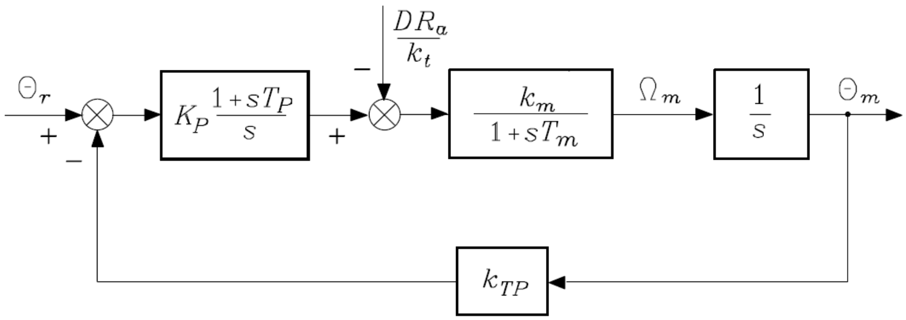
MAE C163C / C263C Homework #3

(Due via Gradescope by 11:59pm on Friday, 4/25)

1. Block diagram algebra



The block diagram above describes general independent joint control with position feedback. Report the transfer functions for the following:

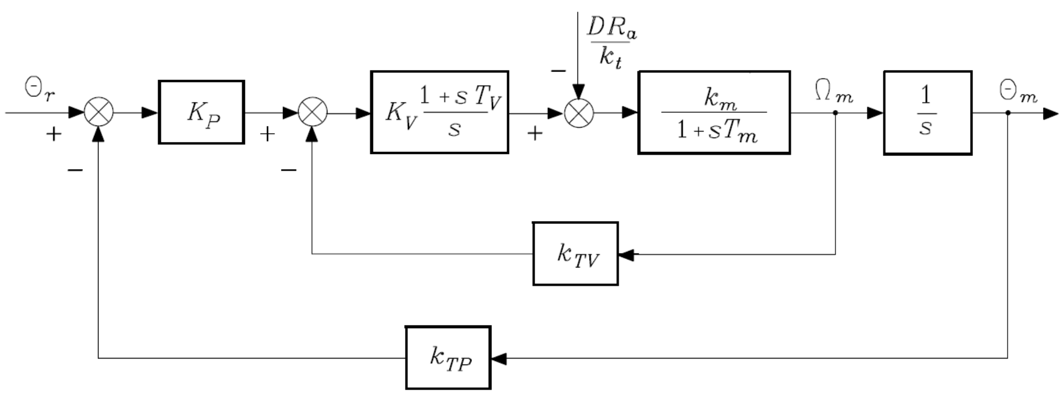
a) Forward path (feedforward)

b) Return path (feedback)

c) Closed-loop input/output transfer function $\frac{\Theta_m(s)}{\Theta_r(s)}$

d) Closed-loop disturbance/output transfer function $\frac{\Theta_m(s)}{D(s)}$

2. General independent joint control with position and velocity feedback



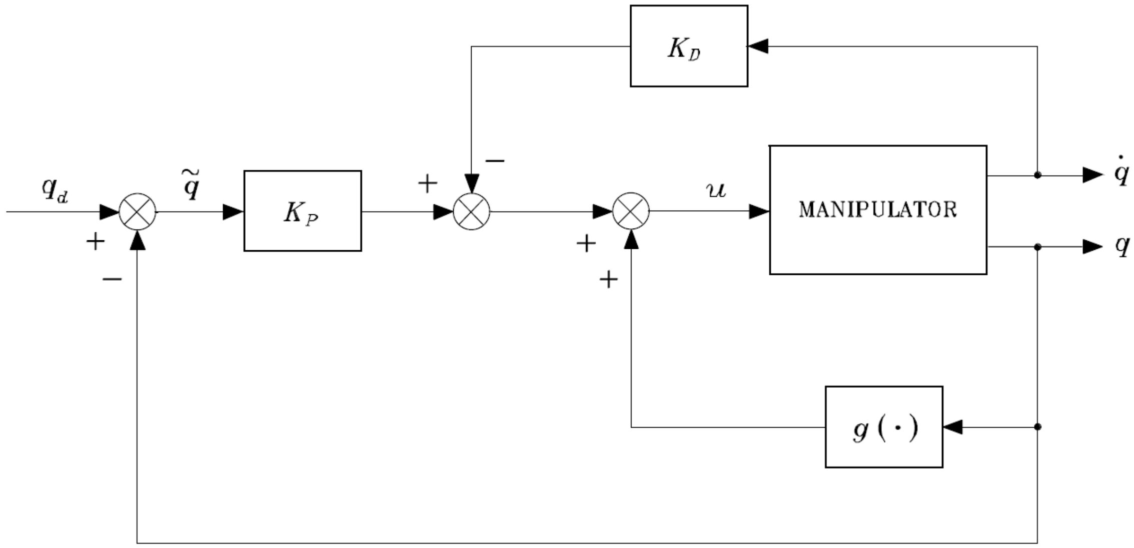
The block diagram above describes general independent joint control with position and velocity feedback. Consider a single joint drive system with the following model parameters: I_m = 6 kg-m^2, R_a = 0.3 Ω, k_t = 0.5 N-m/A, k_v = 0.5 V-s/rad, F_m = 0.001 N-m-s/rad, and unit transducer constants k_{TP} and k_{TV}. You will need to refer to Siciliano et al., Sec. 5.2.1 to assign values to the remaining motor model parameters. For ease of grading, set $T_V = T_m$.

a) Report the symbolic closed-loop input/output transfer function. Design parameters K_P and K_V should be the only variables.

b) Design a position and velocity feedback controller having a closed-loop response with damping ratio ζ = 0.4 and natural frequency ω_n = 20 rad/s. Report your K_P and K_V, along with the poles of the closed-loop system.

c) Report the disturbance rejection factor X_R and output recovery time T_R. Comment on what can be done to improve the output recovery time.

3. Joint space PD control with gravity compensation



The block diagram above describes joint space PD control with gravity compensation.

Consider a two-link planar arm with the following parameters:

$$a_1 = a_2 = 1\,\mathrm{m} \quad \ell_1 = \ell_2 = 0.5\,\mathrm{m} \quad m_{\ell_1} = m_{\ell_2} = 50\,\mathrm{kg} \quad I_{\ell_1} = I_{\ell_2} = 10\,\mathrm{kg{\cdot}m^2}$$

$$k_{r1} = k_{r2} = 100 \qquad m_{m1} = m_{m2} = 5\,\mathrm{kg} \qquad I_{m1} = I_{m2} = 0.01\,\mathrm{kg{\cdot}m^2}$$

The arm is assumed to be driven by two equal actuators with the following parameters:

$$F_{m1} = F_{m2} = 0.01\,\mathrm{N{\cdot}m{\cdot}s/rad} \qquad R_{a1} = R_{a2} = 10\,\mathrm{ohm}$$

$$k_{t1} = k_{t2} = 2\,\mathrm{N{\cdot}m/A} \qquad k_{v1} = k_{v2} = 2\,\mathrm{V{\cdot}s/rad}$$

a) Design a closed-loop stable PD controller with gravity compensation in Drake by completing the inline instructions in *HW3.py*. The code is already set up for a sampling time of 1 ms.

   Your PD controller (comprised of a <u>single set</u> of K_P and K_D gain matrices) must work for both of the following desired poses:

   1) $\boldsymbol{q} = [\,\pi/4 \quad -\pi/2\,]^T$
   2) $\boldsymbol{q} = [\,-\pi \quad -3\pi/4\,]^T$

   <u>Report your K_P and K_D gain matrices.</u> There is no single correct answer. I will accept any single set of K_P and K_D gain matrices that achieves a stable steady state with minimal tracking errors for both desired postures within the time horizon specified in part *b*.

b) Simulate the closed-loop response of the system and verify its closed-loop stability with plots. For each of the two desired poses, plot the time history of the two joint angles in radians in separate subfigures for a time horizon of 2.5 s. The code is already set up for initial joint angles that have been perturbed from the desired values by -0.1 rad. Indicate the desired joint angle in each subfigure by drawing a dashed horizontal line at the desired value and show that your controller drives each joint angle to the desired value within the allotted time.

Summary of deliverables:

Your submission should include:

- Your K_P and K_D gain matrices
- Labeled time history plots
- A plot of your final Drake block diagram
- Your **completed** *HW3.py* file converted to a PDF (To facilitate grading, see the relevant PyCharm help page for how to print a *.py* file to a PDF).

NOTE: Each student must submit their own independent work. **For full credit, you must submit to Gradescope all custom Python code** (e.g. *HW3.py*) **and requested plots with labels.** You may save this content to PDF or take screenshots for electronic submission via Gradescope. Files of the *.py* and *.toml* format cannot be directly uploaded to Gradescope and should not be e-mailed to instructors for grading. The more intermediate results and comments you provide, the greater the opportunity for partial credit.