Linking Cap And Trade Schemes

Roweno J.R.K. Heijmans*

Reyer Gerlagh†

Tilburg University

For the most recent version, click here.

April 12, 2021

Abstract

Recent years have witnessed a rapid increase in the number of cap and trade schemes to mitigate greenhouse gas emissions. With multiple coexisting schemes, their linkage has become a topic of interest. This paper offers a theory of optimal linking under uncertainty. We show that efficient linkage adjusts the joint cap in response to observed trade between the schemes. It lowers allowance price volatility and increases global welfare by efficiently adjusting total emissions in response to private information. Interestingly, while asymmetric information generally harms welfare, asymmetric uncertainty can be exploited to increase welfare. Optimally linked cap and trade schemes expand the range of model parameters for which cap and trade is favored over a carbon tax.

JEL codes: D82, D83, H23, Q52, Q54

Keywords: asymmetric information, policy updating, asymmetric uncertainty, carbon markets, linking

1 Introduction

The number of cap and trade schemes to mitigate greenhouse gas emissions has grown steadily.[1] Reduced to its core, a cap and trade scheme caps CO_2 emissions by allocating

*Email: r.j.r.k.heijmans@uvt.nl

†Email: r.gerlagh@uvt.nl.

[1] The first major emissions trading system (ETS) for greenhouse gases – the European Emissions Trading System (EU ETS) – was established in 2005. To date, there are 20 ETSs in place across five

allowances to emitters who are then allowed to trade their permits; if a firm emits CO_2, it must surrender an equivalent amount of allowances. The policy combines the conservative certainty on emissions offered by direct command-and-control regimes with the efficient allocation of abatement efforts of a carbon tax.

Cap and trade schemes can link. A linkage between two schemes reciprocally enables the use of permits issued in one scheme to meet compliance obligations pursuant to another. Linking is seen as a promising development in cap and trade regulation (Mehling et al., 2018). Article 6 of the Paris Agreement expressly provides for the possibility of linking. Linking has become increasingly prominent in recent years. California's cap and trade system linked with Quebec's on 1 January 2014 and the schemes organize joint auctions. On 1 January 2020, a link between the European Union's Emissions Trading System (EU ETS) and the Swiss Emisions Trading System came into force. Linkages between the Regional Greenhouse Gas Initiative (RGGI) and the Emissions Trading Systems of Virginia and Pennsylvania are currently on their way, as are implicit linkages between California's ETS and Washintong State's Clean Air Rule.[2]

Linking is efficient because it leads to an equalization of marginal abatement costs across jurisdictions. An additional benefit may be that, through their increased cooperation, local planners are less likely to choose their policies noncooperatively (Mideksa and Weitzman, 2019), leading to a more efficient total emission levels. Landry (2021) shows that, even if local planers choose their emissions noncooperatively, competition in caps may restore efficiency.

This paper proposes a theory of optimal linking under aggregate abatement cost uncertainty. Linking two trading systems means that a firm can fulfill compliance obligations pursuant to one jurisdiction by means of emission allowances issued by the other. In practice this means that polluters in either jurisdiction are allowed to trade emission allowances freely and on a one-to-one basis. Free inter-scheme trade of allowances ensures that firms equate marginal abatement costs both within and across schemes. This is necessary for an optimal linkage because as long as marginal abatement costs are not equal across jurisdictions, mutually beneficial exchanges of allowances can be made, contradicting the notion of efficiency. The intuition is essentially the same as that favoring cap and trade over more direct command and control policies in a single jurisdiction. The literature on

continents and covering 27 jurisdictions which produce almost 40 % of global wealth (GDP). With over a dozen more governments considering or having already scheduled an ETS, emissions trading has emerged as a key instrument to cost effectively decarbonize our economies. (ICAP, 2020)

[2]The latter link is mostly hypothetical, as Washinton State's Clean Air Rule was suspended after a 2018 court ruling. Though contested, the ruling was largely upheld by the Washington State Supreme Court on Jan. 16, 2020.

linking has often emphasized this channel of potential efficiency gains (Carbone et al., 2009; Flachsland et al., 2009; Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020).

Allowances are traded between the schemes when marginal abatement costs differ across jurisdictions. The magnitude of net aggregate trade flows provides a sufficient statistic for the gap in marginal abatement costs under the initial allocation. Knowledge of this gap allows the planners to construct a posterior belief on realized abatement costs in both jurisdictions. Since this posterior will generally be different from their prior, the planners thus learn that the initial allocation of emission allowances was inefficient for two reasons. First, the distribution of allowances between the schemes, given the global cap, was suboptimal. This inefficiency is dealt with through trading. But second, the cap itself may turn out ex post inefficient from the point of view of global welfare. An optimal linkage attempts to deal with both of these inefficiencies.

To see how a flow of allowances between jurisdictions signals that the global cap is set inefficiently, note that the level of the cap balances the cost of climate change on the one hand, and the cost of abatement on the other. When posterior beliefs about abatement costs deviate from planners' expectations, that balance is lost. In this sense, emission levels are ex post inefficient, not because of (assumed) perfect information, but because of belief updating based on net trade between the schemes. We therefore propose to adjust the global cap in response to trade between the jurisdictions. To our knowledge, we are the first to study such endogenous cap adjustments in the context of linked cap and trade schemes.

Interestingly, what drives cap adjustments is not the amount of uncertainty per se, but rather the degree to which uncertainty is asymmetric between jurisdictions. This has a clear intuition. Trade flows signal a wedge in jurisdictional marginal abatement costs. Learning about the *absolute* level of abatement costs occurs when planners update beliefs, mostly for the least predictable jurisdiction. In the extreme case where the planners are uncertain about abatement costs in one jurisdiction only, trade flows allow them to pin down abatement costs in the uncertain jurisdiction exactly. Since in this case there is de facto complete information about abatement costs, the planners are able to implement the first-best social optimum. That is, by smartly linking emission trading schemes, aggregate uncertainty can be reduced to match the least uncertain scheme.

Caps become endogenous also when trading is allowed under non-unitary exchange rates. Indeed, optimal linking deviates from emissions traded one-for-one (or, in terms of climate change, ton-for-ton), when jurisdictions have asymmetric uncertainty (Holland

and Yates, 2015). But the efficiency gain associated with cap flexibility comes with a substantial loss of allocative efficiency. To illustrate, suppose we were to contemplate an exchange rate such that 1 allowance issued by jurisdiction N can be traded against 2 allowances issued in jurisdiction S. Then firms in N and S will trade allowances until the marginal cost of reducing emissions in N equals *half* the marginal cost of reducing emissions in S. A non-unitary exchange rate thus drives firms' incentives away from an efficient distribution of abatement efforts.[3]

Cap adjustments based on observed allowance prices is another alternative policy. Compared to quantities, prices are highly efficient information aggregators. Indeed, when the planners observe both emissions and the market price for allowances, they can perfectly back out the abatement cost functions in both jurisdictions. The possibility of using price information hence allows for implementation of the unconstrained best cap on emissions. Our first and foremost recommendation is therefore to adjust the caps in response to carbon prices. Importantly, in practice proposed price-based interventions are often discrete and based on price thresholds; the implementation then remains imperfect and our quantity-based optimal linkage can perform better as it allows for a continuous processing of market information.[4]

Linking cap and trade systems across jurisdictions is related to integrating cap and trade markets over time (Yates and Cronshaw, 2001). Dynamic linking was studied in Heutel (2020) and Pizer and Prest (2020) for flow pollutants, and in Gerlagh and Heijmans (2020) for stock pollutants.[5]

Although we use the language of multiple cap and trade schemes, our analysis also applies to situations where a previously uncovered industry is newly added to an existing scheme, or to cases where multiple unregulated industries are combined into a newly formed cap and trade scheme. Our results are therefore relevant in discussions on such topics as the inclusion of road transport among the industries covered by the EU ETS, or on the extension of RGGI beyond the electricity sector. Similarly, our analysis motivates the question whether clearly identifiable jurisdictions within existing schemes – member states in the EU ETS, or countries in RGGI – are currently "linked" in the most efficient

[3]The same need not apply to local pollutants like NO_X or lead pollution.

[4]An important example to which our theory applies is any linkage involving the EU ETS. Despite repeated calls for a European price floor (Flachsland et al., 2020), the European Union uses only information on quantities for cap-adjustments. Linkages between any cap and trade scheme and the EU ETS will hence benefit from our analysis, "key features for compatibility for linking" being "complications around intervention in price" (EU Commission, 2015, EU ETS Handbook)."

[5]For EU ETS, Gerlagh and Heijmans (2019) and Gerlagh et al. (2021) illustrate several unexpected side-effects of endogenous intertemporal emission caps. This offers an important warning: endogenous cap-adjustments can be efficient, but details matter.

way possible. These are highly relevant policy questions that deserve greater attention from policymakers and academics alike.

The paper is organized as follows. Section 2 introduces our model and the building blocks for welfare analysis. Section 3 discusses different types of (integrated) cap and trade policies and develops our theory of optimal linking. At the end, we revisit the perennial question of instrument choice, prices versus quantities (Weitzman, 1974). Section 4 concludes. Proofs and lengthy derivations are in the Appendix.

2 Model

Given are two jurisdictions, North and South, each operating its own cap and trade scheme. The assumption of two jurisdictions is not restrictive. One may simply consider North a representative jurisdiction for two linked jurisdictions, East and West, where trade between North and South is essentially a reduced-form way of writing trade between East, West, and South.

Each jurisdiction i is populated by firms that produce a composite good the production of which causes emissions. We may write benefits $B_i(\tilde{e}_i;\theta_i)$ as a function of emissions $\tilde{e}_i$, given by:

$$B_i(\tilde{e}_i \mid \theta_i) = (p^* + \theta_i)(\tilde{e}_i - e_i^*) - \frac{b_i}{2}(\tilde{e}_i - e_i^*)^2. \tag{1}$$

Emissions yield benefits because they allow firms to produce goods and save on the cost of abatement. As an umbrella term, we refer to $B_i(\tilde{e}_|\theta_i)$ as abatement costs in jurisdiction i, but other interpretations are possible. For notational convenience, we normalize benefits relative to the ex-ante optimal allocation e_i^* and prices $p_i = p^*$. The parameter θ_i is a fundamental of jurisdiction i's economy and is *private information* of its constituent firms, though it is common knowledge that $\mathbb{E}[\theta_i] = 0$, $\mathbb{E}[\theta_i^2] = \sigma_i^2$, and $\mathbb{E}[\theta_N\theta_S] = \rho\sigma_N\sigma_S$. We interpret σ_i^2 as a measure for the uncertainty about jurisdiction i's economy. We say that uncertainty is asymmetric if $\sigma_N \neq \sigma_S$.

Emissions accumulate in the atmosphere and cause climate change as a global externality. The cost of climate change is given by:

$$C(\tilde{e}_N + \tilde{e}_S) = p^*(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*) + \frac{c}{2}(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*)^2. \tag{2}$$

Costs are also measured relative to the ex-ante optimum e_i^*. We make the simplifying assumption that emissions have local benefits but global costs. One could imagine more complicated settings where emissions also have strictly regional costs like local air pollution

(Caplan and Silva, 2005; Antoniou and Kyriakopoulou, 2019). We abstract from these aspects and focus on the optimal linking of cap and trade schemes to regulate a global externality.

Subtracting the costs of climate change from the benefits due to emissions yields welfare:

$$W = B_N(\tilde{e}_N \mid \theta_N) + B_S(\tilde{e}_S \mid \theta_S) - C(\tilde{e}_N + \tilde{e}_S). \tag{3}$$

Policies are set to align polluters' incentives with the global social cost of carbon (Kotchen, 2018). We interpret optimal linking as the integrated policy for e.g. an international climate treaty like the Paris Agreement or its successor. Yet even strictly local planners may more or less explicitly consider global climate damages when deciding on mitigation policies. There are at least two pieces of suggestive evidence to support our claim. First, most if not all existing cap and trade schemes are policymakers' attempts at meeting mitigation obligations implied by the Paris Agreement. Since the Paris Agreement is global in scope and intention, cap and trade schemes set up to satisfy the associated pledges explicitly or implicitly operate with a measure of global climate welfare in mind.[6] Second, actual linkages often start from a certain amount of cooperation and mutual agreement on the cap. Indeed, among the "essential criteria" to ensure "compatibility between the systems" mentioned up in Annex I of the *Agreement between the European Union and the Swiss Confederation on the linking of their greenhouse gas emissions trading systems* are the "ambition and stringency of the cap". Similarly, the independent but linked cap and trade schemes of California and Quebec organize joint auctions of allowances, implying a high degree of cooperation in determining the linked caps.

We remove tildes for deviations from the ex-ante optimum, $e_i = \tilde{e}_i - e_i^*$, and similar for prices; $\Delta e := \tilde{e} - e^{SO}$ denotes the difference between the value of e and its ex post (after observing θ_i) socially optimal value (see subsection 2.1). For total emissions, we write $\tilde{E} = \tilde{e}_N + \tilde{e}_S$ and for deviations from the ex-ante optimum $E = e_N + e_S$. We use superscripts for policy rules or scenarios.

Firms are profit maximizers. Once a policy k caps emissions at a level e_i^k, individual

[6] As an example, "[...] the [European] Commission proposed in September 2020 to raise the 2030 greenhouse gas emission reduction target, including emissions and removals, to at least 55% compared to 1990. [...] This will enable the EU to move towards a climate-neutral economy and implement its commitments under the Paris Agreement by updating its Nationally Determined Contribution." Moreover, "[t]o achieve the EU's overall greenhouse gas emissions reduction target for 2030, the sectors covered by the EU ETS must reduce their emissions by 43% compared to 2005 levels." Retrieved from https://ec.europa.eu/clima/policies/strategies/2030_en.

firms trade allowances until marginal abatement costs for all are equal to:

$$p_i^k = -b_i e_i^k + \theta_i, \tag{4}$$

which is firms' inverse demand for allowances. In a competitive market for allowances, the price at which permits are traded will be p_i^k in equilibrium.

2.1 Global Social Optimum

In a perfect world without informational frictions – the Social Optimum – a welfare-maximizing planner allocates emissions between the jurisdictions in such a way that marginal benefits are the same in both, i.e. so that $MB_N^{SO} = MB_S^{SO} = MB^{SO}$. Since both jurisdictions by assumption operate a cap and trade scheme, marginal benefits of emissions are also equal to the carbon price, so $p_N^{SO} = p_S^{SO} = p^{SO} = MB^{SO}$. Next, since the level of emissions if efficient if and only if marginal climate costs equal marginal benefits in either jurisdiction, we obtain the following conditions for the socially optimal emission levels:

$$c \cdot (e_N^{SO} + e_S^{SO}) = p^{SO} = -b_i e_i^{SO} + \theta_i. \tag{5}$$

Equation (5) characterizes the Social Optimum and represents three equalities in three unknowns: e_N^{SO}, e_S^{SO}, and p^{SO}. Solving for these, we obtain:

$$p^{SO} = \frac{c(b_S\theta_N + b_N\theta_S)}{cb_N + cb_S + b_N b_S}, \tag{6}$$

$$e_i^{SO} = \frac{b_{-i}\theta_i + c(\theta_i - \theta_{-i})}{cb_N + cb_S + b_N b_S}, \tag{7}$$

$$E^{SO} = \frac{b_S\theta_N + b_N\theta_S}{cb_N + cb_S + b_N b_S}, \tag{8}$$

where $i = N, S$ and $-i$ refers to "the other jurisdiction that is not i". As is intuitive, socially optimal emission levels are higher when abatement is more expensive; E^{SO} increases in θ_i. This basic observation will be useful later on, when we illustrate that trades of allowances between (linked) schemes signal private information about abatement costs to the planners. An optimal linkage exploits this information by adjusting the global cap in response.

For future reference, we note that the variance of prices is given by:

$$\mathbb{E}\left[\left[p^{SO}\right]^2\right] = \left(\frac{c}{c \cdot b_N + c \cdot b_S + b_N b_S}\right)^2 \left[b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho \sigma_N \sigma_S\right]. \tag{9}$$

Thus, abatement cost uncertainty translates into price volatility. We will return to this later.

2.2 Welfare Losses

We rank policies according to their expected welfare levels. Suppose a policy k induces emission levels $\tilde{e}_i^k$ in jurisdiction i. From firms' equilibrium behavior (4), we see that deviations in emissions from the social optimum scale with prices:

$$\Delta^k p_i = -b_i \Delta^k e_i. \tag{10}$$

Expected welfare losses relative to the social optimum are then given by:

$$\begin{aligned} L^k &= \mathbb{E}\left[\Delta^k B_N + \Delta^k B_S - \Delta^k C\right] \\ &= \frac{c}{2}\mathbb{E}\left[\left(\Delta^k E\right)^2\right] + \sum_i \frac{b_i}{2}\mathbb{E}\left[\left(\Delta^k e_i\right)^2\right]. \end{aligned} \tag{11}$$

Throughout the analysis, we interpret a policy as the implementation of a constrained expected welfare maximization problem. We note that the level of welfare in the Social Optimum, W^{SO}, is hypothetical and unaffected by the policy implemented. Thus, expected welfare maximization subject to a set of policy constraints can be treated as the dual problem of expected welfare loss minimization relative to the Social Optimum, subject to the same constraints. Given the equivalence between these two approaches, we may use them interchangeably for convenience.

Linking policies as we study here exhibit equal prices across jurisdictions in equilibrium, as we shall explain below. By (10), emissions (both local and global) under such policies can therefore be expressed in terms of the common price gap Δp. Plugging this into (11), expected welfare losses from a policy featuring equal prices across jurisdictions can be written as:

$$L^k = \frac{1}{2}\frac{(cb_N + cb_S + b_N b_S)(b_N + b_S)}{b_N^2 b_S^2}\mathbb{E}\left[\left(\Delta^k p\right)^2\right]. \tag{12}$$

3 Policies

3.1 Regional cap and trade

The simplest policy operates two separate cap and trade schemes. In this case, the planner of jurisdiction i sets a cap e_i to maximize (3). The first-order condition that determines the policy requires expected marginal benefits in each scheme to equal marginal climate damages. Emissions e_i are chosen such that

$$\mathbb{E}[MB_i \mid e_i] = MC, \tag{13}$$

resulting in emission levels equal to the ex-ante optimum $e_N = e_S = 0$. Plugging these, through (7)-(8), into (11), we obtain expected welfare losses when both jurisdictions operate a regional cap and trade scheme:

$$L^R = \frac{1}{2}\frac{(c + b_S)\sigma_N^2 + (c + b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{14}$$

We note that the marginal damages (the RHS of (13)) are perfectly known when caps are fixed, whereas marginal abatement costs are stochastic variables due to the unobserved fundamentals θ_i. Thus, regional cap and trade implements socially optimal emission levels if and only if abatement costs turn out exactly as expected ($\theta_N = \theta_S = 0$). If abatement costs deviate from expectations, regional cap and trade is inefficient for two reasons. First, the allocation of abatement efforts between the jurisdictions may be ex post inefficient (abatement costs may differ between them). Second, the level of emissions is ex post inefficient. The first of these is remedied by linking schemes across jurisdictions.

3.2 Linking

When North and South link their cap and trade schemes, each planner sets the expected optimal cap for its jurisdiction but allowances issued in one scheme may be used to fulfill abatement obligations pursuant to another. Thus polluters are free to trade their allowances as long as global emissions are not affected:

$$e_N + e_S = E = 0. \tag{15}$$

Linking has attracted a lot of attention in recent years (Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). Just as trade in emission allowances between firms within a jurisdiction is an efficient way to achieve a given amount of local abatement, so the trade of allowances between jurisdictions is an efficient way to achieve a pre-determined amount of global abatement. The intuition is essentially the same: while linking does not affect global emissions, and therefore climatic damages, it does lead to a closer alignment of the benefits and costs of abatement across jurisdictions.

In fact, as long as marginal abatement costs in the jurisdictions are *not* the same, firms can and will profitably exchange permits. The integrated market for emission allowances therefore reaches equilibrium only once the price of an allowance is the same in North and South:

$$p_N = p_S. \tag{16}$$

Moreover, since a firm in either jurisdiction is willing to sell (buy) an allowance against the going market price as long as the price is above (below) its private marginal abatement cost, marginal benefits are also the same across firms and jurisdictions when schemes are linked:

$$-b_N e_N^L + \theta_N = -b_S e_S^L + \theta_S, \tag{17}$$

where e_N and e_S are chosen by the firms conditional on θ_N and θ_S, subject to (15). Plugging this into our welfare loss-function (11), we obtain:

$$L^L = \frac{1}{2}\frac{1}{b_N + b_S}\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho \sigma_N \sigma_S}{cb_N + cb_S + b_N b_S}. \tag{18}$$

Comparing welfare losses, we can now formally state our first substantive result.[7]

Proposition 1. *Linking cap and trade schemes increases global welfare.*

In contrast to regional cap and trade, linking guarantees that marginal abatement costs are equal in both jurisdictions. For this reason, linking is always weakly better for welfare than regional cap and trade. The planners' problem consists of two steps, each with its own an intuitive meaning. First, the planners of North and South cap local emissions at levels that, in expectations, maximize global welfare. When the local caps are set, allowances can be traded on a one-to-one basis between schemes, as long as emissions overall remain fixed at the sum of the two jurisdictional caps. By linking their schemes, the planners of North and South effectively delay the determination of local emission caps until after θ_N and θ_S are known, guaranteeing an ex post efficiency gain through marginal abatement cost equalization

$$MB = MB_N = MB_S \tag{19}$$

In expectations thes equal marginal damages. Cumulative emissions $e_N + e_S$ is chosen

[7]See appendix for derivations and proof.

such that:

$$MB = MB_N = MB_S \tag{20}$$

$$\mathbb{E}[MB \mid e_N + e_S] = MC. \tag{21}$$

Linking benefits global welfare (Proposition 1) but the effects on individual jurisdictions are ambiguous. To see this, note that prices in equilibrium equate marginal benefits, so that the volatility of prices is equal to the volatility of marginal benefits. Hence if abatement costs in South are much less predictable than in North, $\sigma_S > \sigma_N$, North may import part of the price volatility to which South is subject. If this effect is strong enough, North may be harmed by its linkage with South (Holtsmark and Weitzman, 2020; Habla and Winkler, 2018).

Linking suffers from another, more implicit type of inefficiency. Suppose that after trading the planners observe emissions levels e_N^L and e_S^L in jurisdictions N and S, respectively. Since firms will trade allowances between the jurisdictions until marginal abatement costs are equal everywhere, we know that (17) must be satisfied under the observed emission levels e_N^L and e_S^L. Rewriting (17), the planners therefore learn $\mu = \theta_N - \theta_S$. Conditional on μ, planners update their beliefs on the true marginal abatement cost curve in both of the jurisdictions. Since this posterior will generally deviate from their priors, the expected optimal cap on emissions should ideally respond to the observed trade of allowances. It does not under standard linking.

3.3 Optimal Linking

We will now construct our optimal linking policy. We proceed in two steps. First, we derive the expected optimal emission level conditional on the trade flows between the jurisdictions. Second, we formulate a mechanism that is known to all firms and allows the planners to implement the expected optimal cap for any observed trade of allowances.

As we discussed, when post-trading emissions levels are e_N and e_S in jurisdictions N and S, respectively, the planners can back out μ, the vertical distance between regional marginal abatement cost functions:

$$\mu := b_N e_N^L - b_S e_S^L = \theta_N - \theta_S. \tag{22}$$

A key observation is that μ contains more information than the relative position of jurisdictions' abatement cost functions alone – it also signals something about the *absolute* location of the curves. To see this most simply, suppose that the planners are uncertain only about θ_N whereas θ_S is perfectly known (i.e. suppose that $\sigma_S = 0$). In this hypothetical case, observing μ is clearly equivalent to observing θ_N directly. In the more general case where both θ_N and θ_S are unknown, such sharp posteriors are not possible. Still the planners know all the combinations of θ_N and θ_S consistent with μ. Depending on the regional uncertainties σ_N and σ_S, some of these combinations will be more likely than others. The planners can therefore calculate the expected marginal abatement cost in both jurisdictions, conditional on μ and/or post-trade emissions:

$$\begin{aligned}\mathbb{E}[MB \mid \mu] &= \mathbb{E}[\theta_N \mid \mu] - b_N e_N = \mu\frac{\mathbb{E}[\mu\theta_N]}{\mathbb{E}[\mu^2]} - b_N e_N \\ &= \mu\frac{\rho\sigma_N\sigma_S - \sigma_N^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} - b_N e_N \\ &= -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S. \end{aligned} \tag{23}$$

Conditional expected marginal abatement costs in both jurisdictions, as a function of post-trade emission levels e_N and e_S, are given by (23). Since the planners' goal is to achieve that marginal abatement costs, post-trade, are equal to marginal climate damages, the planners need to solve for e_N and e_S that solve:

$$c(e_N + e_S) = -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S. \tag{24}$$

Classic linking, as discussed in the previous section, requires that global emissions do not change in response to trading, so $e_N = -e_S$. This makes the left-hand side of (24) equal to zero by construction. But the right-hand side of (24) is not, generally, equal to zero. Thus, conditional on the information that becomes available after allowances are traded, the initial cap on emissions turns out inefficient. Indeed, it is not difficult to see that an expected efficient cap, given the private information revealed through allowance trading, satisfies the following elegant condition:

$$-\frac{e_S}{e_N} = \delta, \tag{25}$$

with δ a parameter that is endogenous to the structure of our model and given by:

$$\delta = \frac{b_N[\sigma_S^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}{b_S[\sigma_N^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}. \tag{26}$$

We refer to δ as the cap-adjustment rate since it prescribes how the global cap on emissions should be adjusted in response to the information revealed by allowances trading. Interestingly, the cap-adjustment rate δ may be negative. When this happens, higher emissions in one jurisdiction translate into higher emissions in the other jurisdiction as well. The reason is intuitive: if abatement costs in one jurisdiction are very unpredictable yet strongly correlated to those in the other, high abatement costs in the latter are likely matched by equally high costs in the former.

The problem solved by Optimal Linking, is to find the allocation rule that satisfies:

$$MB = MB_N = MB_S \tag{27}$$

$$\mathbb{E}[MB \mid \mu] = MC. \tag{28}$$

The policy is by construction the most efficient type of linkage using information on emissions only. It yields emission levels that are optimal conditional on the most refined information contained in allowance trading. This, indeed, is much more precise – and therefore efficient – than simply solving for a fixed joint cap.

With the preceding discussion in mind, we can formulate our Optimal Linking policy. Two properties are key:

1. Firms exchange emission allowances between jurisdictions on a one-to-one basis. For allocative efficiency, there cannot be a non-unitary exchange rate on allowance trading;

2. Conditional on the net number of allowances traded, the planners buy or auction extra allowances.

Property 1 is crucial because exchange rates according to which an allowance worth one ton of emissions in North is worth $\alpha \neq 1$ tons in South cannot be optimal. Property 2 is needed to efficiently adjust the cap in response to private information revealed through trading. Knowing that the optimal linkage implements (24), we can derive expected welfare losses from an Optimal Linking policy.

Proposition 2. *Optimal Linking is the best possible policy involving only information on quantities. Expected welfare losses under an Optimal Linking policy are given by:*

$$L^O = \frac{1}{2}\frac{b_N + b_S}{cb_N + cb_S + b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{29}$$

The first part of Proposition 2 follows from the fact that an optimal linkage implements (28) by construction.

An optimal linkage performs remarkably well. When uncertainty about abatement costs is strongly asymmetric across jurisdictions ($\sigma_N/\sigma_S \to 0$ or ∞), or when abatement costs are highly correlated ($\rho \to \pm 1$), optimal linking allows for welfare levels very close to the full information Social Optimum. This reflects the planners' great scope for learning in these cases.

Corollary 1. *Where the planners have perfect information about one of the two linked jurisdictions ($\sigma_i = 0$ for $i \in \{N, S\}$), or when abatement costs are perfectly correlated ($\rho = \pm 1$), Optimal Linking implements the first best levels of emissions.*

Even though asymmetric *information* leads to welfare losses, asymmetric *uncertainty* compensates for part (and, in extreme cases, all) of these losses.

Another way to see the great advantage of an optimal linkage is to compare its welfare performance with that of a classic linking policy:

$$\frac{L^O}{L^L} = \frac{(b_N^2 + b_S^2 + 2b_n b_S)(1+\rho)\sigma_N\sigma_S}{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_n b_S \rho\sigma_N\sigma_S} \cdot \frac{(1-\rho)\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{30}$$

As we expect from Corollary 1, an optimal linkage performs much better than classic linking ($L^O/L^L \to 0$) when uncertainty is highly asymmetric or when abatement costs are strongly correlated. If we believe that abatement costs are driven by macroeconomic conditions and available technologies they likely are strongly correlated across jurisdictions indeed – this makes a compelling case for optimal (rather than a standard) linking of the cap and trade schemes. Interestingly, though the absolute levels of welfare losses under both classic an optimal linking depend on climate damages through c, the relative performance of an optimal linkage is independent of the slope of climate damages.

We recall that, though global welfare increases after establishing an optimal linkage, individual jurisdictions may still be worse off. A jurisdiction in which abatement costs are relatively predictable may, through linking, expose itself to the volatile abatement costs in the other jurisdiction and therefore import a variable allowance price (Holtsmark and Weitzman, 2020). To the extent that such concerns are important for real world linkages, our optimal linking policy offers some relief.

Proposition 3. *Optimally linked cap and trade schemes admit lower price volatility than classically linked cap and trade schemes:*

$$\mathbb{E}\left[\left(p^{OL}\right)^2\right] \leq \mathbb{E}\left[\left(p^{L}\right)^2\right]. \tag{31}$$

Thus, even though it remains possible that an individual jurisdiction experiences higher allowance price volatility once an optimal linkage was established, this effect (if it occurs) will be less severe than under standard linking. Note that with inter*temporal* trading of permits, an endogenous cap also reduces price volatility, see Gerlagh et al. (2020).

Optimal Linking is the best the planners can do when using only information on quantities to update their beliefs. From a theoretical point of view, the limitation to quantity-information is arbitrary. If the planners are willing to use both (post-trade) emissions and allowance prices, they can perfectly back out the marginal abatement cost function in each jurisdiction i. To see this, we rewrite firms' inverse demand function (4) to obtain:

$$\theta_i = p + b_i e_i. \tag{32}$$

An ideal policy uses information on both prices and quantities to pin down θ_i in each jurisdiction $i = N, S$ and, given these, adjusts the caps so that emission levels end up in the Social Optimum. Our first and foremost recommendation is therefore to implement a policy along those lines.

It is important to note that the ideal instrument is continuous in prices and emissions. A simple price collar – often proposed in the context of cap and trade policies – will not work. To our knowledge, there is no literature on optimal price collars. We conjecture that an Optimal Linking policy outperforms price-collar-based cap adjustments when uncertainty is highly asymmetric or when abatement costs are strongly correlated (which likely they are since abatement costs are largely driven by technological developments and macroeconomic conditions). In these cases, an optimal linkage implements welfare levels very close to the Social Optimum, see Corollary 1.

3.4 Prices vs. Quantities

We saw how jurisdictions can optimally link their cap and trade schemes. As an alternative, each jurisdiction could instead levy a carbon tax. We now revisit the classic question of Weitzman (1974) on the choice between instruments.

We assume that emissions are taxed at an expected optimal rate – that is, taxes are set to minimize (11). Recall that p^* is defined as the expected welfare-maximizing carbon price in each jurisdiction, so the expected optima tax sets $p_N = p_S = 0$. We may therefore invoke (9) and (12) to derive expected welfare losses when both jurisdictions tax emissions:

$$L^{tax} = \frac{1}{2}\left(\frac{c}{b_N b_S}\right)^2 \frac{b_N + b_S}{cb_N + cb_S + b_N b_S}(b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S). \tag{33}$$

All else equal, the expected welfare loss when jurisdictions tax emissions is increasing in c, the marginal climate damage. We can now compare (33) and (29), which yields the following proposition.

Proposition 4. *Optimally Linked cap and trade schemes are favored over Taxes if and only if:*

$$\frac{(1+\rho)\sigma_N\sigma_S}{b_S^2\sigma_N^2+b_N^2\sigma_S^2+2b_Nb_S\rho\sigma_N\sigma_S}\frac{(1-\rho)\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}<\left(\frac{c}{b_Nb_S}\right)^2 \tag{34}$$

Inequality (34) is likely satisfied when both regions have strongly asymmetric uncertainty ($\sigma_N/\sigma_S \to 0$ or ∞) or when abatement costs are highly correlated ($\rho \to \pm 1$). This should not come as a surprise: Corollary 1 showed that precisely in these cases two optimally linked cap and trade schemes implement welfare levels very close to the complete information social optimum. Proposition 4 once again underlining the real-world relevance of optimal linking regime.

4 Summary

We propose a simple theory of optimal linking. Our results have the potential to greatly increase welfare compared to current practices. The core of an optimal linkage boils down to a basic observation: trading in allowances between schemes signals valuable information about abatement costs in the jurisdictions. An efficient policy aims to incorporate this information and adjusts the linked global cap in response to allowance trading. We pin down a precise analytic formulation for such endogenous policy updating.

There are various ways to adjust the global cap in response to trade flows, but under an optimal linkage, firms are allowed to exchange allowances one-for-one both within and across schemes. Thus, our optimal linking policy cannot rely on "trading ratios" for emissions allowances (Holland and Yates, 2015). While trading ratios on allowances could endogenizes the (global) cap in response to trading (Holland and Yates, 2015), it disturbs individuals firms' incentives away from an exact equalization of marginal abatement costs. A straightforward way to achieve this is to either inject new permits or buy back already issued ones as called for by the observed trade flows. This is not too complicated; Hintermayer (2020) analyzes a buyback policy in a dynamic model of the EU ETS.

An important concept is asymmetric uncertainty. When two schemes trade allowances, information on *relative* abatement costs is revealed. But when the schemes are asymmetrically uncertain, this information on relative abatement costs can be used to make (sharp) predictions about *absolute* abatement costs as well. The same is not possible when trade between symmetrically uncertain schemes is observed. Our results suggest that the study of asymmetric uncertainty deserves a more prominent place in environmental economics.

Cap and trade schemes have become a major policy instrument in the fight against climate change. In Europe alone, roughly 45% of greenhouse gas emissions are regulated by EU ETS, the world's largest market for carbon. Ever more cap and trade schemes being erected, linking is now a prominent policy issue – linkages between local schemes already exist, and more are contemplated. The are linkages between EU ETS and the Swiss ETS, between RGGI and Quebec, between Quebec and California. The up-and-coming carbon markets of China and the post-Brexit UK will create new possibilities for linking. Given the large amount of money and CO_2 involved in cap and trade, and given the increasing prevalence of linkages between jurisdictional schemes, constructive ideas on optimal linking are needed. This paper offers some initial thoughts.

Although are narrative focuses on linking of cap and trade schemes at the level of a jurisdiction, our analysis equally applies to cases where an existing cap and trade scheme is expanded to cover additional sectors, or where a new cap and trade scheme covering several distinct industries is erected. Consider aviation. Passenger flights outside the European Economic Area are not covered by the EU ETS. To nevertheless reduce emissions in the aviation industry, "the International Civil Aviation Organization (ICAO) agreed on a Resolution for a global market-based measure to address CO2 emissions from international aviation as of 2021. The agreed Resolution sets out the objective and key design elements of the global scheme, as well as a roadmap for the completion of the work on implementing modalities. The Carbon Offsetting and Reduction Scheme for International Aviation, or CORSIA, aims to stabilise CO2 emissions at 2020 levels by requiring airlines to offset the growth of their emissions after 2020."[8] Though it is still unclear how exactly CORSIA and EU ETS will interact, our analysis suggests that a direct incorporation of CORSIA into the EU ETS may be suboptimal. Similarly, our results suggest that trade of allowances between sectors or clearly identified jurisdictions within an existing cap and trade scheme – countries in the EU ETS, states in RGGI, industries in the South Korea ETS – can be made more efficient by incorporating a policy along the lines of our optimal linking regime.

Our theory is preliminary and does not address several aspects of real life emission trading. Most large-scale cap and trade schemes are dynamic and allow covered industries to bank (and sometimes borrow) allowances across periods. How two dynamic cap and trade schemes should optimally be linked will likely depend on details of the dynamic policy. Similarly, it remains unclear how two cap and trade schemes, each with their own

[8]Retrieved from https://ec.europa.eu/clima/policies/transport/aviation_en

price collar on allowances, should best be linked. We leave an exploration of these and other aspects for future research.

References

Antoniou, F. and Kyriakopoulou, E. (2019). On the strategic effect of international permits trading on local pollution. *Environmental and Resource Economics*, 74(3):1299–1329.

Caplan, A. J. and Silva, E. C. (2005). An efficient mechanism to control correlated externalities: redistributive transfers and the coexistence of regional and global pollution permit markets. *Journal of Environmental Economics and Management*, 49(1):68–82.

Carbone, J. C., Helm, C., and Rutherford, T. F. (2009). The case for international emission trade in the absence of cooperative climate policy. *Journal of Environmental Economics and Management*, 58(3):266–280.

Doda, B., Quemin, S., and Taschini, L. (2019). Linking permit markets multilaterally. *Journal of Environmental Economics and Management*, 98:102259.

Doda, B. and Taschini, L. (2017). Carbon dating: When is it beneficial to link etss? *Journal of the Association of Environmental and Resource Economists*, 4(3):701–730.

Flachsland, C., Marschinski, R., and Edenhofer, O. (2009). To link or not to link: benefits and disadvantages of linking cap-and-trade systems. *Climate Policy*, 9(4):358–372.

Flachsland, C., Pahle, M., Burtraw, D., Edenhofer, O., Elkerbout, M., Fischer, C., Tietjen, O., and Zetterberg, L. (2020). How to avoid history repeating itself: the case for an eu emissions trading system (eu ets) price floor revisited. *Climate Policy*, 20(1):133–142.

Gerlagh, R. and Heijmans, R. (2020). Regulating stock externalities.

Gerlagh, R. and Heijmans, R. J. (2019). Climate-conscious consumers and the buy, bank, burn program. *Nature Climate Change*, 9(6):431–433.

Gerlagh, R., Heijmans, R. J., and Rosendahl, K. E. (2020). COVID-19 tests the market stability reserve. *Environmental and Resource Economics*, 76(4):855–865.

Gerlagh, R., Heijmans, R. J., and Rosendahl, K. E. (2021). An endogenous emission cap produces a green paradox. *Economic Policy*.

Habla, W. and Winkler, R. (2018). Strategic delegation and international permit markets: Why linking may fail. *Journal of environmental economics and management*, 92:244–250.

Heutel, G. (2020). Bankability and information in pollution policy. *Journal of the Association of Environmental and Resource Economists*, 7(4):779–799.

Hintermayer, M. (2020). A carbon price floor in the reformed EU ETS: Design matters! *Energy Policy*, 147:111905.

Holland, S. P. and Yates, A. J. (2015). Optimal trading ratios for pollution permit markets. *Journal of Public Economics*, 125:16–27.

Holtsmark, B. and Weitzman, M. L. (2020). On the effects of linking cap-and-trade systems for co2 emissions. *Environmental and Resource Economics*, 75(3):615–630.

ICAP (2020). Icap brief 3: Emissions trading at a glance. *International Carbon Action Partnership*.

Kotchen, M. J. (2018). Which social cost of carbon? A theoretical perspective. *Journal of the Association of Environmental and Resource Economists*, 5(3):673–694.

Landry, J. R. (2021). Think globally, cap locally, and trade widely: Efficient decentralized policy making in the presence of spillovers. *Journal of the Association of Environmental and Resource Economists*, 8(1):91–124.

Mehling, M. A., Metcalf, G. E., and Stavins, R. N. (2018). Linking climate policies to advance global mitigation. *Science*, 359(6379):997–998.

Mideksa, T. K. and Weitzman, M. L. (2019). Prices versus quantities across jurisdictions. *Journal of the Association of Environmental and Resource Economists*, 6(5):883–891.

Pizer, W. A. and Prest, B. C. (2020). Prices versus quantities with policy updating. *Journal of the Association of Environmental and Resource Economists*, 7(3):483–518.

Weitzman, M. L. (1974). Prices vs. quantities. *Review of Economic Studies*, 41(4):477–491.

Yates, A. J. and Cronshaw, M. B. (2001). Pollution permit markets with intertemporal trading and asymmetric information. *Journal of Environmental Economics and Management*, 42(1):104–118.

A Derivations and Proofs

DERIVATION OF (18):

Combining the definition with the firms' FOCs, (4), we find the change in permit use by jurisdiction:

$$\Delta^L e_N = \frac{\theta_N - \theta_S}{b_N + b_S} \tag{35}$$

$$\Delta^L e_S = \frac{\theta_S - \theta_N}{b_N + b_S}. \tag{36}$$

PROOF OF PROPOSITION 1:

Proof. We only need to compare welfare losses under a linked cap and trade regime, equation (18), to those under jurisdictional cap and trade, equation (14). Linking outperforms regional cap and trade iff:

$$\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{b_N + b_S} < (c + b_S)\sigma_N^2 + (c + b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S$$

$$\Longleftrightarrow$$

$$\frac{2\rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2} < \frac{cb_N + cb_S + b_N b_S}{cb_N + cb_S},$$

which is always true as the LHS is below and the RHS is above one. □

DERIVATION OF (26):

Regional and global deviations from Socially Optimal permit use are given by:

$$\Delta^O e_N = \frac{b_S}{b_N + \delta b_S} \frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{37}$$

$$\Delta^O e_S = \frac{b_N}{b_N + \delta b_S} \frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{38}$$

$$\Delta^O Q = \frac{b_N + b_S}{b_N + \delta b_S} \frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S}. \tag{39}$$

Define

$$\xi := \frac{b_N + c(1-\delta)}{b_N + \delta b_S} \implies 1 - \xi := \frac{\delta b_S - c(1-\delta)}{b_N + \delta b_S}. \tag{40}$$

Welfare losses can now be written as:

$$\begin{aligned} L^O &= \frac{1}{2}\frac{c(b_N+b_S)^2+b_N^2 b_S+b_N b_S^2}{(cb_N+cb_S+b_N b_S)^2}\mathbb{E}\left[(1-\xi)\theta_N+\xi\theta_S\right]^2 \\ &= \frac{b_N+b_S}{2}\frac{(1-\xi)^2\sigma_N^2+\xi^2\sigma_S^2+2\xi(1-\xi)\rho\sigma_N\sigma_S}{cb_N+cb_S+b_N b_S}. \end{aligned} \tag{41}$$

If for notational convenience, we define:

$$\psi := \frac{1}{2}\frac{b_N+b_S}{cb_N+cb_S+b_N b_S}, \tag{42}$$

it is straightforward to derive:

$$\frac{\partial}{\partial\xi}\frac{L^O}{\psi} = 2\xi\sigma_S^2 - 2(1-\xi)\sigma_N^2 + 2(1-\xi)\rho\sigma_N\sigma_S - 2\xi\rho\sigma_N\sigma_S. \tag{43}$$

The welfare-maximizing ξ^* therefore satisfies:

$$\xi^* = \frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}. \tag{44}$$

From the definition of ξ, the optimal cap-adjustment rate δ^* follows:

$$\delta^* = \frac{(b_N+c)[\sigma_S^2-\rho\sigma_N\sigma_S]+c[\sigma_N^2-\rho\sigma_N\sigma_S]}{(b_S+c)[\sigma_N^2-\rho\sigma_N\sigma_S]+c[\sigma_S^2-\rho\sigma_N\sigma_S]}, \tag{45}$$

as given.

PROOF OF PROPOSITION 2:

Proof. Plugging (44) in (41), we find:

$$\begin{aligned} \frac{L^O}{\psi} &= \left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_N^2 + \left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_S^2 \\ &+ \left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\rho\sigma_N\sigma_S \\ &= \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S} \\ &\Longrightarrow \\ L^O &= \frac{1}{2}\frac{b_N+b_S}{cb_N+cb_S+b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}, \end{aligned}$$

as stated. This is strictly lower than the welfare loss under traditional Trading if and only if:

$$
\begin{gathered}
L^L - L^O \geq 0 \\
\Longrightarrow \\
\frac{1}{b_N + b_S} \frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S} - \frac{b_N + b_S}{cb_N + cb_S + b_N b_S} \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} \geq 0 \\
\Longrightarrow \\
(\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S)(b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S) - (1-\rho^2)(b_N^2 + b_S^2 + 2b_N b_S)\sigma_N^2\sigma_S^2 \geq 0 \\
\Longrightarrow \\
[(b_S\sigma_N^2 - b_N\sigma_S^2) + (b_N - b_S)\rho\sigma_N\sigma_S]^2 \geq 0,
\end{gathered}
$$

which is always true. □

PROOF OF PROPOSITION 3:

Proof. We derived quantity derivations under both policies. Prices are equal in both jurisdictions, so without loss of generality we can solve for price deviations in jurisdiction 1:

$$
\begin{aligned}
\Delta^L p_N &= \frac{b_S\theta_N + b_N\theta_S}{b_N + b_S} \\
\Delta^{OL} p_N &= \frac{\delta b_S\theta_N + b_N\theta_S}{b_N + \delta b_S}.
\end{aligned}
$$

Thus:

$$
\begin{aligned}
\mathbb{E}\left[\left(\Delta^L p\right)^2\right] &= \frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}{b_N^2 + b_S^2 + 2b_N b_S} \\
\mathbb{E}\left[\left(\Delta^{OL} p\right)^2\right] &= \frac{\delta^2 b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2\delta b_N b_S\rho\sigma_N\sigma_S}{b_N^2 + \delta^2 b_S^2 + 2\delta b_N b_S}.
\end{aligned}
$$

Writing these out, we obtain:

$$
\mathbb{E}\left[\left(\Delta^{OL} p\right)^2\right] < \mathbb{E}\left[\left(\Delta^L p\right)^2\right] \iff (\delta - 1)\left[b_S\left(\sigma_N^2 - \rho\sigma_N\sigma_S\right) - b_N\left(\sigma_S^2 - \rho\sigma_N\sigma_S\right)\right] < 0.
$$

This condition is always satisfied. □