7. Coordinates-to-index and Index-to-coordinates

7.1 2-dimension

In a 2-dimensional grid, given the size of the grid is $(L_1,\ L_2)$ and the coordinate is $(\mathrm{x}_1,\ \mathrm{x}_2)$, the mathematical equation of concerting coordinates to index are expressed by:

$$I = x_1 + x_2 \times L_1$$

Given the size of the grid is $(L_1,\ L_2)$ and the index is I, the mathematical equation of concerting index to coordinates are expressed by:

$$x_2 = I \div (L_1)$$

$$x_1 = I - (x_1 \times L_1)$$

7.2 d-dimension

In a d-dimensional grid, given the size of the grid is $(L_1,\ L_2, \cdots, L_d)$ and the coordinate is $(\mathrm{x}_1,\ \mathrm{x}_2, \cdots, x_d)$, the mathematical equation of concerting coordinates to index are expressed by:

$$I = x_1 + x_2(L_1) + x_3(L_1L_2) + \cdots + x_{\mathrm{d}}(L_1L_2 \cdots L_{\text{d-1}})$$

$$= \sum_{\mathrm{i}=1}^{\mathrm{d}} x_d \prod_{\mathrm{j}=\mathrm{i}+1}^{\mathrm{d}} L_{\mathrm{j}}$$

Given the size of the grid is $(L_1,\ L_2)$ and the index is I, the mathematical equation of concerting index to coordinates are expressed by:

$$x_{\mathrm{d}} = \frac{I}{(L_1L_2 \cdots L_{\text{d-1}})}$$

$$x_{\text{d-1}} = \frac{I - x_{\mathrm{d}}(L_1L_2 \cdots L_{\text{d-1}})}{(L_1L_2 \cdots L_{\text{d-2}})}$$

$$\vdots$$

Which can be generalized as

$$x_d = \frac{\mathrm{I} - \sum_{\mathrm{i}=1}^{\text{d-1}} x_{\mathrm{i}} c_i}{c_d} \quad with\ \ c_d = \prod_{\mathrm{j}=\mathrm{i}+1}^{\mathrm{d}} L_{\mathrm{j}}$$