NBD

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```
library(readxl)
library(knitr)
library(xtable)
library(bbmle)
```

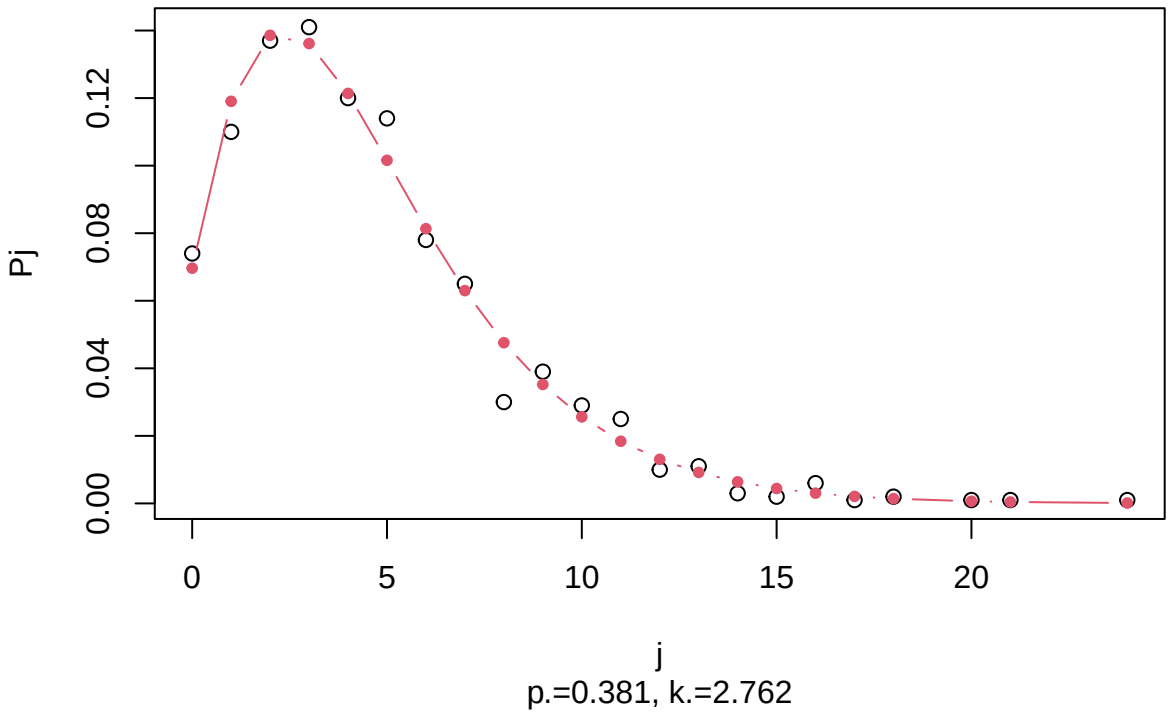
```
## Loading required package: stats4
```

```
library(nleqslv)

EstNB1<-function(X)
{

  minuslogl2 <- function(size,  x) {
    -sum(dnbinom(x = x, size = size, prob = size/(size+mean(x)), log = TRUE))
  }


  k0<-VarG(X)
  M<-mean(X)

  file<-as.numeric(X)
  m <- mle2(
    minuslogl = minuslogl2,
    start = list(size = k0),
    data = list(x = file),
    "L-BFGS-B",
    lower = list(size = 1e-4, prob = 1e-6),
    upper=list(size = 1e+5, prob = 0.9999))

  k <- coef(m)[1]
  p<-k/(k+M)

  V<-c(p=p,k=k)
  return(V)
}

VarG<-function(X)
{
  M<-mean(X)
  p<-M/var(X)
  return(p/(1-p)*M)
}
```

```
Graph<-function(X,res)
{
  y<-table(X)
test<-cbind(y/N,
sapply(as.numeric(names(y)), function(j)dnbinom(j,prob=res[1],size=res[2])))
test
plot(as.numeric(row.names(test)),test[,1],type="p",xlab="j",ylab="Pj")
lines(as.numeric(row.names(test)),test[,2],type="b",xlab="j",ylab="Pj",col=2,pch=20)
p.<-res[1]; k.<-res[2]
a1<-paste(paste("p.",round(p.,3),sep="="),
                paste("k.",round(k.,3),sep="="),sep=", ")
title(sub=a1)
}


N<-1000
p<-0.4; k<-3
X<-rnbinom(N,prob=p,size=k)

res1<-EstNB1(X)
```

```
## Warning in mle2(minuslogl = minuslogl2, start = list(size = k0), data = list(x =
## file), : length mismatch between lower/upper and number of non-fixed parameters:
## # lower=2, # upper=2, # non-fixed=1
```

```
Graph(X,res1)
```



$$\ln \mathcal{L}(x_1, \ldots, x_n | k, p) = \sum_{i=1}^{n} \ln \beta_{-}(x_i | k, p)$$

$$ln\beta_-(x_i|k,p) = \ln\Gamma(k+j) - \ln\Gamma(k) - \ln\Gamma(j+1) + k\ln p + j\ln(1-p)$$

$$\frac{\partial\mathcal{L}(x_1,\ldots,x_n|k,p)}{\partial p} = \sum_{i=1}^{n}\left(\frac{k}{p} - \frac{x_i}{1-p}\right) = 0$$

$$\frac{\partial\mathcal{L}(x_1,\ldots,x_n|k,p)}{\partial k} = \sum_{i=1}^{n}\left(\psi(k+x_i) - \psi(k) + \ln p\right) = 0$$

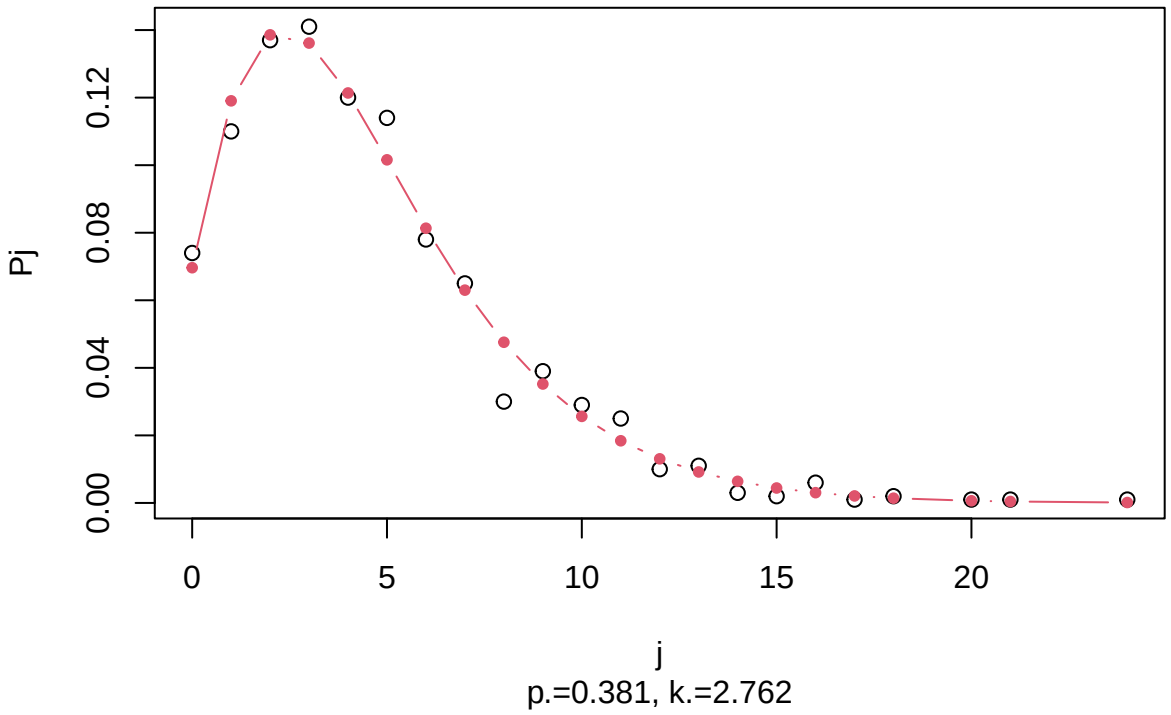
$$\frac{kn}{p} = \frac{n\bar{x}}{1-p} \iff p = \frac{k}{k+\bar{x}}$$

$$\frac{1}{n}\sum_{i=1}^{n}\psi(k+x_i) = \psi(k) - \ln k + \ln(k+\bar{x})$$

```
EstNB2<-function(X)
{
  M<-mean(X);M
f1<-function(x) sum(digamma(x+X))/length(X)-digamma(x)+log(x)-log(x+mean(X))
k.<-nleqslv(VarG(X), f1)$x
p.<-k./(k.+M)
c(p.=p.,k.=k.)
}

res2<-EstNB2(X)
res2
```

```
##         p.         k.
## 0.3811403 2.7622002
```

```
Graph(X,res2)
```

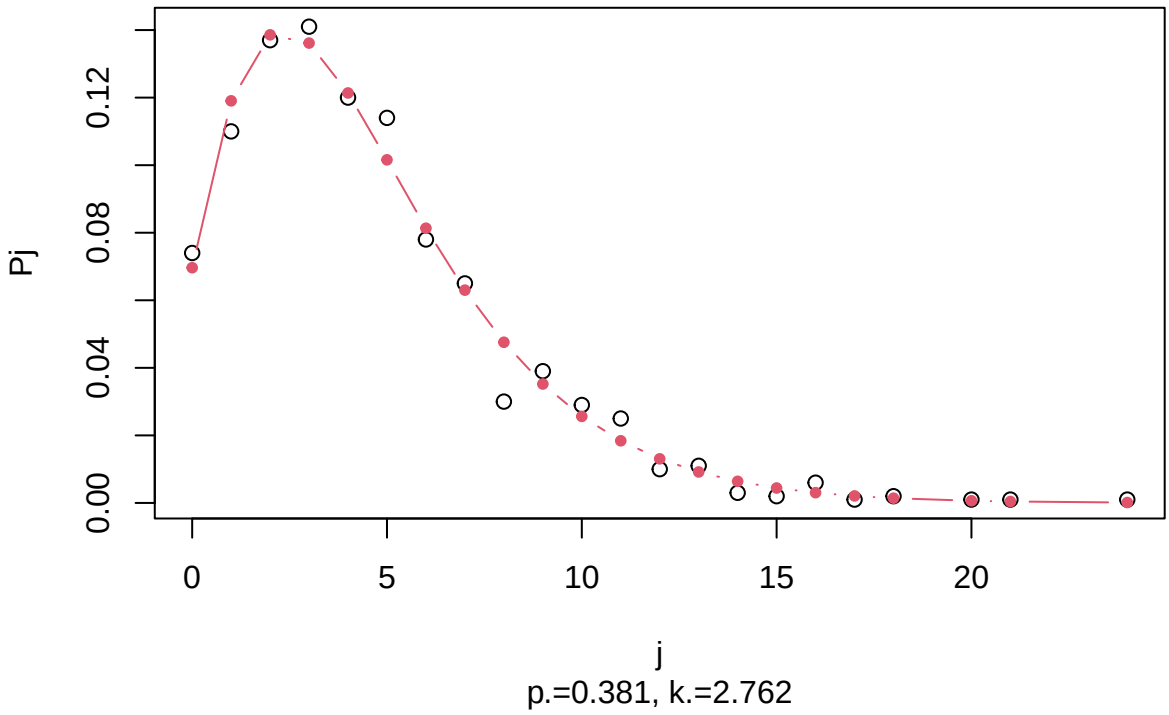


$$\frac{1}{n}\sum_{i=1}^{n}\sum_{l=0}^{x_i-1}\frac{1}{k+l}=\ln(k+\bar{x})-\ln k$$

```
EstNB3<-function(x)
{

M<-mean(X);M


f2<-function(k)
{
  mean(sapply(X,function(x) ifelse(x==0,0,sum(1/(k+c(0:(x-1)))))))-log(k+mean(X))+log(k)
}

k.<-nleqslv(VarG(X), f2)$x
k.
p.<-k./(k.+M)
c(p.=p.,k.=k.)
}
res3<-EstNB3(X)
res3
```

```
##         p.         k.
## 0.3811403 2.7622002
```

```
Graph(X,res3)
```




```
cbind(res1,res2,res3)
```

```
##              res1      res2      res3
## p.size 0.3811408 0.3811403 0.3811403
## k.size 2.7622053 2.7622002 2.7622002
```