**Example** Write a function **safe-sum** such that:

- *If l ⇒ a proper list of numbers, then*
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- *If l ⇒ a proper list whose elements are **not** all numbers, then*
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- Eliminating the LET would produce the function on the next slide, or an equivalent function that uses COND instead of nested IFs. Those functions would be ***extremely inefficient*** when L is a list of numbers: Their running time grows ***exponentially*** with the length of the list.

- Eliminating LET from the 1st version of the definition gives:

**(defun safe-sum (L) ; very inefficient!**<br>
**(if (null L)**<br>
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- These versions never make more than one direct recursive call, as a result of which **(safe-sum '(0 1 … 49))** computes its result using just 50 recursive calls rather than quadrillions!

**Comments on Lisp Assignment 4**

Problems 1 – 13 can be solved by starting with one of the templates below or a dual of the $2^{nd}$ template in which the roles of e1 and e2 are switched. (These are just the templates presented earlier!)

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**Recall that:**

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**Recall that:**

- If there is no case in which **X** is used more than once, then ***eliminate the LET***.
- If the LET isn't eliminated, ***move any case in which X needn't be used out of the*** LET. If the LET ***is*** eliminated but ***there's a case where the recursive call's result isn't needed, deal with such cases as base cases--i.e., without making a recursive call***.

**Debugging Suggestions**

For concreteness, let's assume you are writing a 2-argument function **f** such that, when **e1** $\not\Rightarrow$ **NIL**, **(f e1 e2)** computes its result from **(f (cdr e1) e2)**.

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- However, a similar debugging approach can be used if the definition of f does not use LET (e.g., because the LET has been eliminated) or the definition has more than one base case before the LET.

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   When you find arguments **e1** and **e2** that satisfy (i) & (ii), fix the $\boxed{\dots}$ expr so **(f e1 e2)** $\Rightarrow$ the ***correct*** result.

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   When you find arguments **e1** and **e2** that satisfy (i) & (ii), fix the $\boxed{\dots}$ expr so **(f e1 e2)** $\Rightarrow$ the ***correct*** result.

**Repeat step 2** until you think the definition of **f** is correct.

## A Debugging Example Relating to Assignment 4

Problem 7 asks you to write a function PARTITION such that if *l* ⇒ a proper list of real numbers and *p* is a real number, then (PARTITION *l* *p*) returns a list whose CAR is a list of those elements of the list given by *l* that are ***less*** than *p*, and whose CADR is a list of the other elements of the list given by *l*. So:

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(defun partition (L p)  ; Incorrect definition!
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- (partition () 4) ⇒ **(NIL NIL)**    Correct!
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- This is just ***the consequent form of the 1st COND clause***, so we can make $\boxed{\ldots}$ ⇒ **((3) (5 6))** by fixing that clause's test.

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- This can be viewed as an advantage of recursion that makes it easier to discover bugs by testing: A single test call of **f** can generate very many other (recursive) calls of **f**.

# More Sophisticated Recursion

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  - For Assignment 5, your function SSORT should use this kind of expression to produce the argument value for its recursive call.

## Example of the Use of (cddr L) as a Recursive Call Argument

**Recall from Assignment 4**: If L ⇒ a list then (SPLIT-LIST L) returns a list of two lists, in which the 1st list consists of the 1st, 3rd, 5th, ... elements of the list given by L, and the 2nd list consists of the 2nd, 4th, 6th, ... elements of the list given by L.

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  If L ⇒ (B), we ***want*** (split-list L) ⇒ ((B) NIL) but
  **(list … (cons (cadr L) (cadr X)))**
  ⇒ a list whose 2nd element is a CONS!

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So … can be written:

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  (if (null L)
      '(()())
      (let ((X (split-list (cddr L))))

                                                         )))
```

So ... can be written:

```
(cond ((null (cdr L)) (list L ()))
      (t (list (cons (car L) (car X))
               (cons (cadr L) (cadr X)))))
```

**Example of the Use of (cddr L) as a Recursive Call Argument**

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- As **X** is used twice in the **t** case, we must ***not*** eliminate the LET: The function would be very inefficient if it called **(split-list (cddr L))** twice!
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- As X is used twice in the **t** case, we must ***not*** eliminate the LET: The function would be very inefficient if it called **(split-list (cddr L))** twice!
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- After that case is moved out of the LET, it can be combined with the **(null L)** base case, because **(list L ())** is a good value to return in both cases.

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**Final version:**

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Note that calling **(split-list (cddr L))** instead of **(split-list (cdr L))** ***reduces the depth of recursion***.

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$$\left(1+\frac{1}{n}\right)^{n} < e < \left(1+\frac{1}{n}\right)\left(1+\frac{1}{n}\right)^{n} = \left(1+\frac{1}{n}\right)^{n+1}$$

WolframAlpha computational intelligence.

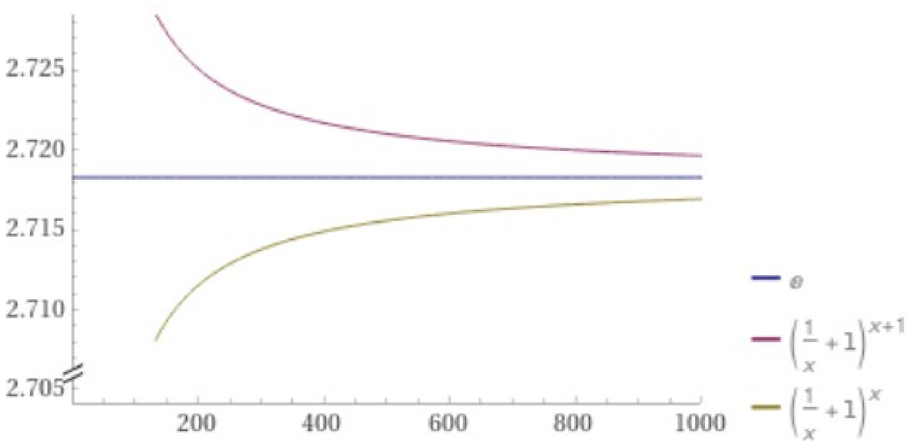
graph of e and (1+1/x)^(x+1) and (1+1/x)^x from 1 to 1000

Extended Keyboard   Upload   Examples

Input interpretation:

| | | |
|---|---|---|
| | $e$ | |
| plot | $\left(1+\frac{1}{x}\right)^{x+1}$ | $x = 1$ to $1000$ |
| | $\left(1+\frac{1}{x}\right)^{x}$ | |

Plot:



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(defun power (z n)    ; far too inefficient!
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```

because when we pass $10^{25}$ to n this function would need a recursion depth of $10^{25}$, which would ***require an impossibly large amount of memory***; and it'd also ***take an impossibly long time*** to execute $10^{25}$ calls of power!

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- A solution is given by the function below, which is based on:

  $z^n = (z^{\lfloor n/2 \rfloor})^2$ if n is ***even***; $z^n = z*(z^{\lfloor n/2 \rfloor})^2$ if n is ***odd***.

  **Examples**: $z^{12} = (z^6)^2$ and $z^{11} = z*(z^5)^2$.

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```

- We get (floor n 2) by ***chopping off the rightmost bit of*** n.
- As $2^{83} < 10^{25} < 2^{84}$, the binary representation of $10^{25}$ has 84 bits: So a call of power with $10^{25}$ as the value of n makes a total of just 84 recursive calls!

This function **power** can now be used in Clisp to compute the number $y$ that satisfies $y < e < (1 + 10^{-25})\,y$.

```
euclid> cl
  i i i i i i i       ooooo    o        ooooooo   ooooo   ooooo
  I I I I I I I      8     8   8           8     8     o  8    8
  I  \ `+' /  I      8         8           8     8        8    8
   \  `-+-'  /       8         8           8      ooooo   8oooo
    `-__|__-'        8         8           8           8  8
        |            8     o   8           8     o     8  8
  ------+------       ooooo    8oooooo  ooo8ooo   ooooo   8

Welcome to GNU CLISP 2.49 (2010-07-07) <http://clisp.cons.org/>

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Type :h and hit Enter for context help.

[1]> (load "power.lsp")
;; Loading file power.lsp ...
;; Loaded file power.lsp
T
[2]> (setf (long-float-digits) 256)
256
[3]> (setf a (+ 1.0L0 (/ 1 (power 10 25))))
1.00000000000000000000000001L0
[4]> (power a (power 10 25))
2.71828182845904523536028733543857107480498532568559840479840654470561981531027L0
[5]>
```

Specifies that Clisp's LONG-FLOAT numbers are to have ⩾ 256 *binary* digits of precision.

1.0L0 means the long-float with value 1.0; this line sets a to the long-float with value $1 + 10^{-25}$.

This and earlier digits are the same as the corresponding digits of *e*.