Time Series

Time is not a line, but a series of new points

Time Series Analysis

- Introduction
- Filter
- ETS Models

Introduction

Time Series Analysis

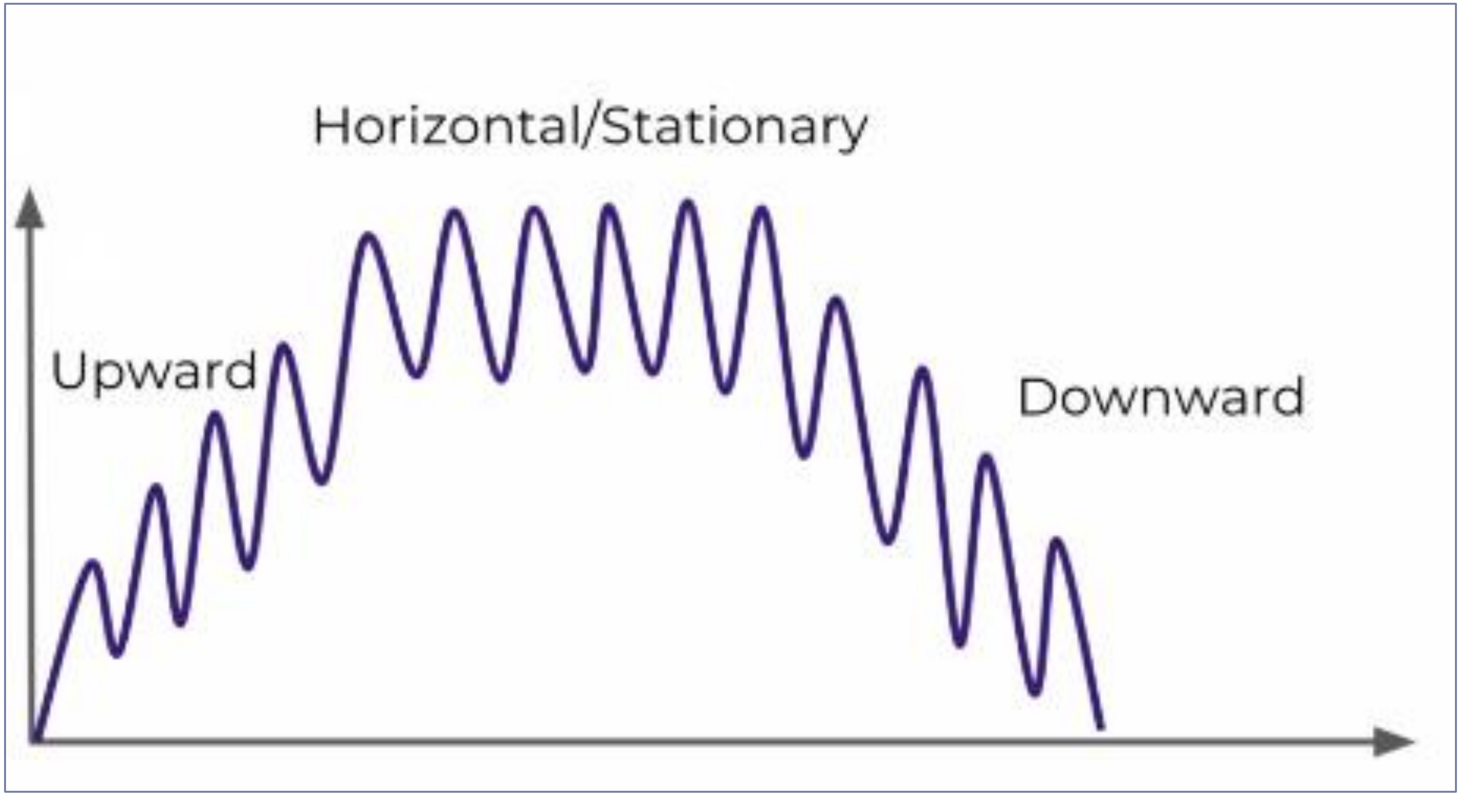
"Time series analysis is a statistical technique that deals with time series data, or trend analysis."

Time series data means that data is in a series of particular time periods or intervals.

Time series data has particular properties, lets take a look at some important terms.

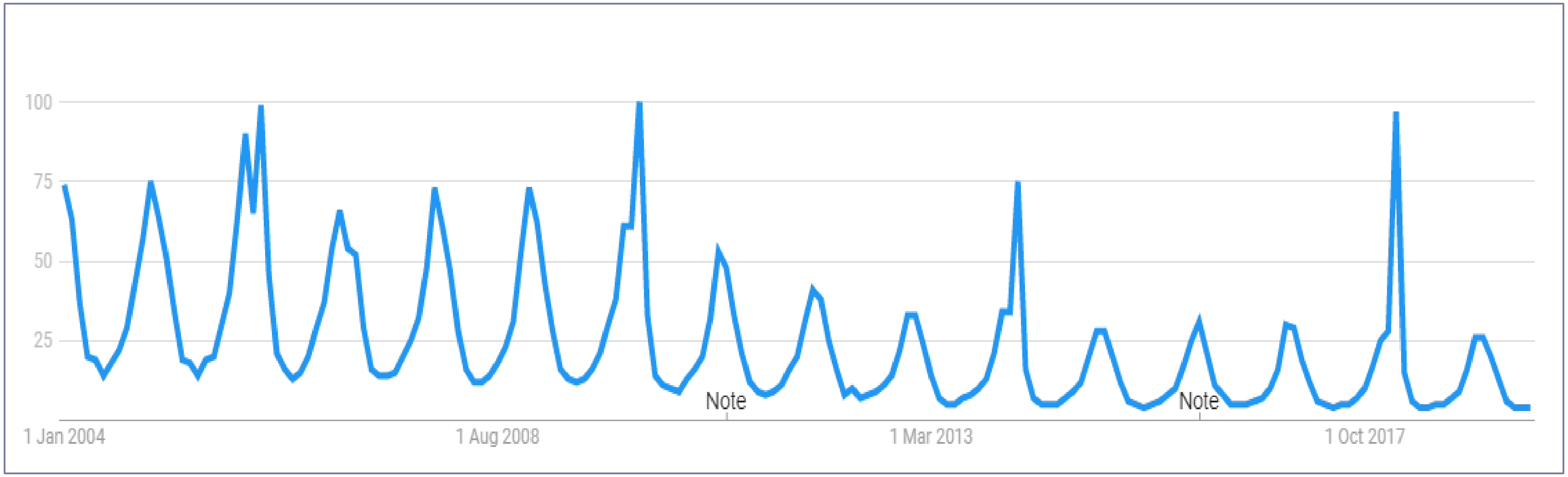
Time Series Analysis

- Trends



Time Series Analysis

- Seasonality – Repeating trends



Google Trends – “Snowboarding”

Time Series Analysis

- Cyclical – Trends with no set repetition

SP500

Filter

Separating the Time Series Components

Time Series Analysis

- Filter

Hodrick – Prescott Filter

The Hodrick-Prescott filter separates:

- A time-series y_t (All our time series data)

Into

- A trend component τ_t
- A cyclical component c_t

$$\boxed{y_t = \tau_t + c_t}$$

So Basically, it tries to find the trend component and the cyclical component of the data.

Time Series Analysis

- Filter

Hodrick – Prescott Filter

The components

- A trend component τ_t
- A cyclical component c_t

Are determined by minimizing the following quadratic loss function:

$$\min_{\tau_t} \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(\tau_t - \tau_{t-1}) - (\tau_{t-1} - \tau_{t-2})]^2$$

Where,

λ = Smoothing parameter, handles variation in growth rate of trend component.

Time Series Analysis

- Filter

Hodrick – Prescott Filter

Default values for λ:

When analyzing quarterly data, use λ = 1600
When analyzing annual data, use λ = 6.25
When analyzing monthly data, use λ = 129,600

ETS Models

Error – Trend - Seasonality

Time Series Analysis

- ETS Models – Time Series Decomposition

ETS Models stands for Error – Trend – Seasonality

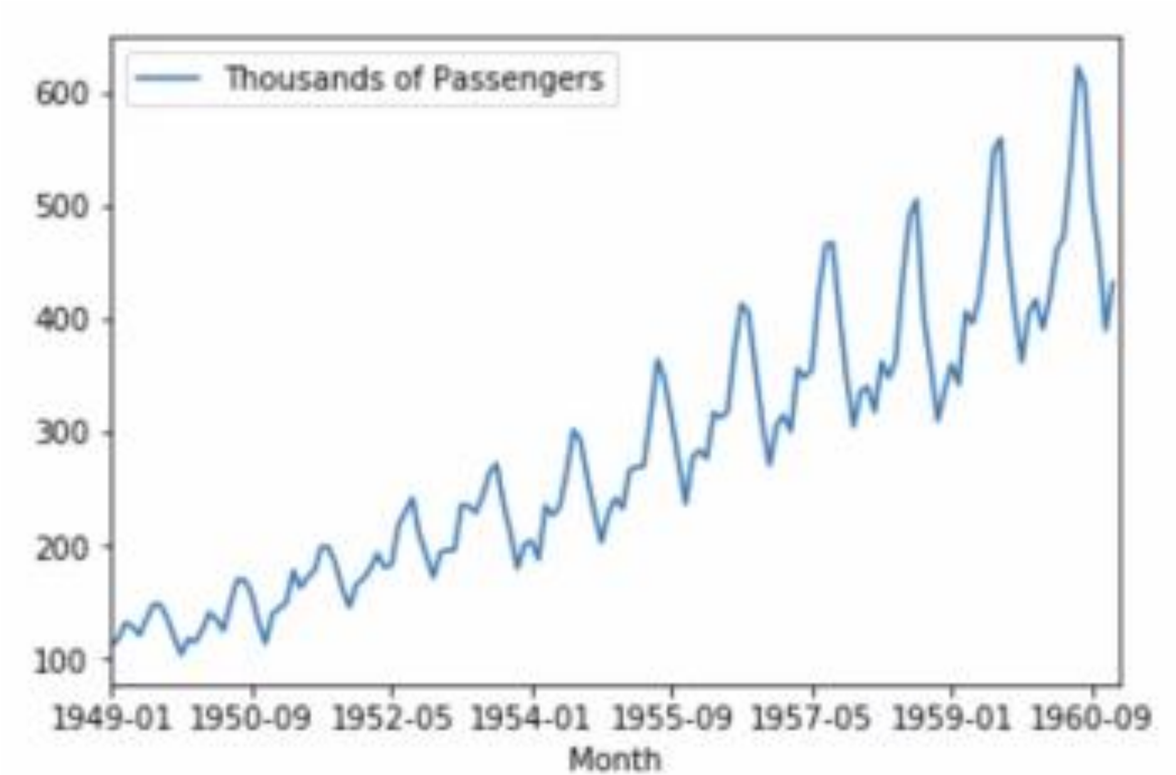
This general term, actually stands for wide variety of ETS models.

- ETS Decomposition
- Exponential Smoothing
- Trend Methods Models

ETS Models takes each of these terms (E-T-S) for 'smoothing' and may add them, multiply them or even just leave some of them out.
Based off these key factors, we can try to create a model to fit our data.

Time Series Analysis

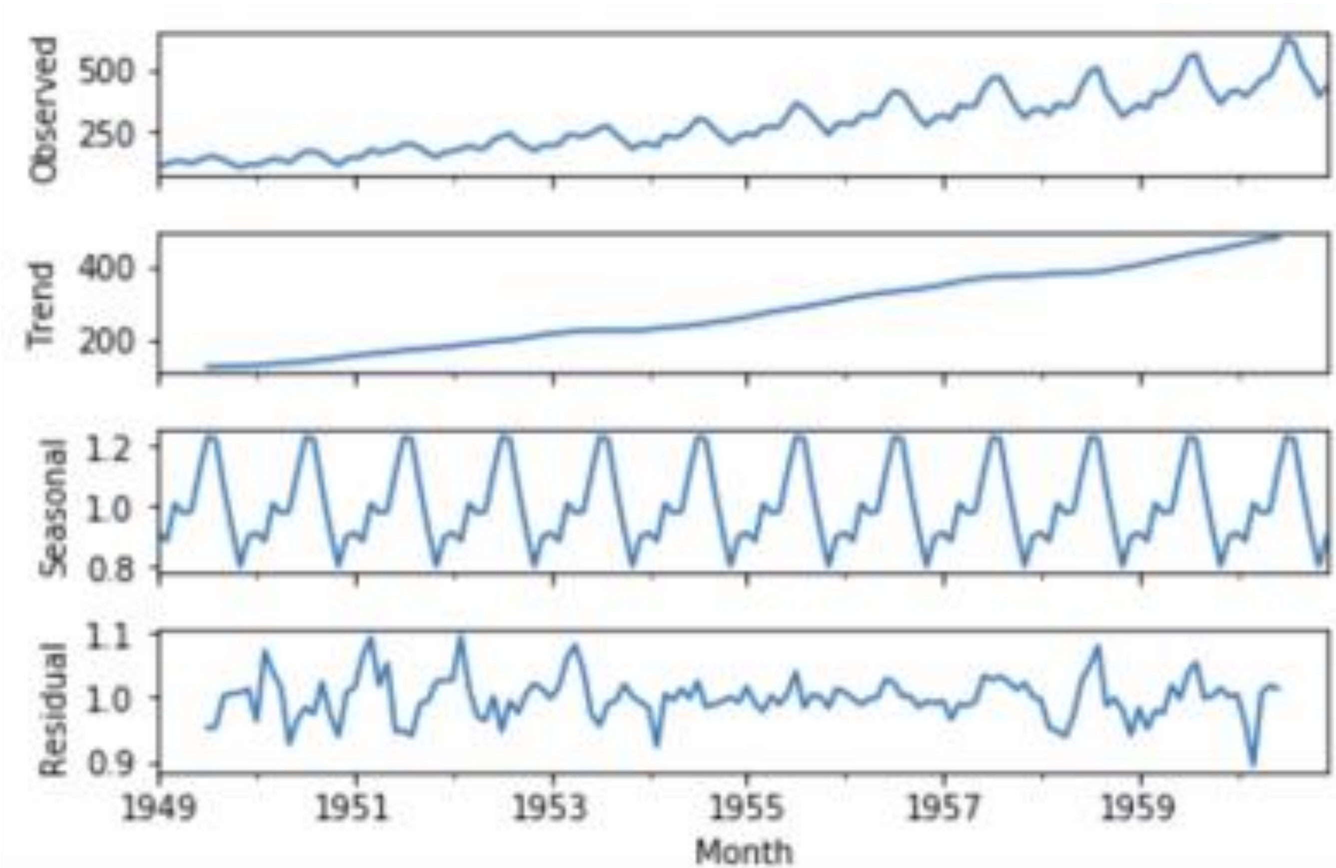
- ETS Models – ETS Decomposition



Airline Passengers

Time Series Analysis

- ETS Models – ETS Decomposition



Time Series Analysis

- ETS Models – ETS Decomposition

Things to remember while doing ETS decomposition
There are two types of ETS models:

- Additive Model
- Multiplicative Model

Additive Mode: We apply additive model when it seems that the trend is more linear and the seasonality and trend components seem to be constant over time.
(e.g. every year we add 1000 passengers)

Multiplicative Model: This is more appropriate when we are increasing at a non-linear rate
(e.g. each year we double the amount of passengers)

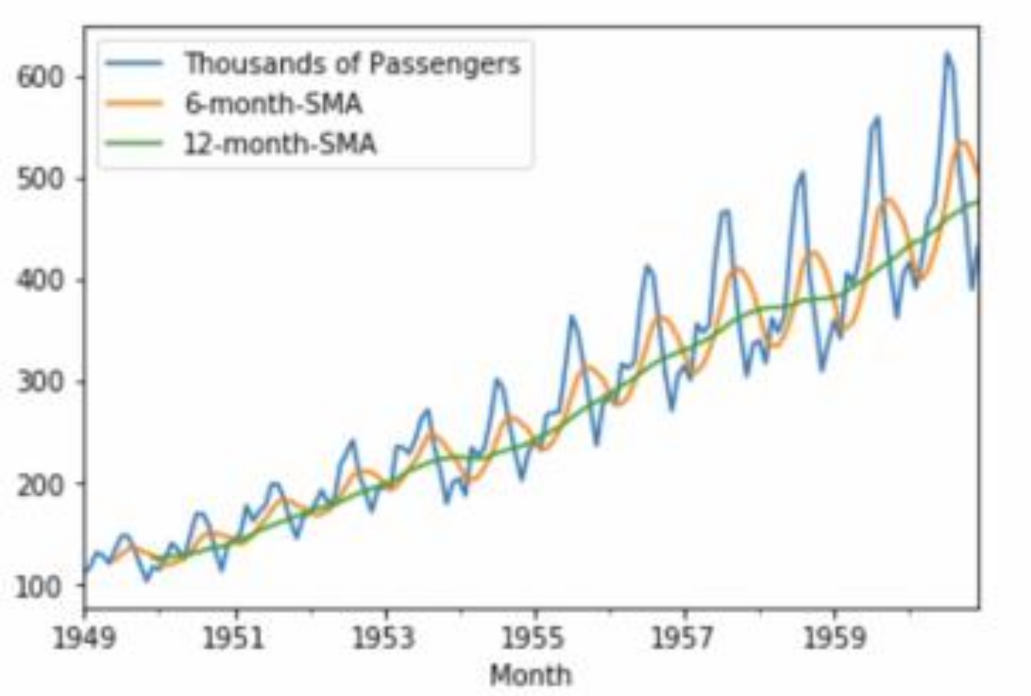
EWMA

Exponentially Weighted Moving Average

Time Series Analysis

- EWMA Models

Before that, we saw SMA (Simple Moving Averages)



Time Series Analysis

- EWMA Models – Problems with SMA

  - **The problem with SMA** is that the entire model is constrained to the same window size (rolling window).
  - Also, smaller window will lead to more noise, rather than signal.
  - SMA will never reach the full peak or valley of the data due to the averaging.
  - Extreme historical values can skew your SMA significantly.

  To help fix these issues, we use **EWMA.**

Time Series Analysis

- EWMA Models

EWMA allows to reduce the lag effect from SMA and it will put more weight on values that occurred more recently
(By applying more weight to the more recent values, thus the name Exponential)

The amount of weight applied to the most recent values will depend on:

- The actual parameters used in EWMA
- The number of periods given a window size

Time Series Analysis

- EWMA Models

EWMA uses **Single Exponential Smoothing**

$$y_0 = x_0$$
$$y_t = (1 - \alpha)y_{t-1} + \alpha x_t$$

Where,

$y_0 = x_0$ means, First input value = First output value

y_t = output value for the rest of the inputs, calculated by formula

Holt – Winters Method

Time Series Analysis

- Holt – Winters Method

Before that…
We saw **Exponential Weighted MA** where we applied **Simple Exponential Smoothing** using just one factor ‘**alpha’**.

This is fine, but it fails to account for other contributing factors like overall trend and seasonality.

This can be fixed using Holt-Winters method.
Note that, Holt-Winters method can be used for forecasting.
But for now, lets see the concept of it.

Time Series Analysis

- Holt – Winters Method

Holt(1957) came up with Double Smoothing method
Winters (1960) extended this method to capture seasonality.

Thus the Holt-Winters method comprise of forecast equation and three additional smoothing equations.

These three smoothing equations are for :

- Level Component = l_t (smoothing parameter is alpha)
- Trend Component = β_t (smoothing parameter is beta)
- Seasonal Component = γ_t (smoothing parameter is gamma)

Time Series Analysis

- Holt – Winters Method

There are two variations to this method depending on the nature of seasonal component

1. **Additive Method:** This is preferred when the seasonal variations are roughly constant through the series.
2. **Multiplicative Method:** This is preferred when the seasonal variations are changing proportional to the levels of the series.

Time Series Analysis

- Holt – Winters Method

Recap:

EWMA uses **Single Exponential Smoothing**

$$y_0 = x_0$$
$$y_t = (1-\alpha)y_{t-1} + \alpha x_t$$

Where,

$y_0 = x_0$ means, First input value = First output value

y_t = output value for the rest of the inputs, calculated by formula

Time Series Analysis

- # Holt – Winters Method

Holt's Method:
We can expand on Single Exponential Smoothing in Holt's method.
This is also know as **Double Exponential Smoothing**.

Here we introduce a new smoothing factor β (beta) that addresses trend:

$$l_t = (1-\alpha)l_{t-1} + \alpha x_t \longleftarrow \text{Level}$$

$$b_t = (1-\beta)b_{t-1} + \beta(l_t - l_{t-1}) \longleftarrow \text{Trend}$$

$$y_t = l_t + b_t \longleftarrow \text{Fitted Model}$$

$$\hat{y}_{t+h} = l_t + hb_t \longleftarrow \text{Forecasting Model (h = \# periods into the future)}$$

Time Series Analysis

- # Holt – Winters Method

Holt's Method (Double Exponential Smoothing)

The reason it is called Double Exponential Smoothing is that we are not dealing with two smoothing parameters alpha and beta.

As we have not considered the seasonal parameter, the forecasting model is essentially just a simple straight extending from the most recent data point.

$$l_t = (1 - \alpha) l_{t-1} + \alpha x_t \longleftarrow \text{Level}$$

$$b_t = (1 - \beta) b_{t-1} + \beta (l_t - l_{t-1}) \longleftarrow \text{Trend}$$

$$y_t = l_t + b_t \longleftarrow \text{Fitted Model}$$

$$\hat{y}_{t+h} = l_t + h b_t \longleftarrow \text{Forecasting Model (h = \# periods into the future)}$$

Time Series Analysis

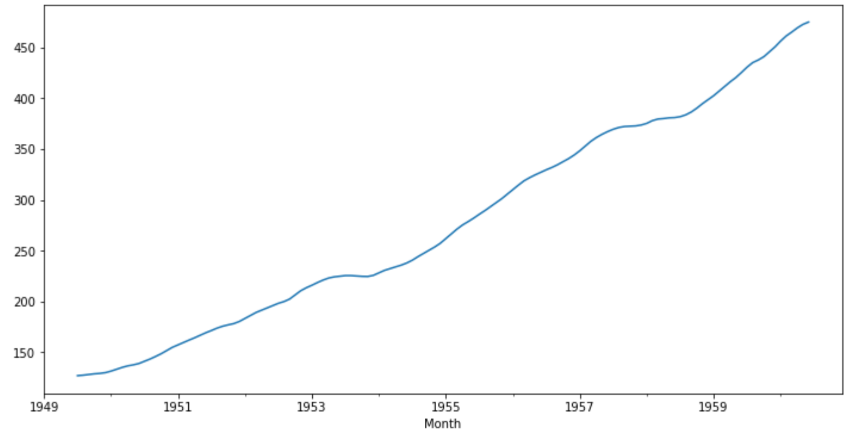
- # Holt – Winters Method

Holt's Method (Double Exponential Smoothing)

As we have not considered the seasonal parameter, the forecasting model is essentially just a simple straight extending from the most recent data point.

What does this mean ?

If we take a look at this model, we are just considering the value at level and the corresponding trend, whether its sloping upwards or downwards. We are **not taking into account the seasonality.**



So, if we use this method to the airlines data, we will be able to predict that number of passengers are increasing.

But we will not be able to predict that it increases more rapidly during the summers than the winters.

Time Series Analysis

- ## Holt – Winters Method

Lets introduce another smoothing factor (gamma) to address the seasonality. This method is **Holt's – Winter Method** also known as **Tripe Smoothing Method**.

$$l_t = (1-\alpha)l_{t-1} + \alpha x_t$$ Level

$$b_t = (1-\beta)b_{t-1} + \beta(l_t - l_{t-1})$$ Trend

$$c_t = (1-\gamma)c_{t-L} + \gamma(x_t - l_{t-1} - b_{t-1})$$ Seasonal

$$y_t = (l_t + b_t)c_t$$ Fitted Model

$$\hat{y}_{t+m} = (l_t + mb_t)c_{t-\boxed{L}+1+(m-1)mod\boxed{L}}$$ Forecasting Model (m = # periods into the future)

Here, L = No of divisions per cycle.

E.g. If we are looking at passengers monthly data, that repeats the pattern each year, we would sat **L=12.**