Deducción Natural en LPO

Lógica Intuicionista

Reglas Básicas

$$\frac{}{\Gamma, \tau \vdash \tau}\, ax \qquad \frac{\Gamma \vdash \bot}{\Gamma \vdash \tau}\, \bot_e$$

$$\frac{\Gamma \vdash \tau \qquad \Gamma \vdash \sigma}{\Gamma \vdash \tau \wedge \sigma}\, \wedge_i \qquad \frac{\Gamma, \tau \vdash \sigma}{\Gamma \vdash \tau \Rightarrow \sigma}\, \Rightarrow_i$$

$$\frac{\Gamma \vdash \tau \wedge \sigma}{\Gamma \vdash \tau}\, \wedge_{e_1} \qquad \frac{\Gamma \vdash \tau \wedge \sigma}{\Gamma \vdash \sigma}\, \wedge_{e_2} \qquad \frac{\Gamma \vdash \tau \Rightarrow \sigma \qquad \Gamma \vdash \tau}{\Gamma \vdash \sigma}\, \Rightarrow_e$$

$$\frac{\Gamma \vdash \tau}{\Gamma \vdash \tau \vee \sigma}\, \vee_{i_1} \qquad \frac{\Gamma \vdash \sigma}{\Gamma \vdash \tau \vee \sigma}\, \vee_{i_2} \qquad \frac{\Gamma, \tau \vdash \bot}{\Gamma \vdash \neg\tau}\, \neg_i$$

$$\frac{\Gamma \vdash \tau \vee \sigma \qquad \Gamma, \tau \vdash \rho \qquad \Gamma, \sigma \vdash \rho}{\Gamma \vdash \rho}\, \vee_e \qquad \frac{\Gamma \vdash \tau \qquad \Gamma \vdash \neg\tau}{\Gamma \vdash \bot}\, \neg_e$$

Reglas Derivadas

$$\frac{\Gamma \vdash \tau}{\Gamma \vdash \neg\neg\tau}\, \neg\neg_i \qquad \frac{\Gamma \vdash \tau \Rightarrow \sigma \qquad \Gamma \vdash \neg\sigma}{\Gamma \vdash \neg\tau}\, MT$$

Cuantificación Universal

$$\frac{\Gamma \vdash \forall X.\sigma}{\Gamma \vdash \sigma\{X := t\}}\, \forall_{\mathrm{E}} \qquad \frac{\Gamma \vdash \sigma \qquad X \notin fv(\Gamma)}{\Gamma \vdash \forall X.\sigma}\, \forall_{\mathrm{I}}$$

Cuantificación Existencial

$$\frac{\Gamma \vdash \sigma\{X := t\}}{\Gamma \vdash \exists X.\sigma}\, \exists_{\mathrm{I}} \qquad \frac{\Gamma \vdash \exists X.\sigma \qquad \Gamma, \sigma \vdash \tau \qquad X \notin fv(\Gamma, \tau)}{\Gamma \vdash \tau}\, \exists_{\mathrm{E}}$$

Lógica Clásica

Regla Básica

$$\frac{\Gamma \vdash \neg\neg\tau}{\Gamma \vdash \tau}\, \neg\neg_e$$

Reglas Derivadas

$$\frac{\Gamma, \neg\tau \vdash \bot}{\Gamma \vdash \tau}\, PBC \qquad \frac{}{\Gamma \vdash \tau \vee \neg\tau}\, LEM$$