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Erratum: Numerical estimation of the relative entropy of entanglement [Phys. Rev. A 82, 052336 (2010)]

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It has come to the attention of the authors that the MATLAB code[1] provided in the original paper does not work as stated. Although the code presented in the original paper was only intended to act as a guide that other researchers might follow for writing their own codes, for the sake of completeness, the present Erratum presents a fully functional code for correctly estimating the relative entropy of entanglement (REE).

The original code relies on SEDUMI [1] as its internal solver for semidefinite programming. Although the code was intended to be used with SEDUMI version 1.1R3, this version is no longer available on the SEDUMI web site. Unfortunately, newer versions of SEDUMI (after version 1.1R3) that are available do not correctly handle Hermitian matrices and complex variables. As a result of this bug, the code in the original paper only works properly for input matrices whose entries are real valued.

To correct these shortcomings, the code for numerical estimation of the REE has been revised to implement the algorithm using CVX [2] (a MATLAB-based modeling system for convex optimization) instead of SEDUMI. The new MATLAB code may be found in Ref. [3]. Other modifications have been made to the code that make it easier to understand. In particular, CVX allows one to more easily define the positive partial transpose (PPT) constraint when setting up a semidefinite program. The requirement $X^\Gamma \geqslant 0$ may be given directly as a constraint, rather than manually defining the semidefinite cone to include only matrices that are positive under partial transpose as is performed in the original paper. The `PartialTranspose` routine [4] from QETLAB [5] is necessary for the code presented here.

It must be shown that the present code does, in fact, correctly compute the REE of input density matrices. One way to check the validity of the numerical estimates is to compare the computed value of the REE to the negativity, another well-known entanglement monotone. This is defined as

$$N(\rho) = \frac{\mathrm{Tr}\,|\rho^\Gamma| - 1}{2}.$$

In Fig. 1, the values of the negativity and the REE are compared for 1000 randomly generated density matrices of two qubits. Here, the random density matrices were generated according to the distribution induced by the Hilbert-Schmidt measure as shown in Refs. [6,7]. The plot in Fig. 1 is compatible with previous works (such as in Ref. [8]).

Additionally, the value of the REE determined by this algorithm can be compared for states whose true value of the REE is "known." This can be performed using methods to determine a closed formula for the inverse problem [9–11]. Random entangled states with known REE can be generated according to the following procedure.

(1) Generate a 4×4 random density matrix A of two qubits with rank 3. This is performed by generating a 4×3 Ginibre random matrix [7] G and setting $A = \frac{GG^\dagger}{\mathrm{Tr}[GG^\dagger]}$. Matrix A will have three positive eigenvalues and one zero eigenvalue.

(2) Take the partial transpose of A. If A^Γ is positive, set $\sigma = A^\Gamma$. Otherwise, repeat step 1. Note that σ generated this way lies on the boundary of the PPT states.

(3) Let $|\varphi\rangle$ be the normalized eigenvector in the null space of σ^Γ. Define the matrix $\phi = I - |\varphi\rangle\langle\varphi|^\Gamma$.

(4) Consider the family of states,

$$\rho(\sigma,\phi,x) = (1-x)\sigma + x L_\sigma^{-1}(\phi), \quad x \in (x, x_{\max}],$$

where $x_{\max}$ is the largest value of x such that $\rho(\sigma,\phi,x)$ is positive semidefinite and L_σ^{-1} is the linear operator defined in Refs. [10,11]. Then each $\rho = \rho(\sigma,\phi,x)$ is an entangled state whose closest separable state is σ. Furthermore, the REE

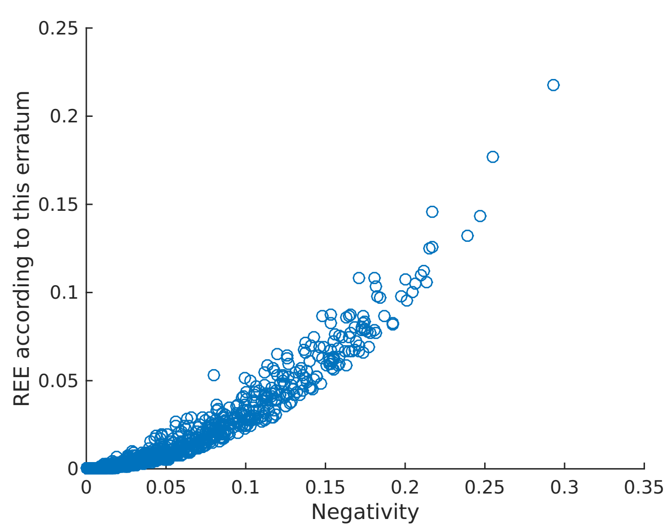

FIG. 1. (Color online) Comparison of negativity and REE (as estimated using the algorithm in the present paper) for 1000 random density matrices of two qubits.

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[1]This is listed as Ref. [12] in the original paper but can presently be found at http://people.ucalgary.ca/∼yzinchen/relEntropy.m

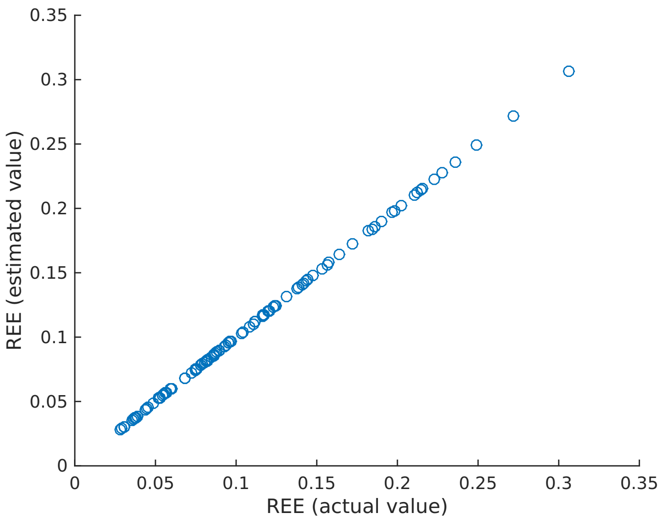

FIG. 2. (Color online) Comparison of actual and estimated values of the REE for 100 randomly generated density matrices of two qubits. Random density matrices with known REE are generated according to the outlined procedure.

of ρ is given by

$$E_R(\rho) = \mathrm{Tr}[\rho \ln \rho] - \mathrm{Tr}[\rho \ln \sigma].$$

Figure 2 shows the actual values of the REE and estimated values determined by the numerical estimation algorithm in the present Erratum for 100 randomly generated density matrices of two qubits where the matrices are generated according the procedure outlined above. In all cases, the estimated value was found to be within 10^{-5} of the actual value.

The present code now correctly implements the algorithm for computing the REE as given in the original paper. This works for a bipartite density matrix with subsystems of any size, not just for systems of two qubits, although computation time increases dramatically with increasing the size of the subsystems. As before, the code is written to be primarily transparent and readable, rather than computationally efficient.

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[1] http://sedumi.ie.lehigh.edu/
[2] Available for free at http://cvxr.com/cvx/
[3] M. W. Girard, https://github.com/markwgirard/relEntropy
[4] https://github.com/nathanieljohnston/QETLAB/blob/master/PartialTranspose.m
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