Lecture 14

STAT 109: Introductory Biostatistics

Lecture 14: Binomial Test — Steps and Examples

For each problem, follow these steps:

1. State the research question.
2. Define the parameter of interest, p, in words.
3. State the null and alternative hypotheses using appropriate symbols.
4. Identify n and the value of p under H_0 (for the null distribution).
5. Verify that the 4 conditions of a Binomial Random Process (BRP) are satisfied by the data collection.
6. Assuming the null hypothesis is true, create a plot of x vs $\text{dbinom}(x, n, p)$ (the null distribution), e.g. by hand or with software. Do **not** shade yet — shading will be done by hand in the direction(s) of the alternative.
7. Identify the observed value of x from the dataset.
8. On the plot, add a vertical line at the observed x and shade the bar(s) in the direction(s) of the inequality in H_a (by hand).
9. Compute the **p-value**: the probability of observing data as or more extreme than the observed x in the direction of H_a, assuming H_0 is true. (For a two-sided H_a, use both tails.)
10. Make a conclusion in context (reject H_0 or fail to reject H_0, and what that means for the research question).

Example 1:

Research question: Is the proportion of left-handed students at our school higher than the national rate of 10%?

Parameter: p = probability that a randomly chosen student from our school is left-handed (i.e., the proportion of left-handers in the population we are sampling).

Hypotheses: $H_0 : p = 0.10$ vs $H_a : p > 0.10$

n and p under H_0: $n = 25$ students, $p = 0.10$

BRP conditions (brief): Binary (left-handed or not), fixed $n = 25$, fixed p under null, independent students.

Data: In a sample of 25 students, $x = 6$ were left-handed.

Plot: Create a bar plot of x vs $\text{dbinom}(x, 25, 0.10)$ for $x = 0, 1, \ldots, 25$. Leave unshaded; then by hand add a vertical line at $x = 6$ and shade the bars for $x \geq 6$ (right tail, in the direction of H_a). Example code to produce the unshaded plot (display only):

```
n <- 25
p <- 0.10
x_vals <- 0:n
df <- data.frame(x = x_vals, prob = dbinom(x_vals, n, p))
library(ggplot2)
ggplot(df, aes(x = x, y = prob)) +
  geom_col() +
  labs(x = "x", y = "P(X = x)")
```

p-value: $P(X \geq 6 \mid H_0) = 1 - P(X \leq 5)$. For example, `1 - pbinom(5, 25, 0.10)` ≈ 0.03. Small p-value.

Conclusion: Reject H_0. The data are inconsistent with a 10% left-handed rate; we have evidence that the proportion of left-handed students at our school is higher than 10%.

Example 2

Research question: Is the proportion of defective items from the new supplier less than 15%?

Parameter: p = probability that a randomly selected item from the new supplier is defective (i.e., the proportion of defectives in the process).

Hypotheses: $H_0 : p = 0.15$ vs $H_a : p < 0.15$

n and p under H_0: $n = 25$ items, $p = 0.15$

BRP conditions (brief): Binary (defective or not), fixed $n = 25$, fixed p under null, independent items.

Data: In a sample of 25 items, $x = 3$ were defective.

Plot: Bar plot of x vs $\text{dbinom}(x, 25, 0.15)$ for $x = 0, 1, \ldots, 25$. By hand, add a vertical line at $x = 3$ and shade the bars for $x \leq 3$ (left tail, in the direction of H_a). Example code to produce the unshaded plot (display only):

```
n <- 25
p <- 0.15
x_vals <- 0:n
df <- data.frame(x = x_vals, prob = dbinom(x_vals, n, p))
library(ggplot2)
ggplot(df, aes(x = x, y = prob)) +
  geom_col() +
  labs(x = "x", y = "P(X = x)")
```

p-value: $P(X \leq 3 \mid H_0) = \text{pbinom}(3, 25, 0.15) \approx 0.47$. Large p-value.

Conclusion: Fail to reject H_0. The data are consistent with a 15% defective rate; we do not have evidence that the proportion of defectives from the new supplier is less than 15%.

Example 3

Research question: Are the two dice fair, in terms of the probability that the sum is 7 or 11? (With fair dice, that probability is 8/36: 6 ways to sum to 7 plus 2 ways to sum to 11, out of 36 equally likely outcomes.)

Parameter: p = probability that a single roll of two dice gives a sum of 7 or 11.

Hypotheses: $H_0 : p = 8/36$ vs $H_a : p \neq 8/36$

n and p under H_0: $n = 25$ rolls of the two dice, $p = 8/36$ (under the null that the dice are fair).

BRP conditions (brief): Binary (sum is 7 or 11, or not), fixed $n = 25$, fixed p under null, independent rolls.

Data: In 25 rolls of the two dice, $x = 2$ times the sum was 7 or 11 (far from the expected $25 \times (8/36) \approx 5.6$ under H_0).

Plot: Bar plot of x vs $\text{dbinom}(x, 25, 8/36)$ for $x = 0, 1, \ldots, 25$. By hand, add a vertical line at $x = 2$ and shade **both** tails: bars for $x \leq 2$ and for $x \geq 9$ (direction of "more extreme than" for a two-sided alternative). Example code to produce the unshaded plot (display only):

```
n <- 25
p <- 8/36
x_vals <- 0:n
df <- data.frame(x = x_vals, prob = dbinom(x_vals, n, p))
library(ggplot2)
ggplot(df, aes(x = x, y = prob)) +
  geom_col() +
  labs(x = "x", y = "P(X = x)")
```

p-value: $P(X \leq 2) + P(X \geq 9)$ under H_0. For example, `pbinom(2, 25, 8/36) + (1 - pbinom(8, 25, 8/36))`

≈ 0.04. Small p-value.

Conclusion: Reject H_0. The data are inconsistent with fair dice having probability 8/36 for sum 7 or 11; we have evidence that $p \neq 8/36$.

Example 4

Research question: Do more than 20% of students at our school use the gym at least once per week?

Parameter: $p =$ probability that a randomly chosen student uses the gym at least once per week (i.e., the proportion of such students in the population).

Hypotheses: $H_0 : p = 0.20$ vs $H_a : p > 0.20$

n and p under H_0: $n = 25$ students, $p = 0.20$

BRP conditions (brief): Binary (uses gym weekly or not), fixed $n = 25$, fixed p under null, independent students.

Data: In a sample of 25 students, $x = 5$ use the gym at least once per week.

Plot: Bar plot of x vs $\text{dbinom}(x, 25, 0.20)$ for $x = 0, 1, \ldots, 25$. By hand, add a vertical line at $x = 5$ and shade the bars for $x \geq 5$ (right tail, in the direction of H_a). Example code to produce the unshaded plot (display only):

```
n <- 25
p <- 0.20
x_vals <- 0:n
df <- data.frame(x = x_vals, prob = dbinom(x_vals, n, p))
library(ggplot2)
ggplot(df, aes(x = x, y = prob)) +
  geom_col() +
  labs(x = "x", y = "P(X = x)")
```

p-value: $P(X \geq 5 \mid H_0) = 1 - P(X \leq 4)$. For example, `1 - pbinom(4, 25, 0.20)` ≈ 0.58. Large p-value.

Conclusion: Fail to reject H_0. The data are consistent with a 20% gym-use rate; we do not have evidence that more than 20% of students use the gym at least once per week.

Summary

Example	H_a direction	Observed x	p-value	Conclusion
1	$p > 0.10$	6	Small	Reject H_0
2	$p < 0.15$	3	Large	Fail to reject
3	$p \neq 8/36$ (two dice, sum 7 or 11)	2	Small	Reject H_0
4	$p > 0.20$	5	Large	Fail to reject

In every case, the plot is x vs $\text{dbinom}(x, n, p)$ with no shading in the code; shade the appropriate tail(s) by hand to match H_a and double-check that the region you shaded compared to the total area of the bars corresponds to the size of the p-value relative to 1.