For the case u = a*x + b*y we have a Cov(u):

$$\left[\begin{matrix}V_{xx} a^{2} + 2 V_{xy} a b + V_{yy} b^{2}\end{matrix}\right] \tag{1}$$

For the case of going from x,y to r,theta(t), using a Rotation

$$\left[\begin{matrix}V_{xx} & V_{xy}\\V_{xy} & V_{yy}\end{matrix}\right] \tag{2}$$

Rotation Matrix:

$$\left[\begin{matrix}\cos{\left (t \right)} & - \sin{\left (t \right)}\\\sin{\left (t \right)} & \cos{\left (t \right)}\end{matrix}\right] \tag{3}$$

Final(destination) covariance Matrix:

$$\left[\begin{matrix}V_{xx} \cos^{2}{\left (t \right)} - 2 V_{xy} \sin{\left (t \right)} \cos{\left (t \right)} + V_{yy} \sin^{2}{\left (t \right)} & V_{xx} \sin{\left (t \right)} \cos{\left (t \right)} - V_{xy} \sin^{2}{\left (t \right)} + V_{xy} \cos^{2}{\left (t \right)} - V_{yy} \sin{\left (t \right)} \cos{\left (t \right)}\\V_{xx} \sin{\left (t \right)} \cos{\left (t \right)} - V_{xy} \sin^{2}{\left (t \right)} + V_{xy} \cos^{2}{\left (t \right)} - V_{yy} \sin{\left (t \right)} \cos{\left (t \right)} & V_{xx} \sin^{2}{\left (t \right)} + 2 V_{xy} \sin{\left (t \right)} \cos{\left (t \right)} + V_{yy} \cos^{2}{\left (t \right)}\end{matrix}\right] \tag{4}$$

Case of cartesian to polar transformation
Cartesian Variance:

$$\left[\begin{matrix}V_{xx} & V_{xy}\\V_{xy} & V_{yy}\end{matrix}\right] \tag{5}$$

Jacobian-cart-to-polar:

$$\left[\begin{matrix}\cos{\left (t \right)} & - r \sin{\left (t \right)}\\\sin{\left (t \right)} & r \cos{\left (t \right)}\end{matrix}\right] \tag{6}$$

Polar Variance:

$$\left[\begin{matrix}V_{xx} \cos^{2}{\left (t \right)} - V_{xy} r \sin{\left (t \right)} \cos{\left (t \right)} + V_{xy} \sin{\left (t \right)} \cos{\left (t \right)} - V_{yy} r \sin^{2}{\left (t \right)} & - V_{xx} r \sin{\left (t \right)} \cos{\left (t \right)} + V_{xy} r^{2} \sin^{2}{\left (t \right)} + V_{xy} r \cos^{2}{\left (t \right)} - V_{yy} r^{2} \sin{\left (t \right)} \cos{\left (t \right)}\\V_{xx} \sin{\left (t \right)} \cos{\left (t \right)} + V_{xy} r \cos^{2}{\left (t \right)} + V_{xy} \sin^{2}{\left (t \right)} + V_{yy} r \sin{\left (t \right)} \cos{\left (t \right)} & - V_{xx} r \sin^{2}{\left (t \right)} - V_{xy} r^{2} \sin{\left (t \right)} \cos{\left (t \right)} + V_{xy} r \sin{\left (t \right)} \cos{\left (t \right)} + V_{yy} r^{2} \cos^{2}{\left (t \right)}\end{matrix}\right] \tag{7}$$

Case of polar to cartesian transformation
Polar Variance:

$$\left[\begin{matrix}V_{rr} & V_{rt}\\V_{rt} & V_{tt}\end{matrix}\right] \tag{8}$$

Jacobian-rad-to-cart:

$$\left[\begin{matrix}\frac{x}{\left|{r}\right|} & \frac{y}{\left|{r}\right|}\\- \frac{y}{r^{2}} & \frac{x}{r^{2}}\end{matrix}\right] \tag{9}$$

Cartesian Variance:

$$\left[\begin{matrix}\frac{V_{rr} x^{2}}{r^{2}} + \frac{2 V_{rt}}{r^{2}} x y + \frac{V_{tt} y^{2}}{r^{2}} & - \frac{V_{rr} x y}{r^{2} \left|{r}\right|} + \frac{V_{rt} x^{2}}{r^{2} \left|{r}\right|} - \frac{V_{rt} y^{2}}{r^{2} \left|{r}\right|} + \frac{V_{tt} x y}{r^{2} \left|{r}\right|}\\- \frac{V_{rr} x y}{r^{2} \left|{r}\right|} + \frac{V_{rt} x^{2}}{r^{2} \left|{r}\right|} - \frac{V_{rt} y^{2}}{r^{2} \left|{r}\right|} + \frac{V_{tt} x y}{r^{2} \left|{r}\right|} & \frac{V_{rr} y^{2}}{r^{4}} - \frac{2 V_{rt}}{r^{4}} x y + \frac{V_{tt} x^{2}}{r^{4}}\end{matrix}\right] \tag{10}$$

Polar to Cartesian transformation, using multiple points with block matrices.

Input Covariance block matrix:

$$\begin{bmatrix} V_{r11} & V_{r12} & V_{r1t1} & V_{r1t2} \\ V_{r12} & V_{r22} & V_{r2t1} & V_{r2t2} \\ V_{t1r1} & V_{t1r2} & V_{t11} & V_{t12} \\ V_{t2r1} & V_{t2r2} & V_{t12} & V_{t22} \end{bmatrix} \tag{11}$$

Jacobian block matrix:

$$\begin{bmatrix} \frac{x_1}{r_1} & 0 & \frac{y_1}{r_1} & 0 \\ 0 & \frac{x_2}{r_2} & 0 & \frac{y_2}{r_2} \\ -\frac{y_1}{r_1^2} & 0 & \frac{x_1}{r_1^2} & 0 \\ 0 & -\frac{y_2}{r_2^2} & 0 & \frac{x_2}{r_2^2} \end{bmatrix} \tag{12}$$

Cartesian Covariance block matrix:

$$\left[\begin{matrix} \frac{V_{r11} x_1^2}{r_1^2} + \frac{V_{r1t1} x_1}{r_1^2} y_1 + \frac{V_{t11} y_1^2}{r_1^2} + \frac{V_{t1r1} x_1}{r_1^2} y_1 & \frac{V_{r12} x_1 x_2}{r_1 r_2} + \frac{V_{r1t2} x_1 y_2}{r_1 r_2} + \frac{V_{t12} y_1 y_2}{r_1 r_2} + \frac{V_{t1r2} x_2 y_1}{r_1 r_2} & -\frac{V_{r11} x_1}{r_1^3} y_1 + \frac{V_{r1t1} x_1^2}{r_1^3} + \frac{V_{t11} x_1}{r_1^3} y_1 - \frac{V_{t1r1} y_1^2}{r_1^3} & -\frac{V_{r12} x_1 y_2}{r_1 r_2^2} + \frac{V_{r1t2} x_1 x_2}{r_1 r_2^2} + \frac{V_{t12} x_2 y_1}{r_1 r_2^2} - \frac{V_{t1r2} y_1 y_2}{r_1 r_2^2} \\ \frac{V_{r12} x_1 x_2}{r_1 r_2} + \frac{V_{r2t1} x_2 y_1}{r_1 r_2} + \frac{V_{t12} y_1 y_2}{r_1 r_2} + \frac{V_{t2r1} x_1 y_2}{r_1 r_2} & \frac{V_{r22} x_2^2}{r_2^2} + \frac{V_{r2t2} x_2}{r_2^2} y_2 + \frac{V_{t22} y_2^2}{r_2^2} + \frac{V_{t2r2} x_2}{r_2^2} y_2 & -\frac{V_{r12} x_2 y_1}{r_1^2 r_2} + \frac{V_{r2t1} x_1 x_2}{r_1^2 r_2} + \frac{V_{t12} x_1 y_2}{r_1^2 r_2} - \frac{V_{t2r1} y_1 y_2}{r_1^2 r_2} & -\frac{V_{r22} x_2}{r_2^3} y_2 + \frac{V_{r2t2} x_2^2}{r_2^3} + \frac{V_{t22} x_2}{r_2^3} y_2 - \frac{V_{t2r2} y_2^2}{r_2^3} \\ -\frac{V_{r11} x_1}{r_1^3} y_1 - \frac{V_{r1t1} y_1^2}{r_1^3} + \frac{V_{t11} x_1}{r_1^3} y_1 + \frac{V_{t1r1} x_1^2}{r_1^3} & -\frac{V_{r12} x_2 y_1}{r_1^2 r_2} - \frac{V_{r1t2} y_1 y_2}{r_1^2 r_2} + \frac{V_{t12} x_1 y_2}{r_1^2 r_2} + \frac{V_{t1r2} x_1 x_2}{r_1^2 r_2} & \frac{V_{r11} y_1^2}{r_1^4} - \frac{V_{r1t1} x_1}{r_1^4} y_1 + \frac{V_{t11} x_1^2}{r_1^4} - \frac{V_{t1r1} x_1}{r_1^4} y_1 & \frac{V_{r12} y_1 y_2}{r_1^2 r_2^2} - \frac{V_{r1t2} x_2 y_1}{r_1^2 r_2^2} + \frac{V_{t12} x_1 x_2}{r_1^2 r_2^2} - \frac{V_{t1r2} x_1 y_2}{r_1^2 r_2^2} \\ -\frac{V_{r12} x_1 y_2}{r_1 r_2^2} - \frac{V_{r2t1} y_1 y_2}{r_1 r_2^2} + \frac{V_{t12} x_2 y_1}{r_1 r_2^2} + \frac{V_{t2r1} x_1 x_2}{r_1 r_2^2} & -\frac{V_{r22} x_2}{r_2^3} y_2 - \frac{V_{r2t2} y_2^2}{r_2^3} + \frac{V_{t22} x_2}{r_2^3} y_2 + \frac{V_{t2r2} x_2^2}{r_2^3} & \frac{V_{r12} y_1 y_2}{r_1^2 r_2^2} - \frac{V_{r2t1} x_1 y_2}{r_1^2 r_2^2} + \frac{V_{t12} x_1 x_2}{r_1^2 r_2^2} - \frac{V_{t2r1} x_2 y_1}{r_1^2 r_2^2} & \frac{V_{r22} y_2^2}{r_2^4} - \frac{V_{r2t2} x_2}{r_2^4} y_2 + \frac{V_{t22} x_2^2}{r_2^4} - \frac{V_{t2r2} x_2}{r_2^4} y_2 \end{matrix}\right] \tag{13}$$

Input values:

a:

$$(-X_0)\hat{\mathbf{i}}_{\mathbf{N}} + (-Y_0)\hat{\mathbf{j}}_{\mathbf{N}} \tag{14}$$

b:

$$(-X_0 + x_i)\hat{\mathbf{i}}_{\mathbf{N}} + (-Y_0 + y_i)\hat{\mathbf{j}}_{\mathbf{N}} \tag{15}$$

cross:

$$-X_0 y_i + Y_0 x_i \tag{16}$$

dot:

$$X_0^2 - X_0 x_i + Y_0^2 - Y_0 y_i \tag{17}$$

km:

$$-\frac{Rq}{X_0^2 - 2X_0 x_i + Y_0^2 - 2Y_0 y_i + x_i^2 + y_i^2} \tag{18}$$

k:

$$-\frac{Rq}{\left(X_0^2 + Y_0^2\right)\left(X_0^2 - 2X_0 x_i + Y_0^2 - 2Y_0 y_i + x_i^2 + y_i^2\right)} \tag{19}$$

k also as:

$$-\frac{Rq}{dot**2 + cross**2} \tag{20}$$

i.e. k:

$$\frac{km}{X_0^2 + Y_0^2} \tag{21}$$

s:

$$-Rq \operatorname{atan}\left(\frac{cross}{dot}\right) \tag{22}$$

Jacobian [x0, y0, R, xi, yi]:

$$\left[\begin{matrix}k\left(X_0^2 y_i - 2X_0 Y_0 x_i - Y_0^2 y_i + Y_0 x_i^2 + Y_0 y_i^2\right)\\k\left(X_0^2 x_i + 2X_0 Y_0 y_i - X_0 x_i^2 - X_0 y_i^2 - Y_0^2 x_i\right)\\-q \operatorname{atan}\left(\frac{cross}{dot}\right)\\km(Y_0 - y_i)\\-km(X_0 - x_i)\end{matrix}\right] \tag{23}$$

Note that -(b_y - a_y)*dot + (b_x + a_x)*cross is:

$$X_0^2 y_i - 2X_0 Y_0 x_i - Y_0^2 y_i + Y_0 x_i^2 + Y_0 y_i^2 \tag{24}$$

Note that (b_x - a_x)*dot - (-a_y - b_y)*cross is:

$$X_0^2 x_i + 2X_0 Y_0 y_i - X_0 x_i^2 - X_0 y_i^2 - Y_0^2 x_i \tag{25}$$

Note that -a_y*dot - a_x*cross is:

$$\left(X_0^2 + Y_0^2\right)(Y_0 - y_i) \tag{26}$$

Note that a_x*dot-a_y*cross is:

$$-(X_0 - x_i)\left(X_0^2 + Y_0^2\right) \tag{27}$$