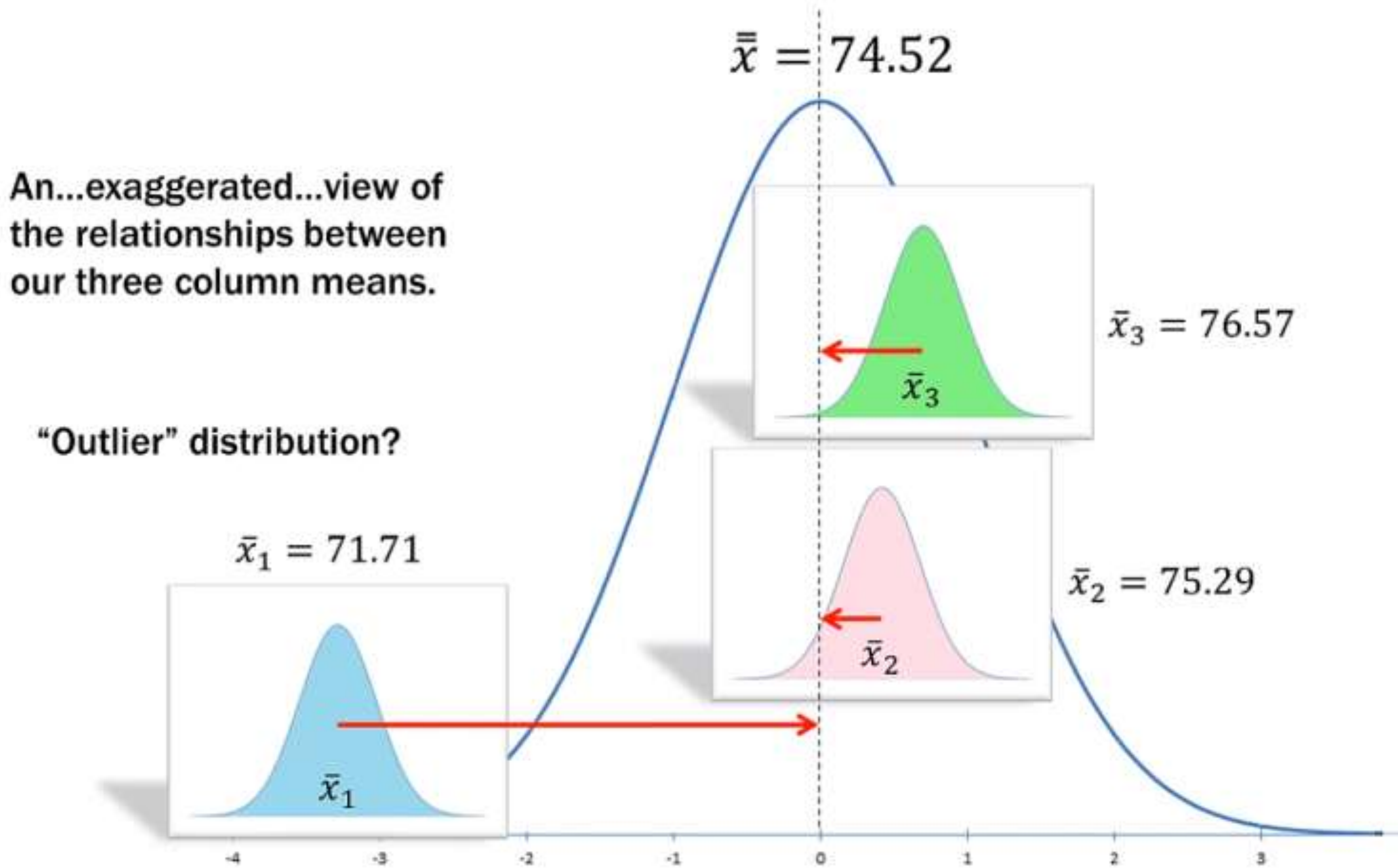


Example – One way ANOVA

Step 3: Calculate Sum of Squares (SST, SSC, SSE)

Example – One way ANOVA

Step 3: Calculate Sum of Squares (SST, SSC, SSE)



Example – One way ANOVA

Step 4: Calculate Degree of Freedom (df), MSC and MSE

SSC $\qquad df\,columns = C - 1$ $\qquad MSC = \frac{SSC}{df_{columns}}$

SSE $\qquad df_{error} = N - C$ $\qquad MSE = \frac{SSE}{df_{error}}$

SST $\qquad df_{total} = N - 1$ $\qquad F = \frac{MSC}{MSE}$

N = total number of observations

C = Number of columns/treatments

Example – One way ANOVA

Step 4: Calculate Degree of Freedom (df), MSC and MSE

SSC $\quad dfcolumns = 3 - 1 = 2 \quad MSC = \frac{SSC}{df_{columns}} = \frac{88.66}{2} =$

44.33

SSE $\quad dferror = 21 - 3 = 18 \quad MSE = \frac{SSE}{df_{error}} = \frac{2812.571}{18} =$

156.254

SST $\quad df_{total} = 21 - 1 = 20 \quad F = \frac{MSC}{MSE}$

MSC = Mean Square Columns/treatments
MSE = Mean Square Error

Example – One way ANOVA

Step 5: Calculate F Ratio

$SSC \qquad dfcolumns = 3 - 1 = 2 \qquad MSC = \frac{SSC}{df_{columns}} = \frac{88.66}{2} = 44.33$

$SSE \qquad dferror = 21 - 3 = 18 \qquad MSE = \frac{SSE}{df_{error}} = \frac{2812.571}{18} = 156.254$

$SST \qquad df_{total} = 21 - 1 = 20 \qquad F = \frac{MSC}{MSE} = \frac{44.33}{156.254} = 0.2837$

MSC = Mean Square Columns/treatments
MSE = Mean Square Error

Example – One way ANOVA

Step 6: Calculate F Critical Value

$SSC \qquad dfcolumns = 3 - 1 = 2 \qquad MSC = \frac{SSC}{df_{columns}} = \frac{88.66}{2} = 44.33$

$SSE \qquad dferror = 21 - 3 = 18 \qquad MSE = \frac{SSE}{df_{error}} = \frac{2812.571}{18} = 156.254$

$SST \qquad df_{total} = 21 - 1 = 20 \qquad F = \frac{MSC}{MSE} = \frac{44.33}{156.254} = 0.2837$

Look up F statistic distribution table for alpha = 0.05 and degree of freedom of numerator (SSC) = 2 and degree of freedom for denominator (SSE) = 18

Example – One way ANOVA

Step 6: Calculate F Critical Value

Look up F statistic distribution table for alpha = 0.05 and degree of freedom of numerator (SSC) = 2 and degree of freedom for denominator (SSE) = 18. Refer F statistics table.

14	4.6001	3.7389	3.3439	3.1122
15	4.5431	3.6823	3.2874	3.0556
16	4.4940	3.6337	3.2389	3.0069
17	4.4513	3.5915	3.1968	2.9647
18	4.4139	3.5546	3.1599	2.9277
19	4.3807	3.5219	3.1274	2.8951
20	4.3512	3.4928	3.0984	2.8661

Example – One way ANOVA

Step 7: Inference

F Ratio = 0.2837
While the F critical value for alpha = 0.05 is
F critical = 3.5546

Is our F statistic (i.e. F Ratio) value larger or beyond F critical ?
No. Hence our NULL Hypothesis holds.
i.e. We fail to reject the H_0

Hence, there is no significant difference in mean test score by Year of Student.

Why ANOVA ?

- Using various tests for Hypothesis, we have been comparing two populations.
  - Independent Samples t-test (random)
  - Matched sample t-test (paired)

- However, this limit us to the comparison of two populations only.

- If you wish to compare the means of more than two populations each containing several levels or subgroups we use ANOVA

- **AN**alysis **O**f **VA**riance

Two-Way ANOVA 'BLOCK' Design

- In one-way ANOVA we selected random sample for each column/treatment group

- Two-way ANOVA allows us to 'account for variation' at the ROW level due to some other factor or grouping.
- i.e. in two-way ANOVA we add another dimension, the row dimension based on certain criteria.

- Here in two-way ANOVA, we attempt to minimize the ERROR variance by saying that some of the ERROR variance is actually due to the variance in the ROWS.

- So here, we now have 4 types of Sum of Squares (Sources of variance):
- **Total Variance = SSC + SSE + SSB (Sum of Square of Rows/Blocks)**

Example – Two way ANOVA – Without Rep

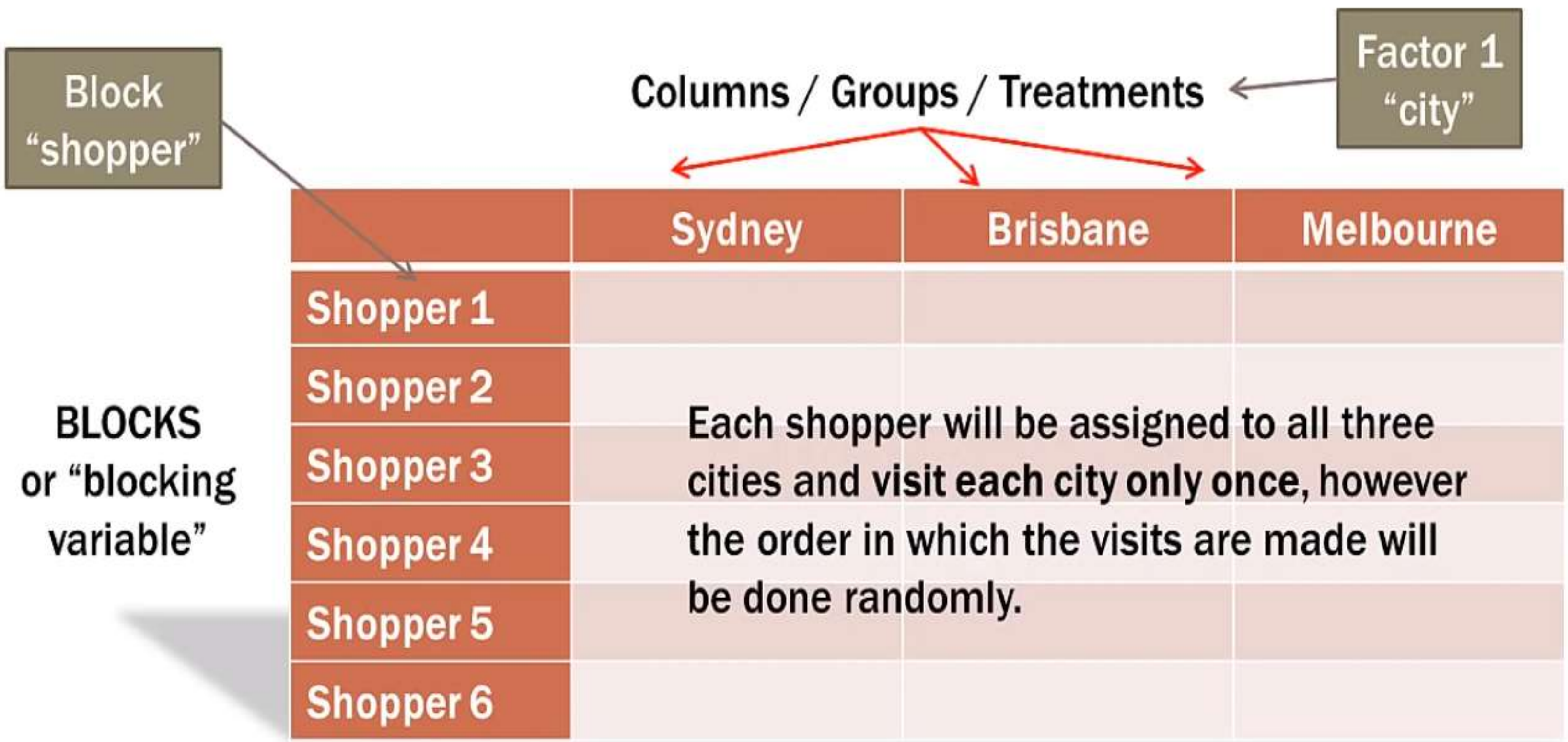
Starbucks under the pressure of Quality Control, sends out 6 Shopper inspectors as regular customers to the Australian cities of Sydney, Brisbane and Melbourne.

These 6 inspectors will visit the same stores in each 3 cities in a random manner. They will do the survey and check how well the store is managed, how good is the service and quality of products etc.

Here, Starbucks Management like to know If a difference in the Inspector ratings exists among the cities. Are they all same ? Is one significantly higher than the other two ?
Are all three different from each other ?

Note: What makes this problem good fit for Two-way ANOVA is that the Inspectors will have their own natural variation.

Example – Two way ANOVA – Without Rep



Example – Two way ANOVA – Without Rep

Step 1: Find Column means, Row Means
Step 2: Find the Overall Mean

Block means

SHOPPER VARIATION

	Sydney	Brisbane	Melbourne	
Shopper 1	75	75	90	$\bar{x}_{R1} = 80$
Shopper 2	70	70	70	$\bar{x}_{R2} = 70$
Shopper 3	50	55	75	$\bar{x}_{R3} = 60$
Shopper 4	65	60	85	$\bar{x}_{R4} = 70$
Shopper 5	80	65	80	$\bar{x}_{R5} = 75$
Shopper 6	65	65	65	$\bar{x}_{R6} = 65$
	$\bar{x}_{C1} = 67.5$	$\bar{x}_{C2} = 65$	$\bar{x}_{C3} = 77.5$	$\bar{\bar{x}} = 70$

CITY VARIATION

Example – Two way ANOVA – Without Rep

Step 3: Calculate Sum of Squares (SST, SSC, SSE, SSB)

$$SS = \sum (x - \mu)^2$$

SST = SSC + SSE + SSB

Where

SST = Sum of square Totals or Total Sum of Squares, which is

Sum of square of (Each item in all samples – Overall Mean)

SSC = Sum of Square of Columns, which is

Sum of square of (Each Group Mean – Overall Mean)

SSE = Sum of Square or Sum of Square of Errors, which is

Sum of square if (Each item in a group – Mean of that group)

SSB = Sum of Square Errors of Blocks, which is

Sum of square of (Each average in block – Overall Mean)

Example – Two way ANOVA – Without Rep

Step 3: Calculate Sum of Squares (SST, SSC, SSE, SSB)

	Sydney	Brisbane	Melbourne
Shopper 1	75	75	90
Shopper 2	70	70	70
Shopper 3	50	55	75
Shopper 4	65	60	85
Shopper 5	80	65	80
Shopper 6	65	65	65
	$\bar{x}_{C1} = 67.5$	$\bar{x}_{C2} = 65$	$\bar{x}_{C3} = 77.5$

SST
(total / overall)
sum of squares

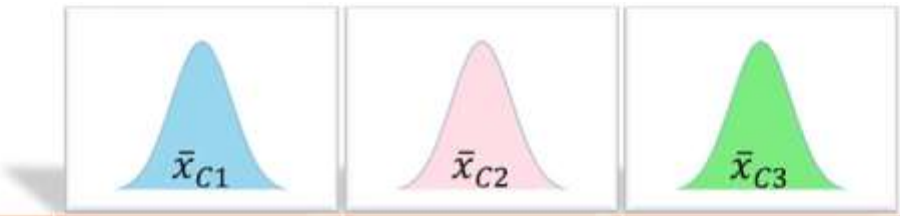
1. Find difference between each data point and the overall mean.
2. Square the difference.
3. Add them up

$$\bar{\bar{x}} = 70$$

SST = 1750

Example – Two way ANOVA – Without Rep

Step 3: Calculate Sum of Squares (SST, SSC, SSE, SSB)



	Sydney	Brisbane	Melbourne
Shopper 1	75	75	90
Shopper 2	70	70	70
Shopper 3	50	55	75
Shopper 4	65	60	85
Shopper 5	80	65	80
Shopper 6	65	65	65
	$\bar{x}_{C1} = 67.5$	$\bar{x}_{C2} = 65$	$\bar{x}_{C3} = 77.5$

SSC
(column/ between) sum of squares

SSC = SSC * No of blocks
SSC = 87.5 * 6 = **525**

$$\bar{\bar{x}} = 70$$

1. Find difference between each group mean and the overall mean.
2. Square the deviations.
3. Add them up.
4. In this case we would have 3 squared deviation

Example – Two way ANOVA – Without Rep

Step 3: Calculate Sum of Squares (SST, SSC, SSE, SSB)

	Sydney	Brisbane	Melbourne	
Shopper 1	82	71	64	$\bar{x}_{R1} = 80$
Shopper 2	93	62	73	$\bar{x}_{R2} = 70$
Shopper 3	61	85	87	$\bar{x}_{R3} = 60$
Shopper 4	74	94	91	$\bar{x}_{R4} = 70$
Shopper 5	69	78	56	$\bar{x}_{R5} = 75$
Shopper 6	70	66	78	$\bar{x}_{R6} = 65$
	$\bar{x}_{C1} = 67.5$	$\bar{x}_{C2} = 65$	$\bar{x}_{C3} = 77.5$	$\bar{\bar{x}} = 70$

SSB

block sum of squares

1. Find difference between each row/block mean and the overall mean.
2. Square each deviation.
3. Add them up the squared deviations.
4. In this case we would have 6 squared deviations.

SSB = SSB * No of Columns/Groups = 250 * 3 = **750**

Example – Two way ANOVA – Without Rep

Step 3: Calculate Sum of Squares (SST, SSC, SSE, SSB)

$$SS = \sum (x - \mu)^2$$

SST = SSC + SSE + SSB

Hence

SSB = SST – SSC – SSE

SSB = 1750 – 525 – 750 = 475

SSB = 475

Example – Two way ANOVA – Without Rep

Step 4: Calculate Degree of Freedom (df), MSC, MSB and MSE

SSC $\qquad df columns = C - 1$ $\qquad MSC = \frac{SSC}{df_{columns}}$

SSB $\qquad df_{blocks} = B - 1$ $\qquad MSB = \frac{SSB}{df_{blocks}}$

SSE $\qquad df error = (C - 1)(B - 1)$ $\qquad MSE = \frac{SSE}{df_{error}}$

SST $\qquad df_{total} = N - 1$ $\qquad MST = \frac{SST}{df_{total}}$

N = total number of observations
C = Number of columns/treatments

Example – Two way ANOVA – Without Rep

Step 4: Calculate Degree of Freedom (df), MSC, MSB and MSE

$SSC \qquad df columns = 3 - 1 = 2 \qquad MSC = \frac{525}{2}$=262.5

$SSB \qquad df_{blocks} = 6 - 1 = 5 \qquad MSB = \frac{750}{5} = 150$

$SSE \qquad df_{error} = (3-1)(6-1) = 10 \qquad MSE = \frac{475}{10}$= 47.5

$SST \qquad df_{total} = 18 - 1 = 17 \qquad MST = \frac{1750}{17} = 102.941$

N = total number of observations
C = Number of columns/treatments

Example – Two way ANOVA – Without Rep

Step 5: Calculate F Ratios

$$F = \frac{MSC}{MSE} = \frac{262.5}{47.5} = 5.526$$

$$F = \frac{MSB}{MSE} = \frac{150}{47.5} = 3.158$$

Example – Two way ANOVA – Without Rep

Step 6: Calculate F Critical Values

$$F = \frac{MSC}{MSE} = \frac{262.5}{47.5} = 5.526$$

$$F_{critical} = \mathrm{F}_{\alpha=0.05,2,10} = 4.1028$$

Is our F statistic larger than $F_{critical}$?

Yes. Reject the H_0

Significant difference in Mean quality score by city.

$$F = \frac{MSB}{MSE} = \frac{150}{47.5} = 3.158$$

$$F_{critical} = \mathrm{F}_{\alpha=0.05,5,10} = 3.3258$$

So there is some difference in the city scores even accounting for the variation in the shopper.