Problem 3

Let i be the index of an example. We can compute all the weight terms immediately by
$w = w + \alpha x^{(i)^T}(y^{(i)} - h_w(x^{(i)}))$

w = $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

i = 1:

$x^{(1)}w = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1$

$h_w(x^{(1)}) = 1$

w = $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} (0 - 1) = \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$

Check if class correctly:

Xw = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 4 \\ 1 & 2 & 4.5 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -0.5 \\ 0 \\ 0.75 \\ 1 \end{bmatrix}$

$h_w(X) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ Not class correctly, continue

i=2:

$x^{(2)}w = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} = -0.5$

$h_w(x^{(2)}) = 0$

w = $\begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} (0 - 0) = \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$

w does not change, continue i=3:

$x^{(3)}w = \begin{bmatrix} 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} = 0$

$$\begin{bmatrix} 0.5 \end{bmatrix}$$

$h_w(x^{(3)}) = 1$

$$\text{w} = \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} (0 - 1) = \begin{bmatrix} -1 \\ -2 \\ -1.5 \end{bmatrix}$$

Check if class correctly:

$$\text{Xw} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 4 \\ 1 & 2 & 4.5 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ -1.5 \end{bmatrix} = \begin{bmatrix} -4.5 \\ -8 \\ -13 \\ -11.75 \\ -16 \end{bmatrix}$$

$h_w(X) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Not class correctly, continue

i=4:

$$x^{(4)} w = \begin{bmatrix} 1 & 2 & 4.5 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ -1.5 \end{bmatrix} = -11.75$$

$h_w(x^{(3)}) = 0$

$$\text{w} = \begin{bmatrix} -1 \\ -2 \\ -1.5 \end{bmatrix} + 0.5 \begin{bmatrix} 1 \\ 2 \\ 4.5 \end{bmatrix} (1 - 0) = \begin{bmatrix} -0.5 \\ -1 \\ 0.75 \end{bmatrix}$$

Check if class correctly:

$$\text{Xw} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 4 \\ 1 & 2 & 4.5 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} -0.5 \\ -1 \\ 0.75 \end{bmatrix} = \begin{bmatrix} -0.75 \\ -1 \\ -0.5 \\ 0.875 \\ 1 \end{bmatrix}$$

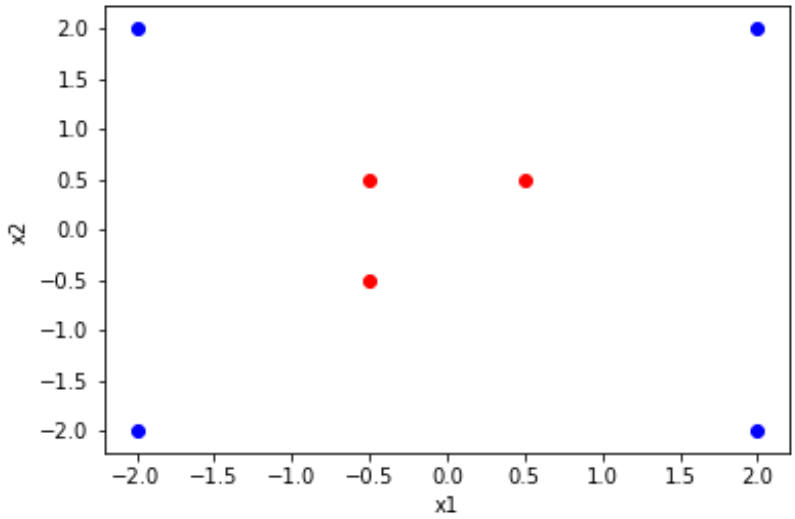
$h_w(X) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ class correctly

Problem 4

(a)

In [195]:

```
plt.scatter([0.5,-0.5,-0.5],[0.5,0.5,-0.5],color='red')
plt.scatter([2,2,-2,-2],[2,-2,2,-2],color='blue')
plt.xlabel('x1')
plt.ylabel('x2')
plt.show()
```



The boundary I choose is a circle $x^2+y^2=1$

(b)Using perceptron updating algorithm. And let + be 1, - be 0

Let w = $\begin{bmatrix} 9 \\ -1 \\ -1 \end{bmatrix}$ i=1:

$$x^{(1)}w = \begin{bmatrix} 1 & 0.25 & 0.25 \end{bmatrix} \begin{bmatrix} 9 \\ -1 \\ -1 \end{bmatrix} = 8.5$$

$h_w(x^{(1)}) = 1$

So $y - h_w(x^{(1)}) = 1 - 1 = 0$, w will not change

Thus we can skip example 2 and 3.

i=4:

$$x^{(4)}w = \begin{bmatrix} 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -1 \\ -1 \end{bmatrix} = 1 \; h_w(x^{(1)}) = 1$$

$$w = \begin{bmatrix} 9 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} (0-1) = \begin{bmatrix} 8 \\ -5 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0.25 & 0.25 \\ 1 & 0.25 & 0.25 \\ 1 & 0.25 & 0.25 \end{bmatrix} \begin{bmatrix} 8 \\ \end{bmatrix} \quad \begin{bmatrix} 5.5 \\ 5.5 \\ 5.5 \end{bmatrix}$$

$$Xw = \begin{bmatrix} 1 & 4 & 4 \\ 1 & 4 & 4 \\ 1 & 4 & 4 \\ 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} -5 \\ -5 \end{bmatrix} = \begin{bmatrix} -32 \\ -32 \\ -32 \\ -32 \end{bmatrix}$$

$$hw(X) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ Class correctly

(c)

$$p(x^{(1)}) = \frac{1}{1 + e^{-5.5}} \approx 0.996$$

$$p(x^{(3)}) = \frac{1}{1 + e^{-5.5}} \approx 0.996$$