Entanglement Rényi α Entropy

Definition

- For a pure state $|\psi\rangle$ with density operator ρ, the entanglement of this pure state can be quantified by the Rényi α entropy of one of the reduced operators [1]:

$$R_\alpha(\psi) \equiv \frac{1}{1-\alpha} \cdot \log_2(\mathrm{tr}(\rho_B^{\alpha}))$$

where α determines which part of the entanglement spectrum is emphasised.

For a mixed state with density operator ρ:

$$R_\alpha(\rho) \equiv \min_{\{p_k, \psi_k\}} \sum_k p_k R_\alpha(\psi_k)$$

- The Rényi Entropy is a measure of entanglement and provides more information on entanglement since it gives a continuous spectrum of entanglement, parametrised by α.

In Relation to the Von Neumann Entropy

- In the $\alpha \to 1$ limit, the Rényi entropy ~~reduces~~ for pure states reduces to the Von Neumann entropy:

$$R_{\alpha \to 1}(\psi) = -\mathrm{tr}(\rho_B \log_2(\rho_B))$$

- Thus, we can see how the Rényi entropy is a general, spectral representation of entanglement.

Two-Qubit Systems

- [1] defines the Rényi entropy for a system of two qubits using Wooters' concurrence. This is even further the evidence of its usefulness due to how general of a definition it is.

$$R_\alpha(\psi) = \frac{1}{1-\alpha} \log_2 (\lambda_+^\alpha + \lambda_-^\alpha)$$

$$\equiv \Omega(C, \alpha)$$

where $\lambda_\pm = (1 \pm \sqrt{1-C^2})/2$, and are the eigenvalues of the reduced density matrix ρ_B.

C is concurrence.

- For a mixed state of two qubits, [1] goes on to give Wooters' definition of concurrence.

<u>Relevance</u>

- In our case, we are currently looking at pure density operators of a TLS-coupled to a Harmonic oscillator. Thus only the definition below applies (since as mentioned before, we are not looking at a two-qubit system):

$$R_\alpha(\psi) = \frac{1}{1-\alpha} \log_2 (\mathrm{tr}(\rho_B^\alpha))$$