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Quantum Collapses and Revivals in a Quantized Trap.

C. A. BLOCKLEY (*), D. F. WALLS (*) and H. RISKEN (**)
(*) *Department of Physics, University of Auckland, Private Bag Auckland, New Zealand*
(**) *Theoretische Physik, Universität Ulm - Ulm, W-7900 Germany*

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Abstract. – A single two-level atom, with quantized centre-of-mass motion, is constrained to move in a one-dimensional harmonic potential, while interacting with a single-mode classical travelling light field. When the atoms's centre-of-mass motion is in a coherent state, we show that the atomic inversion exhibits collapses and revivals. Whereas in the Jaynes-Cummings model this behaviour occurs due to the discrete nature of the light field, in our case the behaviour is due to the discrete nature of the vibrational trap states. The Q-function for the external motion is also calculated and shown to break into two peaks in the collapse region. Finally the parameter ranges under which the collapses and revivals can be observed are discussed, as well as the possibility of an experiment.

In this letter we wish to look at a simple trapping model, in which a two-level atom or ion is constrained to move in a harmonic potential, while interacting with a single-mode classical light field. This simple model approaches the situation of ions in a Penning or Paul trap [1, 2]. Recently [3] it has been shown that a single ion can be cooled down to its zero-point vibrational energy in such a trap. It is important, therefore, that the centre-of-mass motion of the atom or ion be treated quantum mechanically (see Javanainen and Stenholm [4]).

In this model the Schrödinger equation leads to a set of linear differential equations which couple the probability amplitudes for the different vibrational states. In the Lamb-Dicke regime, these equations decouple, as only the interaction between nearest neighbours is significant. The model now takes on a form similar to the Jaynes-Cummings model (JCM) [5] except that the role of the quantized radiation field is replaced by the atom's quantized centre-of-mass motion. If the field driving the atom in the trap is tuned to one of the vibrational sidebands of the atomic transition and, for weak spontaneous emission, there are Rabi oscillations between atomic excited and ground states. We shall show that when the atom's motion in the harmonic potential is in a coherent state, there are collapses and revivals in the atomic inversion, due to dephasing and rephasing between these Rabi oscillations [6].

The trapping model consists of an ideal two-level atom (or ion) of mass m constrained to move in a one-dimensional harmonic potential. x and p are the atom's position and momentum operators, respectively. σ_z, σ^+ and σ^- are the internal atomic operators. The atom is taken to be interacting with a classical single-mode travelling light field of frequency ω_L, which is detuned from the atomic transition (frequency ω_a) by $\Delta = \omega_a - \omega_L$. The Rabi frequency is Ω.

This simple model can be described by the following Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{1}{2} m\nu^2 x^2 + \frac{1}{2}\hbar\omega_a \sigma_z + \\ + \frac{1}{2}\hbar\Omega(\exp[ikx]\sigma^+ \exp[-i\omega_L t] + \exp[-ikx]\sigma^- \exp[i\omega_L t]). \qquad (1)$$

The first two terms correspond to the atom's kinetic and potential energy in the trap, respectively, ν being the trap frequency. The final term gives the interaction between the atom and the light field, where k is the wavevector of the light field.

Because the trap potential is harmonic, the position and momentum operators can be written in terms of creation and annihilation operators for the trap quanta:

$$x = \sqrt{\frac{\hbar}{2\nu m}}(a + a^\dagger), \qquad p = i\sqrt{\frac{\hbar\nu m}{2}}(a^\dagger - a). \qquad (2)$$

In a rotating frame the Hamiltonian now takes the form

$$H = \hbar\nu\left(a^\dagger a + \frac{1}{2}\right) + \frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\Omega(F\sigma^+ + F^*\sigma^-), \qquad (3)$$

where $F \equiv \exp[ikx] = \exp[i\varepsilon(a^\dagger + a)]$.

The parameter ε is given by $\varepsilon = (E_r/E_\nu)^{1/2}$, where $E_r = \hbar^2 k^2/2m$ is the classical recoil energy of the atom and $E_\nu = \hbar\nu$ is the energy of a trap quantum. In the classical limit, with trap states very close together, ε is large, and the absorption or emission of a photon will always cause some change in the vibrational state of the atom. In the nonclassical or Lamb-Dicke limit of small ε, the trap states are well spaced, and many photons may need to be absorbed or emitted before the atom changes vibrational states.

The state vector for the system can be expanded in a superposition of external and internal states, $|\psi\rangle = \sum_{m=0}^{\infty} (g_m|g, m\rangle + e_m|e, m\rangle)$, where g and e label atomic ground and excited states, respectively, and m is the number of vibrational quanta. g_m and e_m are the probabilities that the atom be found in the ground and excited state, respectively, and in the m-th trap state. The equations of motion for the probability amplitudes are, for $m = 0, \ldots, \infty$,

$$\begin{cases} \dot{g}_m = -i\omega_m^g g_m - \dfrac{i}{2}\Omega \sum\limits_{n=0}^{\infty} u_{mn}^g e_n, \\ \dot{e}_m = -i\omega_m^e e_m - \dfrac{i}{2}\Omega \sum\limits_{n=0}^{\infty} u_{mn}^e g_n. \end{cases} \qquad (4)$$

The free oscillation frequencies are $\omega_m^{e,g} = (m + 1/2) \pm (1/2)\Delta$, where the plus (minus) corresponds to the excited (ground) state. The $u_{m,n}^{g,e}$ are matrix elements of F between the number states and give the strength of the coupling between different vibrational states. They can be evaluated by writing F as a series in the operators a and $a^\dagger$, and acting with these left and right on the number states. The result for $n \geq m$ is

$$u_{mn} \equiv u^{\mathrm{e}}_{mn} = \exp[-\varepsilon^2/2](i\varepsilon)^{n-m} \sqrt{\frac{m!}{n!}} L_m^{n-m}(\varepsilon^2), \tag{5}$$

where L_m^{n-m} is an associated Laguerre polynomial. It can be seen that $u^{\mathrm{g}}_{mn} = (u_{nm})^*$ and that u_{mn} is symmetric on interchange of its indicies.

The Q-function can be calculated and this gives information on the position and momentum distributions of the atom. For a system described by the density matrix ρ, the Q-function is given by $Q(\alpha) = (1/\pi)\langle\alpha|\rho_{\mathrm{red}}|\alpha\rangle$, where ρ_{red} is the reduced density matrix for the external degrees of freedom, and $|\alpha\rangle$ is a coherent state. The Q-function can be written as

$$Q(\alpha) = \exp[-|\alpha|^2]\left[\left|\sum_{m=0}^{\infty}\frac{\alpha^m}{\sqrt{m!}}g_m\right|^2 + \left|\sum_{m=0}^{\infty}\frac{\alpha^m}{\sqrt{m!}}e_m\right|^2\right], \tag{6}$$

where the coherent state has been expanded in terms of the number states.

We look further at the analogy between the trapping model and the JCM by considering transitions which involve the exchange of one trap quantum. In such a case the driving field must be tuned to the first vibrational sideband, $\Delta = \pm\nu$. All other transitions can be neglected if they are oscillating with sufficiently high frequencies. The rotating wave approximation, therefore, requires that $\nu \gg \Omega$.

If these limits are satisfied, (4) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} g_m \\ e_{m+1}\end{bmatrix} = -i\begin{bmatrix} \omega^{\mathrm{g}}_m & -(1/2)\,\Omega u^*_{m,m+1} \\ -(1/2)\,\Omega u_{m+1,m} & \omega^{\mathrm{e}}_{m+1}\end{bmatrix}\begin{bmatrix} g_m \\ e_{m+1}\end{bmatrix} \tag{7}$$

for a heating transition. The eigenvalues are $\lambda = \nu(m+1) \pm (1/2)\mu$, where $\mu(m) = \sqrt{(\nu+\Delta)^2 + \Omega^2|u_{m,m+1}|^2}$ is the effective Rabi frequency. The JCM Rabi frequency, by comparison, is $\mu_{\mathrm{JC}}(n) = \sqrt{\Delta^2 + 4\lambda^2(n+1)}$, where Δ is the detuning between atom and field, λ is the coupling constant and n is the photon number. In the Lamb-Dicke regime μ has, to first order, the same dependence on m, as μ_{JC} has on the number of light quanta. The matrix element $u_{m,m+1}$ can be evaluated from (5) and we obtain $\mu(m) = \sqrt{(\nu+\Delta)^2 + \Omega^2\varepsilon^2(m+1)}$ to first order in ε.

In the JCM the time between subsequent revivals in the atomic inversion is given by $t_{\mathrm{R}} = 2\pi\sqrt{\bar{n}}/\lambda$ where $\bar{n}$ is the average number of photons in the coherent state and Δ has been set to zero [6]. If the first revival is to be resolved, the decay time due to spontaneous emission must be greater than the revival time, $1/\Gamma > t_{\mathrm{R}}$. For the trapping model, λ is replaced by $(1/2)\Omega\varepsilon$ in the Lamb-Dicke limit, giving $\Omega > 4\pi\Gamma\sqrt{\bar{m}}/\varepsilon$, where $\bar{m}$ is the average number of vibrational quanta in the coherent field. It should be noted that, if the system is not operating in the Lamb-Dicke regime, $u_{m,m+1}$ is a quickly oscillating function of m and this tends to wash out the collapses and revivals.

The time evolution of the inversion is calculated from (4). For an initial coherent state the probability amplitudes will be peaked at $\bar{m}$ and will become negligible for sufficiently high values of the vibrational quantum number. Therefore the series of equations in (4) can be truncated and the eigenvalues and eigenvectors obtained numerically.

Figure 1 shows the quantum collapses and revivals occuring in the trapping model. Initially the atom is assumed to be in its atomic ground state, while the centre-of-mass motion is in a coherent state with an average vibrational quantum number $\bar{m}$. A scaled time, $\tau = \Omega t/2\pi$, is used, and also $\tilde{\nu} = \nu/\Omega$ and $\tilde{\Delta} = \Delta/\Omega$. It can be seen that the choice of parameters satisfies the condition $\nu \gg \Omega$.

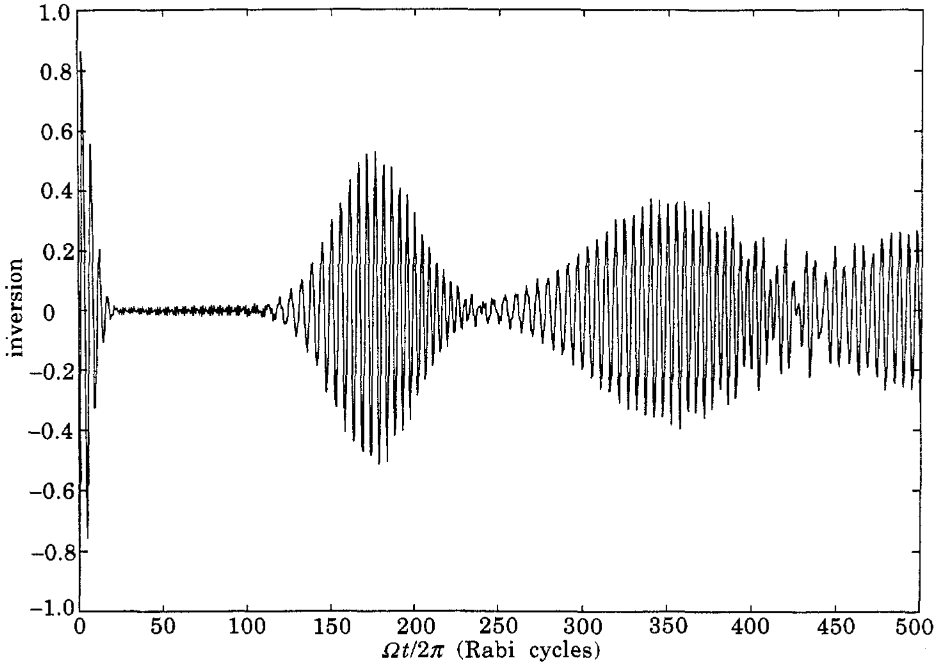

Fig. 1. – Collapses and revivals for the trapping model, for $\varepsilon = 0.05$, $\tilde{\nu} = 50$, $\tilde{\Delta} = -50$, and $\overline{m} = 16$.

Figure 2 shows the Q-function for the external degrees of freedom of the atom in the trap. There is a fast motion in the phase space corresponding to motion at approximately the trap frequency ν (this motion has been removed from the plot). The atom begins in a well localized (coherent) state, but separates into two peaks on the time scale of the first collapse of the inversion. Finally the two peaks begin to interfere and the revival in the inversion occurs. This behaviour has already been investigated for the light field in the JCM [7]. It should be noted that this is an example of a quantum superposition or Schrödinger cat. It has been shown for the JCM that, to a good approximation, the atom and the field are individually in pure states in the middle of the collapse region of the inversion [8]. For the trapping model the splitting of the Q-function shows that the probability distribution has «macroscopically» distinct peaks in the phase space, while [8] shows that this is a true superposition and not a mixture.

In Diedrich *et al.* [3] use is made of the narrow ${}^2S_{1/2}$-${}^2D_{5/2}$ electric-quadrupole transition in ${}^{198}Hg^+$ to carry out sideband cooling. The trap frequency is $\nu = 2.96$ MHz, the decay rate is $\Gamma = 11$ Hz [9] and the wavelength of the transition is $\lambda = 281.5$ nm. These parameters give $\varepsilon \approx 0.07$, and a choice of Ω can be made which satisfies both the conditions $\nu \gg \Omega$ and $\Omega > 4\pi\Gamma\sqrt{\overline{m}}/\varepsilon$.

For an experiment along the lines of Diedrich *et al.* [3] the inversion could be measured by monitoring the spontaneous emission. The spontaneous emission as a function of time would give information on the internal state of the atom. The difficulty with this is that the spontaneous emission is required to be weak in the first place, therefore the measurements would need to be repeated many times. An alternative scheme would utilize quantum multiplication provided by shelving of the ion [10, 11]. In this scheme the atomic ground state is coupled to another excited state $|e'\rangle$ which has a short lifetime. A strong pulse of radiation on this transition will provide a spontaneous emission signal if the ion is initially in

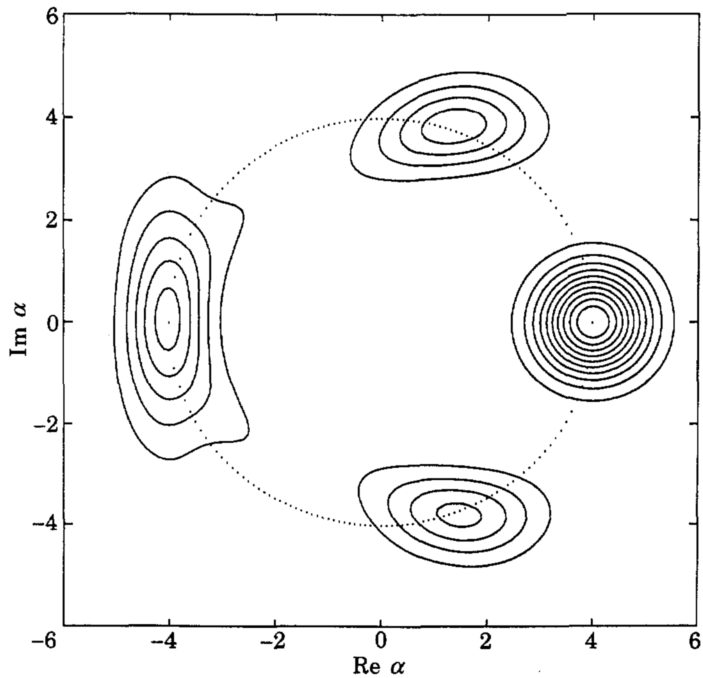

Fig. 2. – Time evolution of the Q-function: $\tau = 0$, 68, 175. Same parameters as for fig. 1.

$|g\rangle$ but no signal if the ion is in $|e\rangle$. The duration of the pulse must be long compared with the lifetime of $|e'\rangle - |g\rangle$, but short compared with the lifetime of $|e\rangle - |g\rangle$. The disadvantage of this scheme is that the ion must be recooled after every measurement. A coherent state in the trap would be prepared by a nonadiabatic change in the origin of the trapping potential for an ion cooled into the ground state of the potential, or by a classical driving field [10].

Before an experiment could be done a full three-dimensional analysis would need to be carried out. In particular the micromotion in the trap would need to be included, and a careful investigation of the effect of using an electric-quadrupole transition.

In this letter we have looked at a simple trapping model for a single atom moving in a one-dimensional harmonic potential, while interacting with a single-mode classical travelling light field. When the atom's motion is initially in a coherent state, the atomic inversion exhibits collapses and revivals, this being due to the discrete nature of the vibrational states in the trap. We have stated and discussed in the limits under which the collapses and revivals could be expected to be seen, and have discussed the possibility of an experiment.

* * *

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