Lecture 3

STAT 109: Introductory Biostatistics

Lecture 3

Probability calculations the "long way"

Learning Objectives

- Define and identify the following terms for a given trial:
  - Sample space
  - Outcome
  - Event
- Use appropriate notation for the probability of an event, $P(A)$
- State and use the formula for the probability of an event if the outcomes in the sample space are equally likely.

Definitions

Let's define the following terms:

- **Sample space:** the set of all possible outcomes of a trial, conventional notation S
- **Event:** a set of one or more outcomes of a trial, usually notated with an uppercase letter, e.g. A or B or a meaningful letter such as H for "heads"

Set Notation and Operator Practice

Fill in the meaning or notation for each of the following:

Suppose that a single six-sided die is rolled.

- $S =$ _____
- Let _____ = be the event that the die lands with the "5" or the "6" side up
- Not A means _____
- Let B be the event that the die is an even number
- A and B is the set _____
- A or B is the set _____
- A given B means "assuming that _____ happened, that the event _____ also happened"

Probability Notation

Translate the following "sentences" to English:

- $P(A) = 0.7$
- $P(A$ and $B) = 0.2$
- $P(\text{not } A) = 0.1$
- $P(A|B) = 0.25$

Three "Axioms" of Probability

Fill in the blanks:

1. For any event A, _____ $P(A)$ _____
2. $P(S) = $ _____
3. For two non-overlapping events A and B, $P(A \text{ or } B) = $ _____

> **Note:** "axiom" is a type of rule that we must merely "agree" is true, rather than being able to "prove" that it is true.

Translate the following sentence to correct probability notation:

"The chance of it raining in the next 10 minutes is 80%"

Probability Rule for Equally Likely Outcomes

If all outcomes in the Sample Space S of a trial are equally likely, then

$$P(A) = \frac{|A|}{|S|}$$

where A is any event that is contained in S, and the notation $|set|$ is the **number** of outcomes in the set.

Coin Toss Example

Suppose a fair coin is tossed 2 times.

1. List the sample space S of this experiment
2. List the outcomes in the sample space that are in the event $B = $ "the coin lands heads at least once"
3. Find $P(B)$.

Six-sided Fair Die Example

Suppose a fair die is rolled twice.

1. List the sample space S of this experiment
2. List the outcomes in the sample space that are in the event C = "the sum of the two rolls is 7"
3. Find $P(C)$

Example: Briefcases in Elevator

Suppose Alice, Bob and Carol are in an elevator, each holding a brown briefcase, when an earthquake occurs. In the resulting chaos, their briefcases are dropped and shuffled around. Assuming Alice, Bob and Carol each pick up one briefcase, find the probability that they each pick up their own briefcase.

Practice Problem: Three Coin Tosses

Suppose a fair coin is tossed three times. Let A be the event "the coin lands heads at least once".

Find $P(A)$.