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(defun f (e1 e2)
  (if
    [ ]
    (let ((X [ (f (cdr e1) e2) or (f (- e1 1) e2) ]))
      [an expression that  $\Rightarrow$  value of (f e1 e2) and
        that involves X and, possibly, e1 and/or e2 ])))
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```
(defun f (e1 e2)
  (if (null e1) or (zerop e1)
      value of (f nil e2) or (f 0 e2)
      (let ((X (f (cdr e1) e2) or (f (- e1 1) e2) ))
        an expression that  $\Rightarrow$  value of (f e1 e2) and
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Example Without using `append`, write a function **append-2** such that:

If $L1 \Rightarrow$ a proper list *and* $L2 \Rightarrow$ a proper list, *then*
 $(\text{append-2 } L1 \ L2) \Rightarrow$ a list that is equal to $(\text{append } L1 \ L2)$

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*If L1 \Rightarrow a proper list and L2 \Rightarrow a proper list, then
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- To solve this problem in the above-mentioned way, we must first decide whether it is the first or the second argument of the recursive call that will have a smaller value than the corresponding argument of the current call.
- Experienced programmers are able to "look ahead" and see which of these two possibilities leads to a good function definition, *but if you can't see which choice is right then just guess*: If your guess doesn't yield a good definition, go back and make the other choice!

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- Experienced programmers are able to "look ahead" and see which of these two possibilities leads to a good function definition, *but if you can't see which choice is right then just guess*: If your guess doesn't yield a good definition, go back and make the other choice!
- We will attempt to write the function by giving the first argument of the recursive call a smaller value than the corresponding argument of the current call.
- This will turn out to be the right choice; we will see later why the other choice would not work.

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(defun append-2 (L1 L2)
```

```
  (if (null L1)
```

```
    value of (append-2 nil L2)
```

What will this be?

```
    (let ((X (append-2 (cdr L1) L2)))
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an expression that \Rightarrow value of $(\text{append-2 } L1 \ L2)$
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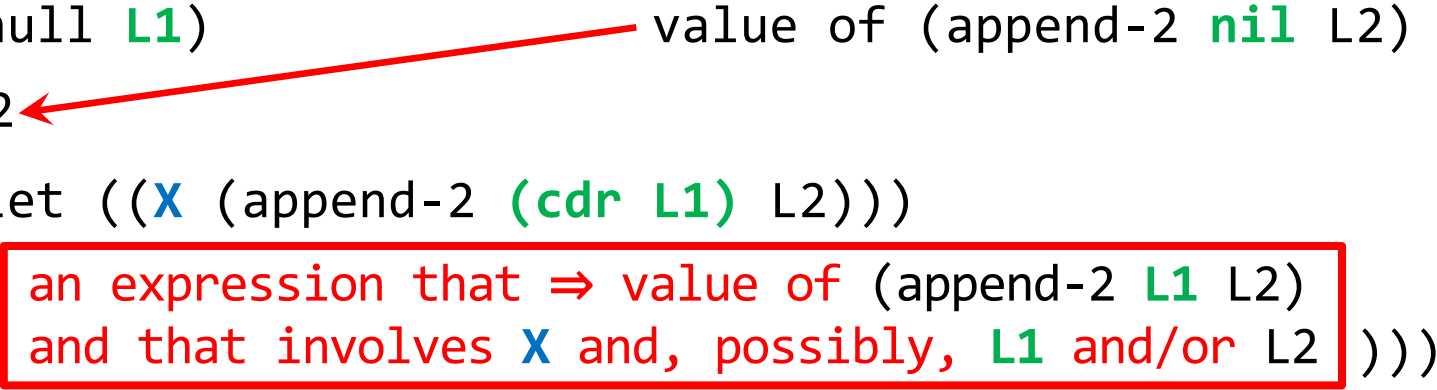


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For this L1 and L2, `...` should \Rightarrow (1 2 3 4 A B C).

Q.

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Q. What expression (involving X and, possibly, L1 and/or L2)
will \Rightarrow (1 2 3 4 A B C)? Ans.:

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Q. Is $(\text{cons } (\text{car } L1) \ X)$ a good ... expression for all

valid values of $L1$ and $L2$ such that $L1 \neq \text{NIL}$?

A. If we're not sure, try *another* pair of values of $L1$ & $L2$.

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(defun append-2 (L1 L2)
```

```
  (if (null L1)
```

```
      L2
```

```
      (let ((X (append-2 (cdr L1) L2)))
```

an expression that \Rightarrow value of $(\text{append-2 } L1 \ L2)$
and that involves X and, possibly, $L1$ and/or $L2$

```
    )))
```

- Suppose $L1 \Rightarrow (1 \ 2 \ 3 \ 4)$ and $L2 \Rightarrow (A \ B \ C)$,

so $(\text{cdr } L1) \Rightarrow (2 \ 3 \ 4)$ and $X \Rightarrow (2 \ 3 \ 4 \ A \ B \ C)$.

For this $L1$ and $L2$, ... should $\Rightarrow (1 \ 2 \ 3 \ 4 \ A \ B \ C)$.

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Example Without using `append`, write a function `append-2` such that:

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- Suppose $L1 \Rightarrow (A \ B \ C \ D \ E \ F)$ and $L2 \Rightarrow (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7)$,
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For this $L1$ and $L2$, ... should \Rightarrow .

Example Without using append, write a function **append-2** such that:

If L1 \Rightarrow a proper list *and* L2 \Rightarrow a proper list, *then*

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will \Rightarrow (1 2 3 4 A B C)? Ans.: (cons (car L1) X)

- Suppose L1 \Rightarrow (A B C D E F) and L2 \Rightarrow (1 2 3 4 5 6 7),

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○ $(\text{cons } (\text{car } L1) \ X) \Rightarrow (A \ B \ C \ D \ E \ F \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7)$ too. Good!

Example Without using append, write a function **append-2** such that:

If L1 \Rightarrow a proper list and L2 \Rightarrow a proper list, then

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- When we are satisfied that $(\text{cons } (\text{car } L1) \ X)$ is a good ... expression for all valid values of $L1$ and $L2$ such that $L1 \neq \text{NIL}$, we complete the above definition!

Example Without using `append`, write a function `append-2` such that:

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- X is never used more than once, so we *eliminate the LET*:

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(defun append-2 (L1 L2)
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Final version:

Example Without using append, write a function **append-2** such that:

*If L1 \Rightarrow a proper list and L2 \Rightarrow a proper list, then
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- In our definition of `append-2`, the *first* argument of its recursive call has a smaller value than the first argument of the current call, while the *second* argument has the same value in the recursive call as in the current call.

-

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- In our definition of `append-2`, the *first* argument of its recursive call has a smaller value than the first argument of the current call, while the *second* argument has the same value in the recursive call as in the current call.
- Why can't we define `append-2` in the opposite way—i.e., by letting the *second* argument of its recursive call have a smaller value than the second argument of the current call, and letting the *first* argument have the same value in the recursive call as in the current call?

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```
(defun append-2 (L1 L2)
```

```
  (if (null L2)
```

```
    value of (append-2 L1 nil)
```

What will this be?

```
    (let ((X (append-2 L1 (cdr L2))))
```


```
      an expression that  $\Rightarrow$  value of  $(\text{append-2 } L1 \ L2)$   
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Example Without using `append`, write a function `append-2` such that:

If `L1` \Rightarrow a proper list *and* `L2` \Rightarrow a proper list, *then*
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(defun append-2 (L1 L2)
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Value of `(append-2 L1 nil)`.
L1 

- To *try* to write the ... expression, let's consider a *possible pair of values of* `L1` *and* `L2`, *the resulting value of* `X`, and *what* ... *'s value should be in this case:*

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Value of $(\text{append-2 } L1 \ \text{nil})$.
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- To try to write the `...` expression, let's consider a *possible pair of values of L1 and L2, the resulting value of X*, and *what `...`'s value should be in this case*:
- Suppose L1 \Rightarrow (1 2 3 4) and L2 \Rightarrow (A B C D E),
so (cdr L2) \Rightarrow (B C D E) and X \Rightarrow (1 2 3 4 B C D E).
For this L1 and L2, `...` should \Rightarrow (1 2 3 4 A B C D E).
-

Example Without using `append`, write a function `append-2` such that:

*If L1 \Rightarrow a proper list and L2 \Rightarrow a proper list, then
(`append-2` L1 L2) \Rightarrow a list that is equal to (`append` L1 L2)*

```
(defun append-2 (L1 L2)
  (if (null L2)
      L1
      (let ((X (append-2 L1 (cdr L2))))
        an expression that  $\Rightarrow$  value of (append-2 L1 L2)
        and that involves X and, possibly, L1 and/or L2 ))))
```

- To try to write the `...` expression, let's consider a *possible pair of values of L1 and L2, the resulting value of X*, and *what `...`'s value should be in this case*:
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so (`cdr` L2) \Rightarrow (B C D E) and X \Rightarrow (1 2 3 4 B C D E).
For this L1 and L2, `...` should \Rightarrow (1 2 3 4 A B C D E).
- There's no good way to construct (1 2 3 4 A B C D E) from (1 2 3 4 B C D E), (1 2 3 4), and (A B C D E), so there's no good way to write `...`!

Example Without using append, write a function **append-2** such that:

If L1 \Rightarrow a proper list *and* L2 \Rightarrow a proper list, *then*

(append-2 L1 L2) \Rightarrow a list that is equal to (append L1 L2)

```
(defun append-2 (L1 L2)
```

```
  (if (null L2)
```

```
      L1
```

```
      (let ((X (append-2 L1 (cdr L2))))
```

an expression that \Rightarrow value of (append-2 L1 L2)
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- There's no good way to construct (1 2 3 4 A B C D E) from
(1 2 3 4 B C D E), (1 2 3 4), and (A B C D E), so there's
no good way to write ...!
-

Example Without using append, write a function **append-2** such that:

If $L1 \Rightarrow$ a proper list and $L2 \Rightarrow$ a proper list, then
 $(\text{append-2 } L1 \ L2) \Rightarrow$ a list that is equal to $(\text{append } L1 \ L2)$

```
(defun append-2 (L1 L2)
  (if (null L2)
      L1
      (let ((X (append-2 L1 (cdr L2))))
        an expression that  $\Rightarrow$  value of  $(\text{append-2 } L1 \ L2)$ 
        and that involves X and, possibly, L1 and/or L2 ))))
```

- Suppose $L1 \Rightarrow (1 \ 2 \ 3 \ 4)$ and $L2 \Rightarrow (A \ B \ C \ D \ E)$,
so $(\text{cdr } L2) \Rightarrow (B \ C \ D \ E)$ and $X \Rightarrow (1 \ 2 \ 3 \ 4 \ B \ C \ D \ E)$.
For this $L1$ and $L2$, ... should $\Rightarrow (1 \ 2 \ 3 \ 4 \ A \ B \ C \ D \ E)$.
- There's no good way to construct $(1 \ 2 \ 3 \ 4 \ A \ B \ C \ D \ E)$ from
 $(1 \ 2 \ 3 \ 4 \ B \ C \ D \ E)$, $(1 \ 2 \ 3 \ 4)$, and $(A \ B \ C \ D \ E)$, so there's
no good way to write ...!
- So our original decision to let the second (rather than the first) argument of **append-2** have the same value in the recursive call as in the current call was the right decision!

Example Write a function **all-numbers** such that:

If $L \Rightarrow$ a proper list, then

$(\text{all-numbers } L) \Rightarrow T$ if every element of the list is a number

$(\text{all-numbers } L) \Rightarrow \text{NIL}$ otherwise.

So: $(\text{all-numbers } '(6\ 2\ 6)) \Rightarrow T$; $(\text{all-numbers } '(7\ 1\ \text{DOG}\ 9)) \Rightarrow \text{NIL}$

Example Write a function `all-numbers` such that:

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- We'll solve this problem in the way that was described above:

```
(defun all-numbers (L)
```

```
  (if (null L)
```

```
    value of (all-numbers nil)
```

```
    (let ((X (all-numbers (cdr L))))
```

```
      an expression that  $\Rightarrow$  value of (all-numbers L)
```

```
      and that involves X and, possibly, L      )))
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Example Write a function **all-numbers** such that:

If $L \Rightarrow$ a proper list, then

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- We'll solve this problem in the way that was described above:

```
(defun all-numbers (L)
```

```
  (if (null L)
```

```
    T
```

We see from the spec that
 $(\text{all-numbers nil}) \Rightarrow T$.

```
    (let ((X (all-numbers (cdr L))))
```

an expression that \Rightarrow value of $(\text{all-numbers } L)$
and that involves **X** and, possibly, **L**

```
    )))
```

•

Example Write a function `all-numbers` such that:

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```

```
      T
```

```
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```

```
        an expression that  $\Rightarrow$  value of  $(\text{all-numbers } L)$ 
```

```
        and that involves X and, possibly, L      )))
```

- We also see from the spec that $(\text{and } X\ (\text{numberp } (\text{car } L)))$ would be a correct ... expression, so we can now complete the definition!

Example Write a function **all-numbers** such that:

If $L \Rightarrow$ a proper list, then

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```
(defun all-numbers (L)
  (if (null L)
      T
      (let ((X (all-numbers (cdr L))))
        (and X (numberp (car L))) )))
```

•

Example Write a function `all-numbers` such that:

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      T
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```

- `X` is never used more than once, so we eliminate the LET:

```
(defun all-numbers (L)
  (if (null L)
      T
      (let ((X (all-numbers (cdr L))))
        (and X (all-numbers (cdr L)) (numberp (car L)))))))
```

Example Write a function `all-numbers` such that:

If $L \Rightarrow$ a proper List, then

$(\text{all-numbers } L) \Rightarrow T$ if every element of the List is a number

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```
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```

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- **X** is never used more than once, so we eliminate the LET:

```
(defun all-numbers (L)
  (if (null L)
      T
      (and (all-numbers (cdr L)) (numberp (car L)))))
```

RECALL:

Example Write a function `all-numbers` such that:

If $L \Rightarrow$ a proper list, then

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(defun all-numbers (L)
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RECALL:

- If the LET isn't eliminated, move any case in which **X** needn't be used out of the LET. If the LET is eliminated but there's a case where the recursive call's result isn't needed, deal with such cases as base cases--i.e., without making a recursive call.

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In the case $(\text{numberp } (\text{car } L)) \Rightarrow \text{NIL}$, the result of the recursive call $(\text{all-numbers } (\text{cdr } L))$ isn't needed, as the function will return NIL regardless of what that call returns!
So we rewrite the code to deal with that case without the call.

Example Write a function `all-numbers` such that:

If $L \Rightarrow$ a proper list, then

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- **X** is never used more than once, so we eliminate the LET:

```
(defun all-numbers (L)
  (if (null L)
      T
      (and (numberp (car L)) (all-numbers (cdr L)))))
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RECALL:

- If the LET isn't eliminated, move any case in which **X** needn't be used out of the LET. If the LET is eliminated but there's a case where the recursive call's result isn't needed, deal with such cases as base cases--i.e., without making a recursive call.

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```
(defun all-numbers (L)
  (if (null L)
      T
      (and (numberp (car L)) (all-numbers (cdr L)))))
```

- Final cleanup:

Example Write a function `all-numbers` such that:

If $L \Rightarrow$ a proper list, then

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```
(defun all-numbers (L)
  (if (null L)
      T
      (and (numberp (car L)) (all-numbers (cdr L)))))
```

- **Final cleanup:**

Since $(\text{if } c\ T\ e) = (\text{or } c\ e)$ if the value of c is always either T or NIL , we can simplify the above definition to:

```
(defun all-numbers (L)
  (or (null L)
      (and (numberp (car L)) (all-numbers (cdr L)))))
```

Example Write a function **safe-sum** such that:

- *If $l \Rightarrow$ a proper list of numbers, then
(safe-sum l) \Rightarrow the sum of the elements of that list.*
- *If $l \Rightarrow$ a proper list whose elements are not all numbers, then
(safe-sum l) \Rightarrow the symbol **ERR!**.*

So: (safe-sum '(7 2 4 0 9)) \Rightarrow 22; (safe-sum '(7 2 A 9)) \Rightarrow ERR!

Example Write a function **safe-sum** such that:

- If $L \Rightarrow$ a proper list of numbers, then
 $(\text{safe-sum } L) \Rightarrow$ the sum of the elements of that list.
- If $L \Rightarrow$ a proper list whose elements are not all numbers, then
 $(\text{safe-sum } L) \Rightarrow$ the symbol **ERR!**.

So: $(\text{safe-sum } '(7\ 2\ 4\ 0\ 9)) \Rightarrow 22$; $(\text{safe-sum } '(7\ 2\ A\ 9)) \Rightarrow \text{ERR!}$

```
(defun safe-sum (L)
  (if (null L)
```

```
    value of (safe-sum nil)
```

```
    (let ((X (safe-sum (cdr L))))
```

```
      an expression that  $\Rightarrow$  value of (safe-sum L)
      and that involves X and, possibly, L
```

```
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```

Example Write a function **safe-sum** such that:

- If $L \Rightarrow$ a proper list of numbers, then
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So: $(\text{safe-sum } '(7\ 2\ 4\ 0\ 9)) \Rightarrow 22$; $(\text{safe-sum } '(7\ 2\ A\ 9)) \Rightarrow \text{ERR!}$

```
(defun safe-sum (L)
  (if (null L)
```

0

```
    (let ((X (safe-sum (cdr L))))
```

an expression that \Rightarrow value of $(\text{safe-sum } L)$
and that involves **X** and, possibly, **L**

```
    )))
```

We see from the spec that
 $(\text{safe-sum nil}) \Rightarrow 0$.

Example Write a function **safe-sum** such that:

- If $L \Rightarrow$ a proper list of numbers, then
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- To write the **...** expression, let's first consider
a possible value of L, the resulting value of X,
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```

- To write the **...** expression, let's first consider
a possible value of L, the resulting value of X,
and *what ...'s value should be for that value of L:*
- Suppose $L \Rightarrow (7\ 2\ 4\ 9\ 3)$, so $(\text{cdr } L) \Rightarrow (2\ 4\ 9\ 3)$.
Then $X \Rightarrow 2+4+9+3 = 18$ and **...** should $\Rightarrow 7+2+4+9+3 = 25$.
 - We see $(+ (\text{car } L) X)$ is a good **...** expression for this L!

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- If $L \Rightarrow$ a proper list of numbers, then
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(defun safe-sum (L)
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```

```
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an expression that \Rightarrow value of `(safe-sum L)`
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```
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Q. For what non-null values of L is $(+ (\text{car } L) X)$ a good `...`?

A. $(+ (\text{car } L) X)$ is a good `...` when

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Q. For what non-null values of L is $(+ (\text{car } L) X)$ a good `...`?

A. $(+ (\text{car } L) X)$ is a good `...` when $(\text{car } L)$ and $X \Rightarrow$ numbers
(equivalently, when $(\text{car } L) \Rightarrow$ a number and $X \not\Rightarrow \text{ERR!}$).

Example Write a function **safe-sum** such that:

- If $L \Rightarrow$ a proper list of numbers, then
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Q. For what non-null values of L is $(+ (\text{car } L) X)$ a good ... ?

A. $(+ (\text{car } L) X)$ is a good ... when $(\text{car } L)$ and $X \Rightarrow$ numbers
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Example Write a function **safe-sum** such that:

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- In this case **X** is used as the argument of (numberp X),
and used again as the argument of (+ (car L) X)!

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 - The value of `'ERR!` is returned, and `'ERR!` doesn't use `X`.

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1st version
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