# Online Speed Optimization for Sailing Yachts Using Extremum Seeking

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***Abstract*—This paper briefly presents the main points on the development and testing of an extremum seeking controller used to maximize the longitudinal velocity of surface sailing vehicles by changing the angle of the sail. The algorithm is suitable for sailing purposes since it requires only the measurements of the vehicle's velocity and the sail angle. As an illustration, we present a few simulation results on our previously-obtained sailing yacht simulator, which was developed based on a 4 DOF nonlinear dynamic model for surface sailing vehicles, showing that the proposed extremum seeking controller is capable of maximizing the sailing yacht's speed performance through online sail tuning. Furthermore, the proposed sail optimization algorithm is tested at sea on an experimental platform, i.e. a small scale autonomous sailboat, illustrating the potential of the controller.**

## I. Introduction

Surface sailing vehicles are a class of surface vehicles that are using the wind as their only means of propulsion and are normally rigged with sails to catch the wind, such as the sailboats and sailing yachts. They are exposed to a highly dynamic and ever changing environment, hence it can require great skills for sailors to simultaneously steer the rudder along a specific heading (autonomous heading control for sailing vehicles are already dealt with in [1]) and adjust the sail to get the propulsion. Because of the challenges that sailing presents, introducing automation in sail control would reduce the demands on sailors and increase the safety at sea. At the same time, one of the main concerns of most sailors is to optimize the longitudinal (or surge) velocity towards a direction by trimming the sail in the best possible way, especially in sailing races like in the American's Cup. Consequently, sail optimization is a topic of interest which would assist in maximizing the vehicle's velocity in sailing.

However, there were only a few studies dedicated to the autonomous speed (sail) control for such kinds of wind-propelled surface sailing vehicles, and most of these research work were based on a fuzzy control scheme (refer to [2][3]). In [4], Stelzer et al. proposed to determine a desired heel angle for the boat according to the speed and direction of the apparent wind, and then a feedback-loop implemented as a fuzzy inference system controls the sail angle towards the heel value. In the present paper, we propose a feedback non-model-based real-time optimization approach for speed control and sail optimization, i.e. belonging to the class of Extremum Seeking Control (ESC) algorithms (see [5] and references therein), and design a generalized optimal sail controller used for a wide variety of surface sailing vehicles and can perform well with a minimum knowledge of the system dynamics driving the output of the system to its extremum.

Roughly speaking, ESC is an adaptive control method, which optimizes the observable signal of a continuous system by trial-and-error adjustment of the controlled variables. The most commonly used method is to make use of a periodic external perturbation on the plant input [6], which has been applied to various control problems, amongst others to container ships speed and heading control in the presence of roll parametric resonance [7]. It also has applications in flight formation control [8], PID parameters tuning [9] and etc. In addition to its adaptivity, the most promising feature of the extremum seeking approach in the context of sailing is that no explicit system model is required, it requires only the measurements of the vehicle's speed and the sail angle.

Nevertheless, the ESC algorithm presented in this paper does not need a perturbation signal and is computationally simpler. There are two non-perturbative extremum seeking control approaches according to [10]. One is based on the introduction of sliding modes [11][12][13], while the second one necessitates the knowledge of gradients of the function to be optimized [14] and changes the direction of the seeking input depending on the sign of the gradient. Our proposed algorithm is analogous to the latter and is a modified *hill climbing* method [15], which would be elaborated later in the controller design. When implementing this extremum seeking (ES) controller, the control input of the objective function is tuned as close as possible to an unknown value resulting in the optimal performance of the system, however, it will be noticed that the input variable will oscillate in the end thus resulting in the oscillation of the output of the system around an estimated maximum (minimum) value.

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The structure of this paper is as follows. In section II-A, we describe the extremum seeking controller and present the controller design. Section II-B would then show a series of simulation results of the proposed sail optimization algorithm applied to a specific 12-m class sailing yacht model, here, a few analysis on the control algorithm and parameters tuning are discussed. Next, we introduce the well-known performance polar diagram in section II-C, and compare the speed polar derived by applying the ESC method with the theoretically computed polar plot. Finally, section II-D is dedicated to the description of the experimental platform and the experimental tests on sail optimization using ESC. Then, a few concluding remarks end the paper.

## II. Extremum Seeking Applied to Sail Control

### A. Extremum Seeking Controller Design

In the present analysis, the sail angle and surge are the input and output of the dynamic system, respectively. We aim to derive an extremum seeking controller which iteratively tunes the sail angle of the surface sailing vehicle such that the surge speed reaches the optimum along a specific heading.

For energy considerations, i.e. the online optimization scheme should not run out the batteries of the vehicle, we consider the controller is expressed in a discrete-time setting, where the control input, represented by the angle of the main sail, is made to change in discrete steps. In this way, the system's dynamic model is not included in the controller design, which is referred to the problem of static optimization, hence showing the simplicity of the controller.

Besides the sampling frequency (with $T_s$ the sampling time), the controller is tuned using a single gain $k$, which, roughly speaking, represents the increments of the sail angle used to perform a search in the so-called polar performance plot, which characterizes the static performance of a sailing vehicle, and is well-known to sailors.

More specifically, the controller adjusts the sail angle, which is denoted as $\delta_s$, in discrete steps of constant size. After each step, the output $u$, i.e. the longitudinal speed, is measured. The direction of stepping is reversed whenever $u$ appears to have decreased over its value after the previous step. The control law can then be written as [14]

$$\begin{aligned} \delta_s(t+1) &= \delta_s(t) + k * sign(\Delta_s(t)) * sign(\Delta_u(t)), \\ \Delta_s(t) &= \delta_s(t-1) - \delta_s(t-2), \\ \Delta_u(t) &= u(t) - u(t-1), \end{aligned} \tag{1}$$

with $k$ positive constant, $t+1$ the time after one step and similar meanings for $t-1$ and $t-2$, $sign(x)$ is a function such that

$$sign(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{otherwise.} \end{cases} \tag{2}$$

The newly proposed sail controller composed of (1) and (2) is simpler in terms of tuning parameters compared to ES controller with sliding mode [13], for which we have to define a continually increasing function as the reference trajectory, a search signal and a gain to be tuned. Here, we regulate only two parameters, i.e. $T_s$ and $k$. In addition, different from the controller represented in [14], we defined a special signum function in (2), i.e. $sign(x)$ is either $1$ or $-1$. In this case, the sail angle is forced to change every $T_s$ seconds, which ensures a continuous tracking of the maximum speed. Moreover, as seen in (1), the difference in sail angle is computed by subtracting the value recorded at time $t-2$ from the measurement one step after, due to the fact that the speed at time $t$ is obtained by setting the sail angle as $\delta_s(t-1)$ for $T_s$ seconds. Similarly, fix the sail angle to $\delta_s(t-2)$ for one sampling period, leading to the speed $u(t-1)$.

Additionally, the proposed speed optimization technique is reminiscent of the so-called *hill climbing* algorithm [15], which has various applications in genetic algorithms [16], analysis of animal behaviors [17], MPPT in photovoltaic power systems [18], and etc. Our ES controller is a modified hill climbing algorithm, with the sail-surge curve, i.e. the graph of the controlled variable versus the performance, characterizing the "hill". The multiplication of two $sign$ functions gives value either $1$ or $-1$, indicating the direction that must follow on the "hill" in order to climb up to the peak. Once it is getting close enough to the optima, the operating point moves around the "hilltop".

### B. Simulation Results and Analysis

As an illustration, the results of our ESC scheme in (1) and (2) are simulated in Matlab Simulink for a 12-m class sailing yacht with the 4 DOF dynamic model derived in [1] as the simulation model, whose behavior is visualized in a 3D graphic environment (see Fig. 1), with parameters of the yacht taken from [19]. In addition, more explanations on the ESC algorithm and parameters tuning are presented.

Fig. 1. Virtual representation of our yacht model.

We start the simulation with $u(0) = 0\,\mathrm{m/s}$ and $\delta_s(0) = 20°$, and the initial heading is set to $\psi(0) = 120°$ with the desired heading $\psi_d = 60°$. The wind is assumed to be coming from the North and has a fixed velocity $10\,\mathrm{m/s}$ (approximately $20\,\mathrm{knots}$). At the very start, the sampling period is set to $T_s = 10\,\mathrm{s}$ and $k = 1°$ is the incremental step in sail angle. It can be seen from Fig. 2(a)(b), our ES controller behaves reasonably well, which tracks the optimal value within approximately 3 minutes. Note that after the vehicle reaches the target heading, by gradually pulling the sail, i.e. the sail angle is decreased $1^\circ$ every 10 seconds, the longitudinal velocity of the sailing yacht increases to find its optimum. In Fig. 2(c), $\psi_r$ denotes the reference heading used in heading control [1]. The results presented in Fig. 2 also indicates that the proposed surge controller can be used in conjunction with the heading control algorithm that determines the heading whereas the surge controller tracks the optimal sail angle for the established heading.

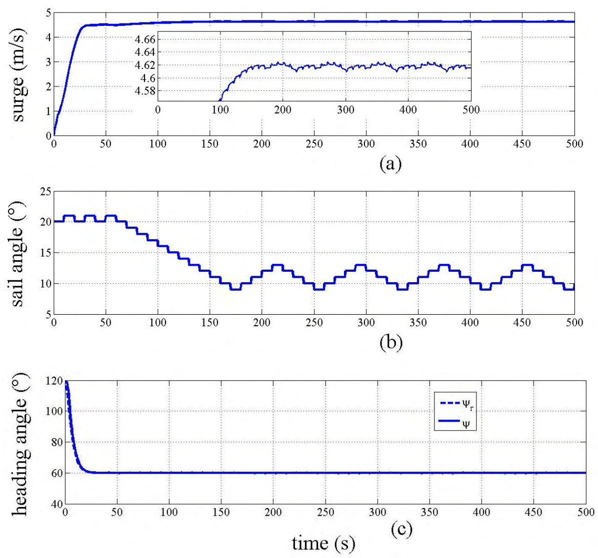


Fig. 2. Optimization of sailing yachts surge speed (a) and sail angle (b) along with the heading evolution (c) ($k = 1^\circ, T_s = 10\,\mathrm{s}$).

However, an oscillation of the input signal as shown in Fig. 2(b) is caused by the inherent discontinuous property of the controller design in (1) and (2), thus resulting in an oscillating behavior of the output around approximately $4.62\,\mathrm{m/s}$ (as seen in the amplified figure in Fig. 2(a)), which is known as *chattering* [20][21]. As the case shown in Fig. 2, and it reveals from (1) that $\delta_s$ keeps decreasing such that $u$ climbs up towards its maximum. Ideally, $\delta_s$ remains at the value when the maximum speed is reached. However, due to the signum function defined in (2), $\delta_s$ is made to move every sampling period. As a result, $u$ drops down from the maximum value, resulting in the direction of stepping is reversed, i.e. $\delta_s$ is getting bigger for the next sampling period. After the control switches, it keeps searching for the maximum speed by increasing the sail angle. Once again the maximum is reached, $\delta_s$ can not keep unchanged, thus leading to the oscillation depicted in Fig. 2.

As an index to indicate the steady-state performance of the extremum seeking controller, the average of the output is recorded, whose notation takes the subscript $avg$. In the present simulation, $u_{avg} = 4.6176\,\mathrm{m/s}$, $\delta_{s,avg} = 10.9263^\circ$, and $\psi_{avg} = 59.9887^\circ$, with all the numbers the averaged values of the last 250 seconds.

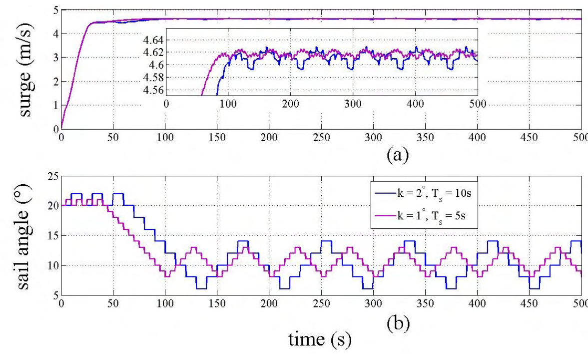


Fig. 3. Parameters tuning in the optimization of sailing yachts surge speed (a) and sail angle (b).

In the following, we discuss the performance of the ES controller by tuning the parameters $k$ and $T_s$. At first, the simulation was running with $k = 2^\circ$ and $T_s = 10\,\mathrm{s}$, whose performance is depicted in blue in Fig. 3, with the wind and other initial conditions the same as in Fig. 2. Afterwards, we did the simulation with $k = 1^\circ$ and $T_s = 5\,\mathrm{s}$, as represented in violet in Fig. 3. Comparing Fig. 3 with Fig. 2, we note that the convergence speed is faster by either increasing $k$ or decreasing $T_s$, with the convergence time are approximately 2 and 1.5 minutes, respectively. Nevertheless, it presents less oscillation of the input signal in Fig. 2(b), causing less chattering phenomenon of the output (as compared to that shown in the amplified figure in Fig. 3(a)), thus resulting in a higher control accuracy. Therefore, a tradeoff between the convergence speed and control accuracy must be considered when regulating the controller parameters.

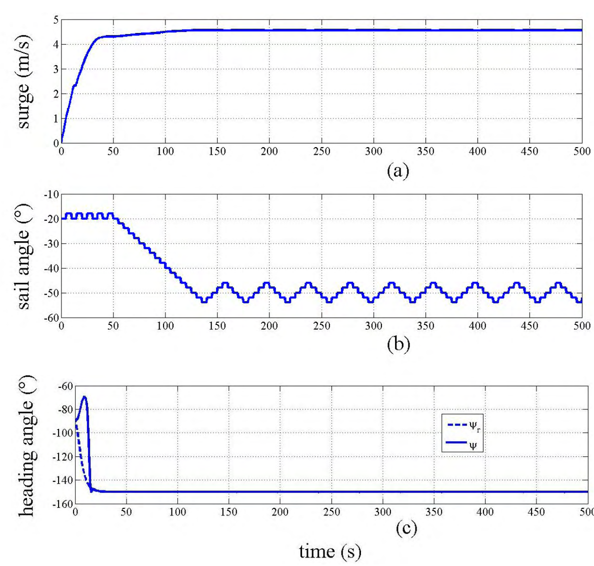


Fig. 4. Optimization of sailing yachts surge speed (a) and sail angle (b) along with the heading evolution (c) ($k = 2^\circ, T_s = 5\,\mathrm{s}$).

The control law in (1) and (2) is also tested along different headings, as the results plotted in Fig. 4 is another verification on the optimization of sail angle. Fig. 4(c) shows that the sailing yacht starts at $\psi(0) = -90°$ and is heading to $\psi_d = -150°$, so the initial sail angle is $-20°$. Here, $\psi$ is not able to follow the reference $\psi_r$ at the beginning because of the zero starting speed. In this case, the vehicle has to move ahead and accumulate speed before turning. It can be seen in Fig. 4(b), by slowly releasing the sail, using $2°$ every 5 seconds as the step size, the surge speed is ascending to reach its maximum. Eventually, in the steady-state, we have $u_{avg} = 4.5569\,\mathrm{m/s}$, $\delta_{s,avg} = -50.1166°$, and $\psi_{avg} = -150.0004°$. So, the simulation results in Fig. 4 validate the performance of the ES controller at different headings.

In addition, we introduce a change of wind direction in the simulation and test the robustness of the controller by adding output noises. Here, the desired heading is still chosen as $60°$. Assuming that the wind is firstly coming from the North and it changes direction in one simulation step (i.e. $0.2\,\mathrm{s}$) when the simulation runs to $300\,\mathrm{s}$. The wind is assumed to be coming from the West between $300\,\mathrm{s}$ and $600\,\mathrm{s}$, afterwards, it changes again to be coming from the northwest. Accordingly, the sail angle is adjusted (as described in Fig. 5(b)) by using the controller in (1) and (2) so as to track the optimal velocity along the heading $\psi = 60°$ under different wind directions. The surge evolution in Fig. 5(a) illustrates that the ES controller ensures fast convergence and optimizes the longitudinal speed of the vehicle in the presence of wind direction changes.

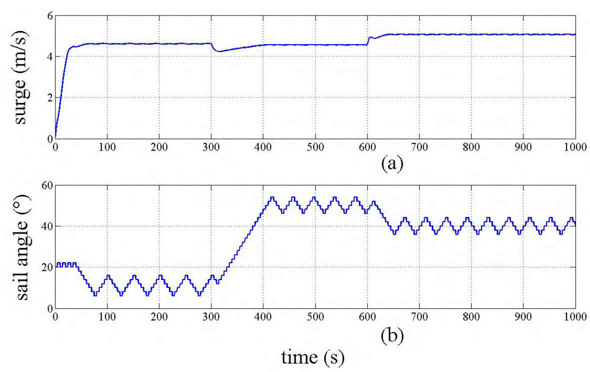


Fig. 5. Optimization of sailing yachts surge speed (a) and sail angle (b) in the presence of wind direction changes ($k = 2°, T_s = 5\,\mathrm{s}$).

Furthermore, considering the measurement errors and environmental factors, the noise represented by a Gaussian random variable $u_\xi$ is added to the output $u$. Consequently, the controller operates not with true values of $u$ but with estimates $u + u_\xi$. Thus, in the controller expressions, we have

$$\Delta_u(t) = (u + u_\xi)(t) - (u + u_\xi)(t - 1). \tag{3}$$

The error $u_\xi$ in successive estimates is assumed to be independent, with zero mean value and the covariance is chosen as 0.1. The performance of the controller consisted of (1) and (3) is represented in Fig. 6, indicating that the ES controller is robust to output noises.

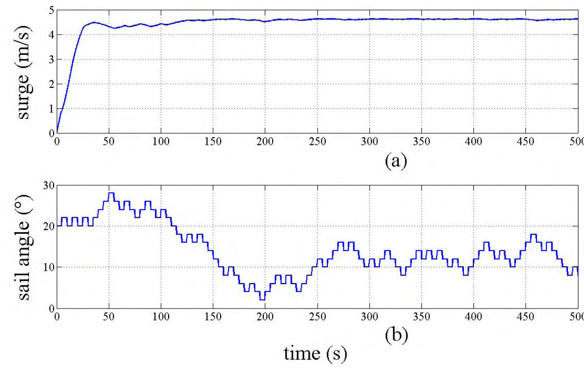


Fig. 6. Optimization of sailing yachts surge speed (a) and sail angle (b) in the presence of noise ($k = 2°, T_s = 5\,\mathrm{s}$).

### C. Polar Performance

The simulation results have shown clearly that our proposed sail optimization algorithm is working well and tracks the velocity to an optimal operation. In this subsection, we aim to compute the performance polar diagram for a specific 12-m class sailing yacht, which was used as the simulation environment model, and compare the maximum attainable speed from the ES controller with the value in the polar plot at certain headings, so as to see the control accuracy of our surge controller.

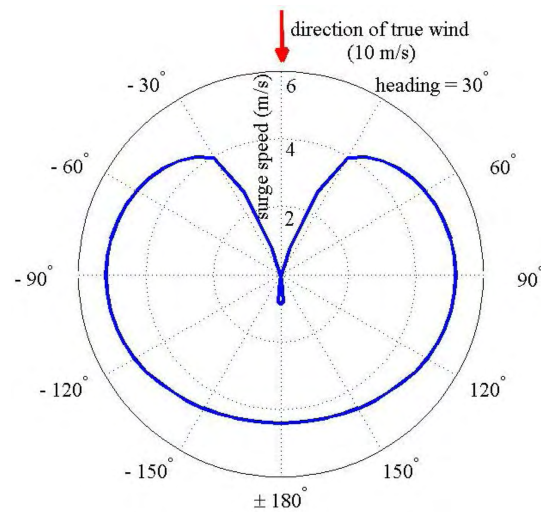


Fig. 7. Performance polar diagram of a 12-m class sailing yacht.

The polar performance plot describes the speed performance of a specific sailing vehicle in static state under a certain wind direction and strength, which predicts the maximum steady-state speed of the vehicle at a particular angle to the wind [22]. In order to obtain this polar diagram numerically, we established an implicit function $\dot{u} = f = 0$, with explicit expression of function $f$ calculated from the 4 DOF model in [1]. And then, solved for equilibriums of $f = 0$ and searched for $u_{max}$ for a set of headings within the interval $[-\pi, \pi]$. As a consequence, Fig. 7 gives a good overall picture of the performance of a 12-m class sailing yacht. The red arrow in

Fig. 7 expresses the direction of true wind, with speed $10\,\mathrm{m/s}$. The resulted curve represents a distinct "no-go zone" in the upwind directions, i.e. the area between approximated $-30°$ and $30°$. Moreover, the negative velocities come from the sail characteristics on drag coefficient, i.e. there still exists drag force from the sail even if the sail is luffing with the apparent wind, meaning that the sailing vehicles might go backwards in this case. However, as the model tests for aero-hydrodynamic data and the modeling of vehicles' dynamic motion are both implemented under a series of assumptions, the curve can only be regarded as approximate.

Fig. 8 presents both the theoretically computed polar diagram (blue curve) and the results of the extremum search (red marks) in simulations. Although it presents small mismatches in the figure, delicate tuning of the parameter $k$ in the ES controller could result in better performances. Since the values in most directions are matching, indicating that the results obtained from the ES controller are in reasonable agreement with the theoretic performance, meaning that our surge controller ensures a good control accuracy.

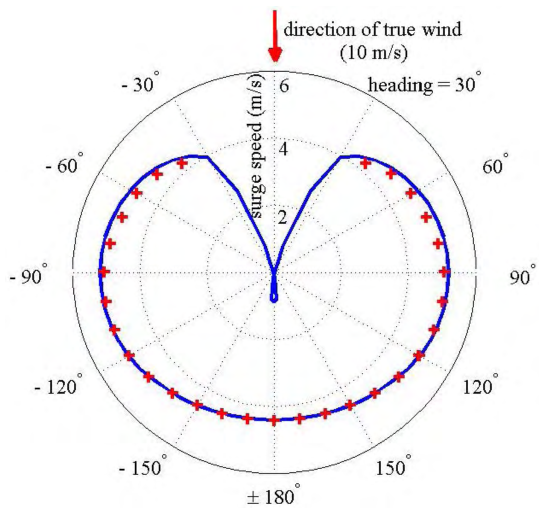


Fig. 8. Performance polar diagram (blue), together with the results from ESC (red).

## D. Experimental Results

As a validation of the controller design in (1) and (2), an experimental platform is currently undergoing a series of tests to investigate the potential of application of this method, with Fig. 9 the picture of the small scale unmanned autonomous sailboat.

The FEUP (Faculty of Engineering at the University of Porto) autonomous sailboat (FASt) was designed and built at the Department of Electrical and Computer Engineering of FEUP, in Portugal [23]. The design length was set to $2.5\,\mathrm{m}$, and with a displacement of $40\,\mathrm{kg}$. To increase stability, the boat includes a deep keel with a lead ballast of $20\,\mathrm{kg}$. The rig is a standard Marconi configuration with the main sail mounted on a boom, as used in small radio-controlled sailing boats. The sail area is $3.15\,\mathrm{m}^2$, and the mast is $3.4\,\mathrm{m}$ in height.


Fig. 9. FASt - the FEUP Autonomous Sailboat.

A lot more information about the software and hardware architecture, the sensors used in FASt, including the wind vane and anemometer, boom position sensor, digital compass, GPS, and etc., are explicitly introduced in [23].

We tested the sail optimization algorithm at the sea, where is located in front of the Matosinhos beach and is south of the Leixoes harbor in Portugal. On the day we did the tests with FASt, the wind was coming from the southeast, with true wind speed around $7\,\mathrm{m/s}$, i.e. approximately $13.5\,\mathrm{knots}$. The boat performed a series of tacks with different heading references and simultaneously with the sail being adjusted by our proposed surge controller. Here, we set the sail optimization frequency to $0.2\,\mathrm{Hz}$, i.e. the sampling time is $T_s = 5\,\mathrm{s}$, and $k = 5°$ such that the convergence would be faster. We plotted the logged data for one tack with the sail performance and surge time evolution shown in Fig. 10.

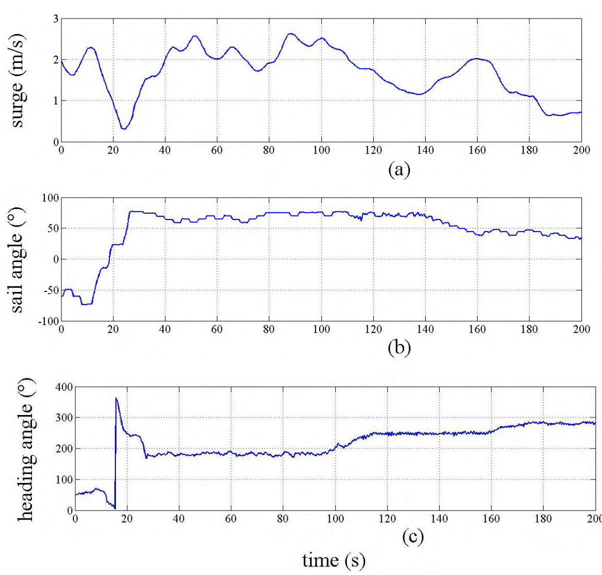


Fig. 10. Performance of the ES controller on the experimental platform. (a) surge (b) sail angle (c) heading

It indicates in Fig. 10(c) that the boat is sailing upwind, with the true wind direction approximately $310°$. Moreover, the sailing boat is tacking as its heading changed from approximately $60^\circ$ to $180^\circ$ during the time from $15\,\mathrm{s}$ to $25\,\mathrm{s}$, while the sail changes direction from negative to positive as seen in Fig. 10(b). At the same time, the velocity of the boat is decreased because of the crossing of the no-go zone. Afterwards, as shown in Fig. 10(a), the boat's velocity keeps increasing while the sail is regulated such that the sail angle is decreased step by step. And then, we can see the periodic adjustment of the sail angle and the oscillation of the surge speed until the reference heading is changed again. In Fig. 10(a), from the time around $140\,\mathrm{s}$ the boat is accelerated again from $1\,\mathrm{m/s}$ to $2\,\mathrm{m/s}$ by gradually pulling the sail.

It should be noticed in Fig. 10(a), there is a significant drop before $80\,\mathrm{s}$ while the boat's heading is not changed much. This might be influenced by a series of unstable variables, including the wind direction and speed, the wave at sea (as seen in the picture Fig. 9), and the sensors and actuators. However, the periodic adjustment in sail angle and the corresponding speed performance are able to demonstrate that the ES controller designed in (1) and (2) is working on our small autonomous sailboat, and is shown to search for the boat's maximum velocity with robustness to rapidly changing environmental conditions.

## III. Concluding Remarks

In this paper, an extremum seeking controller for surface sailing vehicles was designed to achieve speed optimization by adjusting the boom position (sail angle) in discrete steps of constant size. In the control algorithm, no external perturbation is needed. The simulation results certify that the controller is capable of tracking the optimal speed of the sailing yacht along different headings, despite changing wind conditions and unknown output noise. The control performances can be improved by tuning the parameters in the controller design.

The results obtained from the ESC are compared to the polar performance diagram for a specific sailing yacht, demonstrating that our proposed surge controller has a high control accuracy. In addition, the data logged from the experimental tests show that the controller was not developed for a specific sailing yacht, the same methodology can be applied to design sail controllers for other surface sailing vehicles, only the measurements on sail angle and surge are required.

Future research work could include the automatic online tuning of the controller parameters in order to improve both the dynamic and steady-state control performances.

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