Section 1.4 Separable Equation

General Solution

A first-order differential equation that can be written in the form

$$p(y)\frac{dy}{dx} = q(x) \text{ or } g(y)dy = f(x)dx$$

is called a **separable** differential equation.

How to solve:

1. Try to simplify and rewrite the equation in the form $g(y)dy = f(x)dx$.

2. Integrate both sides, $\int p(y)dy = \int q(x)dx + c$

Example 1. Solve the differential equation

$\frac{dy}{dx} = \frac{x-5}{y^2}$.

Example 2. Solve the differential equation

$$\frac{dy}{dx} = ky$$

where k is constant.

Example 3. Solve the IVP

$$(1 - x^2)\frac{dy}{dx} + xy = 4x, \qquad y(0) = 8.$$

1.6 Linear Differential Equations

Recall that a linear first-order differential equation is in the form

$a_1(x)\frac{dy}{dx} + a_0(x)y = \text{b(x)}$

if we divide both side by $a_1(x)$ we have

General Solution

A first-order ordinary differential equation is linear in the dependent variable y and the independent variable x if it can be written as

$$\frac{dy}{dx} + P(x)y = Q(x)$$

We wish to determine $\mu(x)$ so that the left hand of the product rule

$$\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)Q(x).$$

1^{st}-order Linear DE: Integrating Factor Method

Step 1: Write the equation in the standard form
$\frac{dy}{dx} + P(x)y = Q(x)$.

Step 2: Calculate the integrating factor

$\mu(x) = e^{\int P(x)dx}$.

Step 3: Multiply both sides of the equation in step 1, write the left side as a product rule and integrate.

Example 1. Find the general solution to

$$\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = x\cos x, \qquad x > 0$$

Example 2. Find the general solution to

$$\frac{dy}{dx} + \frac{2x+1}{x} y = e^{-2x}, \qquad x > 0$$

Example 3. Solve the IVP

$$(x^2 + 1)\frac{dy}{dx} + 4xy = x, \qquad y(0) = 10$$

Example 4. Solve

$$y' = \sin x(y \sec x - 2)$$

Example 4. Is the differential equation $y^2 dx + (3xy - 1)dy = 0$ linear? Find the general solution.