MECH&AE 263F Homework 2 Deliverables

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Abstract— **This document contains the deliverables as required by the second homework of the MECH&AE 263F course at UCLA. These deliverables display results from simulating an elastic rod, where deformation and node coordinate data are kept.**

I. DELIVERABLE I

For this deliverable, a simulation of an elastic rod is required. The following parameters are to be used for the simulation:

- Total rod length (l) = $20cm$
- Natural radius (R_n) = $2cm$
- Density (ρ) = $1000\frac{kg}{m^3}$
- Cross-sectional radius (r_0) = $1mm$
- Young's modulus (E) = $10MPa$
- Shear modulus (G) = $\frac{E}{3}$
- Gravitational acceleration (g) = $9.81\frac{m}{s^2}$

The following simulation parameters were also to be used within this deliverable:

- First two nodes and twist angle were fixed throughout the simulation
- Location of nodes at ($t = 0$):
$$x_k = \begin{bmatrix} R_n \cos((k-1)\Delta\theta) \\ R_n \sin((k-1)\Delta\theta) \\ 0 \end{bmatrix}$$
where $\Delta\theta = \frac{1}{R_n}\frac{1}{N-1}$
- Twist angles (θ^k) at $t = 0$ are 0

Under these considerations, the following are items asked within this deliverable:

1) Description of the discrete elastic rod simulation implementation
2) Simulation of the rod deformation under gravity from $t = 0$ to $t = 5$
3) Plot of z-coordinate for the end node of the rod from $t = 0$ to $t = 5$

Description of discrete elastic rode implementation (1)

Below with figure 1 is a diagram of the implementation of the discrete elastic rod simulation.

Within the code, the initial physical parameters and frames are coded in the the user. The physical parameters are those listed in the problem statement, and the geometrical parameters of the rod are determined by the initial orientation of it (whether it has a certain shape or is straight). This then gives the coordinates necessary for calculating the movement of the rod. The frames can be chosen for direction initially, while also using the tangent and transport functions to calculate the frames for each node at the initial time step. An

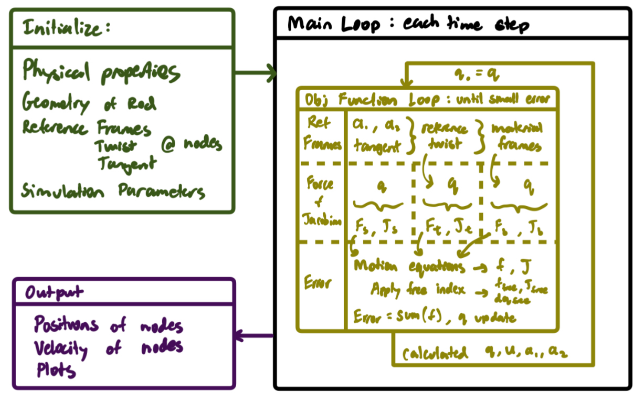

Fig. 1: Flow diagram of the discrete elastic rod simulation code

important item to consider is that the frames are calculated with space parallel transport because there isn't a second time step to use for the time parallel transport.

Upon starting of the main loop, the objective function loop intakes the frames, positions, and velocity values from the previous step to use as a starting point in calculating the new position values at the current time step. From the reference frames and positions of the nodes, the reference twist angles between nodes and the material frames are computed.

These values then can be used when deriving the associated force values and Jacobians for the types of energy present: bending, twisting, and stretching. Bending force needs the material frames calculated at each node along with stiffness properties from inertia. Twisting force uses the reference twist values calculated at each node along with the stiffness properties from polar moment of inertia. Stretching, along with bending and twisting, utilizes the positions from the q vector to get the associated stretching force.

Using the associated functions for each type of source of energy to get the force and jacobians, the forces can be summed and used within the equation of motion for the model. Along with the mass matrix and time step parameters of the simulation the f value used within the newton raphson method can be obtained. Similarly, the jacobian matrix used for the Newton raphson can also be calculated by using the equation of motion. As the beam is restricted to being fixed at the first two nodes and edge, boundary conditions need to be applied to obtain the elements of the position and velocity vector that correspond to nodes that are free to move.

Finally, the delta value within the newton raphson method can be calculated by obtaining the solution vector from the f and J. Adding the delta to the position vector then gives the position at the time step if the error, the sum of the elements in the f vector, is within the set tolerance. The velocity is then calculated from the current and previous position using the time step value. These position values can then be stored to make the plot of the end node coordinates for this deliverable.

Deformation under gravity (2)

Below are the plots of the deformation of the elastic rod over time at certain times. The rod starts off as coiled within a circle as shown in figure 2 from the specifications made for the initial node coordinates within the deliverable statement. Gravity acts on the nodes while the first node in red remains in the same position because of the fixed constraint imposed by the deliverable.

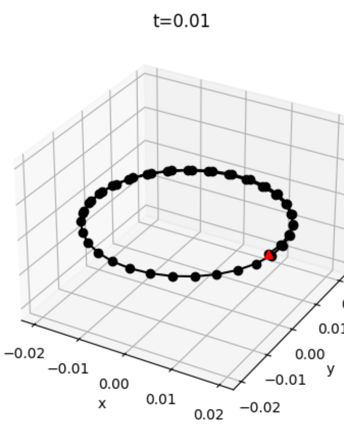

Fig. 2: Displacement at time 0.01 of the rod

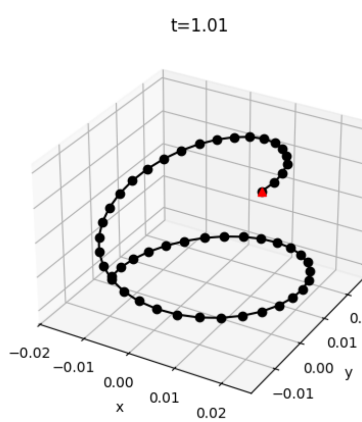

Fig. 3: Displacement at time 1.01 of the rod

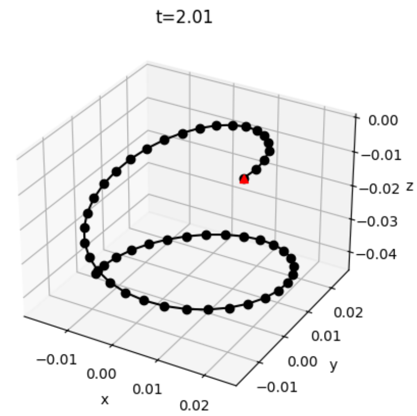

Fig. 4: Displacement at time 2.01 of the rod

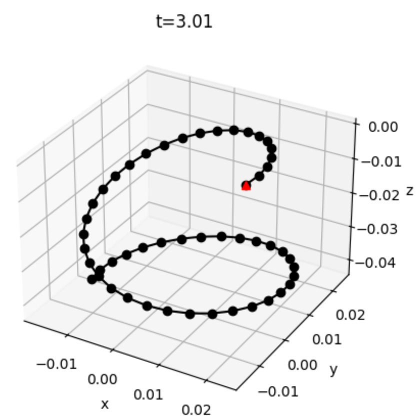

Fig. 5: Displacement at time 3.01 of the rod

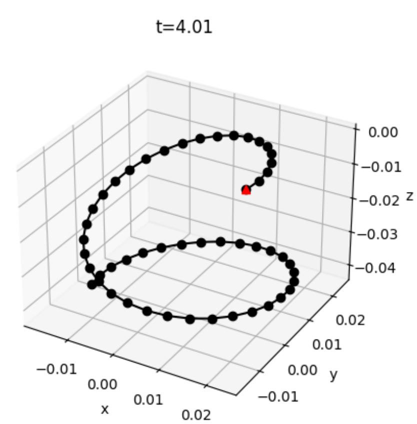

Fig. 6: Displacement at time 4.01 of the rod

Coordinate of last node (3)

As seen with figure 7, the end node's z-coordinate oscillates throughout the run of the system and settles at a steady state value of about $0.04m$ downwards in the simulation. This can also be observed within the figures of part two of this deliverable. Within the later time steps of the simulation such as in figures 5 and 6, the end node's z-coordinate can be seen to be located at $0.04m$ downwards as well.

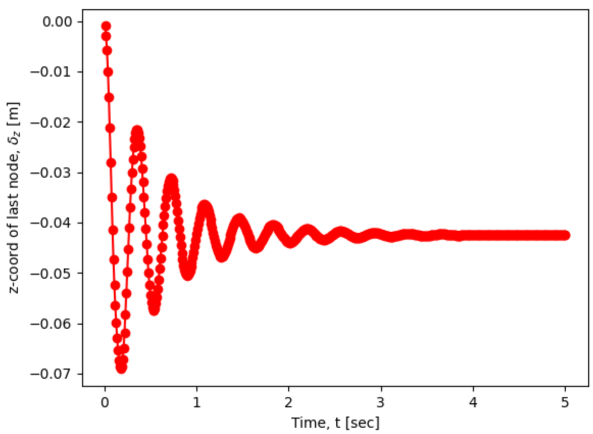

Fig. 7: Plot of the end node's z coordinate

REFERENCES

[1] Khalid Jawed, M, and Sangmin Lim. "Discrete Simulation of Slender Structures". *BruinLearn*, 2022.