Simple Examples of Functional Programming

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// factorial(n) ⇒ n! = 1 * 2 * ... * (n-1) * n if 1 ≤ n ≤ 20
static long factorial (int n)
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Why Factorial Works

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  If factorial(n-1) returns 1 * 2 * ... * (n-1),
  then factorial(n) returns 1 * 2 * ... * (n-1) * n.
- factorial(1) returns the right result, 1, because evaluating (n==1) ? 1 : factorial(n-1) * n when n==1 does **not** cause factorial(n-1) * n to be evaluated.

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Here is a third Java function of this kind:

// returns n^k if k > 0 and $|n|^k < 2^{63}$

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static long pwr(long n, int k)
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When $k > 1$ and $|n|^k < 2^{63}$, pwr(n,k) ⇒ the right value, n^k, if the recursive call pwr(n*n, k/2) ⇒ the right value, $(n*n)^{k/2}$, because:

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When $k = 1$, pwr(n,k) returns the right value, n.

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Examples Suppose `MyClass` is the class that contains the above functions `factorial` and `pwr`. Then:

```
sigma(MyClass::factorial, 3, 7)
```

returns

.

```
sigma(i->MyClass.pwr(i,5), 3, 7)
```

returns .

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- **recursion**

Functional programming also makes use of ***functions that take functions as arguments***:

As an illustration of this, consider a function with header

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static long sigma(Function<Integer,Long> g, int m, int n)
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that returns the *sum* of the results of applying the function given by its parameter g to each integer i, m ≤ i ≤ n.

Examples Suppose `MyClass` is the class that contains the above functions `factorial` and `pwr`. Then:

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sigma(MyClass::factorial, 3, 7)
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returns 3! + 4! + 5! + 6! + 7!
= 6 + 24 + 120 + 720 + 5040 = 5910.

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Here `i->MyClass.pwr(i,5)` is a "lambda expression": It denotes an unnamed function that maps an integer i to i^5.

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(Java does ***not*** allow this call to be written as g(m)!)

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