Math 2250 **Group Work #5**

Problem 1. Given that

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -6,$$

compute

(a):

$$\det \begin{bmatrix} d & e & f \\ -3a & -3b & -3c \\ g-4d & h-4e & i-4f \end{bmatrix}. = -18$$

(b):

$$\det \begin{bmatrix} a+b & d+e & g+h \\ 2c & 2f & 2i \\ b & e & h \end{bmatrix}. = 12$$

Problem 2. Let A and B be matrices 4×4 matrices such that $\det(A) = 5$ and $\det(B) = 3$. Find

(a): $\det(AB^T)$ $= 15$

(b): $\det(A^2B^5)$ $= 6075$

(c): $\det((A^{-1}B^2)^3)$ $= 729/125$

(d): $\det((5A)(2B))$ $= 150000$

Problem 3. TRUE or FALSE. If true, supply a proof or convincing reason. If false, give a counterexample or convincing reason.

(a): For all $n \times n$ matrices A and B, we have

$$\det(A+B) = \det(A) + \det(B).$$

next page

(b): If A, B, and C are $n \times n$ matrices and A is invertible, then if $AB = AC$, then $B = C$.

next page

(c): Suppose A and B are invertible $n \times n$ matrices such that the inverse of A^2 is B. Then the inverse of A^{10} is B^5.

next page

1a) $R_1 \leftrightarrow R_2$

$\det(B) = 6$

$-3R_2$

$\det(B) = -18$

$-4R_1 + R_3$

$\boxed{\det(B) = -18}$

1b) $R_2 \leftrightarrow R_3$

$\det(B) = 6$

$R_1 + R_3$

$\det(B) = 6$

$2R_2$

$\boxed{\det(B) = 12}$

2a) $\det(AB^T)$

$= \det(A)\det(B^T)$

$= \det(A)\det(B)$

$= 5(3)$

$\boxed{= 15}$

2b) $\det(A^2 B^5)$

$= \det(A^2)\det(B^5)$

$= (5^2)(3^5)$

$= 25(243)$

$\boxed{= 6075}$

2c) $\det(A^{-1}B^2)^3$

$= \det(A^{-3}B^6)$

$= \det(A^{-3})\det(B^6)$

$= (5^{-3})(3^6)$

$= \dfrac{3^6}{5^3} = \boxed{\dfrac{729}{125}}$

2c) $\det(5A \cdot 2B)$

$= \det(5A)\det(2B)$

$= 5^4 \det(A) 2^4 \det(B)$

$= (625 \cdot 5)(16 \cdot 3)$

$\boxed{= 150000}$

3a) False, lets say,

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\quad A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\det(A) = 1 \quad \det(B) = 1 \quad \det(A+B) = 0$

$\det(A+B) = \det(A) + \det(B)$

$0 = 1 + 1$

$0 \neq 2$ $\blacksquare$

3b) True, since A is invertible,

$$AB = AC$$

$$A^{-1}(AB) = A^{-1}(AC)$$

$$IB = IC$$

$$B = C \quad \blacksquare$$

3c) True,

$$(A^2)^{-1} = B$$

$$(A^{-1})^2 = B$$

$$A^{-1} = B^{1/2}$$

$$(A^{10})^{-1} = B^5$$

$$(A^{-1})^{10} = B^5$$

$$A^{-1} = B^{1/2} \quad \checkmark \quad \blacksquare$$