Quiz 6

STAT 109: Introductory Biostatistics

Quiz 6

Quiz 6 Practice Problems

These practice problems cover:

- **Lecture 12:** Introduction to the binomial test (ESP example), hypotheses H_0 vs H_a, the 4 conditions of a Binomial Random Process (BRP), and the steps of the test.
- **Lecture 13:** ESP worksheet — stating hypotheses, verifying BRP conditions, identifying n and p, and finding the p-value.
- **Lecture 14:** Binomial test steps and examples (left-handed, defective items, dice, gym use): research question, parameter, hypotheses, null distribution plot, **adding the observed data and shading the p-value** on the graph, and drawing conclusions.

Instructions

- Work **without notes** first; then check your answers against the solutions in Canvas.
- For **graph questions:** The figures show the **null distribution** (bar plot of x vs $P(X = x)$) with **no shading**. On each figure, **by hand**: (1) draw a **vertical line** at the observed value of x given in the problem, and (2) **shade the bar(s)** that correspond to the p-value (in the direction of H_a).
- The in-person quiz in lab will be created from these practice problems.

Part A: Lecture 12 — Hypotheses and BRP (ESP)

A1. ESP hypotheses

Research question (in words): We want to test whether a person has *Extra Sensory Perception (ESP)* — the ability to predict outcomes better than chance. To do this, we use an online test: on each of 25 trials, the computer randomly shows one of **5 possible symbols** (e.g. circle, cross, star), and the person guesses which symbol it is. If they do *not* have ESP, they are just guessing among 5 options on each trial. If they *do* have ESP, we expect them to get more than chance correct.

Let p = probability of a correct prediction on a single trial (when the person makes one guess).

1. Write the **null hypothesis** H_0 and **alternative hypothesis** H_a in symbols.
2. In words, what does it mean if H_0 is true? What are we claiming if we believe H_a?

A2. Four conditions of a Binomial Random Process

Setting: The same ESP test as in A1: 25 trials, 5 possible symbols per trial, the computer picks one symbol at random each time ("open deck"), and the person sees the card (or result) after each guess. We record the number of correct predictions out of 25.

List the **4 conditions** of a Binomial Random Process and briefly state why each is satisfied in this setting.

Part B: Lecture 13 — ESP worksheet style

B1. n and p under the null

Setting: The same ESP test as in Part A: 25 trials, 5 symbols, the person guesses on each trial. Under the null hypothesis, the person is just guessing (no ESP).

- What is n?
- What is p (the probability of a correct guess on one trial under H_0)?

B2. Observed data and p-value direction

Setting: Same ESP test: 25 trials, 5 symbols. Under the null, $p = 0.2$ (just guessing). We test $H_0 : p = 0.2$ vs $H_a : p > 0.2$. Suppose you took the test and got **7** correct out of 25.

1. What is the observed value of x?
2. The p-value is the probability, assuming H_0 is true, of observing data **as or more extreme than** the observed x in the direction of H_a. Write the p-value as a probability involving X (e.g. $P(X \geq ...)$ or $P(X \leq ...)$).
3. Would you need the **right tail**, the **left tail**, or **both tails** of the null distribution to compute this p-value?

Part C: Graphs — add the data and shade the p-value

Each figure below shows the **null distribution** for a scenario (bar plot of x vs $P(X = x)$). The plots are **unshaded**. For each problem:

1. **On the figure:** Draw a **vertical line** at the **observed value of** x (given in the problem).
2. **On the figure: Shade the bar(s)** that represent the **p-value** (i.e. the region in the direction of H_a: right tail for $H_a : p > ...$, left tail for $H_a : p < ...$, or both tails for $H_a : p \neq ...$).
3. **In the blank:** State whether the p-value is **small** or **large** and give the **conclusion** (reject H_0 or fail to reject H_0) in context.

C1. Left-handed students

Research question: Is the proportion of left-handed students at our school higher than the national rate of 10%?

Parameter: p = proportion of students at our school who are left-handed.

Hypotheses: $H_0 : p = 0.10$ vs $H_a : p > 0.10$. Under H_0, we use $n = 25$ and $p = 0.10$ for the null distribution.

Data: In a random sample of 25 students, $x = 6$ were left-handed.

The plot below shows the null distribution (x vs $P(X = x)$ for $X \sim \text{Binomial}(25, 0.10)$).

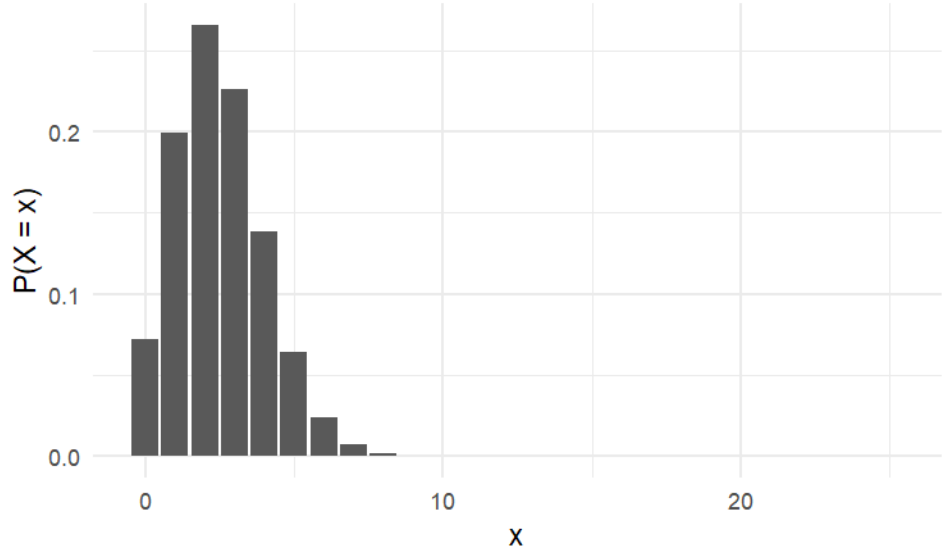

Figure 1: Null distribution for Example 1 (left-handed): $n = 25$, $p = 0.10$.

Your tasks:

1. On the figure, add a **vertical line at the observed** $x = 6$ and **shade the bars** that represent the p-value (right tail, in the direction of H_a).
2. p-value $= P(X \geq 6 \mid H_0) =$ _____ ("small" or "large").
3. Conclusion: _____ H_0. In context: _____.

C2. Defective items

Research question: Is the proportion of defective items from the new supplier less than 15%?

Parameter: $p =$ proportion of items from the new supplier that are defective.

Hypotheses: $H_0 : p = 0.15$ vs $H_a : p < 0.15$. Under H_0, we use $n = 25$ and $p = 0.15$ for the null distribution.

Data: In a random sample of 25 items from the new supplier, $x = 3$ were defective.

The plot below shows the null distribution (x vs $P(X = x)$ for $X \sim \text{Binomial}(25, 0.15)$).

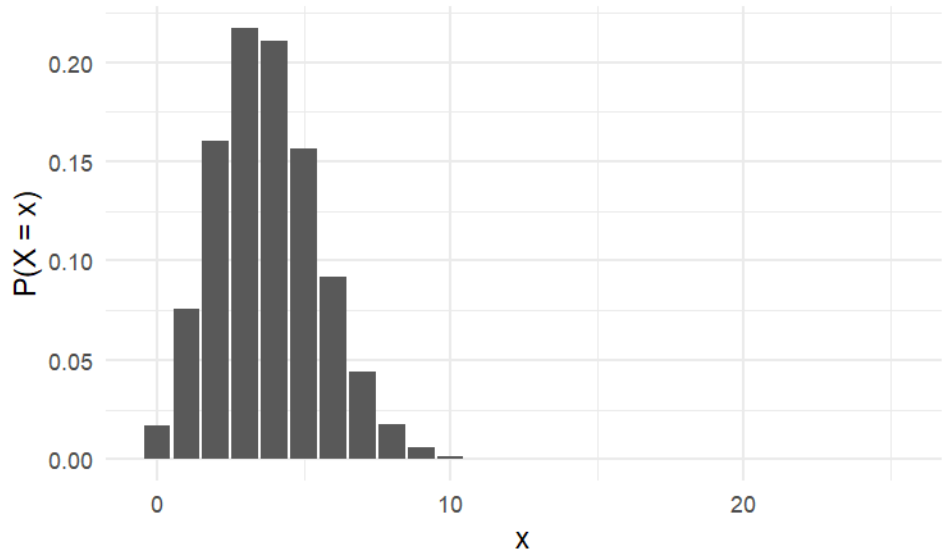

Figure 2: Null distribution for Example 2 (defective items): $n = 25$, $p = 0.15$.

Your tasks:

1. On the figure, add a **vertical line at the observed** $x = 3$ and **shade the bars** that represent the p-value (left tail, in the direction of H_a).
2. p-value $= P(X \leq 3 \mid H_0) =$ _____ ("small" or "large").
3. Conclusion: _____ H_0. In context: _____.

C3. Dice (sum 7 or 11)

Research question: When we roll two dice, is the probability that the sum is 7 or 11 equal to the fair-dice value? (With fair dice, that probability is 8/36: 6 ways to get sum 7 plus 2 ways to get sum 11, out of 36 equally likely outcomes.)

Parameter: p = probability that a single roll of two dice gives a sum of 7 or 11.

Hypotheses: $H_0 : p = 8/36$ vs $H_a : p \neq 8/36$. Under H_0, we use $n = 25$ rolls and $p = 8/36$ for the null distribution.

Data: We rolled the two dice 25 times. The sum was 7 or 11 on $x = 2$ of those rolls.

The plot below shows the null distribution (x vs $P(X = x)$ for $X \sim \text{Binomial}(25, 8/36)$).

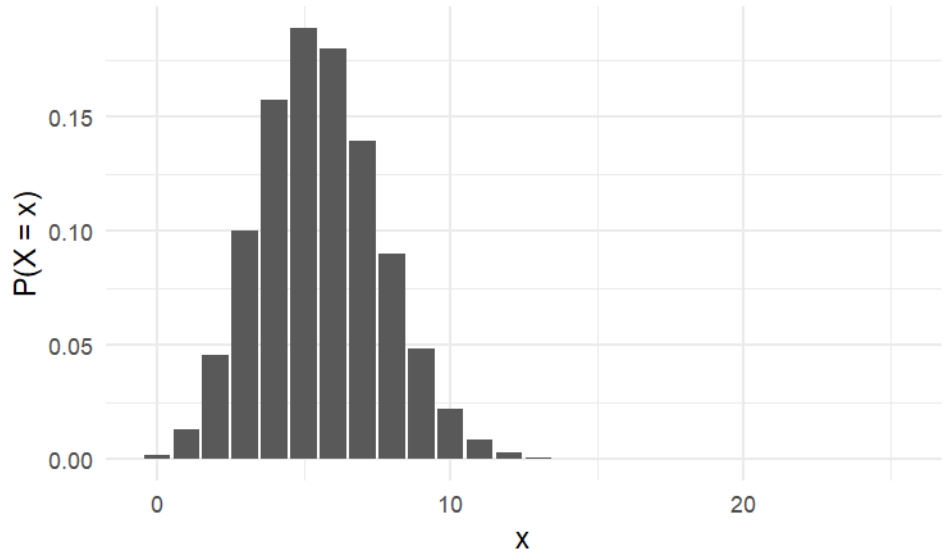

Figure 3: Null distribution for Example 3 (dice, sum 7 or 11): $n = 25$, $p = 8/36$.

Your tasks:

1. On the figure, add a **vertical line at the observed** $x = 2$ and **shade the bars** that represent the p-value. For a **two-sided** H_a, shade **both** tails (the bars for $x \leq 2$ and for $x \geq 9$, or the symmetric "more extreme" region).
2. p-value $= P(X \leq 2) + P(X \geq 9)$ under H_0 = _____ ("small" or "large").
3. Conclusion: _____ H_0. In context: _____.

C4. Gym use

Research question: Do more than 20% of students at our school use the gym at least once per week?

Parameter: p = proportion of students at our school who use the gym at least once per week.

Hypotheses: $H_0 : p = 0.20$ vs $H_a : p > 0.20$. Under H_0, we use $n = 25$ and $p = 0.20$ for the null distribution.

Data: In a random sample of 25 students, $x = 5$ use the gym at least once per week.

The plot below shows the null distribution (x vs $P(X = x)$ for $X \sim \text{Binomial}(25, 0.20)$).

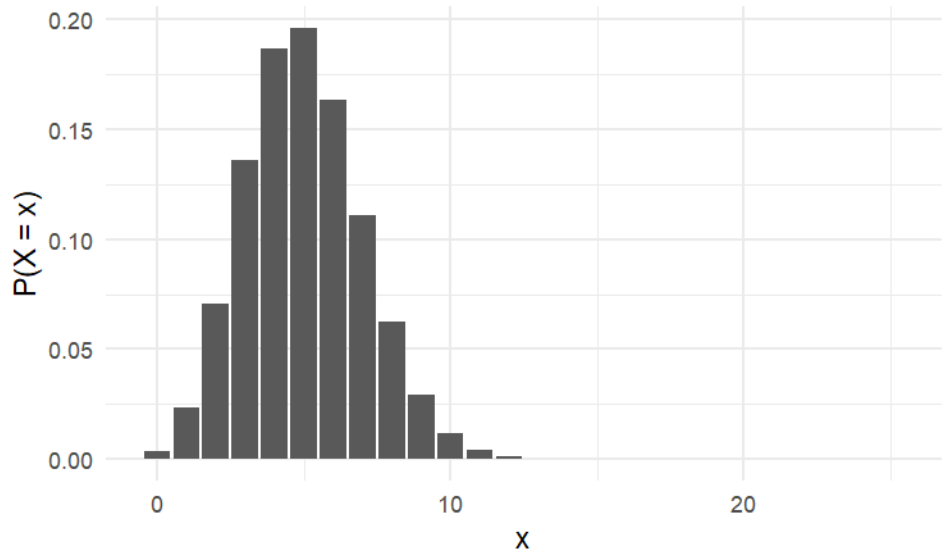

Figure 4: Null distribution for Example 4 (gym use): $n = 25$, $p = 0.20$.

Your tasks:

1. On the figure, add a **vertical line at the observed** $x = 5$ and **shade the bars** that represent the p-value (right tail, in the direction of H_a).
2. p-value $= P(X \geq 5 \mid H_0) =$ _____ ("small" or "large").
3. Conclusion: _____ H_0. In context: _____.

Part D: Full steps and one- vs two-sided

D1. Order the steps

Below are **4 key steps** of the binomial test, given in **random order**. Number them **1, 2, 3, 4** in the order we perform them when conducting the test.

- **(A)** Verify that the 4 conditions of a Binomial Random Process (BRP) are satisfied.
- **(B)** State the research question, define the parameter p, state H_0 and H_a, and identify n and p under H_0.
- **(C)** Create a plot of the null distribution (x vs $P(X = x)$ for the n and p under H_0).
- **(D)** Collect the data (observe the value of x) and compare it with the null distribution: on the plot, add the observed x and shade the bar(s) that represent the p-value; compute the p-value and state the conclusion in context.

Your ordering: 1 = ____ , 2 = ____ , 3 = ____ , 4 = ____

D2. One-sided vs two-sided

Consider a binomial test with null distribution plotted as a bar chart.

1. If the alternative hypothesis is **one-sided** (e.g. $H_a : p > 0.2$ or $H_a : p < 0.15$), do we shade **one tail** or **both tails** of the null distribution to find the p-value?
2. If the alternative hypothesis is **two-sided** (e.g. $H_a : p \neq 8/36$), do we shade **one tail** or **both tails**?

End of Quiz 6 practice problems.