# Examining Formulations of The Phillips Curve

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## I. Introduction

**Context and Motivation**:

The Phillips Curve represents a foundational concept in macroeconomics, establishing a relationship between inflation and unemployment. In its simplest form, it suggests that when unemployment falls below its natural rate, upward pressure on wages and production costs can lead to an acceleration in inflation, and conversely, when unemployment remains persistently high, inflationary pressures tend to subside. Although the initial formulations of the Phillips Curve were grounded in empirical observation, subsequent theoretical work integrated expectations into the equation, underscoring that forward-looking agents, anticipating future policy actions and economic conditions, influence the dynamics between inflation and unemployment. Over time, the Phillips Curve has evolved from a purely empirical relationship into a critical tool for understanding macroeconomic trade-offs, modeling inflation dynamics, and shaping the lens through which policymakers evaluate labor market conditions and price stability.

A comprehensive understanding of the Phillips Curve is pivotal for effective macroeconomic policymaking and forecasting. Central banks, for instance, frequently rely on Phillips Curve estimates to inform decisions on interest rates, recognizing the delicate balance between keeping inflation in check and promoting full employment. Similarly, fiscal policymakers use these insights to gauge the likely effects of stimulus measures, labor market reforms, or tax changes on overall economic stability. By identifying when and how inflation responds to shifts in unemployment and expectations, policy analysts and forecasters can better anticipate emerging economic trends, mitigate the risks of runaway inflation or persistent stagnation, and guide the economy toward sustainable growth. In short, a nuanced understanding of the Phillips Curve's behavior offers policymakers and economists essential guidance as they navigate the complexities of modern macroeconomic environments.

**Research Goals**:

The central objective of my research is to partially replicate and compare the findings of Ball et al. (2021), who analyzed various formulations of the Phillips Curve and examined where traditional models may have faltered, particularly during the pandemic period. By closely following their methods and data sources - while updating or adjusting certain specifications -I attempt to provide insights into the robustness and flexibility of different Phillips Curve estimations. This will involve testing multiple functional forms, some of which incorporate measures of long-term inflation expectations drawn from surveys, such as the Survey of Professional Forecasters (SPF), to determine which specification best captures the empirical relationship between inflation, unemployment, and expectations over time.

In examining these different model variants, the approach will offer a broader perspective on the Phillips Curve's underlying mechanisms and the role of expectations in shaping inflation dynamics. Specifically, the analysis will explore how well models that feature lagged inflation terms, forward-looking expectations, or alternative definitions of labor market slack reproduce the patterns observed in historical data and align with current economic conditions. By doing so, this study not only provides a contemporary assessment of the Phillips Curve but also contributes to the ongoing debate regarding the stability, relevance, and policy implications of this foundational macroeconomic relationship.

**Dependent Variable & Key Conceptual Variables**:

In my analysis, the primary dependent variable is the inflation rate, a measure capturing the change in overall price levels across the economy. Inflation is central to the Phillips Curve framework because it reflects how wage and price pressures emerge and evolve under varying labor market conditions. To better understand these dynamics, incorporating long-term inflation expectations is crucial, as it connects current inflation outcomes to agents' views about future price developments. Drawing on forecasts from the SPF allows me to incorporate forward-looking information about where professional analysts believe inflation will stand in the long run. These expectations shape wage-setting behavior, pricing decisions, and ultimately the inflation-unemployment relationship, making them an integral component of contemporary Phillips Curve analyses (Ball et al., 2021).

At the heart of this relationship between inflation and labor market conditions is the unemployment gap, defined as the difference between the actual unemployment rate and its natural rate. The natural rate of unemployment represents the level of joblessness consistent with stable inflation; when actual unemployment falls below this level, employers face increasing difficulty finding qualified workers, pushing wages and, by extension, prices upward. Conversely, when actual unemployment exceeds the natural rate, labor markets are slack, and firms have less incentive to raise wages. Including the unemployment gap variable as a primary explanatory factor reflects the idea that imbalances in labor supply and demand are a key driver of inflationary pressures. Thus, the unemployment gap serves as the key independent variable, allowing the empirical analysis to test how well different Phillips Curve specifications capture the relationship between labor market slack and observed inflation outcomes.

**Roadmap of the Paper**:

The remainder of this paper proceeds as follows. Section II presents a comprehensive literature review, situating the Phillips Curve within its historical context and highlighting key theoretical and empirical contributions from seminal studies through to Ball et al. (2021). Section

III then provides an overview of the data sources, transformations, and any irregularities in the dataset. Section IV details the model specification, describing the baseline Phillips Curve equations and various alternative functional forms, including those incorporating lagged inflation and different measures of slack.

Following the setup of the empirical approach, Section V presents the results of the regressions, comparing different specifications and identifying the model configurations that most accurately capture historical dynamics. Finally, Section VI provides a summary of key findings, policy implications, and avenues for further research. The paper concludes with a bibliography that includes all cited references, and an appendix containing full regression outputs, additional robustness checks, and any additional graphs and figures.

## II. Literature Review

**Foundational Phillips Curve Literature**:

In 1958, economist A.W. Phillips published a seminal paper titled "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957," in which he observed an inverse relationship between unemployment and the rate of wage inflation in the UK over the specified period (Phillips, 1958). This empirical finding suggested that lower unemployment rates were associated with higher rates of wage increases, and vice versa. Phillips' work provided a statistical foundation for understanding the trade-off between unemployment and inflation, influencing macroeconomic policy discussions in subsequent decades.

Building upon Phillips' findings, economists Paul Samuelson and Robert Solow extended the analysis to the relationship between unemployment and price inflation, rather than just wage inflation. In their 1960 work, they suggested that policymakers could exploit this inverse relationship to target lower unemployment at the expense of higher inflation, effectively suggesting a menu of policy options along the Phillips Curve (Samuelson & Solow, 1960). This interpretation aligned with Keynesian economic thought, which advocated for active government intervention to manage economic cycles.

However, the apparent stability of the Phillips Curve relationship came into question during the 1970s, a period characterized by stagflation - simultaneously high inflation and high unemployment - which contradicted the traditional Phillips Curve model.For instance, in the United States, between 1969 and 1975, inflation rose from 6% to 9%, while unemployment increased from under 4% to 8.5% (ROM Economics, n.d.). This phenomenon exposed the limitations of the original Phillips Curve, indicating that the relationship between inflation and unemployment was more complex than previously understood.

In response to the breakdown of the Phillips Curve during the stagflation era, economists Milton Friedman and Edmund Phelps introduced the expectations-augmented Phillips Curve. They argued that the traditional Phillips Curve failed to account for the role of inflation expectations in influencing wage-setting and price-setting behavior. Friedman and Phelps posited that when individuals and firms anticipate higher inflation, they adjust their behavior accordingly, leading to a vertical Phillips Curve in the long run at the natural rate of unemployment. This implies that there is no long-term trade-off between inflation and unemployment; attempts to reduce unemployment below its natural rate would only result in accelerating inflation (Gordon, 2018). Their work emphasized the importance of expectations in macroeconomic analysis and policy, highlighting that credible and consistent policies are essential for managing inflation without causing undue fluctuations in unemployment.

The incorporation of rational expectations further refined the understanding of the Phillips Curve. Robert Lucas and Thomas Sargent critiqued traditional Keynesian models, arguing that if economic agents form expectations rationally, systematic monetary policy would be ineffective in influencing real economic variables like unemployment (Lucas, 1976; Sargent & Wallace, 1975). This perspective led to the development of New Classical economics and underscored the importance of credible and rule-based policy frameworks.

In response, New Keynesian economists introduced models incorporating price and wage stickiness, as well as imperfect competition, to explain short-term deviations from the natural rate of unemployment. The New Keynesian Phillips Curve (NKPC) emerged, emphasizing forward-looking expectations and the role of real marginal costs in determining inflation dynamics (Clarida, Galí, & Gertler, 1999). This framework reconciled the short-term trade-off observed by Phillips with the long-term neutrality posited by Friedman and Phelps, providing a more comprehensive tool for analyzing monetary policy.

Over time, the Phillips Curve has evolved from a simple empirical observation to a complex theoretical construct, integrating expectations, credibility, and microeconomic foundations. Its development reflects the dynamic nature of macroeconomic thought and the continuous interplay between theory and real-world economic phenomena.

**Modern Approaches & Empirical Findings**:

Recent studies have revisited the Phillips Curve to assess its relevance and accuracy in contemporary economic contexts. Ball et al. (2021) examined various measures of core inflation during the COVID-19 pandemic, highlighting that traditional metrics might not fully capture underlying inflation trends during such unprecedented times. Their analysis suggests that alternative measures, such as weighted median inflation, could provide more stable indicators of core inflation, thereby offering a refined perspective on the inflation-unemployment relationship

Similarly, Hazell et al. (2022) utilized state-level data to estimate the slope of the Phillips Curve within the United States. Their findings indicate that the curve remains relatively flat, with only a modest decline in its slope since the 1980s. This suggests that the sensitivity of inflation to unemployment has not diminished as significantly as previously thought, emphasizing the continued importance of labor market conditions in influencing inflation dynamics

The COVID-19 pandemic posed significant challenges to traditional Phillips Curve predictions. The simultaneous occurrence of supply chain disruptions, shifts in consumer demand, and unprecedented fiscal and monetary interventions led to inflationary pressures that deviated from historical patterns. Furman (2022) discusses how the pandemic-era inflation was not fully anticipated by standard models, underscoring the need to incorporate factors such as supply constraints and changes in labor force participation into inflation forecasting frameworks
Furthermore, Crump et al. (2022) analyzed the unemployment-inflation trade-off during the pandemic, documenting a rapid increase in the natural rate of unemployment and its implications for inflation dynamics. Their study suggests that while temporary supply factors played a significant role in the inflation surge, negative unemployment gaps also contributed, highlighting the complex interplay between labor market conditions and inflation during the pandemic
These contemporary analyses highlight the evolving nature of the Phillips Curve and the necessity for models to adapt to changing economic landscapes. The pandemic era, in particular, has underscored the importance of considering a broader set of variables, including supply-side factors and shifts in expectations, to accurately capture the dynamics between inflation and unemployment.

**Rationale for Multiple Specifications & Expectations Measures**:

A central challenge in estimating the Phillips Curve is accurately capturing the role of inflation expectations and labor market conditions in determining inflation dynamics. Using different data sources and model specifications allows researchers to assess how robust the Phillips Curve is across varying contexts and to identify which variables most effectively explain observed inflation trends. For instance, inflation expectations derived from the Survey of Professional Forecasters (SPF) offer a forward-looking perspective on how market participants anticipate inflation over different horizons. These expectations play a crucial role in shaping wage and price-setting behavior, making them a vital component in models that seek to reflect real-world dynamics (Ball et al., 2021).

Including lagged variables, such as past inflation rates or previous unemployment gaps, provides additional insights by accounting for the inertia and delayed effects that often characterize inflation dynamics. For example, lagged inflation can capture how persistent inflationary pressures influence current outcomes, reflecting both structural rigidities and adaptive expectations. Similarly, incorporating lagged unemployment measures can help assess whether labor market slack impacts inflation with a time delay. These lagged variables enrich the analysis by allowing models to account for historical dependencies, thus improving their explanatory and predictive power (Furman, 2022).

Moreover, the choice of data sources for inflation expectations, such as the SPF or alternative measures like the Michigan Survey of Consumers, can significantly affect the interpretation of results. While professional forecasts tend to reflect forward-looking market dynamics, consumer surveys often highlight backward-looking expectations shaped by recent price changes. Testing models with different expectation measures provides a more comprehensive understanding of how diverse economic agents influence inflation. By employing multiple specifications and expectations measures, this study aims to evaluate the robustness of the Phillips Curve and uncover the most reliable methods for explaining and predicting inflation in varying economic conditions.

## III. Data and Methodology

**Data Description**:

**Inflation Rate (CPI_YOY)**: Measured as the percentage change in the Consumer Price Index (CPI) over the previous year, this variable reflects the overall increase in price levels within the economy. Data for the CPI is sourced from the FRED_QD data set, and the year over year percentage change is generated manually.

**Expected Inflation (INFCPI10YR)**: Derived from the Survey of Professional Forecasters (SPF), this variable represents the anticipated rate of inflation over a specified future period. The SPF, is conducted quarterly by the Federal Reserve Bank of Philadelphia, which aggregates forecasts from professional economists regarding key economic indicators, including inflation expectations. I will be using both the 10 year and the 1 year forecasts for inflation.

**Unemployment Rate (UNRATE)**: This variable indicates the percentage of the labor force that is unemployed and actively seeking employment. Data on the unemployment rate is obtained from the Bureau of Labor Statistics (BLS) and is accessible through the FRED_QD data set.

**Natural Rate of Unemployment (NROU)**: Also known as the Non-Accelerating Inflation Rate of Unemployment (NAIRU), this represents the unemployment rate at which inflation is stable, implying no upward or downward pressure on inflation. Estimates for the natural rate are provided by the Congressional Budget Office (CBO) and are available via FRED.

**Data Processing**

There are several transformations and modifications being made to the data set. To align the quarterly publications of each of the variables, NROU was shifted back one month. This is due to the fact that CPI and UNRATE come from the FRED_QD data set, while NROU comes as a stand alone variable from the FRED website. FRED_QD is dated on the first of Mar, Jun, Sep, and Dec while NROU is dated on the first of Apr, Jul, Oct, Jan. In order to combine these data sets the dates NROU were shifted by one month before merging. Additionally, CPI_YOY was calculated manually with the following equation: $CPI\_YOY_t = (CPI_t - CPI_{t-1})/CPI_{t-1}$. Finally a slack variable is generated $SLACK_t = UNRATE_t - NROU_t$ . Additional derivative variables for lags and inflation slack are generated as needed.

There are several cells with missing values, as INFCPI10YR began in 1991. These missing values are ignored while estimating models which do not include inflation expectations. When analyzing the models which do include inflation expectations, all years containing NA values are dropped.

**V. Model Specification**

**Baseline Phillips Curve Equation (Samuelson & Solow 1960)**:

This is the most simple model for estimating the Phillips curve as it relates to price inflation. The model is specified as follows:

$$\pi_t = \beta_0 - \beta_1 U_t + \epsilon_t$$

where:

$\pi_t$ = $CPI\_YOY_t$: The observed percentage price change from a year ago, at time t.

$U_t$ = $UNRATE_t$

The expected sign of $\beta_1$ is negative according to the relationship posited by Phillips and Samuelson & Solow, but this relationship has shifted over the decades, and recent estimates from Hazell et al. (2022) suggest it remains relatively flat.

**Alternative Specifications**:

**Ball et al.**

The second specification incorporates inflation expectations as well as the slack expectations and is copied directly from Ball et al. The specification is as follows:

$$\pi_t - \pi_t^e = \beta_0 - \beta_1 \tilde{u}_t + \epsilon_t$$

Where:

$\pi_t^e$ = INFCPI10YR$_t$ : $\pi_t - \pi_t^e$ = INFDIFF10$_t$ The difference between observed inflation and the 10 ahead year inflation prediction taken from the survey of economic forecasters.

$\tilde{u}_t$ = SLACK$_t$ : Difference between unemployment and the long-run

From now on, to simplify model specification and regression results , $\pi_t - \pi_t^e$ will be displayed simply as $\Delta_t$.

The expected sign of $\beta_1$ is negative because when the inflation gap ($\tilde{u}_t$) is positive, meaning unemployment is above its natural rate, it indicates slack in the labor market. This slack reduces inflationary pressures because firms face lower wage growth and cost pressures. Alternatively when $\tilde{u}_t$ is negative (unemployment is below its natural rate), tighter labor market conditions lead to upward wage pressures and higher inflation.

**Lagged model.**

The final specification is a lagged model which is based on the previous model but incorporates lags to the slack and inflation variables to capture lagged effects on inflation. The basic specification is as follows:

$$\Delta_t = \beta_0 - \beta_1 \tilde{u}_t - \beta_2 \tilde{u}_{t-1} + \beta_3 \Delta_{t-1} + \beta_4 \Delta_{t-2} + \epsilon_t$$

Where:

$\Delta_t$ = INFDIFF10

$\tilde{u}_t$ = SLACK

Including lags in this model is important because they account for the delayed effects of labor market slack (L1_SLACK) and capture inflation persistence through past values of inflation (L1_INFDIFF and L2_INFDIFF). This helps reflect the dynamic nature of inflationary pressures and improves the model's explanatory power. The expected signs of $\beta_1$ and $\beta_2$ are negative for the same reasons as described for the previous model. With the additional lags capturing the distributed, delayed effect labor market slack has on inflation at time t. The expected signs of $\beta_3$ and $\beta_4$ are positive because inflation in previous periods is likely positively correlated with observed inflation at time t.

**V. Results and Analysis**

**Presentation of Estimated Specifications**:

**Baseline:** (Appendix, Table 1)

$$\pi_t = 3.13 + .107 U_t$$

$$(.6319) \quad (.1030)$$

$$\text{adj-}R^2 = 0.0003, \quad N = 259$$

As we can see from this baseline estimation of the model, the coefficient for $U_t$ is positive, which is different from our expectation. Additionally, the standard errors are very high, resulting in a low t-score of 1.04, meaning the estimated coefficient is not statistically significant. Furthermore, the adj-$R^2$ is very low, meaning variations in unemployment rate do not explain variation in inflation. These results suggest there is likely bias in our model due to one or more excluded independent variables. Moreover, the result is as expected, due to the fact that this model estimation is very basic and has since been developed to account for other variables. A simple scatter plot of this simple model (Figure 1) shows the weak relationship between inflation and unemployment. Additionally we can see that the relationship between the variables has shifted significantly over the decades.

**Ball et al:** (Appendix, Table 2.1)

$$\Delta_t = .2485 - .3042\tilde{u}_t$$

$$(.1312) \; (.0679)$$

$$\text{adj-}R^2 = 0.1269, \quad N = 132$$

Here we can see a significant improvement in the estimation of the model. The sign of the coefficient is negative, as expected. Additionally, low standard errors result in a t-score of 4.48 which is statistically significant at a 95% confidence level. Furthermore, the adj-$R^2$ has improved, although not to a level at which we can say the model has strong predictive power. When the two variables are graphed against each other, we can see the points are tightly grouped in the center with larger variations as slack goes negative as well as increases past 4, suggesting heteroskedasticity in this model. Indeed when the residuals are graphed against date (Figure 4), we can see evidence of both heteroskedasticity as well as pure positive serial correlation.

To test for serial correlation, I ran both the Durbin-Watson test for first order serial correlation as well as an LM test. The d-statistic result (Table 2.2) of 0.2789518 strongly suggests the presence of positive first-order serial correlation in the residuals. Additionally the result for Breusch–Godfrey LM test for autocorrelation (Table 2.3 & 2.4) $chi^2$ values for 4 lags, and 8 lags (1 year and 2 year) are $chi^2 = 99.947$ and $chi^2 = 103.934$ which is strong evidence of serial correlation at up to 2 years of lag.

To test for heteroskedasticity, I ran both the Breusch–Pagan test and the White test. The results of the Breusch–Pagan test (Table 4.5), Prob > chi2 = 0.0890, suggest that there is no strong evidence of heteroskedasticity in my model. Additionally, the result of the White test (Table 4.6) suggests no significant evidence of heteroskedasticity with a p-value of 0.3012. However the test does suggest significant evidence of skewness and kurtosis in the residuals with p-values of 0.0003 and 0.0229 respectively. The overall model indicates potential issues with normality, driven by skewness and kurtosis with a p-value of 0.0004. Although skewness and kurtosis are concerning, addressing these issues are not essential because they do not violate any of the six classical assumptions of OLS.

However, correcting for serial correlation is essential to preserve classical assumption IV. To correct for both issues I ran a GLS model with robust standard errors, additionally I could have run a lagged model, but we will explore a lagged model in the next section. The results of the GLS model (Table 2.7) are as follows:

$$\Delta_t = .252 - .2419\tilde{u}_t$$

$$(.4285)\ (.0658)$$

$$\text{adj-}R^2 = 0.0907,\ N = 131,\ rho = .8609489$$

show an improved DW of 1.55 which is closer to the ideal of 2, suggesting serial correlation has been substantially reduced. $\tilde{u}_t$ remains highly remains highly significant (p=0.000), confirming a strong negative relationship with $\Delta_t$. The constant term is no longer significant (p=0.558), indicating it may not meaningfully contribute to the model. Adj-$R^2$ is reduced to 0.0907, reflecting that slack alone explains only about 9% if the variation in $\Delta_t$. In order to further improve our model I will next examine a lagged variation of the model.

**Lagged** (Table 3.1):

$$\Delta_t = .0247 - .233\tilde{u}_t + .207\tilde{u}_{t-1} + 1.08\Delta_{t-1} - .262\Delta_{t-2}$$

$$(.0670)(.066)\ (.067)(.083)(.082)$$

$$\text{adj-}R^2 = 0.7862,\ N = 130$$

The regression model I developed demonstrates a strong overall fit, with a significant F-statistic and a high R-squared value, indicating that the included variables effectively explain the variability in $\Delta_t$. Specifically, both current and lagged values of $\tilde{u}_t$ and $\Delta_t$ are statistically significant, and influence the dependent variable. This highlights the dynamic nature of the relationship as specified by the alternate Phillips curve model with lags. This model is a striking improvement over the previous model.

I conducted a Ramsey RESET test (Table 3.2 ) to assess the presence of omitted variables or incorrect functional form in the regression model. The Ramsey RESET test results show an F-statistic = 3.83 and Prob > F = 0.0116, this suggests that the model may suffer from omitted variables or an incorrect functional form. This implies that there may be additional factors influencing $\Delta_t$ that are not captured by the current specification, or that the relationship between the variables may not be purely linear. Although addressing this issue could enhance the model's validity and explanatory power, for a simple Phillips curve, this specification is supported by the literature. Other more advanced models such as DSGE can capture the influence of a wider set of variables on inflation. Therefore I will not be modifying the model further.

The residuals from the regression model (Table 3.3 and Figure 3.1) have a mean of 1.91e-09, which is practically zero. The one-sample t-test (Table 3.4) further confirms that the mean of the residuals is not statistically different from zero (p-value = 1.0000). This supports the classical OLS assumption that the error term has a zero population mean, indicating that the model does not systematically overpredict or underpredict the dependent variable.

Based on the results presented in Table 3.5 and Table 3.6: The Durbin–Watson statistic of 2.071 suggests that there is no substantial evidence of autocorrelation in the residuals of the regression model. The Breusch–Godfrey LM test yields a p-value of 0.2726, which is greater than 0.05, leading to the failure to reject the null hypothesis of no serial correlation. Both the Durbin–Watson and Breusch–Godfrey tests indicate that the regression model does not suffer from serial correlation. This satisfies one of the classical OLS assumptions, ensuring that the residuals are uncorrelated over time and that the standard errors of the estimates are unbiased. This is also a significant improvement over the previous model.

Despite the Breusch–Pagan/Cook–Weisberg (Table 3.7) not indicating heteroskedasticity ($p = 0.2945$), White's test (Table 3.8) strongly suggests its presence ($p = 0.0024$). The Cameron & Trivedi's decomposition reinforces this conclusion, showing significant heteroskedasticity ($chi^2 = 33.49$, $p = 0.0024$) with negligible evidence from skewness and kurtosis. Although these results appear to be concerning, when examining the graph of the residuals in Figure 3.1, we can see no significant pattern other than during the 2008 recession and during the 2021 inflationary spike, which both can be interpreted as structural breaks from the normal relationship.

The Variance Inflation Factors (VIF) (Table 3.9) reveal that all explanatory variables ( $\Delta_{t-1}$ = 4.17, $\Delta_{t-2}$ = 4.10, $\tilde{u}_t$ = 3.88, and $\tilde{u}_{t-1}$ = 3.82) have VIF values below 5, with a mean VIF of 3.99. These results indicate that there is no severe multicollinearity among the predictors in the regression model. Consequently, the estimated coefficients are unlikely to be unduly inflated due to high correlations between the independent variables, ensuring the reliability of the regression estimates.

The diagnostic tests for normality (Table 3.10 and Table 3.11), demonstrate that the residuals (ehat) from the regression model do not follow a normal distribution. Specifically, the Shapiro–Wilk test in Table 3.10 produced a W statistic of 0.86928 with a p-value of 0.00000, leading to the rejection of normality. Additionally, the Skewness and Kurtosis tests in Table 3.11 yielded p-values of 0.0227 and 0.0000 for skewness and kurtosis, respectively, and a joint $chi^2$ statistic of 25.55 with a p-value of 0.0000, further confirming the absence of normality in the residuals. This violation suggests that the standard errors of the regression coefficients may be biased, potentially affecting inference; however, with a sample size of 130 observations, the Central Limit Theorem may alleviate some concerns regarding the reliance on normality for large samples.

The unit root tests conducted on $\Delta_t$ and $\tilde{u}_t$, as detailed in Table 3.12 and Table 3.13, consistently indicate that both variables are stationary. Specifically, the Augmented Dickey–Fuller test shows that $\Delta_t$ (test statistic = -3.863, p-value = 0.0023) is stationary at both the 1% and 5% significance levels, while $\tilde{u}_t$ (test statistic = -3.421, p-value = 0.0103) is stationary at the 5% level. Similarly, the Phillips–Perron test confirms these findings, with $\Delta_t$ (test statistic = -3.492, p-value = 0.0082) and $\tilde{u}_t$ (test statistic = -3.645, p-value = 0.0050) both rejecting the null hypothesis of a unit root at the 5% significance level. Therefore, both variables are suitable for regression analysis without concerns of non-stationarity leading to spurious correlation.

Overall this model is a dramatic improvement over both of the previous two models. It passes almost all of the tests, and could only see improvements from including additional independent variables or reworking its functional form.

## VIII. Summary and Conclusion

**Summary:**

The regression analysis explored the relationship between unemployment rates and inflation through multiple model iterations. The initial baseline model revealed a weak and statistically insignificant positive relationship, with an almost negligible explanatory power (adj-$R^2$=0.0003), suggesting model bias from omitted variables. Incorporating additional variables in the Ball et al. model improved the results, yielding a significant negative coefficient for unemployment and a higher adjusted adj-$R^2$=0.1269, although issues with heteroskedasticity and serial correlation persisted. Applying a Generalized Least Squares (GLS) approach mitigated serial correlation, enhancing the Durbin-Watson statistic to 1.55 and maintaining a significant negative relationship, albeit with a reduced adj-$R^2$=0.0907. The final lagged model introduced dynamic elements, substantially increasing the adjusted adj-$R^2$ to 0.7862 and demonstrates a strong fit with significant coefficients for both current and lagged variables. Diagnostic tests confirmed improvements in autocorrelation and multicollinearity, though some concerns with heteroskedasticity and residual normality remained, primarily due to structural economic events.

**Conclusion:**

The step-by-step improvement of the regression models made them better aligned with economic theories and more reliable. We started with a simple model that didn't explain much, then added more factors and fixed statistical problems in each new version. This process led to a final model that effectively shows how unemployment and inflation influence each other over time. Although the last model works well and meets most standard assumptions, there are still small issues with data variability and distribution that could be improved in the future. Overall, the final model successfully explains the relationship between unemployment and inflation, highlighting how economic factors interact and change over time.

**Appendix:**

**Baseline model:**

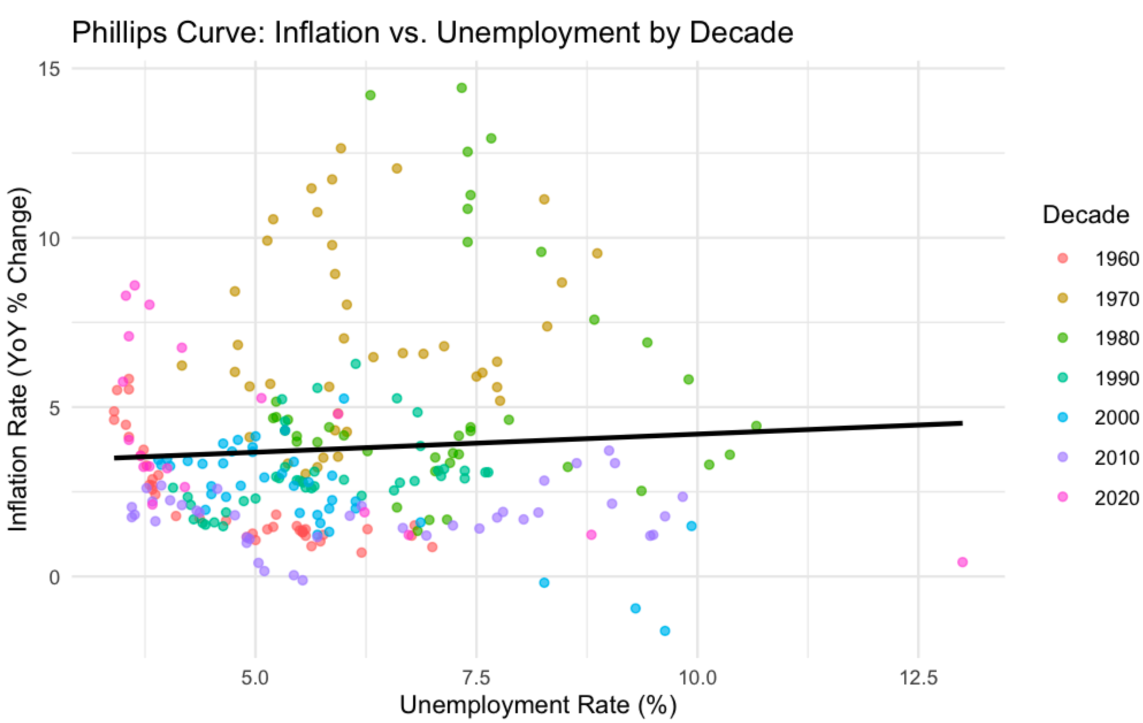
Table 1.

```
. reg CPIAUCSL_yoy UNRATE
```

| Source | SS | df | MS | | | |
|---|---|---|---|---|---|---|
| Model | 8.44055543 | 1 | 8.44055543 | Number of obs | = | 259 |
| Residual | 2014.11448 | 257 | 7.83702133 | F(1, 257) | = | 1.08 |
| | | | | Prob > F | = | 0.3003 |
| Total | 2022.55504 | 258 | 7.83936061 | R-squared | = | 0.0042 |
| | | | | Adj R-squared | = | 0.0003 |
| | | | | Root MSE | = | 2.7995 |

| CPIAUCSL_yoy | Coefficient | Std. err. | t | P>\|t\| | [95% conf. | interval] |
|---|---|---|---|---|---|---|
| UNRATE | .1069599 | .1030649 | 1.04 | 0.300 | -.0959994 | .3099192 |
| _cons | 3.133022 | .6318874 | 4.96 | 0.000 | 1.888685 | 4.377358 |

Figure 1.



**Ball et al:**

Table 2.1

```
. reg INFDIFF10 SLACK

      Source |       SS           df       MS      Number of obs   =       132
-------------+----------------------------------   F(1, 130)       =     20.04
       Model |  38.1815213         1  38.1815213   Prob > F        =    0.0000
    Residual |  247.634058       130  1.90487737   R-squared       =    0.1336
-------------+----------------------------------   Adj R-squared   =    0.1269
       Total |  285.815579       131  2.18179831   Root MSE        =    1.3802

------------------------------------------------------------------------------
   INFDIFF10 | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       SLACK |  -.3042154   .0679498    -4.48   0.000     -.438646   -.1697849
       _cons |   .2485296   .1312488     1.89   0.061    -.0111305    .5081897
------------------------------------------------------------------------------
```

Table 2.2.

```
. estat dwatson

Durbin-Watson d-statistic(  2,   132) =  .2789518
```

Table 2.3.

```
. estat bgodfrey, lags(4)

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       4     |         99.947               4                   0.0000
---------------------------------------------------------------------------
                        H0: no serial correlation
```

Table 2.4

```
. estat bgodfrey, lags(8)

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       8     |        103.934               8                   0.0000
---------------------------------------------------------------------------
                        H0: no serial correlation
```

Table 2.5

```
. estat hettest

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of INFDIFF10

H0: Constant variance

    chi2(1) =   2.89
Prob > chi2 = 0.0890
```

Table 2.6

```
. estat imtest, white

White's test
H0: Homoskedasticity
Ha: Unrestricted heteroskedasticity

    chi2(2) =   2.40
Prob > chi2 = 0.3012

Cameron & Trivedi's decomposition of IM-test
```

| Source | chi2 | df | p |
|---|---|---|---|
| Heteroskedasticity | 2.40 | 2 | 0.3012 |
| Skewness | 12.89 | 1 | 0.0003 |
| Kurtosis | 5.17 | 1 | 0.0229 |
| Total | 20.46 | 4 | 0.0004 |

Table 2.7

```
. prais INFDIFF10 SLACK, corc robust

Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.8592
Iteration 2:  rho = 0.8609
Iteration 3:  rho = 0.8609
Iteration 4:  rho = 0.8609

Cochrane–Orcutt AR(1) regression with iterated estimates

Linear regression                               Number of obs     =        131
                                                F(1, 129)         =      13.51
                                                Prob > F          =     0.0003
                                                R-squared         =     0.0907
                                                Root MSE          =     .70293

------------------------------------------------------------------------------
             |               Semirobust
   INFDIFF10 | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       SLACK |  -.2419293   .0658127    -3.68   0.000    -.3721414   -.1117172
       _cons |   .2519666   .4285765     0.59   0.558    -.5959825    1.099916
-------------+----------------------------------------------------------------
         rho |   .8609489
------------------------------------------------------------------------------
Durbin–Watson statistic (original)    = 0.278952
Durbin–Watson statistic (transformed) = 1.552622
```

Figure 2.1

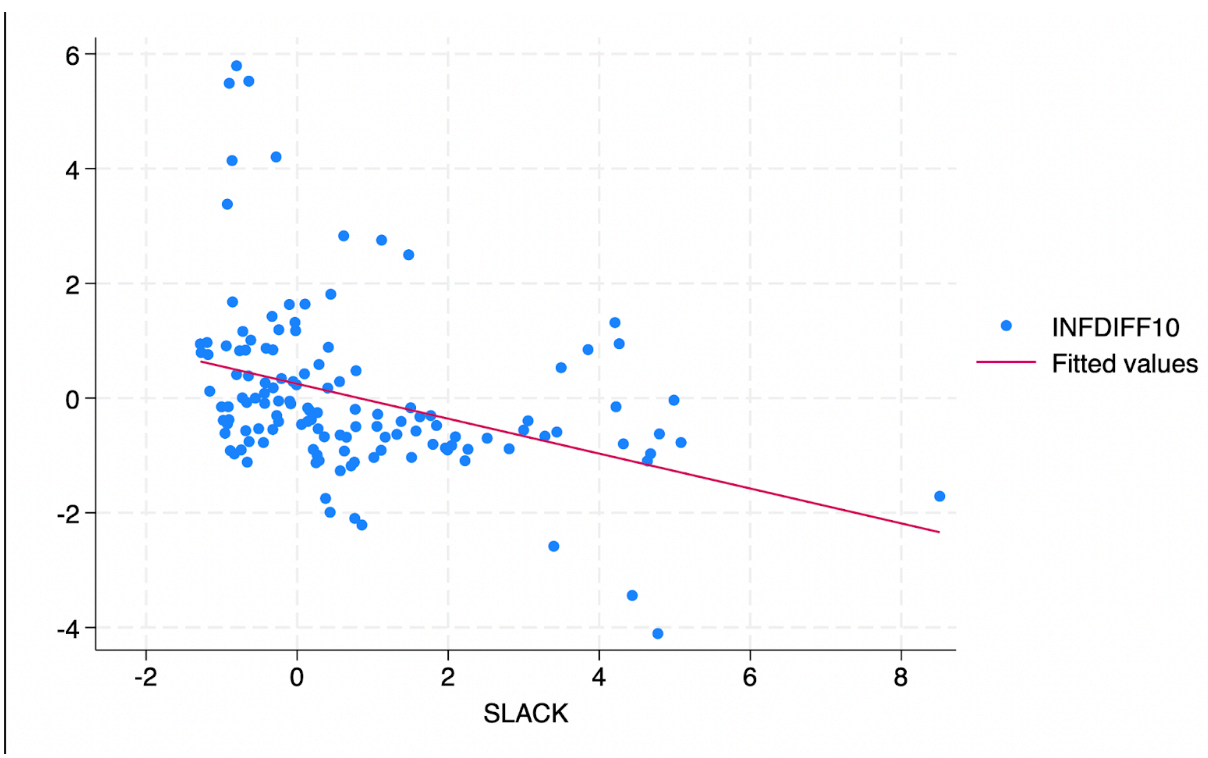
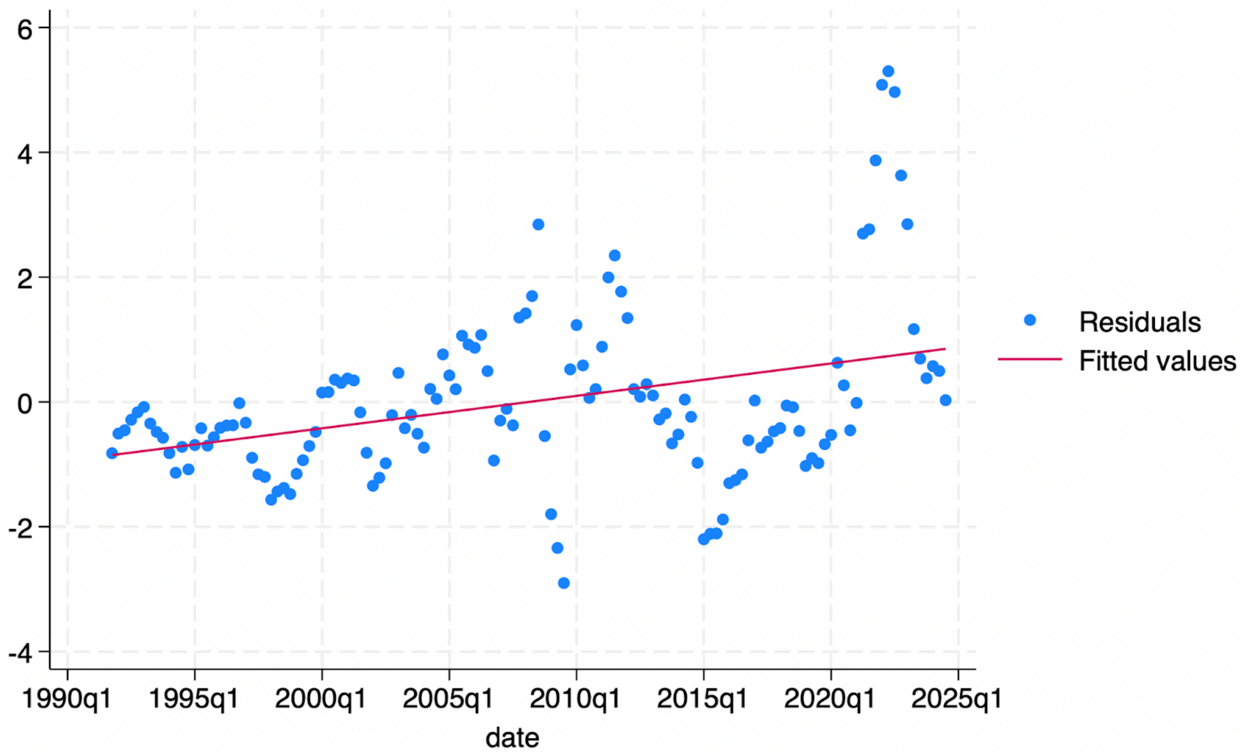


Figure 2.2.



6
4
2
0
-2
-4
1990q1
1995q1
2000q1
2005q1
2010q1
2015q1
2020q1
2025q1
date
Residuals
Fitted values

**Lagged:**

Table 3.1

```
. reg INFDIFF10 SLACK L1_SLACK L1_INFDIFF L2_INFDIFF
```

| Source | SS | df | MS |
|---|---|---|---|
| Model | 225.171404 | 4 | 56.292851 |
| Residual | 58.8532402 | 125 | .470825921 |
| Total | 284.024644 | 129 | 2.20174143 |

| | | |
|---|---|---|
| Number of obs | = | 130 |
| F(4, 125) | = | 119.56 |
| Prob > F | = | 0.0000 |
| R-squared | = | 0.7928 |
| Adj R-squared | = | 0.7862 |
| Root MSE | = | .68617 |

| INFDIFF10 | Coefficient | Std. err. | t | P>\|t\| | [95% conf. | interval] |
|---|---|---|---|---|---|---|
| SLACK | -.2333138 | .0661474 | -3.53 | 0.001 | -.3642277 | -.1024 |
| L1_SLACK | .2072815 | .0666625 | 3.11 | 0.002 | .0753481 | .3392148 |
| L1_INFDIFF | 1.087517 | .0830075 | 13.10 | 0.000 | .923235 | 1.251799 |
| L2_INFDIFF | -.2621203 | .0823099 | -3.18 | 0.002 | -.4250217 | -.0992188 |
| _cons | .0247114 | .0670307 | 0.37 | 0.713 | -.1079506 | .1573734 |

Table 3.2

```
. estat ovtest

Ramsey RESET test for omitted variables
Omitted: Powers of fitted values of INFDIFF10

H0: Model has no omitted variables

F(3, 122) =   3.83
 Prob > F = 0.0116
```

Table 3.3

```
. summarize ehat
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|---|---|---|---|---|---|
| ehat | 130 | 1.91e-09 | .6754455 | -3.214939 | 2.960174 |

Table 3.4

```
. ttest ehat==0

One-sample t test
```

| Variable | Obs | Mean | Std. err. | Std. dev. | [95% conf. interval] | |
|---|---|---|---|---|---|---|
| ehat | 130 | 1.91e-09 | .0592405 | .6754455 | -.1172088 | .1172088 |

```
    mean = mean(ehat)                                         t =   0.0000
H0: mean = 0                                 Degrees of freedom =      129

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.5000         Pr(|T| > |t|) = 1.0000          Pr(T > t) = 0.5000
```

Table 3.5

```
. estat dwatson

Durbin-Watson d-statistic(  5,   130) =  2.071177
```

Table 3.6

```
. estat bgodfrey

Breusch-Godfrey LM test for autocorrelation
```

| lags(*p*) | chi2 | df | Prob > chi2 |
|---|---|---|---|
| 1 | 1.204 | 1 | 0.2726 |

```
                        H0: no serial correlation
```

Table 3.7

```
Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of INFDIFF10

H0: Constant variance

    chi2(1) =   1.10
Prob > chi2 = 0.2945
```

Table 3.8

```
. estat imtest, white

White's test
H0: Homoskedasticity
Ha: Unrestricted heteroskedasticity

   chi2(14) =  33.49
Prob > chi2 = 0.0024

Cameron & Trivedi's decomposition of IM-test
```

| Source | chi2 | df | p |
|---|---|---|---|
| Heteroskedasticity | 33.49 | 14 | 0.0024 |
| Skewness | 9.28 | 4 | 0.0544 |
| Kurtosis | 3.49 | 1 | 0.0618 |
| Total | 46.26 | 19 | 0.0005 |

Table 3.9

```
. estat vif
```

| Variable | VIF | 1/VIF |
|---|---|---|
| L1_INFDIFF | 4.17 | 0.240092 |
| L2_INFDIFF | 4.10 | 0.243877 |
| L1_SLACK | 3.88 | 0.257754 |
| SLACK | 3.82 | 0.261841 |
| Mean VIF | 3.99 | |

Table 3.10

```
. swilk ehat

                   Shapiro-Wilk W test for normal data
```

| Variable | Obs | W | V | z | Prob>z |
|---|---|---|---|---|---|
| ehat | 130 | 0.86928 | 13.462 | 5.850 | 0.00000 |

Table 3.11

```
. sktest ehat

Skewness and kurtosis tests for normality
                                                     ------ Joint test ------
    Variable         Obs   Pr(skewness)   Pr(kurtosis)   Adj chi2(2)  Prob>chi2

        ehat         130         0.0227         0.0000         25.55     0.0000
```

Table 3.12

```
. dfuller INFDIFF10, lags(1)

Augmented Dickey–Fuller test for unit root

Variable: INFDIFF10                          Number of obs  = 130
                                             Number of lags =   1

H0: Random walk without drift, d = 0

                                       Dickey–Fuller
                   Test          -------- critical value ---------
              statistic           1%           5%          10%

 Z(t)            -3.863       -3.500       -2.888       -2.578

MacKinnon approximate p-value for Z(t) = 0.0023.
```

Table 3.13

```
. dfuller SLACK, lags(1)

Augmented Dickey–Fuller test for unit root

Variable: SLACK                              Number of obs  = 261
                                             Number of lags =   1

H0: Random walk without drift, d = 0

                                       Dickey–Fuller
                   Test          -------- critical value ---------
              statistic           1%           5%          10%

 Z(t)            -3.421       -3.459       -2.880       -2.570

MacKinnon approximate p-value for Z(t) = 0.0103.
```

Table 3.14

```
. pperron INFDIFF10

Phillips–Perron test for unit root       Number of obs   = 131
Variable: INFDIFF10                      Newey–West lags =   4

H0: Random walk without drift, d = 0

                                         Dickey–Fuller
                   Test          ———————— critical value —————————
              statistic             1%            5%            10%
--------------------------------------------------------------------
 Z(rho)         -23.194        -19.903       -13.762        -11.041
 Z(t)            -3.492         -3.500        -2.888         -2.578
--------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0082.
```

Table 3.15

```
. pperron SLACK

Phillips–Perron test for unit root       Number of obs   = 262
Variable: SLACK                          Newey–West lags =   4

H0: Random walk without drift, d = 0

                                         Dickey–Fuller
                   Test          ———————— critical value —————————
              statistic             1%            5%            10%
--------------------------------------------------------------------
 Z(rho)         -25.658        -20.310       -14.000        -11.200
 Z(t)            -3.645         -3.459        -2.880         -2.570
--------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0050.
```

Figure 3.1

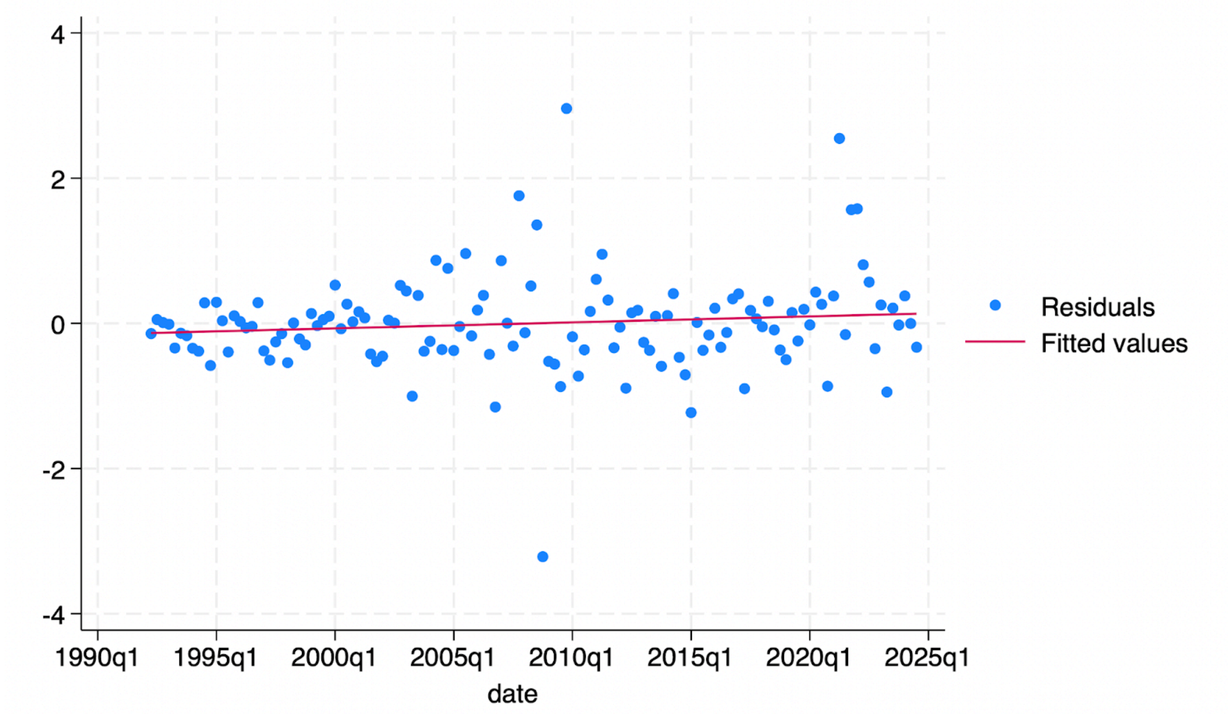
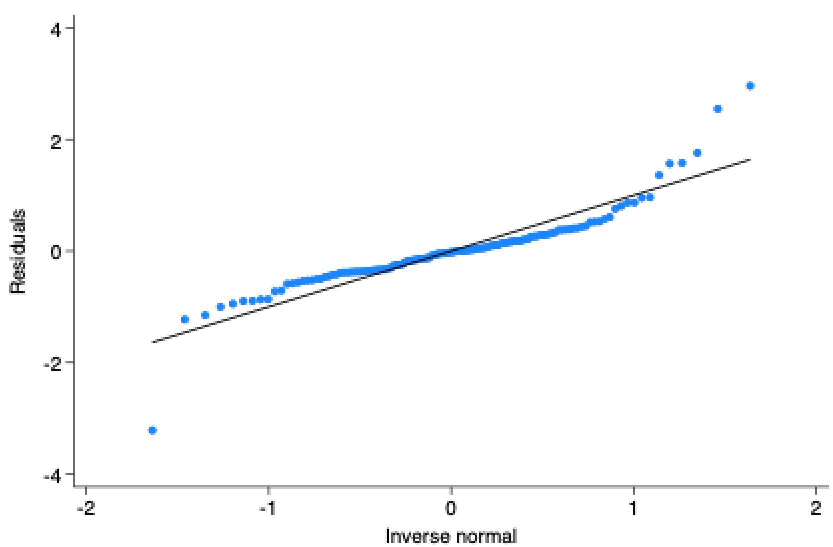



Figure 3.2



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