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Let e.value denote the value of e. Then:

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  However, a prefix or postfix expression may be ambiguous if you don't know the arities of operators.
- In prefix and postfix notations, operators are ***not*** divided into different precedence classes.
- In prefix and postfix notations, there is no concept of left- or right-associativity.

Abstract Syntax Trees

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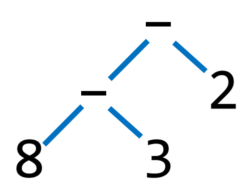
For example, the Lisp expression `(- (- 8 3) 2)` and the two Java expressions `8 - 3 - 2` and `((8 - 3) - 2)` all have the following AST:



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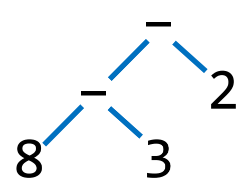

- Two expressions are equivalent (i.e., have the same semantics) ***if and only if*** they have the same AST.

Thus the above three expressions are equivalent.

Note that ASTs do ***not*** have parentheses as nodes!

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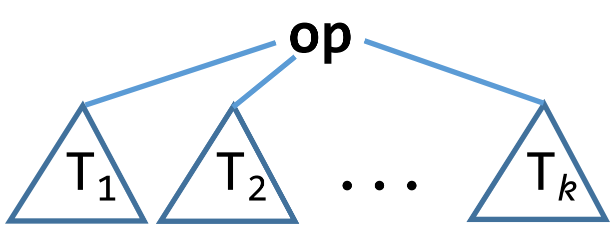
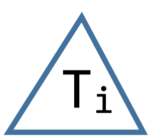


where 

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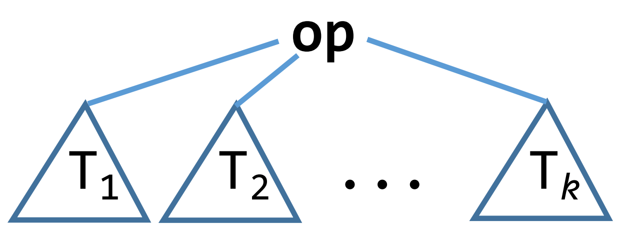
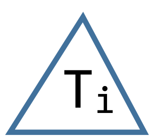


where 

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ASTs of *infix* expressions are binary trees, because infix notation doesn’t allow operators of arity > 2.

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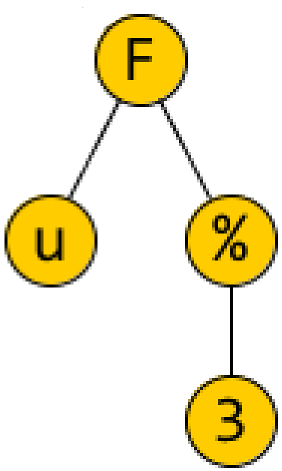

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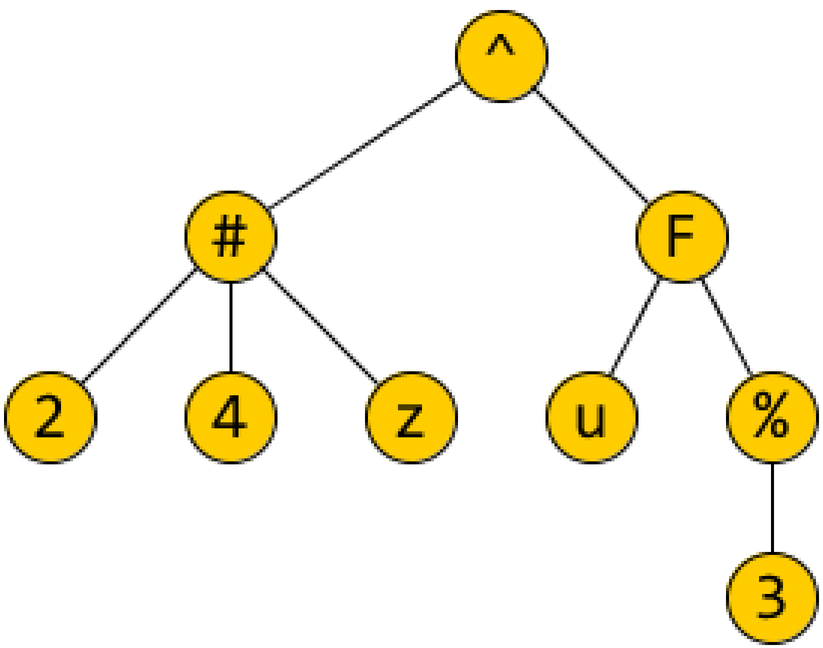

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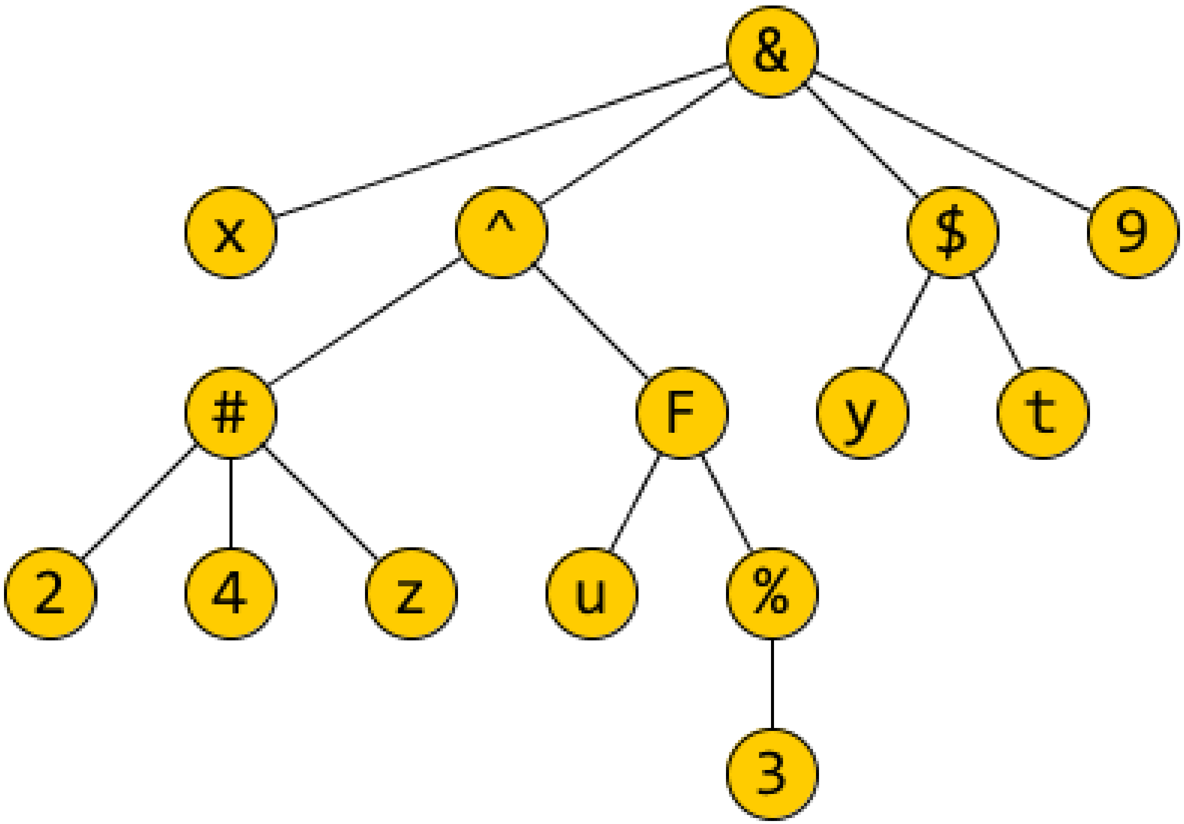

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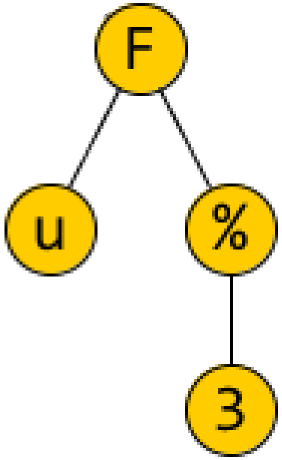

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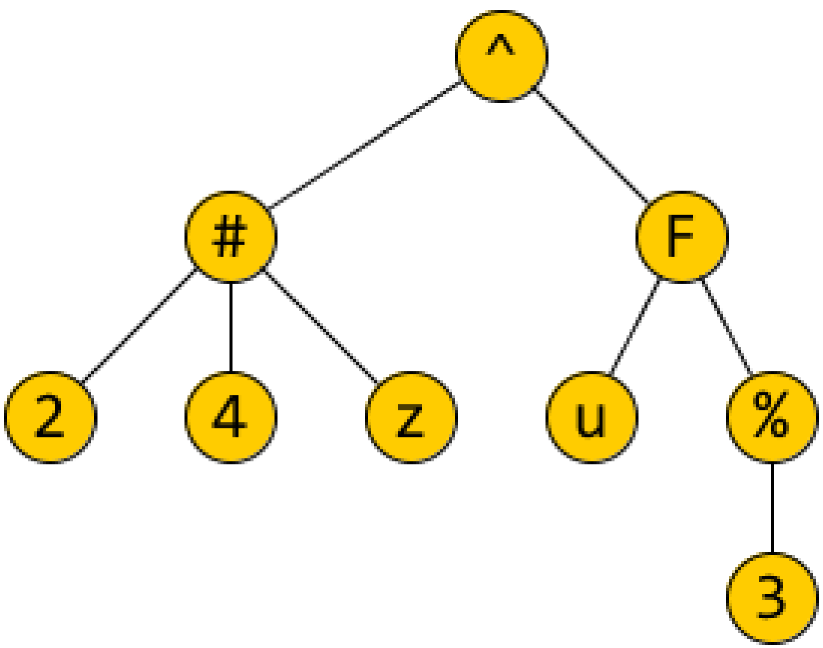

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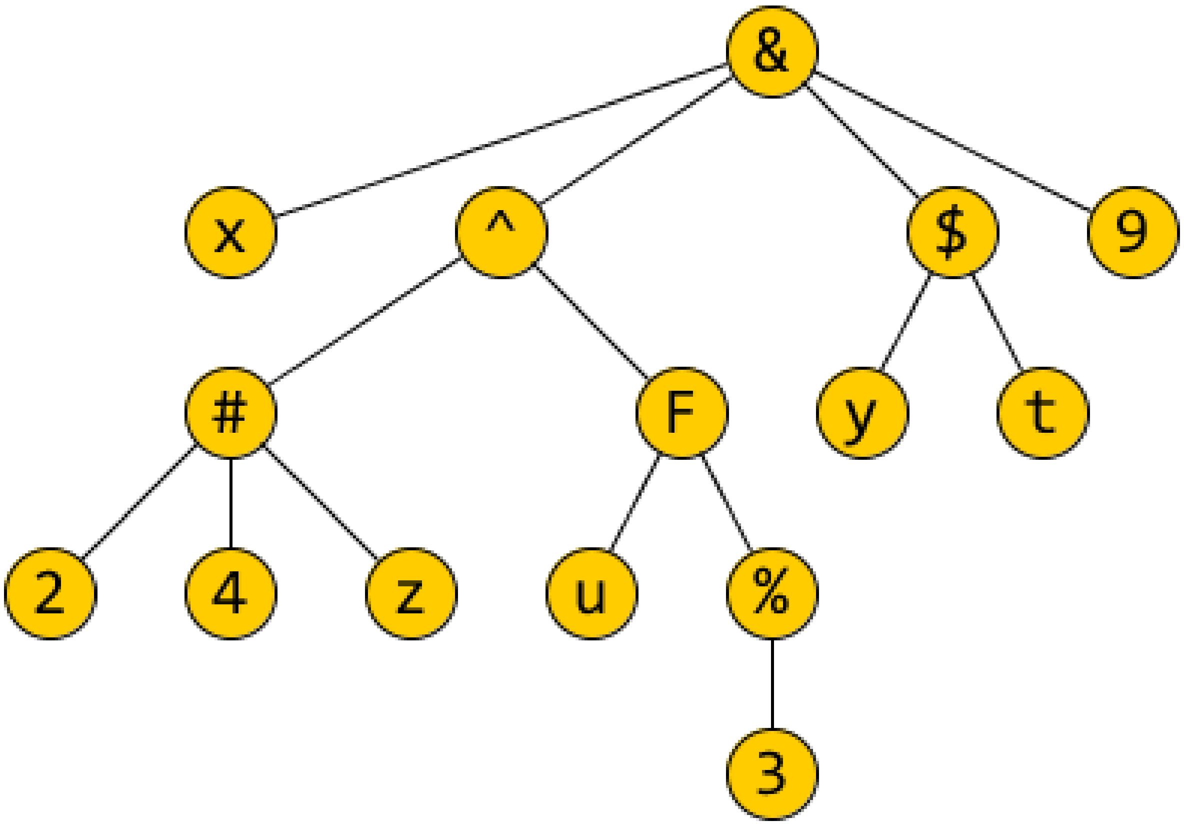

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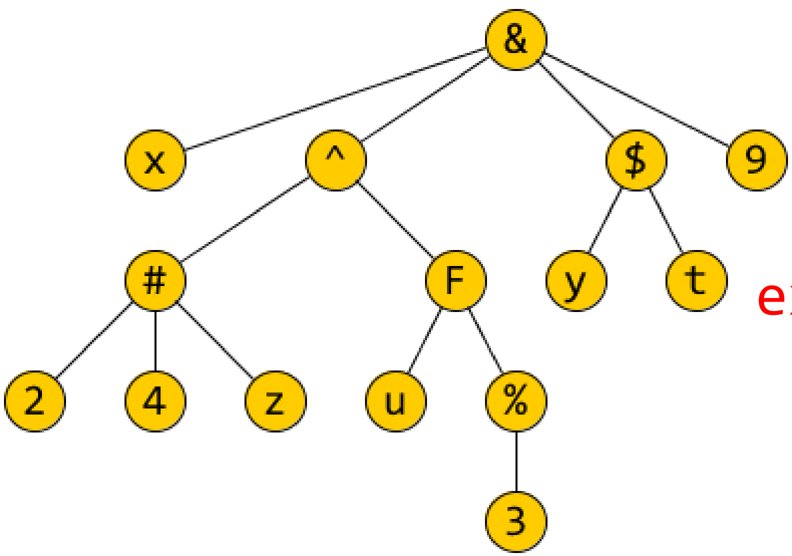

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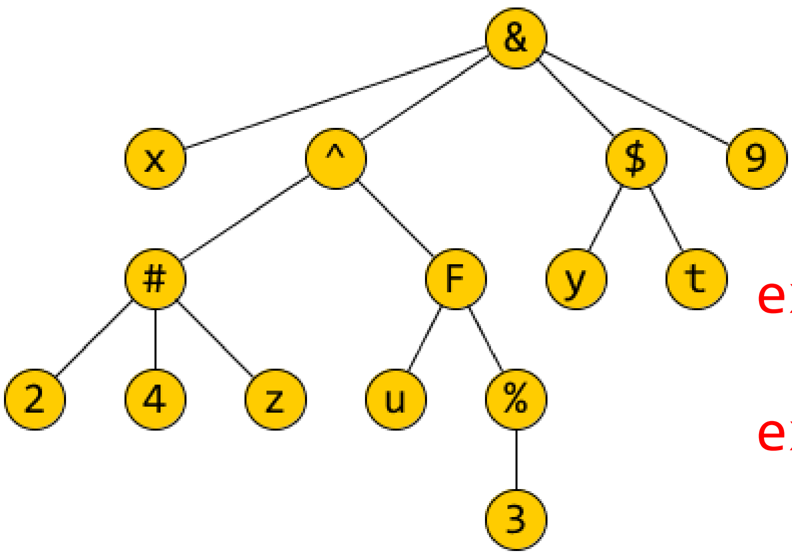

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x 2 4 z # u 3 % F ^ y t $ 9 &

Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y − 1

assuming the operators' precedence classses are as follows:

	prefix unary ops	**binary ops**	**associativity**
Class 1	~		*right*-associative
Class 2	+ −	+ −	*left*-associative
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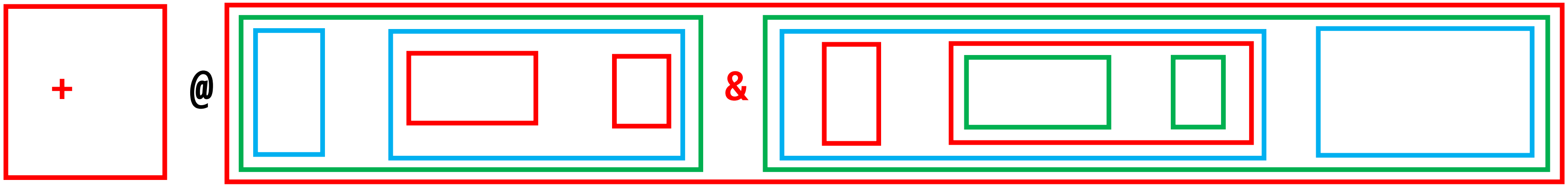

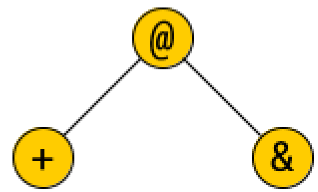

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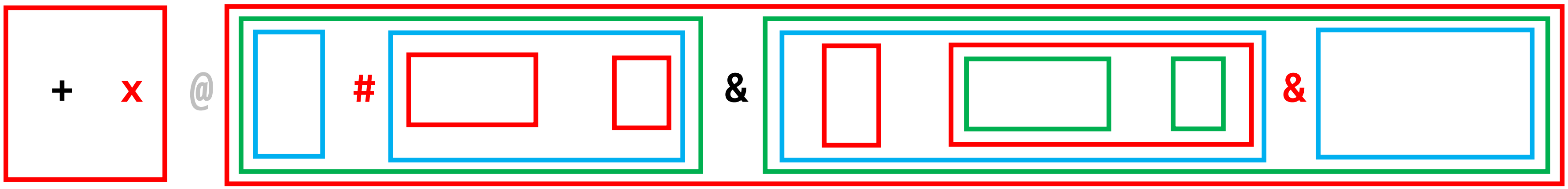

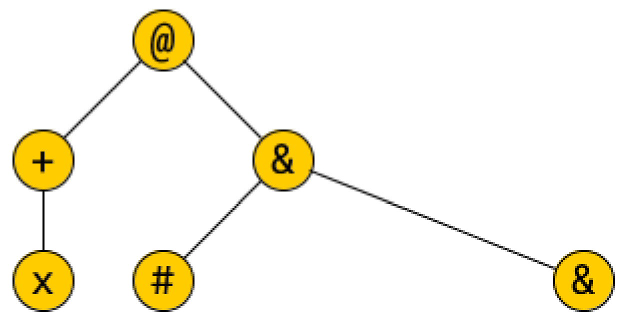

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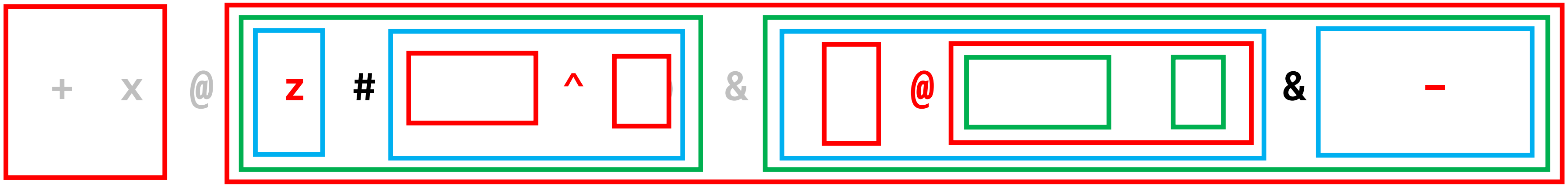

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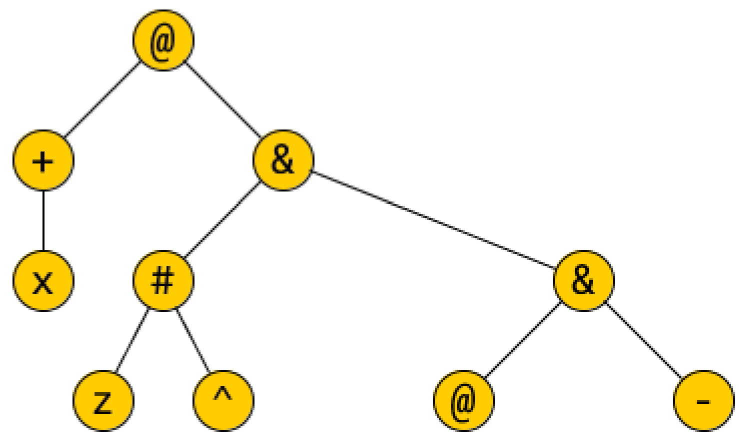

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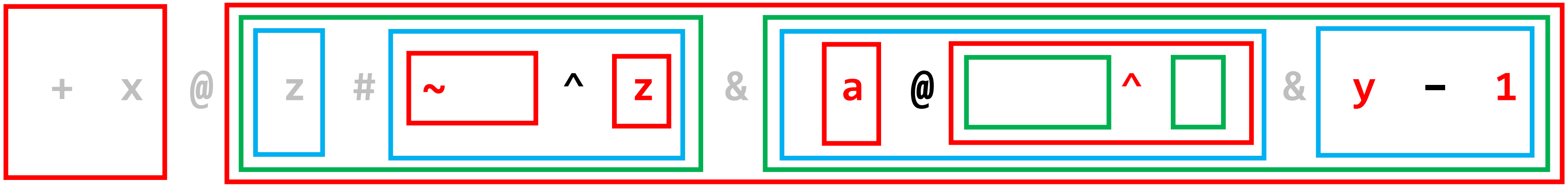

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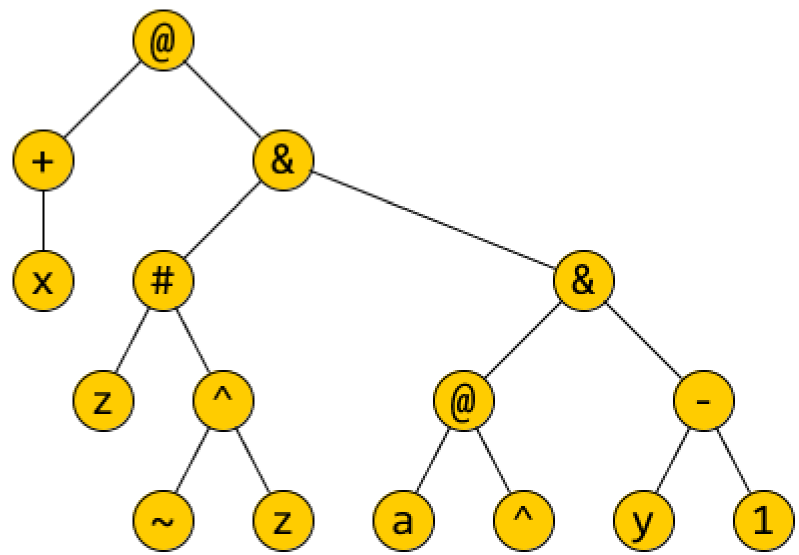

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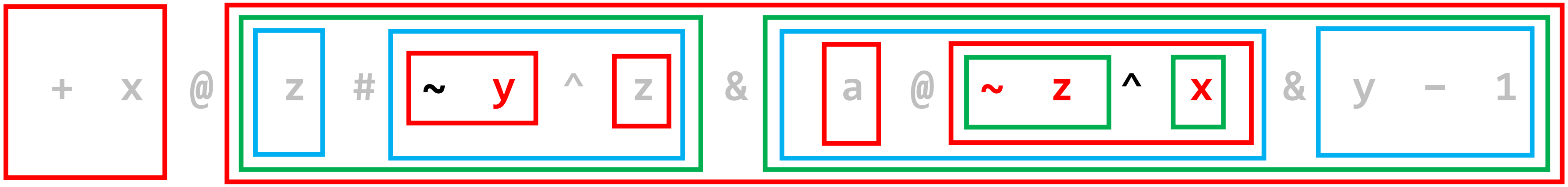

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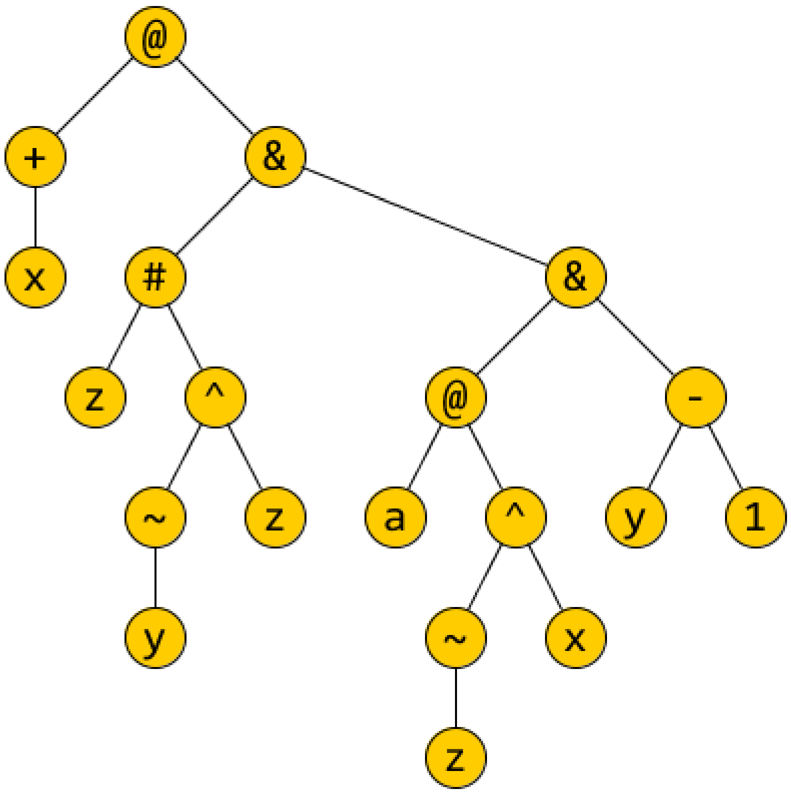

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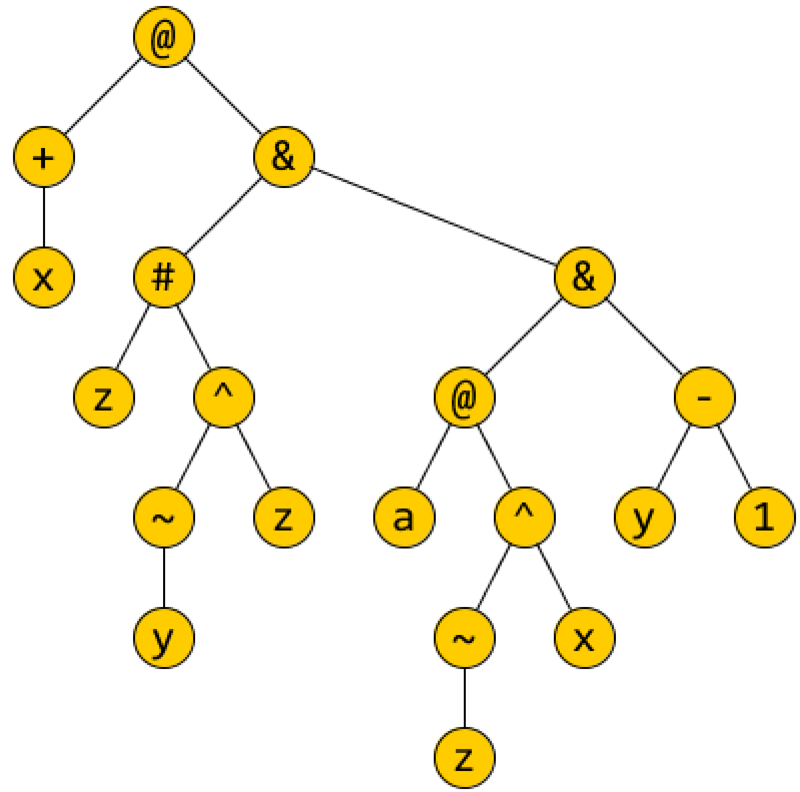

ASTs of Language Constructs Other Than Expressions

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- The subtrees of the root are ASTs of substructures whose meanings determine the meaning of the construct.

Example of a Possible AST of a Java Statement

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if (a+1 > b) {
  x = 2;
  b = a;
}
else {
  while (c < 0) {
    a += x;
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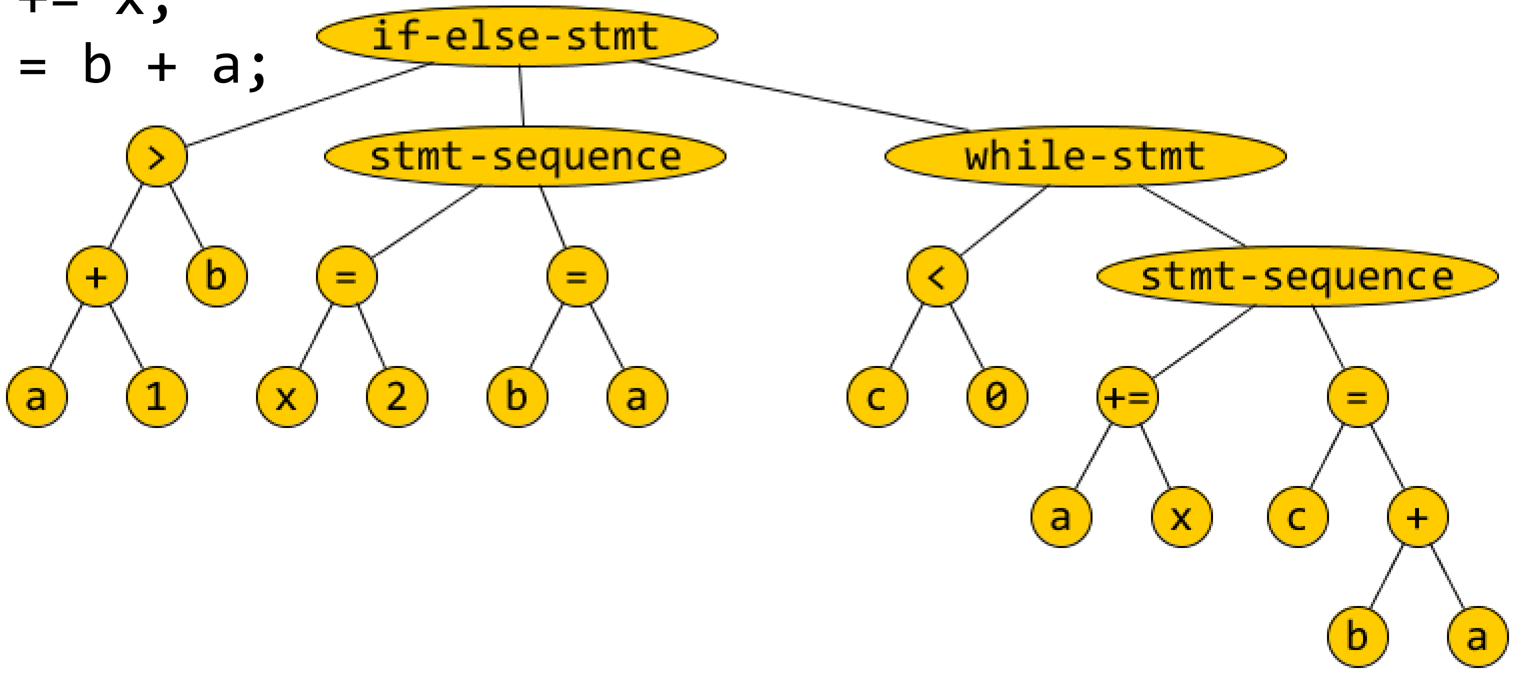

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The last homework exercise in section A of https://euclid.cs.qc.cuny.edu/316/Syntax-Reading-and-Exercises.pdf asks you to evaluate a postfix expression in this way!

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Prefix expressions can be evaluated in a similar way, if we read the expression from ***right to left***.

Translating Prefix/Postfix Notations to Lisp/“rpnLisp”

Recall:

- ***Prefix*** notation = ***Lisp*** *notation* *without parentheses*.
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Lisp: (*$_3$ x (-$_2$ (+$_3$ 2 3 y) (*$_2$ w x)) 5)

rpnLisp: (x ((2 3 y +$_3$) (w x *$_2$) -$_2$) 5 *$_3$)

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Q. Given a prefix / postfix expression, how can we insert parentheses to produce an equivalent Lisp / rpnLisp expression?

A.

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A. We can use variants of the stack-based algorithms for evaluating prefix / postfix expressions.

Translating Prefix/Postfix Notations to Lisp/"rpnLisp"

Recall:

- ***Prefix*** notation = ***Lisp*** *notation without parentheses*.
- ***Postfix*** notation = *"**rpnLisp**" notation without parentheses*.

Lisp: $(*_3\ x\ (-_2\ (+_3\ 2\ 3\ y)\ (*_2\ w\ x))\ 5)$

Prefix notation: $*_3\ x\ -_2\ +_3\ 2\ 3\ y\ *_2\ w\ x\ 5$

rpnLisp: $(x\ ((2\ 3\ y\ +_3)\ (w\ x\ *_2)\ -_2)\ 5\ *_3)$

Postfix notation: $x\ 2\ 3\ y\ +_3\ w\ x\ *_2\ -_2\ 5\ *_3$

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Notation: We will write $\boxed{\textbf{op}\ e_1\ \ldots\ e_k}$ and $\boxed{e_1\ \ldots\ e_k\ \textbf{op}}$ for the Lisp and rpnLisp expressions $(\textbf{op}\ e_1\ \ldots\ e_k)$ and $(e_1\ \ldots\ e_k\ \textbf{op})$.

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The Lisp expression $(*_3\ x\ (-_2\ (+_3\ 2\ 3\ y)\ (*_2\ w\ x))\ 5)$

will be written $\boxed{*_3\ x\ \boxed{-_2\ \boxed{+_3\ 2\ 3\ y}\ \boxed{*_2\ w\ x}}\ 5}$

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The rpnLisp expression $(x\ ((2\ 3\ y\ +_3)\ (w\ x\ *_2)\ -_2)\ 5\ *_3)$

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Example Translate the following postfix expression into rpnLisp: $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

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***UNREAD* INPUT:** $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

STACK:

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STACK: **x**

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Example Translate the following postfix expression into rpnLisp: **x 2 3 $+_3$ y $-_2$ u x 5 $*_2$ $*_3$ $-_1$**

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** **2 3 $+_3$ y $-_2$ u x 5 $*_2$ $*_3$ $-_1$**

STACK: **x 2**

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Example Translate the following postfix expression into rpnLisp: $\mathbf{x \ \ 2 \ \ 3 \ \ +_3 \ \ y \ \ -_2 \ \ u \ \ x \ \ 5 \ \ *_2 \ \ *_3 \ \ -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $\mathbf{3 \ \ +_3 \ \ y \ \ -_2 \ \ u \ \ x \ \ 5 \ \ *_2 \ \ *_3 \ \ -_1}$

STACK: **x 2 3**

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***UNREAD* INPUT:** $+_3 \ \ y \ \ -_2 \ \ u \ \ x \ \ 5 \ \ *_2 \ \ *_3 \ \ -_1$

STACK: $\boxed{x \ \ 2 \ \ 3 \ \ +_3}$

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***UNREAD* INPUT:** $\mathbf{y \;\; -_2 \;\; u \;\; x \;\; 5 \;\; *_2 \;\; *_3 \;\; -_1}$

STACK: $\boxed{\mathbf{x \;\; 2 \;\; 3 \;\; +_3}}$ $\mathbf{y}$

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***UNREAD* INPUT:** $\mathbf{-_2\quad u\quad x\quad 5\quad *_2\quad *_3\quad -_1}$

STACK: $\boxed{\boxed{\mathbf{x\quad 2\quad 3\quad +_3}}\quad \mathbf{y\quad -_2}}$

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***UNREAD* INPUT:** $\mathbf{u \ x \ 5 \ *_2 \ *_3 \ -_1}$

STACK: $\boxed{\boxed{\mathbf{x \ 2 \ 3 \ +_3}} \ \mathbf{y \ -_2}} \ \mathbf{u}$

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***UNREAD* INPUT:** x **5 $*_2$ $*_3$ $-_1$**

STACK: $\boxed{\boxed{\mathbf{x\ \ 2\ \ 3\ \ +_3}}\ \mathbf{y\ \ -_2}}$ **u x**

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***UNREAD* INPUT:** $\mathbf{5 \quad *_2 \quad *_3 \quad -_1}$

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Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $*_2 \quad *_3 \quad -_1$

STACK: $\boxed{\boxed{x \quad 2 \quad 3 \quad +_3} \quad y \quad -_2} \quad u \quad \boxed{x \quad 5 \quad *_2}$

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***UNREAD* INPUT:** $*_3 \quad -_1$

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***UNREAD* INPUT:** $-_1$

STACK: $\boxed{\boxed{\boxed{\boxed{x \quad 2 \quad 3 \quad +_3} \quad y \quad -_2} \quad u \quad \boxed{x \quad 5 \quad *_2} \quad *_3} \quad -_1}$

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After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

rpnLisp equivalent of the expression

$$\boxed{\boxed{\boxed{\boxed{x \quad 2 \quad 3 \quad +_3} \quad y \quad -_2} \quad u \quad \boxed{x \quad 5 \quad *_2} \quad *_3} \quad -_1}$$

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After the entire expression has been processed in this way, the Lisp equivalent of the prefix expression will be the only thing on the stack.

Example Translate the following prefix expression into Lisp.

$$*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x \quad 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

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***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x \quad 5$

STACK:

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$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3\quad x\quad -_2\quad +_3\quad 2\quad 3\quad y\quad *_2\quad w\quad x\quad 5$

STACK: **5**

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After the entire expression has been processed in this way, the Lisp equivalent of the prefix expression will be the only thing on the stack.

Example Translate the following prefix expression into Lisp.

$*_3$ x $-_2$ $+_3$ 2 3 y $*_2$ w x 5

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3$ x $-_2$ $+_3$ 2 3 y $*_2$ w x

STACK: x 5

We can use a stack as follows to translate a **prefix** expression to **Lisp**:

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$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w$

STACK: **w x 5**

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After the entire expression has been processed in this way, the Lisp equivalent of the prefix expression will be the only thing on the stack.

Example Translate the following prefix expression into Lisp.

$$*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x \quad 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2$

STACK: $\boxed{*_2\ w\ x}\quad 5$

We can use a stack as follows to translate a **prefix** expression to **Lisp**:

- Read the expression *from* *right to left*.
- *Push* each variable or constant that is seen.
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***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2$

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- The structure of the Lisp / rpnLisp equivalent of a prefix / postfix expression does ***not*** depend on the names and semantics of the operators, but only depends on the *arities* of the operators.

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x 2 3 $@_3$ y $\#_2$ u x 5 $\hat{}_2$ $!_3$ $\sim_1$

Here $@_3$ and $!_3$ are 3-ary, $\hat{}_2$ and $\#_2$ are binary, and $\sim_1$ is unary.

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> Translate the following postfix expression into rpnLisp:
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> Here $@_3$ and $!_3$ are 3-ary, $\wedge_2$ and $\#_2$ are binary, and $\sim_1$ is unary.

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that we solved above: Substituting $@_3$, $!_3$, $\wedge_2$, $\#_2$, and $\sim_1$ for $+_3$, $*_3$, $*_2$, $-_2$, and $-_1$ in our solution to the latter problem gives a solution to the former problem.

Functions That Return Functions as Their Results

7.16 FUNCTIONS THAT MAKE FUNCTIONS From Touretzky's book

It is possible to write a function whose value is another function. Suppose we want to make a function that returns true if its input is greater than a certain number N. We can make this function by constructing a lambda expression that refers to N, and returning that lambda expression:

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(defun make-greater-than-predicate (n)
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The value returned by MAKE-GREATER-THAN-PREDICATE will be a lexical closure. We can store this value away somewhere, or pass it as an argument to FUNCALL or any applicative operator.

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> (setf pred (make-greater-than-predicate 3))
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Any function that takes a function as an argument or returns a function as its result is called a ***higher-order function***.

More Examples of Functions That Return Functions

A function that takes two single argument functions as its arguments and returns their composition:

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[1]> (defun compose (f g) (lambda (x) (funcall f (funcall g x))))
COMPOSE
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A function that takes a 2-argument function h as argument and returns a function h′ such that (h′ x y) = (h y x):

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((D E F G) A B C)
[9]> (funcall (flip-arguments #'list) '(A B C) '(D E F G))
((D E F G) (A B C))
[10]>
```