MECH&AE 263F Homework 1 Deliverables

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***Abstract*— This electronic document contains the plots and answers to questions within the first homework assignment of the MECH&AE 263F class within UCLA. Sections were created appropriately for different problem statements to be answered.**

I. Deliverable 1

This deliverable required the creation of an executable script for simulating falling connected spheres within a liquid, both with specified physical properties. The simulation time ran for 10 seconds, had timesteps of 0.01 seconds, and simulated three connected spheres. Within this deliverable, two forms of calculations were used: implicit and explicit. Implicit form utilizes both old and new steps in time to iterate towards a solution, while explicit methods directly solve for the value by assuming a small value for the time step used in the simulation. Within this deliverable, the following equation was used for the explicit solution:

$$q(t_{k+1}) = q(t_k) + \left(\frac{\Delta t^2}{m}\right)\left[\left(\frac{m}{\Delta t}\right)u(t_k) - C\big(u(t_k)\big) - \left(\frac{\partial E}{\partial q}\right)\right] \quad (1)$$

The following were asked to have responses:

1. Plots for the position and velocity of the middle sphere at times: 0s, 0,01s, 0.05s, 0.10s, 1.0s, 10.0s.
2. The observable terminal velocity of the simulation
3. Observed behavior of the turning angle when the radii are the same
4. Benefits and downsides of solving through implicit versus explicit methods within the simulation

A. *Plots for specified times (1)*

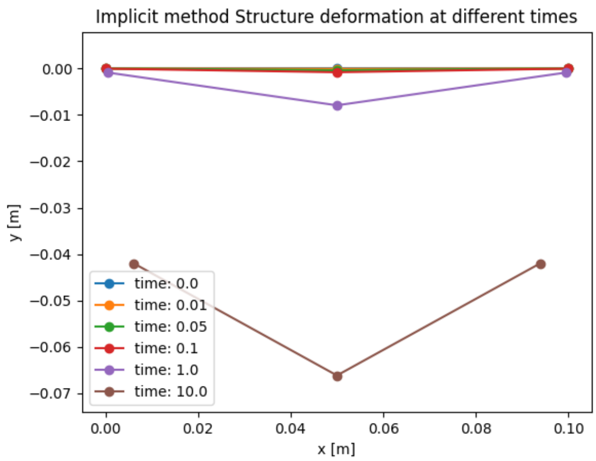

Figure 1. Collective plots of structure over different time steps through the application of implicit methods

The figure above shows the overall displacement of the structure at the time stamps specified when the implicit method was applied at a time step of 0.01s. The figures 2 through 7 below are individual plots of each time step for clarity.

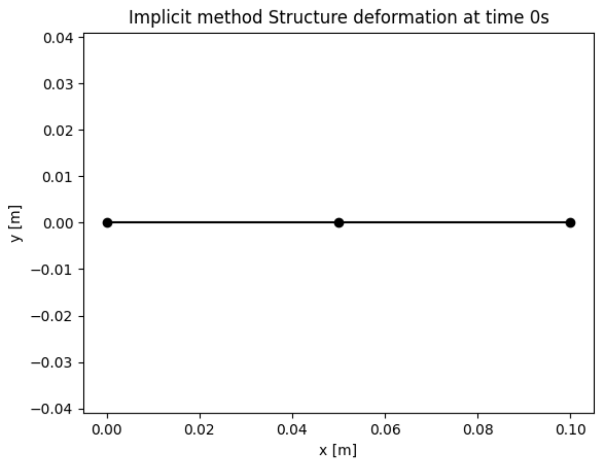

Figure 2. Plot of structure at 0s of the implicit method

*Generation of the graphs through usage of python. Algorithms for calculating energy gradients, Jacobians, and simulation code were based off scripts provided by Professor M. Khalid Jawed

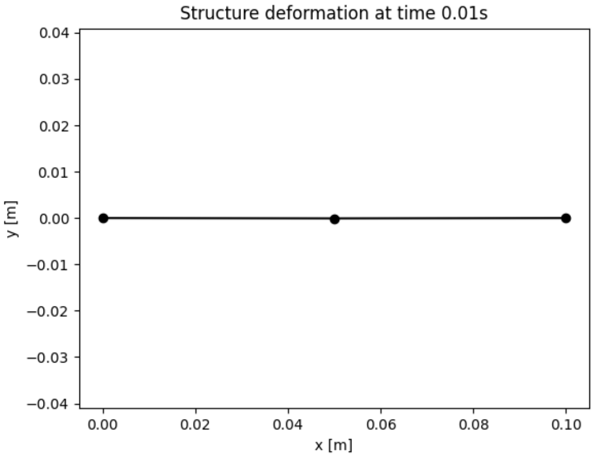

Figure 3. Plot of structure at 0.01s of the implicit method

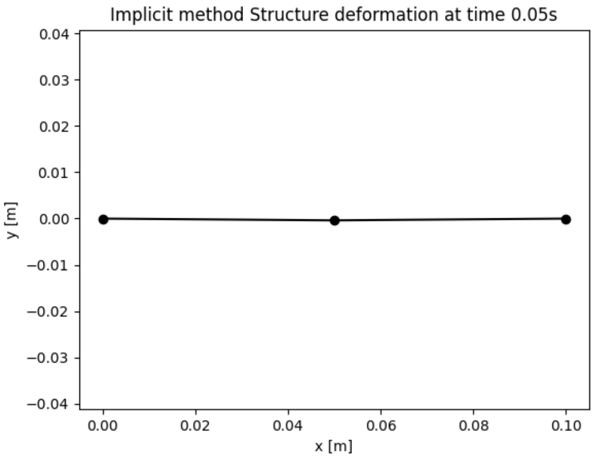

Figure 4. Plot of structure at 0.05s of the implicit method

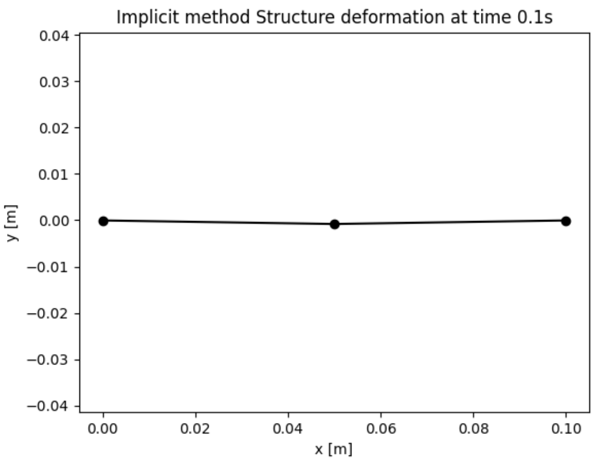

Figure 5. Plot of structure at 01s of the implicit method

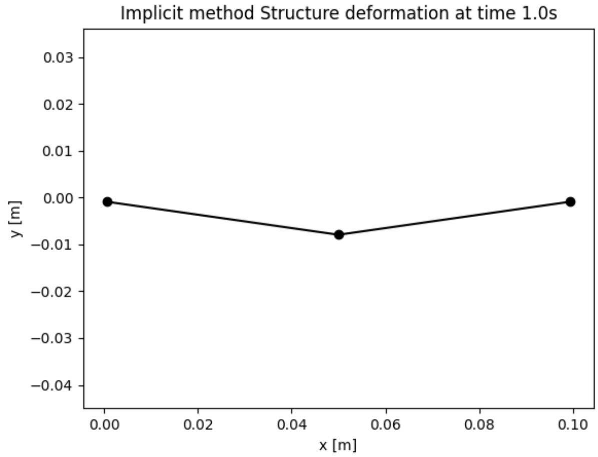

Figure 6. Plot of structure at 1.0s of the implicit method

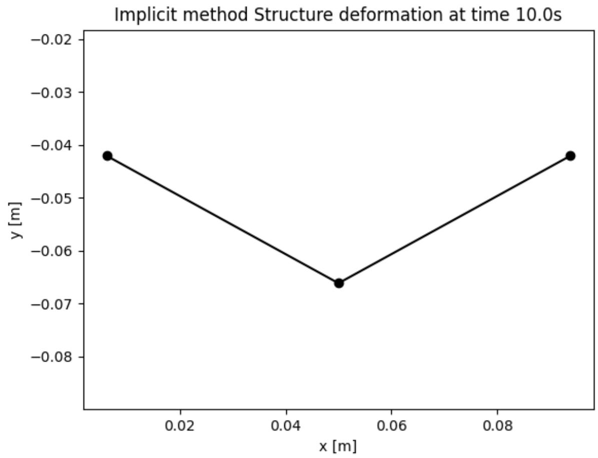

Figure 7. Plot of structure at 10.0s of the implicit method

Below in figure 8 is another plot of displacements of the structure at different time stamps, but this case involves the usage of explicit methods within the simulation calculations.

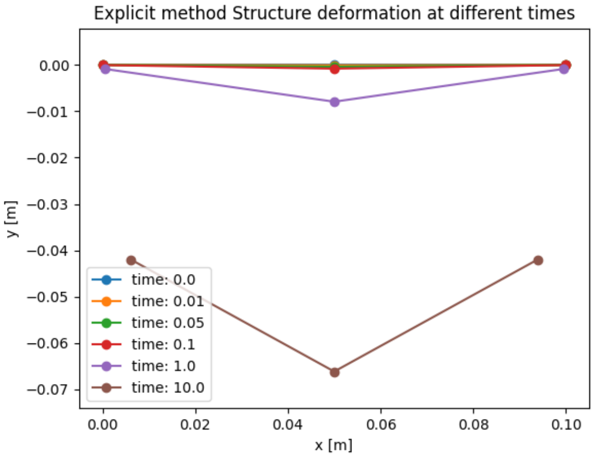

Figure 8. Collective plots of structure over different time steps through the application of explicit methods

Along with the implicit method plots, below in figures 9 through 14 are individual plots of the displacements at different timesteps.

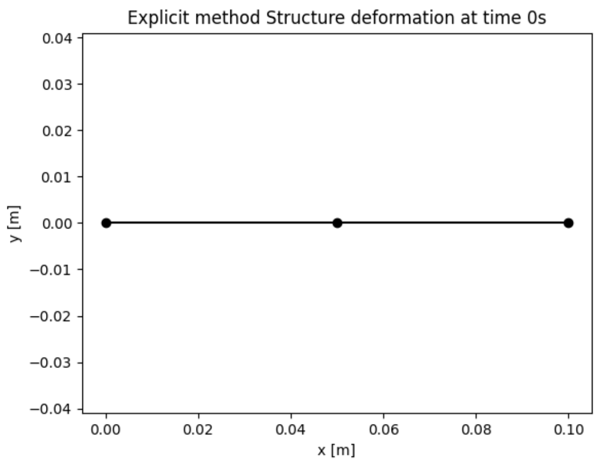

Figure 9. Plot of structure at 0s of the explicit method

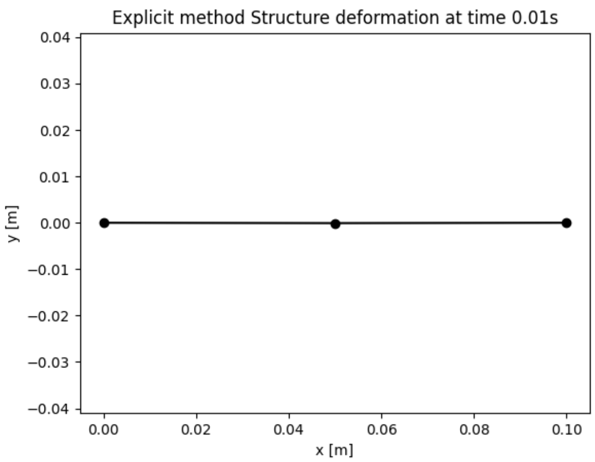

Figure 10. Plot of structure at 0.01s of the explicit method

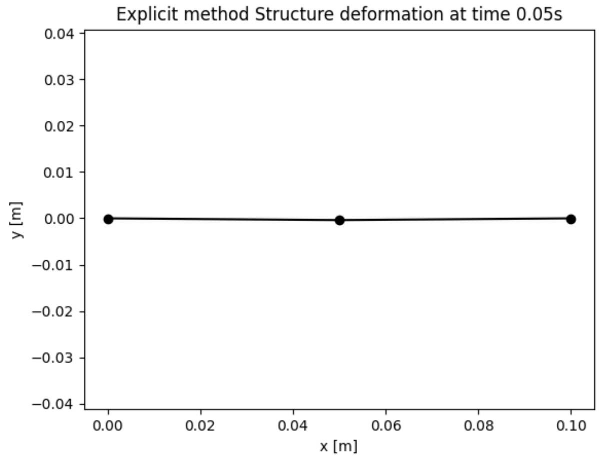

Figure 11. Plot of structure at 0.05s of the explicit method

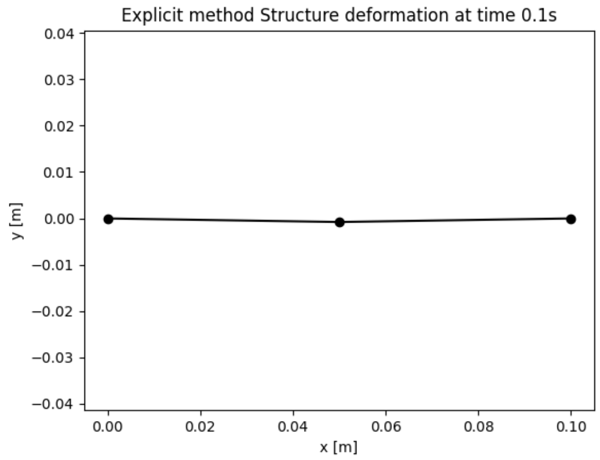

Figure 12. Plot of structure at 0.1s of the explicit method

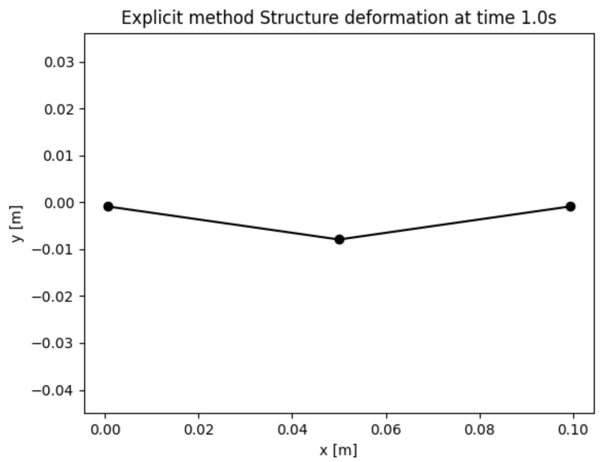

Figure 13. Plot of structure at 1.0s of the explicit method

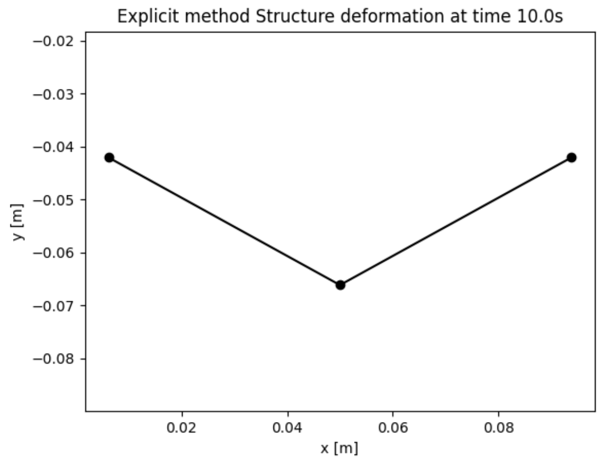

Figure 14. Plot of structure at 10s of the explicit method

B. Observed Terminal Velocity (2)

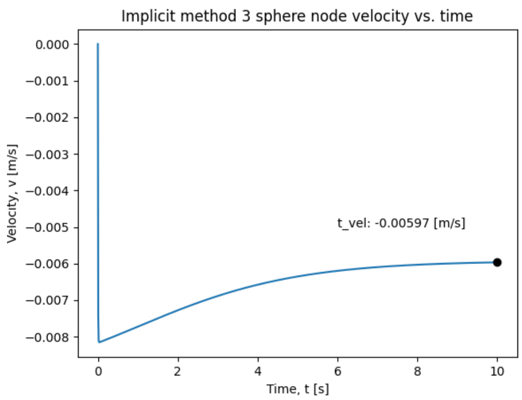

Figure 15. Plot of structure vertical velocity from the implicit method

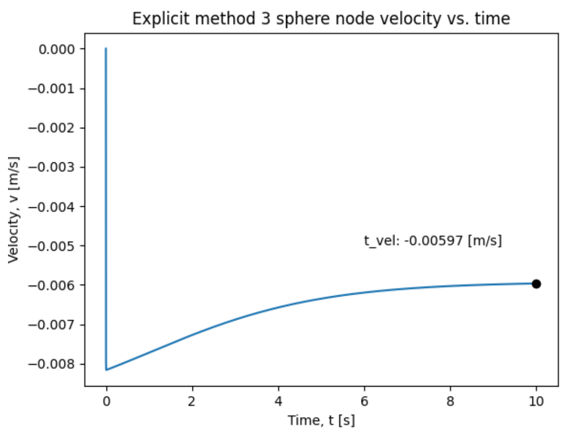

Figure 16. Plot of structure vertical velocity from the explicit method

From the above figure, the terminal velocity can be seen to be around 0.006 m/s, as it is the value the velocity plateaus at towards the end of the simulation

C. Turning Angle Behavior (3)

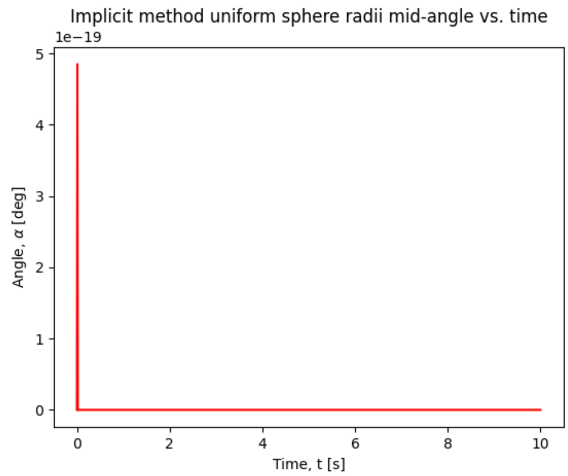

Figure 17. Plot of structure turning angle at the middle node

From the above figure, the turning angle can be seen to be 0 throughout the simulation, as the peak in the beginning has a relatively small scale. This is expected as the equal node radii gives equal amounts of viscous forces from the surrounding liquid, giving equal forces applied to all nodes or spheres, thus giving the same behavior in displacement and velocity. This then gives a resulting value of zero for the angle, as the displacement of the spheres is the same, keeping the structure in a straight line throughout the simulation.

D. Implicit versus Explicit Methods (4)

Explicit methods are prone to giving results not as representable of the actual situation if the time step isn't small enough when chosen. In cases where the time step is relatively large, the simulation may not converge. However, if too small of time steps are chosen for the method, the added time of computation of the simulation may cancel out the benefits of its simpler calculations, while also being more prone to errors from precision within the computational machine used for the simulation.

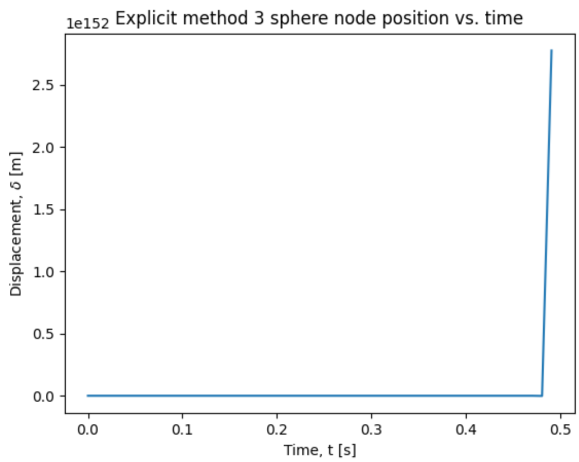

Figure 18. Displacement from explicit method utilizing 0.01s timestep

The figure above is an example of errors from using larger timesteps within the explicit method of solving. In this case, the same time step of 0.01s from the implicit method was used, propagating error within the resulting simulation. This is seen with the cut off in values around halfway through the simulation time.

The implicit method is the more involved form of the calculation, where an exact form of the movement equation is given for the computation, requiring the use of root finding methods such as Newton-Raphson to obtain the solution of the position and velocity at specified times. The usage of this method then requires larger computational time in exchange for better overall accuracy for a specific time step. When choosing a method of simulation, factors such as amount of simulation and complexity of the model involved. Explicit methods may be beneficial with mass amounts of less complex simulations executed, as the simulation needs to be easier to both set up and execute. More complex cases would highly benefit from the use of implicit methods, as they would then give higher accuracy for the setup made.

II. Deliverable 2

This deliverable required the creation of a simulation of 21 nodes within a time of 50s and a time step of 0.01s. The following were specifically asked for responses:

1. Plots for the position and velocity of the middle sphere, along with the observed terminal velocity
2. The final deformed shape of the system.

A. *Plots for position and velocity of the middle sphere*

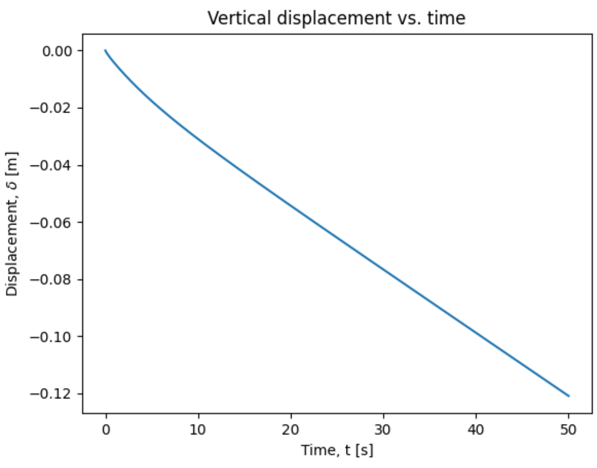

Figure 19. Vertical displacement of a 21 node system over 50s

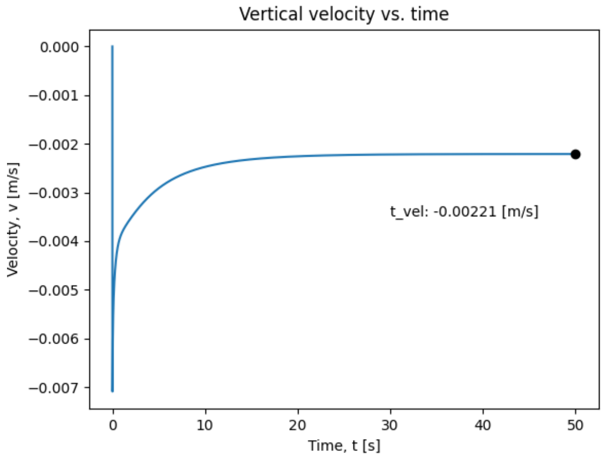

Figure 20. Vertical velocity of the 21 node system within 50s

From the plot of vertical velocity seen within figure 20, the terminal velocity of the system can then be approximated as around 0.00221 m/s, as the velocity plateaus towards the end of the simulation.

B. *Final Deformed Shape of the Beam (2)*

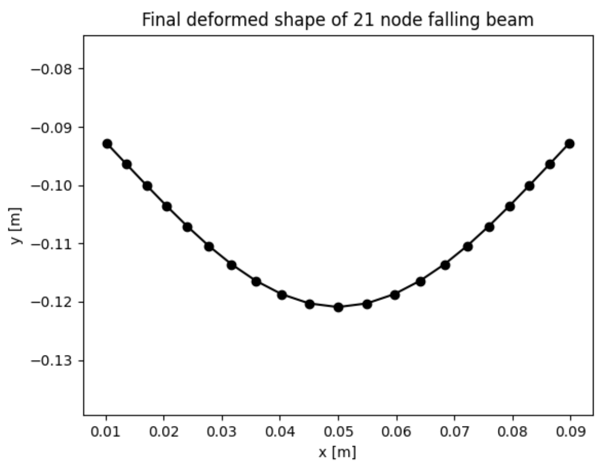

Figure 21. Final deformed shape of a 21 node system over 50s

III. Deliverable 3

This deliverable required the simulation of a 50-node beam over 1s with a time step of 0.01s. Comparisons were made with Euler's beam theory, which calculates displacement of simply supported beams with the following equations:

$$y_{max} = \frac{Pc\left(l^2 - c^2\right)^{1.5}}{9\sqrt{3}EIl} \tag{2}$$

$$c = \min\left(\mathrm{d}, \mathrm{l} - \mathrm{d}\right) \tag{3}$$

Where y_{max} is the maximum displacement of the beam with a load P applied at a location d away from the edge of the beam of length l, modulus of elasticity E , and inertia I.

For this deliverable, the beam is specified to be a circular pipe with an outer diameter of 0.013m and an inner diameter of 0.011m. The following was asked for this deliverable:

1. Plot of the maximum vertical displacement of the system and comparison against Euler Beam theory with a load of 2000N applied at 0.75m from the beam edge
2. The possible benefits of the simulation compared to the beam theory

A. *Maximum Vertical Displacement Comparison to Theory (1)*

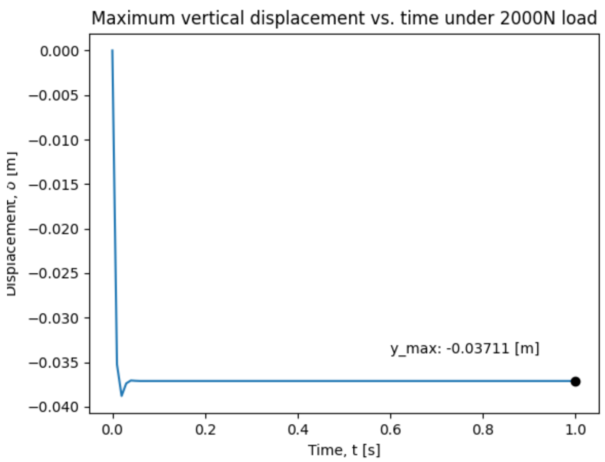

Figure 22. Maximum Vertical displacement of a 50 node system over 50s

While there is a sight fluctuation towards the simulation beginning, the maximum displacement within the beam after the simulation was determined as around 0.038m as shown in figure 22.

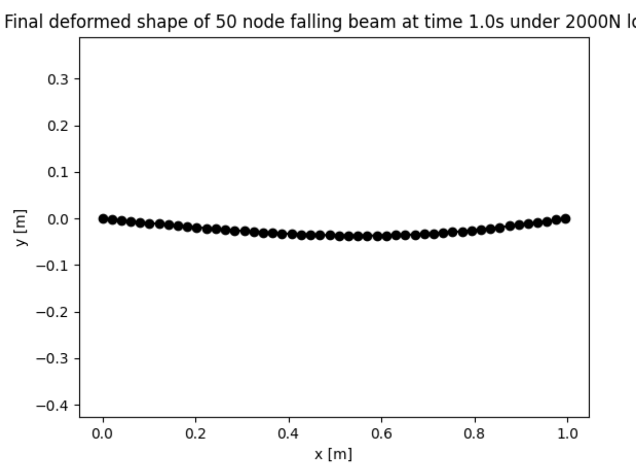

Figure 23. Final deformation of a 50 node system over 50s

When compared with the value calculated from Euler's theory from the script utilized for the problem, the resulting value is about 0.038. As also seen with figure 23 above, the load of 2000N causes a relatively small deformation and curvature in the beam, thus allowing for a reasonably accurate approximation with Euler's method (with a discrepancy of about 0.001m).

B. *Possible Benefits of Simulation Compared to Theory (2)*

The equation shown with Euler's beam theory assumes a small deformation within the beam to be close to the actual situation, so when larger loads are applied to the mean, the displacement values from Euler's theory would get less accurate, while the simulation results match more closely. This is because the simulation directly derives the values from the deformation and other properties without simplification.

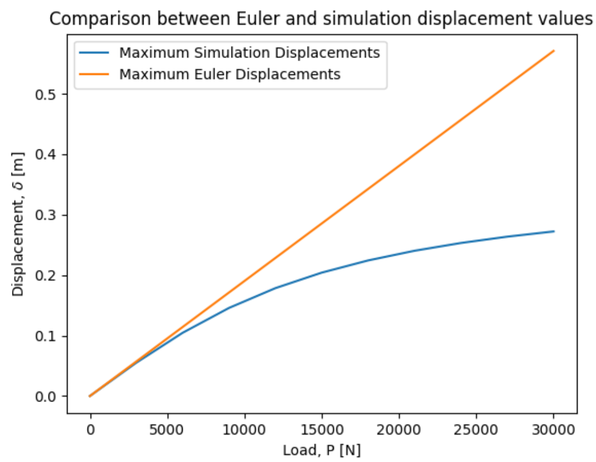

Figure 24. Relation between calculations of displacement from Euler's theory comapred to simulation results

Shown with figure 24 above, Euler theory generates a linear relation between the load applied to the beam with the maximum displacement, where the simulation shows displacement to have a lower rate of increase with larger loads applied. Within the simulation, the break-off point for the Euler's deformation approximation and the simulation results was calculated as around 5000N for a difference of 0.005m between the two methods. As the plot in figure 23 was generated over increasing segments of 1000N from 0 to 30000N, the exact force value for the break off point would fall within 1000N of the 5000N value obtained. This value can then be utilized as threshold in determining the effectiveness of the calculations, and when to use the methods

This is likely caused by the non-linear material properties of the beam becoming more prevalent with larger force loads. With a larger load such as 20000N, there was a significant discrepancy between Euler beam theory and simulation results, which was around 0.15m (Euler's theory gives 0.38m and simulation results give 0.235m).

As such, the benefits of simulation would mainly be within the approximation of beam geometry within larger loads applied. For relatively smaller load cases, Euler's theory can be applied to obtain quick estimations of geometry as necessary. Along with displacement, the overall curvature of the beam is also obtainable within the simulation through calculations at the nodes within the simulation.

REFERENCES

[1] M. Khalid Jawed, S. Lim, DISCRETE SIMULATION OF SLENDER STRUCTURES [p. 4-26].