Lecture 12

STAT 109: Introductory Biostatistics

Lecture 12: Introduction to the Binomial Test

A motivating question: Do I have ESP?

Extra Sensory Perception (ESP) is the ability to accurately predict what will happen using supernatural methods. Do I have it? If so, I could make lots of money by betting on the stock market! To find out, we can take an online ESP test and then analyze the data using the **Binomial Test**.

The online ESP test

We will collect data from the **Advanced ESP (Zener Cards) test** at PsychicScience.org.

- The test uses **5 possible symbols** (circle, cross, waves, square, star).
- On each trial, the computer randomly selects one symbol; your task is to guess it.
- We will use **25 attempts** (25 cards).
- Choose: **Clairvoyance**, **Open deck**, **Cards seen**, and **25 cards**.

From question to hypotheses

Let p be the probability of a correct prediction.

- If I **don't** have ESP, I'm just guessing among 5 options, so $p = 1/5 = 0.2$.
- If I **do** have ESP, I should get more than chance, so $p > 0.2$.

In conventional notation:

$$H_0 : p = 0.2 \quad \text{vs.} \quad H_a : p > 0.2$$

Data collection and the four conditions

When we record the number of correct predictions out of 25 attempts, we treat this as a **Binomial Random Process**. We check the 4 conditions:

1. **Binary outcome:** Each prediction was either correct or not.
2. **Fixed number of trials:** We made a fixed number of attempts, $n = 25$.
3. **Fixed probability:** The probability of a correct guess on each trial doesn't change (open deck: each card is chosen at random from 5 options each time).
4. **Independent trials:** We don't use information from previous attempts to inform the next guess (and the computer randomizes each card independently).

So the number of correct predictions in 25 attempts can be modeled by a **binomial random variable** with $n = 25$ and $p = 0.2$ under the null hypothesis.

Sample project structure (Project 1)

The following is an example of how you might write up a **Binomial Test** in your one-page Project 1 summary. This example uses the ESP question and data from the online test.

Introduction

Extra Sensory Perception (ESP) is the ability to accurately predict what will happen using supernatural methods. Do I have it? If so, I could make lots of money by betting on the stock market! To find out, I took an online ESP test and then analyzed the data using the Binomial Test.

Let p be the probability that I have an accurate prediction. In the online ESP test, there were 5 possible predictions, so if I don't have ESP, then $p = 1/5$. If I do have ESP, then $p > 1/5$. These hypotheses in conventional notation are:

$$H_0 : p = 0.2 \quad \text{vs.} \quad H_a : p > 0.2$$

Data Collection

I recorded the number of correct predictions out of 25 attempts in the online ESP test at Advanced ESP card guessing (Zener Cards) test — PsychicScience.org.

I believe this data collection satisfied the 4 conditions of a Binomial Random Process:

1. Each prediction was correct or not (**binary outcome**).
2. I made a fixed number of attempts, $n = 25$ (**fixed number of trials**).
3. My probability of getting a prediction correct shouldn't change between attempts (**fixed probability of success** in a single trial).
4. I didn't use information from my previous attempts to inform my predictions (**independent trials**).

Data Analysis

Descriptive statistics: In 25 attempts, I correctly predicted the shape on 4 cards (16%).

Inferential statistics: Let X be the number of correct predictions in 25 attempts. Assuming I'm just guessing, X is a Binomial random variable with $n = 25$ and $p = 0.2$. I computed the probability of each value of X and the probability of seeing my data (4) **or more** correct predictions using R. This **p-value** is 0.76.

Conclusion

There is a 76% chance of seeing my results of 4 correct predictions or more in another set of 25 attempts if I don't have ESP and am just guessing. Since this is a large probability, I've concluded I don't have ESP and will find other ways besides betting to make money.

Takeaway

- The **null model** is "no ESP" ($p = 0.2$). Under that model, we compute the probability of data **as extreme as** what we observed (or more extreme).

- That probability is the **p-value**. A **small** p-value suggests the null model is not consistent with the data; a **large** p-value (as in this example) suggests the data are consistent with just guessing.
- Your Project 1 report should follow this same structure: Introduction (question, p, hypotheses), Data Collection (how you collected data and why the 4 BRP conditions hold), Data Analysis (descriptive and inferential statistics, including p-value), and Conclusion.

Summary: 7 steps of the Binomial Test

1. **Define the parameter of interest, p:** Identify and define p. Here p is the **theoretical** probability of a "success" in a single trial for the population or process (e.g., the probability of a correct guess on one ESP card). p is a parameter of the process, not a value observed in your sample.
2. **State the null and alternative hypotheses using appropriate symbols:**
   - **Alternative hypothesis (H_a):** The **direction** (inequality) of your best guess for the range of values of p — your research question in mathematical form (e.g., $p > 0.2$, $p < 0.2$, or $p \neq 0.2$).
   - **Null hypothesis (H_0):** Take the alternative and change the inequality to an **equality** (e.g., if $H_a : p > 0.2$, then $H_0 : p = 0.2$). This is the baseline or "no effect" assumption.
3. **Assuming the null hypothesis is true, generate the null distribution of x for sample size n:** Build the probability distribution of the number of successes, x, you would expect to see in n trials if H_0 were true. This null distribution is used to compute the p-value.
4. **Verify that the 4 conditions of a Binomial Random Process (BRP) are satisfied by the data collection:** Confirm that (a) each trial has a **binary** outcome (success or failure), (b) the number of trials n is **fixed**, (c) the probability of success p is **fixed** on every trial, and (d) trials are **independent**.
5. **Identify the value of x from the dataset:** Record the **observed** number of successes in your n trials (e.g., number of correct ESP guesses out of 25).
6. **Determine whether the data are consistent with the null hypothesis by computing the p-value:** The **p-value** is the probability of observing **new** data (in n trials) that is equal to or **more extreme** (in the direction of the alternative) than the data you actually observed, **assuming the null hypothesis is true**. A small p-value suggests the observed data are inconsistent with the null; a large p-value suggests they are consistent.
7. **Make a conclusion in context:** Interpret the p-value and state a conclusion about your research question (e.g., whether there is evidence for ESP or whether the results are consistent with chance).

R code: Colab notebook

The R code for the binomial test and plots in this lecture is in this Google Colab notebook:

Lecture 12 — Binomial Test (R code)