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that can be used to compute $(1 + 10^{-25})^{10^{25}}$?

- A solution is given by the function below:

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(defun power (z n)
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(defun power-mod (m n k) ; computes m^n mod k
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        (t (let ((X (power-mod m (floor n 2) k)))
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(defun pwr (z n)
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- The following function performs ***modular*** exponentiation in an analogous way:

```
(defun pwr-mod (m n k) ; computes m^n mod k
  (cond
    ((zerop n) 1)
    ((evenp n) (pwr-mod (mod (* m m) k) (/ n 2) k))
    (t (mod (* m (pwr-mod (mod (* m m) k) (floor n 2) k)) k))))
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- (merge-lists L1 L2) returns a list that is equal to the list we would get if we sorted the list returned by (append L1 L2) into ascending order.

Two obvious recursive strategies to consider are:

1. Compute (merge-lists L1 L2) from (merge-lists (cdr L1) L2).
2. Compute (merge-lists L1 L2) from (merge-lists L1 (cdr L2)).

Example: L1 = (2 3 3 5 9 12) L2 = (8 10 11 14)

Strategy **1** is right ***in this example***, because **(car L1) < (car L2)**.

Example: L1 = (8 10 11 14) L2 = (2 3 3 5 9 12)

Strategy **2** is right ***in this example***, because **(car L2) < (car L1)**.

Example: L1 = (2 8 10 11 14) L2 = (2 3 3 5 9 12)

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(merge-lists (cdr L1) L2) and (merge-lists L1 (cdr L2)) are ***equal*** as **(car L2) = (car L1)**. Both strategies are good!

Using Different Recursive Calls for Different Argument Values

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Note: The MERGE-LISTS, UNREPEATED-ELTS, and REPEATED-ELTS functions are expected to make different direct recursive calls in different cases, but ***there should be no case in which in which these functions make more than one direct recursive call***!

Multiple Recursion

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Sethi gives an example of such a function (written in Scheme) in Example 10.1 of the course reader:

Example 10.1 We get a *flattened* form of a list if we ignore all but the initial opening and final closing parenthesis in the written representation of a list. The flattened form of

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((a) ((b b)) (((c c c))))
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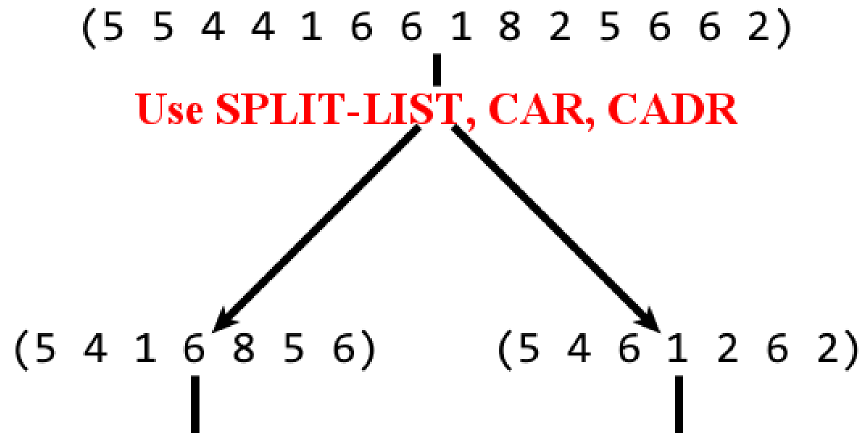

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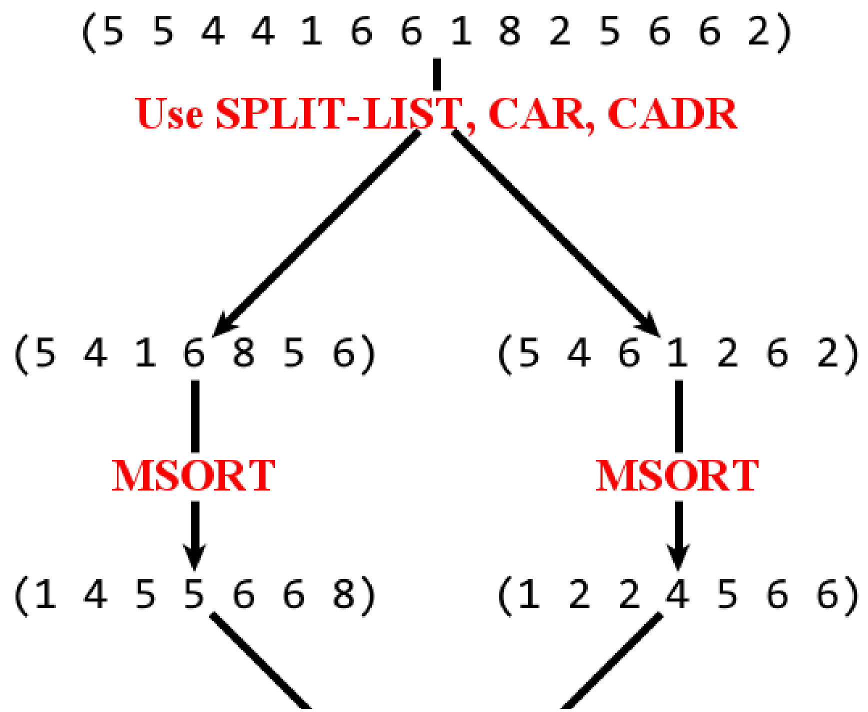

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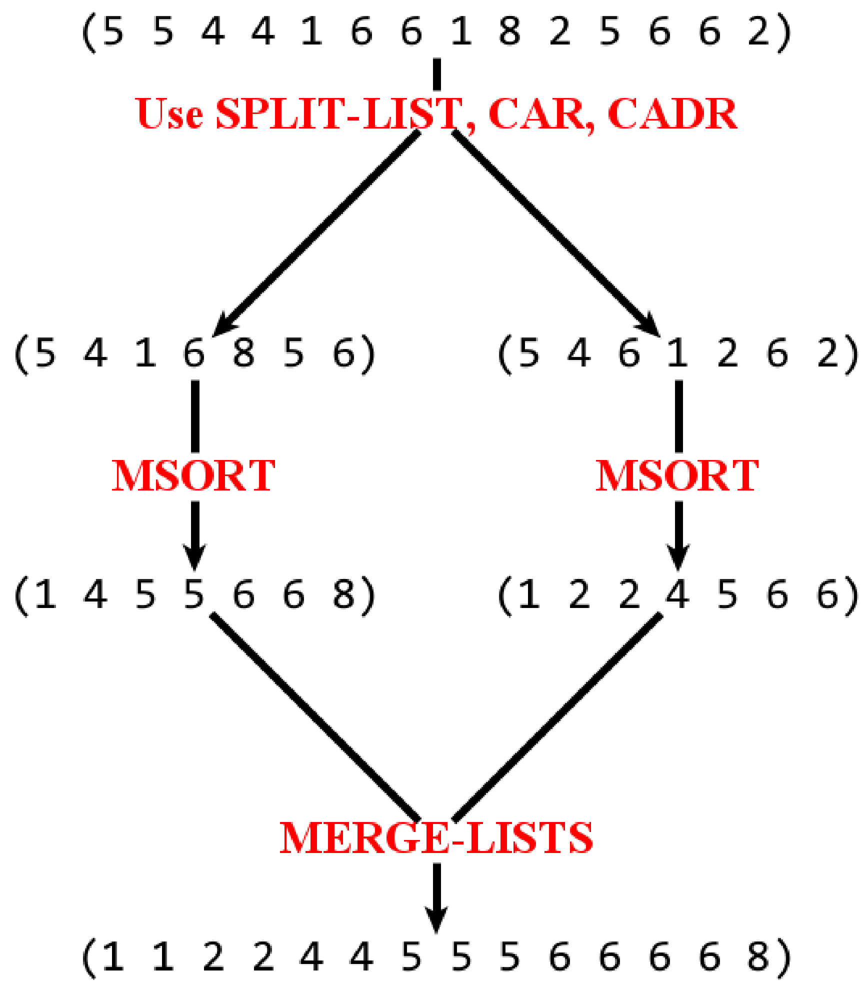

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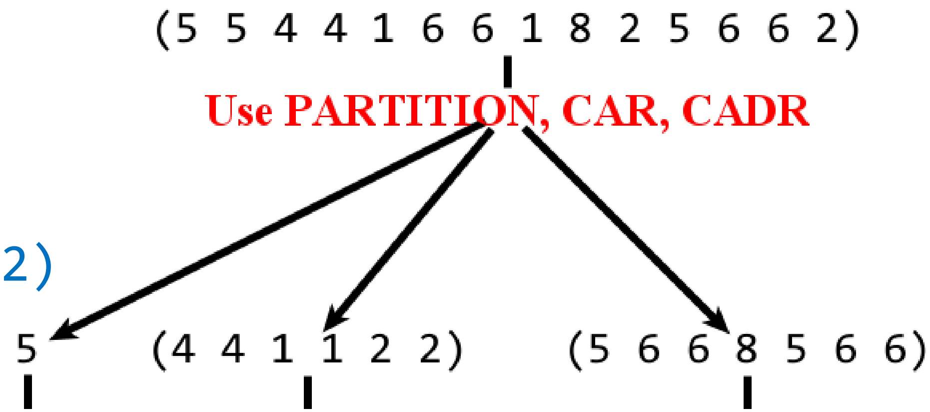

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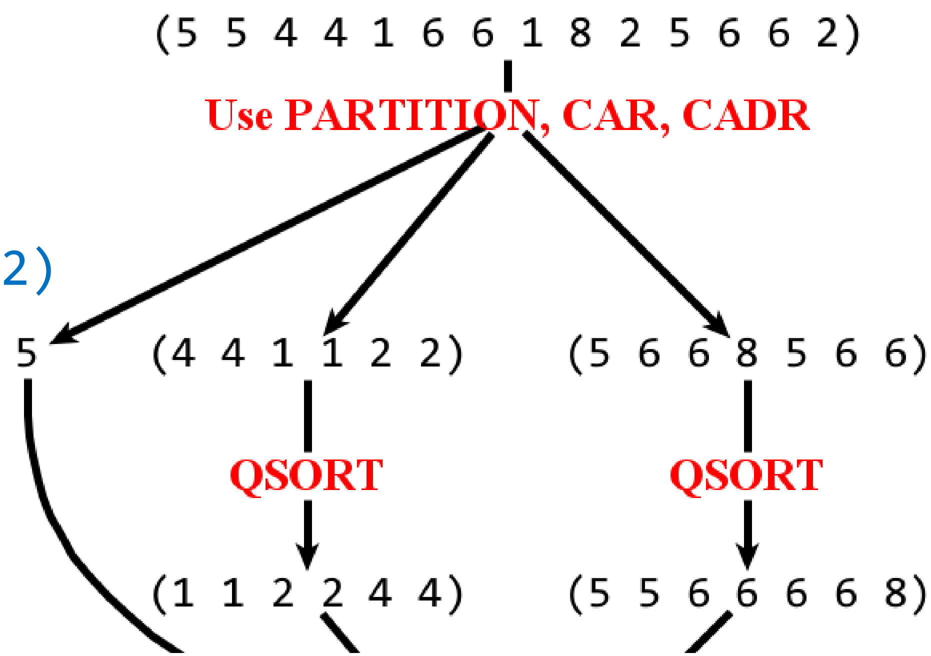

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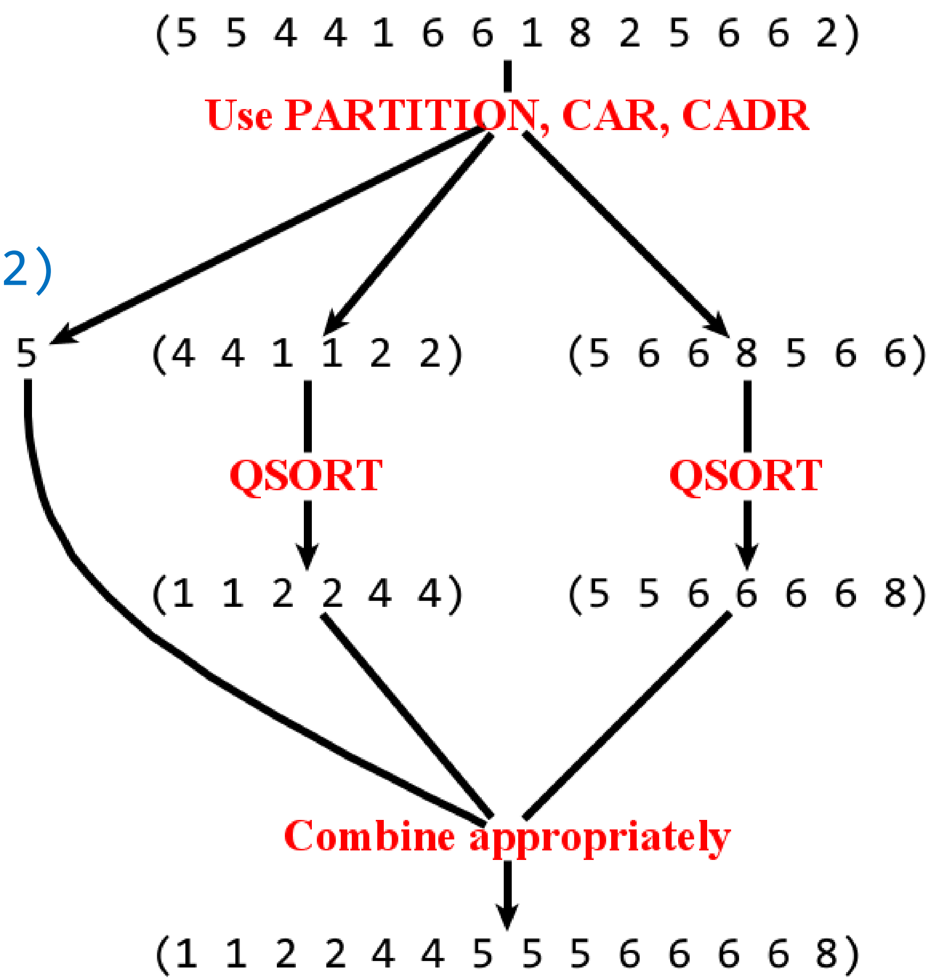

Functions That Take Functions as Arguments

Examples of Functions That Take Functions as Arguments

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Consider a function SIGMA such that, if
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then (sigma g j k) => g(j) + g(j+1) + ... + g(k).
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2. How do we *write* functions like SIGMA that take functions as arguments?

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To allow students to quickly test examples in clisp we first consider three *built-in* functions,

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NOTES: Scheme has built-in functions **map** and **filter** that are analogous to MAPCAR and REMOVE-IF-NOT (though **filter** isn't provided by kawa Scheme).

Problem 11 of Lisp Assignment 5 asks you to write a function **SUBSET** that behaves like the built-in function **REMOVE-IF-NOT**.

7.3 THE MAPCAR OPERATOR

From Touretzky's book.

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(defun square (n) (* n n))

(square 3)  ⇒  9

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With MAPCAR we can apply SQUARE to each element of the list individually. To pass the SQUARE function as an input to MAPCAR, we quote it by writing #'SQUARE.

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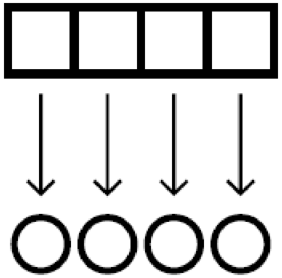
```

Here is a graphical description of the MAPCAR operator. As you can see, each element of the input list is mapped independently to a corresponding element in the output.



When MAPCAR is used on a list of length n, the resulting list also has exactly n elements. So if MAPCAR is used on the empty list, the result is the empty list.

```
(mapcar #'square '())  ⇒  nil
```

Some exercises from Touretzky's book:

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7.2. Let the global variable DAILY-PLANET contain the following table:

```
((olsen jimmy 123-76-4535 cub-reporter)
 (kent  clark 089-52-6787 reporter)
 (lane  lois  951-26-1438 reporter)
 (white perry 355-16-7439 editor))
```

Each table entry consists of a last name, a first name, a social security number, and a job title. Use MAPCAR on this table to extract a list of social security numbers.

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7.3. Write an expression to apply the ZEROP predicate to each element of the list (2 0 3 4 0 -5 -6). The answer you get should be a list of Ts and NILs.

7.4. Suppose we want to solve a problem similar to the preceding one, but instead of testing whether an element is zero, we want to test whether it is greater than five. We can't use > directly for this because > is a function of two inputs; MAPCAR will only give it one input. Show how first writing a one-input function called GREATER-THAN-FIVE-P would help.

If we don't expect to use the GREATER-THAN-FIVE-P function of exercise 7.4 elsewhere, we can give a more concise solution to the exercise: We can *use a* ***lambda expression*** to *create the function without naming it.*

7.5 LAMBDA EXPRESSIONS

From Touretzky's book.

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There are two ways to specify the function to be used by an applicative operator. The first way is to define the function with DEFUN and then specify it by #’*name*, as we have been doing. The second way is to pass the function definition directly. This is done by writing a list called a **lambda expression**. For example, the following lambda expression computes the square of its input:

```
(lambda (n) (* n n))
```

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Since lambda expressions are functions, they can be passed directly to MAPCAR by quoting them with #'. This saves you the trouble of writing a separate DEFUN before calling MAPCAR.

```
> (mapcar #'(lambda (n) (* n n)) '(1 2 3 4 5))
(1 4 9 16 25)
```

The #' before (lambda is now optional!

From sec. 7.5 of Touretzky's book:

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Here is another example of the use of MAPCAR along with a lambda expression. We will turn each element of a list of names into a list (HI THERE *name*).

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```
> (mapcar #'(lambda (x) (list 'hi 'there x))
          '(joe fred wanda))
((HI THERE JOE) (HI THERE FRED) (HI THERE WANDA))
```

7.8 REMOVE-IF AND REMOVE-IF-NOT

From Touretzky's book.

REMOVE-IF is another applicative operator that takes a predicate as input. REMOVE-IF removes all the items from a list that satisfy the predicate, and returns a list of what's left.

```
> (remove-if #'numberp '(2 for 1 sale))
```

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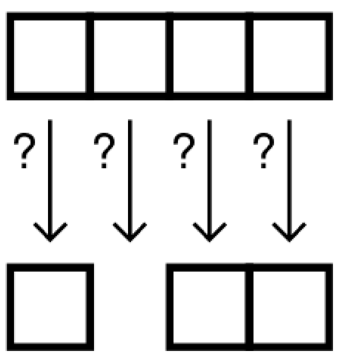
```
> (remove-if #'numberp '(2 for 1 sale))
(FOR SALE)

> (remove-if #'oddp '(1 2 3 4 5 6 7))
(2 4 6)
```

Here is a graphical description of REMOVE-IF:



The REMOVE-IF-NOT operator is used more frequently than REMOVE-IF. It works just like REMOVE-IF except it automatically inverts the sense of the predicate. This means the only items that will be removed are those for which the predicate returns NIL. So REMOVE-IF-NOT returns a list of all the items that *satisfy* the predicate. Thus, if we choose PLUSP as the predicate, REMOVE-IF-NOT will find all the positive numbers in a list.

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```
> (remove-if-not #'plusp '(2 0 -4 6 -8 10))
(2 6 10)
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From Touretzky's Book

Here are some additional examples of REMOVE-IF-NOT:

```
> (remove-if-not #'(lambda (x) (> x 3))
                 '(2 4 6 8 4 2 1))
(4 6 8 4)

> (remove-if-not #'numberp
     '(3 apples 4 pears and 2 little plums))
(3 4 2)

> (remove-if-not #'symbolp
      '(3 apples 4 pears and 2 little plums))
(APPLES PEARS AND LITTLE PLUMS)
```

Using Functions That Take Functions as Arguments and Lambda Expressions to Define Your Own Functions

Here is a function, COUNT-ZEROS, that counts how many zeros appear in a list of numbers. It does this by taking the subset of the list elements that are zero, and then taking the length of the result.

```
(remove-if-not #'zerop '(34 0 0 95 0))  ⇒  (0 0 0)

(defun count-zeros (x)
    (length (remove-if-not #'zerop x)))

(count-zeros '(34 0 0 95 0))  ⇒  3
```

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EXERCISES

7.11. Write a function to pick out those numbers in a list that are greater than one and less than five.

Solution (on p. C-39):

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Note: (lambda (x) (< 1 x 5)) could be changed to (lambda (y) (< 1 y 5))

Solution (on p. C-39):

7.11.
```
(defun pick (x)
    (remove-if-not #'(lambda (x) (< 1 x 5))
                   x))
```