Lecture 10

STAT 109: Introductory Biostatistics

Lecture 10: From Coin Tosses to the Binomial Random Process

Learning Outcomes:

1. Define **random variable** and its **probability distribution function (pdf)** (for discrete random variables).
2. Define a **binomial random variable** and a **binomial random process**.
3. Match the notation (n, x, p) and **four conditions** for a binomial process with the process of tossing a coin.
4. Express probabilities for binomial random variables via the binomial pdf formula.

Random variable

A **random variable** is defined as...

Probability distribution function (pdf)

For a discrete random variable X, the **probability distribution function** (or **probability mass function**) gives the probability that X takes each possible value. We write $f(x) = P(X = x)$ for each possible value x.

Examples: Coin tosses

Example 1. A coin is tossed $n = 1$ time; p = probability of heads. Find an expression for $f(x) = P(X = x)$ for $x = 0$ and $x = 1$.

Example 2. A coin is tossed $n = 2$ times; p = probability of heads. Find an expression for $f(x)$ for $x = 0, 1, 2$.

Example 3. A coin is tossed $n = 3$ times; p = probability of heads. Find an expression for $f(x)$ for $x = 0, 1, 2, 3$.

Example 4. A coin is tossed $n = 4$ times; p = probability of heads. Find an expression for $f(x)$ for $x = 0, 1, 2, 3, 4$.

General case. A coin is tossed n times; p = probability of heads. Find an expression for $f(x) = P(X = x)$ for $x = 0, 1, 2, ..., n$.

.

Binomial random variable and binomial random process

A **binomial random variable** X counts the number of "successes" in a **binomial random process**.

Four conditions for a binomial random process:

1. **Fixed number of trials** n (e.g., n coin tosses).
2. **Two outcomes** on each trial: "success" or "failure" (e.g., heads vs tails).
3. **Independent trials** — the outcome of one trial does not affect the others.
4. **Constant probability** p of success on each trial (e.g., same p for every toss).

Notation: n = number of trials, p = probability of success on one trial, X = number of successes, and x = a specific value of X (0, 1, 2, ..., n).

Binomial formula

If X is binomial with n trials and success probability p, then for $x = 0, 1, 2, \dots, n$,

$$P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x} = {}_n C_x \, p^x (1-p)^{n-x}$$

where $\binom{n}{x} = {}_n C_x = \dfrac{n!}{x!(n-x)!}$ is the number of ways to choose x successes out of n trials.
