Problem 1. Find the solution set for the given system:

$$2x_1 - x_2 - x_3 = 0,$$
$$5x_1 - x_2 + 2x_3 = 1,$$
$$x_1 + x_2 + 4x_3 = 0.$$

Problem 2. Find the solution set for the given system:

$$2x_1 - 4x_2 + 6x_3 = 0,$$
$$3x_1 - 6x_2 + 9x_3 = 0,$$
$$x_1 - 2x_2 + 3x_3 = 0,$$
$$5x_1 - 10x_2 + 15x_3 = 0.$$

Problem 3. Decide whether each statement below is **TRUE** or **FALSE.** If true, give a brief reason. If false, give an example or explanation to show that it is false.

(a) If a matrix A has more rows than a matrix B, then $\operatorname{rank}(A) \geq \operatorname{rank}(B)$.

(b) The columns of the row-echelon of $A^{\#}$ that contain the leading 1s correspond to the free variables.

(c) For $n \times n$ matrices A and B, we have

$$(A + B)^2 = A^2 + 2AB + B^2.$$

Problem 4.

(a): Show that if A is an $n \times n$ matrix such that $A^4 = 0_n$, then $I_n - A$ is an invertible matrix with

$$(I_n - A)^{-1} = I_n + A + A^2 + A^3.$$

(b): Give an example to illustrate that if A and B are invertible matrices of the same size, $A + B$ need not be invertible.