Online Covering

Secretaries, Prophets, and Universal Maps

FOCS 2021 + Forthcoming Work

Roie Levin

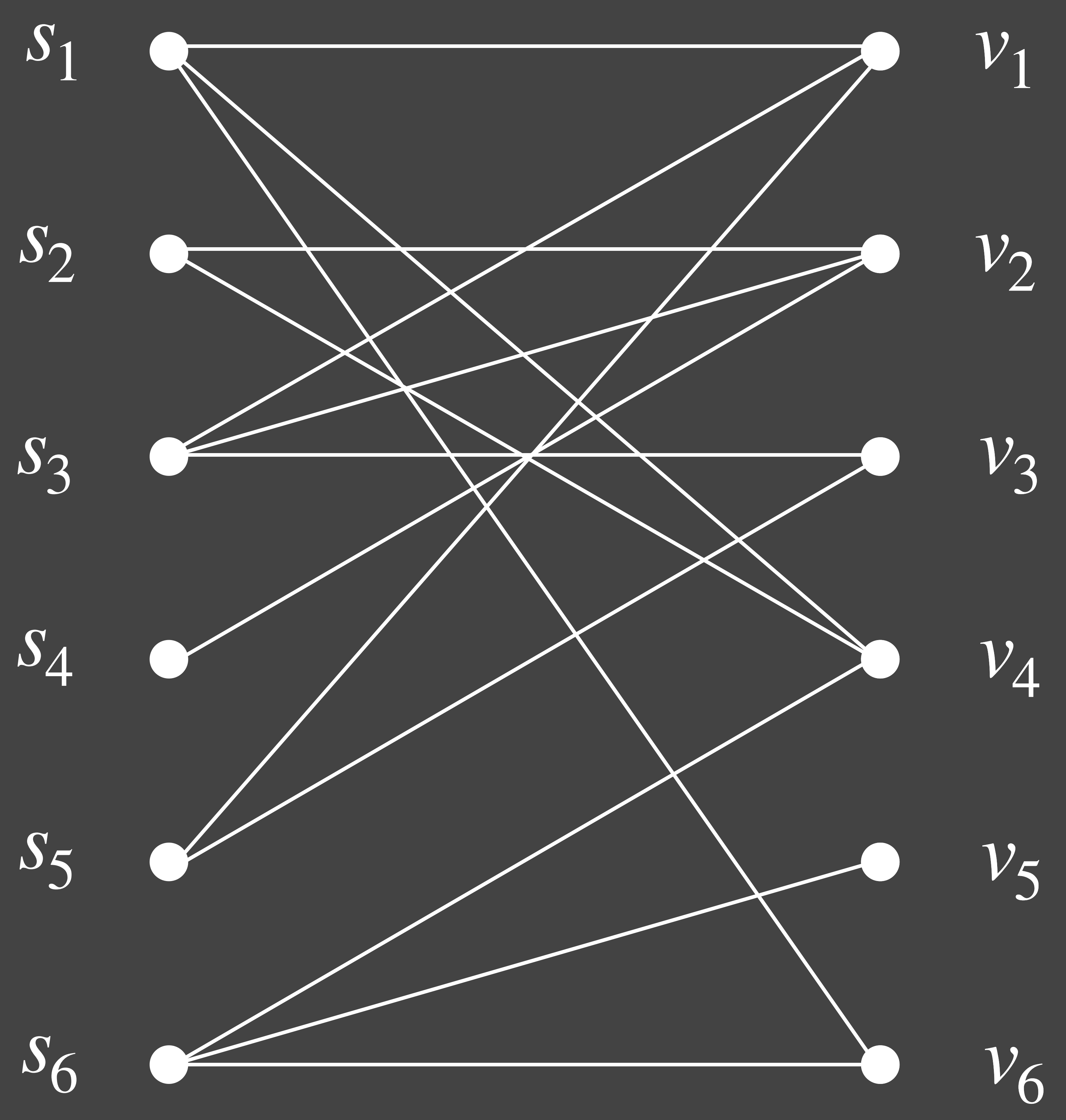
Anupam Gupta (CMU)

Gregory Kehne (Harvard)

Set Cover

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

Set Cover

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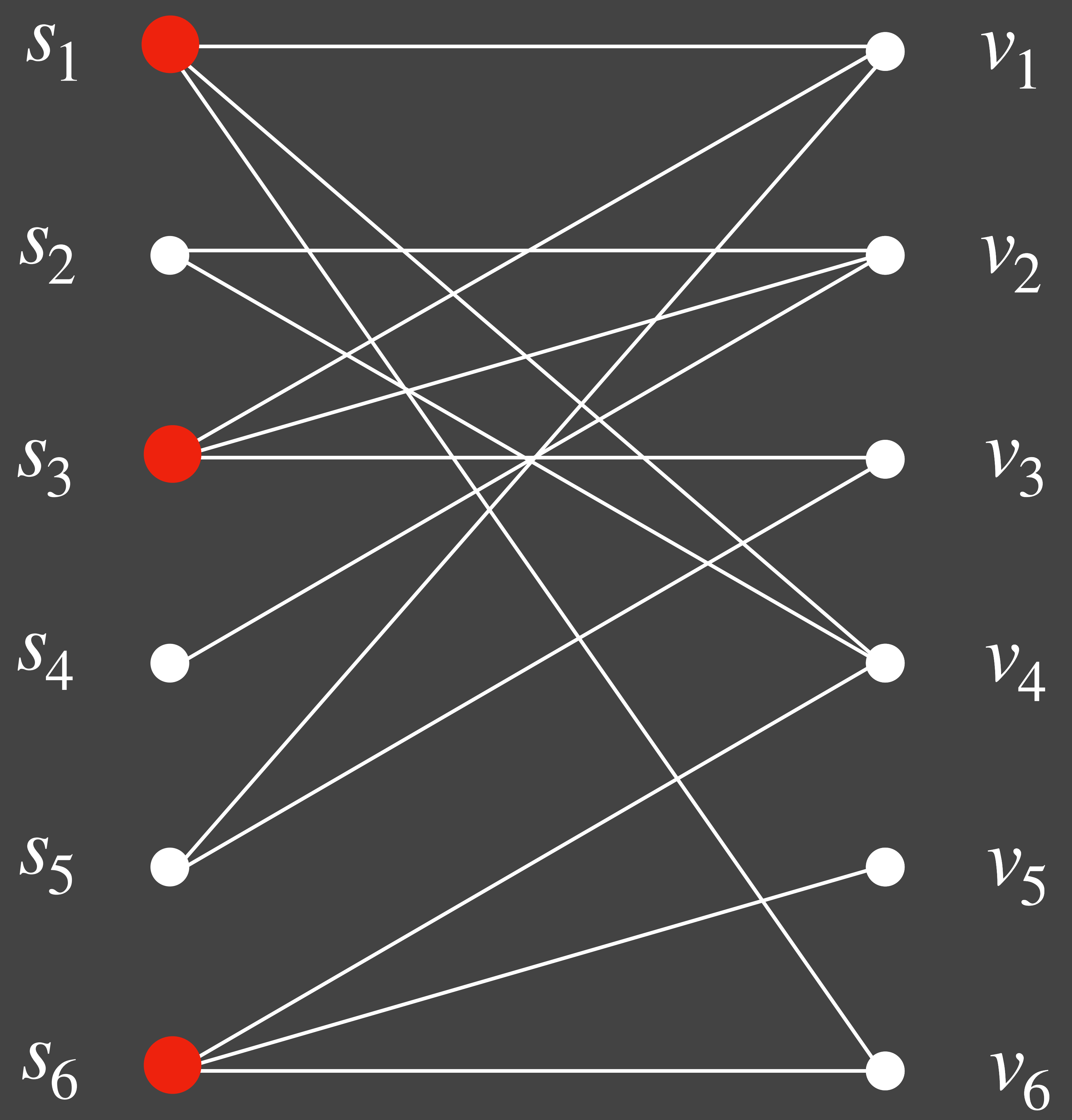

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Set Cover

Apx: $\log n + 1$
[Johnson 74],[Lovasz 75],
[Chvatal 79]

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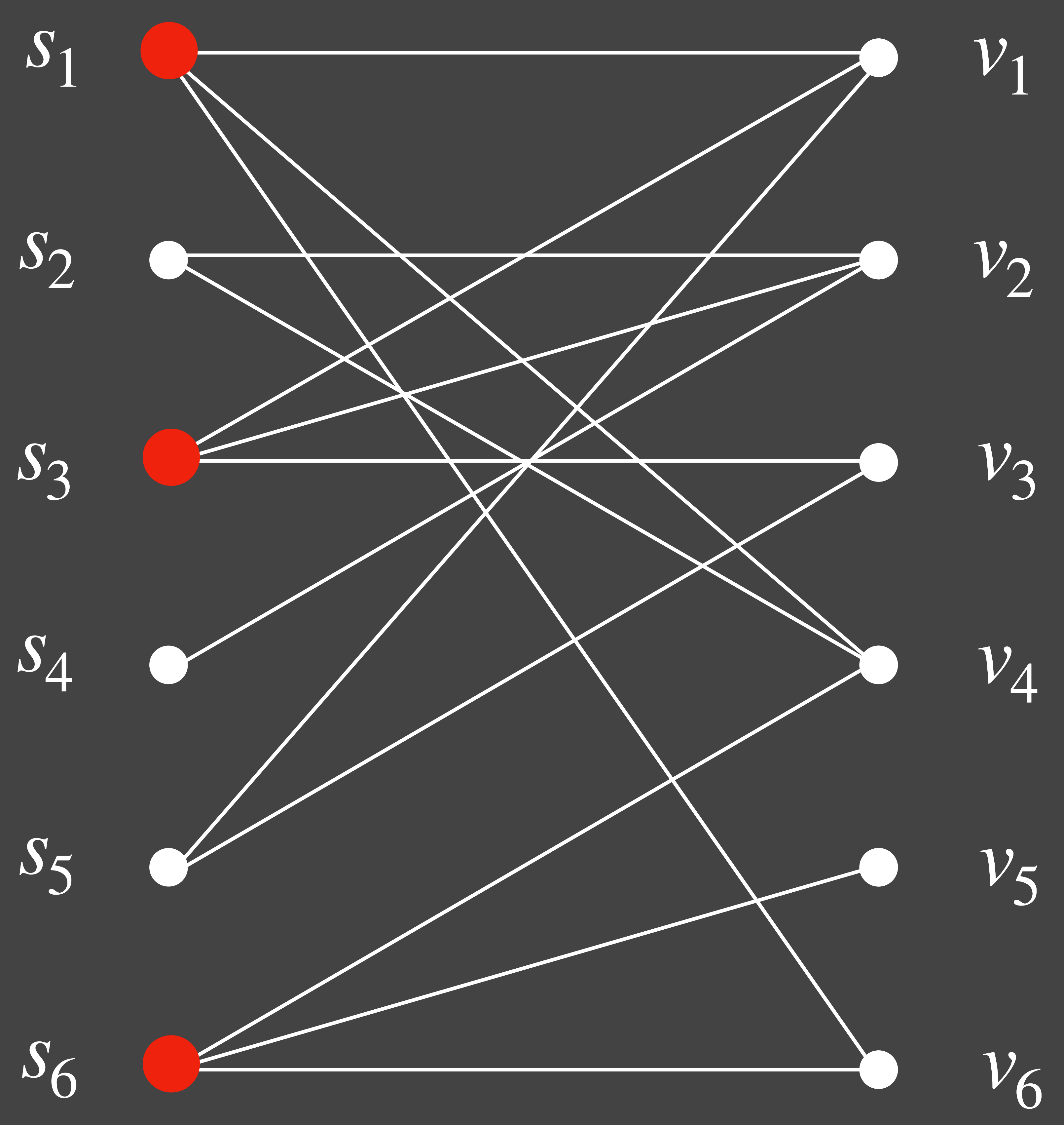

$\mathcal{U}$
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Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

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s_1

s_2

s_3

s_4

s_5

s_6

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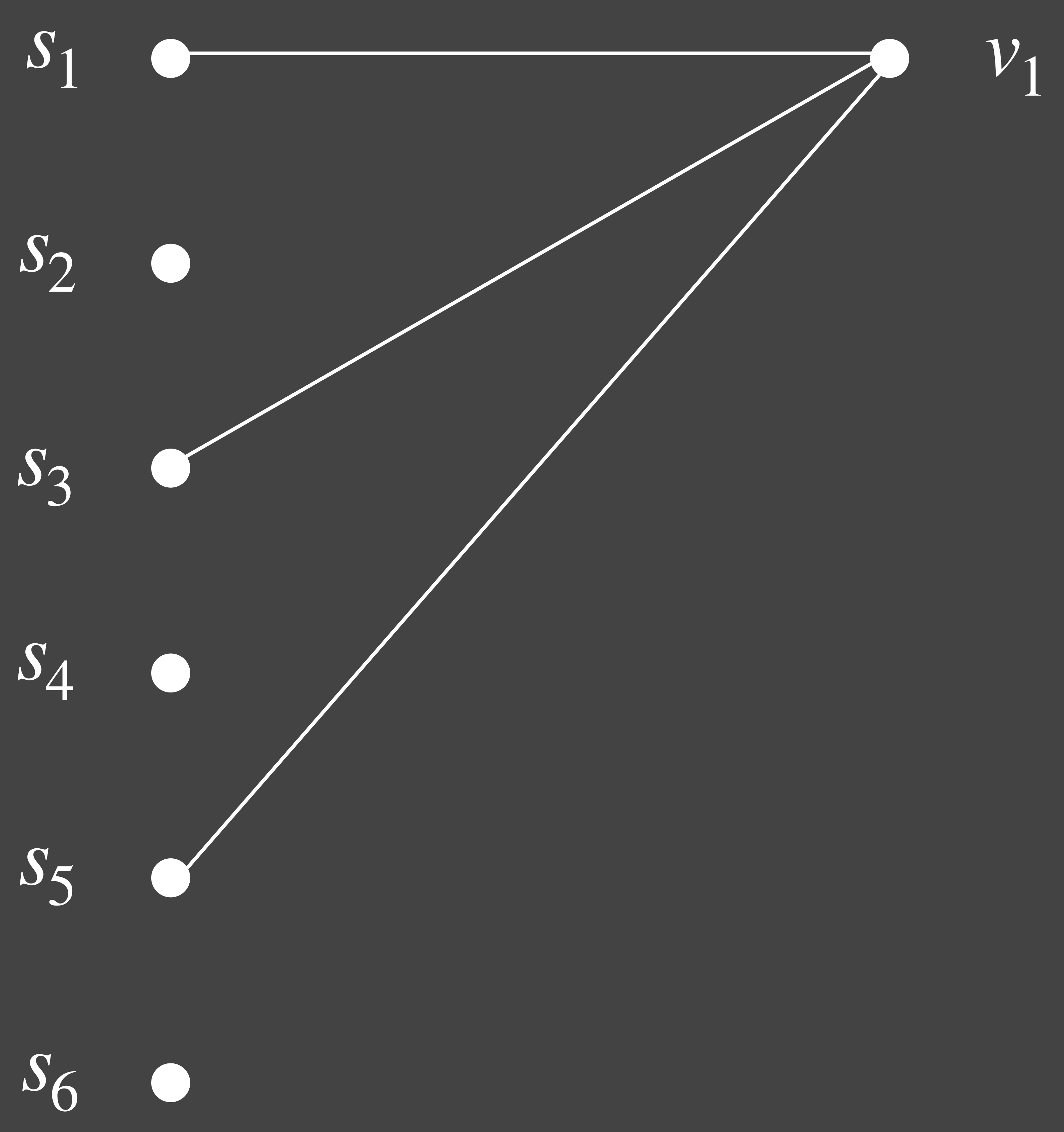

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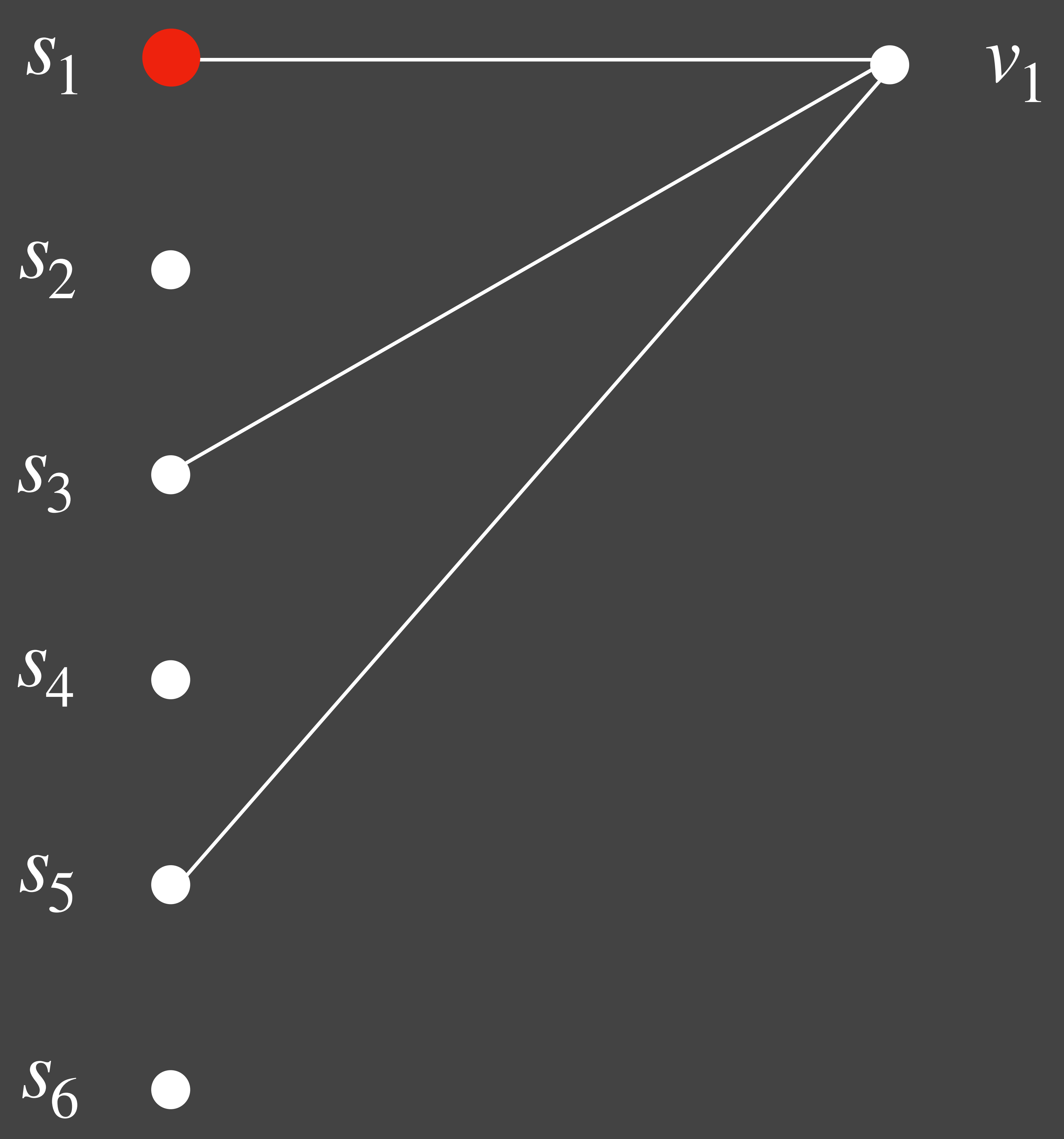

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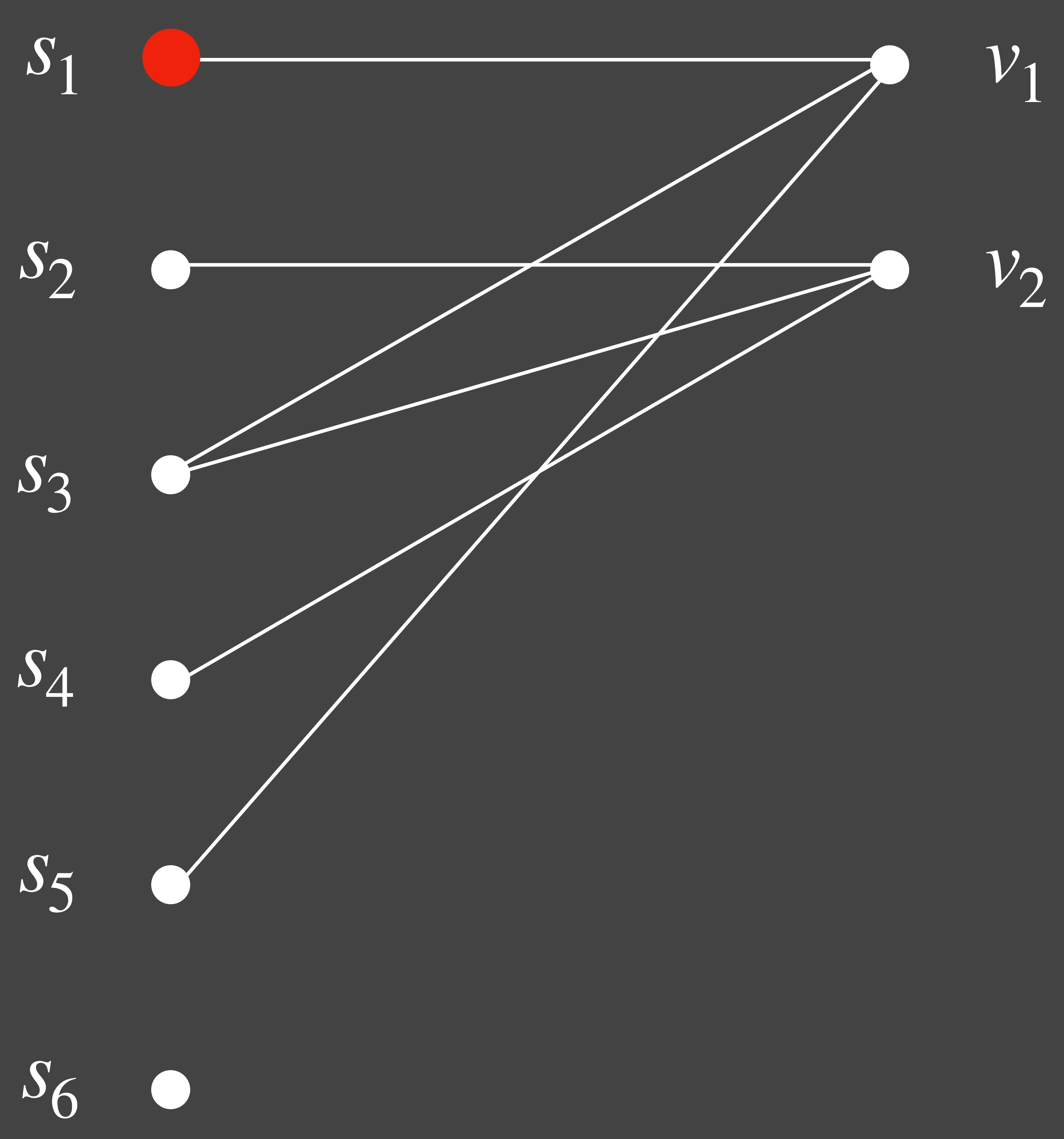

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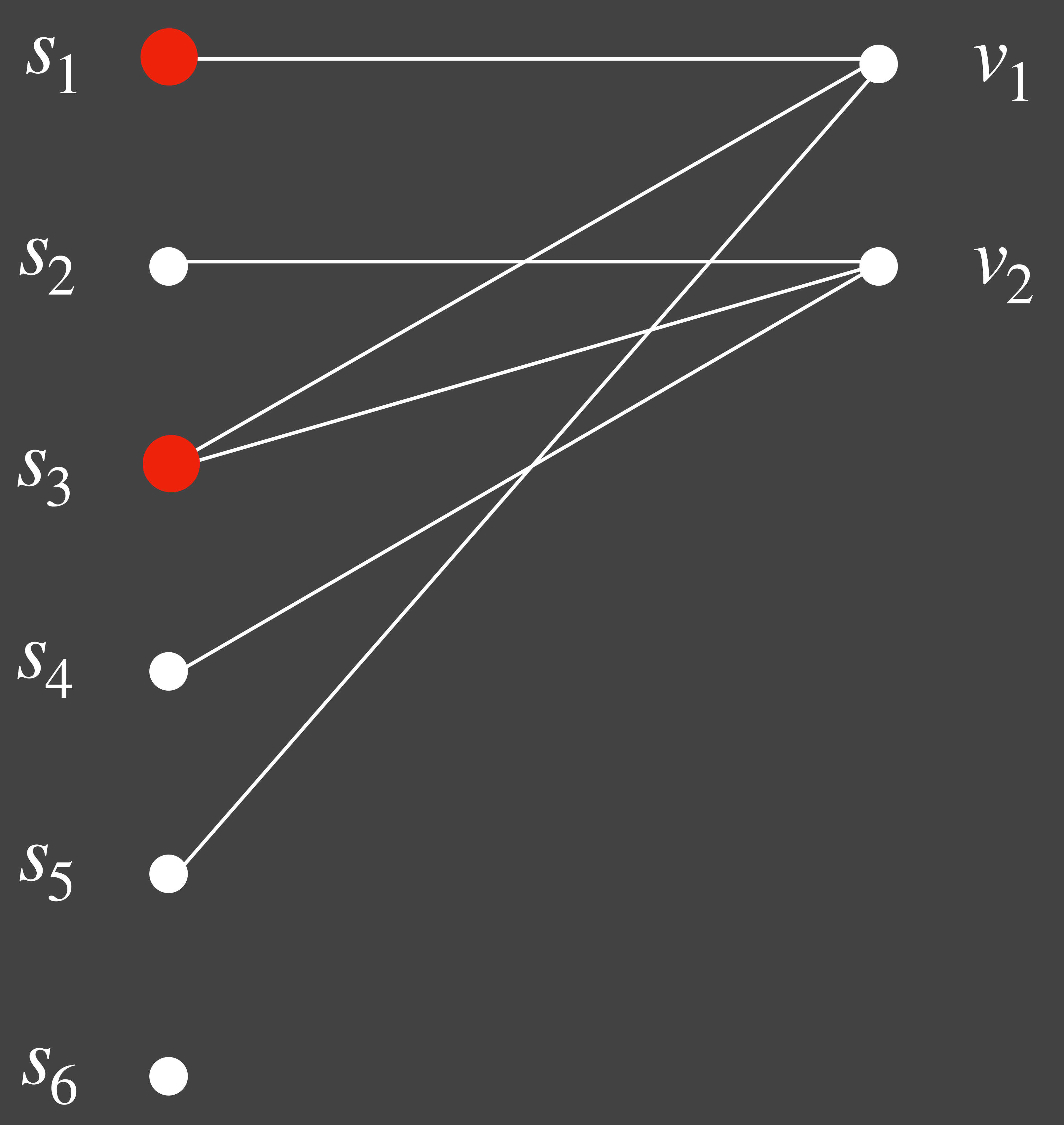

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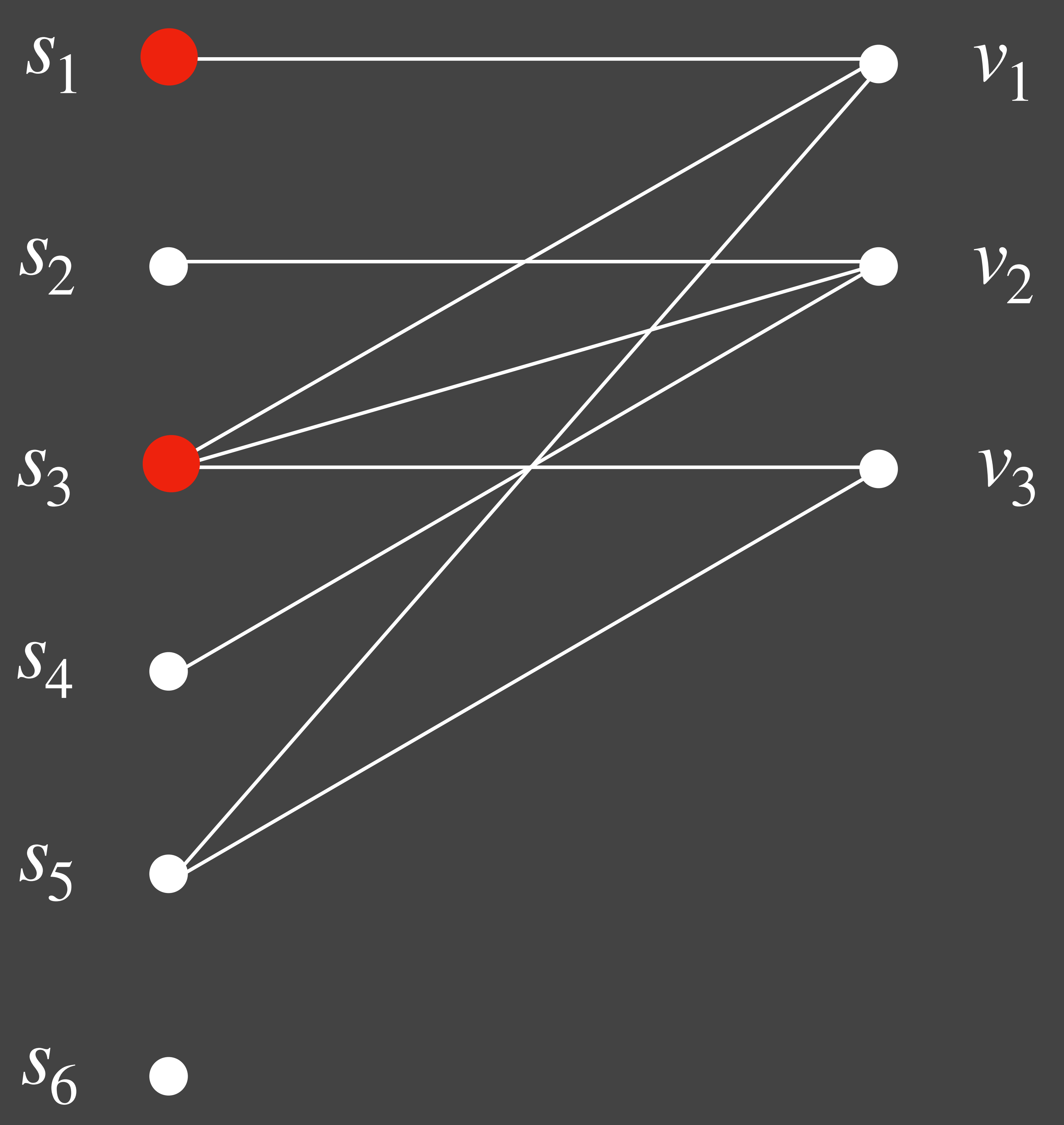

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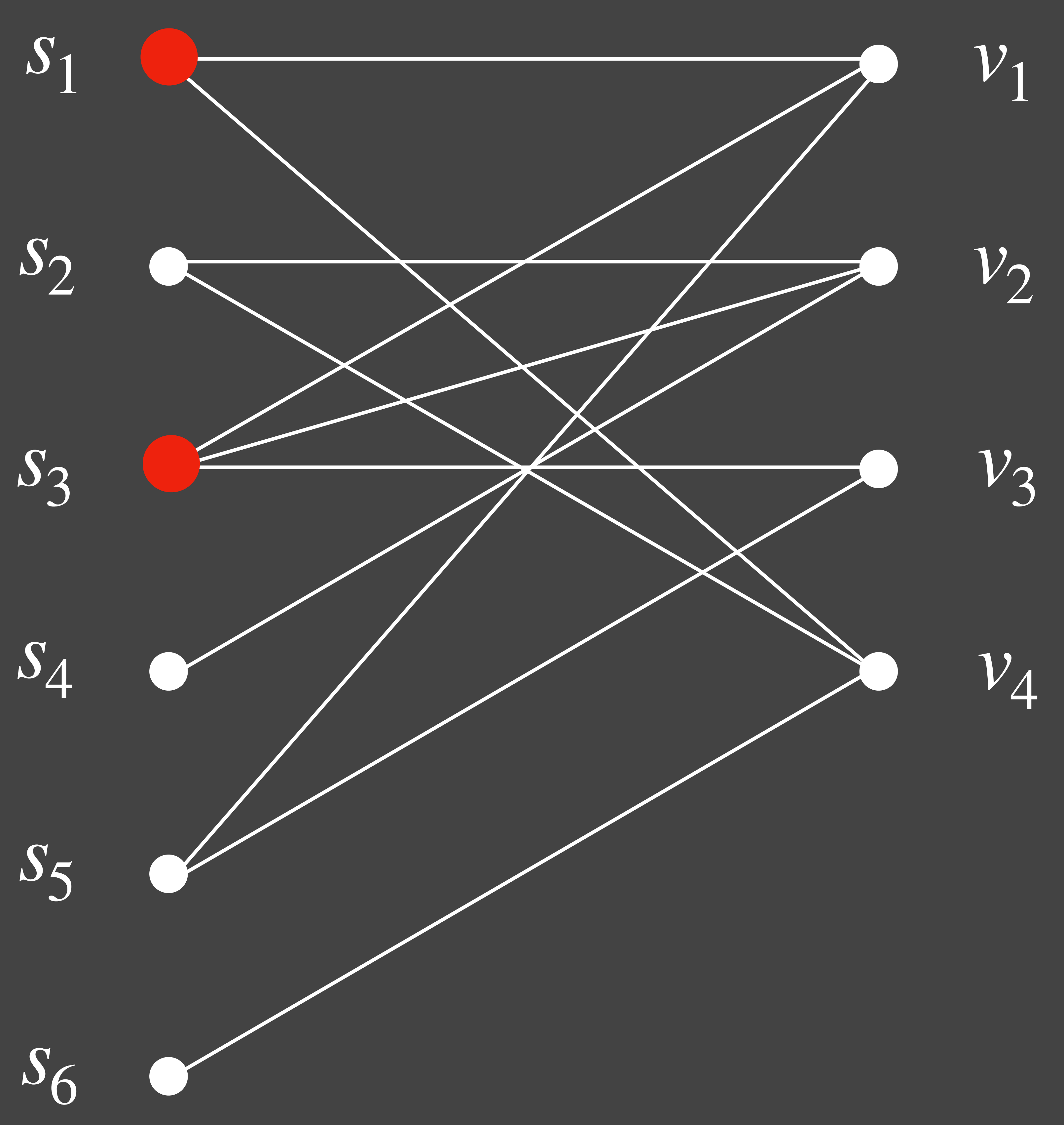

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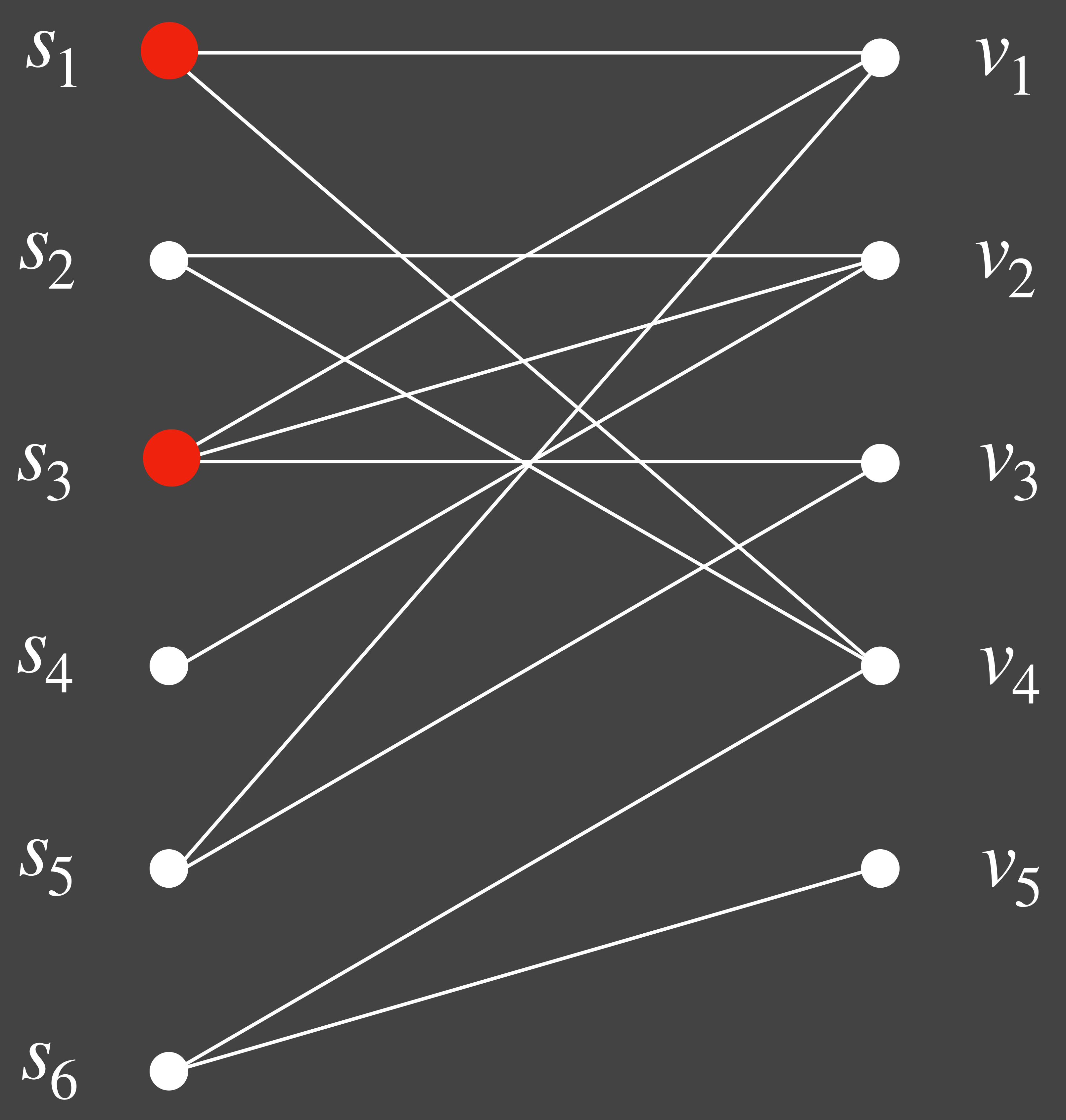

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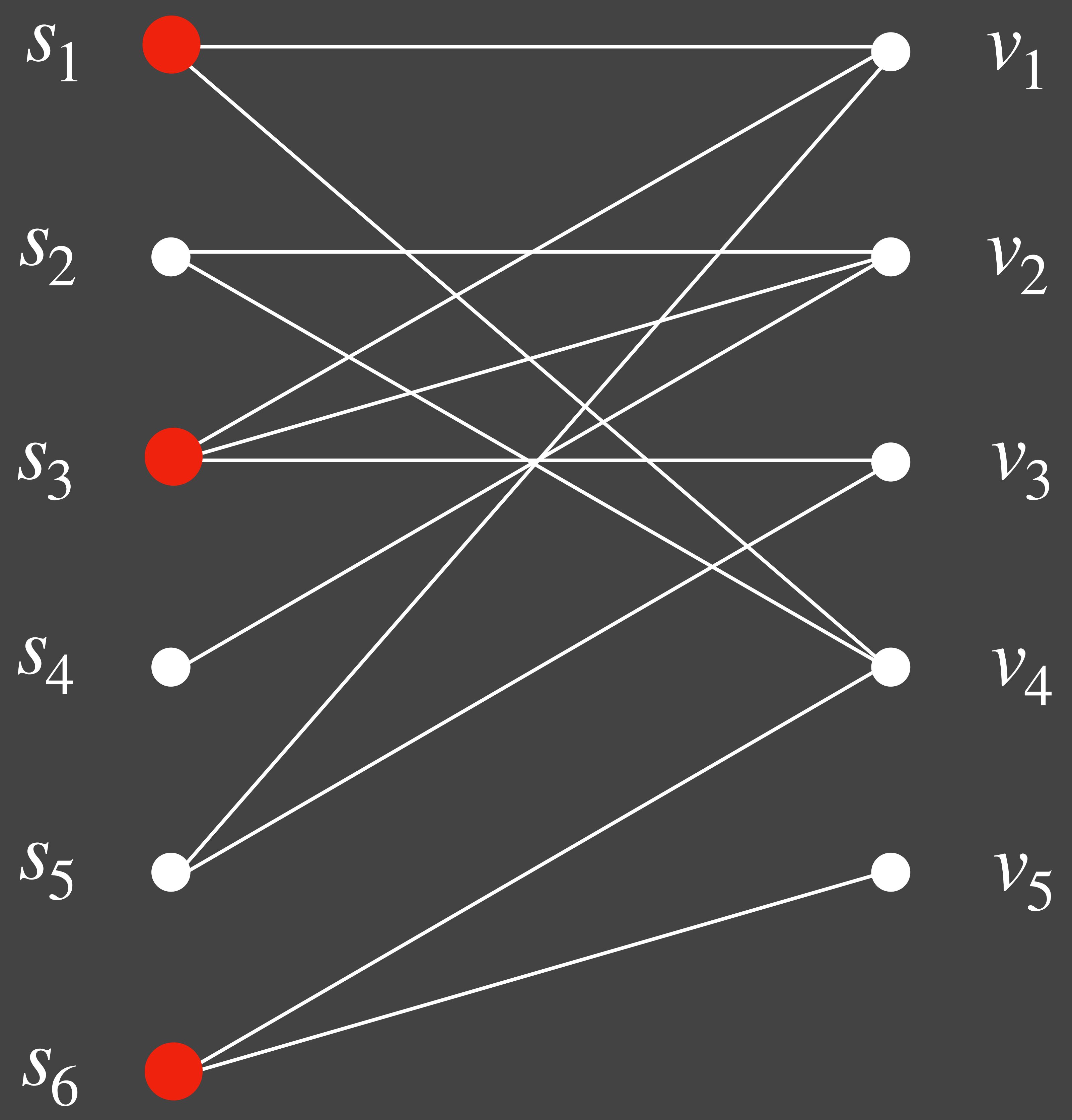

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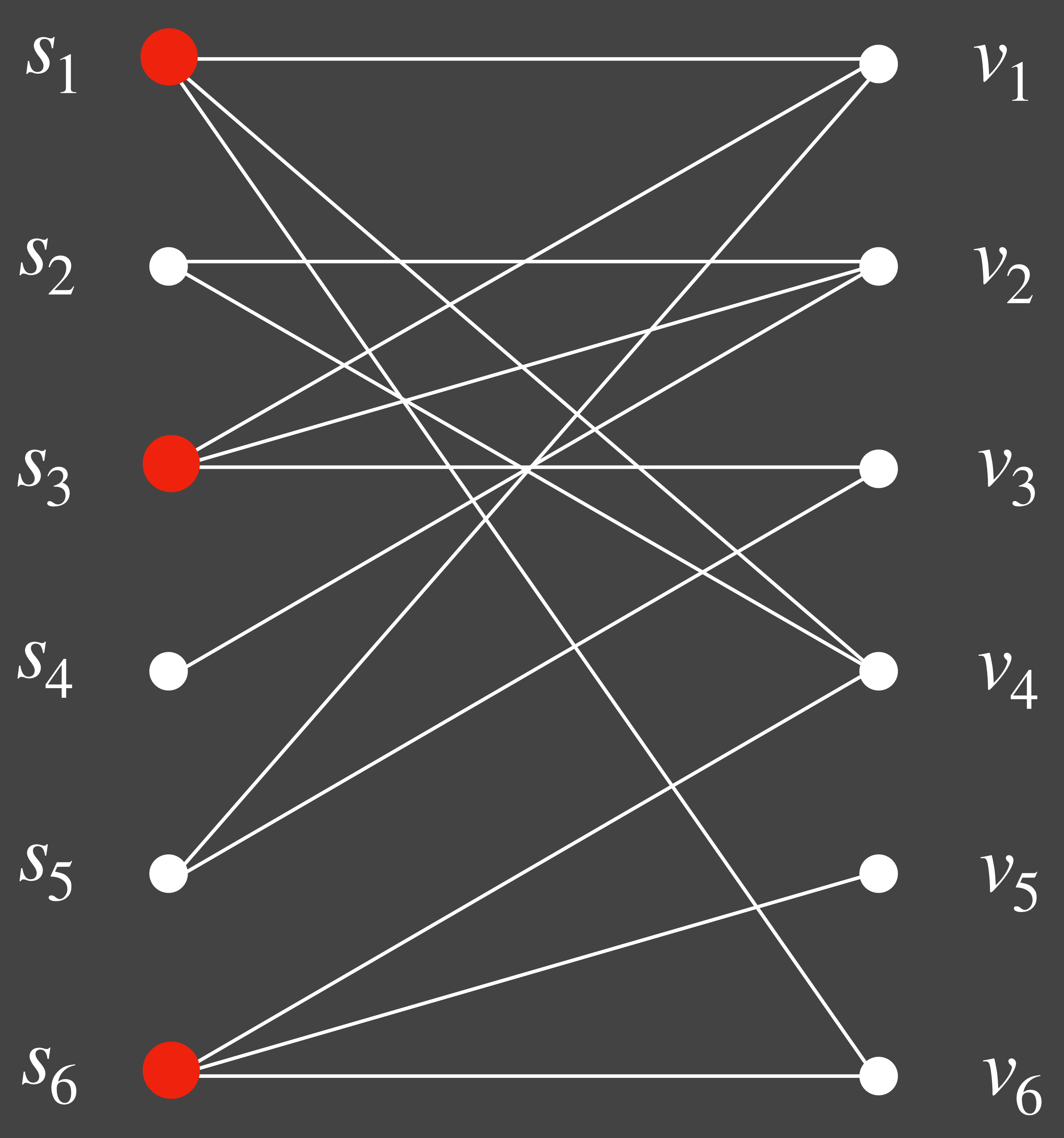

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Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

CR: $O(\log n \log m)$

[Alon+ 03]
[Buchbinder Naor 09]

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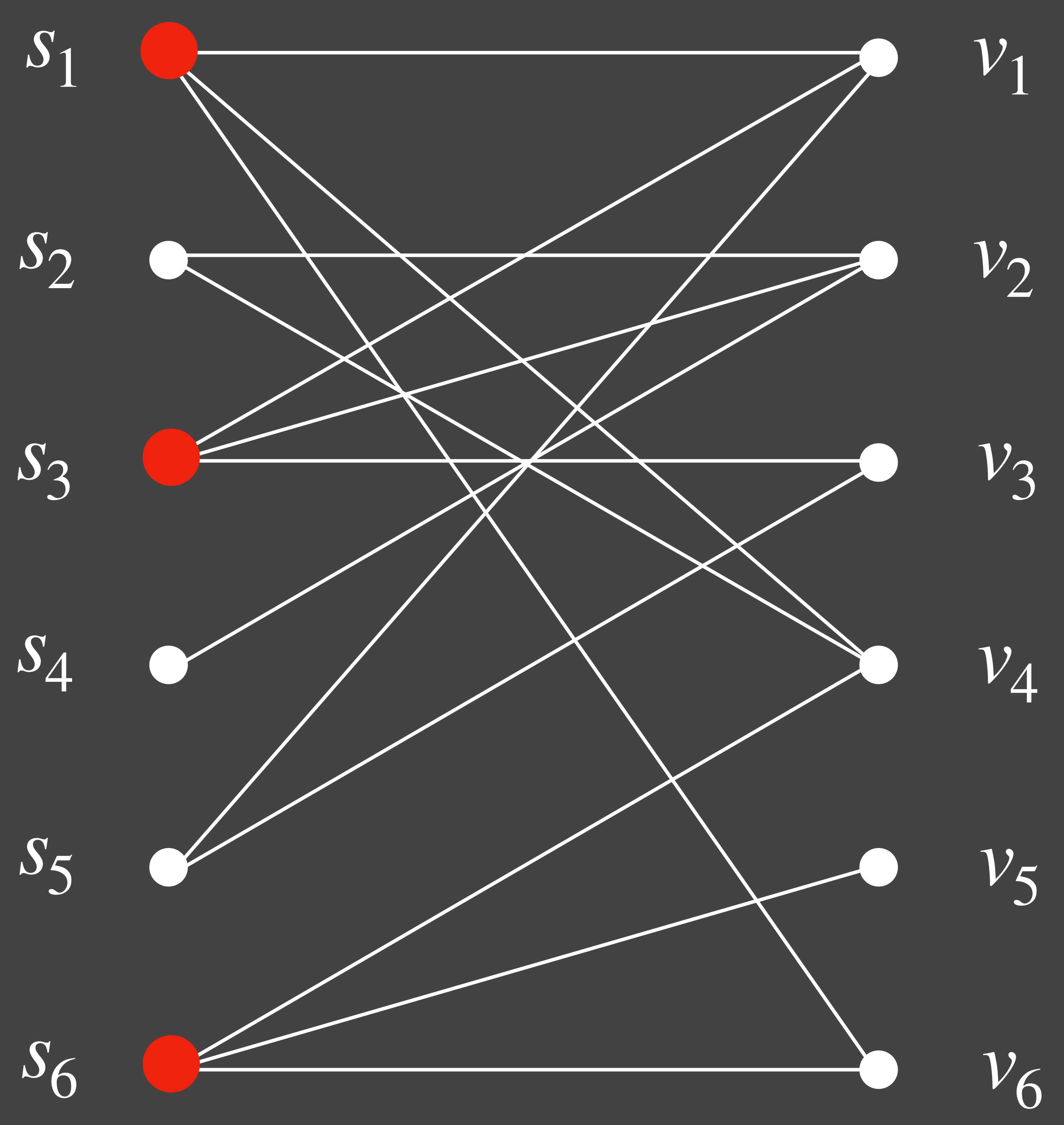
m sets



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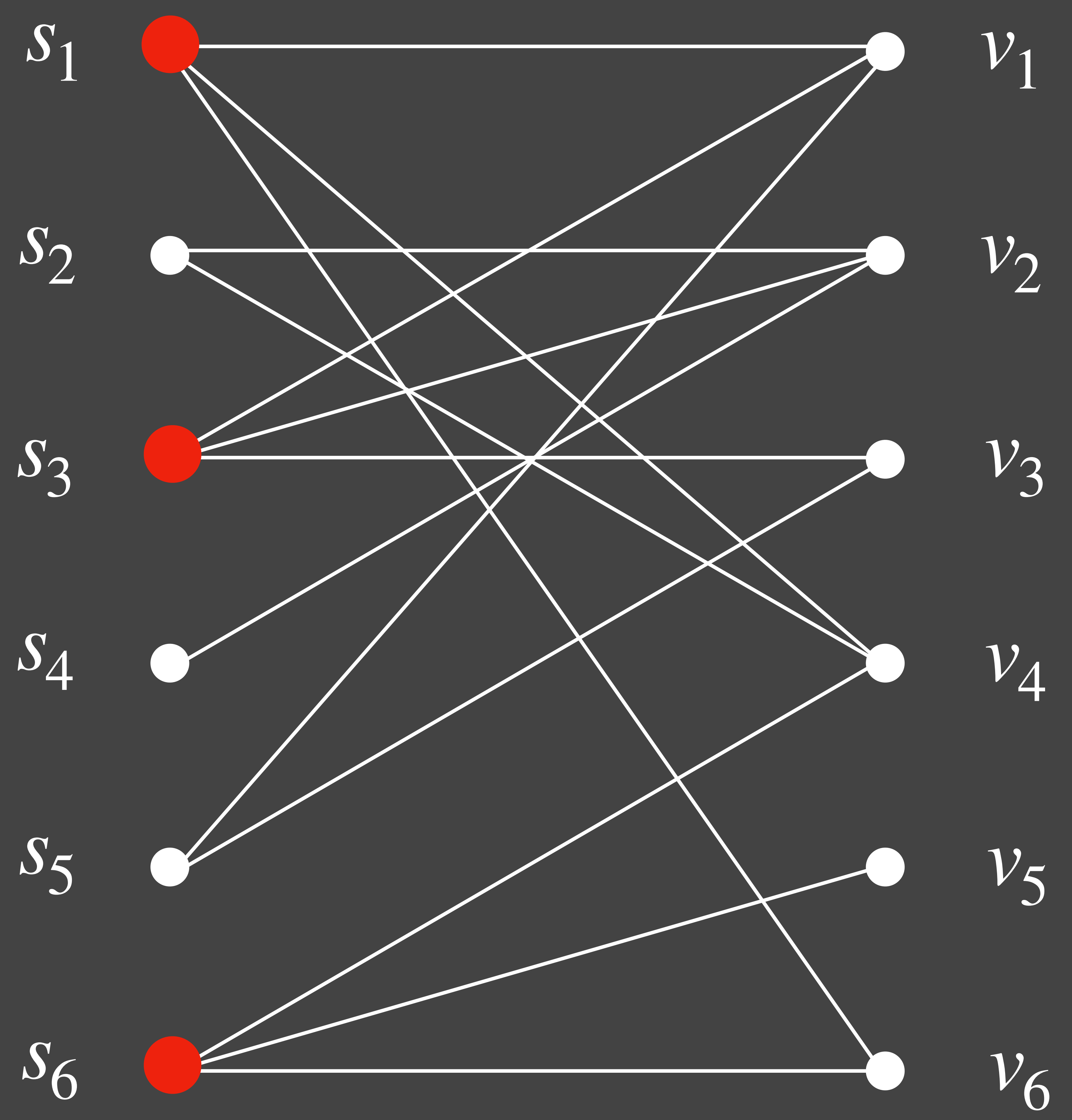


$\mathcal{U}$

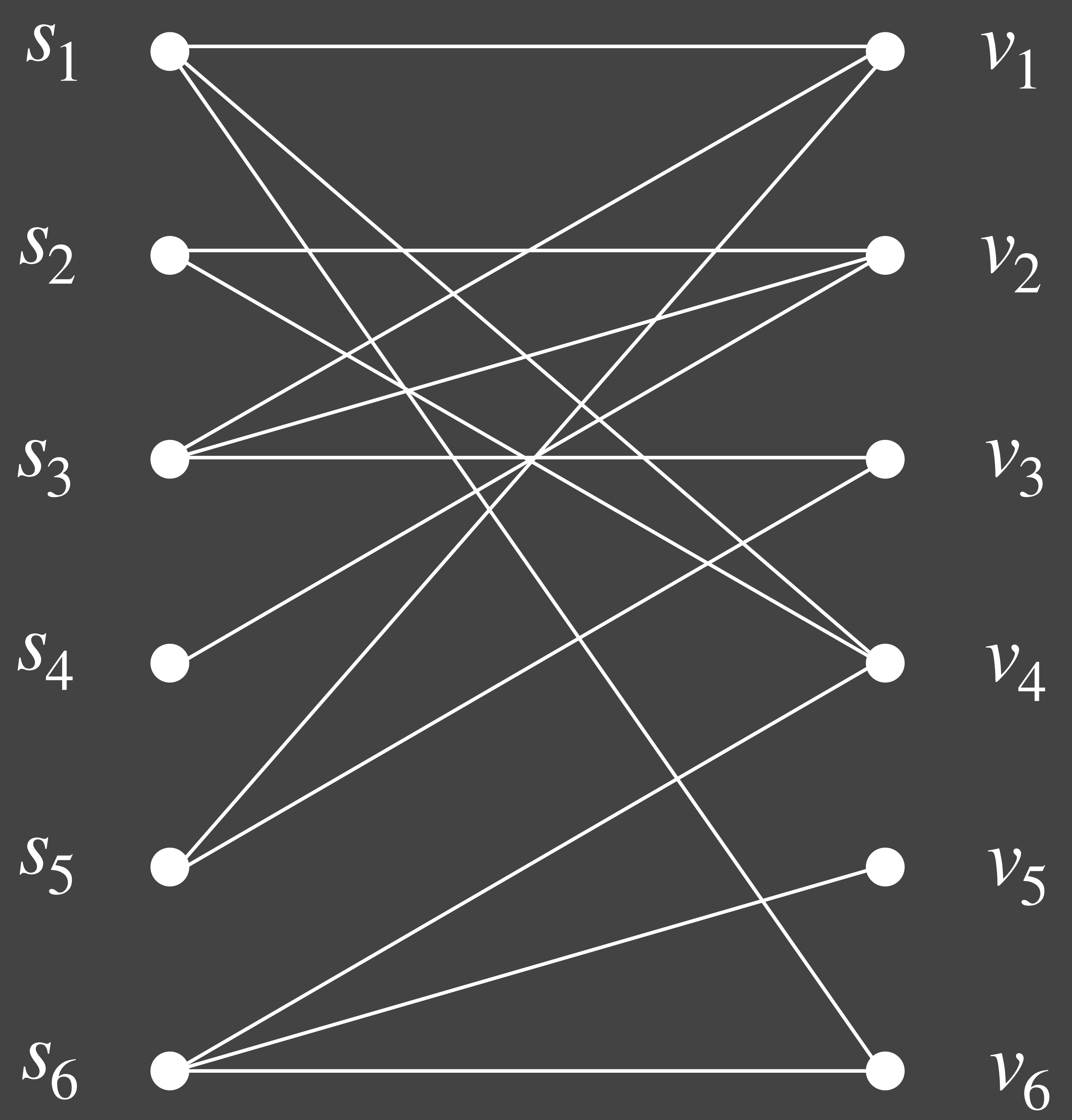
n elements

Q: What happens beyond the worst case?

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets

s_1

s_2

s_3

s_4

s_5

s_6

$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets

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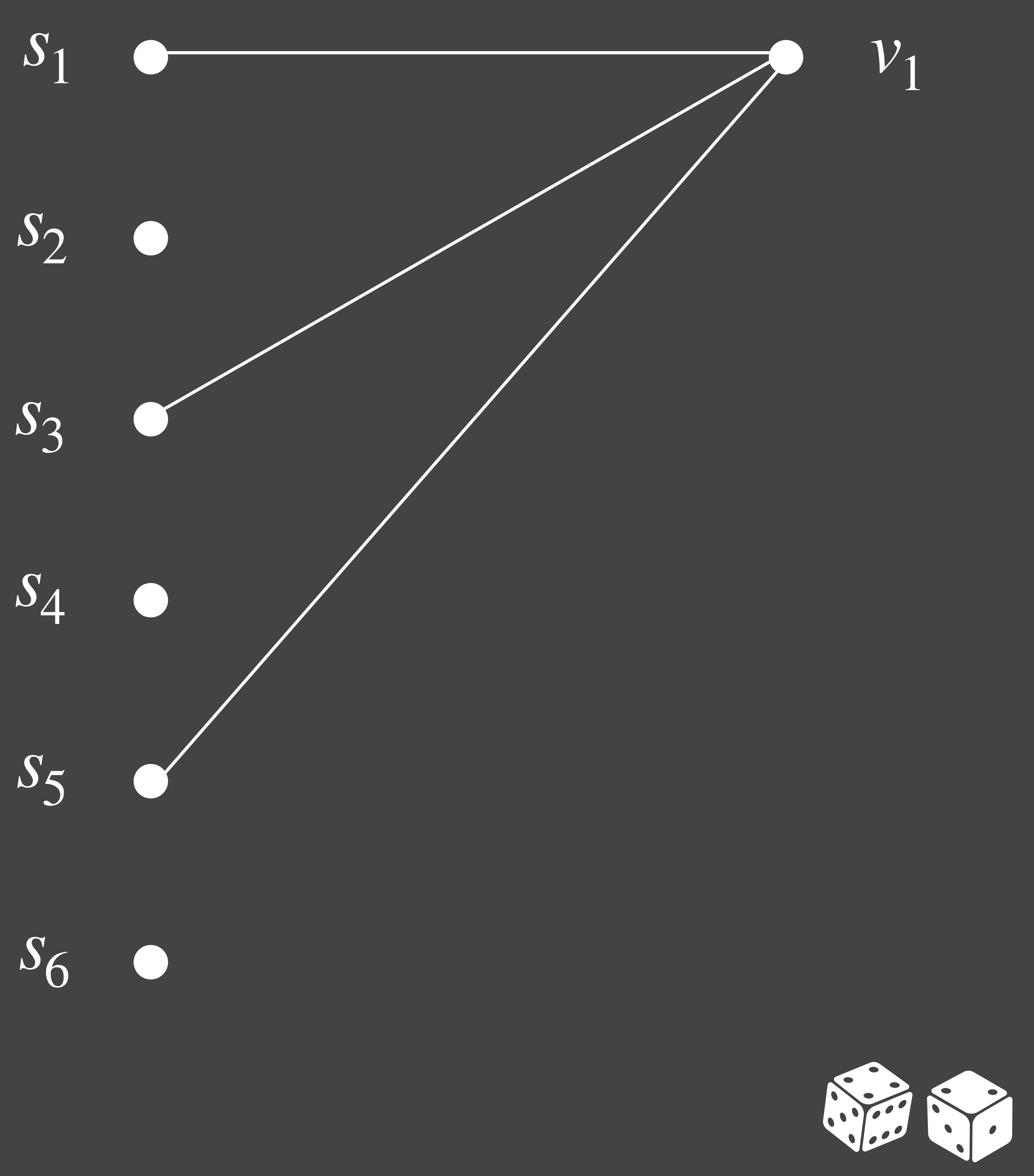
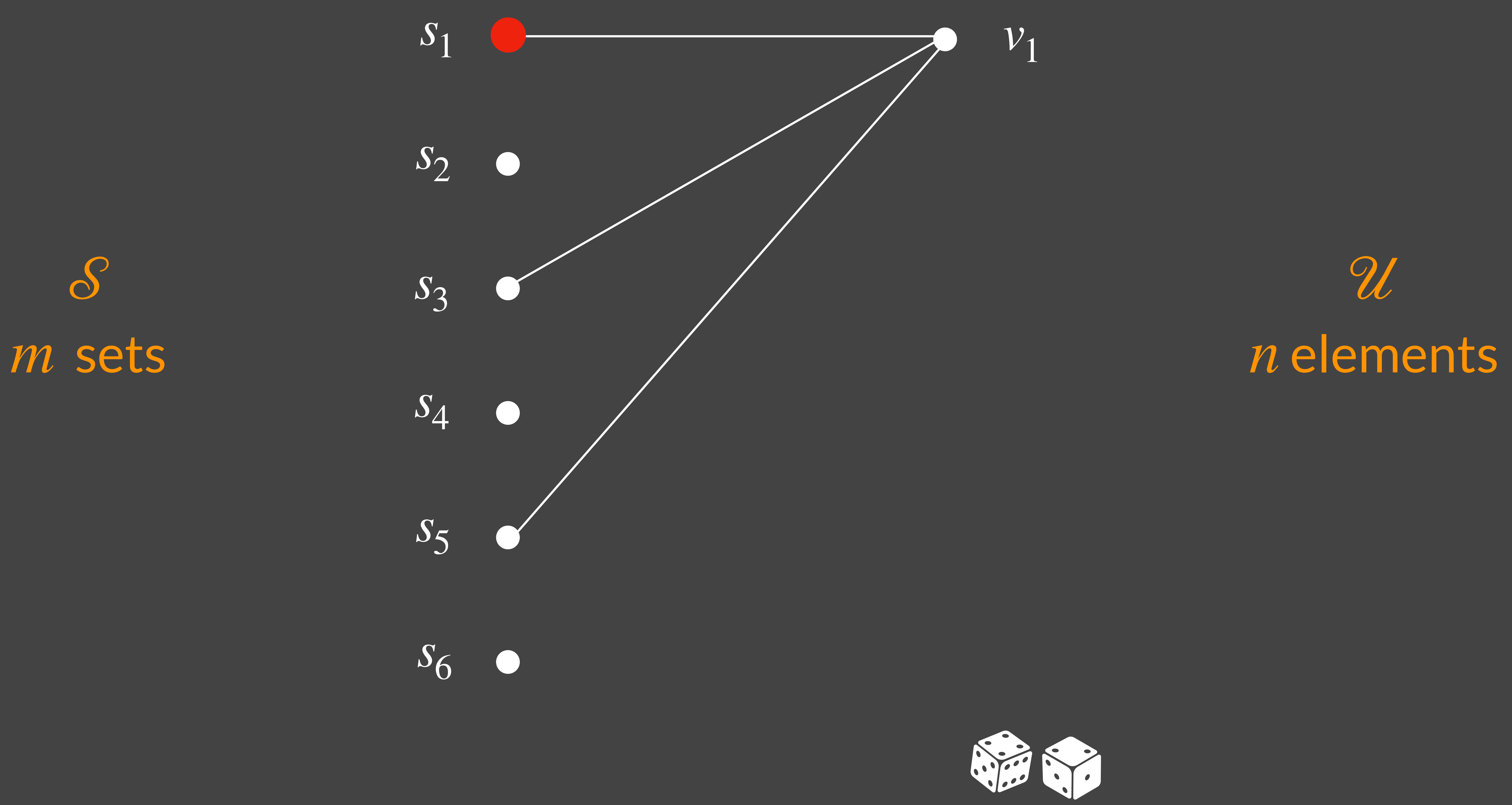


$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)



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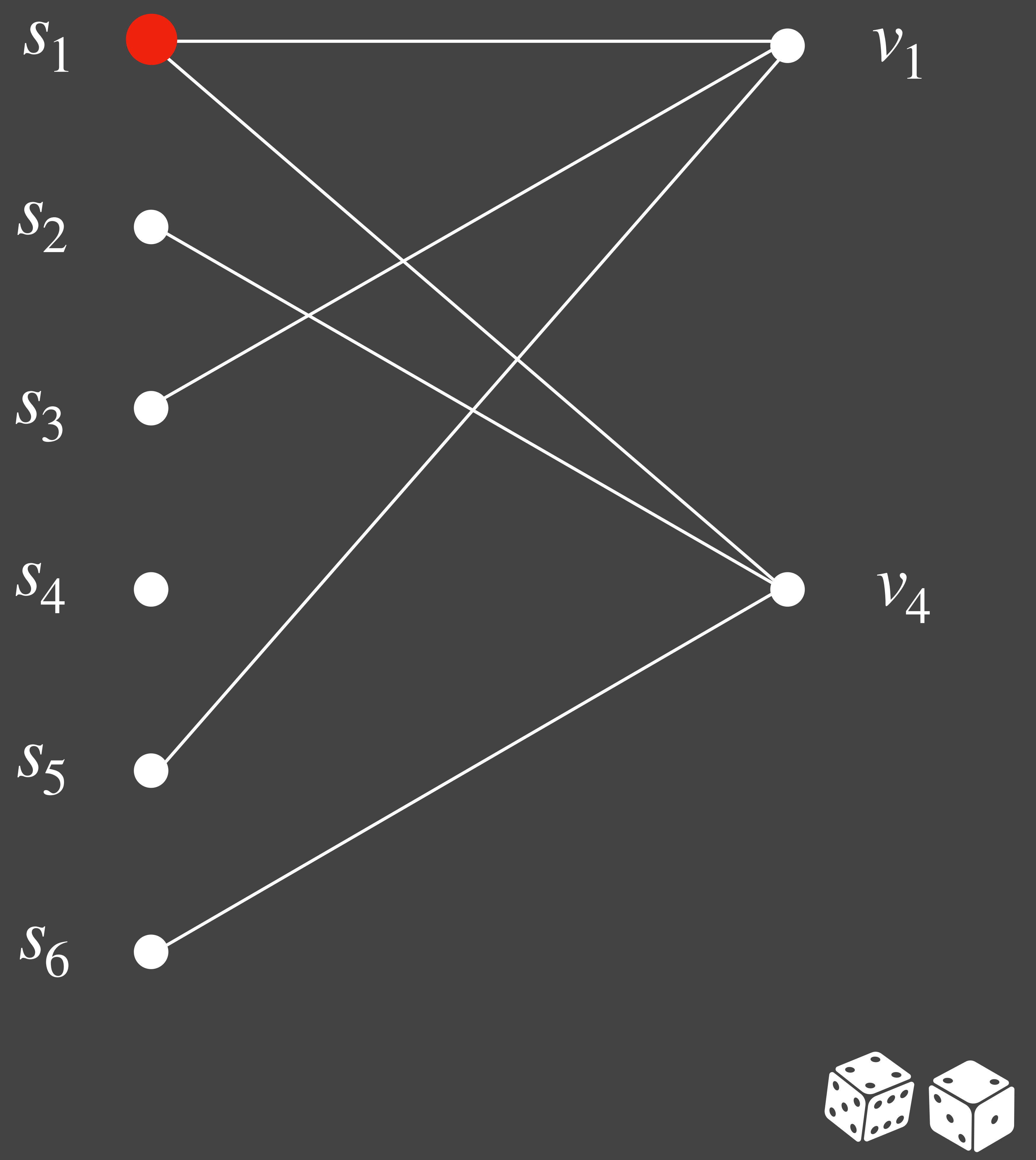
$\mathcal{S}$

m sets



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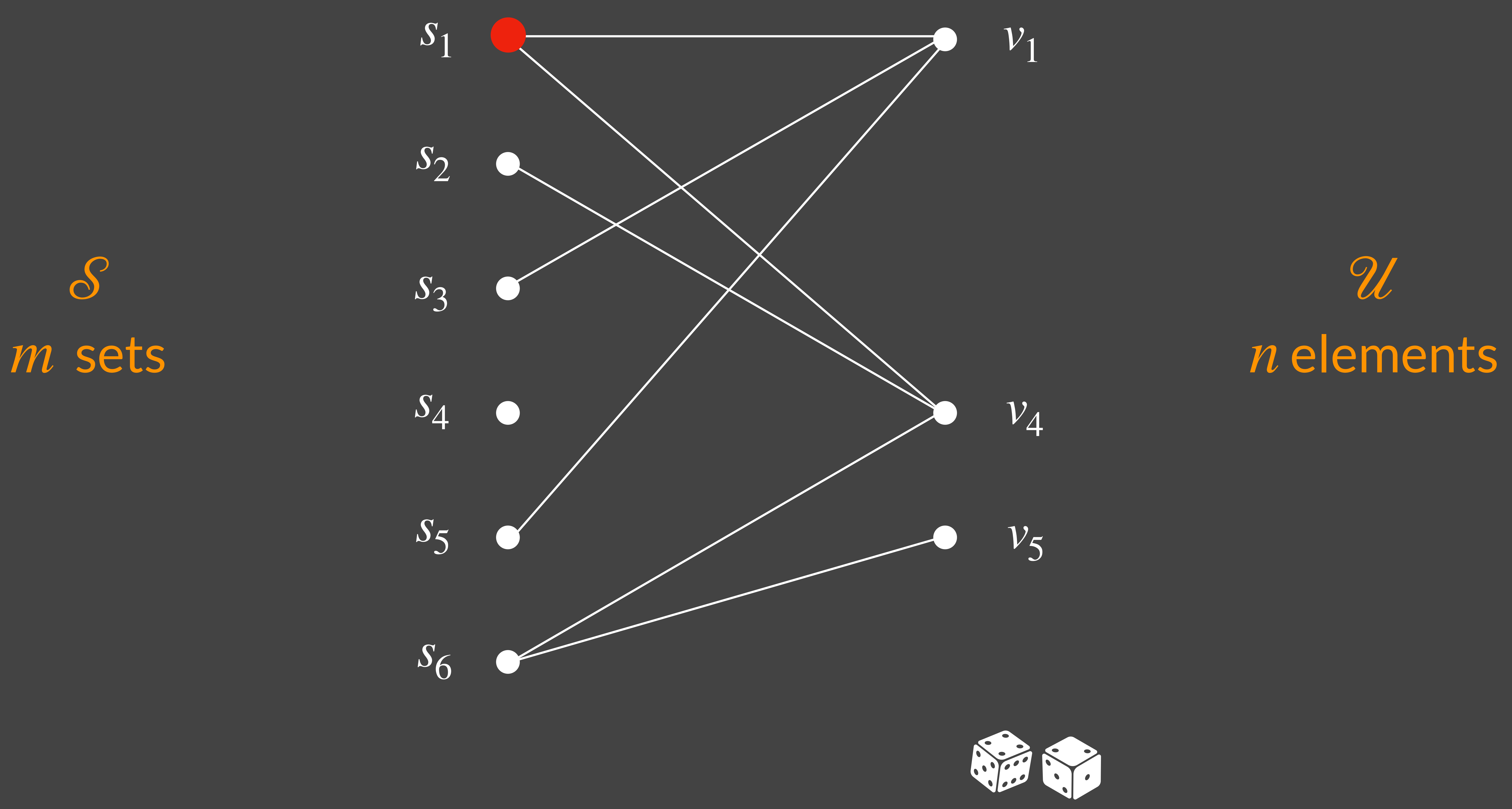

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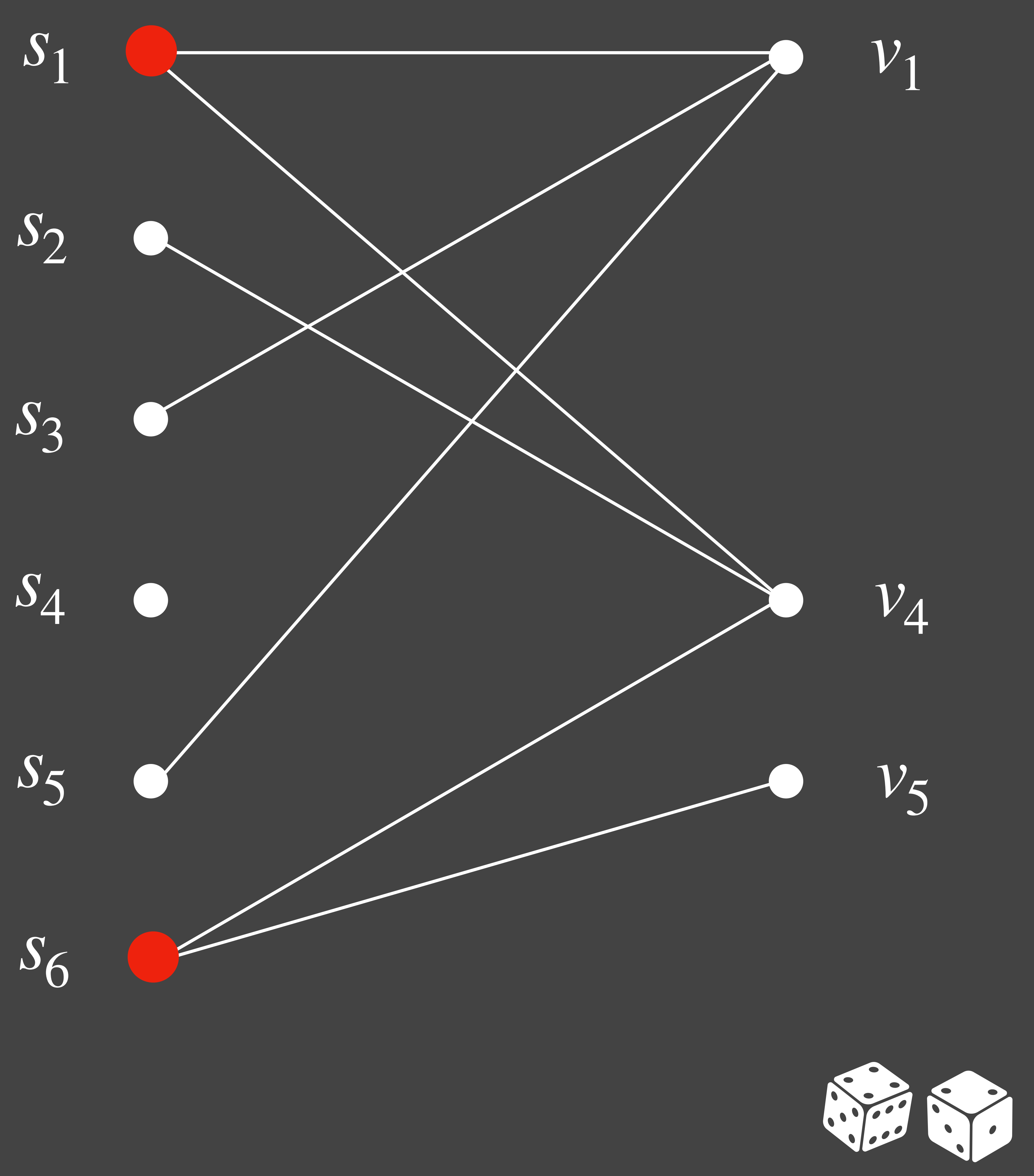

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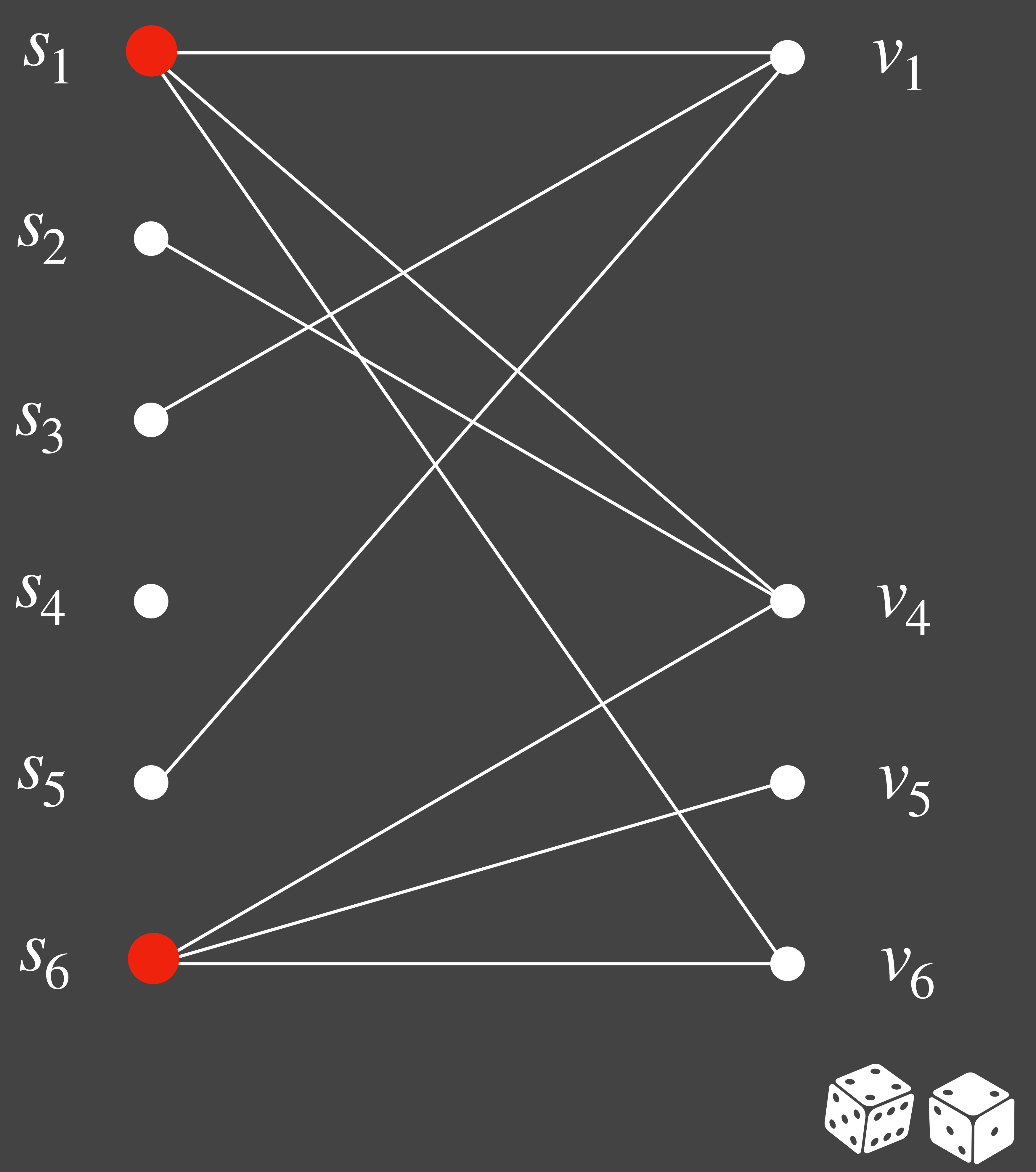

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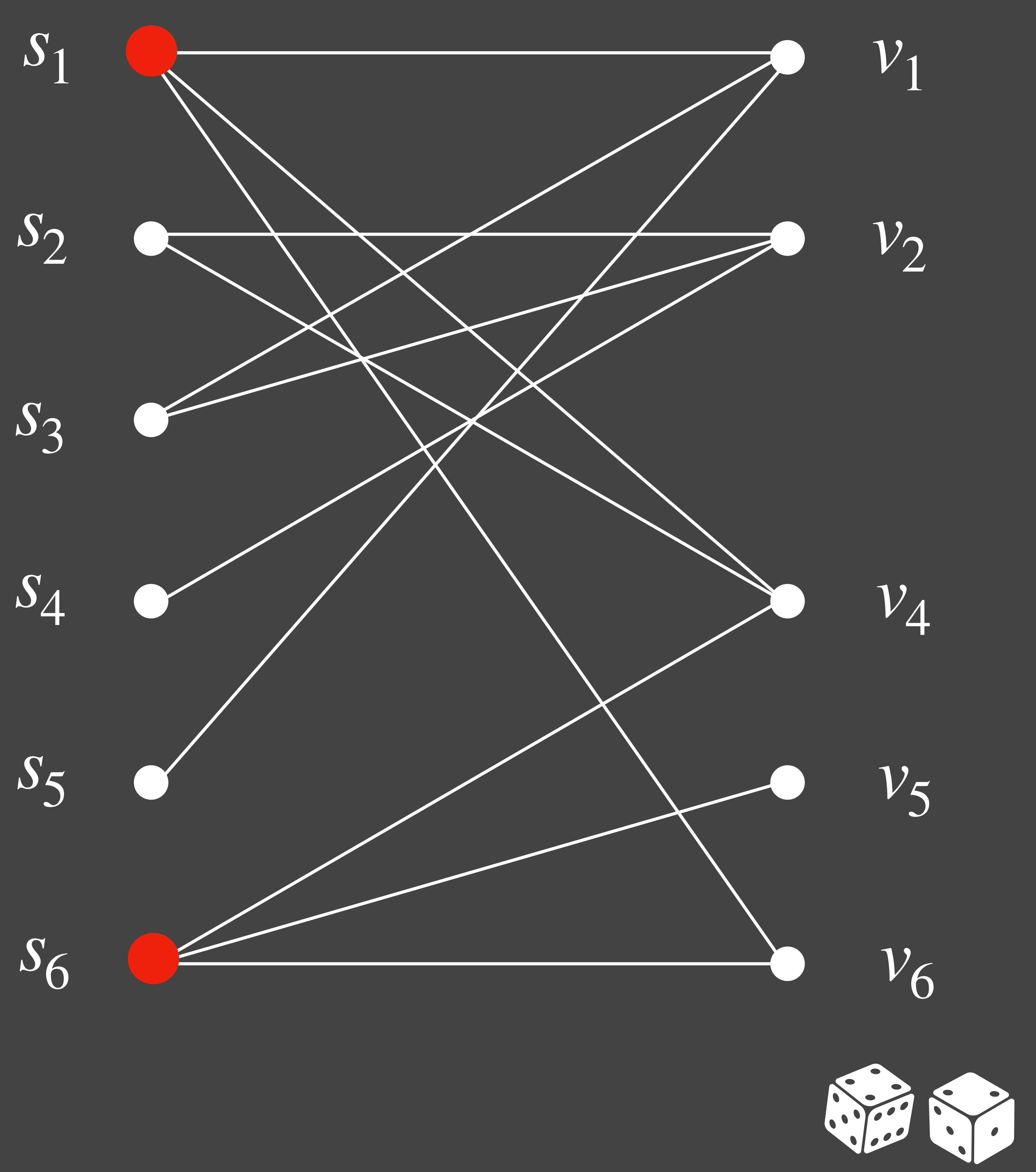

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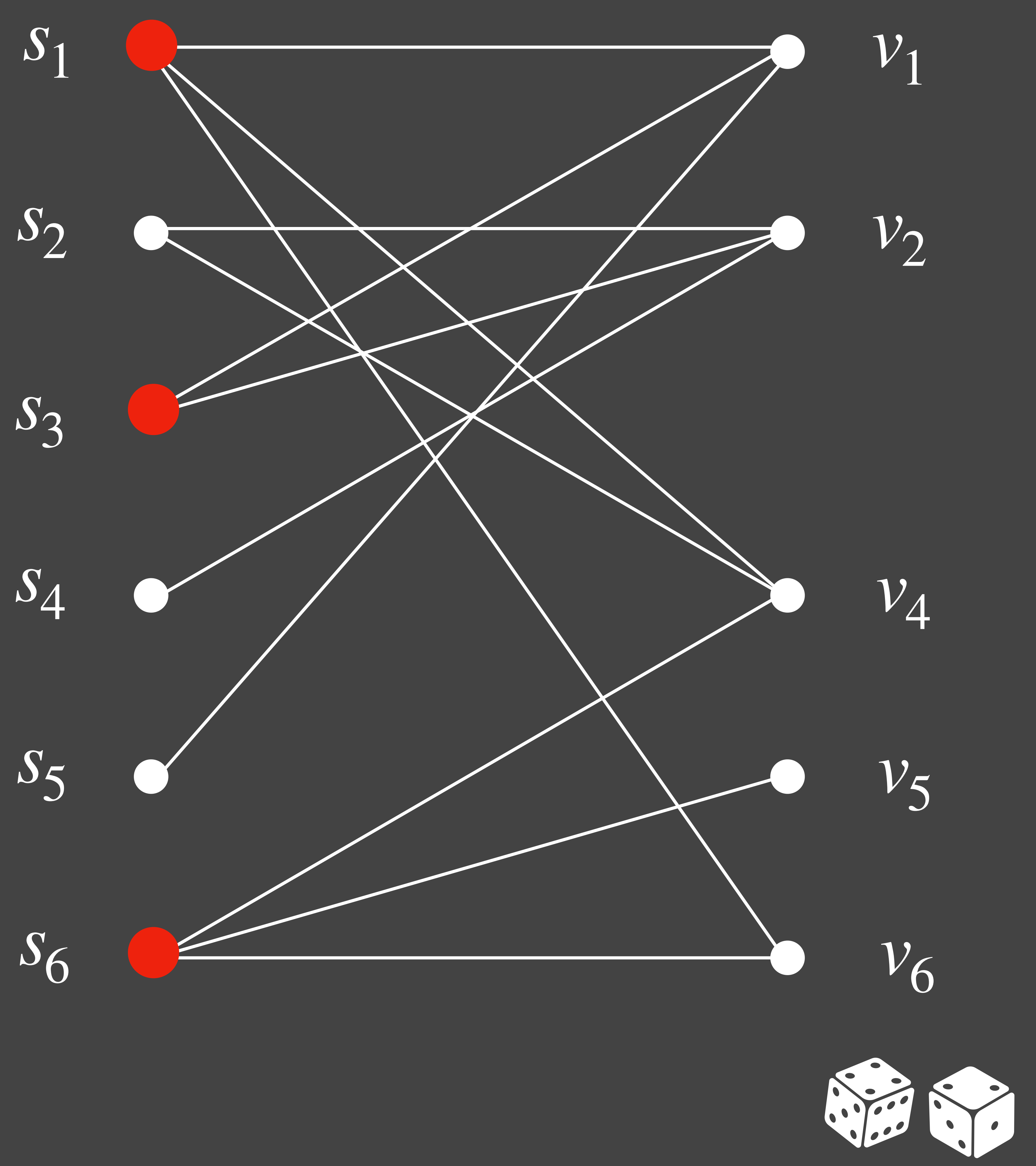

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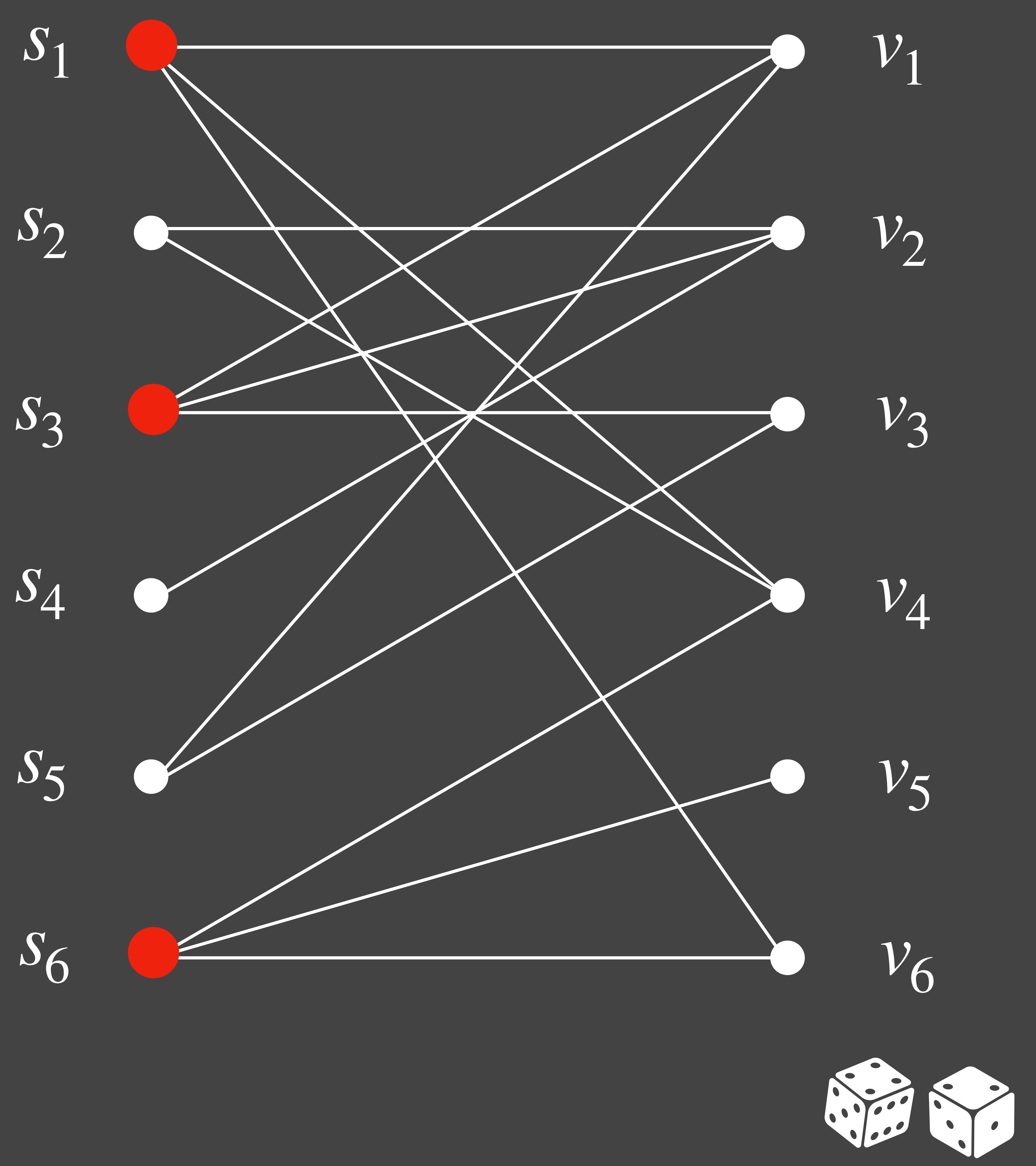

$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets

s_1

s_2

s_3

s_4

s_5

s_6

$\mathcal{U}$

n elements

Relaxation 2: Random Instance

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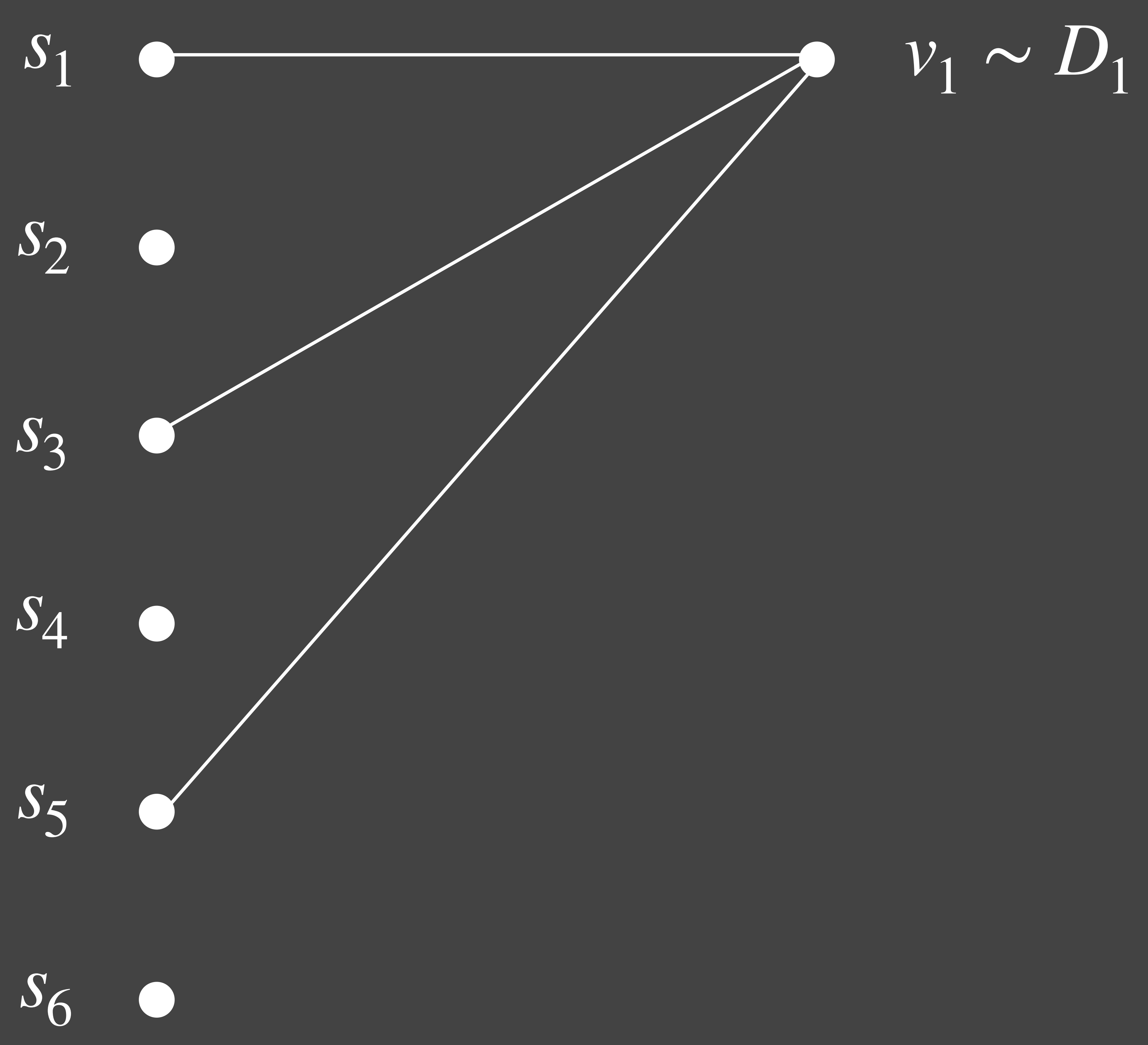

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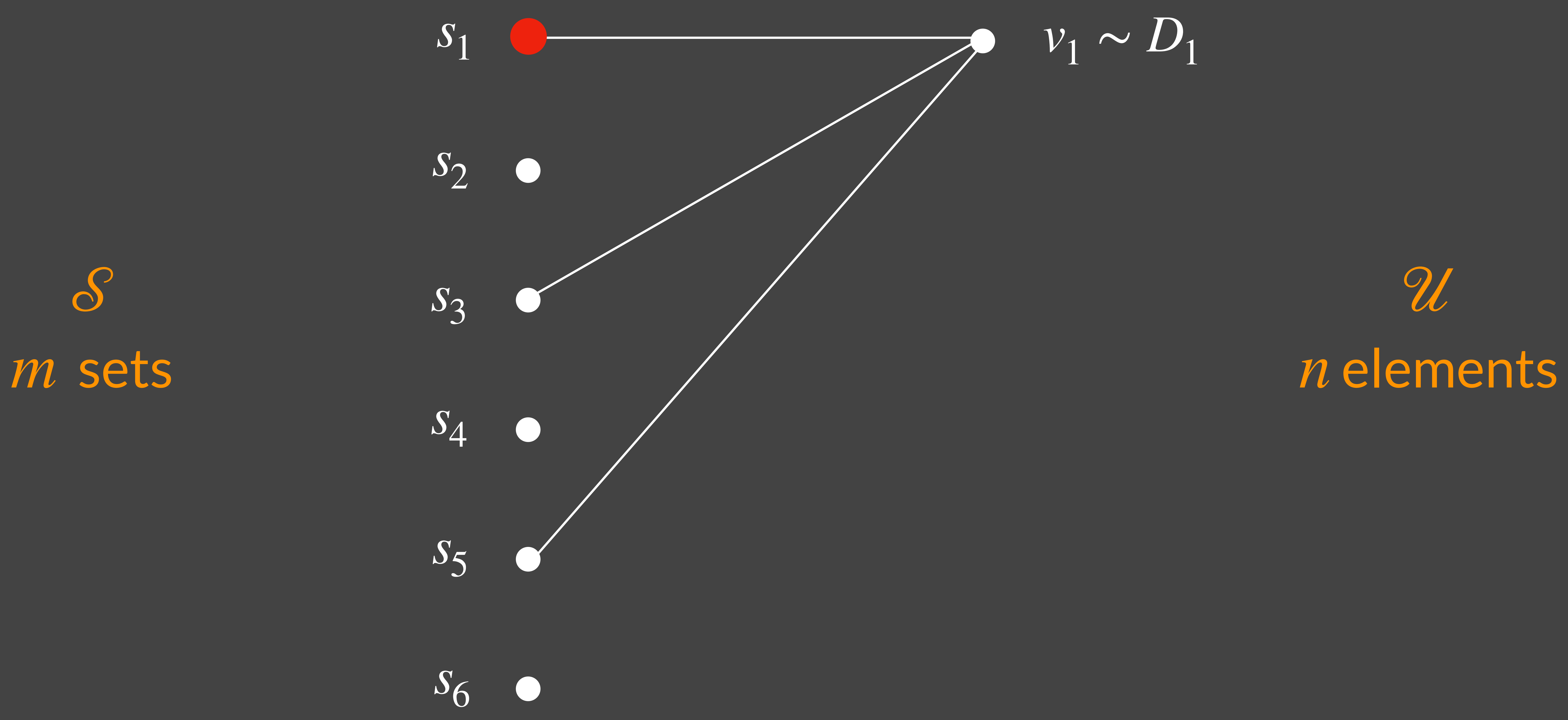

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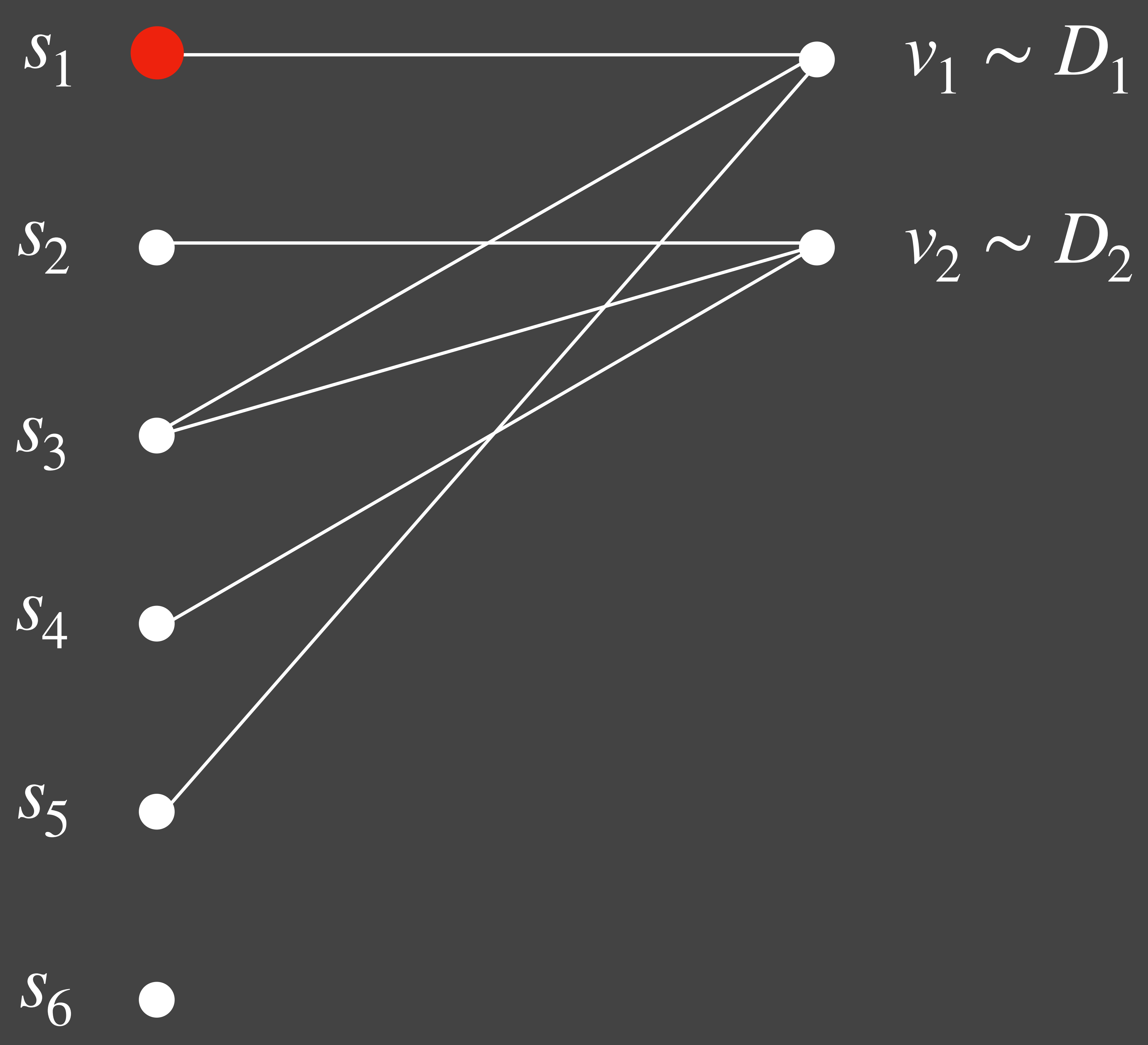

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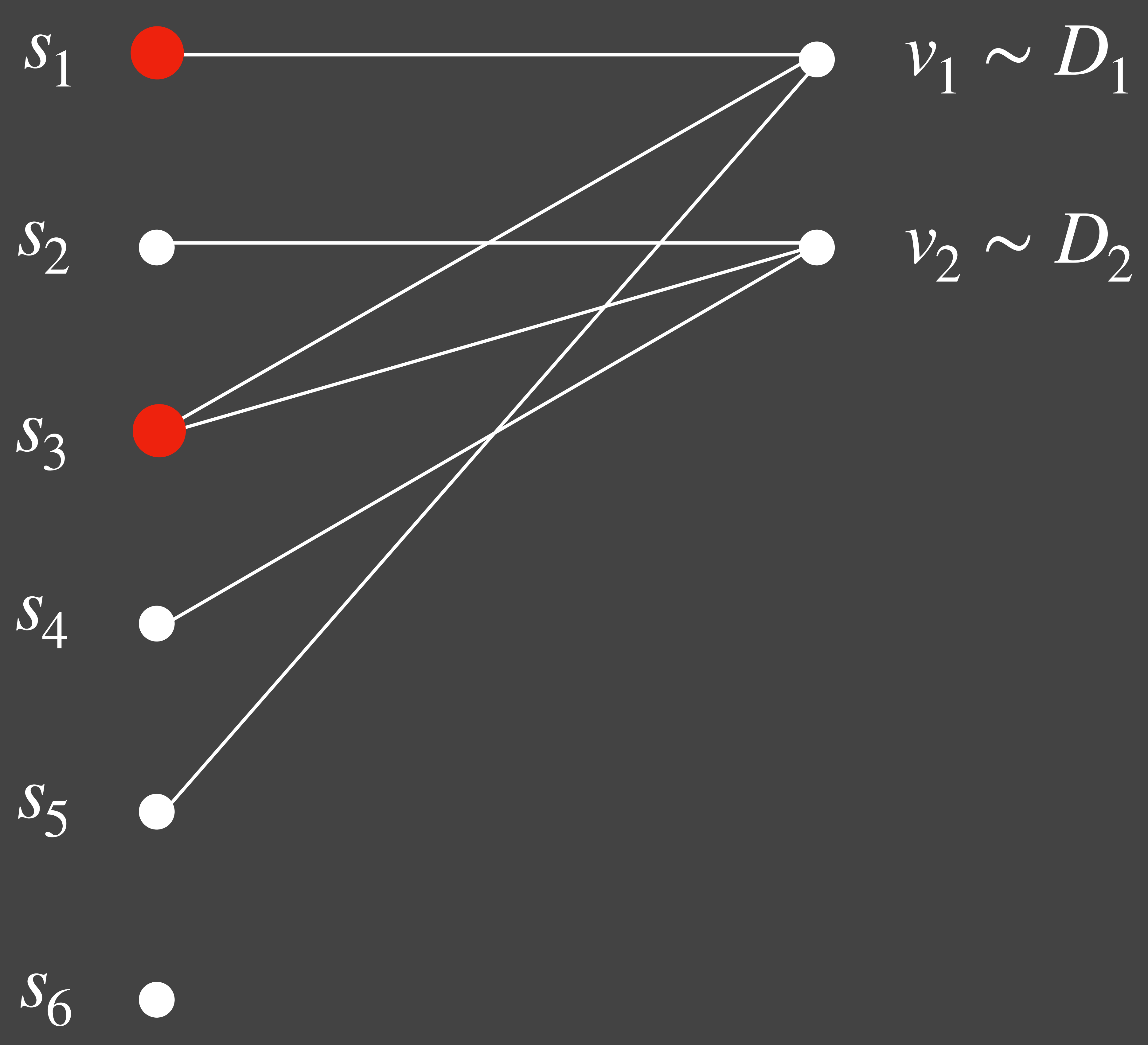

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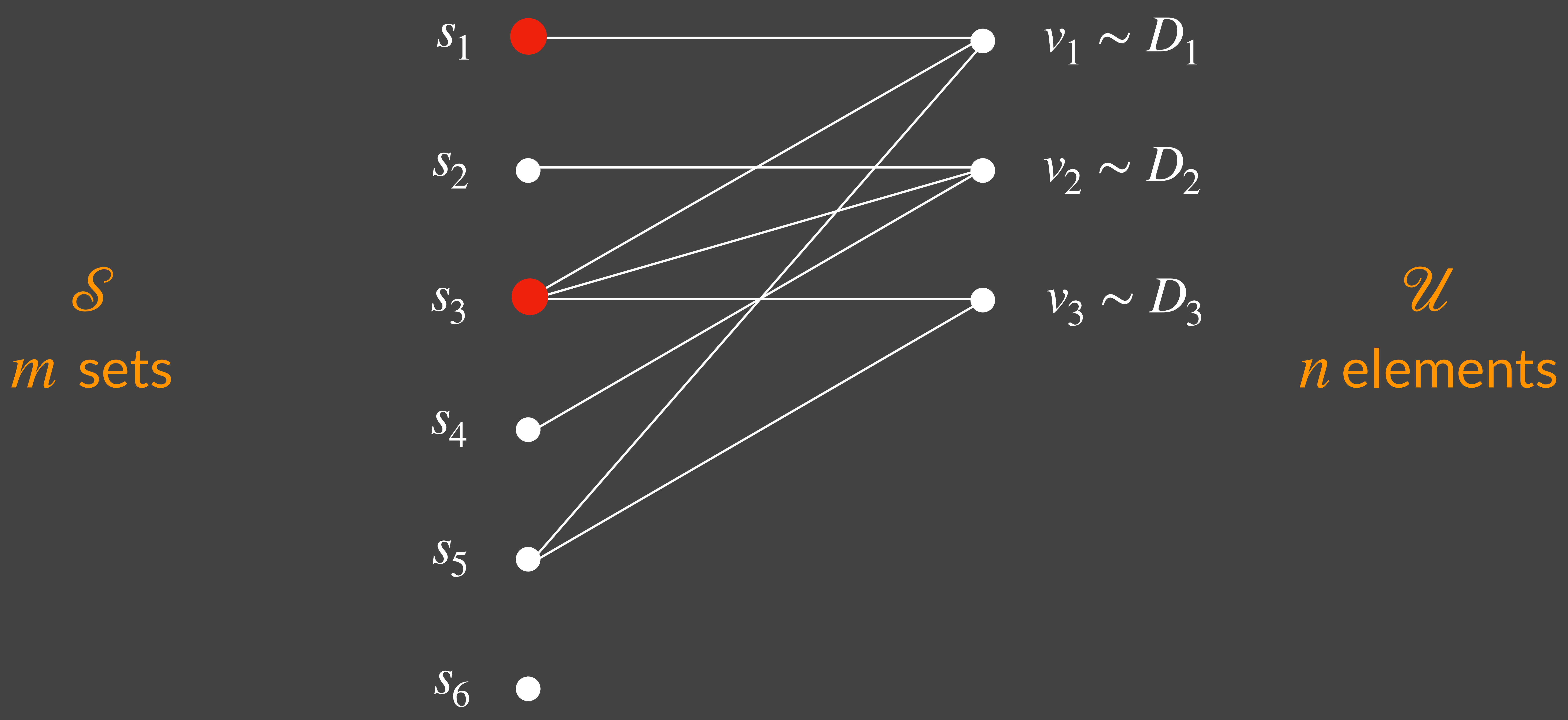

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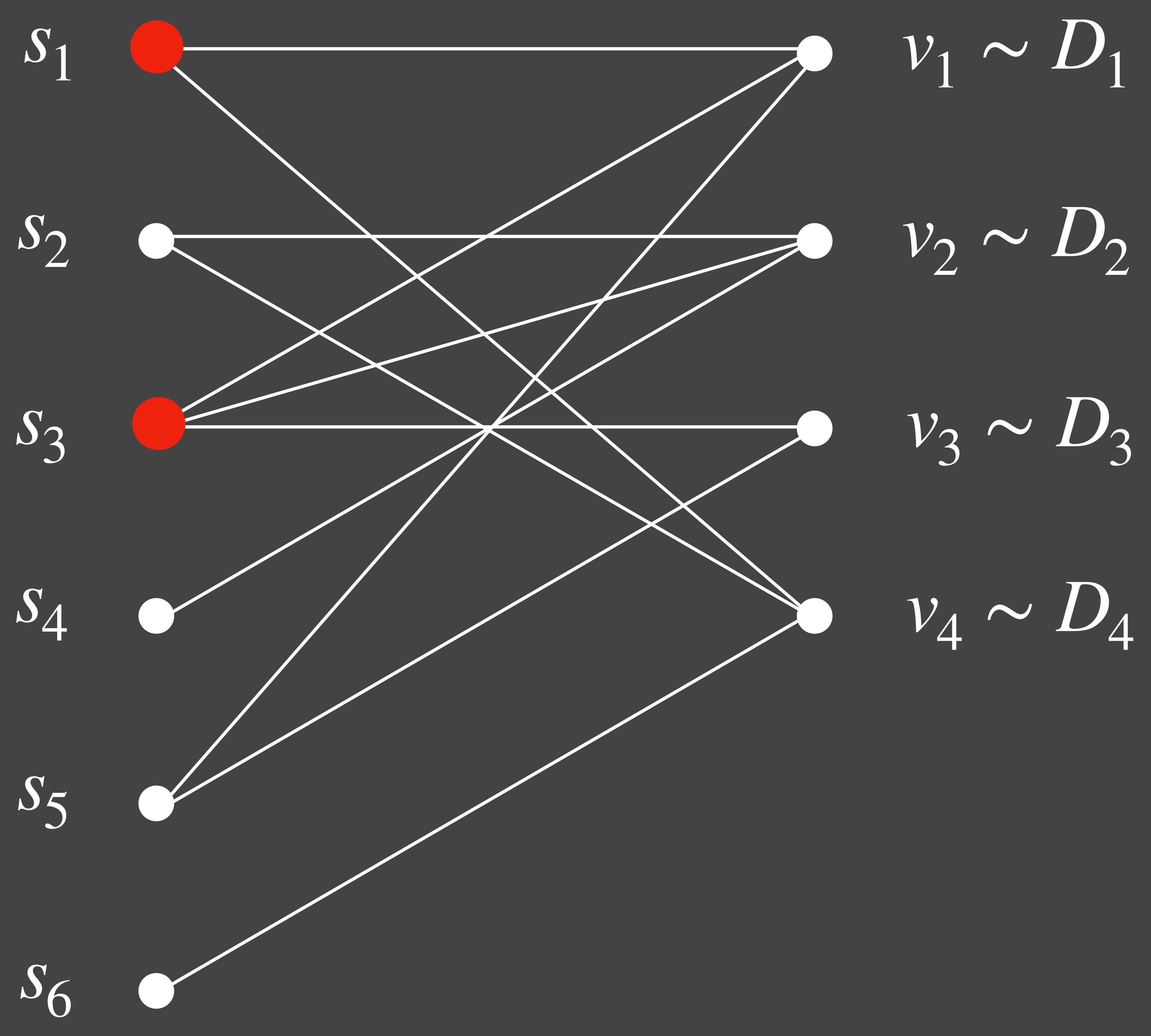

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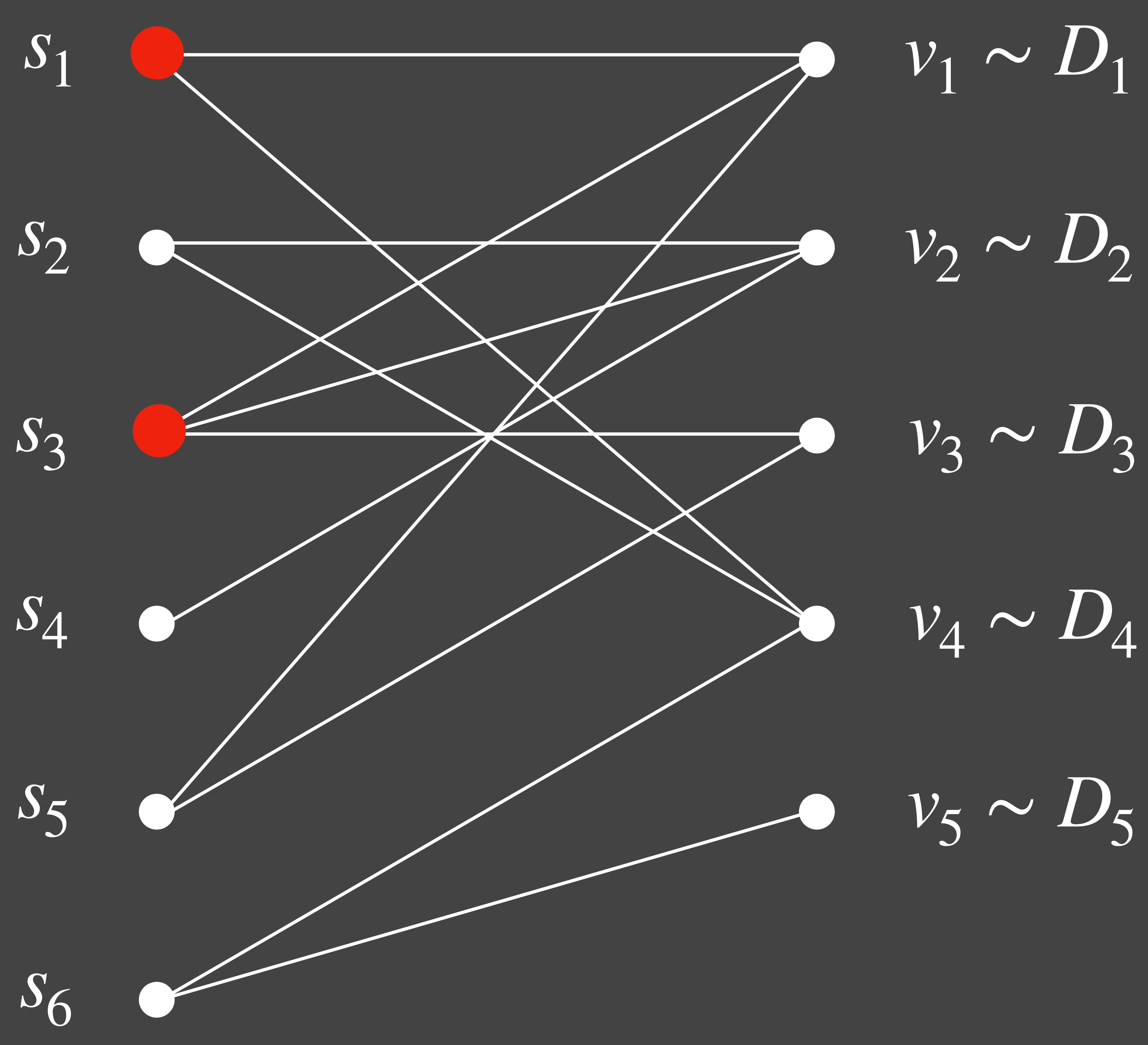

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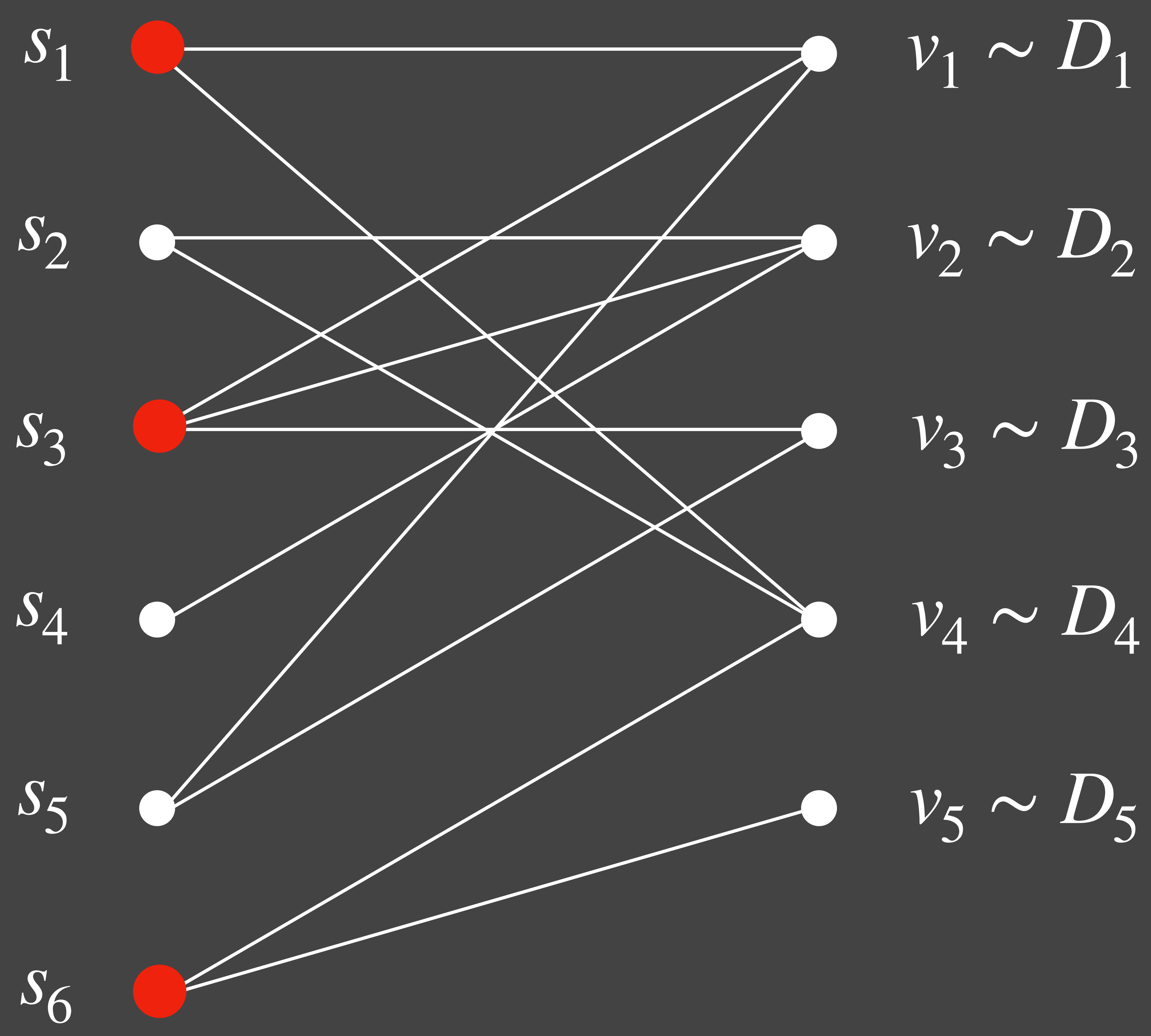

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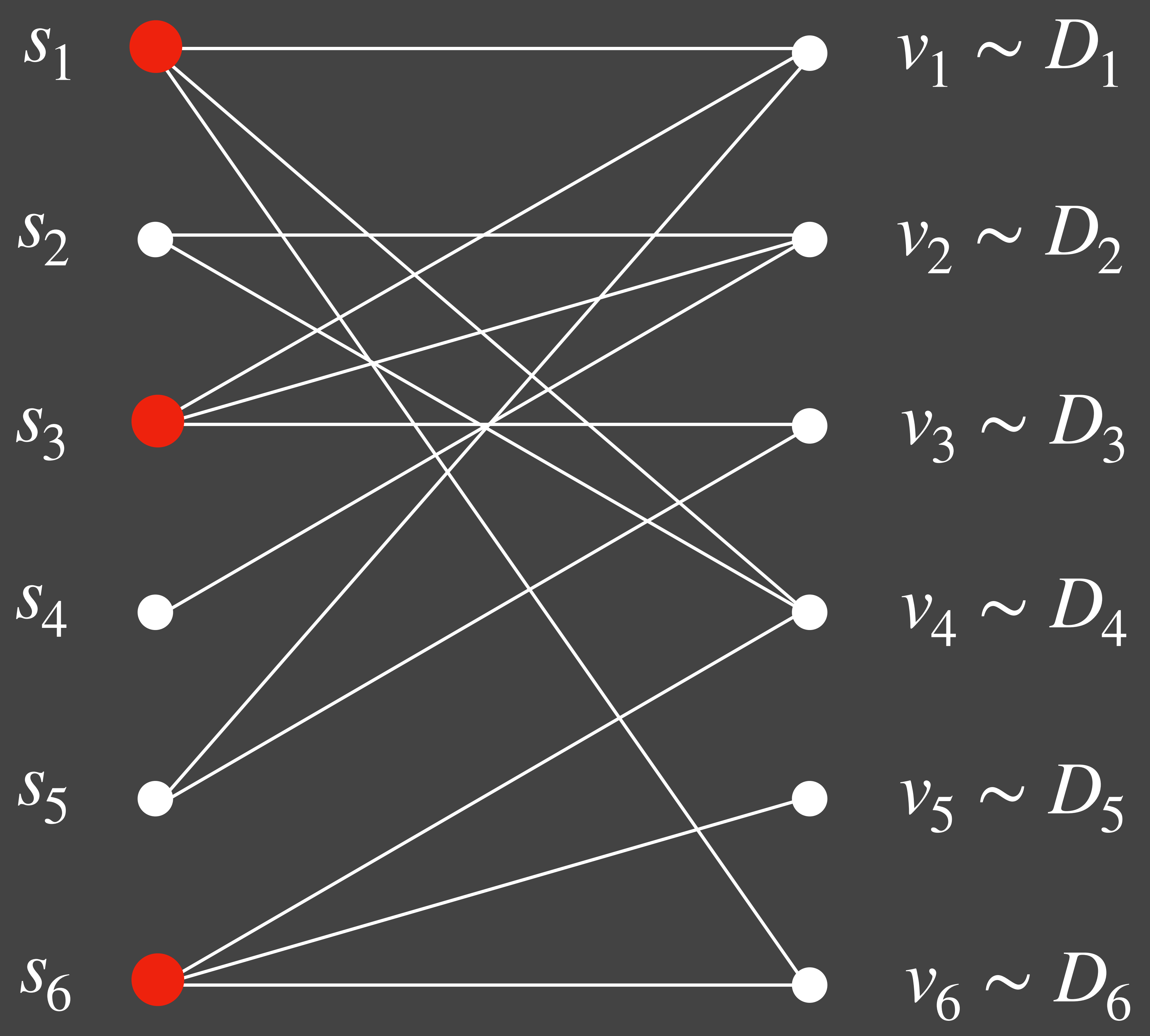

$\mathcal{U}$

n elements

The Landscape

$m =$ # sets
$n =$ # elements

The Landscape

$m =$ # sets
$n =$ # elements

Instance

Arrival Order	Random	Adversarial
Random		
Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

The Landscape

$m =$ # sets
$n =$ # elements

Instance

Arrival Order	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	
Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

The Landscape

m = # sets
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Instance

Arrival Order	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary
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The Landscape

m = # sets
n = # elements

Arrival Order \ Instance	Random	Adversarial
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Adversarial	Prophet	O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

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Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary
Adversarial	Prophet	O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

Some reasons to believe $o(\log n \log m)$ not possible...

The Landscape

Instance

Arrival Order	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary: $\boxed{\text{O(log mn) Our work}}$
Adversarial	Prophet: $\boxed{}$	O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

$m =$ # sets
$n =$ # elements

Theorem [Gupta Kehne **L.** **FOCS** 21]:

There is a poly time algorithm for ***secretary*** *Covering IPs with competitive ratio* $O(\log mn)$.

The Landscape

Instance

Arrival Order

	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	O(log mn) Our work
Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

Secretary

Prophet

m = # sets
n = # elements

Theorem [Gupta Kehne **L.** **FOCS** 21]:

There is a poly time algorithm for ***secretary*** *Covering IPs with competitive ratio* $O(\log mn)$.

New algorithm! We show how to learn distribution & solve at same time.

The Landscape

$m =$ # sets
$n =$ # elements

Instance

Arrival Order	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary: O(log mn) Our work
Adversarial	Prophet: New! O(log mn) Our work	O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

Theorem [Gupta Kehne **L.** FOCS 21]:

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Theorem [Gupta Kehne **L.** 22]:

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The Landscape

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Instance

Arrival Order	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary: O(log mn) Our work
Adversarial	Prophet: New! O(log mn) Our work	O(log n log m) [Alon+ 03] [Buchbinder Naor 09]

Bonus Properties!

1. Only need single sample from each D_i!
2. Universal! Gives sample complexity bound $O(n)$.

Theorem [Gupta Kehne **L.** FOCS 21]:

There is a poly time algorithm for ***secretary*** *Covering IPs with competitive ratio* $O(\log mn)$.

Theorem [Gupta Kehne **L.** 22]:

There is a poly time algorithm for ***prophet*** *Covering IPs with competitive ratio* $O(\log mn)$.

Talk Outline

➡ Intro

Secretary

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Talk Outline

Intro

➔ Secretary

- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Set Cover via Random Rounding

Set Cover via Random Rounding

2 Stage algorithm!

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

(II) Round.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_S x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(II) Round.

Set Cover via Random Rounding

2 Stage algorithm!

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$$\forall v \in \mathscr{U} : \quad \sum_{S \ni v} x_S \geq 1$$

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This is relaxation, so $c(x) \leq c(\text{OPT})$.

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Buy S with probability x_S.

Set Cover via Random Rounding

2 Stage algorithm!

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This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Expected cost is $c(x)$!

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\boxed{\begin{array}{ll} & \min \sum_{S} x_S \\ \forall v \in \mathcal{U}: & \sum_{S \ni v} x_S \geq 1 \\ \forall S \in \mathcal{S}: & x_S \geq 0 \end{array}}$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Expected cost is $c(x)$!

Can show $\forall v \in \mathcal{U}$, covered with constant prob.

Set Cover via Random Rounding

2 Stage algorithm!

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Repeat $O(\log n)$ times, union bound.

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Expected Cost: $O(\log n) \cdot \mathrm{OPT}$

How [Alon+ 03] works

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Same 2 Stages!

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(I) Solve LP Online.

(II) Round Online.

How [Alon+ 03] works

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$$\forall v \in \mathcal{U}: \quad \sum_{S \ni v} x_S \geq 1$$

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How [Alon+ 03] works

Same 2 Stages!

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Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.

How [Alon+ 03] works

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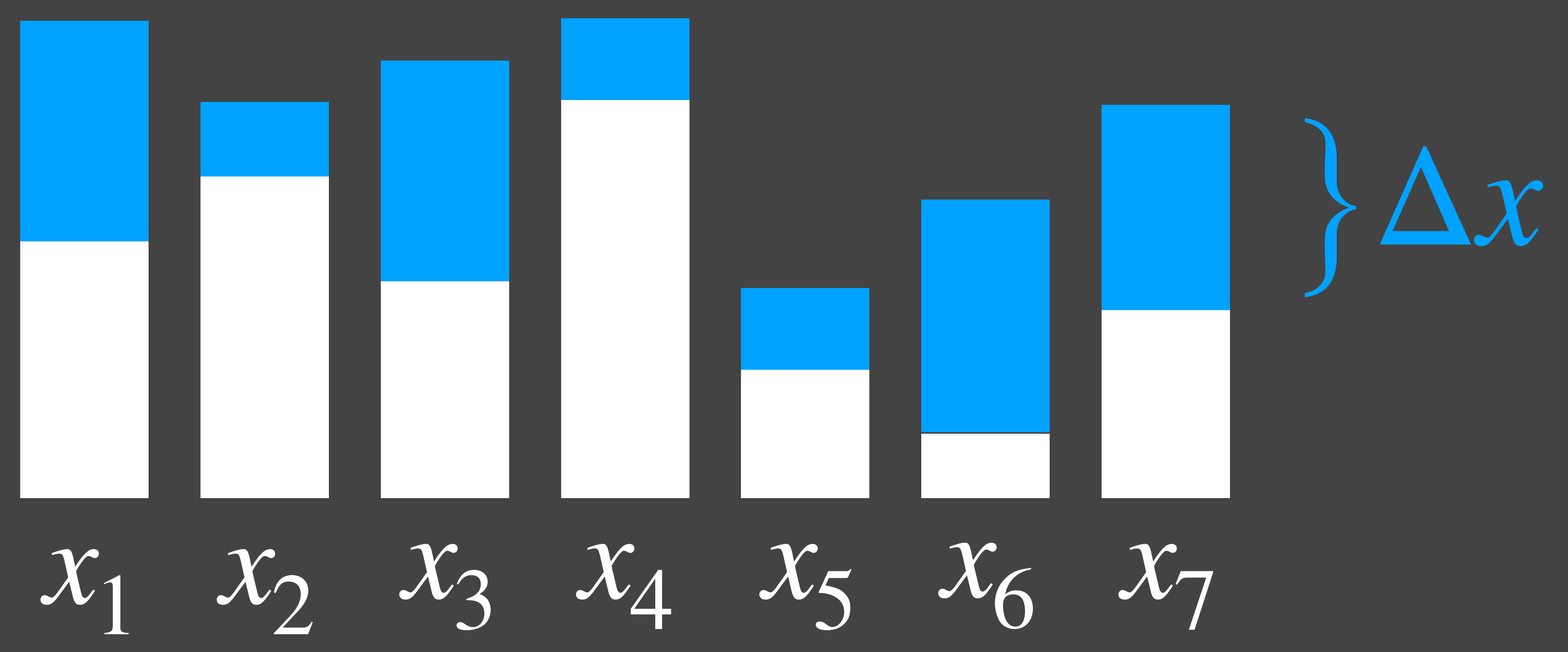

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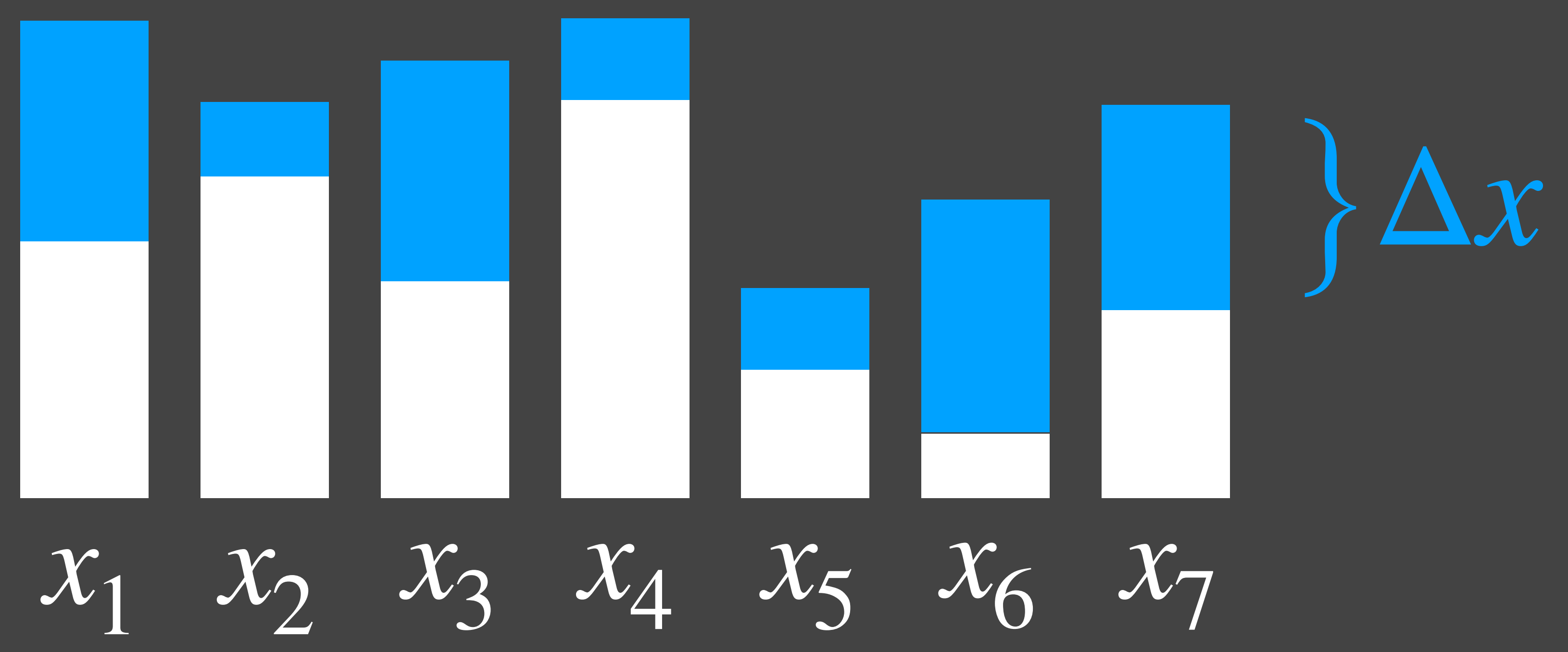

Take S with prob. $\propto \Delta x_S$.

How [Alon+ 03] works

Same 2 Stages!

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$$\min \sum_{S} x_S$$
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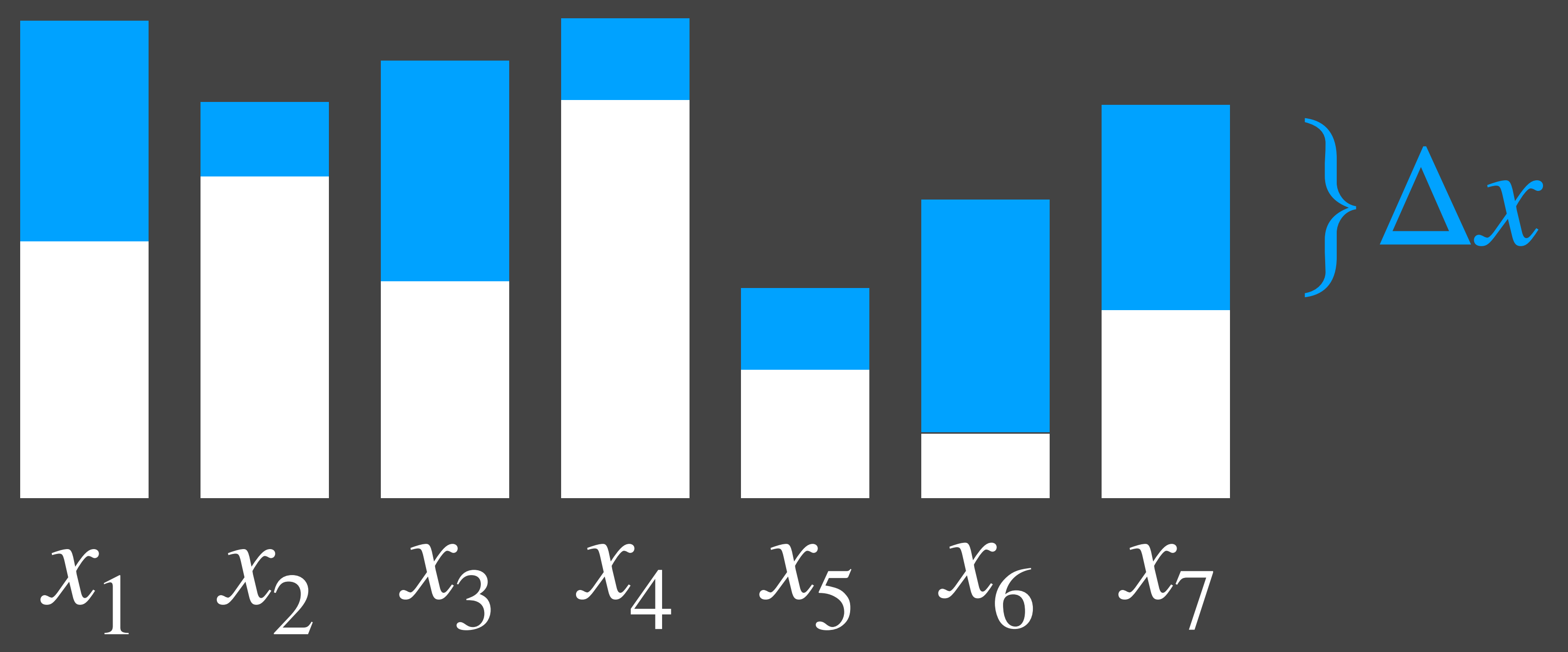

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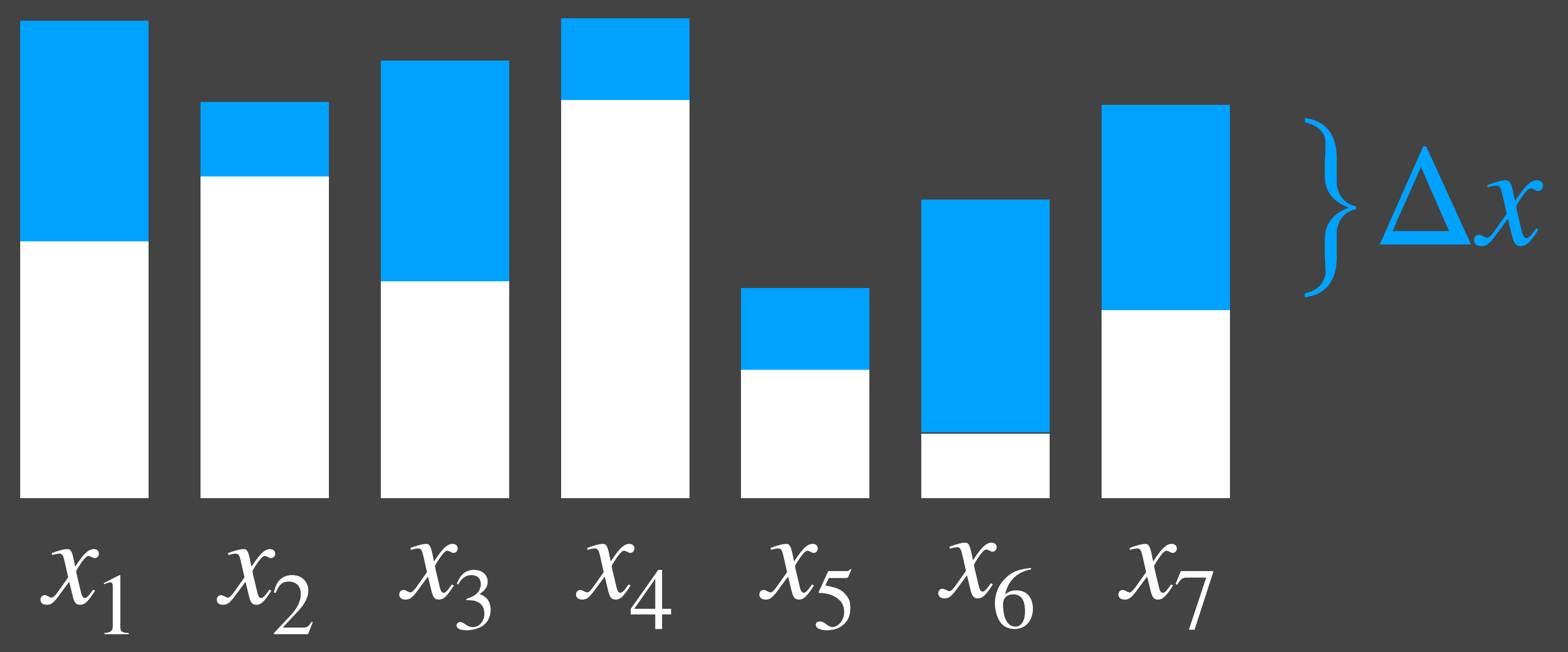

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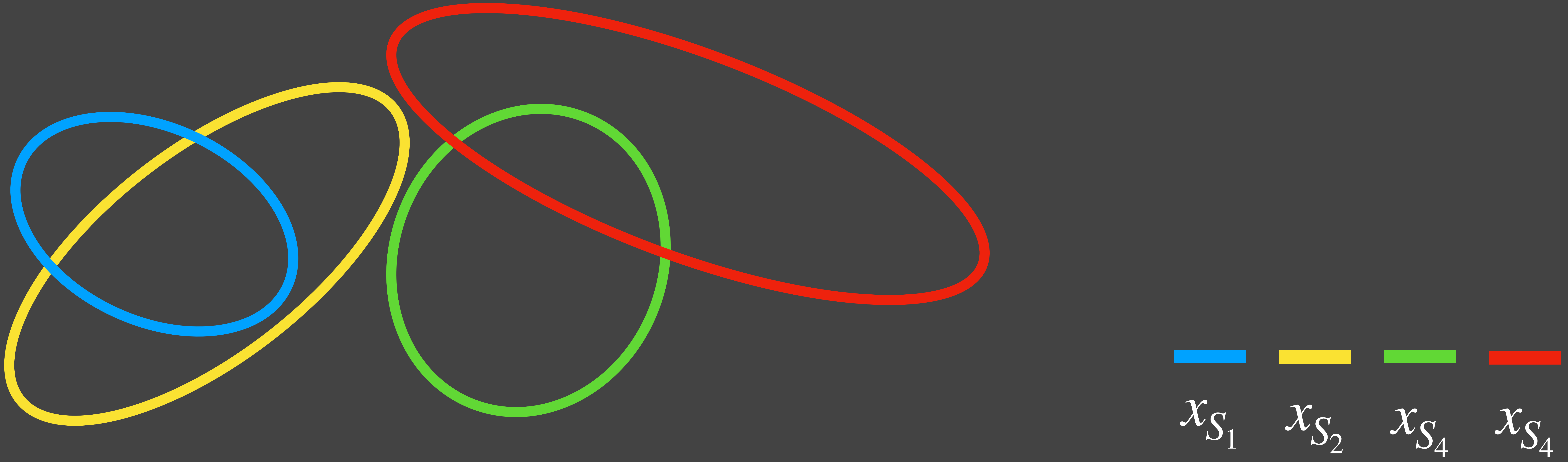
Expected Cost: $O(\log n \log m) \cdot$ OPT

Online LP Solver of [Alon+ 03]



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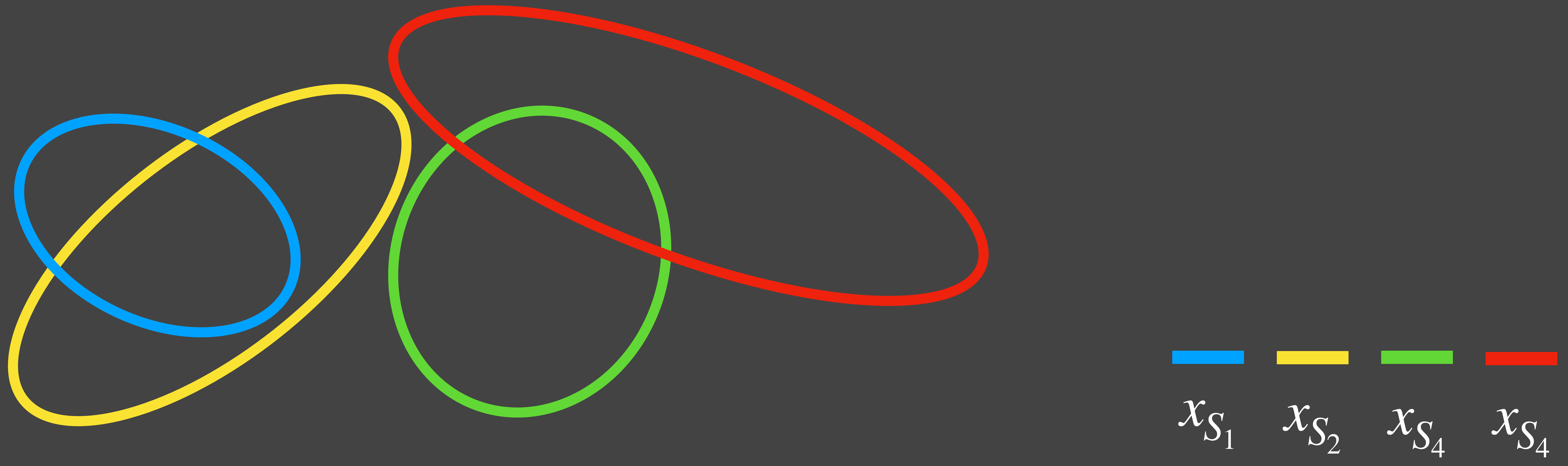

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While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

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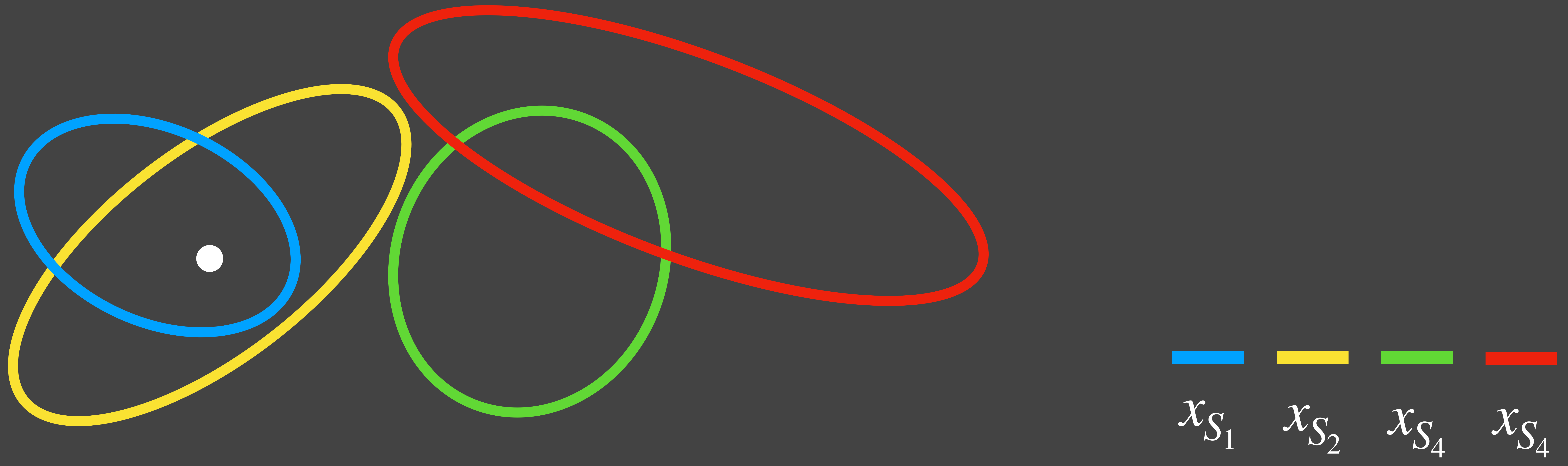

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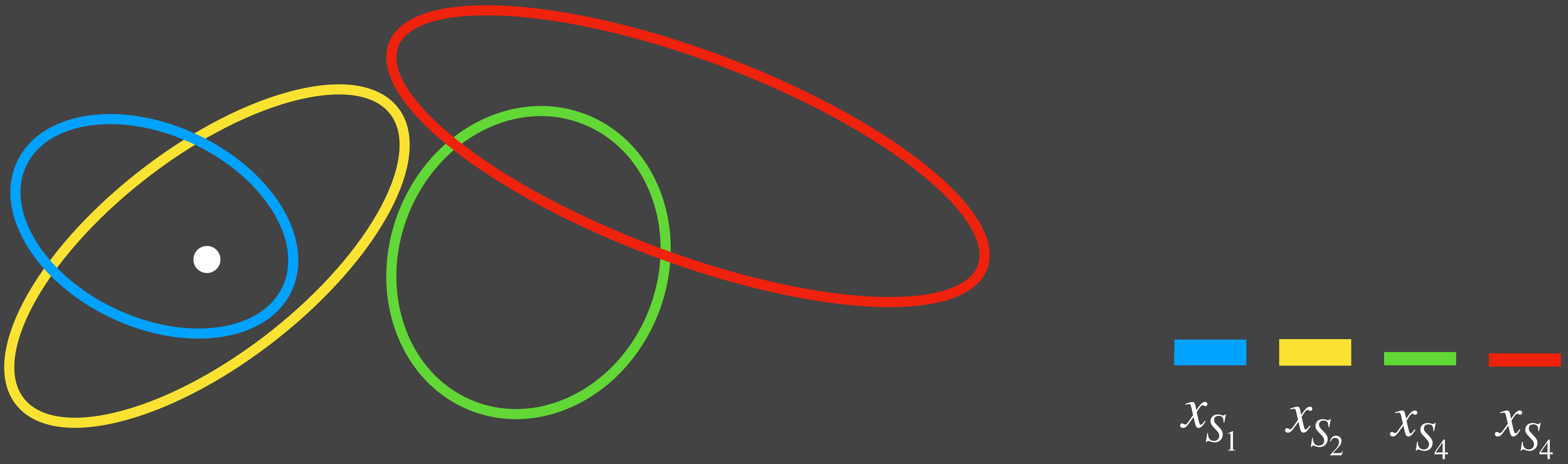

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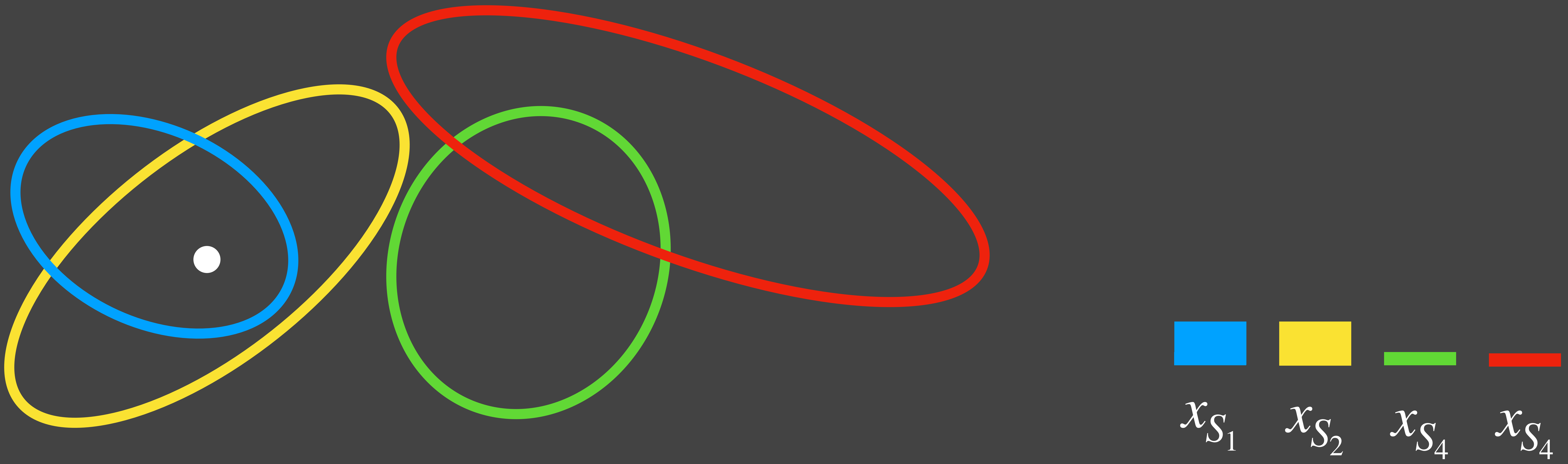

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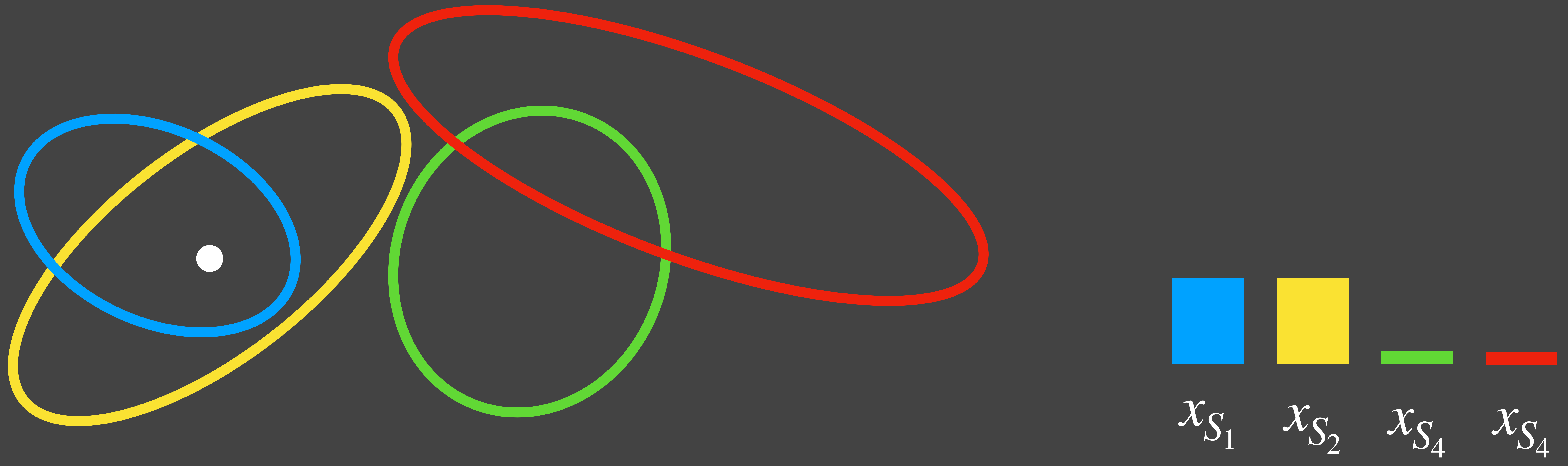

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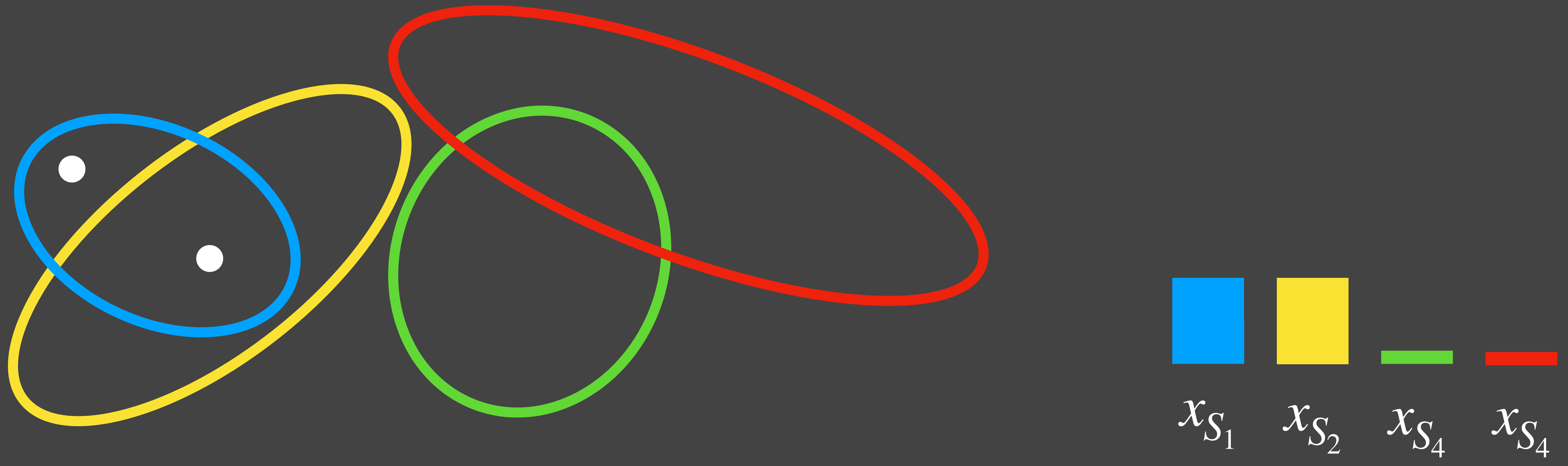

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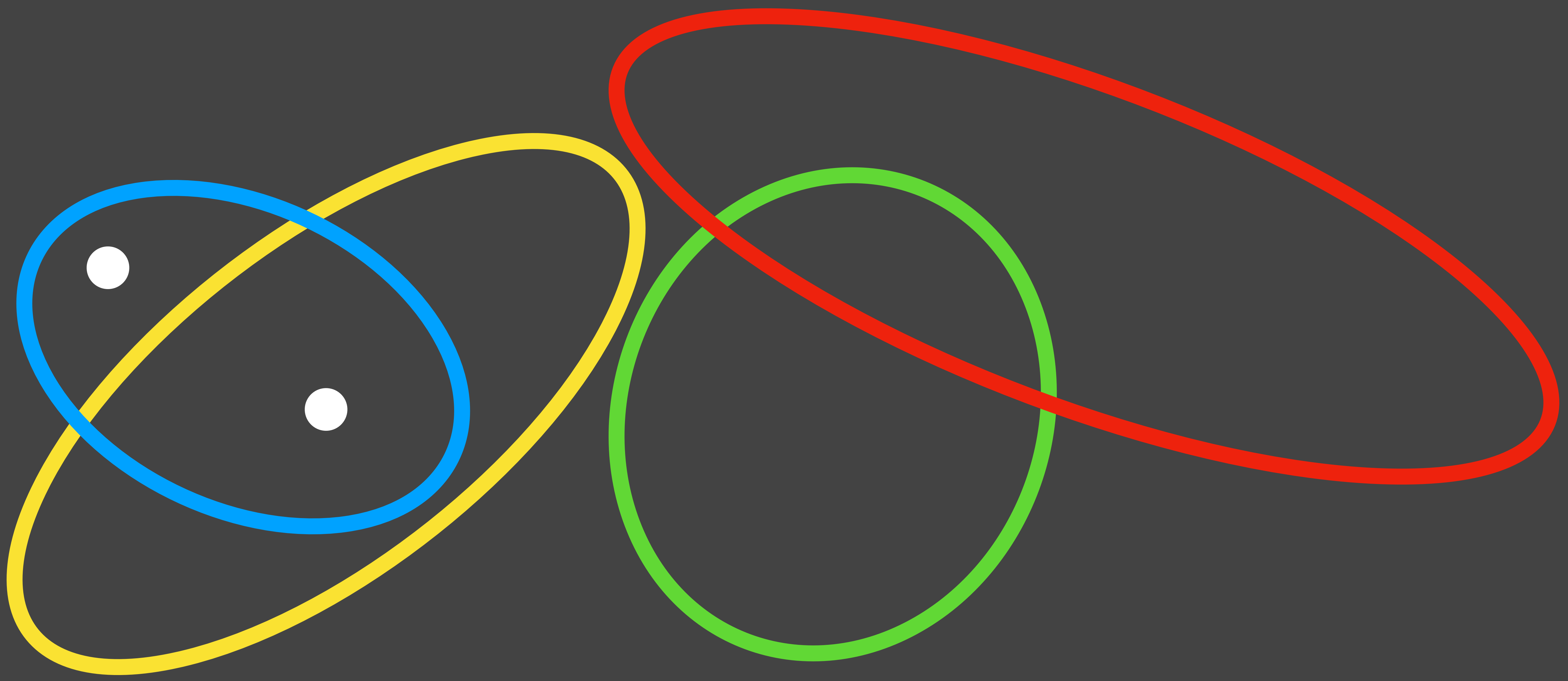

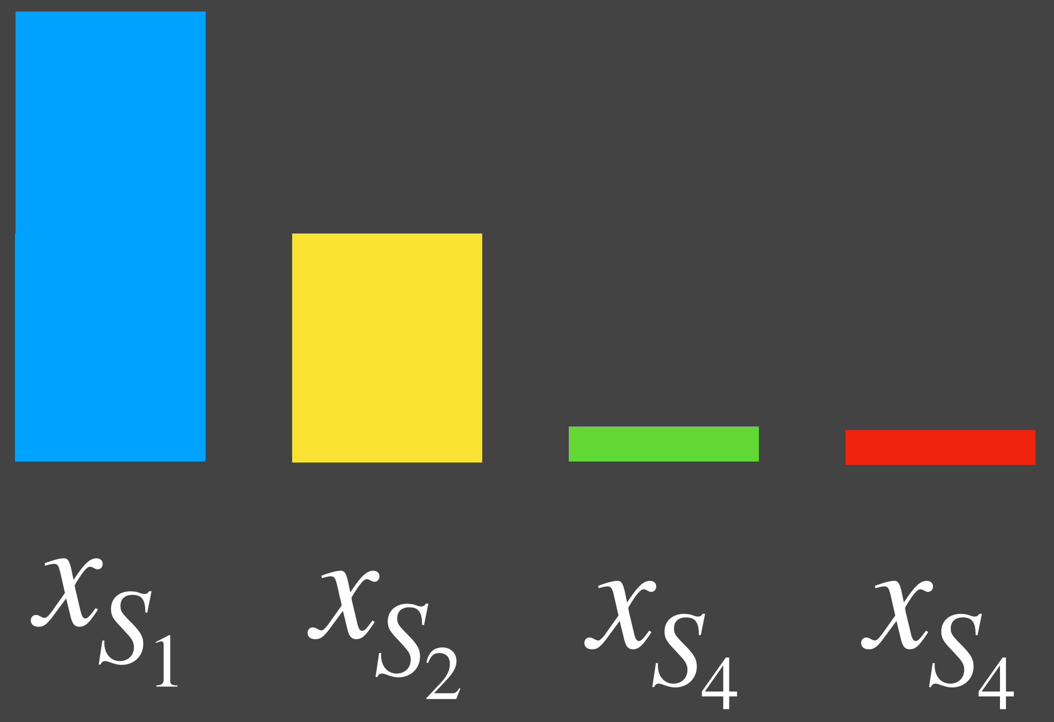

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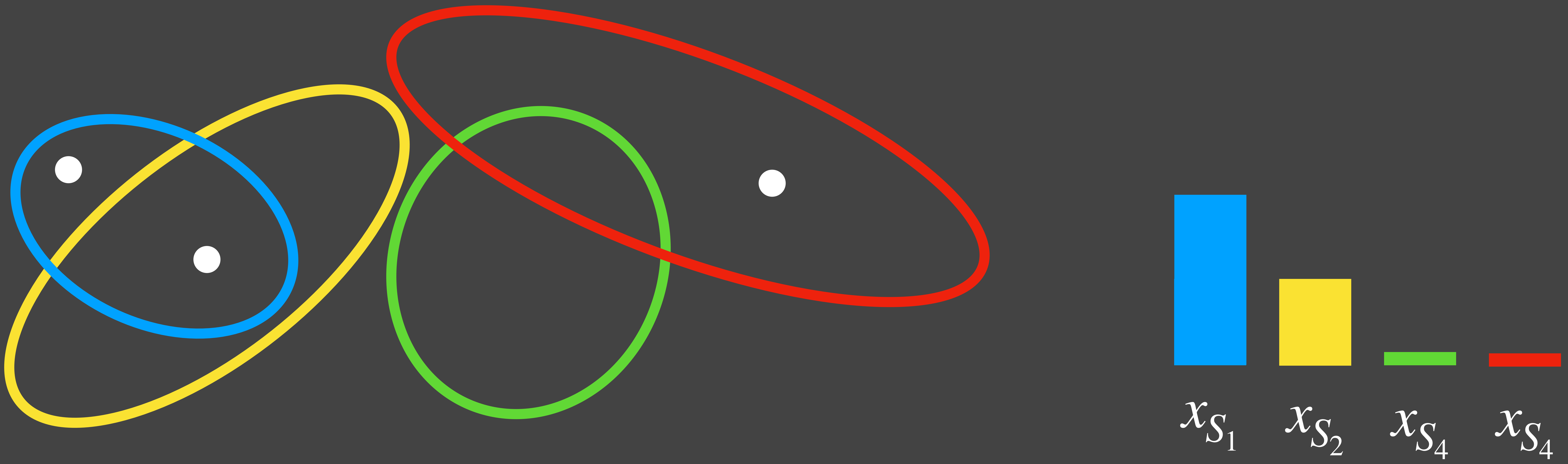

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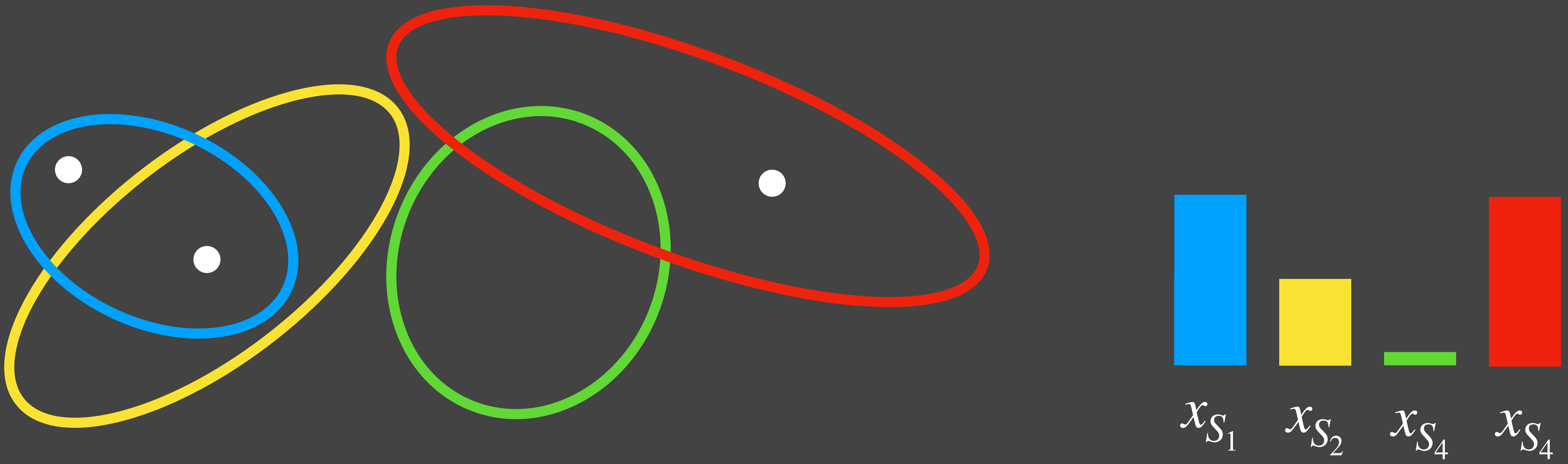

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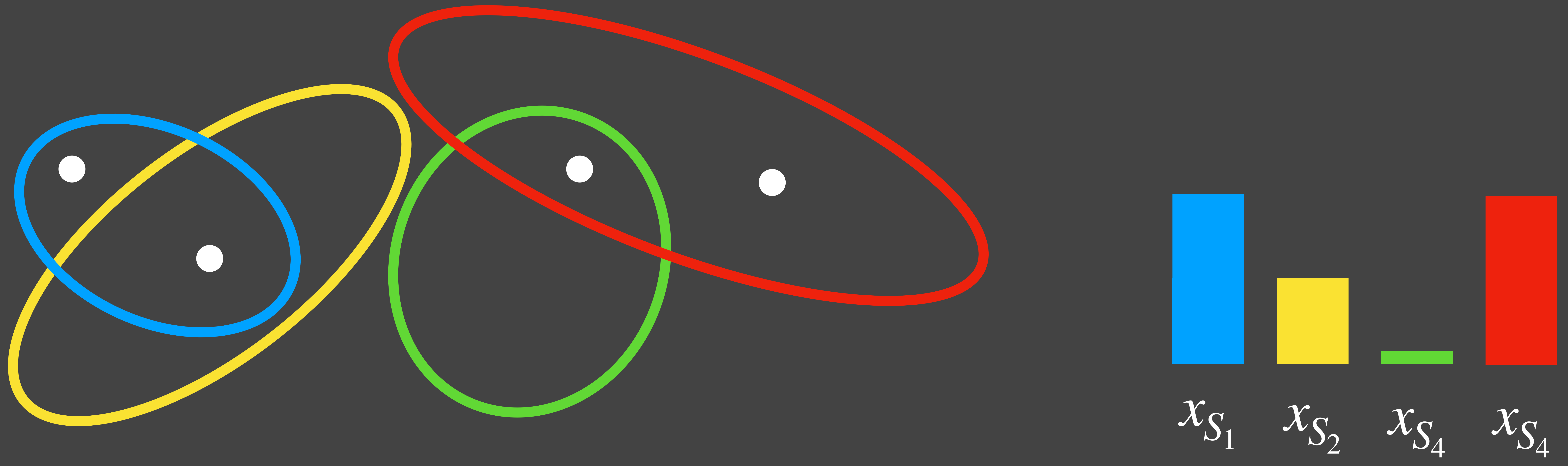

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Claim 3: $y/\log m$ feasible for (**D**).

Neither stage of [Alon+ 03] can be improved!

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New algorithm needed!

We maintain coarse solution x, neither feasible nor monotone,
but round x anyway…

Talk Outline

Intro

➡ Secretary

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

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LearnOrCover

(Unit cost, exp time warmup)

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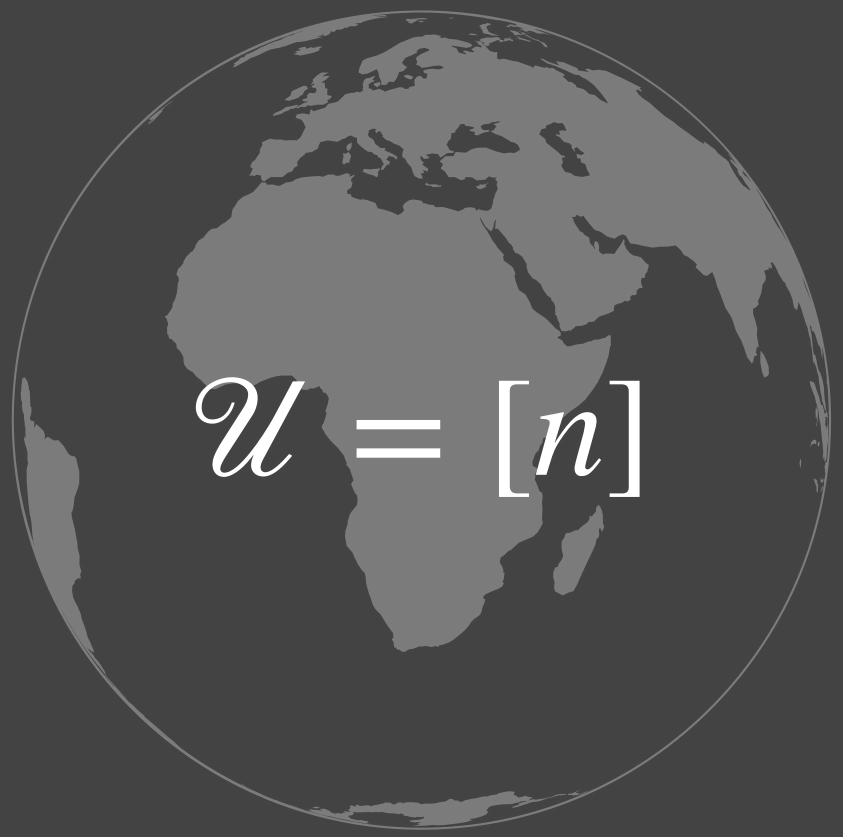
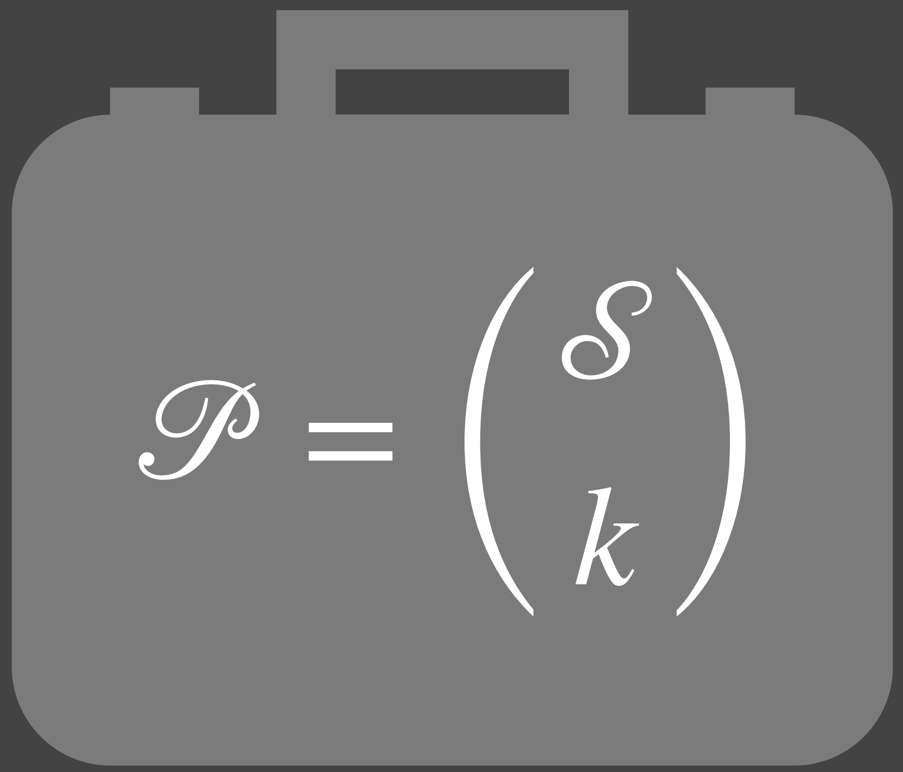
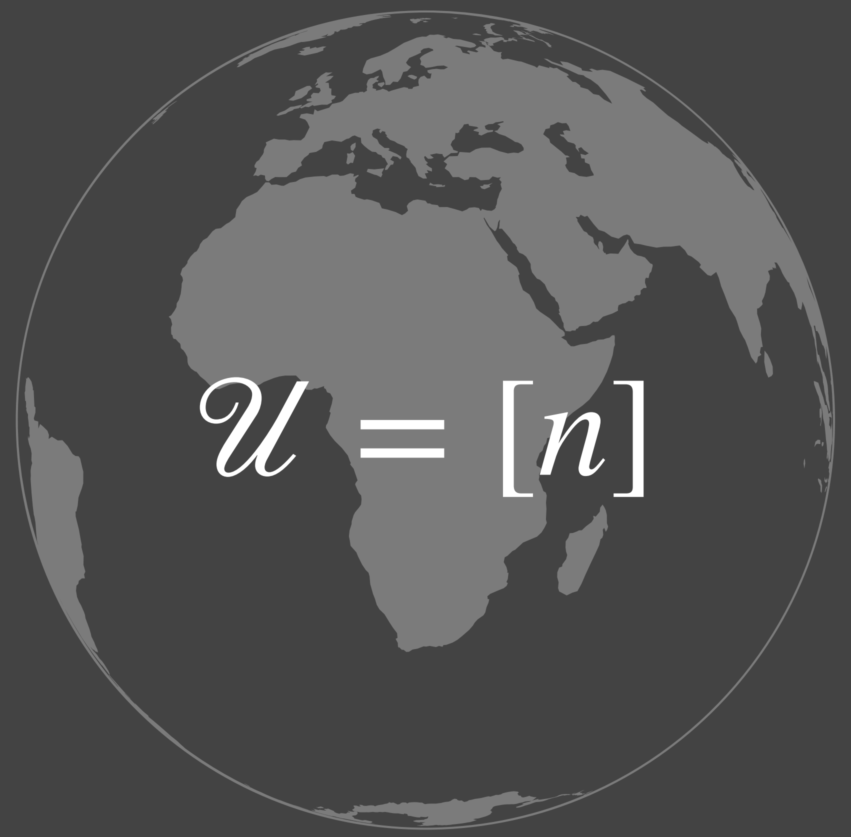


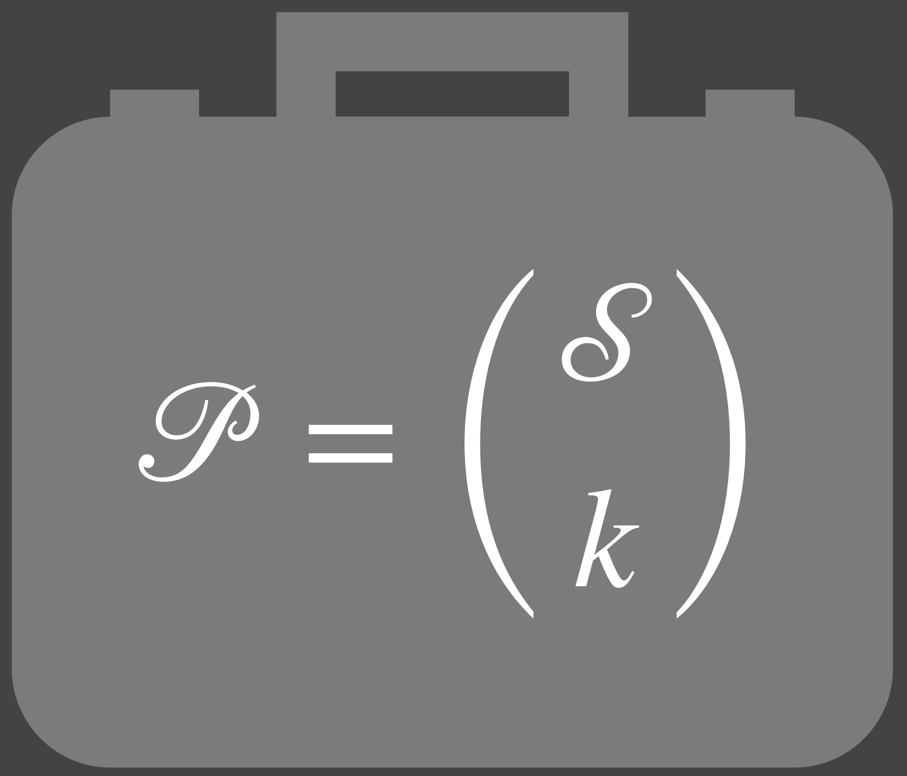


$k := |OPT|$

LearnOrCover
(Unit cost, exp time warmup)



$\mathscr{U} = [n]$



$\mathscr{P} = \binom{\mathscr{S}}{k}$

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@ time t, element v arrives:

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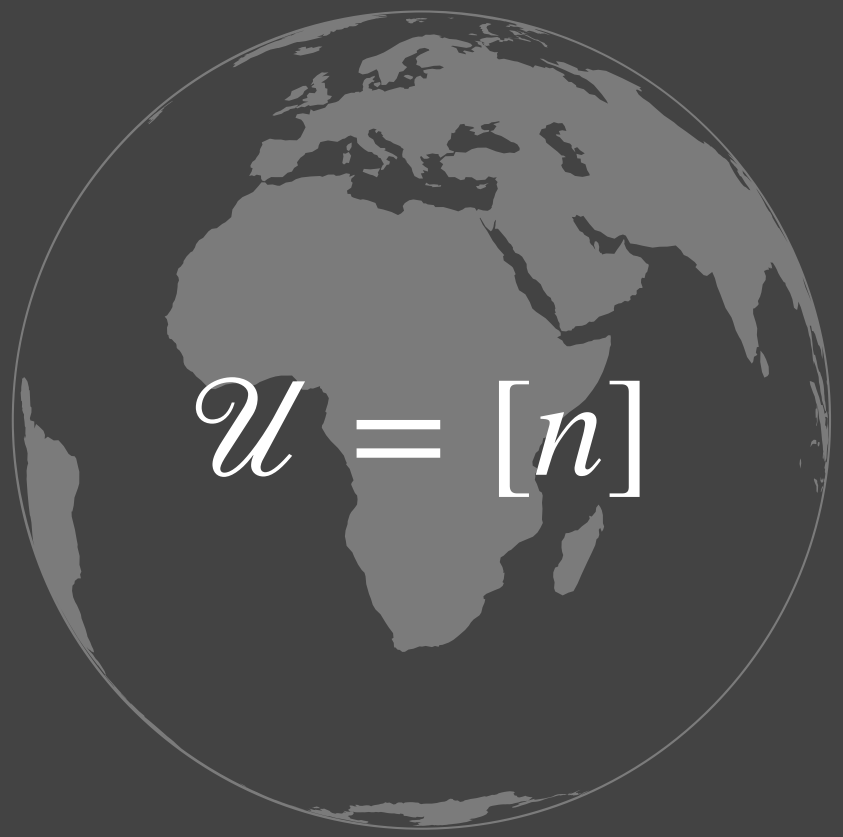

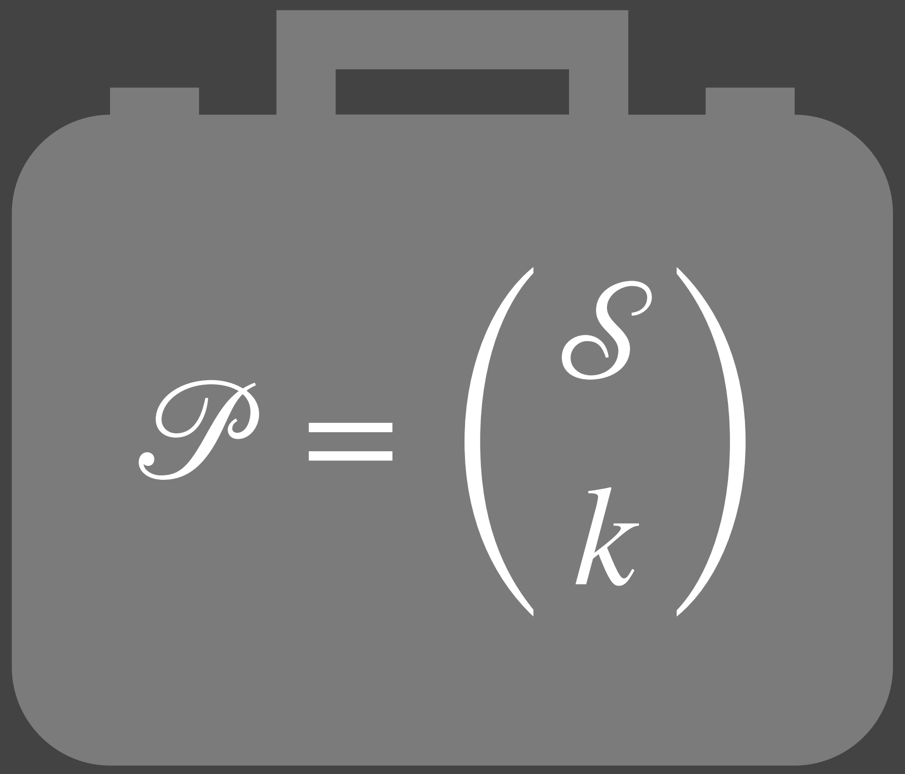

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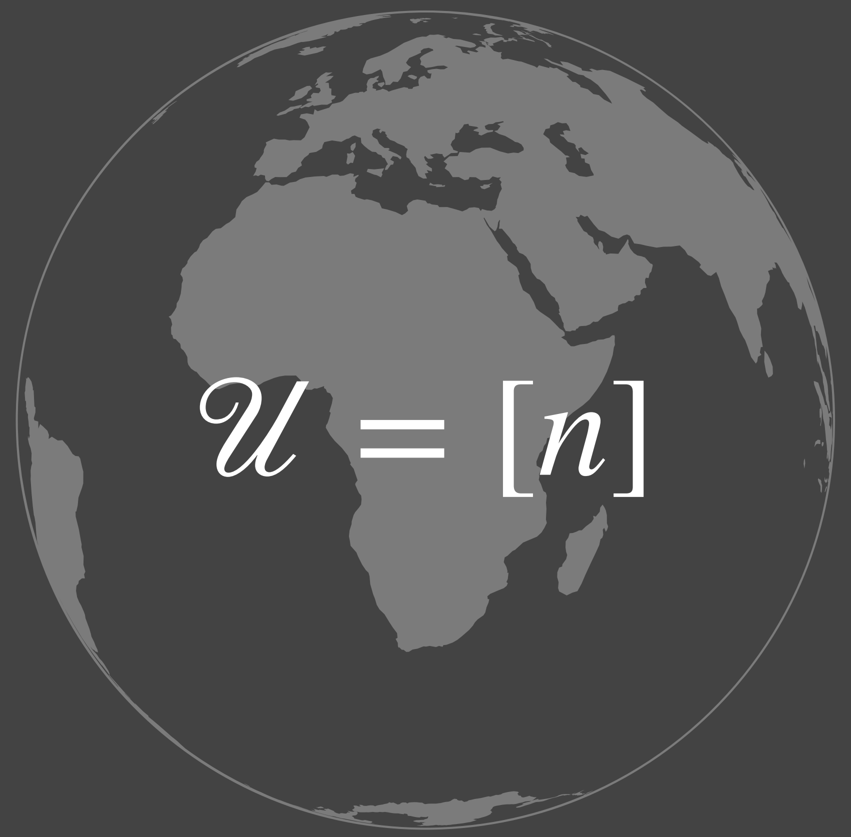

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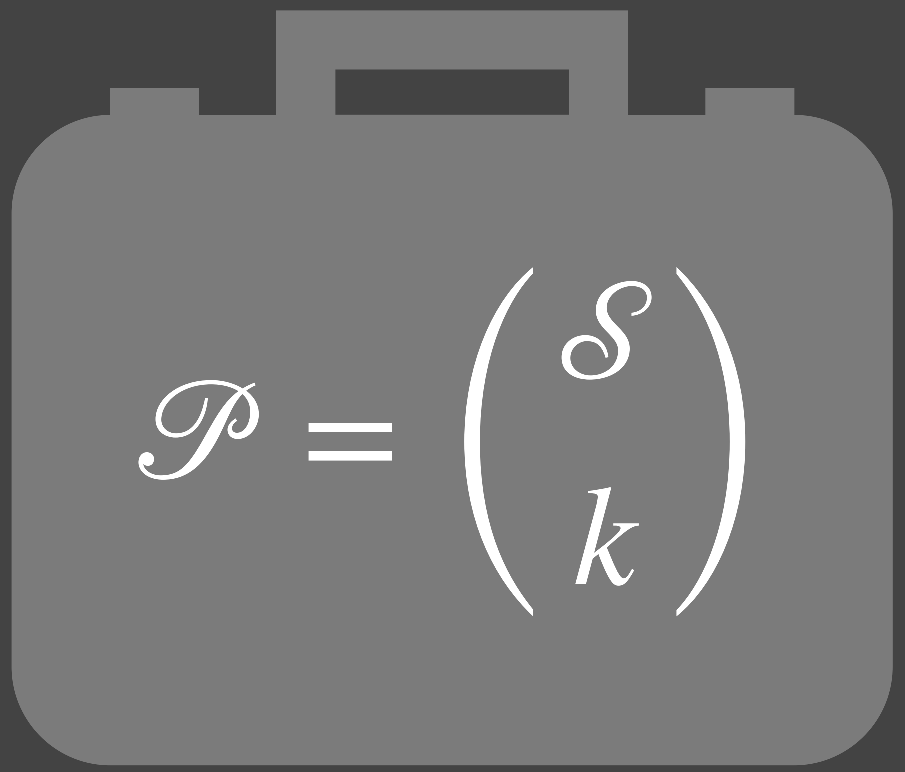

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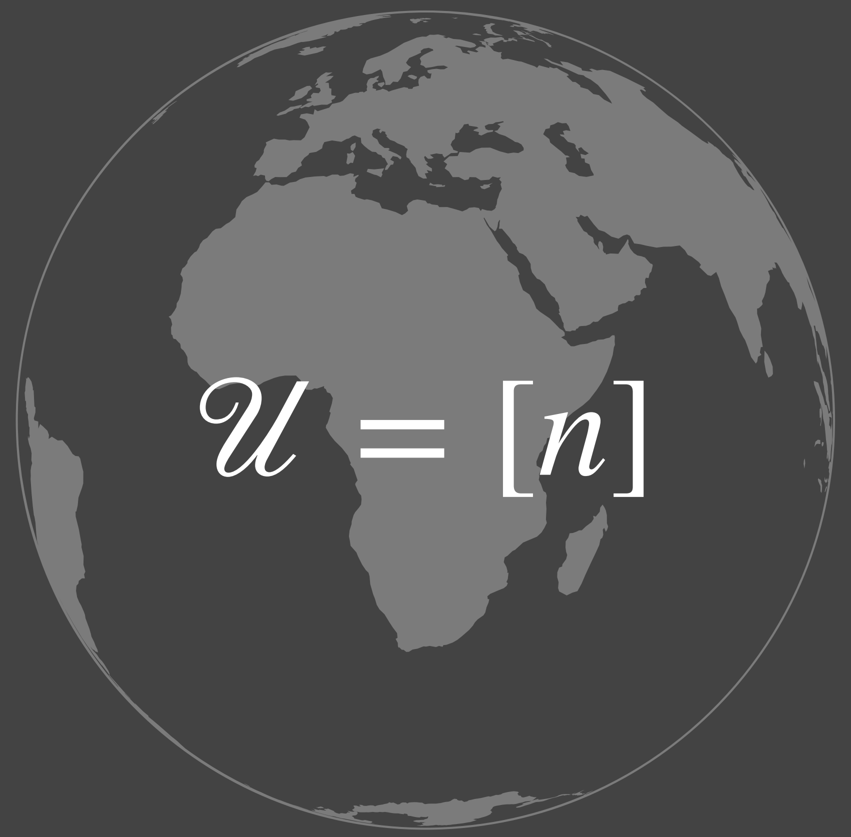

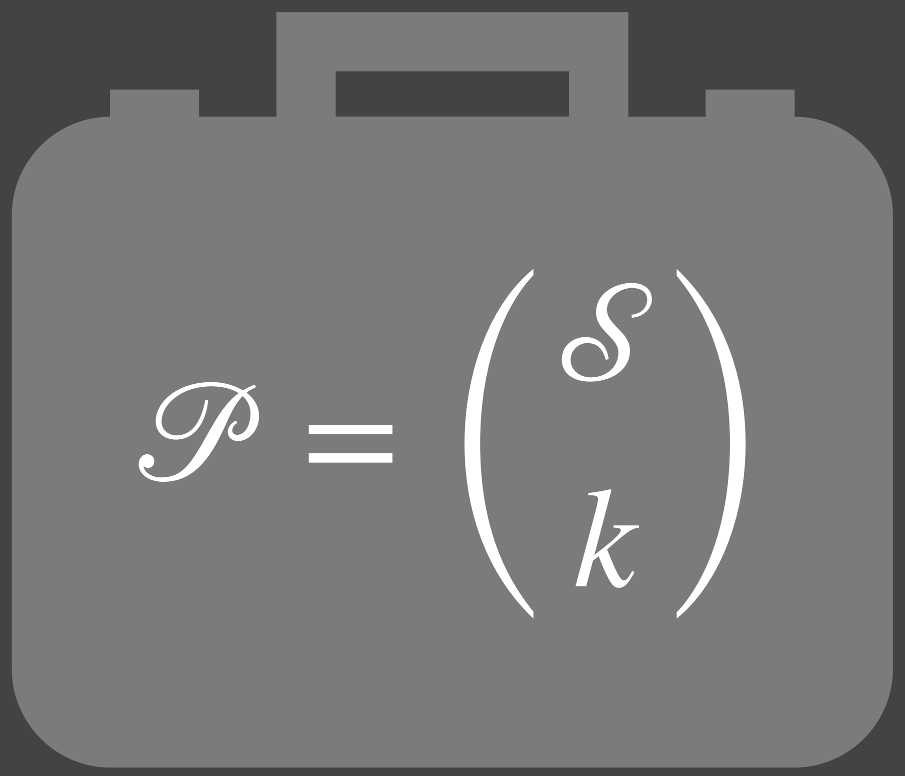

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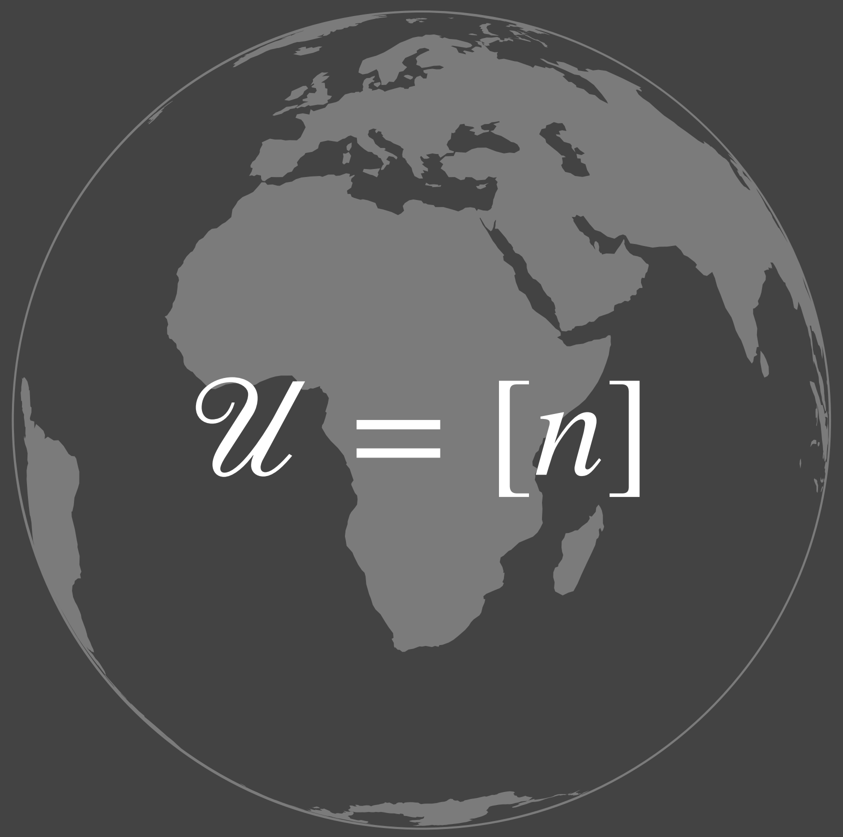

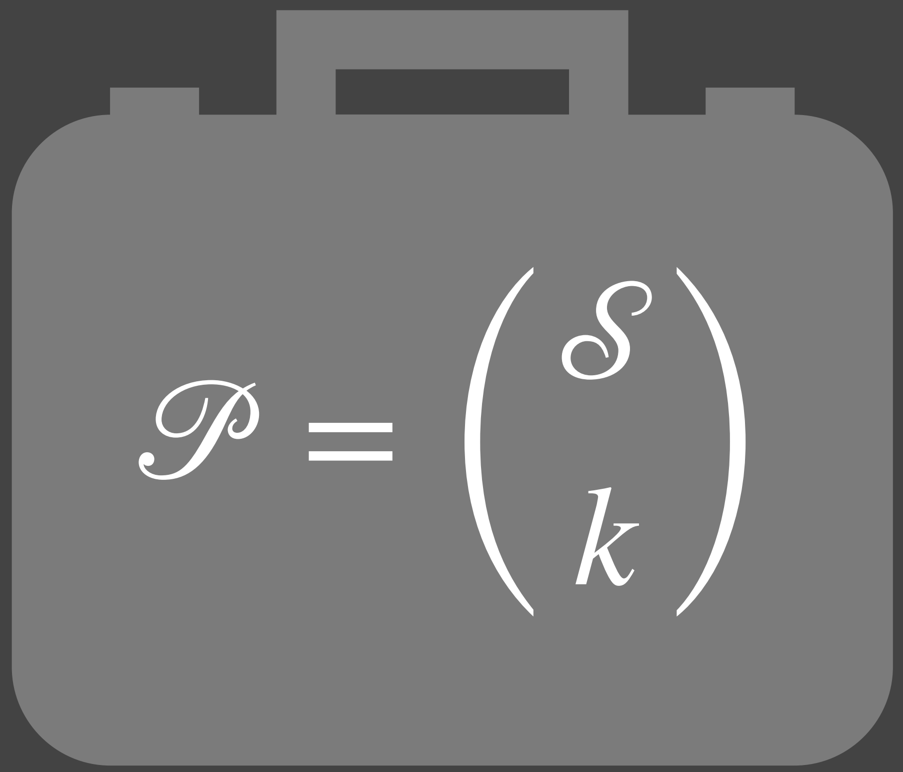

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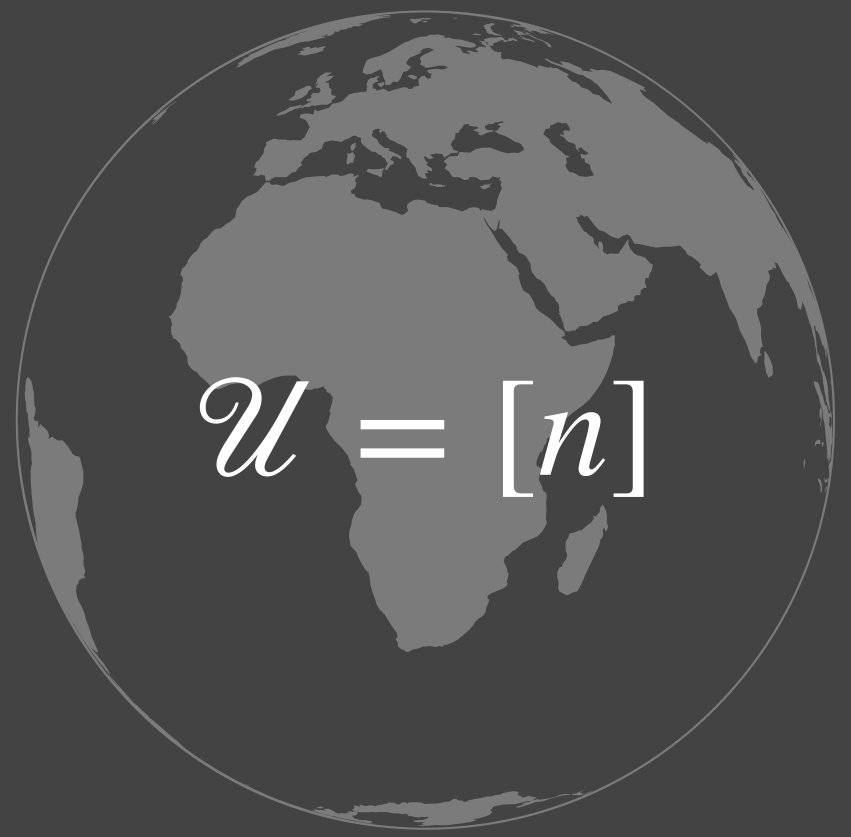
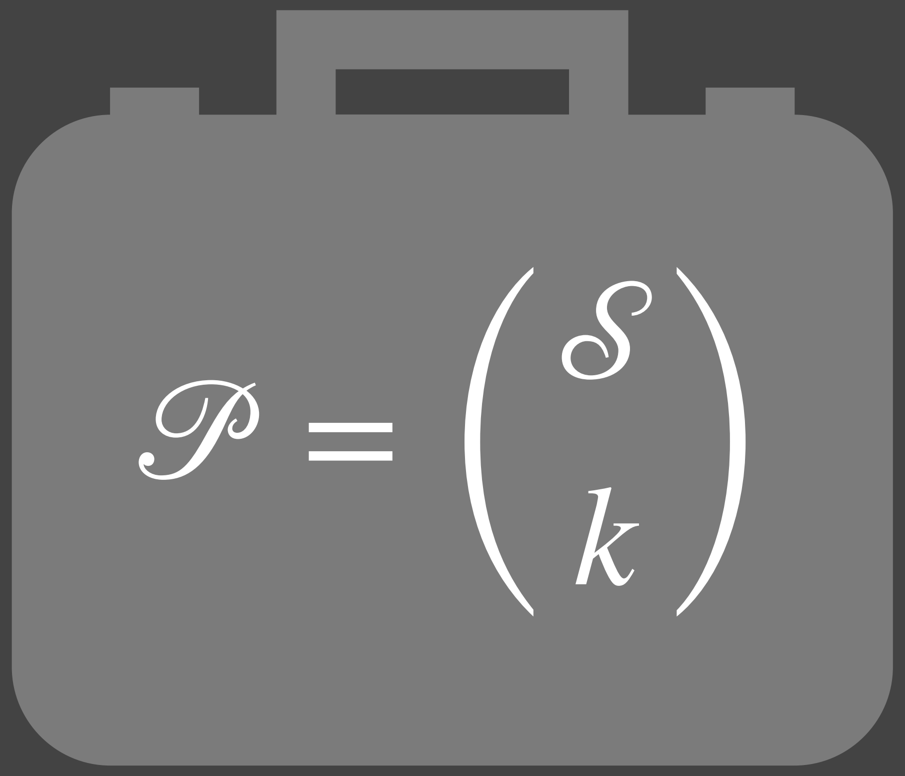
LearnOrCover

(Unit cost, exp time warmup)





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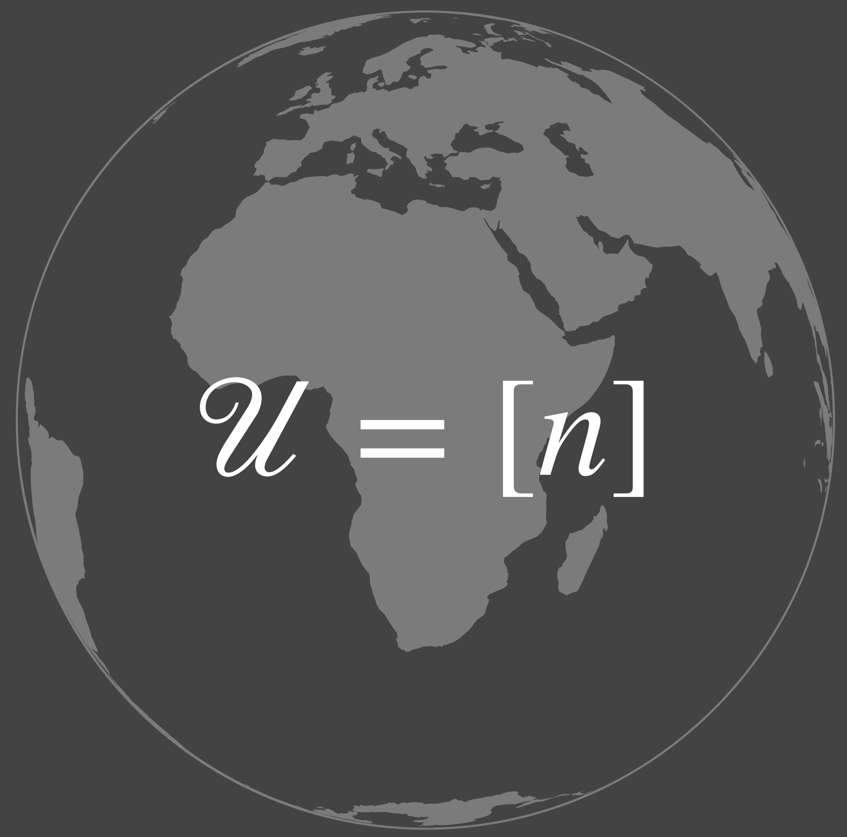

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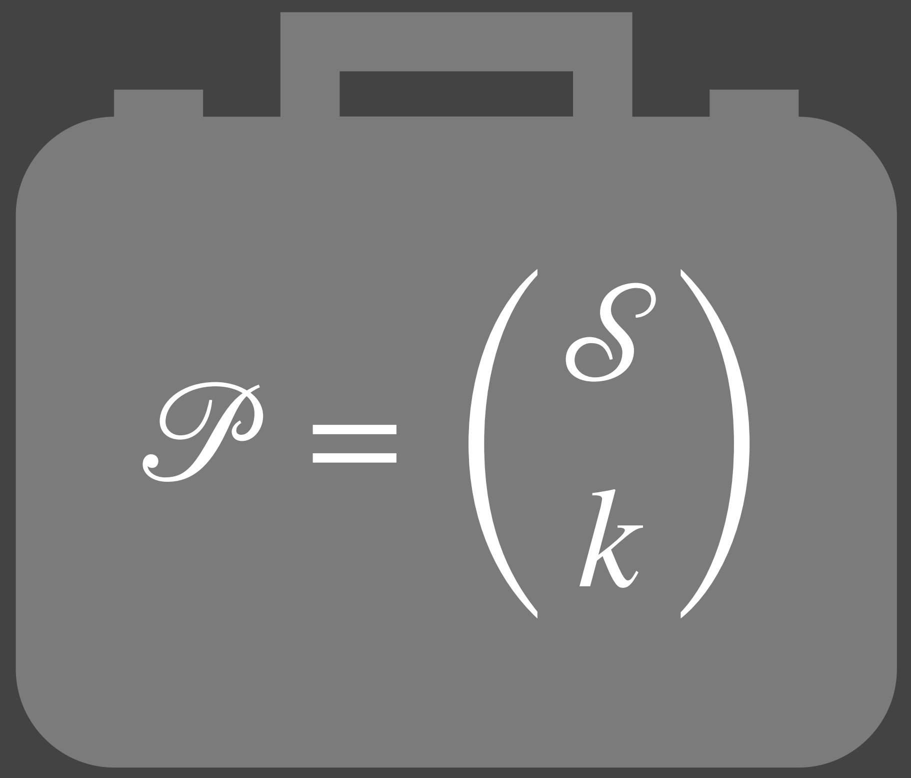

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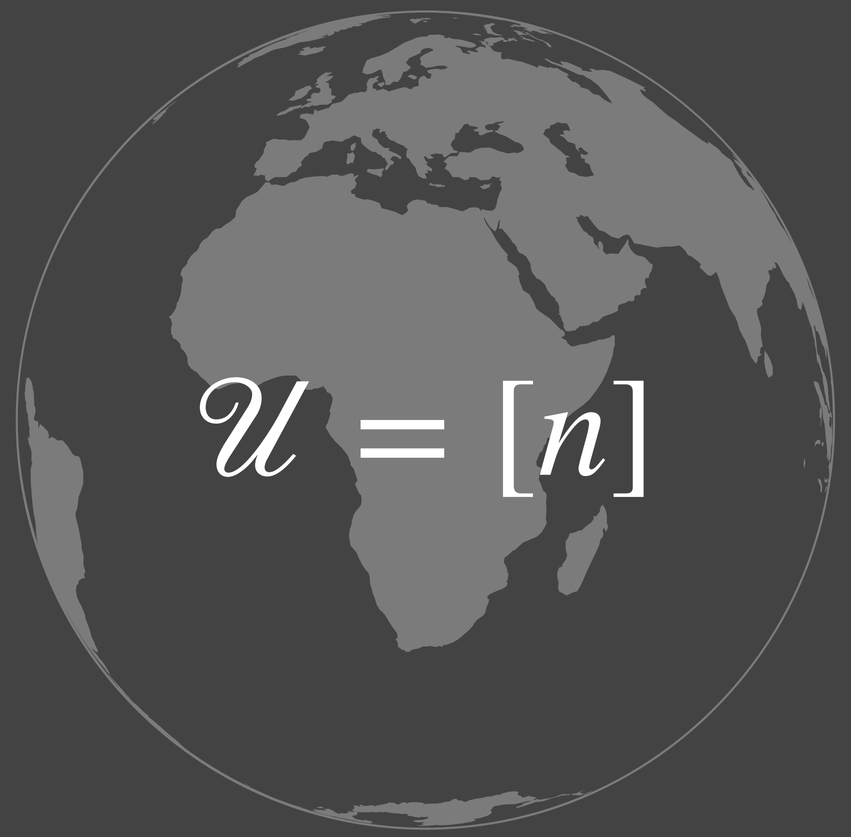

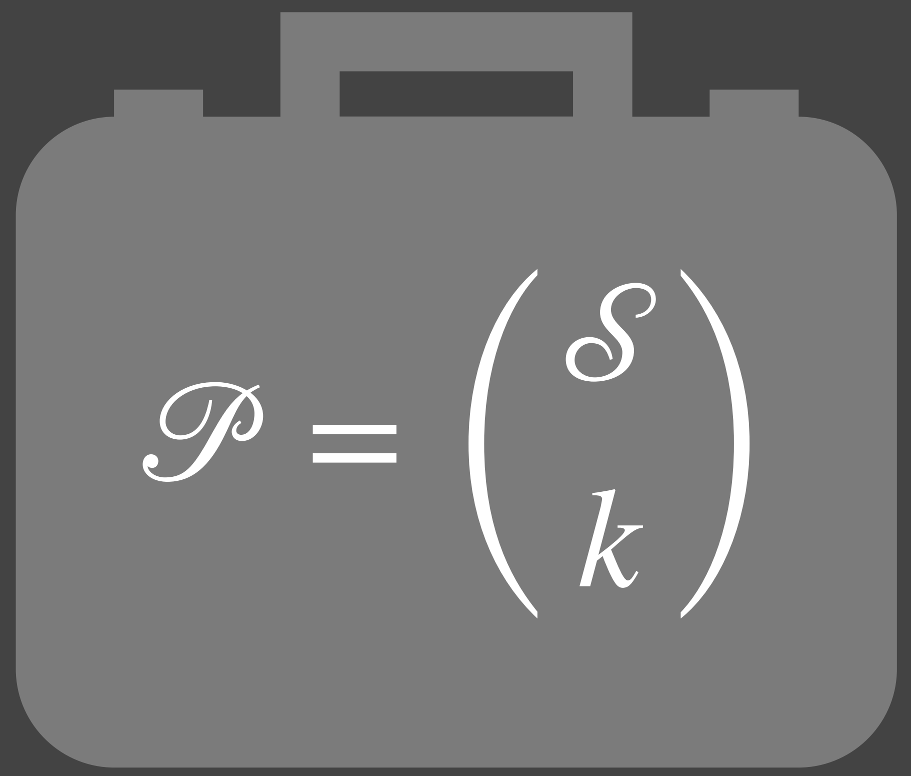

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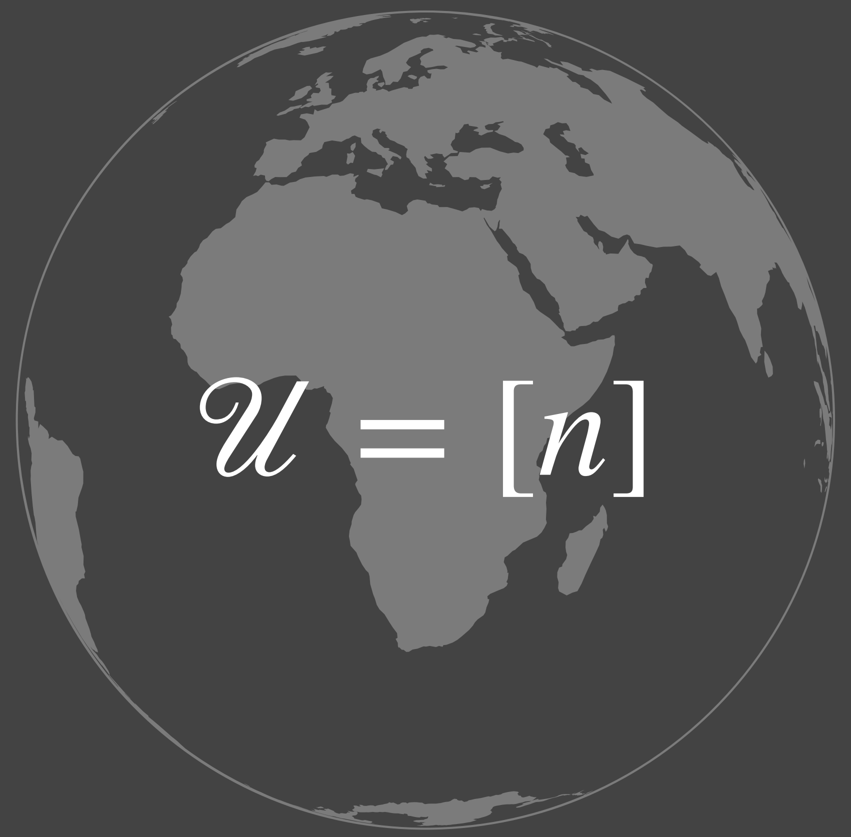

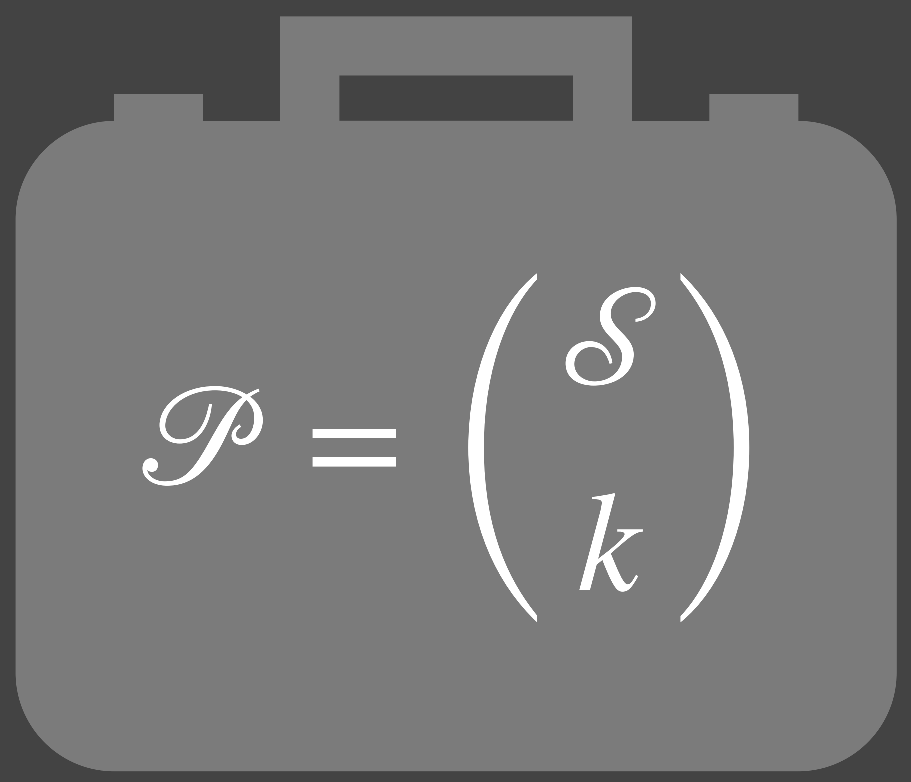

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LearnOrCover

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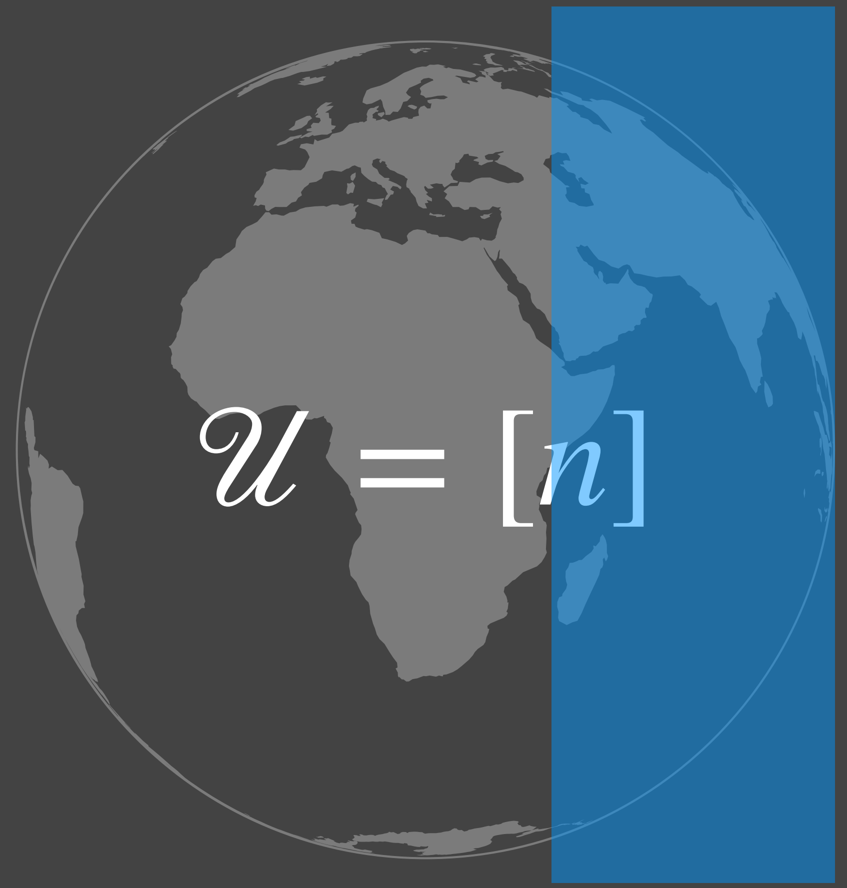

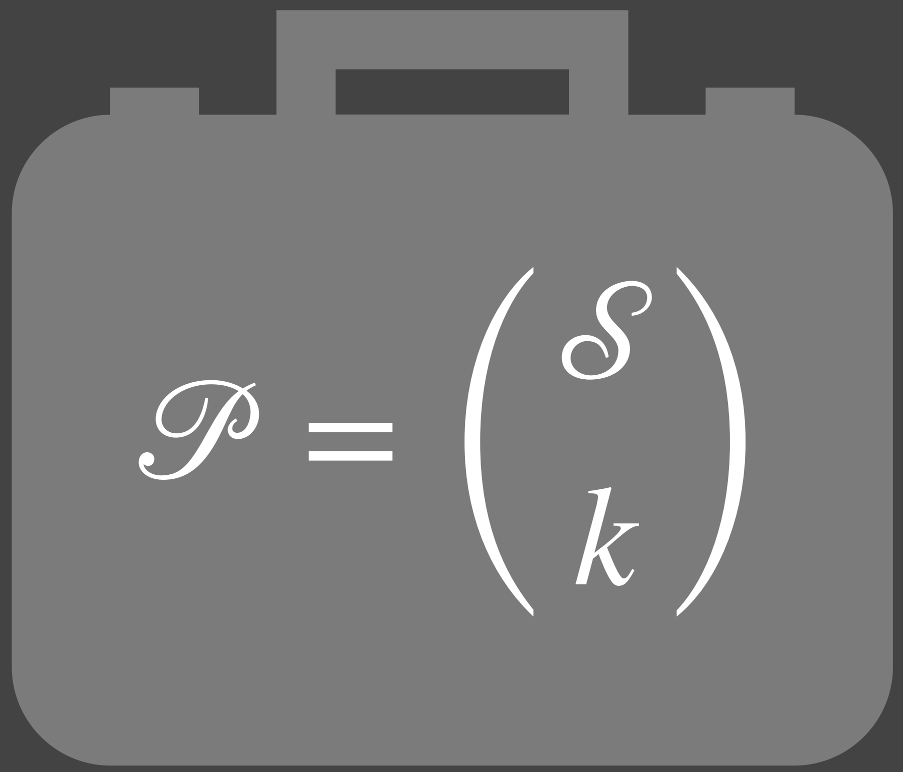

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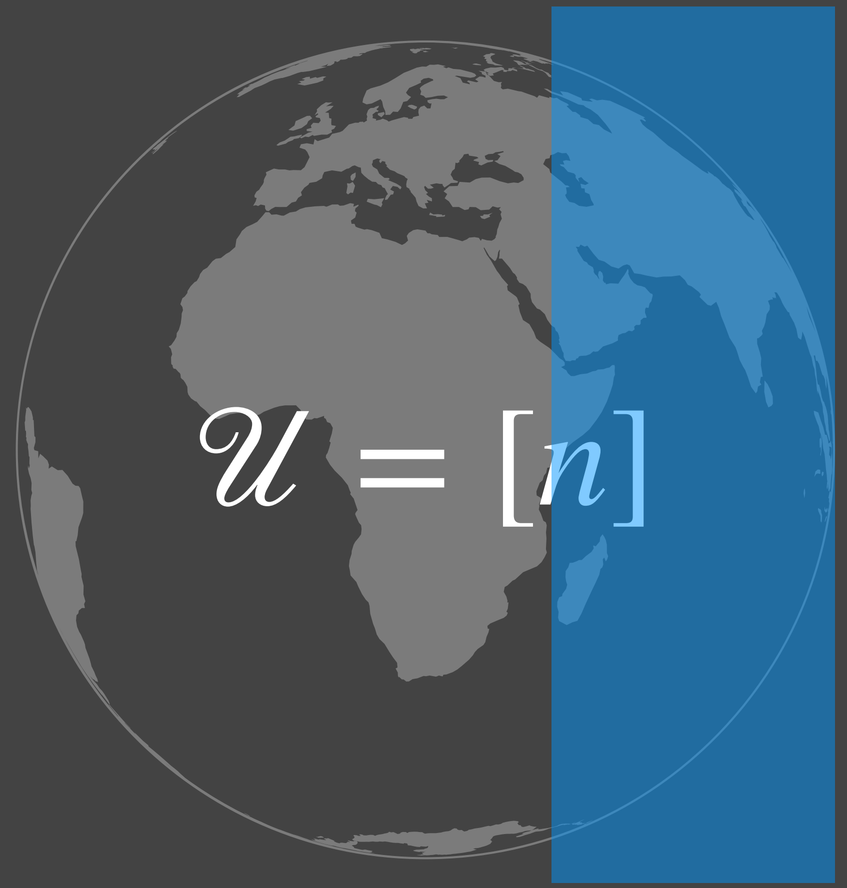
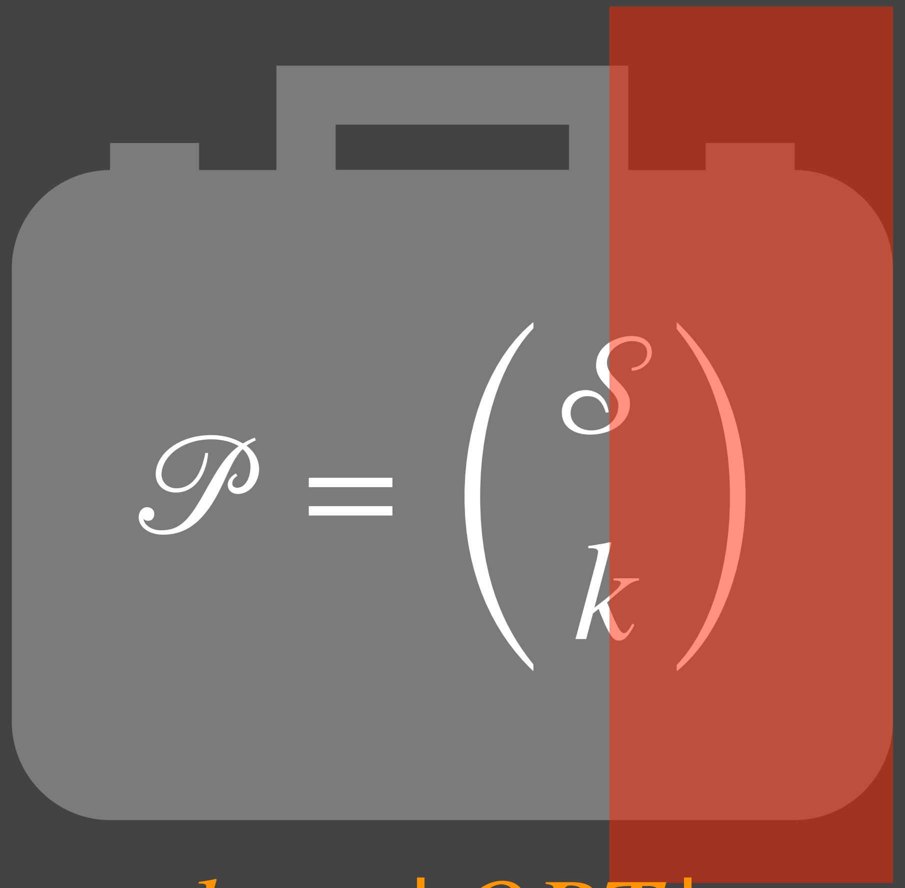
LearnOrCover

(Unit cost, exp time warmup)





$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

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(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.

(II) "Prune" $T \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

Case 1: $\geq 1/2$ of $T \in \mathscr{P}$ cover $\geq 1/2$ of $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

Case 2: $> 1/2$ of $T \in \mathscr{P}$ cover $< 1/2$ of $\mathscr{U}$.

$\geq 1/2$ of $T \in \mathscr{P}$ pruned w.p. $1/2$.

$\mathscr{P}$ shrinks by $3/4$ in expectation.

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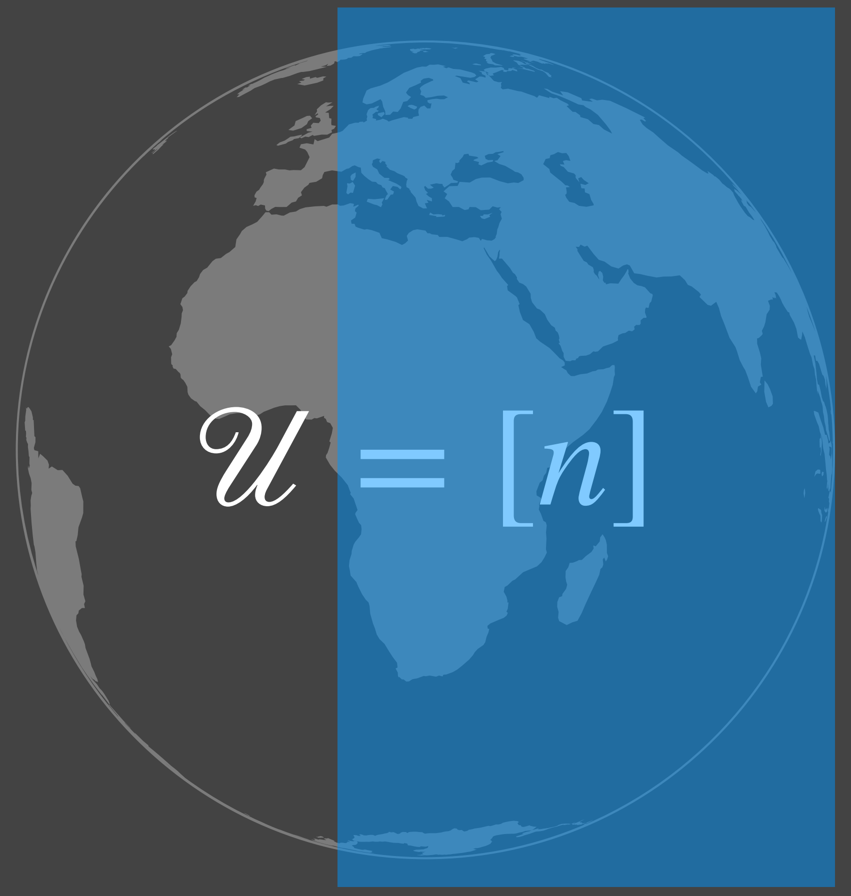

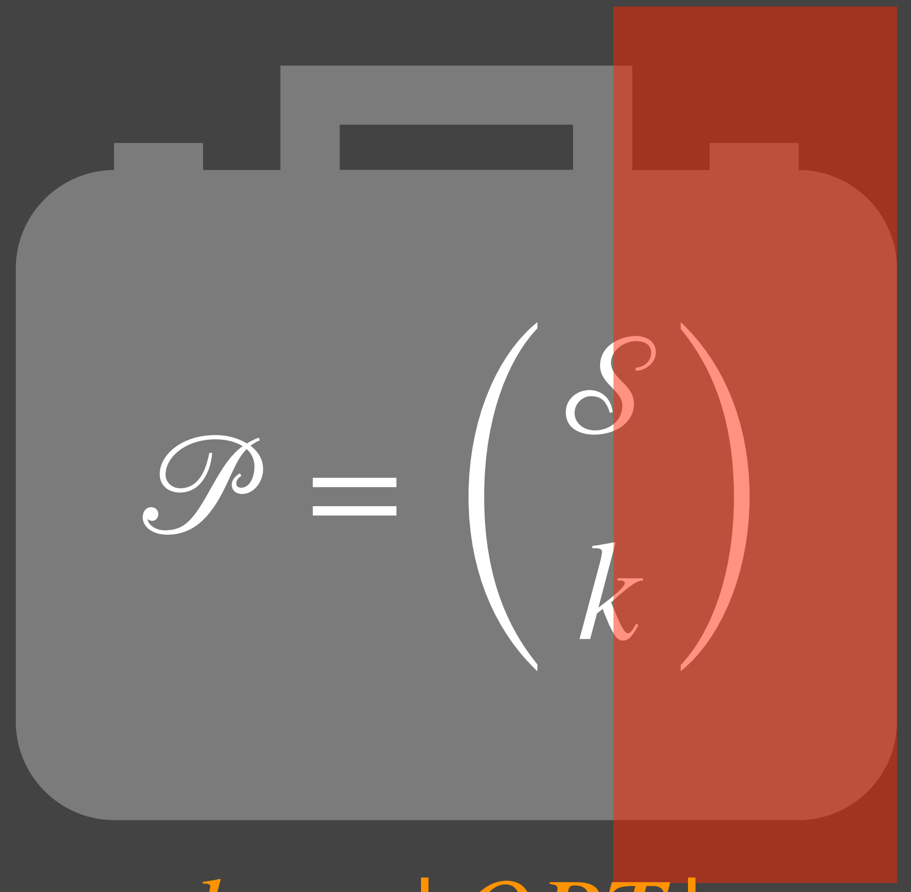

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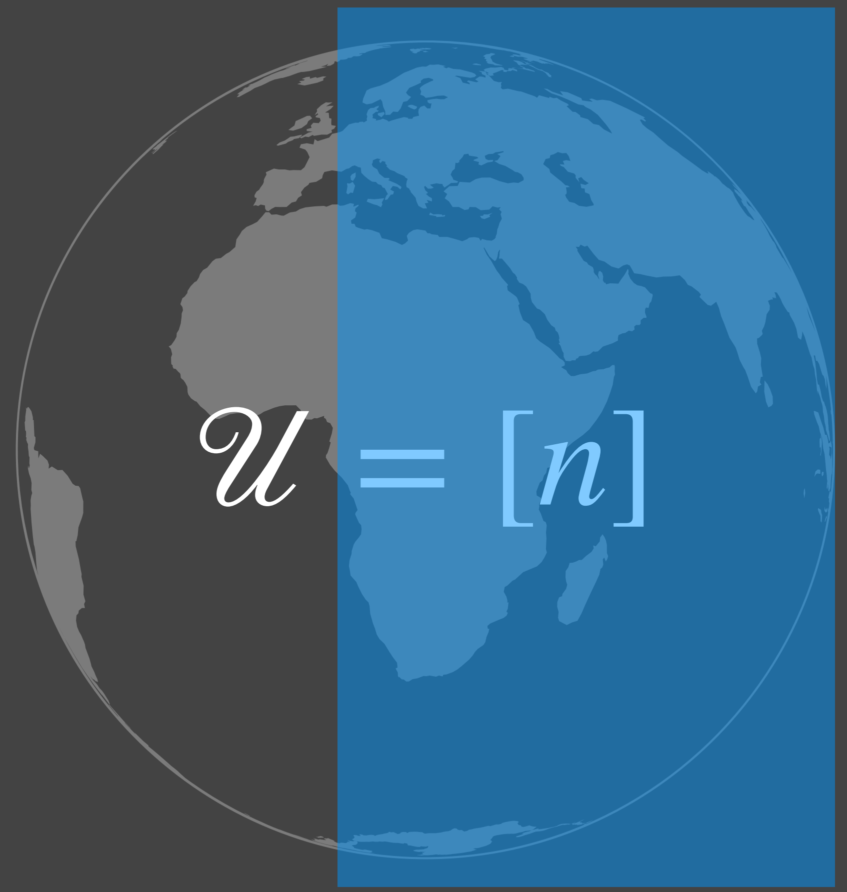

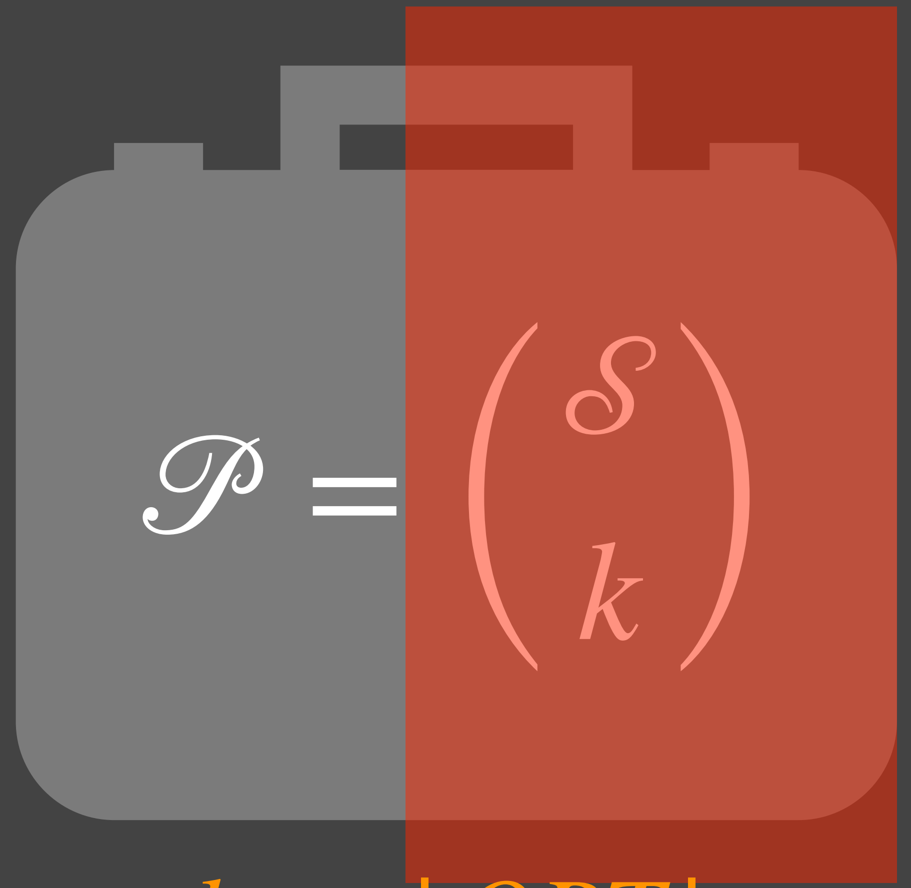

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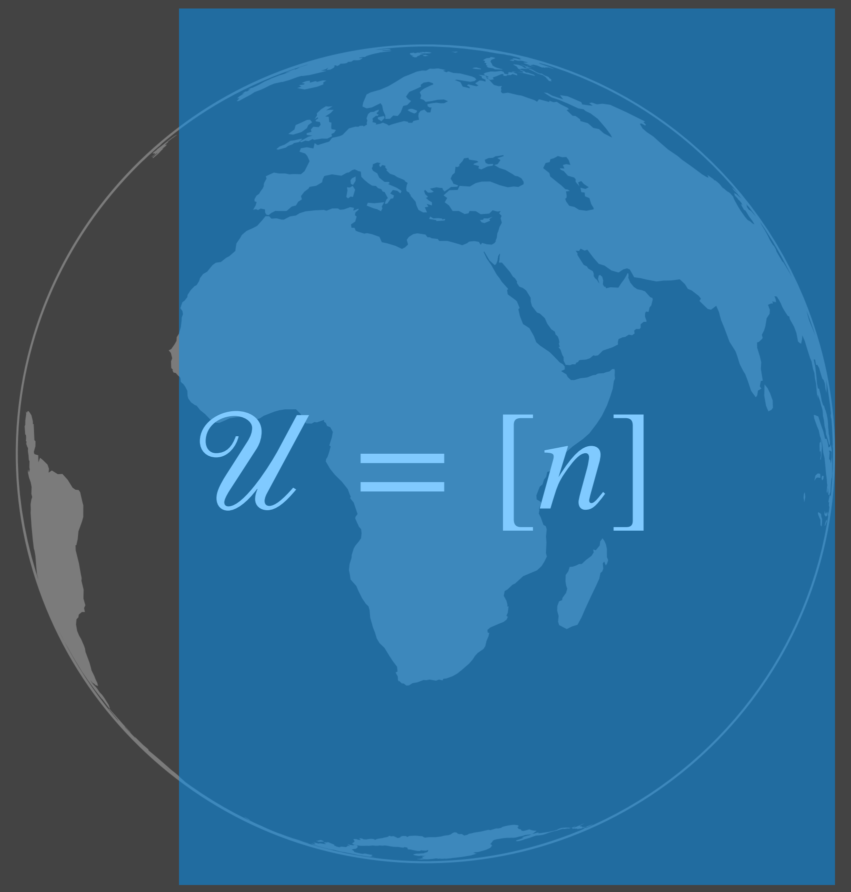

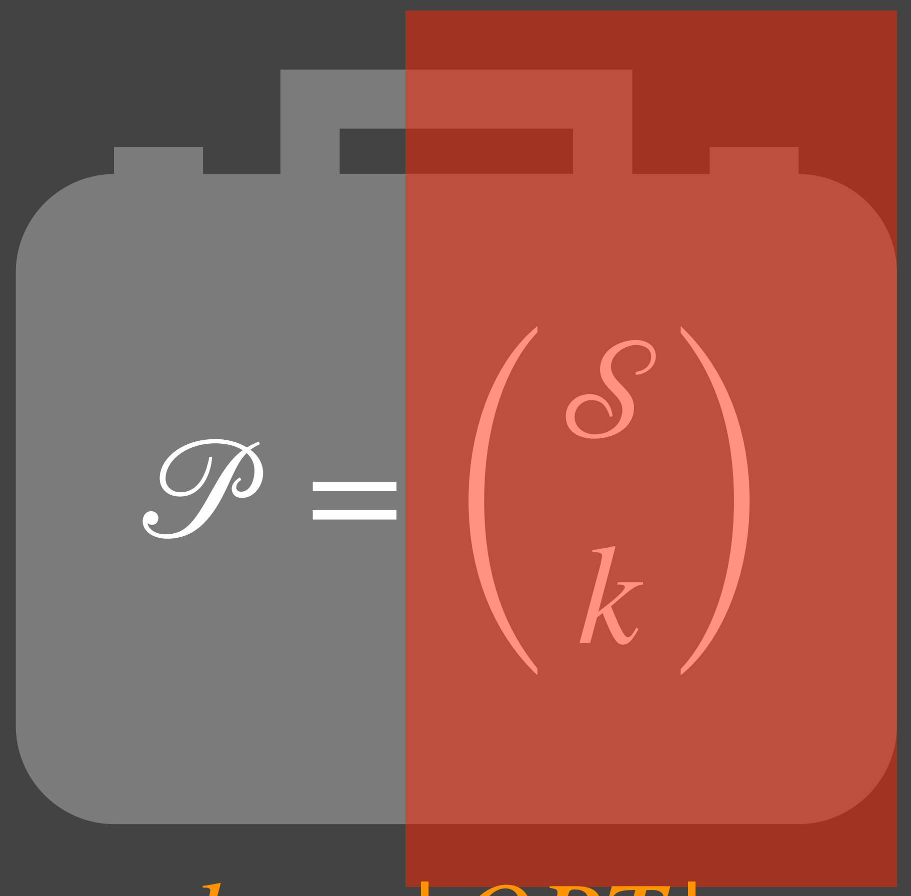

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RO Set Cover

(Exponential Time Warmup)

Case 1: (COVER)

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Case 2: (LEARN)

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But how to make polytime?

Can we reuse LEARN/COVER intuition?

Talk Outline

Intro

Secretary

➡ LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

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Conclusion & Extensions

LearnOrCover
(Unit cost)

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Idea! Measure convergence with potential function:

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$$\boxed{\Phi(t) = c_1\,\mathrm{KL}(x^* \,||\, x^t) + c_2 \log|\mathcal{U}^t|}$$

$\mathcal{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

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(Recall $k = |OPT|$)

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Bound $E_v[\Delta KL]$ over randomness of v.

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Bound $E_v[\Delta KL]$ over randomness of v. $\longleftarrow$ This is where we use RO!

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

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Claim 2b: If v^t uncovered,

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$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1) \cdot E_R[\Delta \log |\mathcal{U}^t|] \le -\frac{1}{k}$$

Claim 2a: If v^t uncovered,

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$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1) \cdot E_R[\Delta \log |\mathcal{U}^t|] \le -\frac{1}{k}$$

Since $\Phi(0) = O(\log(mn))$, expected total cost is $k \log(mn)$.

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Proof:

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

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Proof:

$$\mathbf{KL}(x^* \,||\, x^t) - \mathbf{KL}(x^* \,||\, x^{t-1})$$

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$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

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$$E_v[\Delta \mathrm{KL}] \le (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

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$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S \ni v} (e-1) \cdot x_S^{t-1}\right) - \underbrace{\sum_{S \ni v} x_S^*}_{\ge 1/k}$$

$$\le \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \le z$, take expectation over v, ■.

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \le -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

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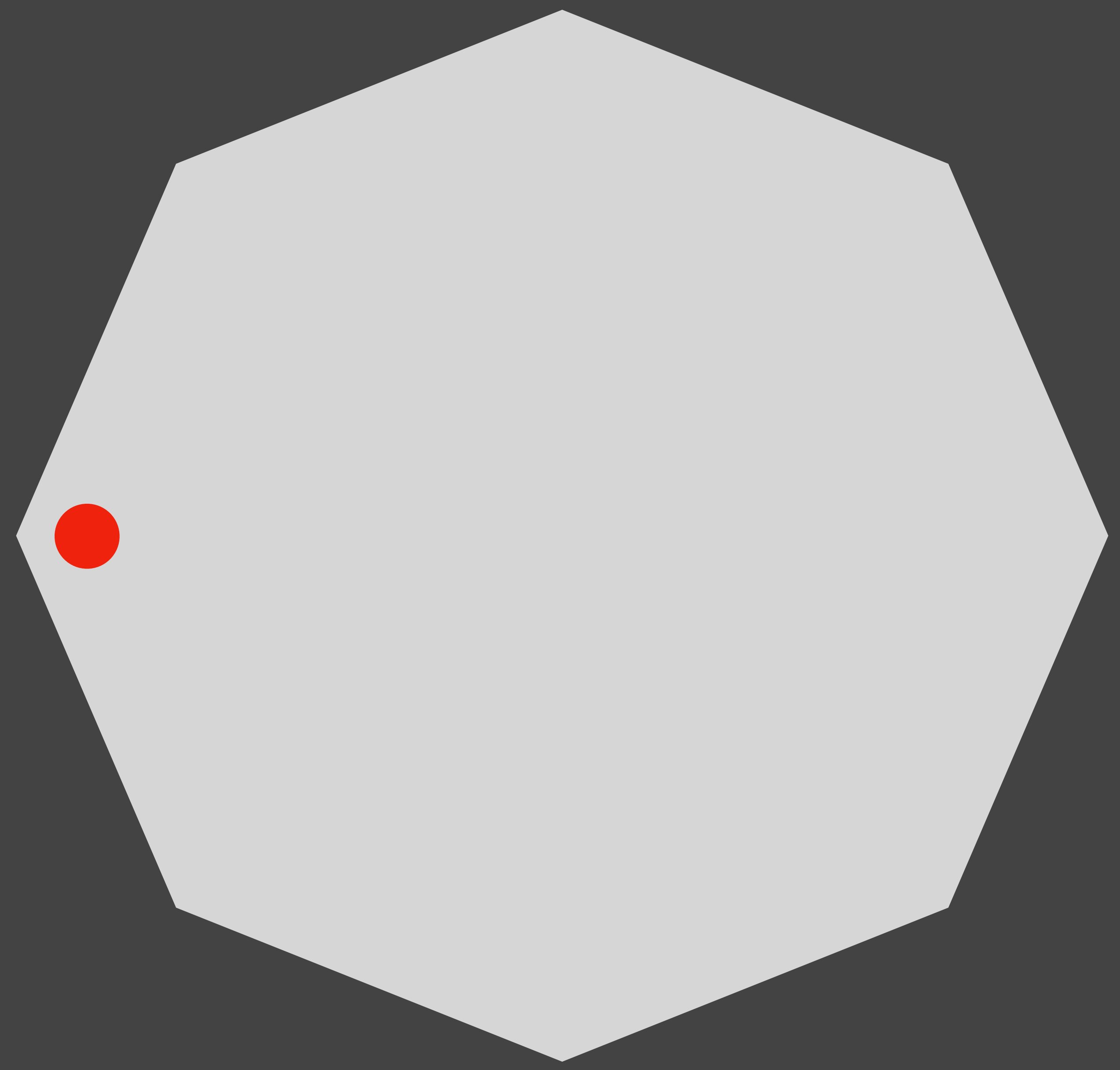
Perspective 1:



[Alon+ 03]
[Buchbinder Gupta Molinaro Naor 19]

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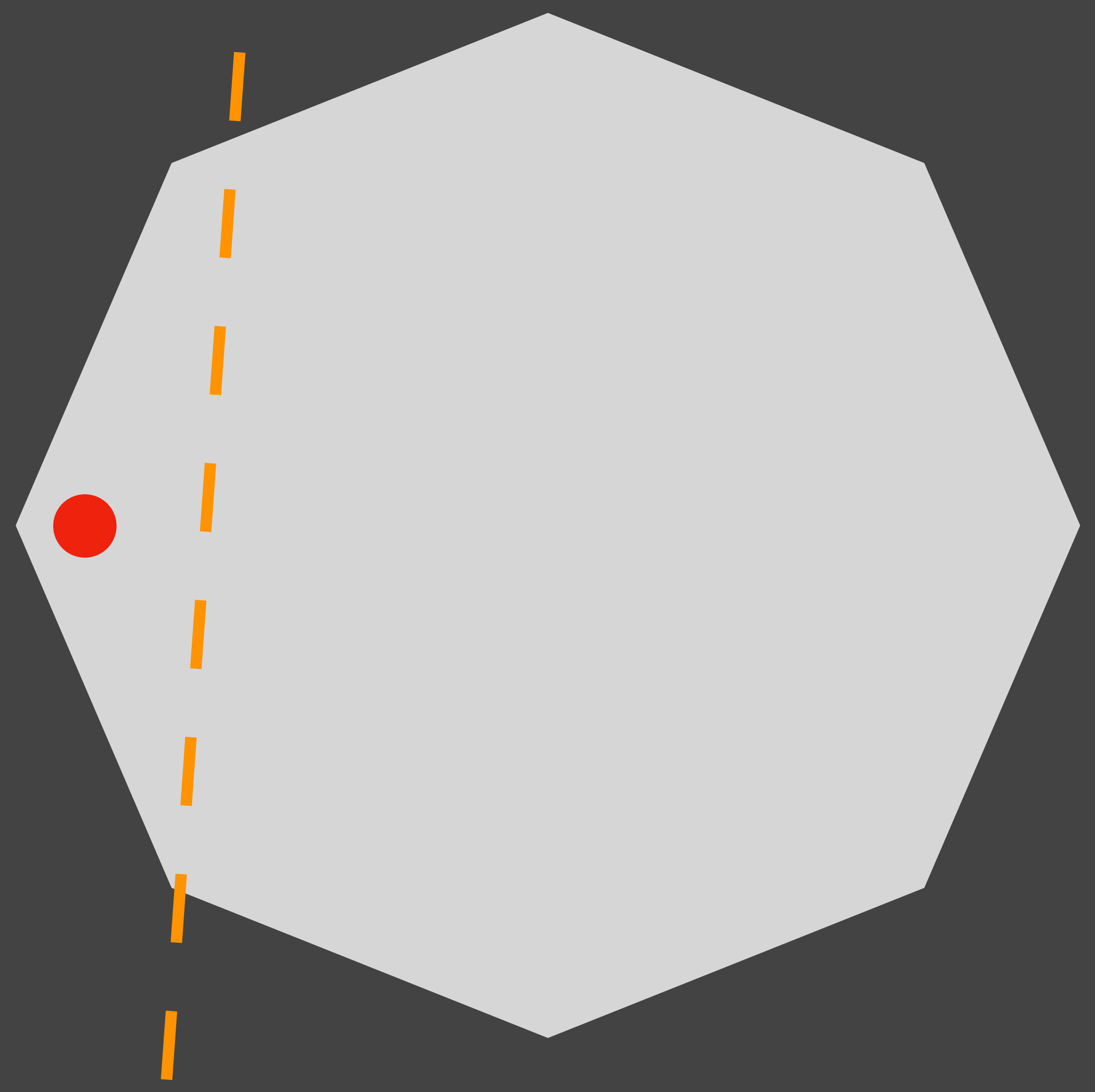

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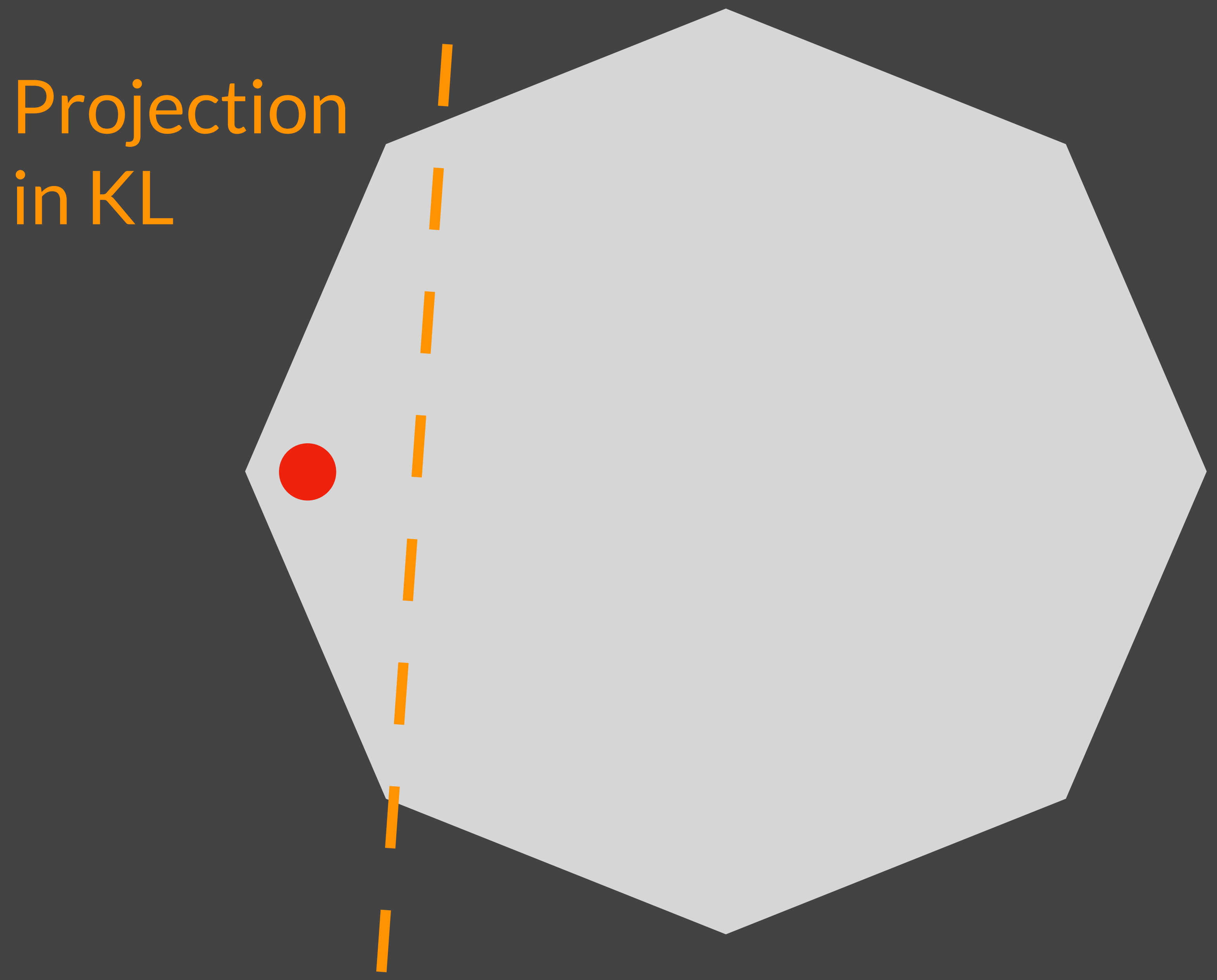

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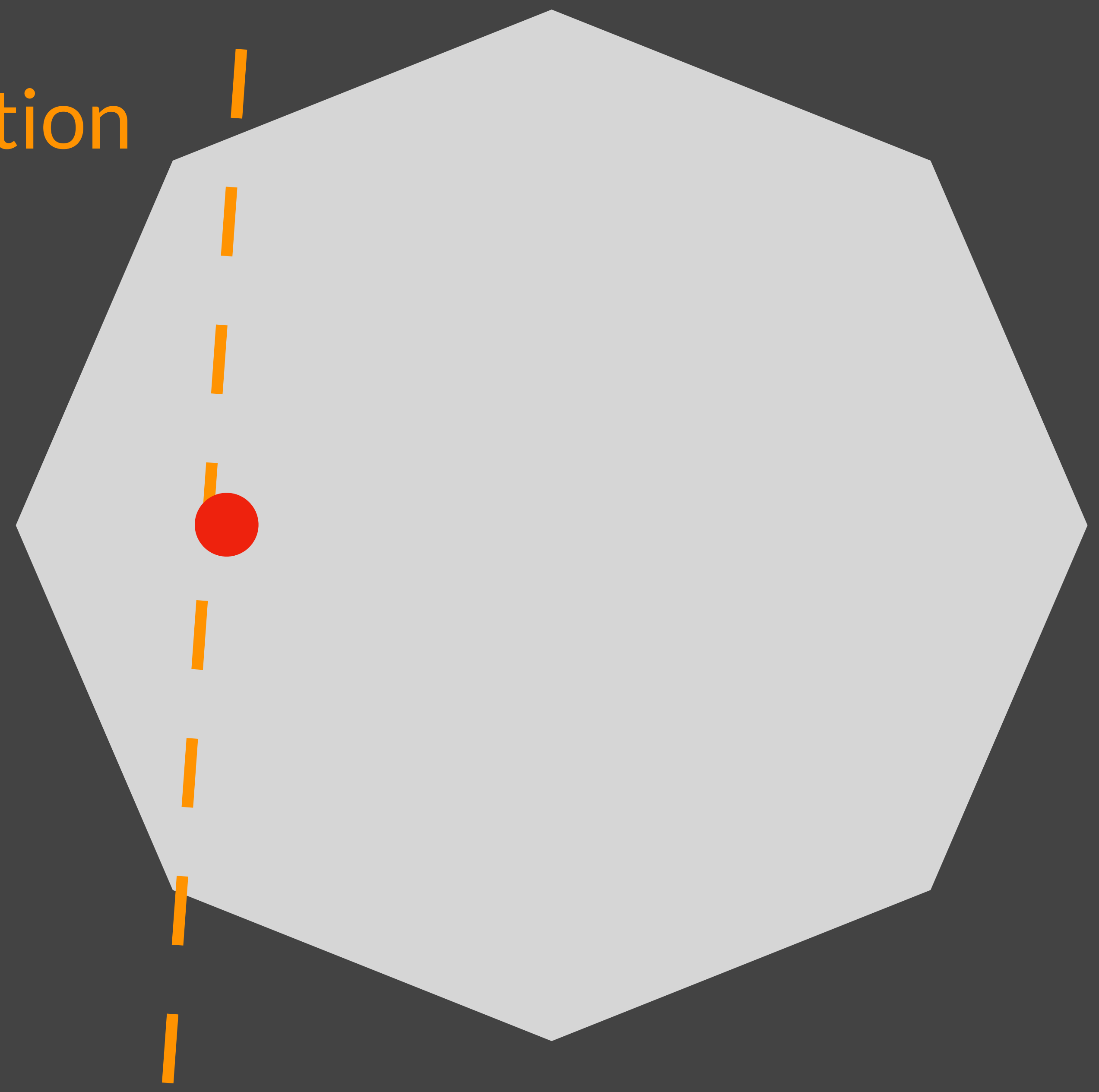

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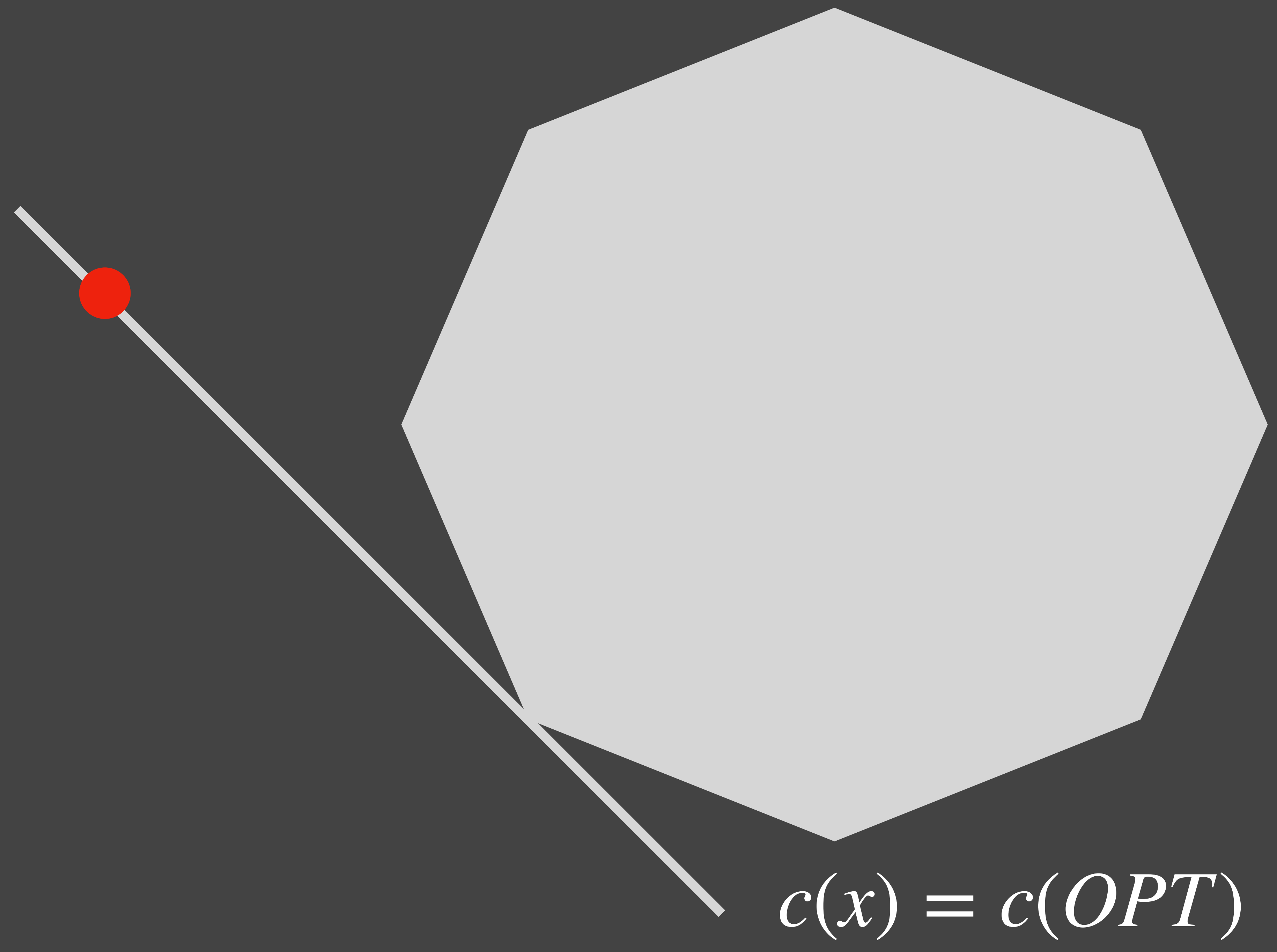

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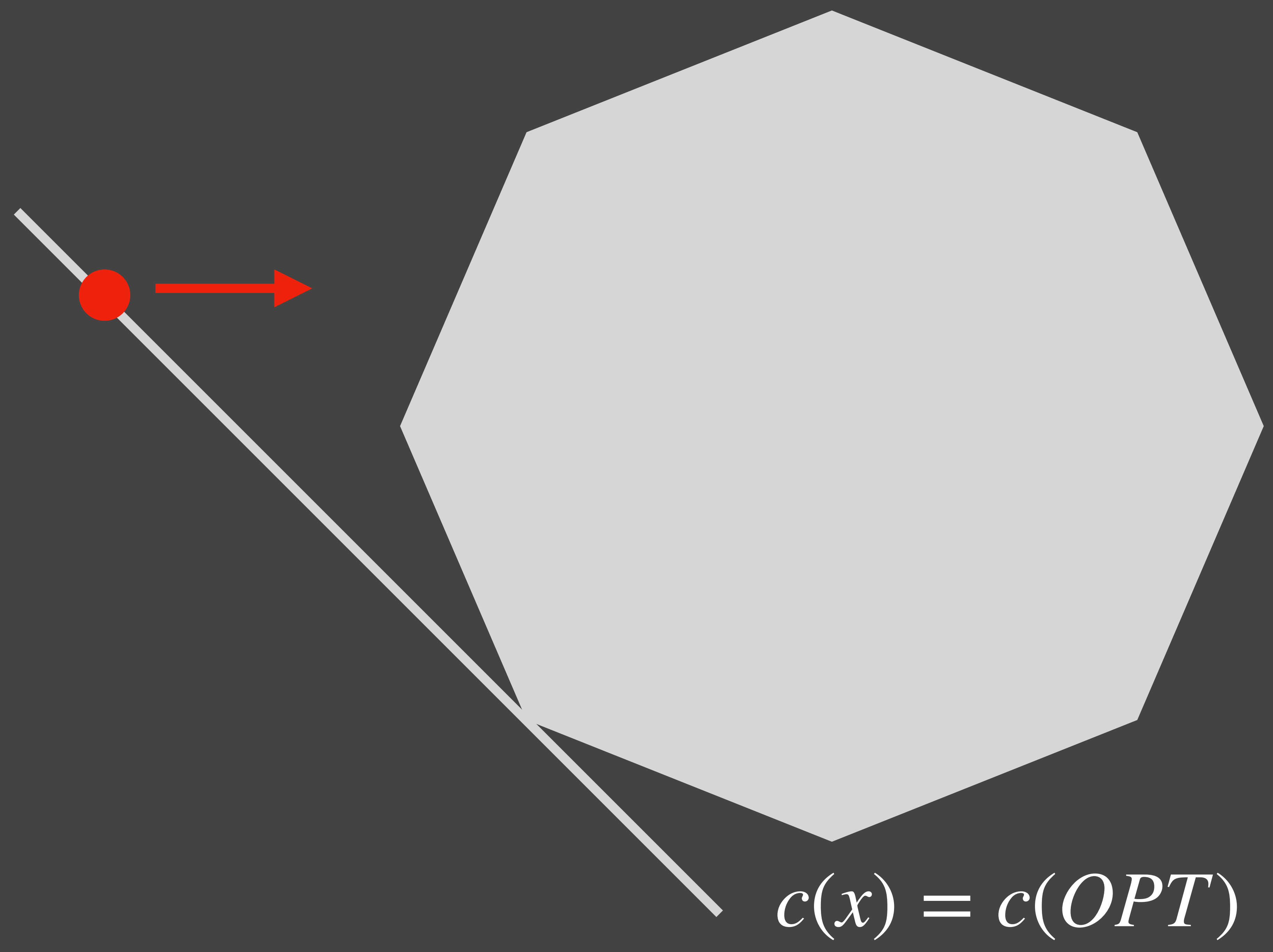

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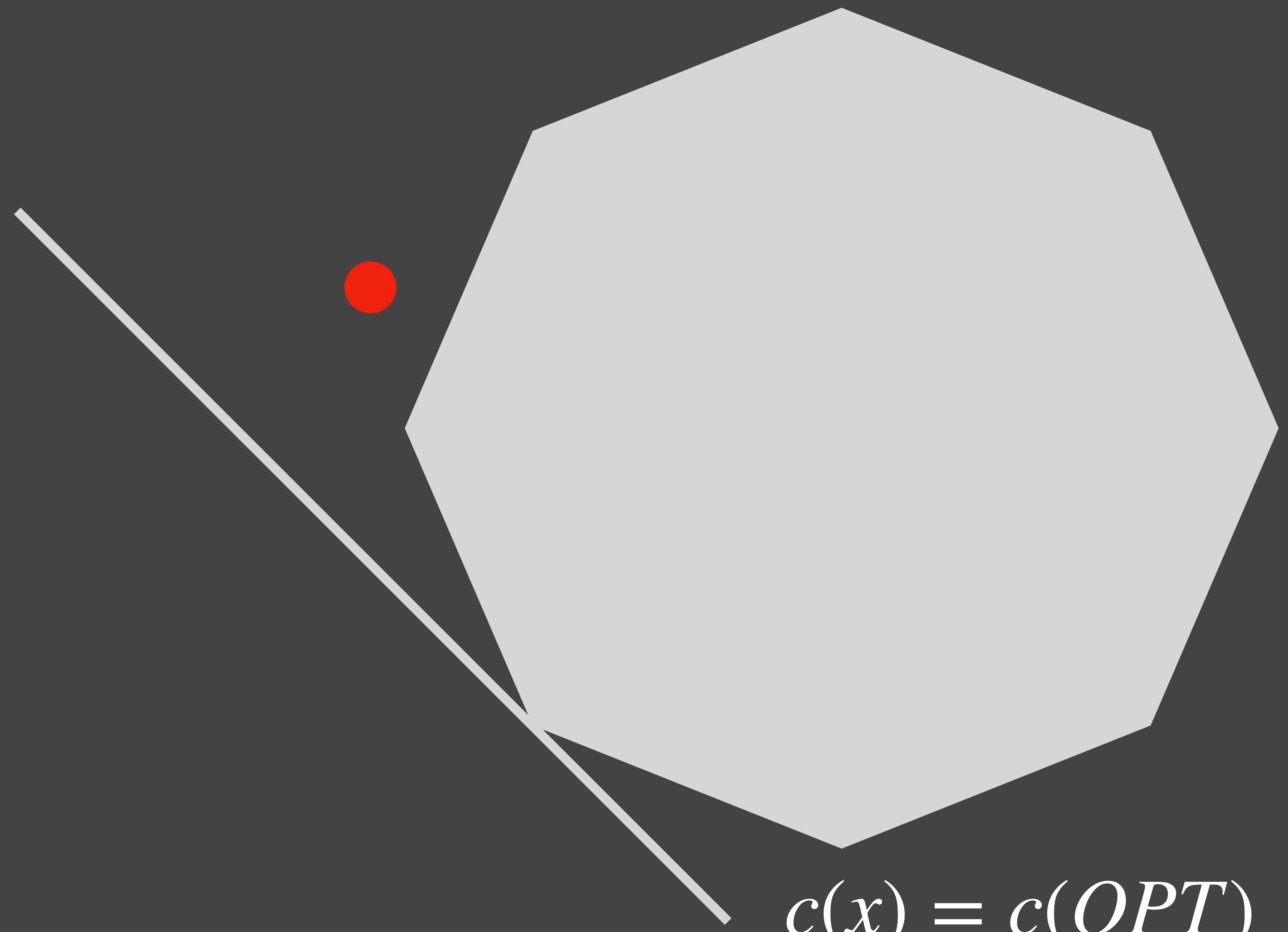

$c(x) = c(OPT)$

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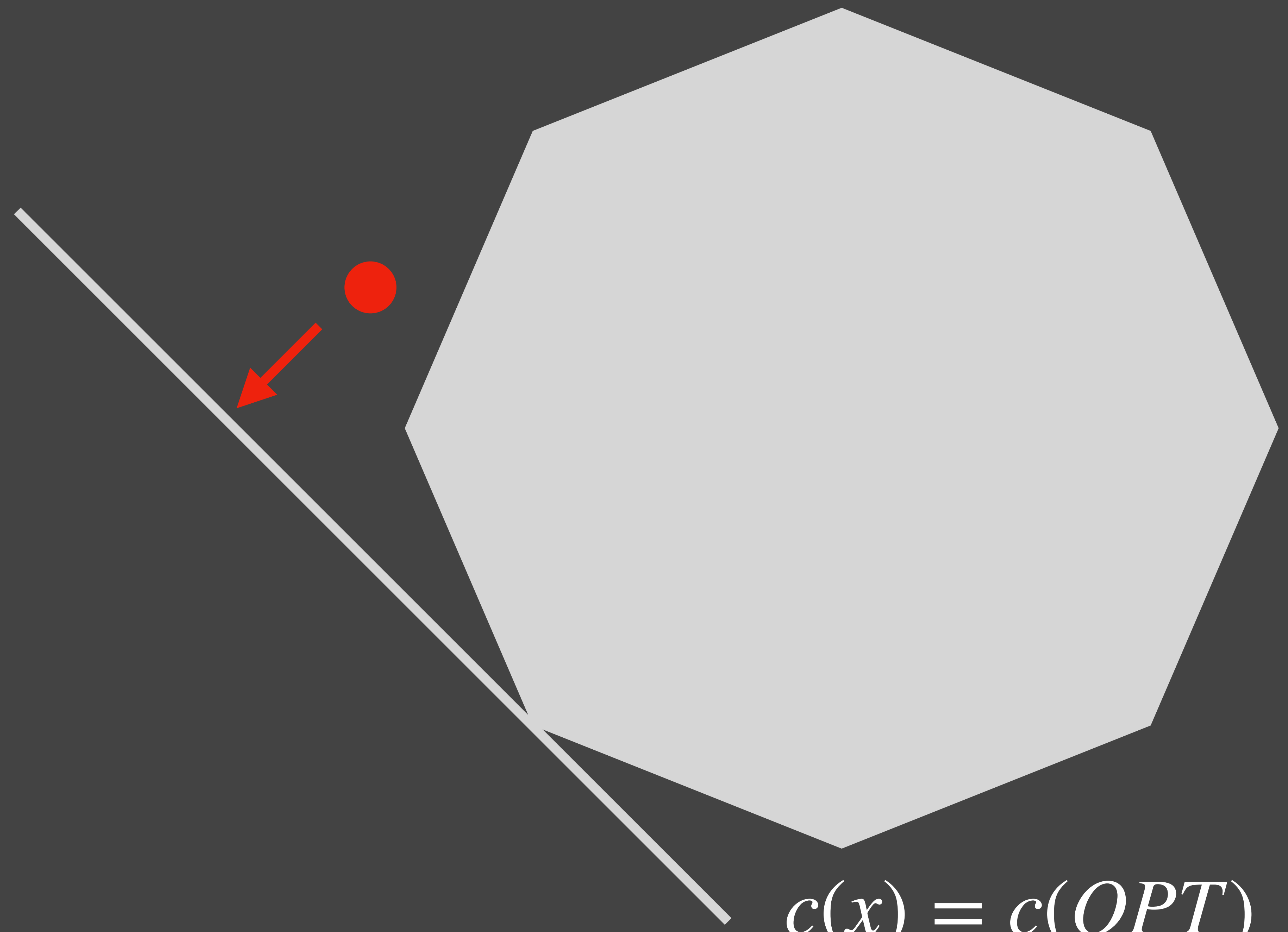

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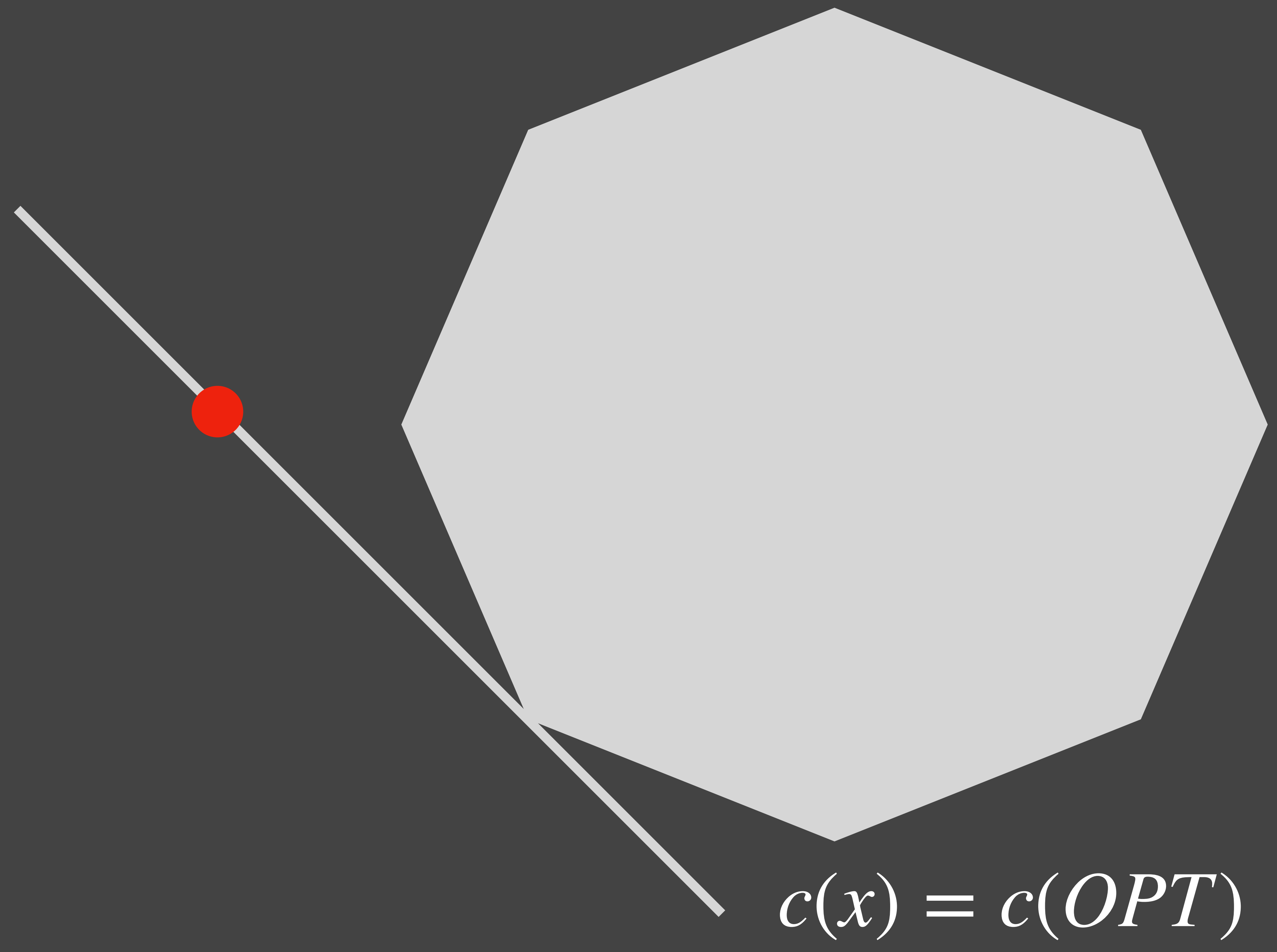

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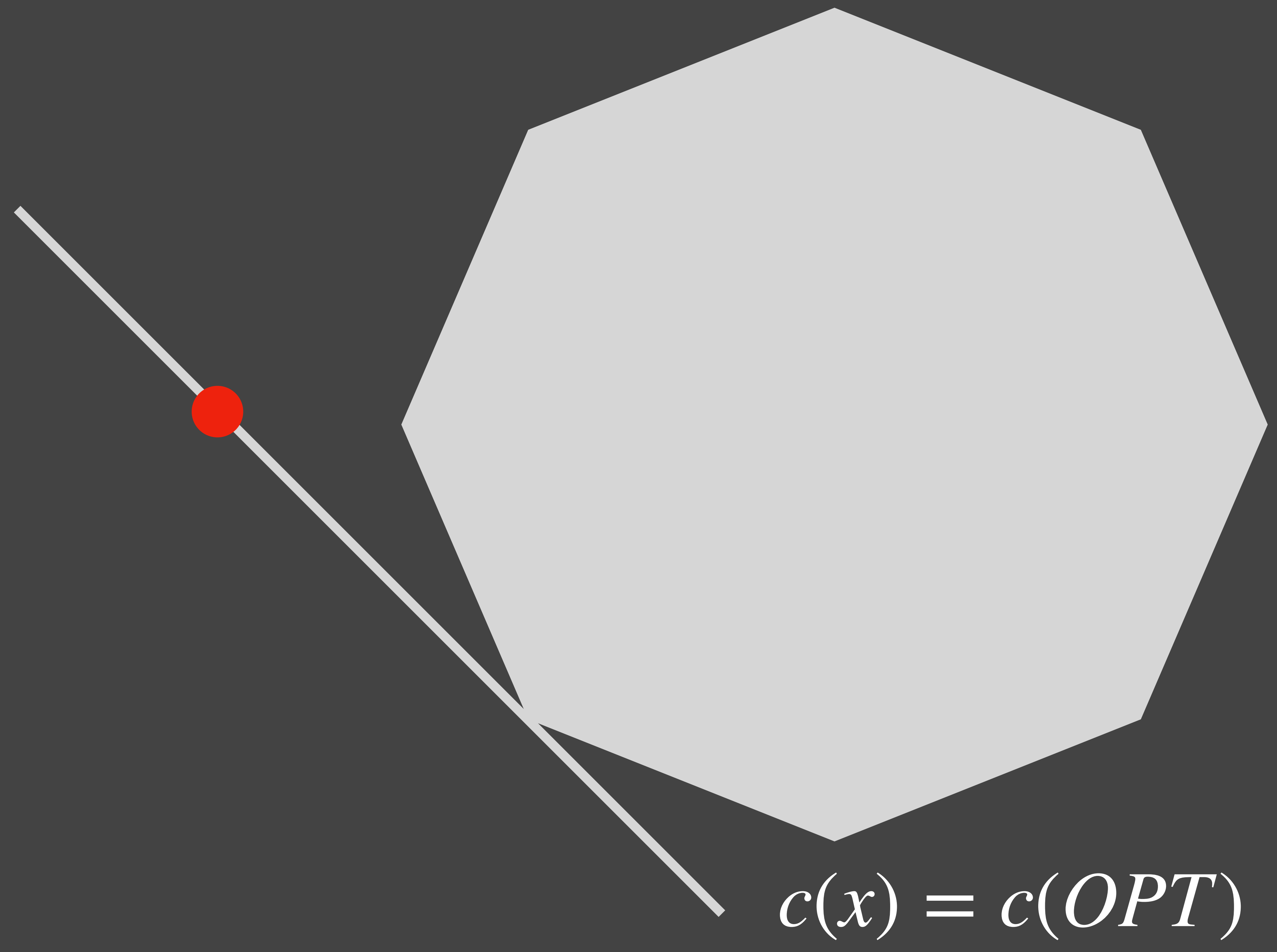

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Perspective 2:

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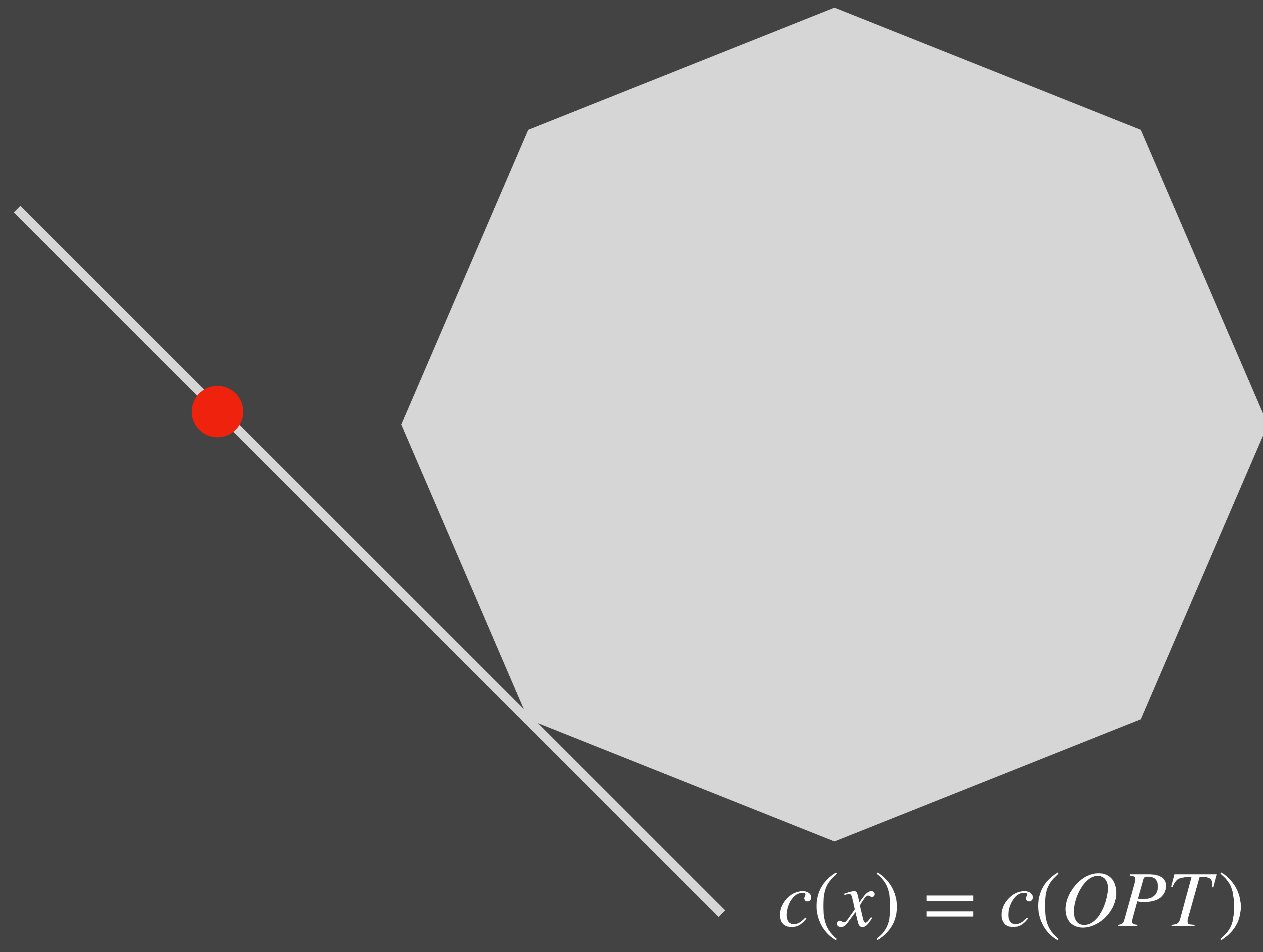

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<u>Perspective 2:</u>

Define

$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

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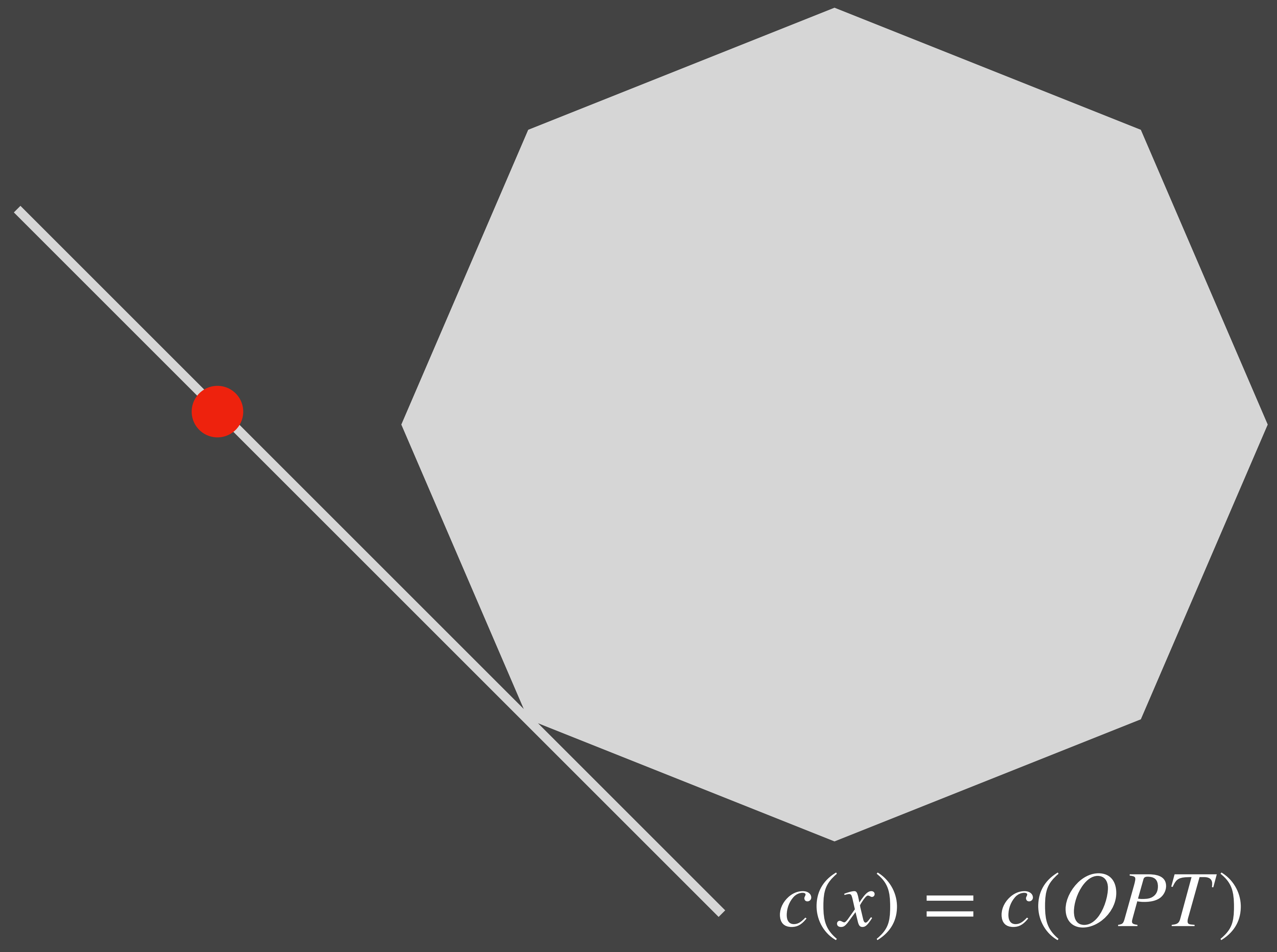

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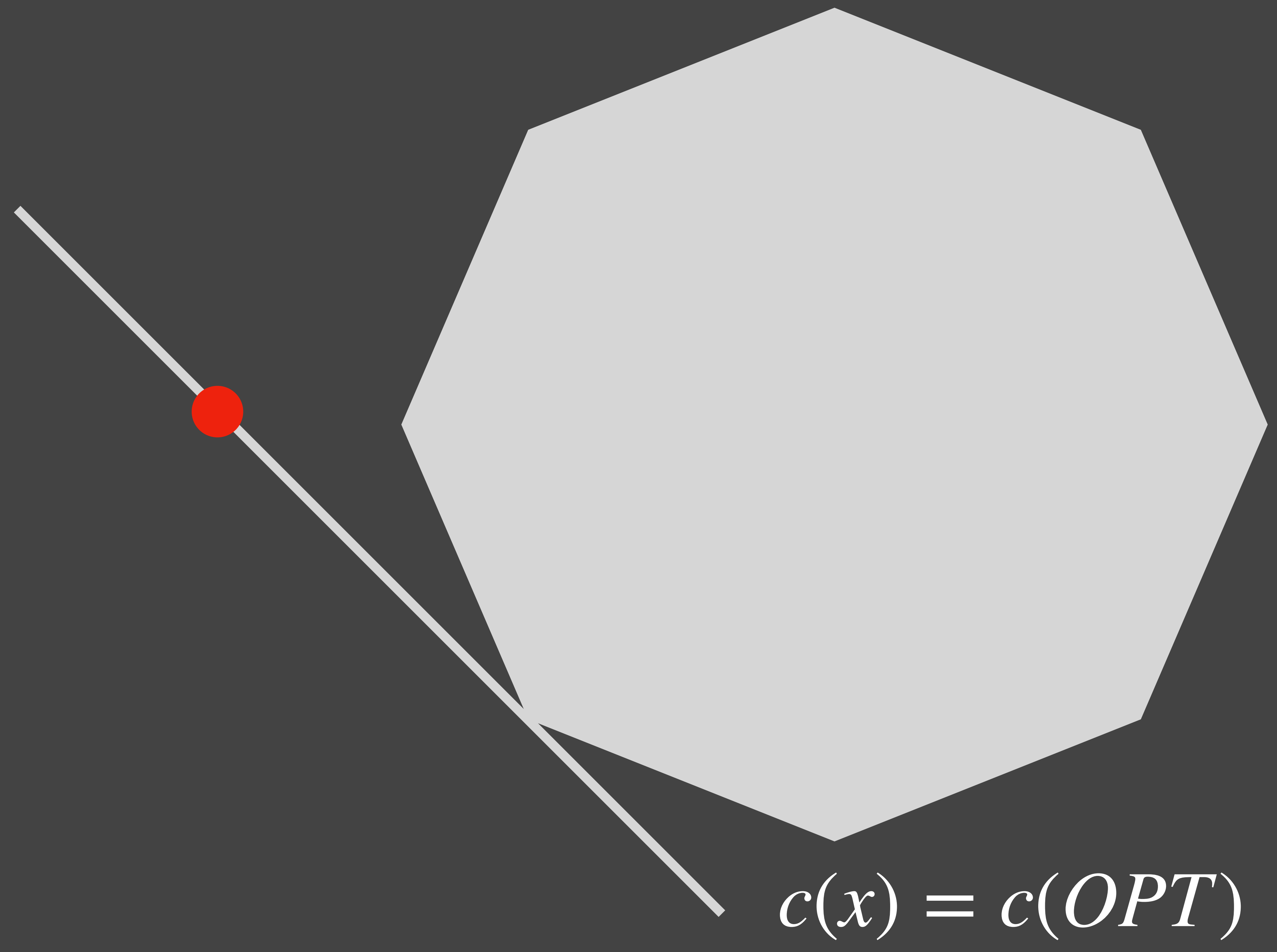

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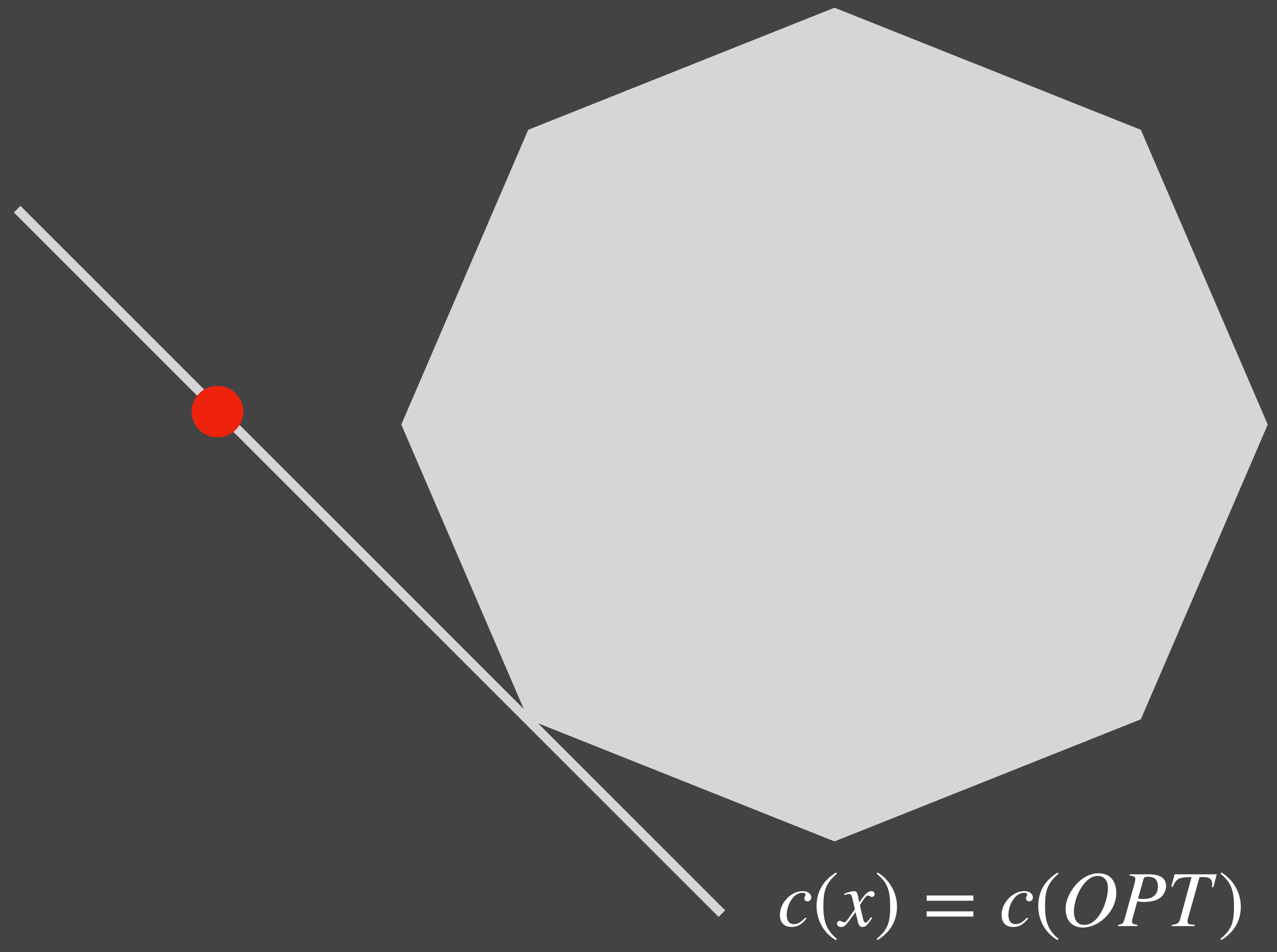

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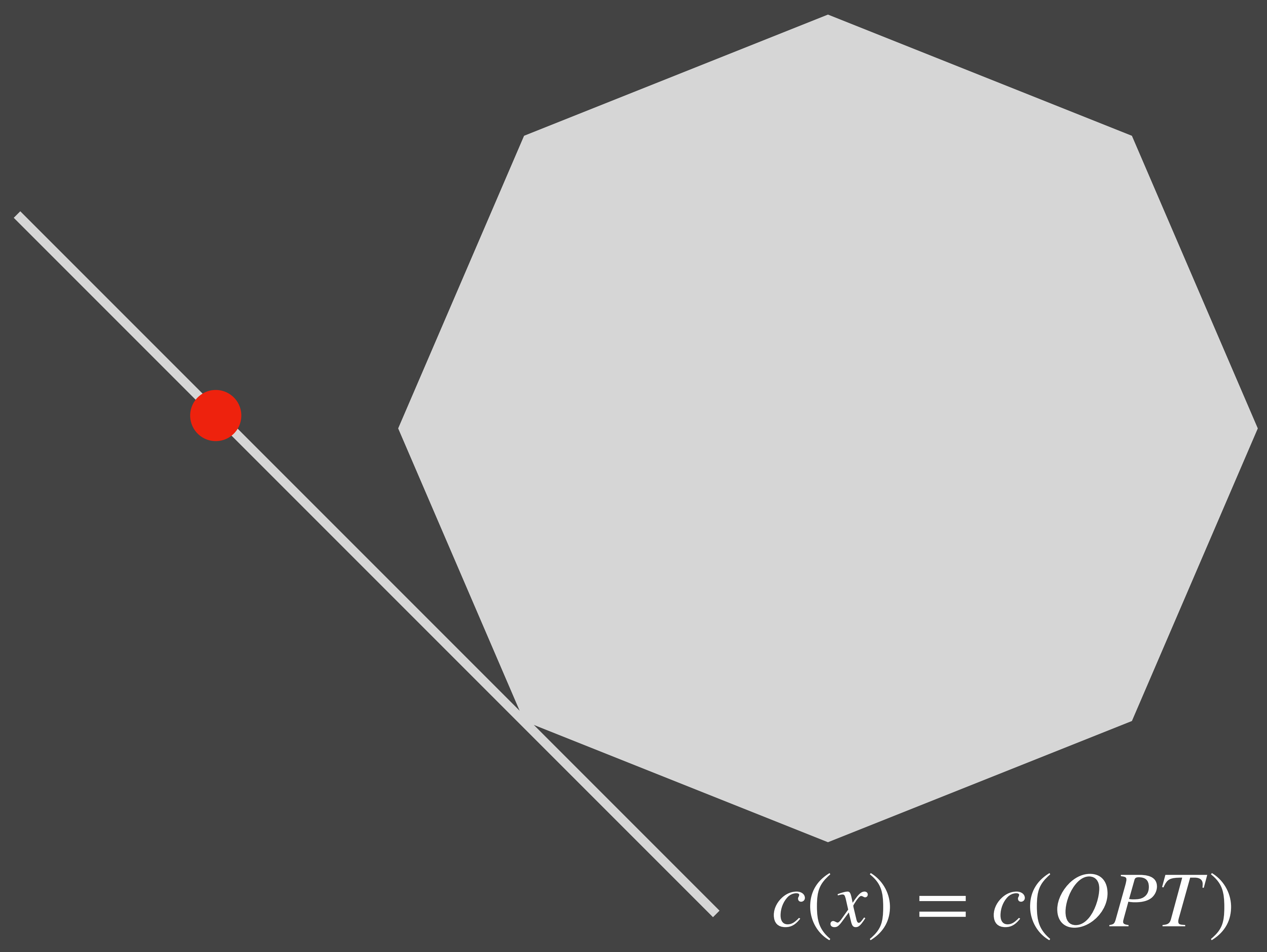

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RO reveals stochastic gradient…

Talk Outline

Intro

Secretary

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Talk Outline

Intro

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➡ (Single Sample) Prophet

Conclusion & Extensions

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Online set cover, but random 1/2 of elements known upfront (see [Kaplan Naori Raz 21]).

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S_1 ●

S_2 ●

S_3 ●

S_4 ●

S_5 ●

S_6 ●

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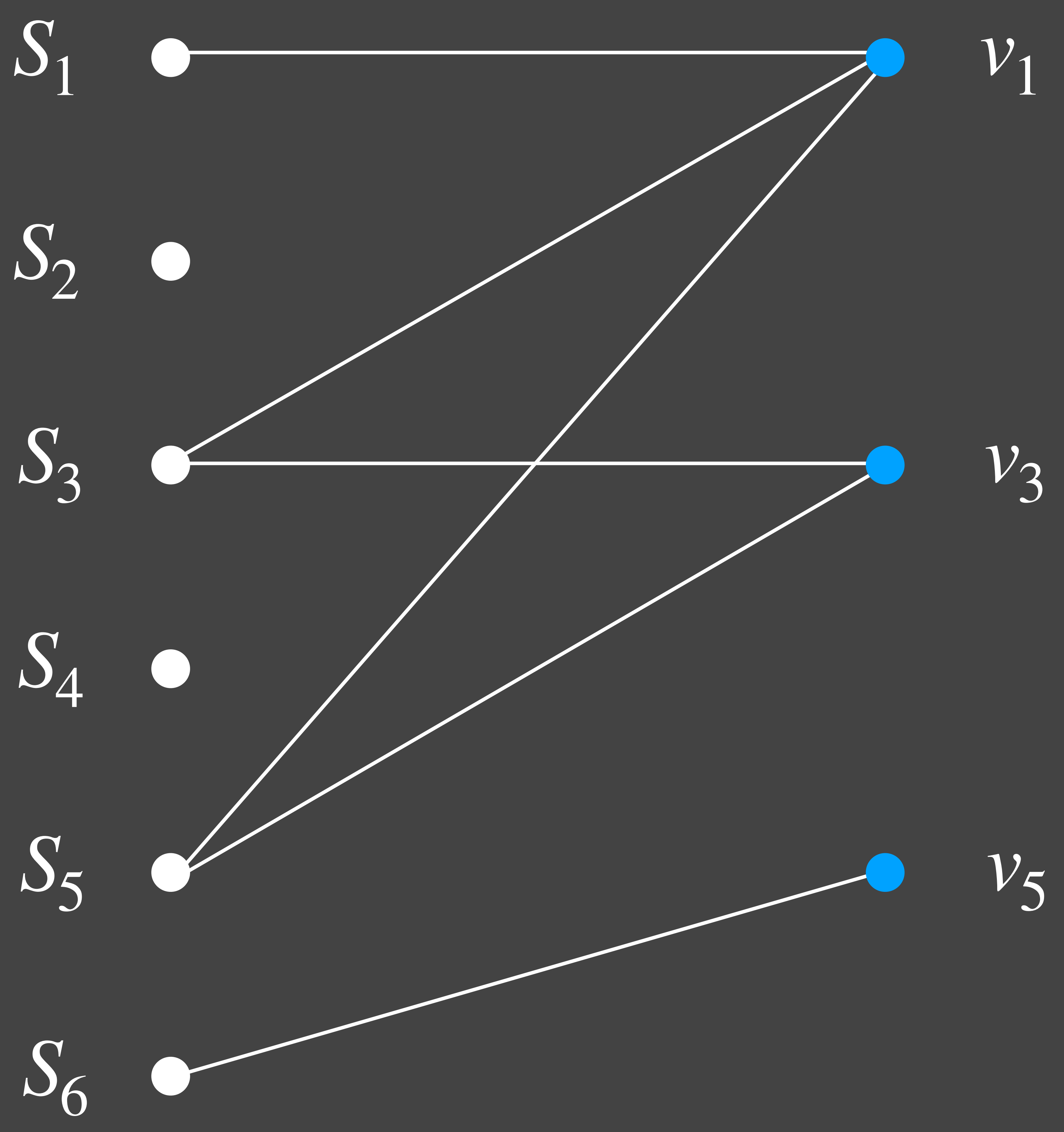

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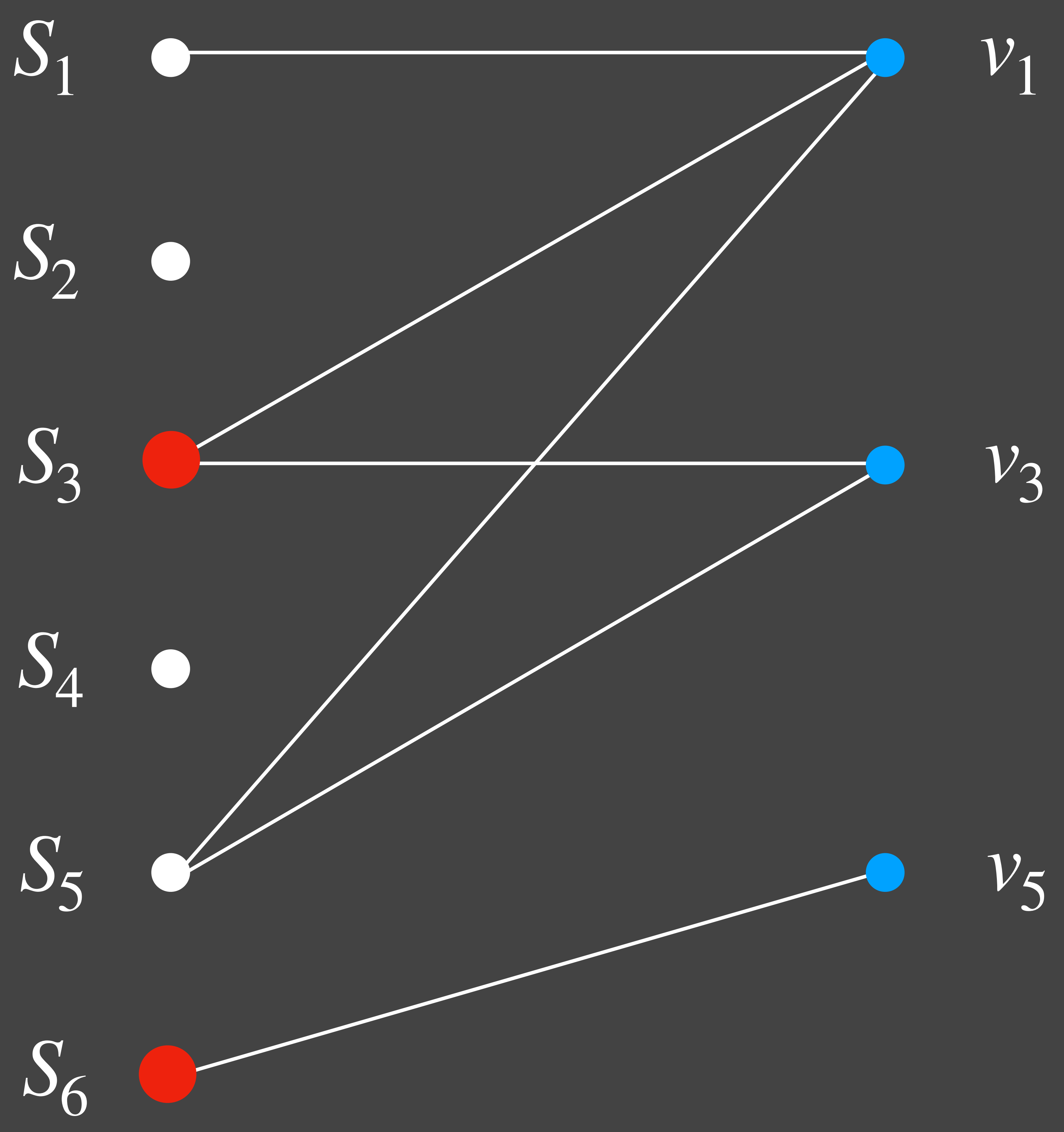

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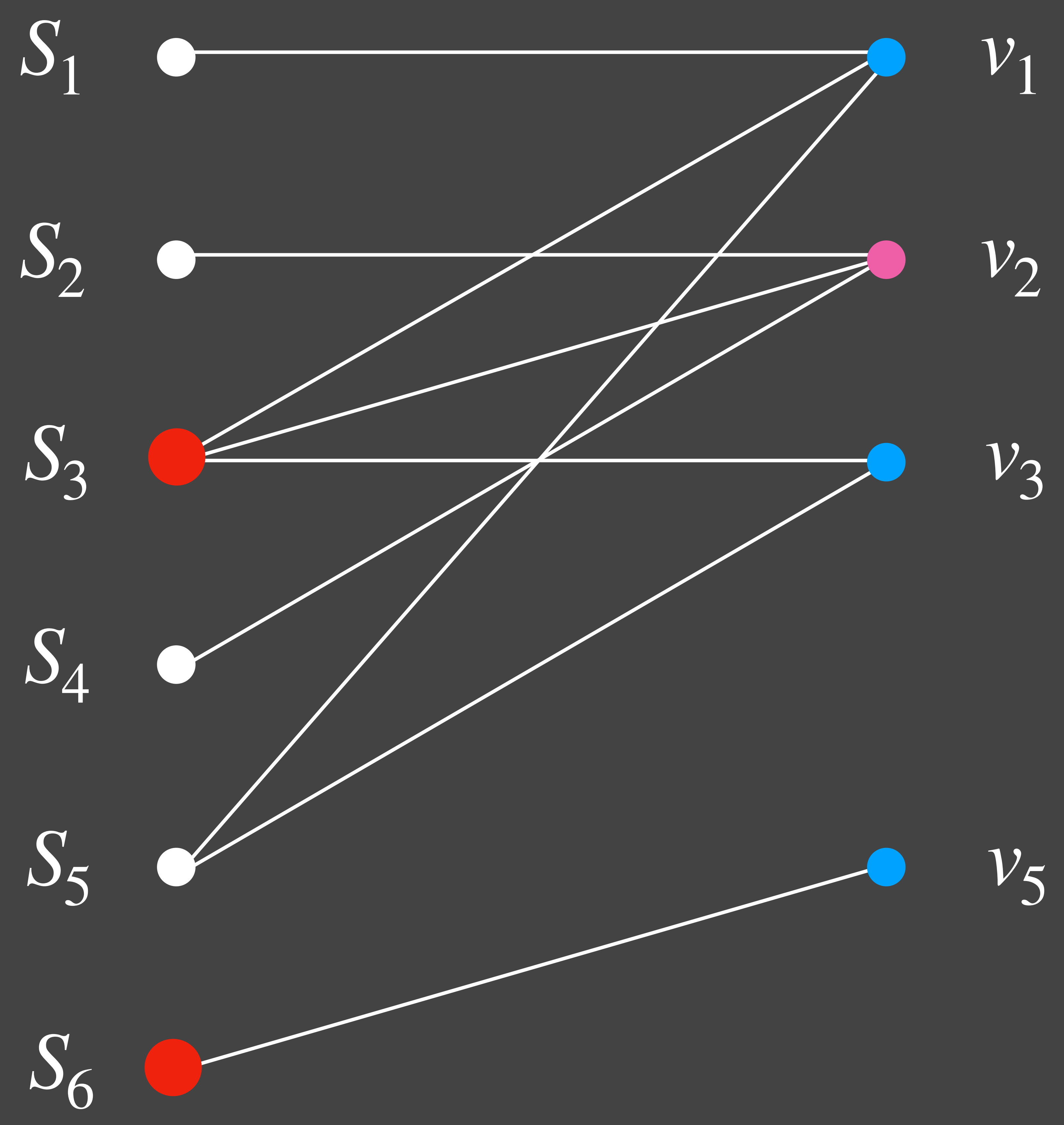

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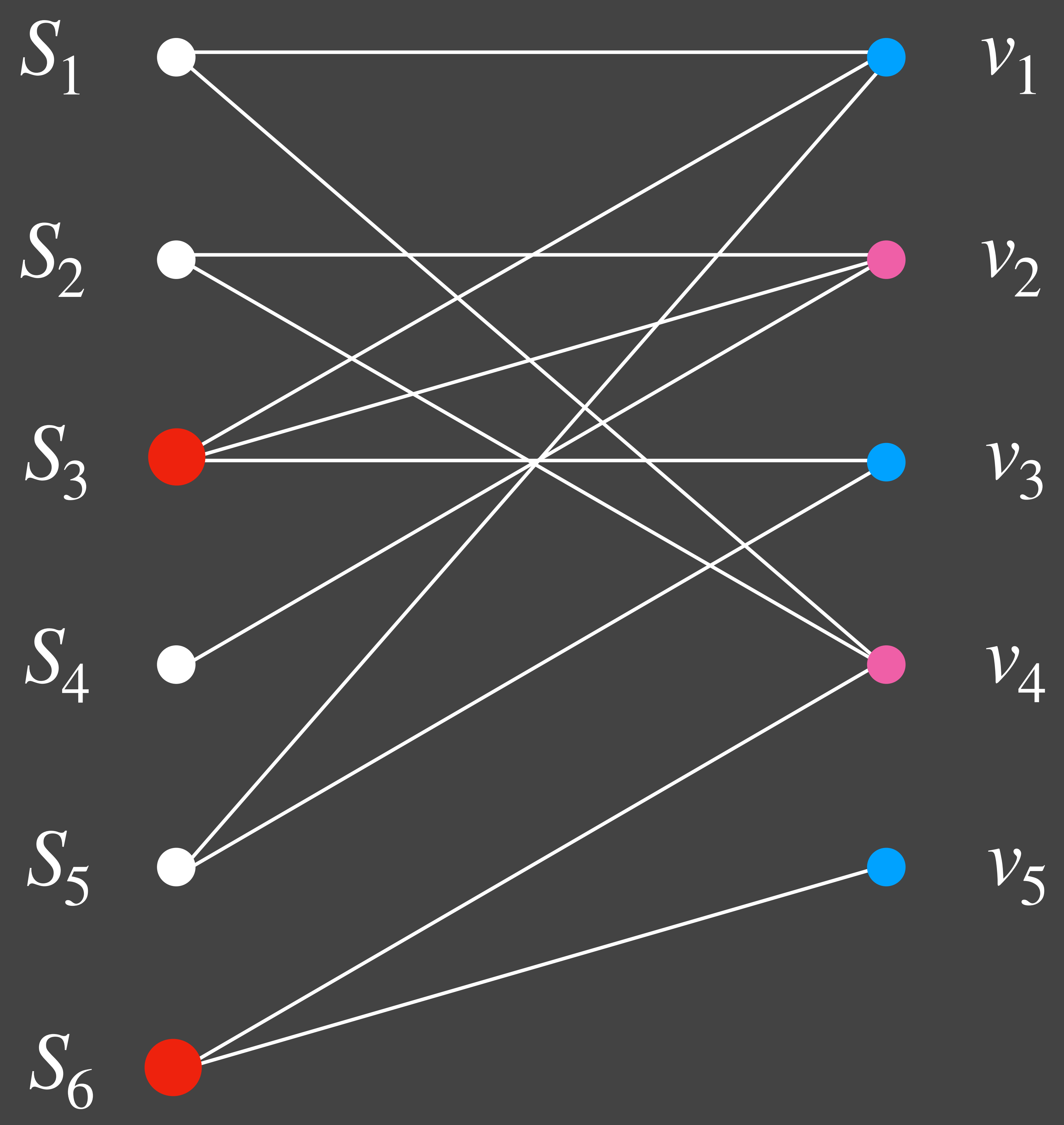

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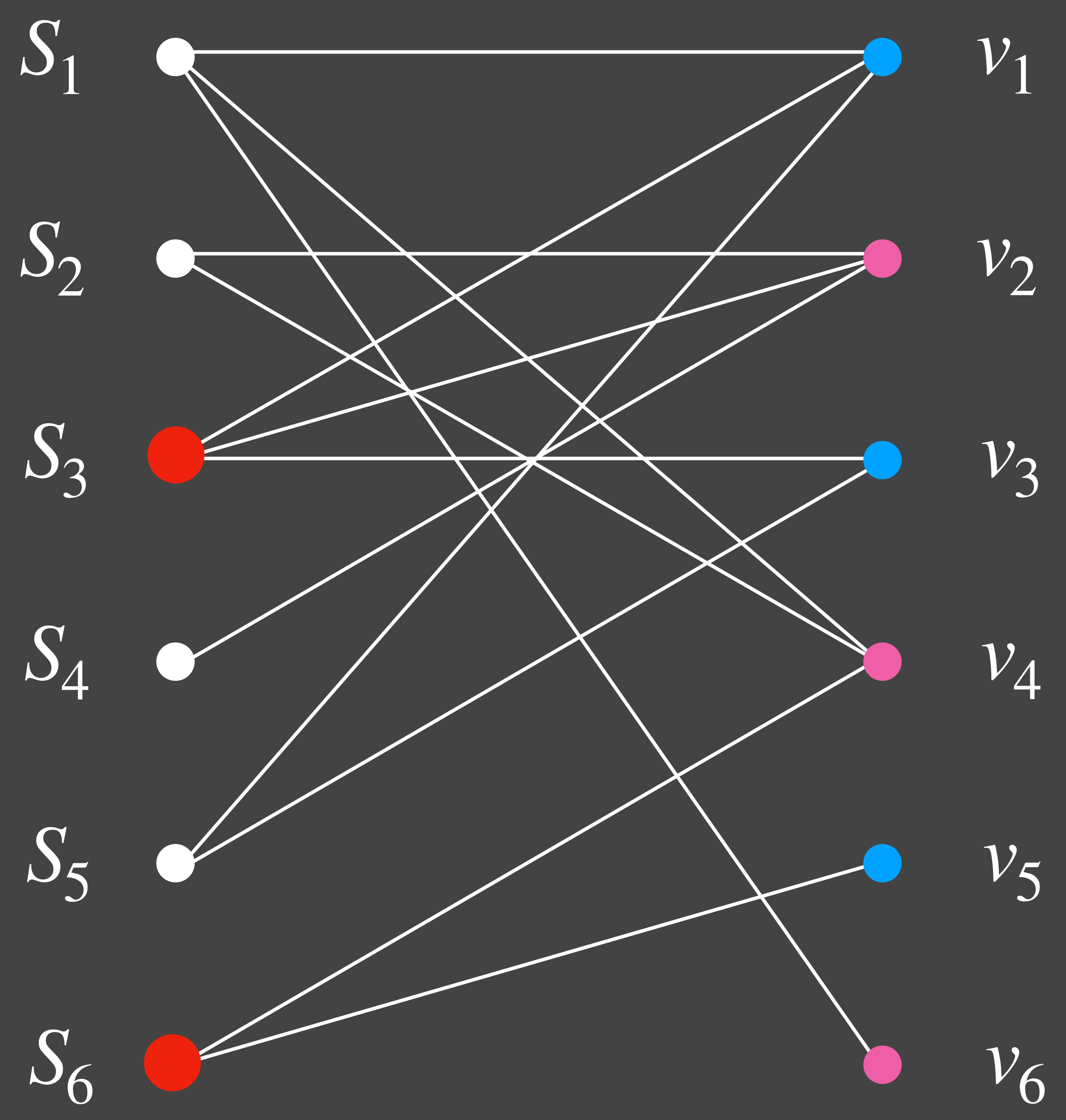

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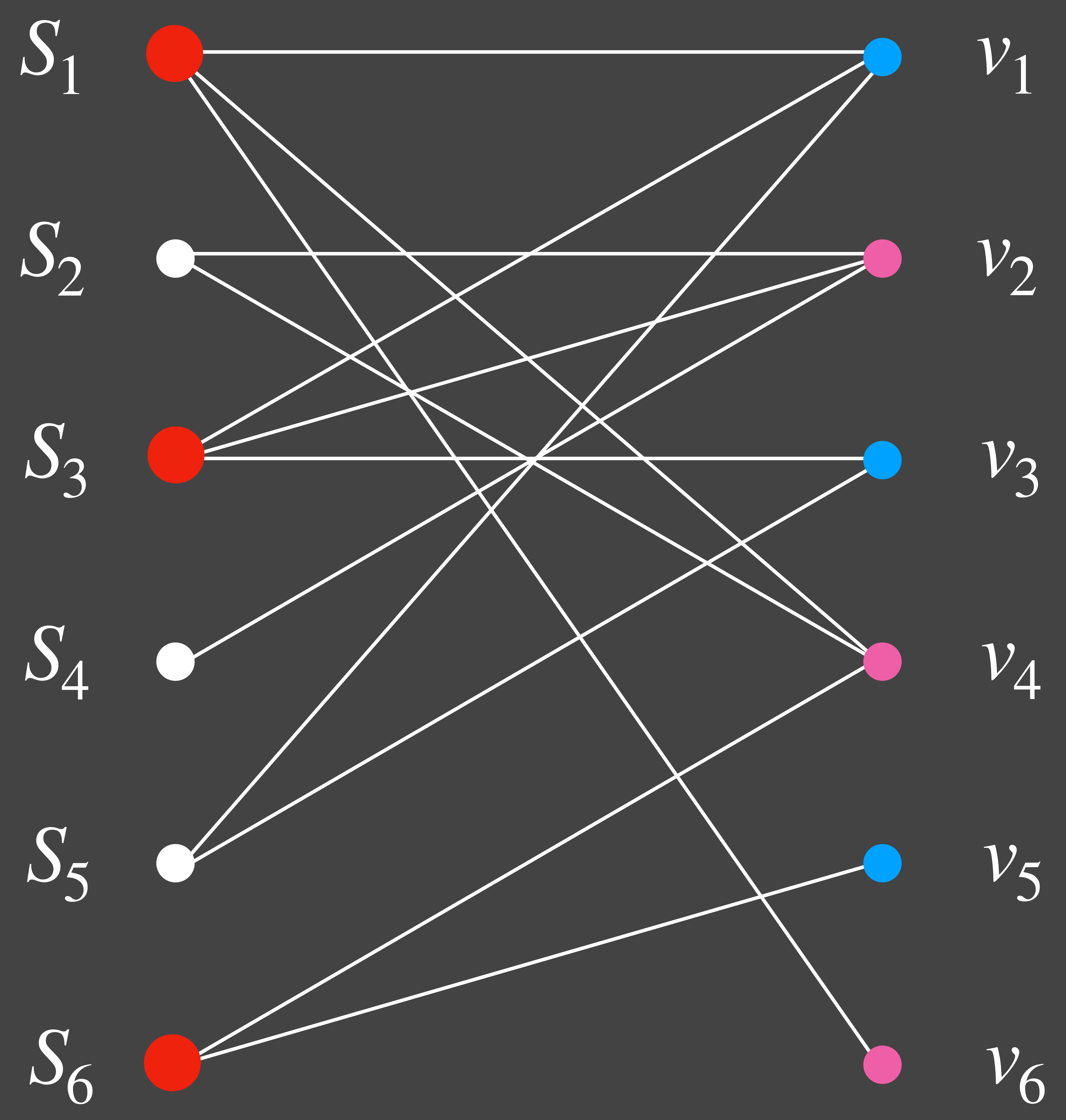
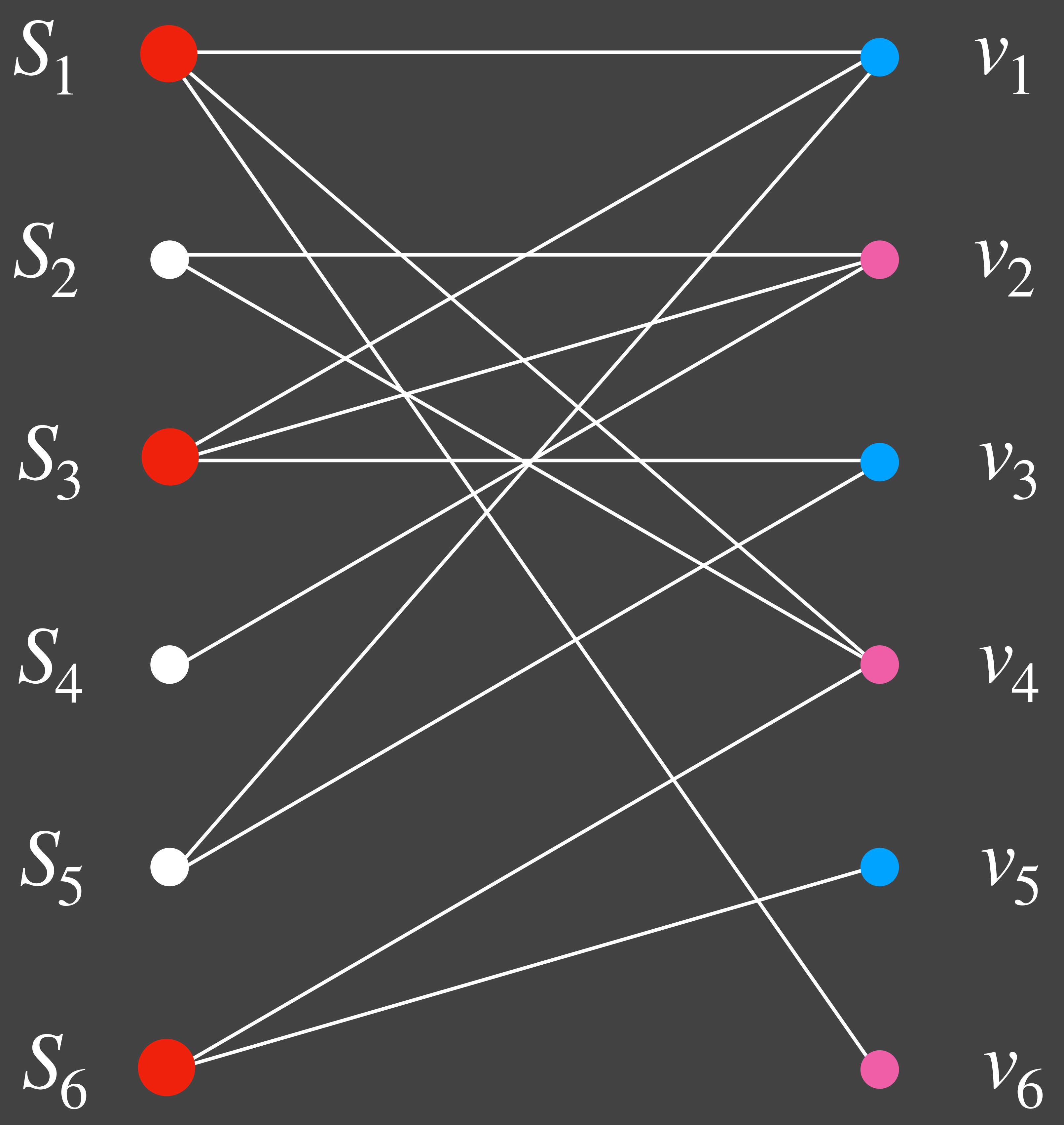


Special Case: the With-a-Sample model

Online set cover, but random 1/2 of elements known upfront (see [Kaplan Naori Raz 21]).

Remaining fraction revealed in adversarial order.



Theorem:

There is a poly time algorithm for Online ***Set*** *Cover* ***With-a-Sample*** *with competitive ratio* $O(\log(mn))$.

Reduction to LearnOrCover!

S_1 •

S_2 •

S_3 •

S_4 •

S_5 •

S_6 •

Reduction to LearnOrCover!

Idea:

1. Run LearnOrCover on samples.
2. Buy arbitrary sets for remaining elements.

S_1 ●

S_2 ●

S_3 ●

S_4 ●

S_5 ●

S_6 ●

Reduction to LearnOrCover!

Idea:

1. Run LearnOrCover on samples.
2. Buy arbitrary sets for remaining elements.

Pretend colored pink (sampled)/blue (adversarial) on arrival.

S_1 ●

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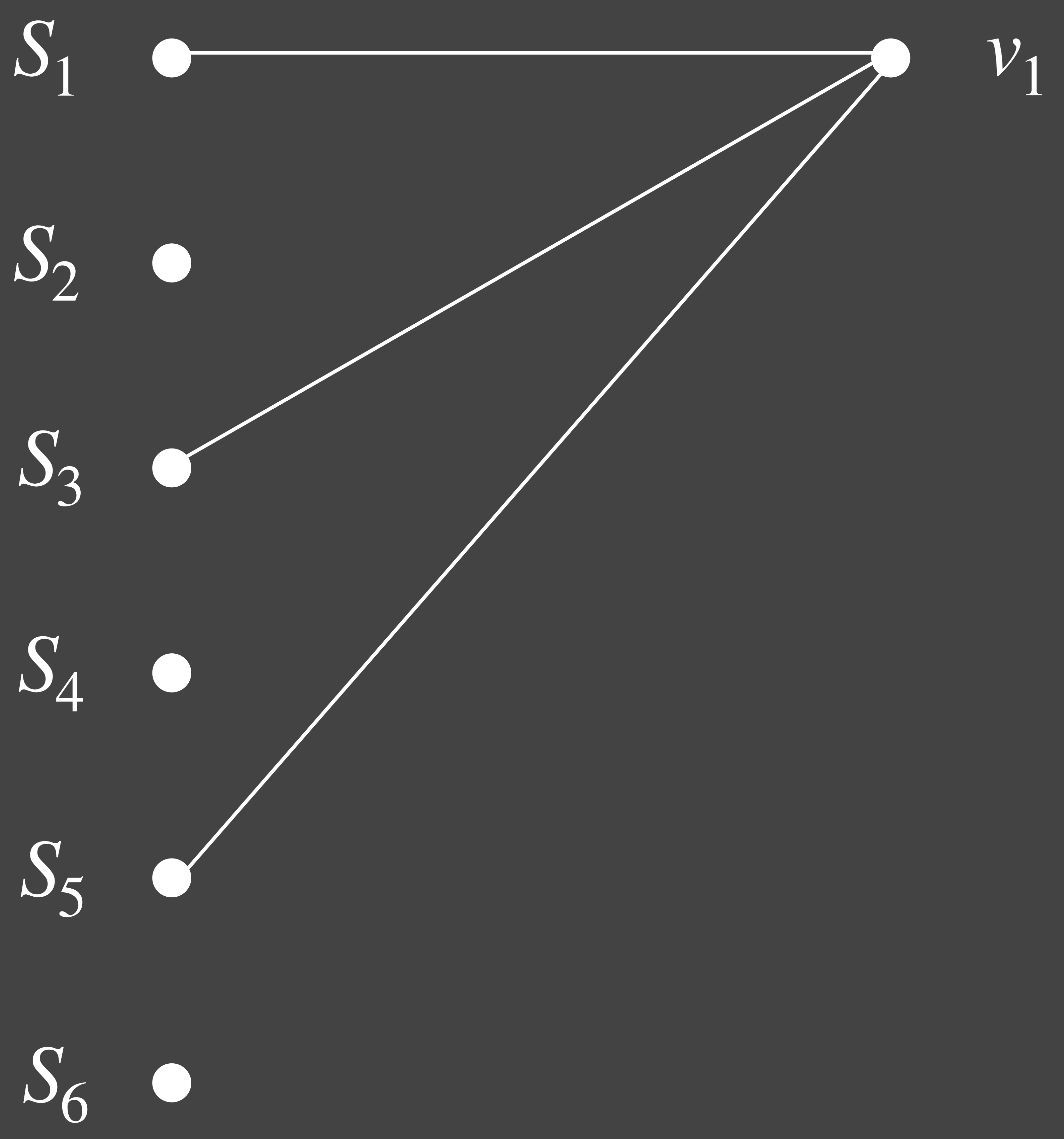

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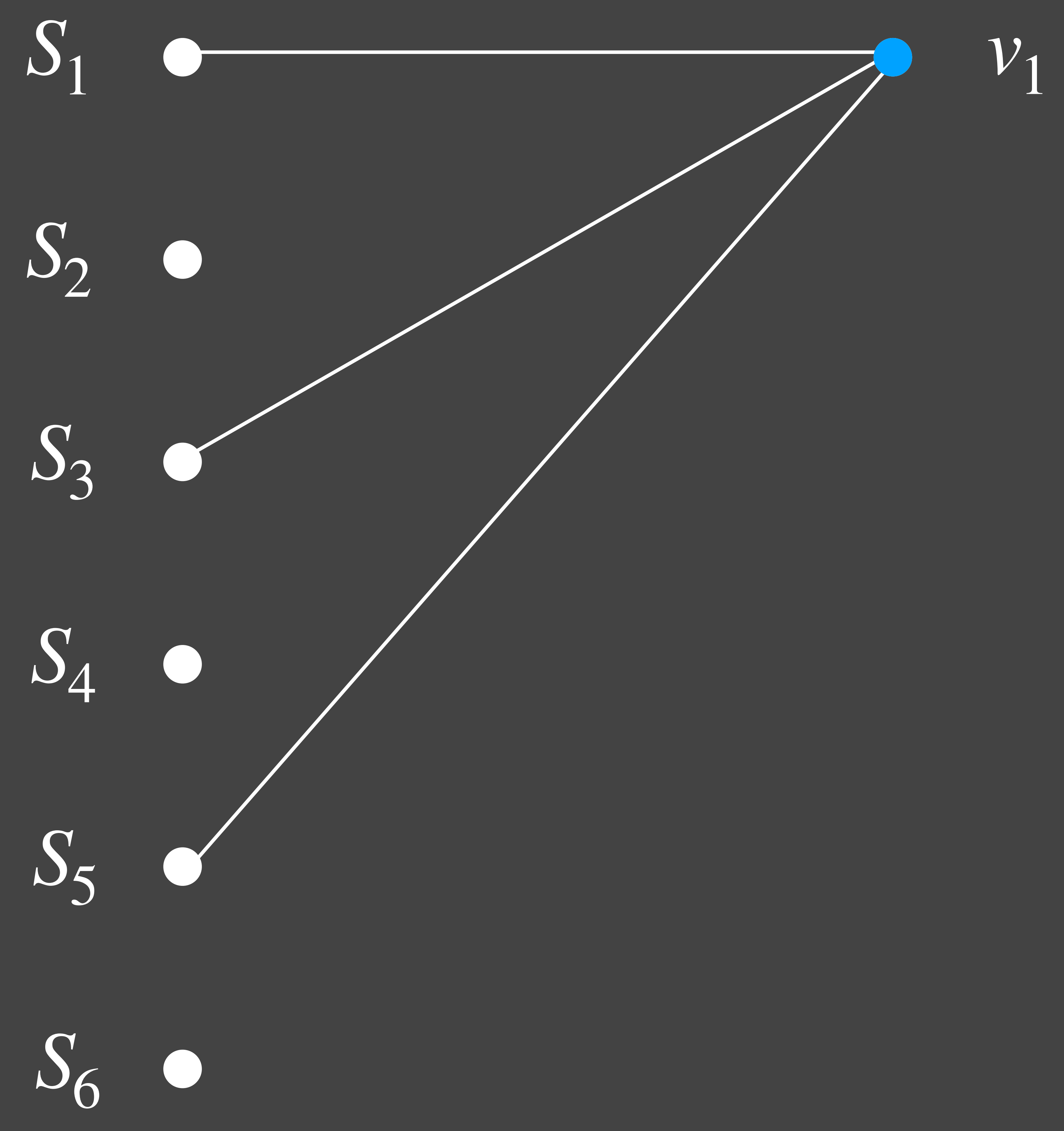

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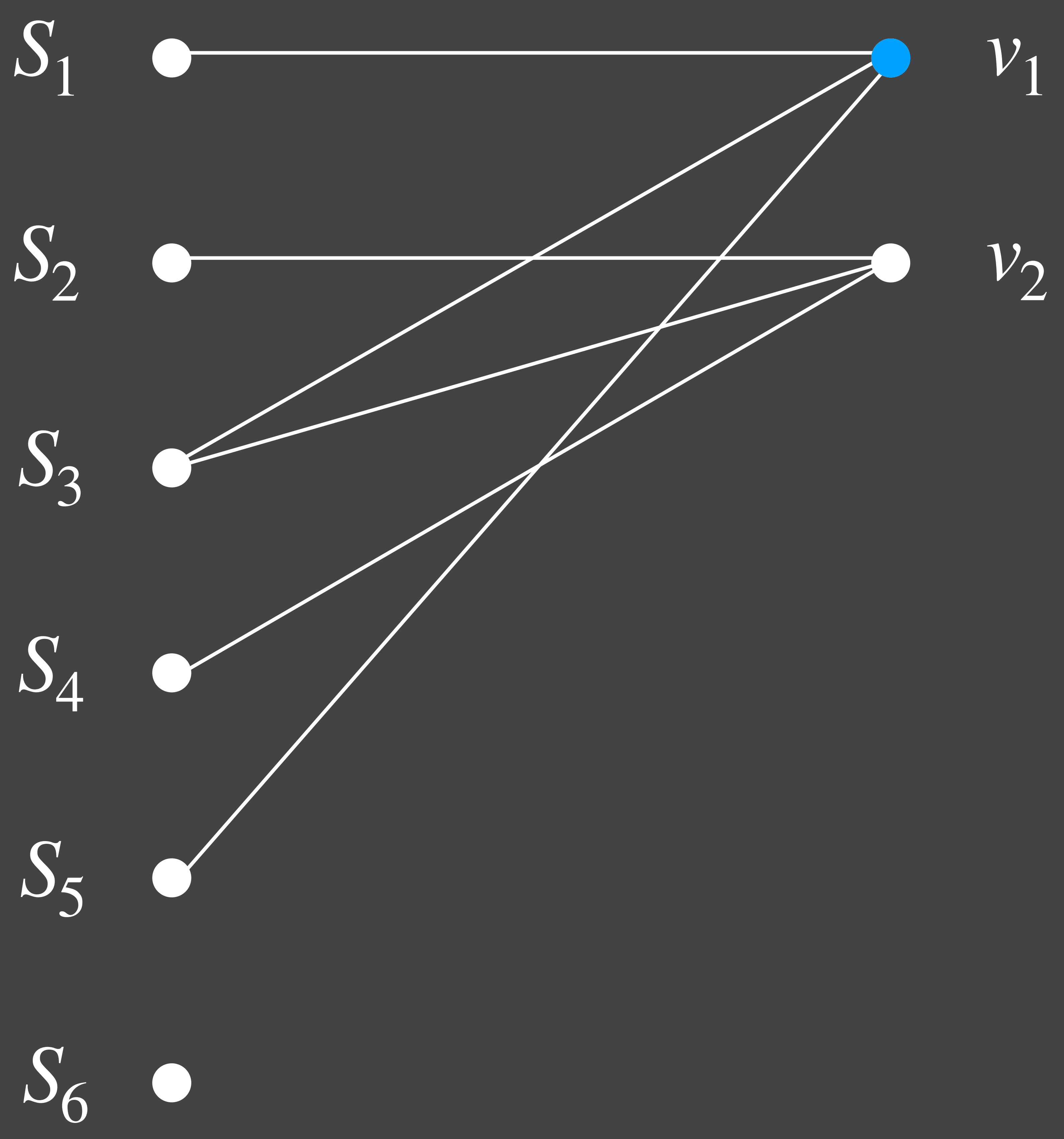

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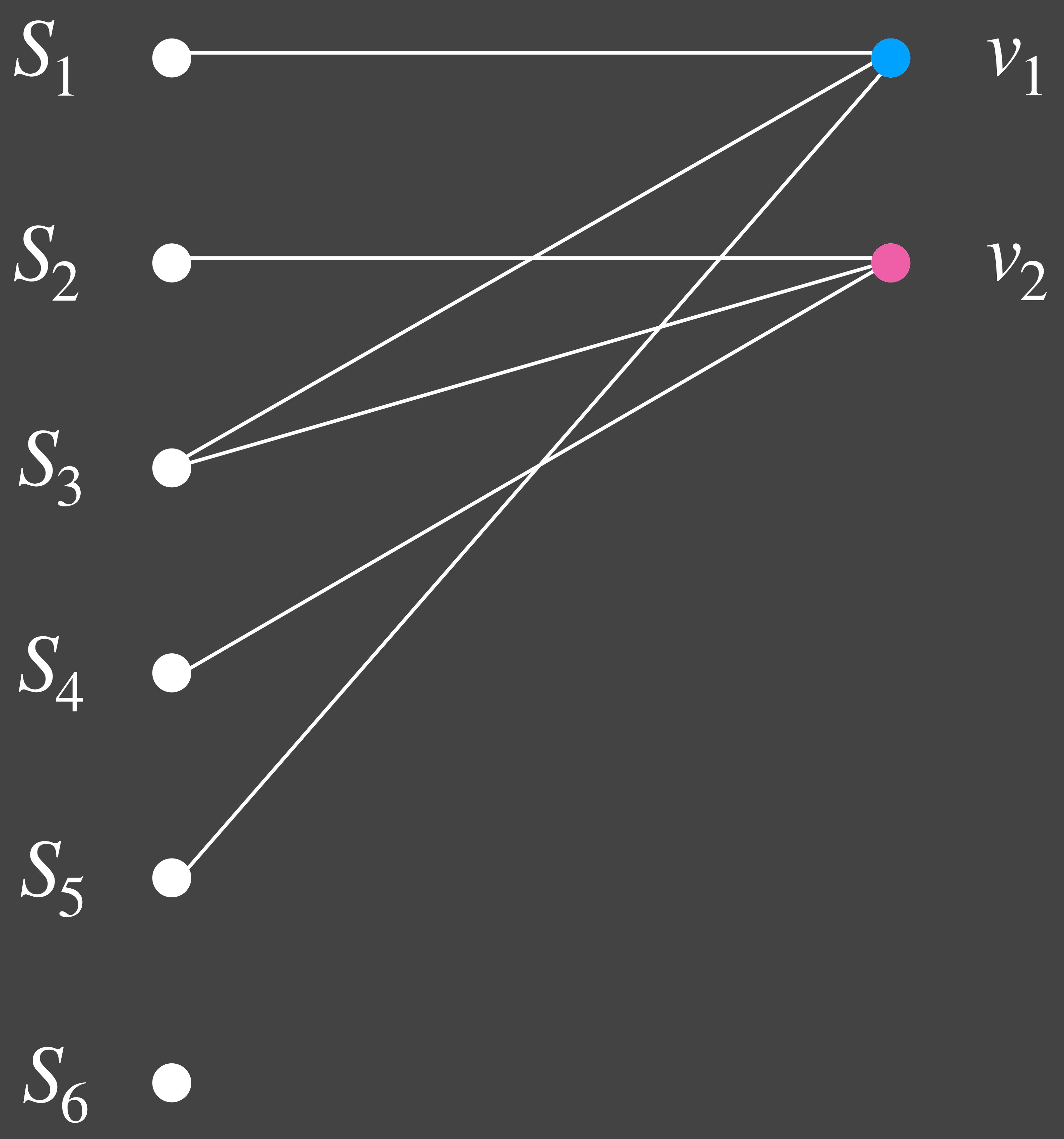

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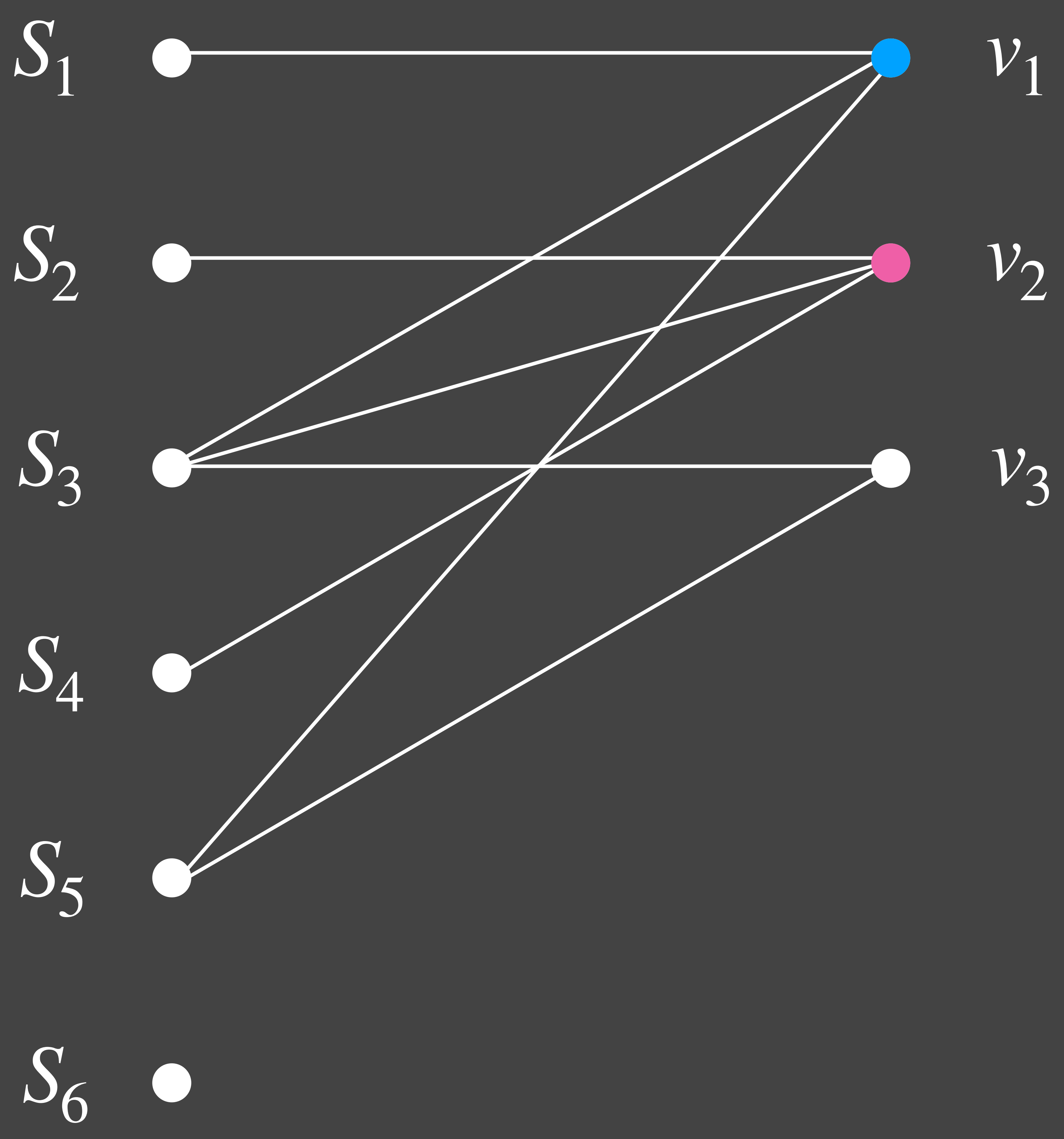

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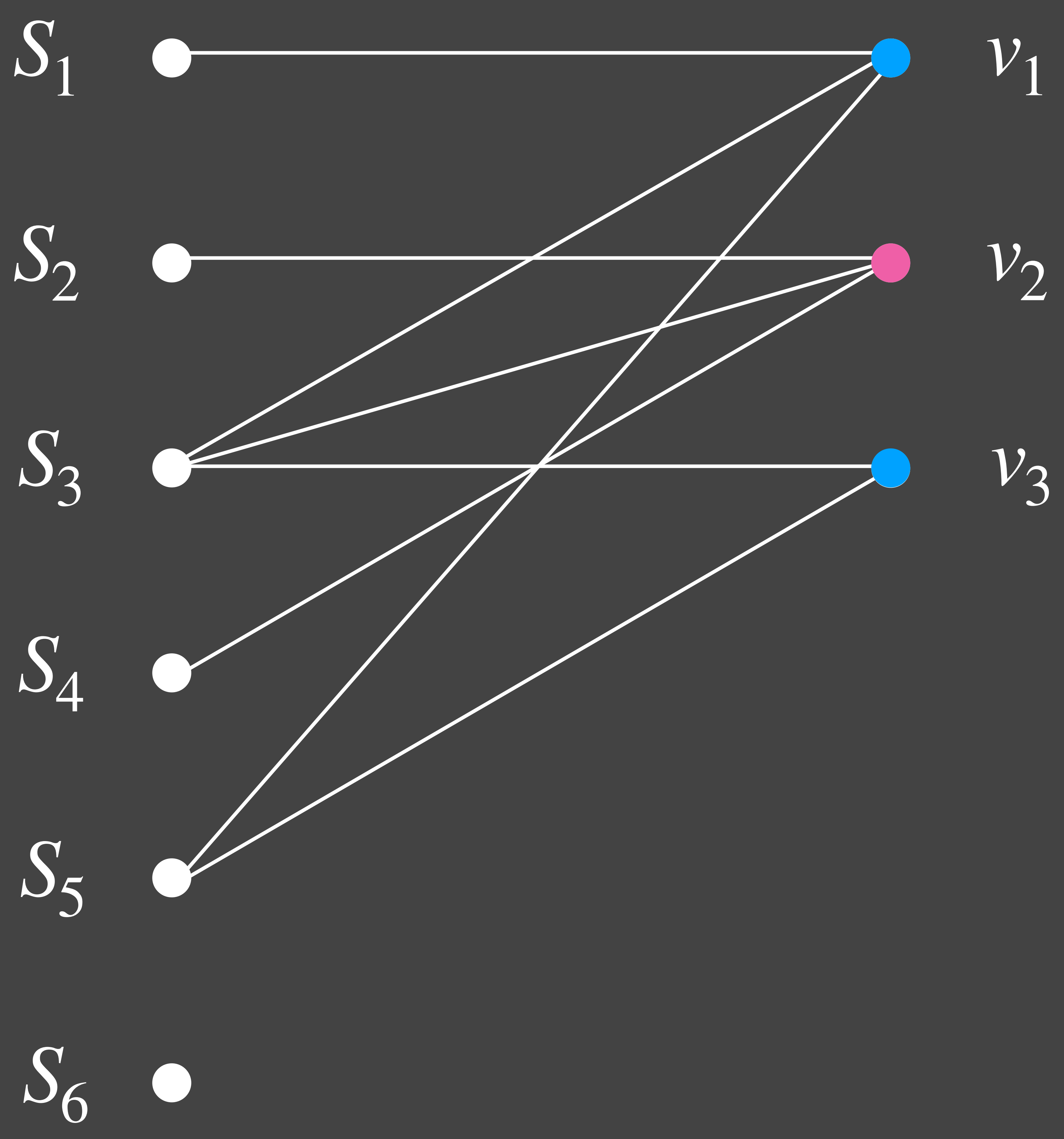

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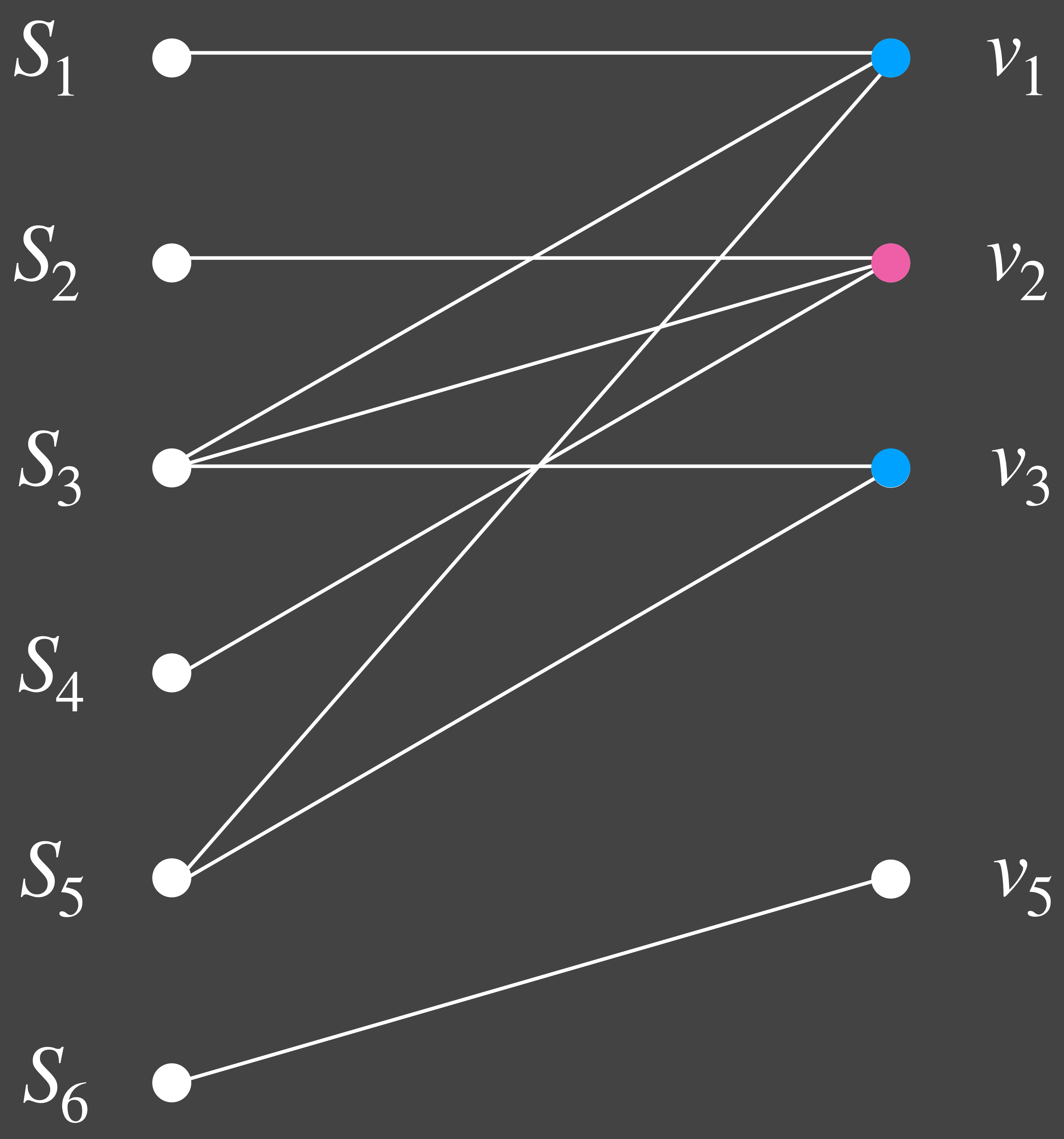

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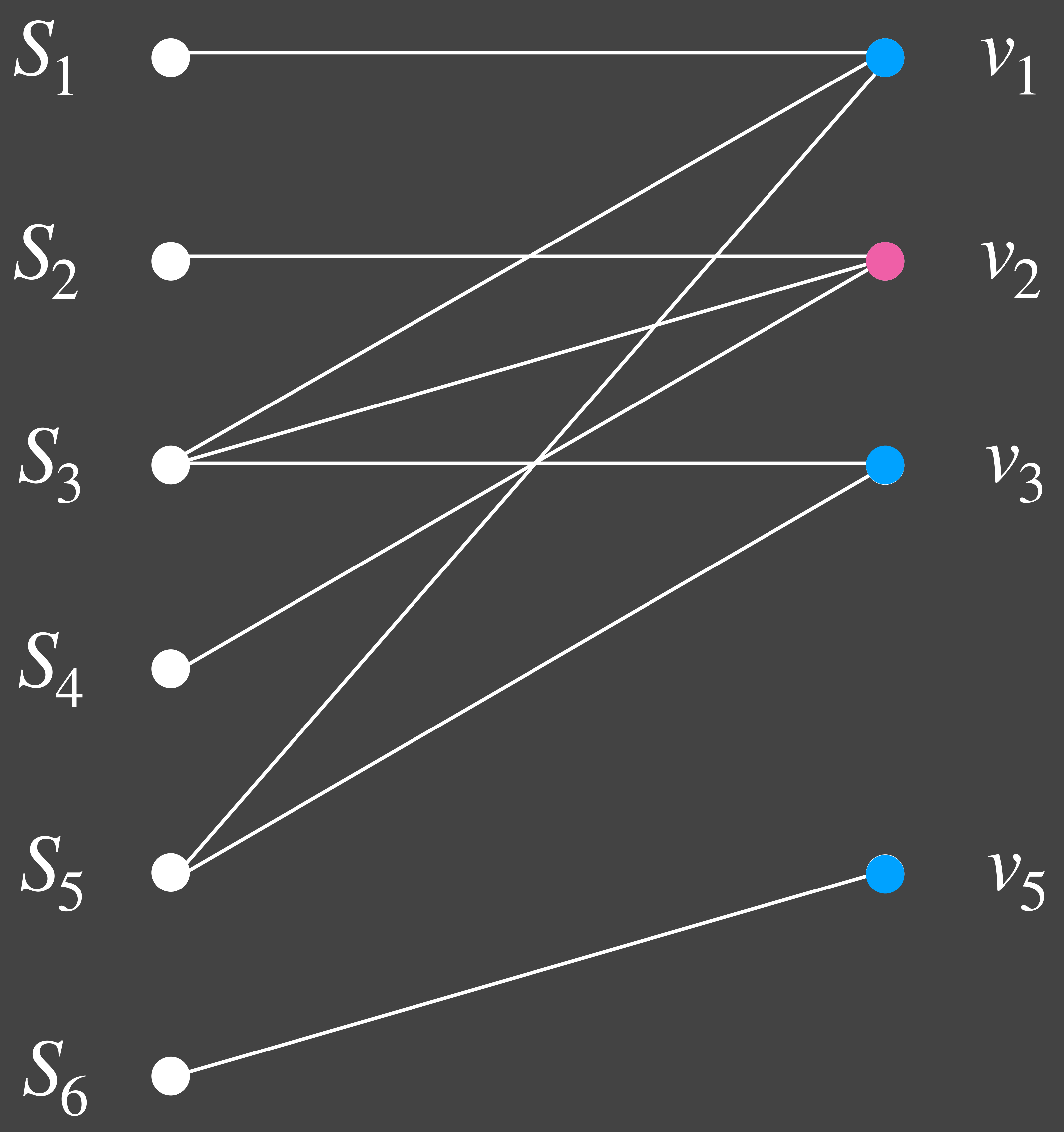

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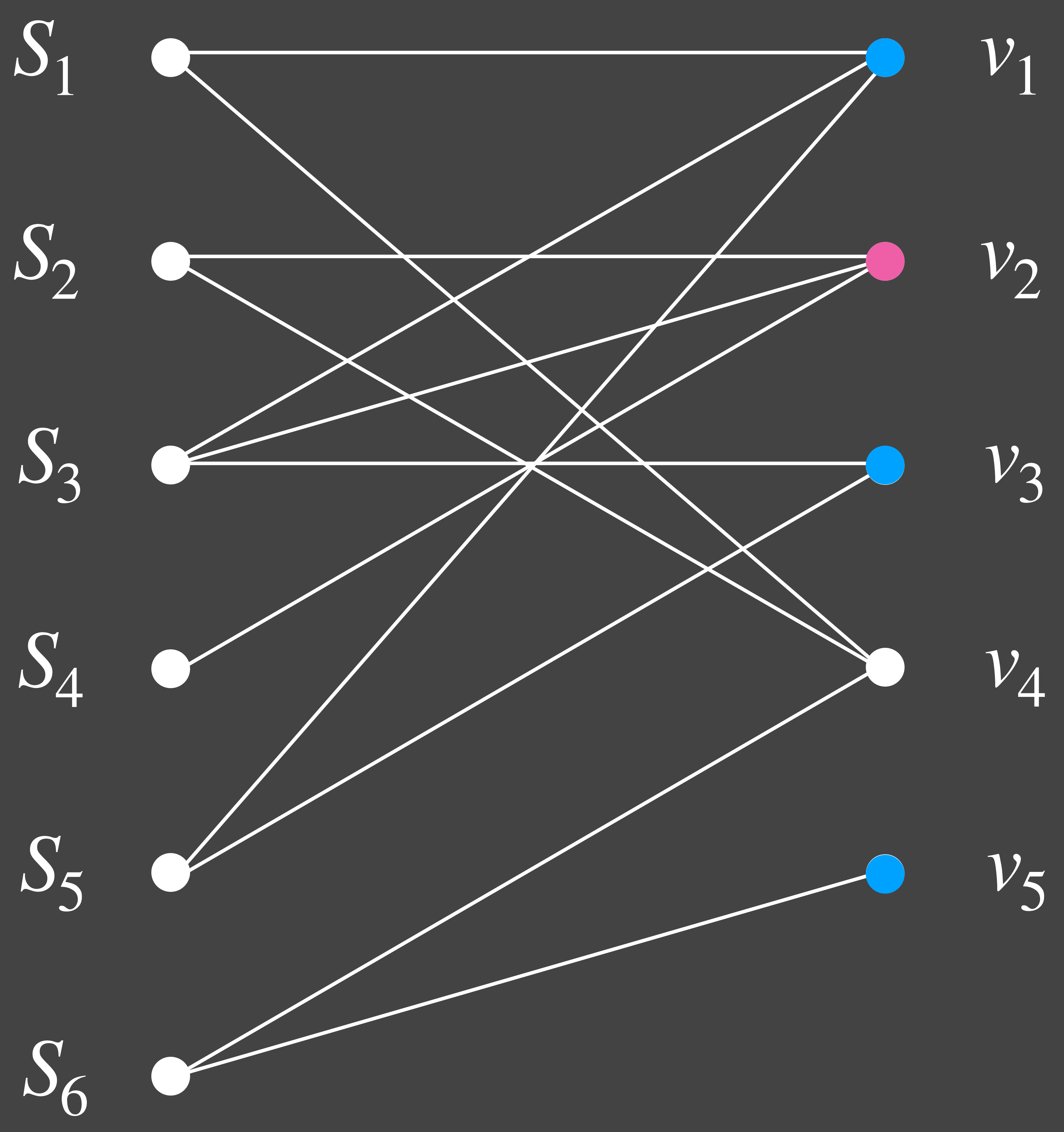

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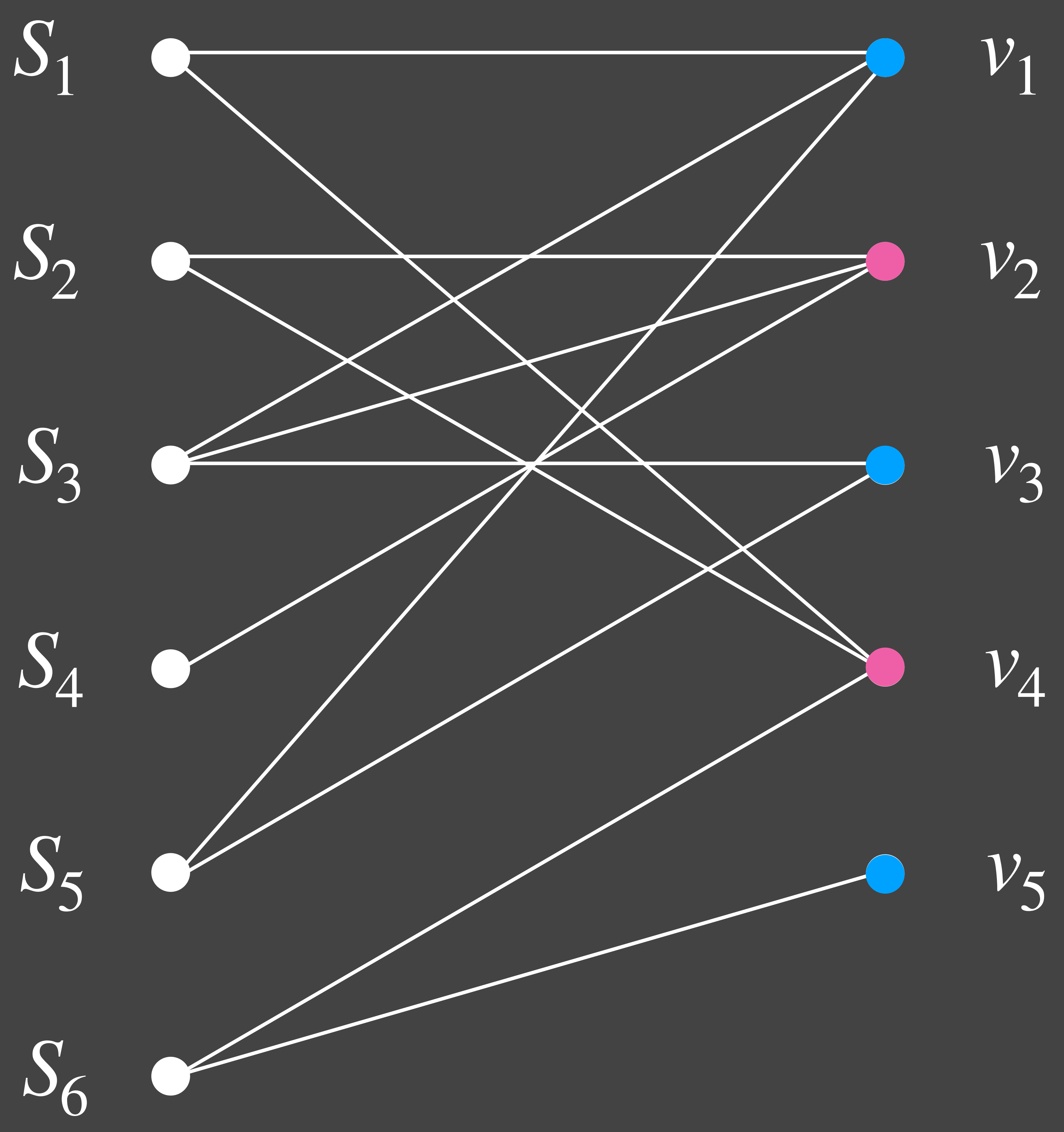

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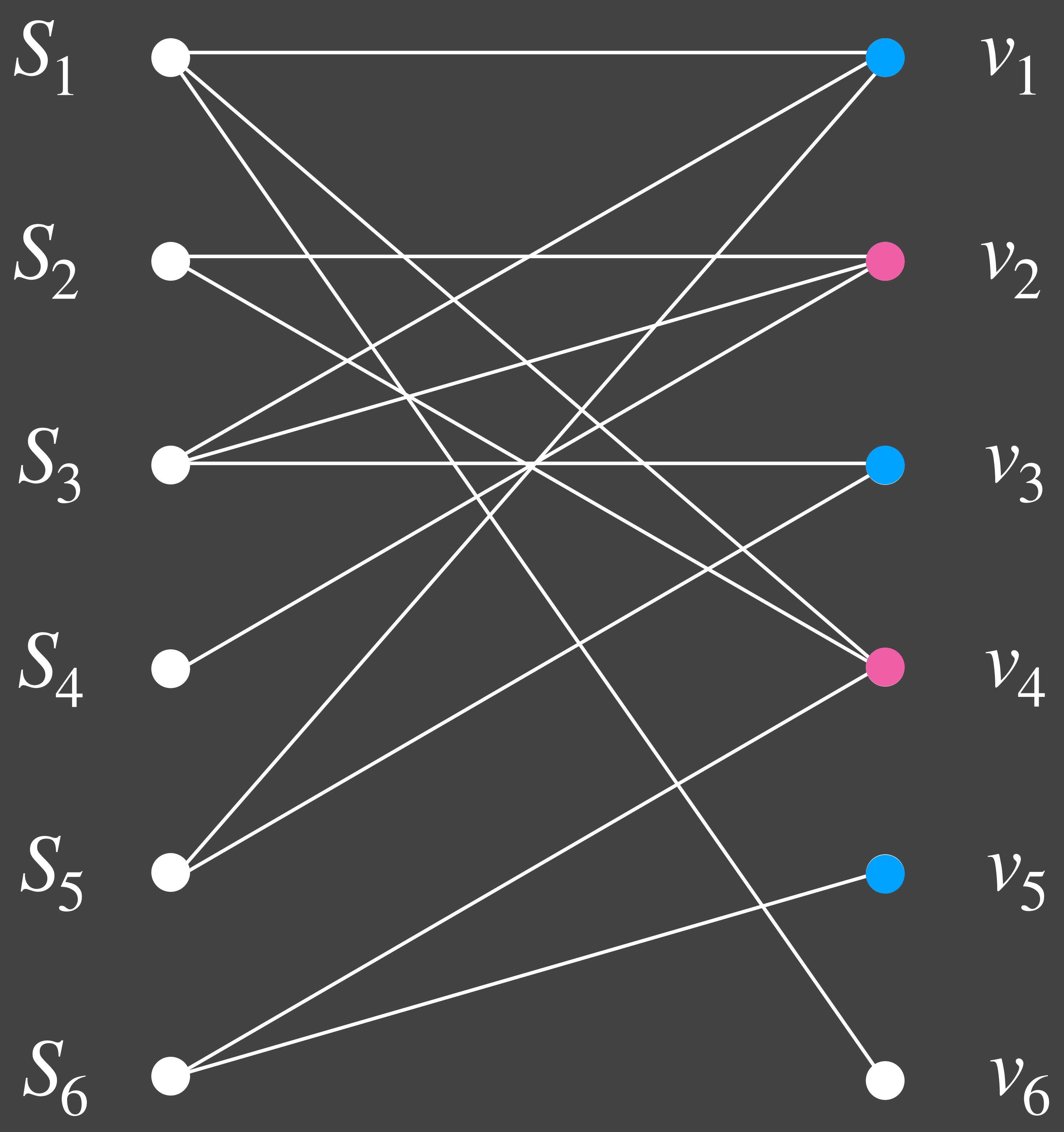

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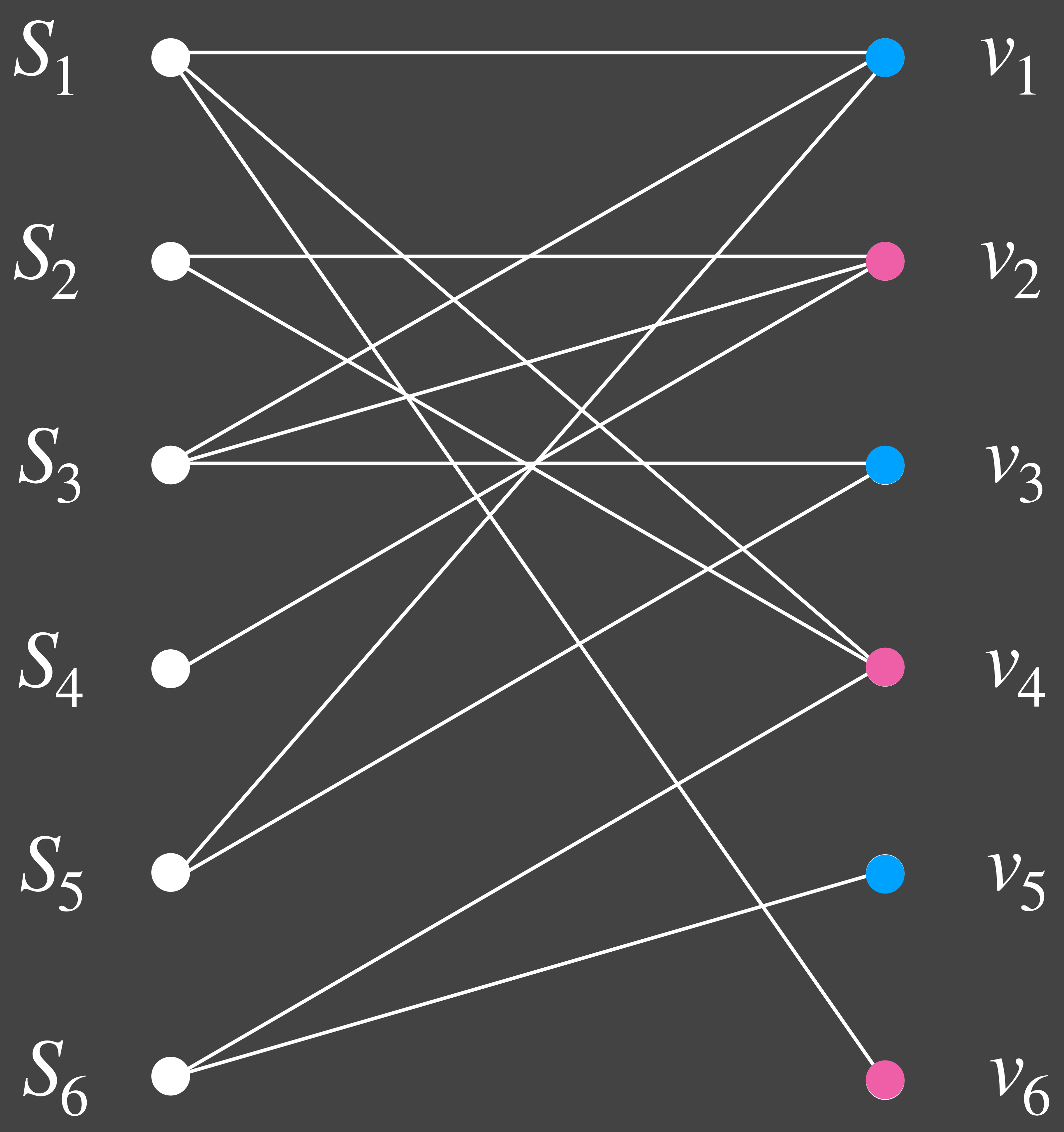

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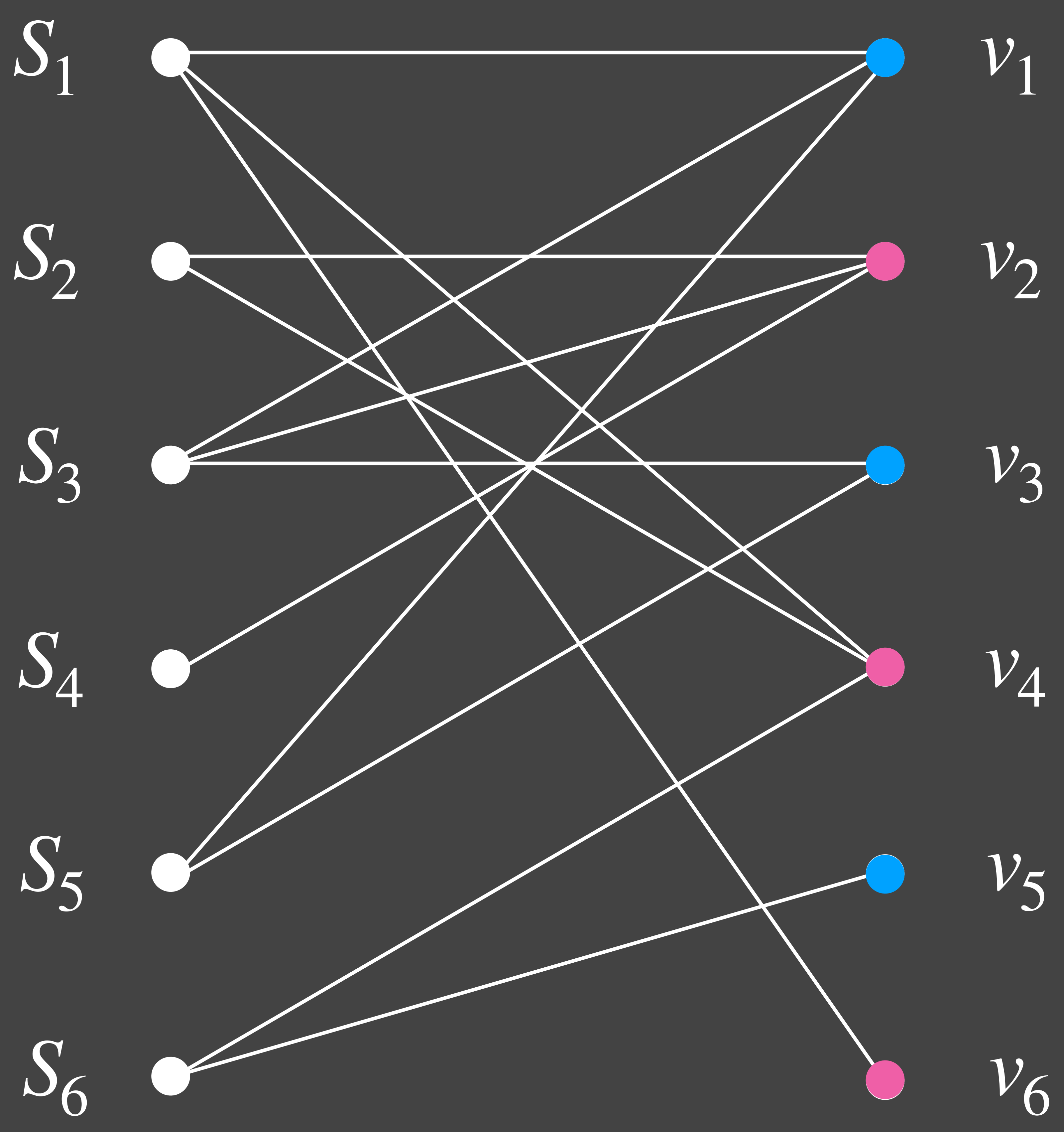

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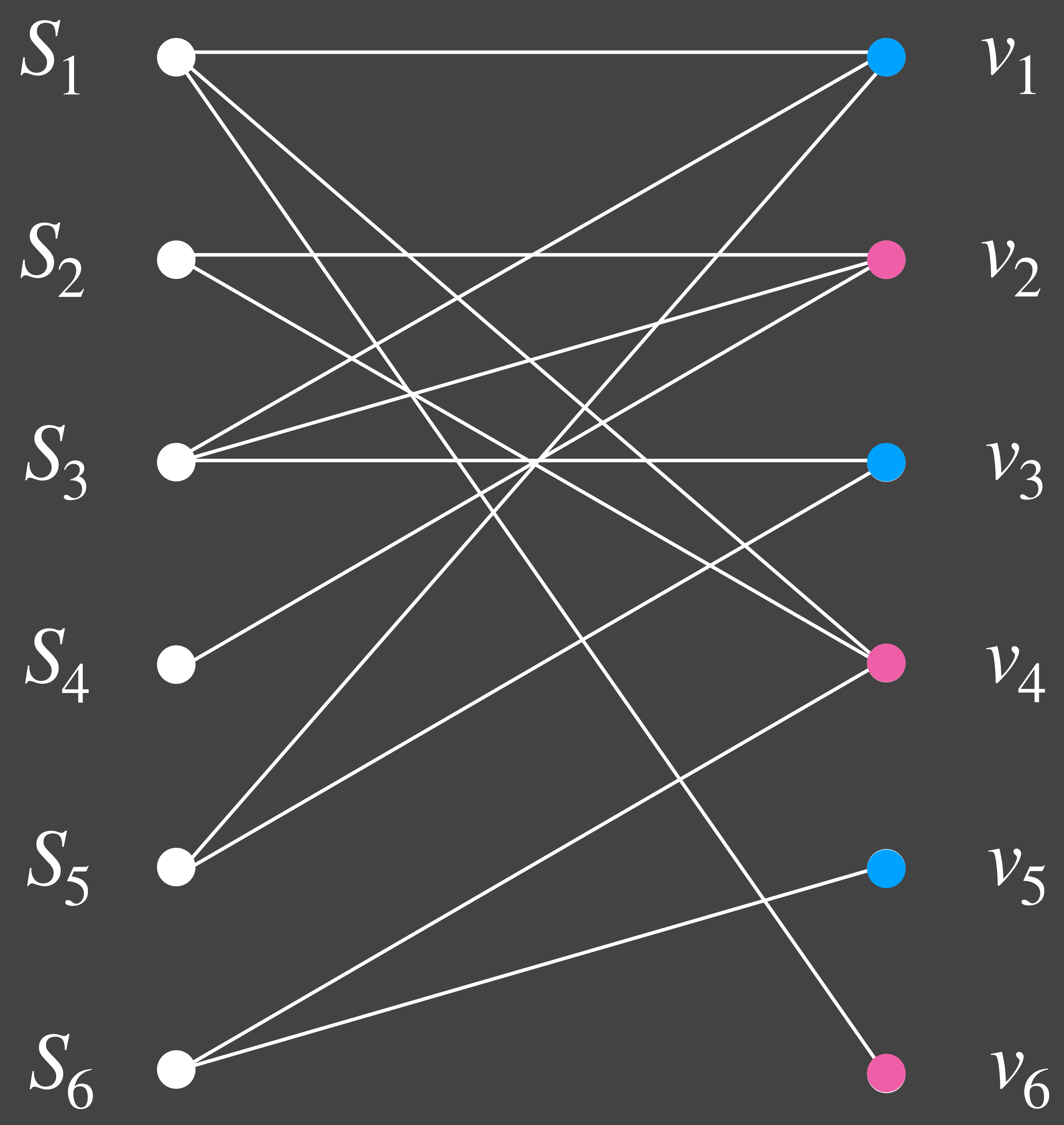

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If v^t pink, feed to LearnOrCover.

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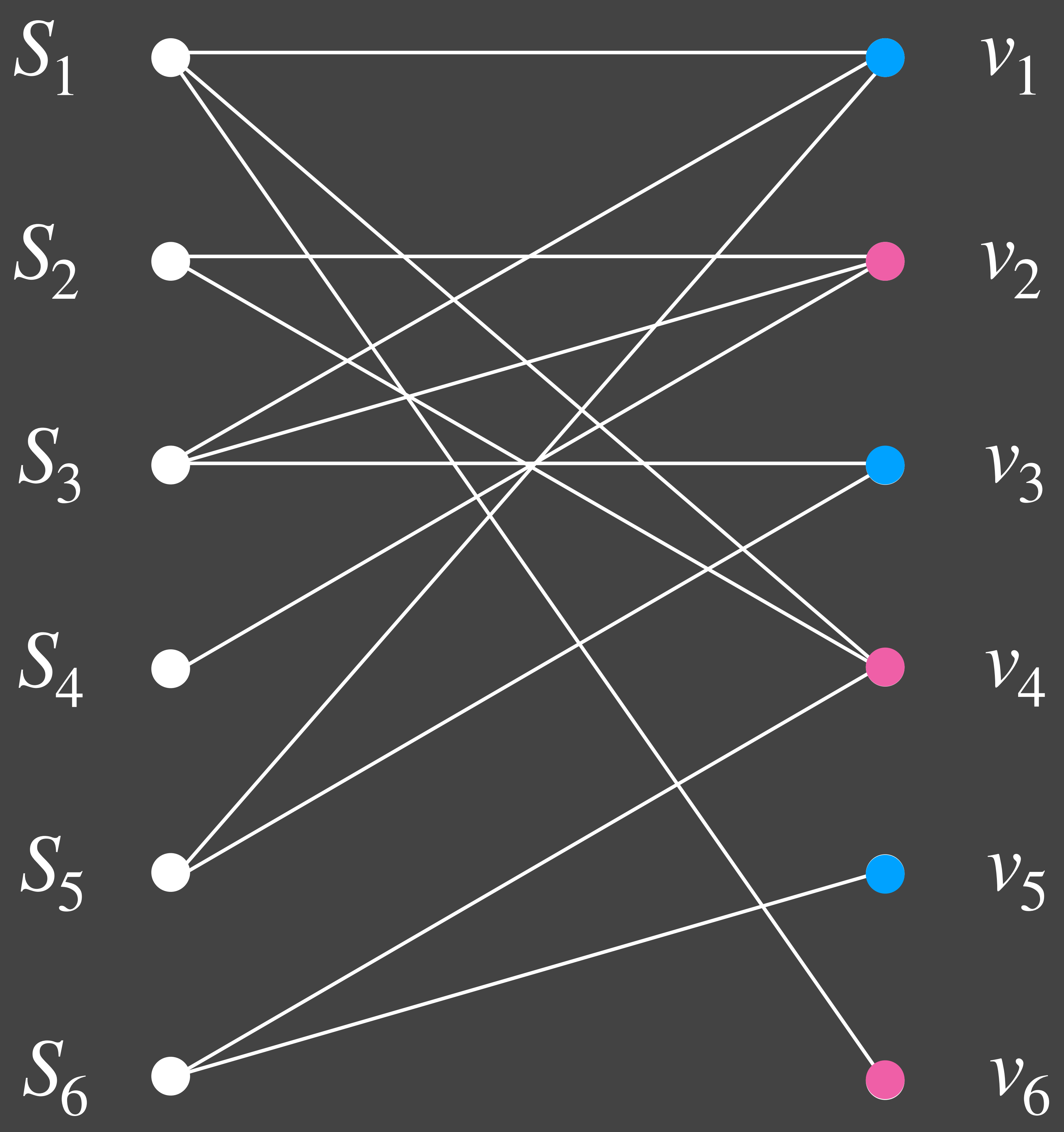

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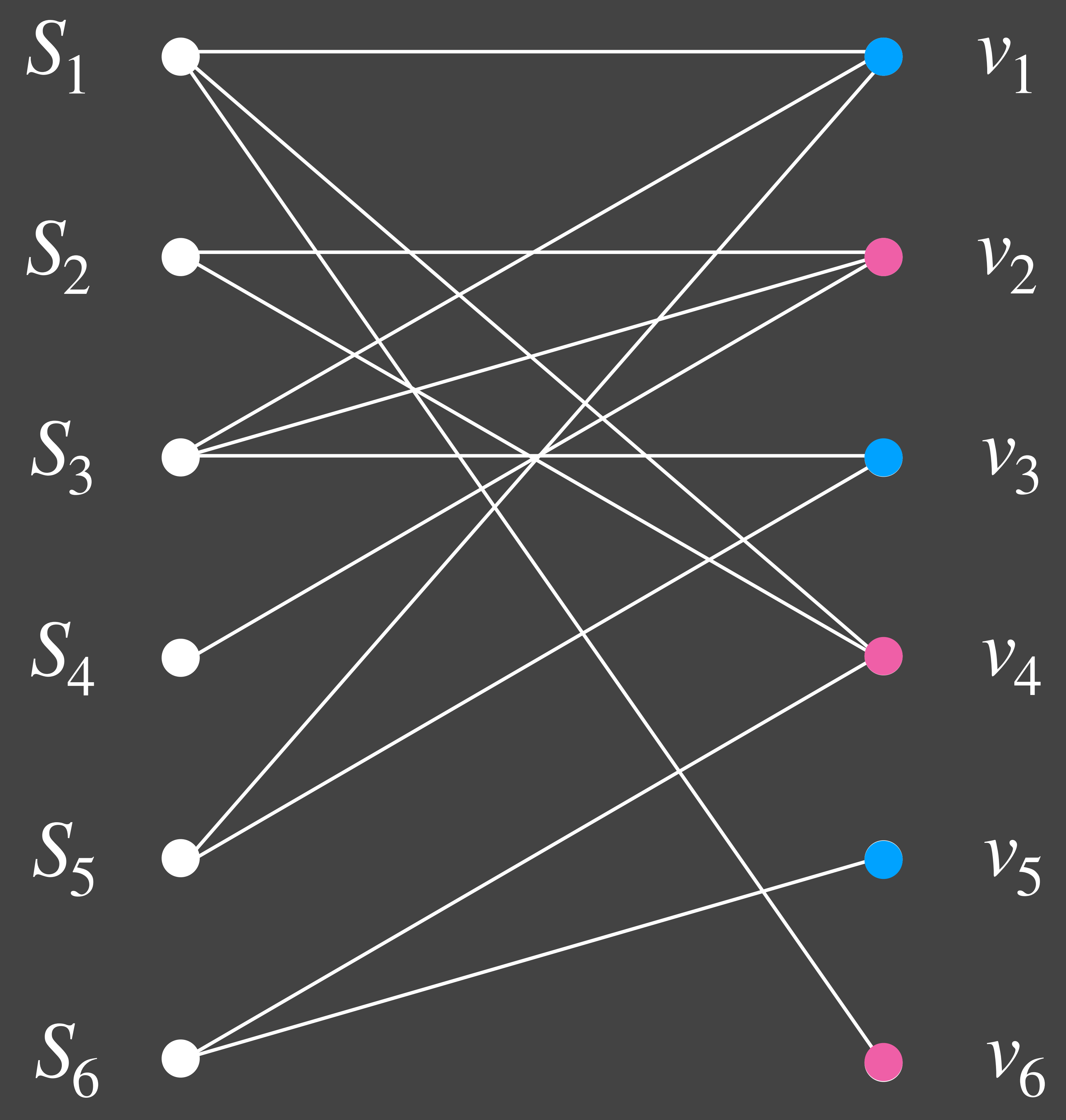

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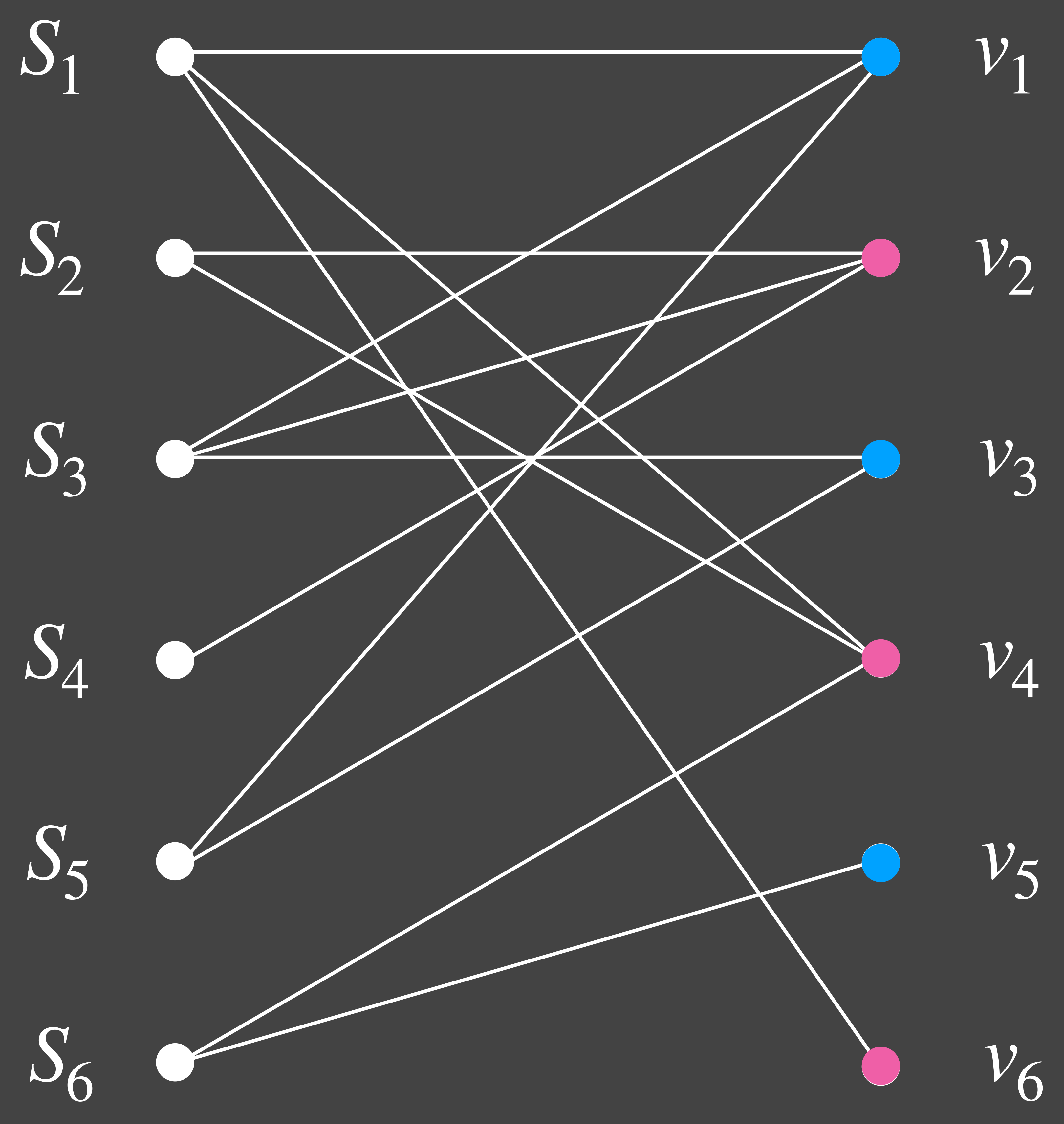

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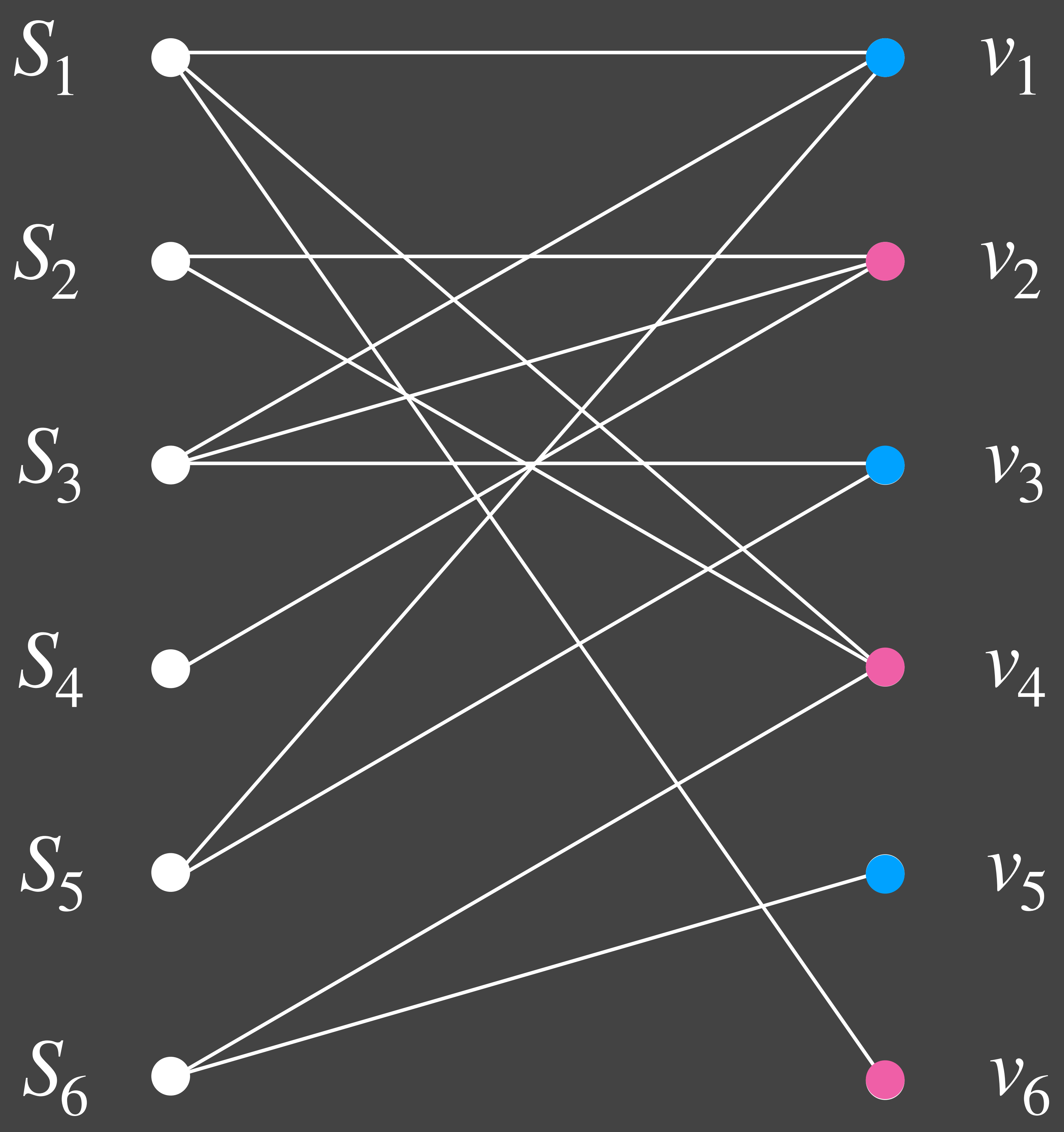

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Claim 2: If v^t uncovered, then $E[\Delta\Phi] \leq -\Omega\left(\frac{1}{k}\right)$.

Reduction to LearnOrCover!

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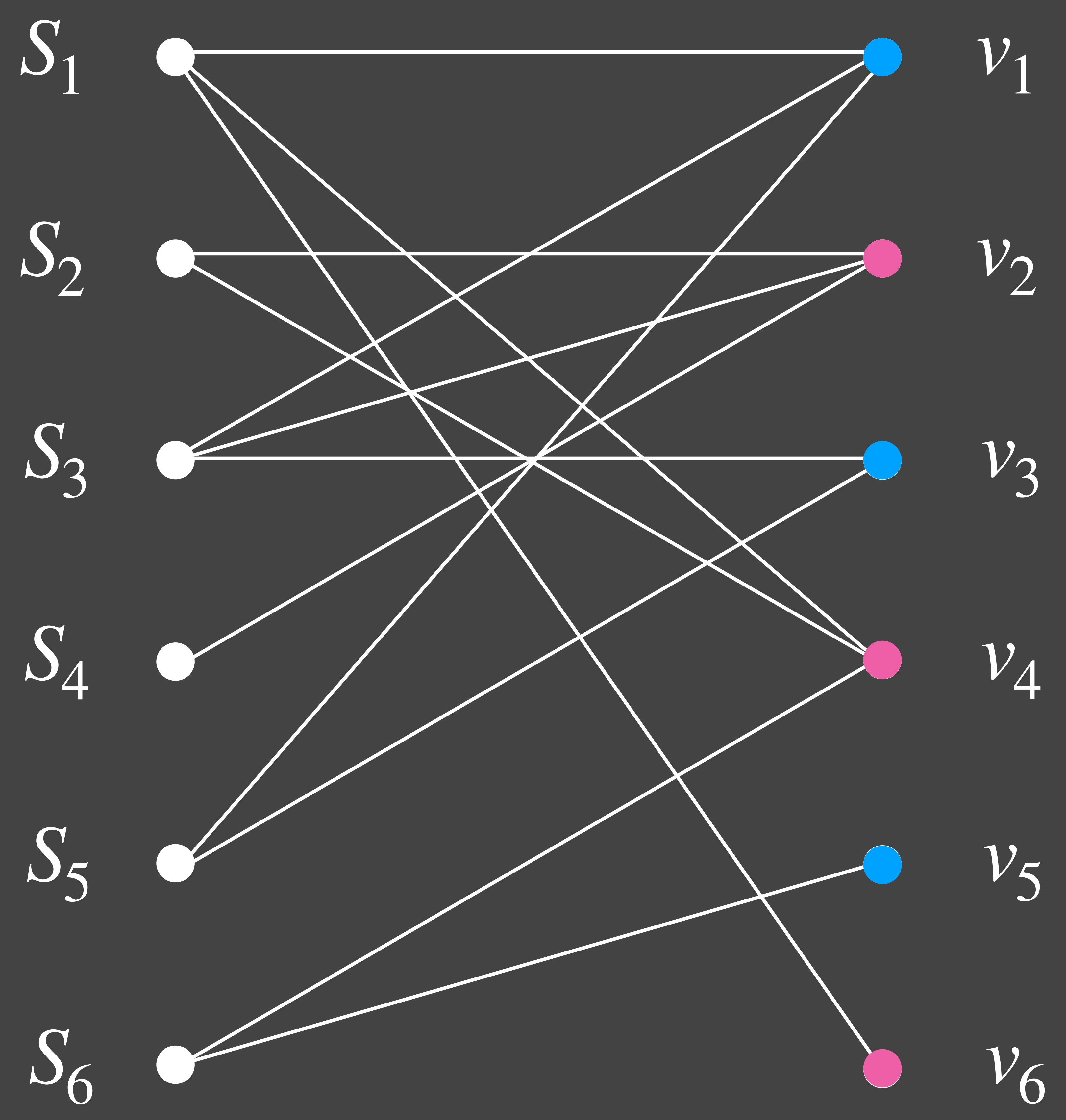

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Φ only deceases during pink steps (so with prob. 1/2), but still $E[\Delta\Phi] \leq -\Omega\left(\frac{1}{k}\right)$.

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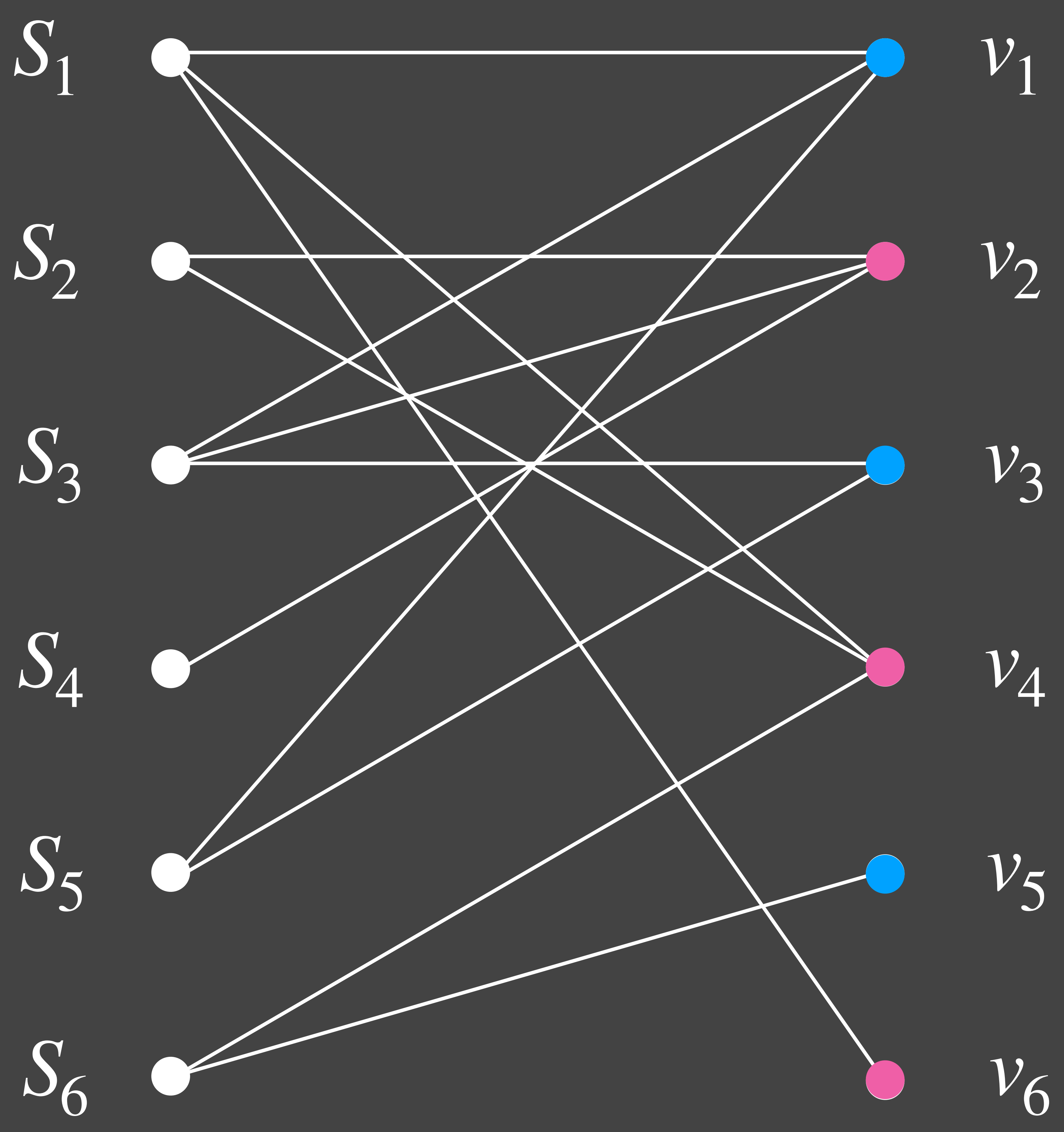

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Universality

<u>Idea</u>: Reduction!

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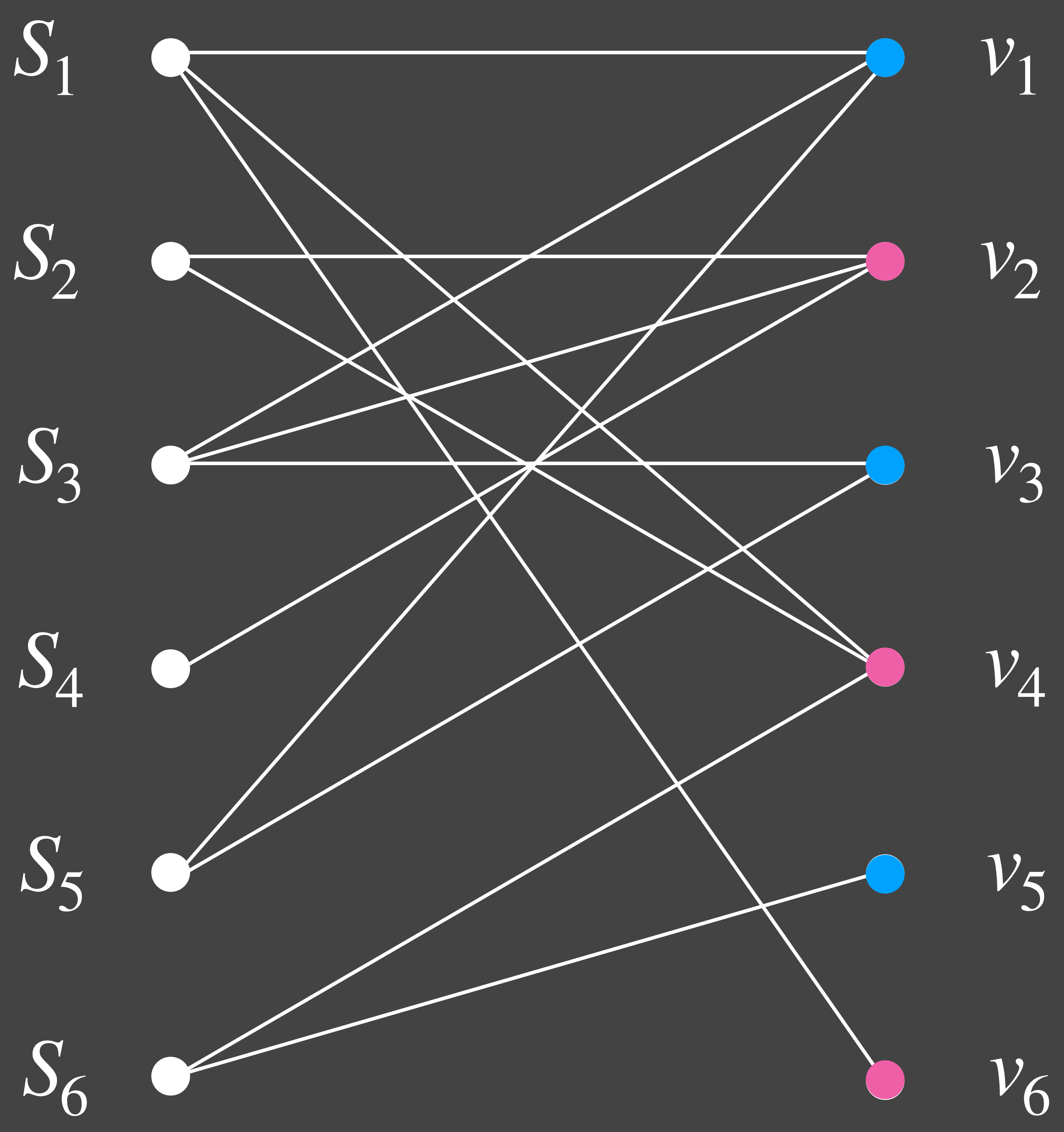
2. Buy arbitrary sets for remaining elements.



Universality

Idea: Reduction!

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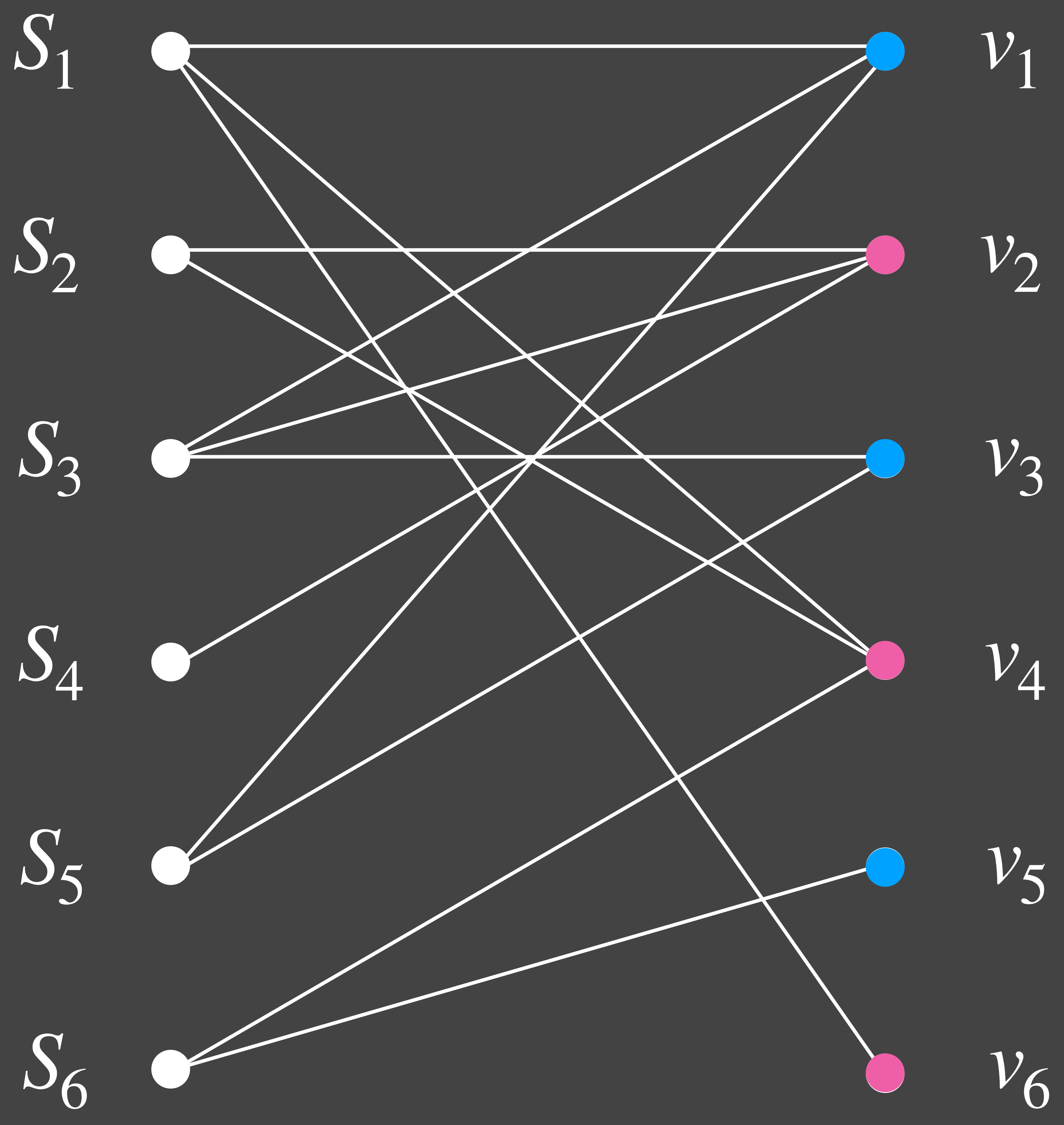

Can build map $f : \mathscr{U} \to \mathscr{S}$ ***before*** we see any actual elements.

Universality

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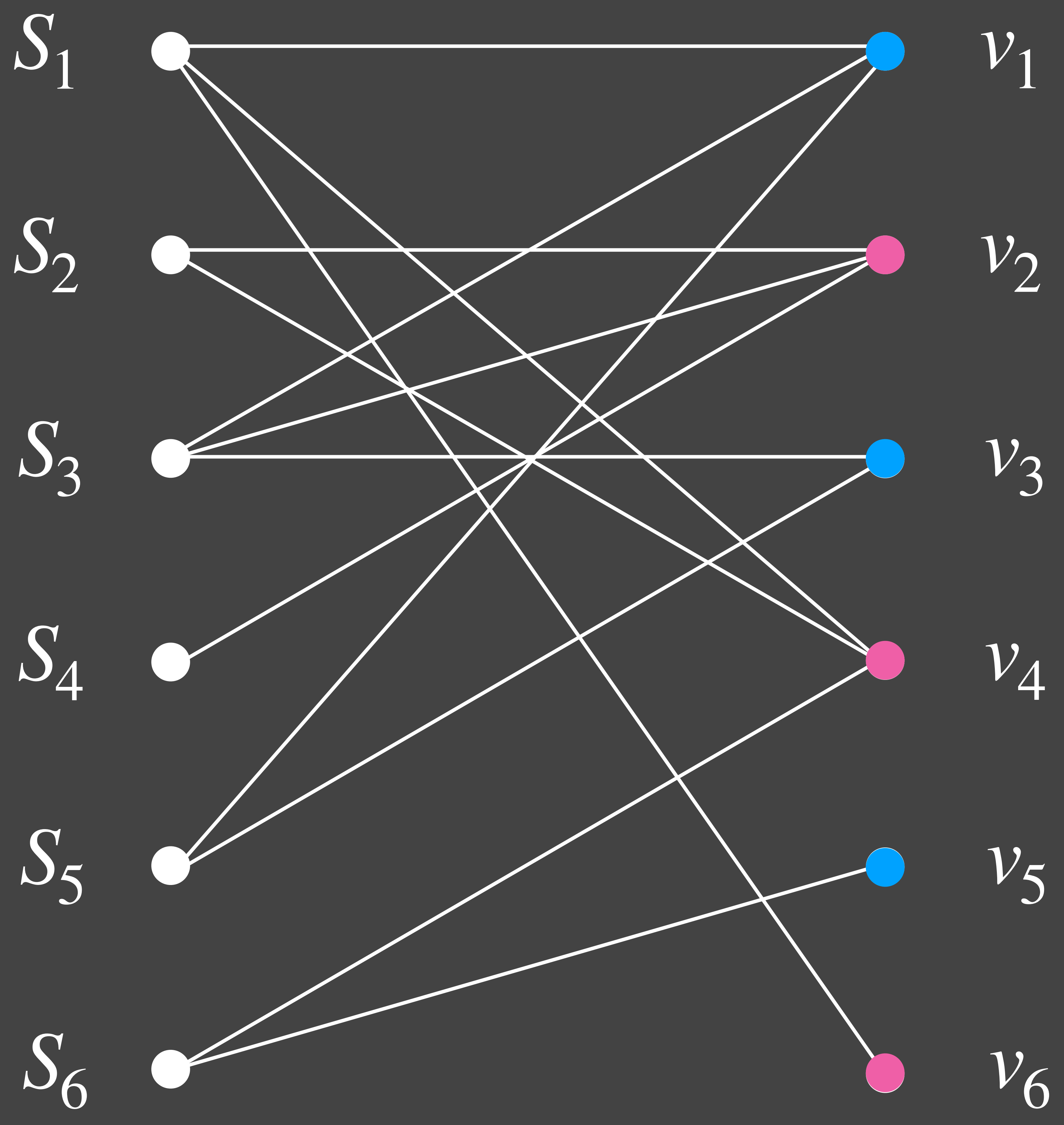

Can build map $f : \mathcal{U} \to \mathcal{S}$ **before** we see any actual elements.

When $u \in \mathcal{U}$ arrives, commit to buying $f(u)$!

Universality

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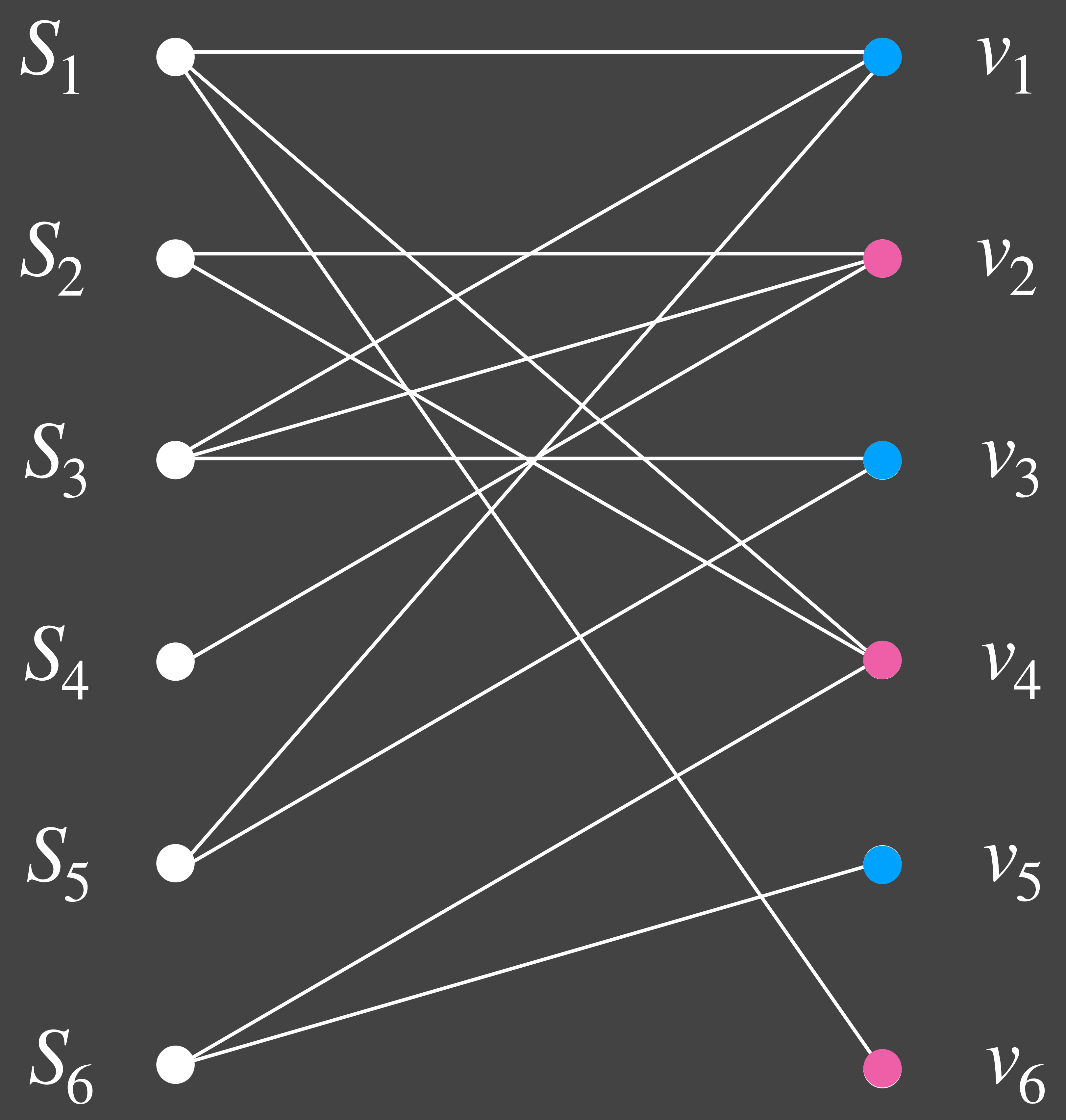

Can build map $f : \mathscr{U} \to \mathscr{S}$ ***<u>before</u>*** we see any actual elements.

When $u \in \mathscr{U}$ arrives, commit to buying $f(u)$!

Our result shows only need $O(n)$ samples to build this map.

Talk Outline

Intro

Secretary

- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time

➡ (Single Sample) Prophet

Conclusion & Extensions

Talk Outline

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➡ Conclusion & Extensions

Conclusion

Conclusion

<u>Theorem</u>: $O(\log mn)$*-comp. algo for RO Covering IPs.*

Conclusion

Theorem: $O(\log mn)$*-comp. algo for RO Covering IPs.*

Theorem: $O(\log mn)$*-comp. algo for Prophet Covering IPs.*

Conclusion

<u>Theorem</u>: *$O(\log mn)$-comp. algo for RO Covering IPs.*

<u>Theorem</u>: *$O(\log mn)$-comp. algo for Prophet Covering IPs.*

\+ Single-Sample!
\+ Universal!

Conclusion

<u>Theorem</u>: $O(\log mn)$*-comp. algo for RO Covering IPs.*

<u>Theorem</u>: $O(\log mn)$*-comp. algo for Prophet Covering IPs.*

<u>Theorem</u>: Same results *for Non-metric facility location.*

+ Single-Sample!
+ Universal!

Conclusion

Theorem: $O(\log mn)$*-comp. algo for RO Covering IPs.*

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Theorem: $\Omega(\log m \log f(\mathcal{N}))$ for RO submodular cover.

+ Single-Sample!
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Open Questions:

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Open Questions:

Does the LearnOrCover idea lend itself to other problems?

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Harder covering problems? Covering IPs w/ box constraints?

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Theorem: $\Omega(\log m \log f(\mathcal{N}))$ for RO submodular cover.

Open Questions:

Does the LearnOrCover idea lend itself to other problems?
Harder covering problems? Covering IPs w/ box constraints?
Unified theory? Reinterpret old RO results as LearnOrCover?

Thanks!

Backup Slides

Online Covering IPs

$$\min \; c^\intercal x$$

$$a_1^\intercal x \geq 1$$
$$a_2^\intercal x \geq 1$$
$$a_3^\intercal x \geq 1$$
$$a_4^\intercal x \geq 1$$
$$a_5^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\min \quad c^\intercal x$$

$$x \in \mathbb{Z}^m_{\geq 0}$$

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Goal: Maintain feasible solution x that is *monotonically* increasing.

Online Covering IPs

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<ins>Goal</ins>: Maintain feasible solution x that is *monotonically* increasing.

Set Cover is the special case where constraint matrix A is 0/1.

LearnOrCover for non-unit costs

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Main issue: # uncovered elements not good proxy for cost.

LearnOrCover for non-unit costs

Main issue: # uncovered elements <u>not</u> good proxy for cost.

LearnOrCover

Init. $x_S \leftarrow 1/m$.

@ time t, element v arrives:

  If v covered, do nothing.

  Else:

    (I) Buy every set R w.p. x_R.

    (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.

    Renormalize $x = x/\|x\|_1$.

    Buy arbitrary set to cover v.

LearnOrCover for non-unit costs

Main issue: # uncovered elements <u>not</u> good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

LearnOrCover

Init. $x_S \leftarrow 1/(c_S \cdot m)$.

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) Buy every set R w.p. $\kappa_v x_R$.

(II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.

Renormalize $x \leftarrow x/\langle c, x \rangle$.

Buy cheapest set to cover v.

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

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<u>Claim 1:</u> $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

LearnOrCover for non-unit costs

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<u>**Claim 2**</u>: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

LearnOrCover for non-unit costs

Main issue: # uncovered elements <u>not</u> good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune <u>learning</u> & <u>sampling</u> rates as a function of κ_v.

LearnOrCover

Init. $x_S \leftarrow 1/(c_S \cdot m)$.

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) Buy every set R w.p. $\kappa_v x_R$.

(II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v / c_S} \cdot x_S$.

Renormalize $x \leftarrow x / \langle c, x \rangle$.

Buy cheapest set to cover v.

<u>**Claim 1:**</u> $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

<u>**Claim 2**</u>: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

<u>**Claim 3**</u>: $E[\Delta \text{cost(ALG)}] = O(\kappa_v)$.

LearnOrCover for non-unit costs

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$$\Rightarrow \quad E[\Delta\Phi + \Delta\text{cost(ALG)}] = 0.$$

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$$E[\text{cost(ALG)}] \leq \Phi(0).$$