Lecture 9

STAT 109: Introductory Biostatistics

Lecture 9: Permutations and Combinations

Learning Outcomes:

1. Distinguish between situations where **order matters** (permutations) and where **order does not matter** (combinations).
2. Use the formulas for ${}_nP_r$ and ${}_nC_r$ to solve counting problems.

Motivating Example: Ice Cream

Problem (a). Find the number of ways to distribute 3 distinct flavors of ice cream to 3 distinct bowls.

Problem (b). Find the number of ways to choose 2 distinct flavors of ice cream from 26 flavors to create an ice cream cone with one scoop on the top and one scoop on the bottom.

Problem (c). Find the number of ways to choose 2 distinct flavors of ice cream from 26 flavors to create a dish with two scoops of ice cream, where the order of the scoops doesn't matter.

Key Idea: Does Order Matter?

- **Permutation:** An **arrangement** of objects where **order matters**. (Chocolate on top, vanilla on bottom $\neq$ vanilla on top, chocolate on bottom.)
- **Combination:** A **selection** of objects where **order does not matter**. (Chocolate + vanilla in a dish = vanilla + chocolate.)

Factorial Notation

The **factorial** of a nonnegative integer n, written $n!$, is the product of all positive integers from n down to 1:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

Convention: $0! = 1$. (This is defined so that our permutation and combination formulas work correctly when $r = n$ or $r = 0$.)

Examples:

- $3! = 3 \times 2 \times 1 = 6$
- $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- $1! = 1$

Definitions and Formulas

Permutations ${}_nP_r$ (or $P(n,r)$): The number of ways to arrange r objects chosen from n distinct objects, where **order matters**.

$${}_nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots(n-r+1)$$

Combinations ${}_nC_r$ (or $\binom{n}{r}$ or $C(n,r)$): The number of ways to choose r objects from n distinct objects, where **order does not matter**.

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{{}_nP_r}{r!}$$

Relationship: ${}_nC_r = \frac{{}_nP_r}{r!}$ — we divide by $r!$ because each combination of r objects can be arranged in $r!$ different orders, and we want to count them once.

Second Example: Choosing 3 Letters from {A, B, C, D, E}

Suppose we have 5 letters: A, B, C, D, E. We want to choose 3 of them.

Part 1 (order matters): How many ways can we choose 3 letters if the order of our choice matters? (e.g., for a 3-letter code)?

Part 2 (order doesn't matter): How many ways can we choose 3 letters if the order of our choice does not matter (e.g., for a committee of 3)?

Summary

Rule of thumb: Ask yourself: "If I swap two of the chosen items, do I get a different outcome?" If yes → permutation. If no → combination.