Wooters' Concurrence and the Entanglement of Formation

See end of document for references.

The entanglement of formation is a measurement of entanglement, and is defined as "the resources needed to create a given entangled state". [1]

Wooters defines the entanglement of formation in terms of a quantity called "Concurrence" [2] as :

$$E(\psi) = \mathcal{E}(C(\psi)).$$

where $E(\psi)$ is the entanglement of formation
$C(\psi)$ is the concurrence

$$\text{and } \mathcal{E}(C) = h\left(\frac{1+\sqrt{1-C^2}}{2}\right), \cancel{h(x)=x}$$

$$h(x) = -x\log_2 x - (1-x)\log_2(1-x).$$

This can be extended to a mixed state of two qubits as

$$E(\rho) = \mathcal{E}(C(\rho))$$

where ρ is the density matrix.

Wooters states [1] that $\mathcal{E}(C)$ ranges from 0 to 1, and C ranges from 0 to 1, so <u>concurrence is effectively its own measure of entanglement</u>. Thus, we shall calculate concurrence based off this exact formula Wooters provides for a system of two qubits:

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \ldots\}$$

where λ are the sqrt of eigenvalues of the matrix $\rho\tilde{\rho}$ (for a non-hermitian ρ.

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nb λ_i are non-negative, real.

· $\tilde{\rho}$ is the spin-flipped state;

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

· A very important point is that Wooters' calculates concurrence (and hence entanglement of formation) for a system of two qubits. Our system is a TLS coupled to a Harmonic Oscillator; there is only one qubit in this system (the TLS) as the HO acts effectively as an environment.

· Thus, Wooters' exact formula for concurrence does not apply to our system — a more general formulation needs to be found.

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[1] W. Wooters. "Entanglement of formation of an Arbitrary State of Two Qubits." *Physical Review Letters*, 1997.

[2] W. Wooters. "Entanglement of Formation and Concurrence. Quantum Information and Computation, Vol 1, No 1 27-44, 2001.