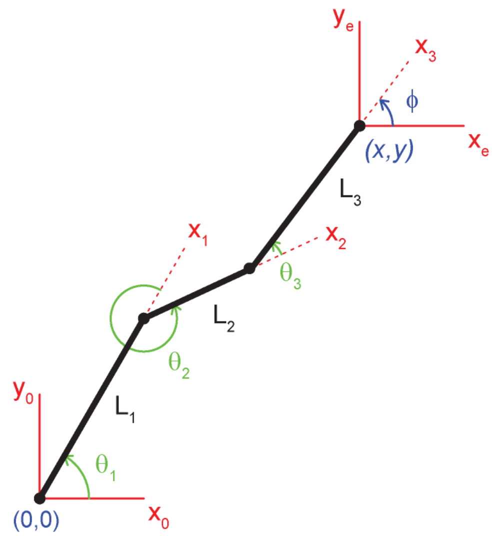
MAE C163C / C263C Homework #2

*(Due via Gradescope by **11:59pm PT on Saturday, 4/19**)*

Consider the following planar 3R manipulator with joint coordinates θ_1, θ_2, θ_3 and end-effector coordinates x, y, and ϕ:



The Jacobian for the planar 3R manipulator is shown in the velocity equation below,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -(L_1 s_1 + L_2 s_{12} + L_3 s_{123}) & -(L_2 s_{12} + L_3 s_{123}) & -L_3 s_{123} \\ (L_1 c_1 + L_2 c_{12} + L_3 c_{123}) & (L_2 c_{12} + L_3 c_{123}) & L_3 c_{123} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Assume the following link lengths (arbitrary units):

L1 = 2

L2 = 1

L3 = 0.75

Use the following three manipulator configurations (joint angles in degrees) for the subsequent analyses.

Configuration	**θ_1 [deg]**	**θ_2 [deg]**	**θ_3 [deg]**
Configuration 0	0	-0.05	0
Configuration 1	-22.5	-22.5	-45
Configuration 2	-45	-67.5	-67.5

Velocity Analysis:

1. Complete the `calc_jacobian` function in *HW2.py* to calculate the Jacobian matrix (J) given link lengths and a configuration as inputs.

2. Report the three singular values of the Jacobian matrix (J) for each of the three configurations. See *HW.2.py* for details on how to perform singular value decomposition in Python.

3. Provide a **single** x-y (2D) plot displaying (i) the manipulator in all three configurations and (ii) the velocity manipulability ellipses (x-dot and y-dot values only) such that each ellipse is centered at the origin of the end-effector frame (i.e. at the distal end of link 3). The comments in *HW2.py* explain how to generate velocity ellipses by transforming a unit sphere joint angle velocity input into the velocity manipulability ellipsoid and projecting the resulting ellipsoid onto the x-dot - y-dot plane. **To facilitate grading, use the default colors as well as axis limits of [-5, 8] for the x-axis and [-5, 5] for the y-axis.**

Force Analysis:

4. Report the three singular values of the Jacobian inverse transpose matrix (J^{-T}) for each of the three configurations.

5. Provide a **single** x-y (2D) plot displaying (i) the manipulator in all three configurations and (ii) the force manipulability ellipses (f_x and f_y values only) such that each ellipse is centered at the origin of the end-effector frame (i.e. at the distal end of link 3). The comments in *HW2.py* explain how to generate force ellipses by transforming a unit sphere joint torque input into the force manipulability ellipsoid and projecting the resulting ellipsoid onto the f_x-f_y plane. **To facilitate grading, use the default colors as well as axis limits of [-5, 8] for the x-axis and [-5, 5] for the y-axis.**

6. For each of the three configurations, provide a single x-y-z (3D) plot that includes *both* the velocity and force manipulability ellipsoids for a given configuration. You should have three separate plots (one for each configuration). The velocity ellipsoid coordinates will be x-dot, y-dot, and phi-dot. The force ellipsoid coordinates will be f_x, f_y, and M_z. Note that you can check your answers for items 3 and 5 above by viewing the x-y plane of each 3D plot (while taking into account differences in axis limits). **To facilitate grading, use the default axis limits, viewpoint, and colors for each of your three plots.**

Summary of deliverables:

Your submission should include:

- All command window outputs and plots requested in items 2-6 above.
- Your **completed** *HW2.py* file converted to a PDF (To facilitate grading, see the relevant PyCharm help page for how to print a *.py* file to a PDF).

NOTE: Each student must submit their own independent work. **For full credit, you must submit to Gradescope all custom Python code** (e.g. *HW2.py*)**, requested command window outputs, and requested plots with labels.** You may save this content to PDF or take photographs for electronic submission via Gradescope. Files of the *.py* and *.toml* format cannot be directly uploaded to Gradescope and should not be e-mailed to instructors for grading. The more intermediate results and comments you provide, the greater the opportunity for partial credit.