

$$1) a) y(t) = e^{rt}$$

$$y'(t) = r e^{rt}$$

$$y''(t) = r^2 e^{rt}$$

$$y'' + 3y' - 10y = 0$$

$$r^2 e^{rt} + 3r e^{rt} - 10 e^{rt} = 0$$

$$e^{rt} (r^2 + 3r - 10) = 0$$

$$(r + 5)(r - 2) = 0$$

$$r = -5, 2$$

$$b) y'''(t) = r^3 e^{rt}$$

$$y''' + 2y'' - y' - 2y = 0$$

$$r^3 e^{rt} + 2r^2 e^{rt} - r e^{rt} - 2 e^{rt} = 0$$

$$e^{rt} (r^3 + 2r^2 - r - 2) = 0$$

$$r^3 + 2r^2 - r - 2 = 0$$

$$r^2(r + 2) - (r + 2) = 0$$

$$(r^2 - 1)(r + 2) = 0$$

$$r = \pm 1, -2$$

$$2) y(x) = x^r$$

$$y'(x) = r x^{r-1}$$

$$y''(x) = r(r-1) x^{r-2}$$

$$x^2 y'' + 3x y' - 8y = 0$$

$$x^2 (r(r-1) x^{r-2}) + 3x (r x^{r-1}) - 8(x^r) = 0$$

$$(r^2 - r) x^r + 3r x^r - 8 x^r = 0$$

$$x^r (r^2 + 2r - 8) = 0$$

$$(r + 4)(r - 2) = 0$$

$$\cancel{r = -4}$$

$$r = 1$$

not in
bound

$$3) x^2 y^2 - x \sin y + 4y - 3 \ln(x) = 7$$

$$2xy^2 + x^2(2y) \frac{dy}{dx} - (\sin y + x \cos y \frac{dy}{dx}) + 4 \frac{dy}{dx} - 3 \left(\frac{1}{x}\right) = 0$$

$$2yx^2 \frac{dy}{dx} - x \cos y \frac{dy}{dx} + 4 \frac{dy}{dx} = -2xy^2 + \sin y + 3/x$$

$$\frac{dy}{dx} (2yx^2 - x \cos y + 4) = \frac{3 + x \sin y - 2x^2 y^2}{x}$$

$$\frac{dy}{dx} = \frac{3 + x \sin y - 2x^2 y^2}{x(2yx^2 - x \cos y + 4)}$$