Syntax of Programming Languages

Syntax and Semantics of Expressions

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 (- x y), 3, and (* z z).

Overview of Some Expression Notations We will say that two expressions are <u>equivalent</u> if they have the same semantics.

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In Lisp, a function call is written as a list whose *first* element is the function name.

Now consider a notation we'll call *rpnLisp* that's the same as Lisp except in that a function call is written as a list whose *Last* element is the function name.

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In rpnLisp, rpn stands for "reverse polish notation".

The above Java expression f(g(h(1,2), f(3,4)), 5)is equivalent to the Lisp expression and this rpnLisp expression:

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Rules 2 - 5 give decompositions of e into two (rules 4 & 5) or three (rule 2 & 3) substructures, but some of these decompositions may <u>violate</u> the following important principle of syntax specification:

 The semantics of a structure should be easily definable in terms of the semantics of its syntactic substructures.

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 Sec. 2.5 of Sethi (assigned reading after Exam 1) gives another way to specify syntactically valid infix expressions that does not have this drawback.

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If e is e₁ op e₂,
 e.value = result of applying op with e₁.value and e₂.value as the 1st and 2nd arguments.

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Otherwise, let op be the operator of e that should be applied <u>last</u>. Then:

- If e is e₁ op e₂,
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The semantics of e tells you how e can be evaluated.

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Key Question: How can we determine which operator of e should be applied <u>last</u>?

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(c)
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An example of precedence and associativity rules (from the course reader).

assignment	= ,
logical or	11
logical and	&&
inclusive or	
exclusive or	^
and	&
equality	== !=
relational	< <= >= >
shift	<< >>
additive	+ -
multiplicative	* / %

Figure 2.9 A partial table of binary operators in C, in order of increasing precedence; that is, the assignment operator = has the lowest precedence and the multiplicative operators *, /, and % have the highest precedence. All operators on the same line have the same precedence and associativity. The assignment operator is right associative; all the other operators are left associative.

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Using these rules, we can find the operator of e that should be applied last as follows:

1.

2.

2.1

2.2

2.3

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Example: Find the operator that should be applied last in this C expression: x*(y+(z+3)-2)+w-u/t

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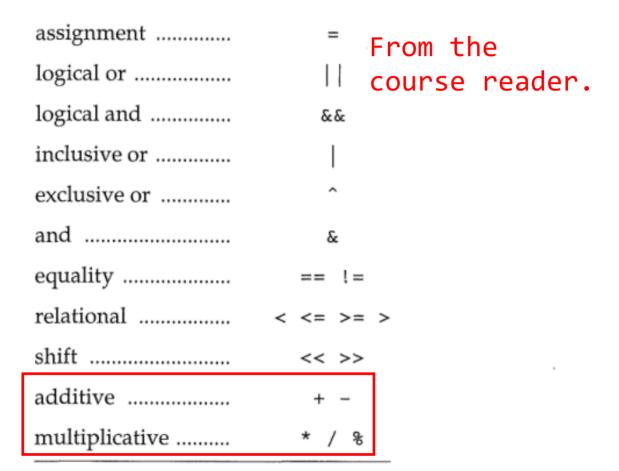


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so int y = 4; cout << y / 2 * --y; may print 3 or 6.