Section 2.4 Row-Echelon Matrices and Elementary Row Operations

Row-Echelon Matrix

An $m \times n$ matrix is said to be in **row-echelon form** if it satisfies the following three conditions:

1. if there are any rows consisting entirely of zeros, they are grouped together at the bottom,
2. the first nonzero element of any nonzero row is a 1 (called the **leading 1**), and
3. the leading 1 of any row below the first row is to the right of the leading 1 of the row above it.

Pivot Column= leftmost nonzero column
Pivot Position= the topmost position in the pivot column

Example 1. Examples of row-echelon matrices are

Elementary Operations for Equivalent Systems:

The general $m \times n$ system of linear equations is of the form

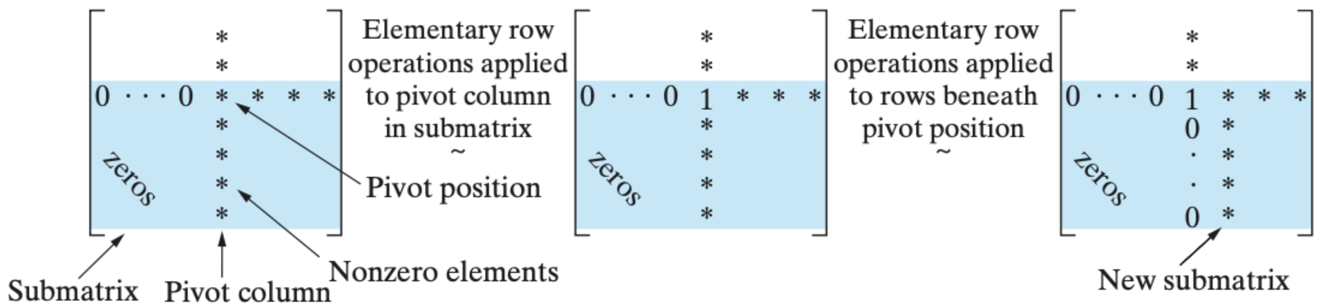
1.	$R_i \leftrightarrow R_j$	P_{ij}	Permute (interchange) row $i$and row j
2.	$kR_i \to R_i$	$M_i(k)$	Multiply row i by a nonzero constant k
3.	$kR_i + R_j \to R_j$	A_{ij}	Multiply row i by a nonzero constant k

Furthermore, the notation $A \sim B$ will mean that matrix B has been obtained from matrix A by a sequence of elementary row operations

Example 2. Use elementary row operations to reduce the given matrix to row-echelon form.

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 1 \\ -4 & 6 & -7 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$$

Algorithm for Reducing an $m \times n$ Matrix A to Row-Echelon Form



Example 3. Use elementary row operations to reduce the given matrix to row-echelon form, and determine its rank.

$$\begin{bmatrix} 3 & 2 & -5 & 2 \\ 1 & 1 & -1 & 1 \\ 1 & 0 & -3 & 4 \end{bmatrix}$$

Reduced Row-Echelon Matrix

An $m \times n$ matrix A is said to be in **reduced row-echelon form** if it satisfies the following three conditions:

1. It is a row-echelon matrix.
2. Any column that contains a leading 1 has zeros everywhere else.
3. the leading 1 of any row below the first row is to the right of the leading 1 of the row above it.

Rank of A= The number of nonzero rows in any row-echelon form of a matrix A denoted $rank(A)$.

Example 4. Examples of reduced row-echelon matrices are

Theorem: An $m \times n$ matrix is row-equivalent to a unique reduced row-echelon matrix.

Example 5. Determine the reduced row-echelon form of A and find $rank(A)$, where

$$A = \begin{bmatrix} 3 & -2 & -1 & 17 \\ 2 & 2 & -4 & 8 \\ -1 & 4 & -3 & 1 \end{bmatrix}$$