

Lecture 11

STAT 109: Introductory Biostatistics

Lecture 11: Practice Problems — Binomial Random Process

Definitions and formula summary

Notation:

- n = number of trials
- p = probability of “success” on a single trial
- x = observed number of successes in n trials
- X = random variable that counts the number of successes in n trials

Factorial: $n! = n(n - 1)(n - 2) \cdots (3)(2)(1)$

Counting:

- ${}_n C_x = {}^n \text{C}_x = \frac{n!}{x!(n-x)!}$
- ${}_n P_x = \frac{n!}{(n-x)!}$

Four conditions for a binomial random process (BRP):

1. Each trial results in either “success” or “failure”.
2. Outcomes between trials are **independent**.
3. The probability of success p is the same on every trial.
4. A **fixed** number of trials n are observed.

Binomial random variable: X = number of successes in n trials when data are collected from a BRP.

Probability distribution function (pdf) for a binomial random variable X :

$$f(x) = P(X = x) = {}^n \text{C}_x p^x (1-p)^{n-x} = {}_n C_x p^x (1-p)^{n-x}$$

Cumulative probability: $P(X \leq x) = f(0) + f(1) + f(2) + \cdots + f(x)$

Note: $P(X > x) = 1 - P(X \leq x) = f(x+1) + f(x+2) + \cdots + f(n)$

Mean of X : $\mu = np$

Standard deviation of X : $\sigma = \sqrt{np(1-p)}$

Practice problems

1. Which situation is binomial?

Which of the following situations is an example of data being collected from a **binomial random process**? Be prepared to justify your answer by discussing the validity of each of the four conditions.

- a. Rolling a six-sided die 15 times and counting the number of times a “3” was obtained.

- b. Picking 10 gummy vitamins from a bottle that contains 100 vitamins of various animal shapes. Count the number of lions, tigers, and bears in the set of 10.
 - c. Drawing cards one at a time from a deck of 52, counting the number of kings drawn. After each card is drawn, it is **replaced** in the deck, the deck is shuffled, and another card is drawn.
 - d. Drawing cards one at a time from a deck of 52, counting the number of kings drawn. After each card is drawn, it is **not** replaced in the deck.
 - e. Giving a class activity to 20 students and counting the number of these students who ask a question about it during class.
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2. Identify n , p , and X ; write probability expression

For each situation below, identify n , p , and the appropriate value to compare to X , then write an expression for the probability using the pdf of a binomial random variable.

Problem 2. Suppose Bob flips a fair coin 4 times. What is the probability he gets exactly 2 heads?

Problem 3. Suppose Neal has an unfair coin: it has an 80% chance of landing “heads”. What is the probability that he gets exactly 2 heads if he tosses the coin 4 times?

Problem 4. Suppose that in a week of 7 days, there is a 65% chance of rain on any single day. What is the

probability of it raining on **at least** 2 of the 7 days? On **at most** 2 of the 7 days?

Problem 5. Suppose customers at a coffee shop can order either iced or hot coffee. Sharon observes 6 customers ordering one day: of these, only 2 order iced coffee. If 10% of all customers order iced coffee, what is the probability of observing 2 or more iced orders (under this model)?

Problem 6. Compute the mean and standard deviation of the random variable X that is the number of “heads” in 25 tosses of a fair coin.