

Lecture 6

STAT 109: Introductory Biostatistics

Lecture 6: Conditional Probability & Multiplication Rule

Learning Outcomes:

1. State the definition of **conditional probability** and compute $P(A | B)$.
2. Use the **multiplication rule** to compute $P(A \text{ and } B)$.
3. Determine whether two events are **independent** (using equivalent criteria).

Conditional Probability

Notation: $P(A | B)$ means “the probability event A occurs **given** event B occurred.”

Definition: For events A and B with $P(B) > 0$,

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$$

Example 1 (die roll): Consider the trial of rolling a fair six-sided die once.

Let A be the event “the roll is a 3” and let B be the event “the roll is odd.”

- a. List the outcomes in A , in B , and in A and B .
- b. Compute $P(A)$, $P(B)$, and $P(A \text{ and } B)$.
- c. Find $P(A | B)$: the probability of getting a 3 **given** the roll resulted in an odd number.
- d. Find $P(B | A)$: the probability of getting an odd number **given** the roll resulted in a 3.

Practice Problem 1: A jar contains 10 candies: 6 are red and 4 are blue. Two candies are selected **without replacement**.

Let A be the event “the first candy is red” and B be the event “the second candy is red.”

- a. Find $P(A)$.
- b. Find $P(B | A)$.

c. Use the multiplication rule (below) to find $P(A \text{ and } B)$.

Multiplication Rule

For any two events A and B with $P(B) > 0$,

$$P(A \text{ and } B) = P(A | B) P(B)$$

Equivalently (if $P(A) > 0$),

$$P(A \text{ and } B) = P(B | A) P(A)$$

Example 2 (two cards, no replacement): A card is drawn from a standard 52-card deck, then a second card is drawn **without replacement**.

Find the probability the **first** card is an Ace and the **second** card is a King.

Independence

Events A and B are **independent** if knowing that one event occurred does not change the probability of the other.

Equivalent ways to check independence:

- $P(A | B) = P(A)$ (when $P(B) > 0$)
- $P(B | A) = P(B)$ (when $P(A) > 0$)
- $P(A \text{ and } B) = P(A) P(B)$

Example 3 (king vs spade): Suppose a card is randomly dealt from a standard 52-card deck. Is the event “the card is a king” independent of the event “the card is a spade”?

Multiplication Rule for Two Independent Events

If A and B are independent, then

$$P(A \text{ and } B) = P(A) P(B)$$

Example 4 (two coin tosses): Suppose a fair coin is tossed twice. Find the probability of getting “heads” on the first toss **and** “tails” on the second toss.

Practice Problem 2: Suppose a fair six-sided die is rolled twice.

Let A be the event “the first roll is even” and let B be the event “the second roll is 6.”

- Are A and B independent? (Explain briefly.)
- Compute $P(A \text{ and } B)$.