Lecture 15

STAT 109: Introductory Biostatistics

Lecture 15: Binomial Mean and Standard Deviation

For a binomial random variable X with n trials and success probability p ($X \sim \text{Binomial}(n, p)$):

Mean (expected value):

$$\mu = np$$

Standard deviation:

$$\sigma = \sqrt{n\,p\,(1-p)}$$

Interpretation

- **Mean μ:** The **center** of the distribution of X. If we observed X many, many times (each time from a new binomial process with the same n and p), the **average** of those values would be $\mu = np$.
- **Standard deviation σ:** A **typical deviation** of X from the center. It is the square root of the average squared distance of (many) observations of X from the mean μ. So σ answers: "Roughly how far from μ do we expect a single observation of X to fall?"

Examples: Identify n and p, then compute μ and σ

For each scenario, (1) state what X counts and verify the setting is a binomial random process (binary outcome, fixed n, fixed p, independent trials); (2) identify n and p; (3) compute the mean $\mu = np$ and the standard deviation $\sigma = \sqrt{np(1-p)}$.

Example 1: ESP test

We run an ESP test with **25 trials**. On each trial the person guesses one of **5 symbols**; the computer picks one at random. So by chance the probability of a correct guess on any one trial is $p = 1/5 = 0.2$. Let X = number of correct guesses out of 25.

- X **counts:** number of correct guesses in 25 trials.
- **Binomial?** Binary (correct or not), fixed $n = 25$, fixed $p = 0.2$, independent trials. Yes.
- n **and** p**:** $n = 25$, $p = 0.2$.
- **Mean:** $\mu = np = 25 \times 0.2 = 5$.
- **SD:** $\sigma = \sqrt{np(1-p)} = \sqrt{25 \times 0.2 \times 0.8} = \sqrt{4} = 2$.

So we expect about 5 correct by chance, with a typical deviation of about 2.

Example 2: Left-handed students

We take a random sample of **30 students** from our school. About **10%** of people are left-handed nationwide; we use $p = 0.10$ as the probability that a randomly chosen student is left-handed. Let X = number of left-handed students in the sample.

- X **counts:** number of left-handed students in 30 students.

- **Binomial?** Binary (left-handed or not), fixed $n = 30$, fixed $p = 0.10$, independent students. Yes.
- **n and p:** $n = 30$, $p = 0.10$.
- **Mean:** $\mu = np = 30 \times 0.10 = 3$.
- **SD:** $\sigma = \sqrt{np(1-p)} = \sqrt{30 \times 0.10 \times 0.90} = \sqrt{2.7} \approx 1.64$.

We expect about 3 left-handed students, with a typical deviation of about 1.64.

Example 3: Defective items

A supplier claims that **12%** of items are defective. We inspect **20 items** at random. Let X = number of defective items among the 20.

- **X counts:** number of defective items in 20 inspected.
- **Binomial?** Binary (defective or not), fixed $n = 20$, fixed $p = 0.12$ (under the supplier's claim), independent items. Yes.
- **n and p:** $n = 20$, $p = 0.12$.
- **Mean:** $\mu = np = 20 \times 0.12 = 2.4$.
- **SD:** $\sigma = \sqrt{np(1-p)} = \sqrt{20 \times 0.12 \times 0.88} = \sqrt{2.112} \approx 1.45$.

We expect about 2.4 defectives, with a typical deviation of about 1.45.

Example 4: Free throws

A basketball player shoots **40 free throws**. Her long-run probability of making any single free throw is **0.75**. Let X = number of made free throws out of 40.

- **X counts:** number of made free throws in 40 attempts.
- **Binomial?** Binary (make or miss), fixed $n = 40$, fixed $p = 0.75$, independent shots. Yes.
- **n and p:** $n = 40$, $p = 0.75$.
- **Mean:** $\mu = np = 40 \times 0.75 = 30$.
- **SD:** $\sigma = \sqrt{np(1-p)} = \sqrt{40 \times 0.75 \times 0.25} = \sqrt{7.5} \approx 2.74$.

We expect about 30 makes, with a typical deviation of about 2.74.

Summary

Example	X counts	n	p	$\mu = np$	$\sigma = \sqrt{np(1-p)}$
ESP	correct guesses	25	0.20	5	2
Left-handed	left-handed in sample	30	0.10	3	≈ 1.64
Defective	defectives in 20 items	20	0.12	2.4	≈ 1.45
Free throws	made shots in 40	40	0.75	30	≈ 2.74

In each case, μ is the center of the distribution (the average of many observations of X), and σ is a typical deviation from that center (square root of the average squared distance from μ).