

# **Lecture 6**

**STAT 109: Introductory Biostatistics**

# Lecture 6: Conditional Probability & Multiplication Rule

Learning Outcomes:

1. State the definition of **conditional probability** and compute  $P(A | B)$ .
  2. Use the **multiplication rule** to compute  $P(A \text{ and } B)$ .
  3. Determine whether two events are **independent** (using equivalent criteria).
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## Conditional Probability

**Notation:**  $P(A | B)$  means “the probability event  $A$  occurs **given** event  $B$  occurred.”

**Definition:** For events  $A$  and  $B$  with  $P(B) > 0$ ,

$$P(A | B) = \frac{P(\text{A and B})}{P(B)}$$

**Example 1 (die roll):** Consider the trial of rolling a fair six-sided die once.

Let  $A$  be the event “the roll is a 3” and let  $B$  be the event “the roll is odd.”

- a. List the outcomes in  $A$ , in  $B$ , and in  $A$  and  $B$ .
- b. Compute  $P(A)$ ,  $P(B)$ , and  $P(A \text{ and } B)$ .
- c. Find  $P(A | B)$ : the probability of getting a 3 **given** the roll resulted in an odd number.
- d. Find  $P(B | A)$ : the probability of getting an odd number **given** the roll resulted in a 3.

**Practice Problem 1:** A jar contains 10 candies: 6 are red and 4 are blue. Two candies are selected **without replacement**.

Let  $A$  be the event “the first candy is red” and  $B$  be the event “the second candy is red.”

- a. Find  $P(A)$ .
- b. Find  $P(B | A)$ .

- c. Use the multiplication rule (below) to find  $P(A \text{ and } B)$ .
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### Multiplication Rule

For any two events  $A$  and  $B$  with  $P(B) > 0$ ,

$$P(A \text{ and } B) = P(A | B) P(B)$$

Equivalently (if  $P(A) > 0$ ),

$$P(A \text{ and } B) = P(B | A) P(A)$$

**Example 2 (two cards, no replacement):** A card is drawn from a standard 52-card deck, then a second card is drawn **without replacement**.

Find the probability the **first** card is an Ace and the **second** card is a King.

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### Independence

Events  $A$  and  $B$  are **independent** if knowing that one event occurred does not change the probability of the other.

Equivalent ways to check independence:

- $P(A | B) = P(A)$  (when  $P(B) > 0$ )
- $P(B | A) = P(B)$  (when  $P(A) > 0$ )
- $P(A \text{ and } B) = P(A) P(B)$

**Example 3 (king vs spade):** Suppose a card is randomly dealt from a standard 52-card deck. Is the event “the card is a king” independent of the event “the card is a spade”?

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### Multiplication Rule for Two Independent Events

If  $A$  and  $B$  are independent, then

$$P(A \text{ and } B) = P(A) P(B)$$

**Example 4 (two coin tosses):** Suppose a fair coin is tossed twice.

Find the probability of getting “heads” on the first toss **and** “tails” on the second toss.

**Practice Problem 2:** Suppose a fair six-sided die is rolled twice.

Let  $A$  be the event “the first roll is even” and let  $B$  be the event “the second roll is 6.”

- Are  $A$  and  $B$  independent? (Explain briefly.)
- Compute  $P(A \text{ and } B)$ .