

[Probability Problem Exercises] (YT - Feel Free to Learn)

① Formulae:

$$\text{Probability} = \frac{\text{Sum of observation}}{\text{Total possibility}}$$

Combination formula:

"n choose r"

$${}^nC_r \text{ or } C(n, r) = \frac{n!}{(n-r)! r!}$$

② And / Or.

In probability problems:

→ Whenever you find the word 'AND' → you need to multiply it.

→ Whenever you find the word 'OR' → you need to add it.

And / OR
x +

Example:

① Suppose there are three balls ① ② and ③. If I have to choose 2 balls out of it, how many I do it?

Solⁿ:

$$\left. \begin{array}{l} 1 - 2 \\ 2 - 3 \\ 1 - 3 \end{array} \right\} 3 \text{ possible} = {}^3C_2 = \frac{3!}{(3-2)! 2!} = \frac{3!}{1! 2!} = \frac{1 \times 2 \times 3}{1 \times 1 \times 2} = \boxed{3}$$

↑
3 possible ways.

② try the same with 4 balls:

Solⁿ: 4C_2

③ Assume we have 40 balls and we want to pick 2 balls. In how many ways we can do that?

Solⁿ:

Total balls → 40
Target to pick → 2 balls

∴ ${}^{40}C_2 = \frac{40 \times 39}{1 \times 2}$

→ shortcut for calculation.
reduce 40 2 times so 40 × 39
divide by 2 factored ∴ 1 × 2

Q. Stat hand: $50C_8 \rightarrow$ so should be reduced 8 times $\therefore \frac{50 \times 49 \times \dots \times 43}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}$
 $50C_8 \rightarrow$ divide by 8 factorial

[[Exercise - 1: Coin based]]

* If one coin is tossed:

we have only 2 possible outcomes $\{H, T\} = \underline{2}$

* If two coins are tossed:

we have 4 possible outcomes $\{HH, HT, TH, TT\} = \underline{4}$

* If three coins are tossed:

we have 8 possible outcomes: $= \underline{8}$

HHH	TTT
HHT	TTH
HTH	THT
THH	HTT

* If four coins are tossed:

we have 16 possible outcomes $= \underline{16}$

HHHH	TTTT
HHHT	TTTH
HHHT	TTHT
HTHH	THTT
THTH	HTTT
HMTT	TTHHT
HTTH	HTHT
TTTH	THTH

* If five coins are tossed then outcome?

So here is the trick,
 If a coin is tossed
 we know that it
 can either be
 heads or tails

\therefore 1 coin tossed = 2^1
 2 coins tossed = 2^2
 3 coins tossed = 2^3
 4 coins tossed = 2^4
 5 coins tossed = 2^5

and so on

n coins tossed = 2^n

Q1: In a simultaneous toss of 2 coins, find the probability of 2 tails.

Solⁿ:

probability set of 2 coins = $\{HH, HT, TH, TT\} = 4$

probability = $\frac{\text{Sum of observations}}{\text{Total probability}}$

\therefore ~~Probability~~ Total probability = 4

Sum of observations = probability of 2 tails = only one probability of 2 tails

\therefore Sum of observations = 1

$$\therefore \boxed{\text{Probability} = \frac{1}{4}}$$

Q2 In a simultaneous toss of 2 coins, find the probability of exactly 1 tail.

Soln:

$$2 \text{ coin toss set} = \{ \underset{x}{HH}, \underset{v}{HT}, \underset{v}{TH}, \underset{x}{HH} \} = 4$$

$$\text{exactly one tail condition} = \underline{HT} + \underline{TH} = \underline{2}$$

$$\text{Total probability} = 4$$

$$\therefore \boxed{\text{Probability} = \frac{2}{4} = \frac{1}{2}}$$

Q3 In a simultaneous toss of 2 coins, find the probability of no tail.

Soln:

$$2 \text{ coin set} = \{ \underset{x}{HH}, \underset{x}{HT}, \underset{x}{TH}, \underset{x}{HH} \} = 4$$

$$\text{no tails} = \{ \underset{\uparrow}{HH} \} = 1$$

$$\text{Total probability of 2 coins toss} = 4$$

$$\text{Sum of observation} = 1$$

$$\therefore \boxed{\text{Probability} = \frac{1}{4}}$$

Q3 In a simultaneous toss of 3 coins, find probability of all heads.

Soln:

$$\text{Refer 3 coin toss set} = \begin{Bmatrix} HHH & TTT \\ HHT & TTH \\ HTH & THT \\ THH & HTT \end{Bmatrix}$$

$$\text{Total probability} = 8$$

$$\text{all possible heads} = \{ HHH \} = 1$$

$$\therefore \boxed{\text{Probability} = \frac{1}{8}}$$

Q4 3 coins are tossed. Find the probability of exactly 2 heads.

Soln:

$$\text{Total probability} = 8$$

$$\text{exactly 2 heads} \Rightarrow \text{Refer 3 coin toss set} = \{ HHT, HTH, THH \} = 3$$

$$\therefore \text{probability} = \frac{\text{Sum of observation}}{\text{Total probability}} = \boxed{\frac{3}{8}}$$

Q5) 3 coins are tossed. Find the probability of atleast 2 heads.

Solⁿ: Total possibility = 8

at least 2 heads = minimum 2 head = 2 heads, 3 heads are accepted

$$\therefore \text{at least 2 head} = \{HHH, HHT, HTH, THH\} = 4$$

$$\therefore \text{probability} = \frac{\text{sum of observations}}{\text{Total possibility}} = \frac{4}{8} = \boxed{\frac{1}{2}} \text{ solⁿ}$$

Q6) 3 coins are tossed. Find the probability of no heads.

Solⁿ: Total possibility for 3 coins tossed = 8

$$\text{no heads} = \{TTT\} = 1$$

$$\therefore \text{probability} = \boxed{\frac{1}{8}} \text{ solⁿ}$$

Q7) 3 coins are tossed. Find the probability of atleast 1H & 1T.

Solⁿ: Total possibility = 8 (3 coins tossed $\therefore 2^3 = 8$)

Condition:

$$\text{at least 1H \& 1T} \rightarrow \begin{array}{ll} HHH \times & TTT \times \\ \checkmark HHT & TTH \checkmark \\ \checkmark HTH & THT \checkmark \\ \checkmark THH & HTT \checkmark \end{array}$$

$$\therefore \text{Sum of observations} = 6$$

$$\therefore \text{Probability} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

Q8) 4 coins are tossed. Find the probability of exactly 3 tails.

Solⁿ: Total possibility = $2^4 = 16$

$$\begin{array}{l} \text{4 coins tossed} \left\{ \begin{array}{ll} HHHH \times & TTTT \times \\ HHH T \times & TTH T \checkmark \\ HHT H \checkmark & THT T \checkmark \\ HTT H \checkmark & HTT T \checkmark \\ THHH \times & TTT H \checkmark \\ HHTT \times & THT H \checkmark \\ HTTH \times & THT T \checkmark \\ TTHH \times & TTH T \checkmark \end{array} \right. \end{array}$$

Condition is 3 tails exactly

$$\therefore \text{Sum of observations} = 4$$

$$\therefore \text{Probability} = \frac{4}{16} = \boxed{\frac{1}{4}}$$

Q.9 4 coins are tossed. Find the probability of atleast 1 tail.

Solⁿ Total possibility = $2^4 = 16$

atleast 1 tail = { HHHH, HHHT, HHTH, HTHH, HTHT, HTTH, THTH, TTHH, TTHH, THTH, THTH, THTH, THTH, THTH, THTH, THTH }
 i.e. 1 or more tails \rightarrow TTHH, THTH, THTH, THTH, THTH, THTH, THTH, THTH
 = 15

$$\therefore \boxed{\text{probability} = \frac{15}{16}}$$

Exercise 2: Dice Based

Tips & Tricks:

* If we roll 1 die, then 6 possibilities = $6^1 = 6$
 $\{1, 2, 3, 4, 5, 6\} = 6$

* If we roll 2 dice, then 6^2 possibilities = $6^2 = 36$

{ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
 (2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
 (3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
 (4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
 (5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
 (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

Concept:

1 dice = $6^1 = 6$

2 dice = $6^2 = 36$

3 dice = 6^3

4 dice = 6^4

\vdots

n dice rolled = 6^n

Q1: In a single throw of 2 dice, find the probability of getting a total of 3 or 5.

Solⁿ Probability = $\frac{\text{sum of observation}}{\text{Total possibility}}$

\therefore for 2 dice roll, total possibility = $6^2 = 36$.

sum of observation = getting a total of 3 or 5

= { (1,2) (2,1) (1,4) (4,1) (2,3) (3,2) }

= 6

$$\therefore \boxed{\text{probability} = \frac{6}{36} = \frac{1}{6}}$$

Q2 In a single throw of 2 dice, find the probability of getting a total of 12.

Soln:

Total probability for 2 dice = $6^2 = 36$

Sum of observation = total of 12 = $\{(6, 6)\} = 1$

$$\therefore \boxed{\text{Probability} = \frac{1}{36}}$$

Q3) In a single throw of 2 dice, find the probability of getting a total of 11

Soln:

Total possibilities for 2 dice rolled = $6^2 = 36$

Sum of observations i.e. total of 11 = $\{(6, 5), (5, 6)\} = 2$

$$\therefore \boxed{\text{Probability} = \frac{2}{36} = \frac{1}{18}}$$

Q4) In a throw of 2 dice, what is the probability of a doublet? (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

Soln:

Total probability = $6^2 = 36$

doublet = $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} = 6$

$$\therefore \boxed{P = \frac{6}{36} = \frac{1}{6}}$$

Q5 In a throw of 2 dice, find the probability of getting even ~~sum~~ numbers as a sum of numbers on both die.

Soln: Total probability = $6^2 = 36$

$$\text{even sum} \Rightarrow \text{Refer 2 die set} = \left\{ \begin{array}{lll} (1, 1) & (1, 3) & (1, 5) \\ (2, 2) & (2, 4) & (2, 6) \\ (3, 1) & (3, 3) & (3, 5) \\ (4, 2) & (4, 4) & (4, 6) \\ (5, 1) & (5, 3) & (5, 5) \\ (6, 2) & (6, 4) & (6, 6) \end{array} \right\} = 18$$

$$\therefore \boxed{P = \frac{18}{36} = \frac{1}{2}}$$

Q6 In a single throw of 3 dice, find the probability of getting a total of 5.

Solⁿ 3 die roll, \therefore total possibility = $6^3 = 216$

total of 5 = $\{(1, 1, 3) (3, 1, 1) (2, 1, 2) (2, 2, 1) (1, 3, 1) (1, 2, 2)\} = 6$

$$\boxed{\text{Probability} = \frac{6}{216} = \frac{1}{36}}$$

Exercise 3 CARDS Based

- Total 52 cards in a pack
- 13 cards of diamond } red color
- 13 cards of heart } red color
- 13 cards of club } black color
- 13 cards of spade } black color
- Total 16 face cards in a pack of 52 cards
 - ↳ 4 Aces (one in each type)
 - ↳ 4 kings (one in each type)
 - ↳ 4 Queens (one in each type)
 - ↳ 4 Jacks (one in each type)

Q1: One card is drawn at random from the well shuffled pack of 52 cards. What is a probability of picking a black card?

Solⁿ

$$\text{Probability} = \frac{\text{sum of obs.}}{\text{possibility}}$$

$\boxed{\text{Possibility} = 52}$ as the cards are 52 in a pack

$$\begin{aligned} \text{sum of observation} &= 1 \text{ black colored card} \\ &= \text{total black colored cards} = 13 + 13 = 26 \end{aligned}$$

$$\therefore \text{sum of observations} = 26$$

$$\therefore \boxed{P = \frac{26}{52} = \frac{1}{2}}$$

Q2^{me} Card is drawn from a pack of 52 cards. What is the probability of picking a Ace of spades or Jack of Diamonds?

Solⁿ: Total possibilities = 52

$$\begin{aligned}\text{Sum of obs} &= \text{either Ace of spades or Jack of diamonds} \\ &= 1 (\text{ace of spades is only 1}) + 1 (\text{Jack of diamonds is only 1}) \\ &= 1 + 1 = 2\end{aligned}$$

$$\therefore \boxed{P = \frac{2}{52} = \frac{1}{26}}$$

Q3 One card drawn at random from pack of 52 cards. What is the probability that the card is either a red card or a king?

Solⁿ: Total possibility = 52

$$\begin{aligned}\text{Sum of obs} &= \text{either a red card or king} \\ &= 26 (\text{hearts \& diamonds}) + 2 (\text{as 2 already considered in hearts \& diamonds so remaining king of club \& spades}) \\ &= 26 + 2 = 28\end{aligned}$$

$$\therefore \boxed{\text{Probability} = \frac{28}{52} = \frac{7}{13}}$$

Q4 One card drawn from a pack of 52 cards. What is the probability of picking an Ace?

Solⁿ: Total possibility = 52

$$\text{Sum of obs: Ace picking} = 4 (\text{there are 4 aces in pack of 52})$$

$$\therefore \boxed{\text{probability} = \frac{4}{52} = \frac{1}{13}}$$

Q5 One card is drawn at random from a pack of 52 cards. What is the probability of picking up a club?

Soln:

Total possibilities = 52

Sum of observations = 13 (there are 13 cards of club in pack)

$$\therefore \left[P = \frac{13}{52} = \frac{1}{4} \right]$$

Q6 One card is drawn at random from a well shuffled pack of 52 cards. What is the probability of picking a red Queen?

Soln:

Total possibilities = 52

Sum of obs = picking a red Queen = 2 (one for hearts & one for diamonds)

$$\therefore \left[\text{Probability} = \frac{2}{52} = \frac{1}{26} \right]$$

Q7 One card drawn at random from a pack of 52 cards. What is the probability that the card is either a King or a Spade?

Soln:

Total possibilities = 52

Sum of obs. = a King $\underline{\text{or}}$ a Spade
 $\begin{matrix} 4 & + & 12 \\ \text{(one for each type)} & & \text{(King already included, hence 12 & \text{not } 13)} \end{matrix}$
 $= 16$

$$\therefore \left[P = \frac{16}{52} = \frac{4}{13} \right]$$

Q8 One card drawn at random from pack of 52 cards. What is the probability that the card is either a Heart or a King?

Soln:

Total possibilities = 52

Sum of obs = a Heart $\underline{\text{or}}$ a King
 $\begin{matrix} 13 & + & 3 \\ & & \text{(one King already included in hearts, hence 3 & \text{not } 4)} \end{matrix}$
 $= 16$

$$\therefore \left[P = \frac{16}{52} = \frac{4}{13} \right]$$

Exercise 4 - Color Balls - 2 balls

Q1) Bag contains 6 Red balls & 4 Yellow balls. 4 balls are picked at random. What is the probability that 3 are red and 1 is yellow?

Solⁿ:

$$\text{Probability} = \frac{\text{Sum of observations}}{\text{Total possibilities}}$$

$$\rightarrow \text{Sum of obs.} = \underset{6C_3}{3 \text{ red balls (out of 6)}} \times \underset{4C_1}{1 \text{ yellow ball (out of 4)}}$$

$$\boxed{\text{Sum of obs.} = 6C_3 \times 4C_1}$$

$$\begin{aligned} \rightarrow \text{Total possibility} &= 6 \text{ Red balls, 4 yellow balls \& 4 balls are picked} \\ &= 4 \text{ balls out of 10} \end{aligned}$$

$$\boxed{\text{Total possibilities} = 10C_4}$$

$$\therefore \text{Probability} = \frac{6C_3 \times 4C_1}{10C_4} = \frac{6 \times 5 \times 4 \times 4}{10 \times 9 \times 8 \times 7} = \frac{5 \times 4 \times 4}{10 \times 3 \times 7}$$

$$\boxed{\text{Probability} = \frac{8}{21}}$$

Q2) Bag contains 6 Red, 4 yellow balls. 3 balls are picked at random. What is probability that 1 is red?

Solⁿ:

$$\text{Probability} = \frac{6C_1 \times 4C_2}{10C_3} = \frac{6 \times 4 \times 3}{10 \times 9 \times 8}$$

$$\boxed{\text{Probability} = \frac{6 \times 2 \times 3}{10 \times 3 \times 4} = \frac{3}{10}}$$

\rightarrow Total $6+4 = 10$ balls

\rightarrow We have 3 out of 10

\rightarrow Total possibilities = $10C_3$

\rightarrow 1 is red \rightarrow 1 red char out of 6 red
 $\hookrightarrow 6C_1$

\rightarrow 1 is red & 2 are yellow of 4

$$\therefore \boxed{6C_1 \times 4C_2}$$

Q3) Bag contains 6 Red and 4 yellow balls. 2 balls are picked at random. What is the probability that either 2 are red or 2 are yellow?

Solⁿ:

$$\begin{array}{l|l} \begin{array}{c} 2 \text{ red} \\ 6C_2 \end{array} & \begin{array}{c} \text{or} \\ + \end{array} & \begin{array}{c} 2 \text{ yellow} \\ 4C_2 \end{array} & \left| \begin{array}{l} \text{Total possibilities} = 2 \text{ out of } 10 \\ = {}^{10}C_2 \end{array} \right. \end{array}$$

$$\text{Probability} = \frac{{}^6C_2 + {}^4C_2}{{}^{10}C_2} = \frac{\frac{6 \times 5}{1 \times 2} + \frac{4 \times 3}{1 \times 2}}{\frac{10 \times 9}{1 \times 2}}$$

$$\boxed{\text{Probability} = \frac{15+6}{45} = \frac{7}{15}}$$

Q4) Bag contains 6 Red & 4 yellow balls. 3 balls are picked at random. What is the probability that none is red?

Solⁿ:

non red balls out of 3 selection
 \rightarrow 3 balls selected are from yellow.
 $\&$ \rightarrow Total possibility is selecting 3 out of 10

$$\therefore \text{probability} = \frac{{}^4C_3}{{}^{10}C_3} = \frac{\frac{4 \times 3 \times 2}{1 \times 2 \times 3}}{\frac{10 \times 9 \times 8}{1 \times 2 \times 3}}$$

$$\boxed{\text{Probability} = \frac{1}{30}}$$

Q5) Bag contains 6 Red & 4 yellow balls. 4 balls are picked at random. What is the probability at least one is red?

Solⁿ:

$$\begin{array}{l|l} \text{At least : minimum} \longrightarrow \text{maximum} & \left| \begin{array}{l} \text{Total probability} = 4 \text{ out of } 10 \\ = {}^{10}C_4 \end{array} \right. \\ \rightarrow \left\{ \begin{array}{l} \min(1 \text{ red}) + 2(\text{red}) + 3(\text{red}) + 4(\text{red}) \end{array} \right\} & \end{array}$$

$$\therefore P = \frac{{}^6C_1 \times {}^4C_3 + {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_4}{{}^{10}C_4}$$

$$\boxed{\text{Probability} = \frac{207}{210}}$$

Q6 Bag contains 6 Red & 4 yellow balls. 3 balls are picked at random. What is the probability atmost 2 are red?

Solⁿ:

Atmost: maximum \rightarrow minimum.

$$\therefore \left\{ \begin{array}{l} \text{max (2 red) \& 1 yellow or 1 red \& 2 yellow or no red \& 3 yellow} \\ {}^6C_2 \times {}^4C_1 + {}^6C_1 \times {}^4C_2 + {}^4C_3 \end{array} \right\}$$

$$\text{Probability} = \frac{{}^6C_2 \times {}^4C_1 + {}^6C_1 \times {}^4C_2 + {}^4C_3}{{}^{10}C_3} = \frac{5}{6}$$

Q7 Bag contains 6 Red & 4 yellow balls. 4 balls are picked at random. What is the probability that 3 balls are red and 1 is yellow or 2 are red and two are yellow?

Solⁿ:

$$\begin{array}{c} 3 \text{ red and } 1 \text{ yellow or } 2 \text{ red and } 2 \text{ yellow} \\ \times \qquad \qquad \qquad + \qquad \qquad \qquad \times \\ {}^6C_3 \times {}^4C_1 + {}^6C_2 \times {}^4C_2 \end{array}$$

$$\therefore \text{Probability} = \frac{{}^6C_3 \times {}^4C_1 + {}^6C_2 \times {}^4C_2}{{}^{10}C_4} = \frac{17}{21}$$

Exercise 5 - 3 color balls

Q1) Bag contains 6 Red balls, 4 yellow and 2 Green balls. 4 balls are picked at random. What is the probability that two are red, one is yellow and one is green?

Solⁿ:

Total possibility = choose 4 balls out of total $6+4+2 = 12$ balls.

$$\therefore \boxed{\text{Total possibility} = {}^{12}C_4}$$

$$\begin{array}{ccccc} 2 \text{ red} & \text{and} & 1 \text{ yellow} & \text{and} & 1 \text{ green} \\ {}^6C_2 & \times & {}^4C_1 & \times & {}^2C_1 \end{array}$$

$$\boxed{\text{Probability} = \frac{{}^6C_2 \times {}^4C_1 \times {}^2C_1}{{}^{12}C_4} = \frac{8}{33}}$$

Q2) Bag contains 6 Red, 4 yellow and 2 green balls. 4 balls are picked at random. What is the probability 2 are red?

Solⁿ:

Total possibility = 4 chosen out of $6+4+2 = {}^{12}C_4$

2 are red (and) (hence remaining 2 are among yellow and green)

$$\Rightarrow {}^6C_2 \times (4 \text{ yellow } + 2 \text{ green balls}) \text{ choose 2 out of it}$$

$$\Rightarrow {}^6C_2 \times {}^6C_2$$

$$\therefore \boxed{\text{Probability} = \frac{{}^6C_2 \times {}^6C_2}{{}^{12}C_4} = \frac{5}{11}}$$

Q3) Bag contains 6 Red, 4 yellow and 2 green balls. 5 balls are picked at random. What is the probability that none is yellow.

Solⁿ: \rightarrow Total Possibility = 5 balls chosen out of 12 balls $\rightarrow {}^{12}C_5$

\rightarrow no yellow balls hence 5 balls to be chosen from ~~6 yellow~~^{red} & 2 green

\rightarrow 5 balls to be chosen from total ~~12~~⁸ balls \rightarrow ~~12~~⁸ C_5

$$\boxed{\text{Probability} = \frac{{}^8C_5}{{}^{12}C_5} = \frac{7}{99}}$$

Q4) Bag contains 6 Red, 4 yellow & 2 green balls. 2 balls are picked at random. What is the probability

(i) Either both are red or both are green

(ii) Neither red nor green:

Solⁿ: Total Possibility = 2 balls chosen out of 12 $\rightarrow {}^{12}C_2$

(i) both red or both green
 ${}^6C_2 + {}^2C_2$

$$\boxed{\text{Probability} = \frac{{}^6C_2 + {}^2C_2}{{}^{12}C_2} = \frac{8}{33}}$$

(ii) no red or no green \therefore all yellow $\rightarrow {}^4C_2$

$$\boxed{\text{Probability} = \frac{{}^4C_2}{{}^{12}C_2} = \frac{1}{11}}$$

Q5) Bag contains 6 Red, 4 yellow and 2 green balls.
3 balls are picked at random. What is the possibility that atleast one is red?

Solⁿ: Total possibilities \rightarrow 3 choose out of 12 $\rightarrow {}^{12}C_3$

Atleast : minimum \rightarrow maximum

minimum (1 red) & 2 (green + yellow) $\frac{4+2=6}{+}$ or 2 (red) & 1 (green + yellow) $\frac{+}{+}$ or 3 (red) & 0 (g + yel)

$${}^6C_1 \times {}^6C_2 + {}^6C_2 \times {}^6C_1 + {}^6C_3$$

$$\therefore \text{Probability} = \frac{({}^6C_1 {}^6C_2) + ({}^6C_2 {}^6C_1) + {}^6C_3}{{}^{12}C_3} = \frac{10}{11}$$

Q6) Bag contains 6 Red, 4 yellow and 2 green balls.
3 balls are picked at random. What is the probability that atmost 2 are yellow?

Solⁿ: Total possibilities $\rightarrow {}^{12}C_3$

atmost : maximum \rightarrow minimum

[max (2 yellow) & 1 (red + green) $\frac{6+2=8}{+}$] or [1 (yellow) and 2 (red + green) $\frac{+}{+}$] or [0 (yel) and 3 (red + g)]

$$[{}^4C_2 \times {}^8C_1] + [{}^4C_1 \times {}^8C_2] + [{}^8C_3]$$

$$\text{Probability} = \frac{({}^4C_2 {}^8C_1) + ({}^4C_1 {}^8C_2) + {}^8C_3}{{}^{12}C_3} = \frac{54}{55}$$

Q7) Bag contains 6 Red, 4 yellow and 2 Green balls. 3 balls are picked at random. What is the probability that:

2 red and 1 is green or 1 is yellow and 2 are green

Solⁿ:

[2 red & 1 green] $\frac{+}{+}$ or [1 yellow and 2 green] $\frac{+}{+}$

$$P = \frac{[{}^6C_2 {}^2C_1] + [{}^4C_1 {}^2C_2]}{{}^{12}C_3} = \frac{17}{110}$$

Exercise - 6 - Memory Based

Q1) Bag contains 7 Blue, 5 yellow balls. If 2 balls are selected at random, what is the probability that none is yellow?

Solⁿ: Total possibility \Rightarrow 2 out of 12 $\Rightarrow {}^{12}C_2$

none is yellow i.e. they are blue $\Rightarrow {}^7C_2$

$$\therefore \boxed{\text{probability} = \frac{{}^7C_2}{{}^{12}C_2} = \frac{7}{22}}$$

Q2) Bag contains 8 brown, 4 orange & 5 black balls. Five balls are chosen at random. What is probability of their being: 2 brown, 1 orange and 2 black balls?

Solⁿ: Total possibilities = $8+4+5 = 17$, chose 5 out of 17

$$\therefore \boxed{\text{Total possibilities} = {}^{17}C_5}$$

2 brown and 1 orange and 2 black

$${}^8C_2 * {}^4C_1 * {}^5C_2$$

$$\boxed{P = \frac{{}^8C_2 * {}^4C_1 * {}^5C_2}{{}^{17}C_5} = \frac{40}{221}}$$

Q3) Bag contains 13 white and 7 black balls. 2 balls are drawn at random. What is the probability that they are of same color?

Solⁿ:

Same color i.e. either 2 white or 2 black
$${}^{13}C_2 + {}^7C_2$$

$$\therefore \boxed{P = \frac{{}^{13}C_2 + {}^7C_2}{{}^{20}C_2} = \frac{99}{190}}$$

Q4) Bag contains 3 red and 4 green balls. 3 are chosen at random. What is the probability that 2 are green and 1 is red?

Solⁿ:

2 green and 1 red
 ${}^4C_2 \quad * \quad {}^3C_1$

$$P = \frac{{}^4C_2 {}^3C_1}{{}^7C_3} = \frac{18}{35}$$

Q5) Bag contains 9 red, 7 white and 4 black balls. If 2 balls are picked at random, find the probability that both the balls are red.

Solⁿ:

both are red $\rightarrow {}^9C_2$

Total possibilities $\rightarrow {}^{20}C_2$

$$P = \frac{{}^9C_2}{{}^{20}C_2} = \frac{18}{75}$$

Q6) Bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is the probability that the balls drawn contain no blue ball?

Solⁿ:

no blue ball \therefore 2 balls from red & green
 $2 + 3 = 5$

5C_2

$$P = \frac{{}^5C_2}{{}^7C_2} = \frac{10}{21}$$

Q7) Box contains 4 black, 3 red & 5 green balls. 2 balls to be picked at random. What is the probability the both are of the same color?

Solⁿ:

Total possibility = $4+3+5 = 12$ choose 2 $\rightarrow {}^{12}C_2$

both of same color.

i.e. $2(\text{black}) \frac{{}^4C_2}{+} 2(\text{red}) \frac{{}^3C_2}{+} 2(\text{green}) \frac{{}^5C_2}{+}$

$$\therefore \text{Probability} = \frac{{}^4C_2 + {}^3C_2 + {}^5C_2}{{}^{12}C_2} = \frac{19}{66}$$

Q8) Out of 5 girls & 3 boys, 4 children are to be randomly selected for a quiz contest. What is the probability that all are girls?

Solⁿ:

Total possibility = $5+3 = 8$ choose 4 = $8C_4$

all girls i.e. 4 girls $\rightarrow {}^5C_4$

$$\therefore P = \frac{{}^5C_4}{{}^8C_4} = \frac{1}{14}$$

Q9) Bag contains 3 red, 2 green, 6 blue, 4 yellow balls. 4 balls are picked at random. What is the probability/probability that atleast one is yellow?

Solⁿ Total probability = $3+2+6+4 = 15$ choose 4 $\rightarrow {}^{15}C_4$

[atleast: min \rightarrow max]

min (1 yellow) & 3 from RGB $\frac{{}^1C_1 \cdot {}^8C_3}{+}$ or 2(yellow) & 2(RGB) $\frac{{}^2C_2 \cdot {}^6C_2}{+}$ or 3(yellow) & 1(RGB) $\frac{{}^3C_3 \cdot {}^4C_1}{+}$ or 4 yellow & 0(RGB) $\frac{{}^4C_4 \cdot {}^0C_0}{+}$

$$P = \frac{{}^4C_1 \cdot {}^{11}C_3 + {}^4C_2 \cdot {}^{10}C_2 + {}^4C_3 \cdot {}^9C_1 + {}^4C_4 \cdot {}^0C_0}{{}^{15}C_4} = \frac{69}{91}$$