The Global Climate Game

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May 1, 2021

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Abstract

I study emissions abatement in a global game of technological investments. Players invest in competing technologies. One technology is cheap and dirty, the other expensive but clean, and investments exhibit technological spillovers. The paper makes two main contributions. My first contribution is to resolve complications due to equilibrium multiplicity in games of technological investment by addressing equilibrium selection through the use of global games. In well-identified cases the unique equilibrium is inefficient, motivating policy intervention. This leads to my second contribution, the introduction of network subsidies. A network subsidy allows the policymaker to correct for the entire externality deriving from technological spillovers but does not, in equilibrium, cost anything. Albeit derived in the context of climate change, the concept of a network subsidy is general and contributes to public economics broadly.

1 Introduction

Climate change is a coordination failure of existential proportions. In order to reduce greenhouse gas emissions and prevent dangerous climate change, large-scale investments in clean technologies are necessary. These investments, however, are costly and their benefits imperfectly understood. Thus we face a strategic situation in which clean investment are required on behalf of multiple agents, leading to a coordination problem under uncertainty. In this paper, I present what is perhaps the most bare-bones model to study this type of decision problem. Players invest in competing technologies. One technology is cheap and dirty, the other expensive but clean, and investments exhibit technological spillovers.

My first contribution is to show that uncertainty about the clean technology leads to selection of a unique Bayesian Nash equilibrium. This result is derived using the machinery of global games (Carlsson and Van Damme, 1993; Frankel et al., 2003) and resolves complications caused by equilibrium multiplicity often encountered in the literature on clean technologies. As many authors have acknowledged, a focus on clean technologies often causes equilibrium multiplicity (Barrett, 2006; De Coninck et al., 2008; Hoel and de Zeeuw, 2010; Hong and Karp, 2012; Harstad, 2012; Battaglini and Harstad, 2016). The intuition is that technological investments can exhibit positive spillovers or other kinds of strategic complementarities (Bulow et al., 1985), turning the game into a coordination game with multiple equilibria. There are several reasons why technological investments may exhibit strategic complementarities. Among those discussed in the literature on clean technologies are network effects (Katz and Shapiro, 1985; Greaker and Midttømme, 2016; Li et al., 2017; Mielke and Steudle, 2018; Clinton and Steinberg, 2019); cost sharing: (De Coninck et al., 2008); R&D in breakthrough technologies (Barrett, 2006; Hoel and de Zeeuw, 2010); climate tipping points (Barrett and Dannenberg, 2017); climate clubs (Nordhaus, 2015); technological and knowledge spillovers (Kverndokk and Rosendahl, 2007; Fischer and Newell, 2008; Gerlagh et al., 2009; Aghion and Jaravel, 2015; Harstad, 2016); social norms (Nyborg et al., 2006; Allcott, 2011; Nyborg, 2018b; Kverndokk et al., 2020; Andor et al., 2020); and reciprocity (Nyborg, 2018a).

Equilibrium multiplicity makes the outcome of a game hard to predict and complicates policymaking; moreover, it is dissatisfying for several reasons. First, one would expect economic, environmental, or technological fundamentals to play a role in equilibrium selection. Second, experimental evidence confirms that individuals manage to coordinate their actions in coordination games (Barrett and Dannenberg, 2012, 2017). Third, international treaties like the Montreal Protocol or the Paris Agreement are ratified by overwhelming majorities of countries, suggesting that coordination is possible in the real world as well. The unique equilibrium I derive admits these properties: players coordinate their actions, the equilibrium may be Pareto dominated, and coordination on the clean technology is more likely as the clean investments become inherently more attractive, ceteris paribus.

In much of the environmental literature, equilibrium selection is treated somewhat implicitly and in a way that is not completely satisfactory. Two approaches are especially prevalent. One approach hand-picks, or at minimum focuses on a particular equilibrium. Thus, players may be a priori assumed to pursue symmetric strategies (Harstad, 2012; Harstad et al., 2019), or to coordinate on the Pareto dominant outcome (Barrett, 2006; Hoel and de Zeeuw, 2010). These papers have generated many tried and tested insights, yet the question remains why we should expect real-world players to behave according to the essentially ad hoc assumptions entertained by the authors. Another approach treats the coordination problem as theoretically indecisive and relies on lab experiments to make predictions (Barrett and Dannenberg, 2012, 2014, 2017; Calzolari et al., 2018; Dengler et al., 2018). While these papers, too, have had a lasting influence on the way we think about possible strategies to fight climate change, it is unclear whether and how to generalize their experimental findings to settings outside the laboratory. My explicit focus on equilibrium selection complements these approaches. It provides sharp conditions under which we would expect rational players to coordinate on the Pareto dominant equilibrium of the game.

The unique equilibrium of the game may be inefficient. For intermediately high clean investment benefits, players adopt the dirty technology even though they would be better off were all to adopt the clean technology instead. This result calls for policy intervention.

My second contribution is the introduction of network subsidies. Like standard subsidies, a network subsidy offers adopters of the clean technology a (financial) reward. Yet the amount paid to an individual investor is contingent on total adoption. As I show, it is possible to construct a simple network subsidy scheme that corrects the entire network externality deriving from players' ignorance of the technological spillovers caused by their clean investment but does not, in equilibrium, cost the policymaker anything. Key to this result is the property that the amount of subsidy paid to each individual investor is a function of aggregate clean investments. Since adoption of the clean technology is more attractive when the number of other players adopting it is higher due to the technological spillover, it is quite intuitive that the network subsidy can decrease in aggregate clean technology adoption without negatively affecting players' incentives. My result is to show that this intuition can be exploited smartly: the policymaker can offer a network subsidy scheme such that what is paid when *all* players adopt the clean technology is zero while at the same time creating incentives to adopt the clean technology whenever that is Pareto dominant (in expectations). Intuitively, the network subsidy insures adopters of the clean technology against the event they would enjoy few technological spillovers since many others adopted the dirty technology. In so doing, it boosts clean investments and therefore is never claimed. This result is independent of the application to clean technologies and suggests that network subsidies are worth studying in other contexts where strategic complementarities occur.

In my model, equilibrium selection is catalyzed by uncertainty about the clean technology.[1] The assumption seems realistic. There are many uncertainties surrounding climate change and individual beliefs vary vastly (Hornsey et al., 2016). Within the model, one is free to interpret this uncertainty in different ways. Uncertainty could pertain to the true severity of climate change, the location of a dangerous tipping point, or the true potential of a breakthrough technology. Although many have studied how incomplete information affects the performance of IEAs (Kolstad, 2007; Barrett and Dannenberg, 2012; Martimort and Sand-Zantman, 2016), none consider the type of uncertainty with idiosyncratic, player-specific (posterior) beliefs studied here.

The remainder of the paper is structured as follows. In Section 2, I present the main model and briefly discuss the game of complete information. In Section 3, I add uncertainty to the analysis and show that the global game has a unique equilibrium. Section 4 introduces network subsidies, and Section 5 concludes.

2 Main Model

Consider a world consisting of N players. Each player chooses to invest in either of two technologies. The first, called the dirty technology, is a cheap and dirty technology. If a player does not invest in the dirty technology, s/he invests in the clean technology, an expensive but environmentally-friendly clean technology. One could think of the clean technology as a breakthrough technology (Barrett, 2006; Hoel and de Zeeuw, 2010). Compared to investment in the dirty technology, the environmental benefit of investing in the clean technology is $b > 0$. I represent the investment decision of player i by a binary variable $x_i \in \{0, 1\}$ such that $x_i = 1$ corresponds to investment in the clean

[1] This type of "global uncertainty" turns the game into a global game. However, other approaches toward equilibrium selection also exist. For example, Poisson games (Matsui and Matsuyama, 1995; Myerson, 1998, 2000; Makris, 2008), i.e. games in which players are uncertain about the number of other players playing the game. Poisson coordination games also have a unique equilibrium as the uncertainty becomes arbitrarily small. Another well-known approach derives equilibrium selection as a dynamic outcome of a process of evolutionary selection (Kandori et al., 1993).

technology. Let $x = (x_1, x_2, ..., x_N)$ denote the vector of actions played by all players, and let $x_{-i} = (x_j)_{j \neq i}$ be the vector of actions by all players but i. For each player i, define m to be the total number of *other* players who invest in the clean technology, i.e. $m_i(x) = \sum_{j \neq i} x_j$.

Investments are costly. Let the cost of investing in the dirty technology be constant at d. Given m, let the cost of investment in the clean technology be $c(m)$. I assume that c is a decreasing function of m, so $c(m+1) < c(m)$ for all $m = 0, 1, ..., N-1$.

Combining these elements, the payoff to player i is:

$$\pi_i(x \mid b) = \begin{cases} b \cdot m_i(x) - d & \text{if} \quad x_i = 0 \\ b \cdot (m_i(x) + 1) - c(m_i(x) + 1) & \text{if} \quad x_i = 1 \end{cases}. \tag{1}$$

There are two externalities associated with investment in the clean technology. The first is an *environmental externality* and relates to the parameter b, the positive impact an individual player's investment in the clean technology has on the environment (and hence payoff) for all other players – think of reduced CO2 emissions. The second is a *network externality* and relates to the investment cost function c, i.e. it captures the fact that a player's investment in the clean technology lowers the cost of clean technological investment for all other players – think of technological or knowledge spillovers.

I define the gain from investing in the clean rather than the dirty technology to player i (given b and $m = m_i(x)$) as the difference in payoffs between playing $x_i = 1$ and $x_i = 0$:

$$\begin{aligned} \Delta_i(x \mid b) &= \pi_i(1 \mid b) - \pi_i(0 \mid b) \\ &= b + d - c(m+1). \end{aligned} \tag{2}$$

Inspecting (2), note that investment in the clean technology is a dominant action for all $b > c(1) - d$ as for those bs it holds that $\Delta_i(x \mid b) > 0$ for all $m = 0, 1, ..., N-1$. Alternatively, investment in the dirty technology is a dominant strategy for all $b < c(N) - d$. In between, the game has multiple equilibria.

Proposition 1.

(i) Coordination on $x = (1, 1, ..., 1)$ is a Nash equilibrium of the game for all $b \geq c(N) - d$. It is the unique Nash equilibrium for all $b > c(1) - d$.

(ii) Coordination on $x = (0, 0, ..., 0)$ is a Nash equilibrium of the game for all $b \leq$

$c(1) - d$. It is the unique Nash equilibrium for all $b < c(N) - d$.

(iii) The outcome $x = (1, 1, ..., 1)$ is Pareto dominant for all $b > \frac{c(N)-d}{N}$.

Proof. This follows from the above dominance argument, together with direct payoff comparisons. □

Note that the game has multiple equilibria for all $b \in [c(N) - d, c(1) - d]$. For this range of b, the equilibrium in which players coordinate on $x = (1, 1, ..., 1)$ is Pareto dominant. Moreover, for all $b \in (\frac{c(N)-d}{N}, c(N) - d)$ the unique Nash equilibrium of the game ($x = (0, 0, ..., 0)$) is inefficient, in the sense that it is Pareto dominated by coordination on $x = (1, 1, ..., 1)$.

Environmental economists have long recognized the possibility of equilibrium multiplicity in games of technological investments (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017; Mielke and Steudle, 2018). Due to a lack of sharp theoretical predictions in such games, experimental methods are used to form expectations about outcomes. From a policy maker's point of view, reliance on experiments alone to predict which equilibrium gets selected is somewhat dissatisfying. This motivates the question of equilibrium selection.

3 The Global Climate Game

Strategic complementarities in clean investments drive equilibrium multiplicity under common knowledge of b, the environmental benefit of clean investment. But the assumption of complete information is strong. There are large numbers of uncertainties surrounding many clean technologies's present or future potential.

Uncertainty and signals. For these reasons, I will now study a global game. In the global game G^ε the true parameter b is unobserved. Rather, it is assumed that b is drawn from the uniform distribution on $[\underline{B}, \overline{B}]$ where $\underline{B} < c(N) - d$ and $\overline{B} > c(1) - d$.[2] Each player i in addition receives a private noisy signal b_i of b, given by:

$$b_i = b + \varepsilon_i. \tag{3}$$

[2]In game theory, it is assumed that the game (in this case G^ε) is common knowledge; hence, the structure of the uncertainty (the joint distrubution of b and all the signals b_j), the possible actions and all the payoff functions are commonly known. For a formal treatment of common knowledge, see Aumann (1976).

The term ε_i captures idiosyncratic noise in i's private signal. It is common knowledge that ε_i is an i.i.d. draw from the uniform distribution on $[-\varepsilon, \varepsilon]$. I assume that ε is sufficiently small: $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$. Let b_{-i} denote the vector of signals received by all players but j, i.e. $b_{-i} = (b_j)_{j \neq i}$. Note that player i observes b_i but neither b nor b_{-i}. Thus I write $\Phi^\varepsilon(\cdot \mid b_i)$ for the joint probability function of $(b, b_j)_{j \neq i}$ conditional on b_i. In what follows I will take $\varepsilon > 0$ as given and introduce the concepts used to analyze the global game G^ε.

Strategies and strict dominance. Player i receives a signal b_i prior to choosing an action. A strategy p_i for player i is a function that assigns to any $b_i \in [\underline{B} - \varepsilon, \overline{B} + \varepsilon]$ a probability $p_i(b_i) \geq 0$ with which the player chooses action $x_i = 1$ when s/he observes b_i. I write $p = (p_1, p_2, ..., p_N)$ for a strategy vector. Similarly, I write $p_{-i} = (p_j)_{j \neq i}$ for the vector of strategies for all players but i. Conditional on the strategy vector p_{-i} and a private signal b_i, the expected gain (of choosing $x_i = 1$ rather than $x_i = 0$) to player i is given by:

$$\Delta_i^\varepsilon(p_{-i} \mid b_i) = \int \Delta_i(m(p_{-i}(b_{-i})) \mid b) \, d\Phi^\varepsilon(b, b_{-i} \mid b_i). \tag{4}$$

I say that the action $x_i = 1$ is dominant at b_i if $\Delta_i^\varepsilon(b_i \mid p) > 0$ for all p. Similarly, the action $x_i = 0$ is dominant (in the global game G^ε) at b_i if $\Delta_i^\varepsilon(b_i \mid p) < 0$ for all p.

Lemma 1. *Consider the global game G^ε. (i) The action $x_i = 1$ is dominant at all $b_i \geq \overline{B}$. (ii) The action $x_i = 0$ is dominant at $b_i \leq \underline{B}$.*

Proof. Observe that $\Delta_i(m \mid b) > 0$ for any m given $b \in [\overline{B} - \varepsilon, \overline{B} + \varepsilon]$. Thus, for $b_i = \overline{B}$ the integration in (4) is over positive terms only and $\Delta_i^\varepsilon(p_{-i} \mid \overline{B}) > 0$ for all p_{-i}. This proves part (i) of the Lemma. The proof of part (ii) relies on a symmetric argument and is therefore omitted. □

Recall that a strategy vector p^* is a Bayesian Nash equilibrium of the game if for any strategy $p_i^* \in p^*$ and any b_i it holds that:

$$p_i^*(b_i) \in \arg\max_{x_i \in \{0,1\}} \pi_i^\varepsilon(x_i, m(p_{-i}^*(b_{-i})) \mid b_i), \tag{5}$$

where $\pi_i^\varepsilon(x(p_{-i}^*(b_{-i})) \mid b_i) = \int \pi_i(x_i, m(p_{-i}^*(b_{-i})) \mid b) \, d\Phi^\varepsilon(b, b_{-i} \mid b_i)$.

Lemma 2. *Consider the global game G^ε. (i) If $x_i = 1$ is dominant at b_i then $p_i(b_i) = 1$ in any Bayesian Nash equilibrium p. (ii) If $x_i = 0$ is dominant at b_i then $p_i(b_i) = 0$ in any Bayesian Nash equilibrium p.*

Conditional dominance. Let L and R be real numbers. The action $x_i = 1$ is dominant at b_i conditional on R if $\Delta_i^\varepsilon(p_{-i} \mid b_i) > 0$ for all p_{-i} such that (for all $j \neq i$) $p_j(b_j) = 1$ for all $b_j > R$. Similarly, the action $x_i = 0$ is dominant at b_i conditional on L if $\Delta_i^\varepsilon(p_{-i} \mid b_i) < 0$ for all p_{-i} such that (for all $j \neq i$) $p_j(b_j) = 1$ for all $b_j > L$.

Increasing strategies. For some $X \in \mathbb{R}$, let p_i^X denote the particular strategy such that $p_i^X(b_i) = 0$ for all $b_i < X$ and $p_i^X(b_i) = 1$ for all $b_i > X$. I will call p_i^X the *increasing strategy with switching point* X. By $p^X = (p_1^X, p_2^X, ..., p_N^X)$ I denote the strategy vector of increasing strategies with switching point X, and $p_{-i}^X = (p_j^X)_{j \neq i}$. Note that if $x_i = 1$ is dominant at b_i conditional on R then it must hold that $\Delta_i^\varepsilon(p_{-i}^R \mid b_i) > 0$. Similarly, if $x_i = 0$ is dominant at b_i conditional on L then it must hold that $\Delta_i^\varepsilon(p_{-i}^L \mid b_i) < 0$. The strategy p_i such that $p_i(b_i) = 0$ for any b_i can, with a slight abuse of notation, be written as p_i^∞.

We now have all notation in place to proceed with the core of the analysis.

Iteration from the right. Observe that $\Delta_i^\varepsilon(p^\infty \mid b_i) \leq \Delta_i^\varepsilon(p^{\overline{B}} \mid b_i)$ for all b_i with a strict inequality for all $b_i > \overline{B} - 2\varepsilon$. Moreover, both $\Delta_i^\varepsilon(p^\infty \mid b_i)$ and $\Delta_i^\varepsilon(p^{\overline{B}} \mid b_i)$ are continuous and monotone in b_i. Lastly, recall that $\Delta_i^\varepsilon(p^\infty \mid \overline{B}) > 0$. These observations combined imply that there exists a point $R^1 < \overline{B}$ such that it solves:

$$\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid R^1) = 0. \tag{6}$$

To any player i, the action $x_i = 1$ is dominant at all $b_i > R^1$ conditional on $\overline{B}$.

Lemma 3. *Consider the global game G^ε. For any Bayesian Nash equilibrium p of the game it holds that $p_i(b_i) = 1$ for all $b_i > R^1$.*

Any undominated strategy p_i must satisfy $p_i(b_i) = 1$ for all $b_i > \overline{B}$; that is, a necessary condition for the strategy p_i to be undominated is that $p_i(b_i) \geq p_i^{\overline{B}}(b_i)$ for all b_i. Each player i thus knows that $(j \neq i)$ $p_j(b_j) \geq p_j^{\overline{B}}(b_j)$ for all b_j. Conditional on this, player i can find a point R^1 such that it is expected payoff maximizing for player i to play $x_i = 1$ at all $b_i > R^1$. Knowing that $x_j = 1$ is dominant at all $b_j > R^1$, yet additional strategies may be eliminated as conditionally dominated for player i.

This argument can be repeated and we obtain a sequence $\overline{B} = R^0, R^1, R^2, ...$ For any $k \geq 0$ and R^k such that $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^k) > 0$, there exists a $R^{k+1} < R^K$ such that $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^{k+1}) = 0$. Induction on k allows for the conclusion that (R^k) is a monotone sequence. Moreover, we also know that $R^k \geq \underline{B}$ for all $k \geq 0$ since $x_i = 0$ is dominant at $b_i < \underline{B}$. Any bounded monotone sequence must converge. I let R^* denote the limit of sequence (R^k). By the definition of a limit, R^* must satisfy:

$$\Delta_i^\varepsilon(p_{-i}^{R^*} \mid R^*) = 0. \tag{7}$$

We say that the action $x_i = 1$ is iteratively dominant at $b_i > R^*$, for all i.

Lemma 4. *Consider the global game G^ε. If p is a Bayesian Nash equilibrium of the game, then it must hold that $p_i(b_i) = 1$ for all $b_i > R^*$ and for all i.*

Iteration from the left. Iterative elimination of strictly dominated strategies yields the point R^* when starting from the right, that is, a range of signals b_i for which $x_i = 1$ is unconditionally and strictly dominant. A similar procedure can be executed starting instead from the left, from signals b_i for which $x_i = 0$ is unconditionally and strictly dominant. Since this analysis is symmetric to the procedure discussed above, I will only provide the key steps of the analysis.

From Lemma 1 it is known that $x_i = 0$ is strictly dominant at $b_i < \underline{B}$. That is, $\Delta_i^\varepsilon(p^{-\infty} \mid \underline{B}) < 0$. Since it is common knowledge that no player plays a strictly dominated strategy, a payoff maximizing player i then finds a point L^1 such that $x_i = 0$ is dominant $b_i < L^1$ conditional on $\underline{B}$:

$$\Delta_i^\varepsilon(p_{-i}^{\underline{B}} \mid L^1) = 0. \tag{8}$$

Any expected payoff maximizing player i plays $x_i = 0$ for all $b_i < L^1$. Since this is common knowledge also, we can repeat the argument over and over. What we obtain is a sequence of points (L^k), $k \geq 0$, each term of which is implicitly defined by:

$$\Delta_i^\varepsilon(p_{-i}^{L^k} \mid L^{k+1}) = 0. \tag{9}$$

The sequence (L^k) is monotone increasing. It is also bound from above by $\overline{B}$ (or, taking account of Lemma 4, by R^*). It must therefore converge, and I call its limit L^*. By construction this limit solves:

$$\Delta_i^\varepsilon(p_{-i}^{L^*} \mid L^*) = 0. \tag{10}$$

Given $L^0 = \underline{B}$, a boundary for strict dominance of $x_i = 0$, we say that $x_i = 0$ is iteratively dominant at $b_i < L^*$.

Lemma 5. *Consider the global game G^ε. If p is a Bayesian Nash equilibrium of the game, then it must hold that $p_i(b_i) = 0$ for all $b_i < L^*$ and for all i.*

I have derived two limits L^* and R^* that demarcate iterative dominance regions of the signal space. If p is a Bayesian Nash equilibrium of the game G^ε, it must hold that $p_i(b_i) = 0$ for all $b_i < L^*$ and $p_i(b_i) = 1$ for all $b_i > R^*$. I will now show that this observation implies that the game G^ε has a unique equilibrium – in particular, the points L^* and R^* must coincide.

Unique limit. The following result provides is a key step in proving that there exists a unique strategy vector surviving iterated elimination of strictly dominated strategies.

Lemma 6. *For all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$, the following holds:*

$$\Delta_i^\varepsilon(p_{-i}^X \mid X) = X - \sum_{m=0}^{N-1} \frac{c(m+1)}{N} + d. \tag{11}$$

Proof. First fix $b \in [\underline{N} + \varepsilon, \overline{B} - \varepsilon]$. Each player $j \neq i$ is assumed to play p_j^X, so the probability that $x_j = 1$ is given by

$$\Pr[b_j > X \mid b] = \Pr[\varepsilon_j > X - b] = \frac{\varepsilon - X + b}{2\varepsilon}, \tag{12}$$

while $x_j = 0$ with the complementary probability. Since each ε_j is (conditional on b) drawn independently , the probability that m given players $j \neq i$ play $x_j = 1$ while the remaining $N - m - 1$ players play $x_j = 0$ (given p_{-i}^X and b) is therefore:

$$\left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1}. \tag{13}$$

As there are $\binom{N-1}{m}$ unique ways in which m out of $N - 1$ players j can choose $x_j = 1$, the total probability of this happening, as a function of b, is:

$$\binom{N-1}{m} \left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1}. \tag{14}$$

When m players $j \neq i$ play $x_j = 1$, the cost of playing $x_i = 1$ to player i is $c(m + 1)$.

The derivation so far took b as a known quantity. We now take account of the fact that player i does not observe b directly but only a noisy signal b_i. Given $p_{-i} = p_{-i}^X$ and

$b_i = X$, the expected gain to player i from playing $x_i = 1$ rather than $x_i = 0$ becomes:

$$\Delta_i^\varepsilon(p_{-i}^X \mid X) = \frac{1}{2\varepsilon}\int\limits_{X-\varepsilon}^{X+\varepsilon} b db + d$$
$$- \sum_{m=0}^{N-1} c(m+1)\binom{N-1}{m}\frac{1}{2\varepsilon}\int\limits_{X-\varepsilon}^{X+\varepsilon}\left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m\left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1} db \tag{15}$$
$$= X + d - \sum_{m=0}^{N-1} c(m+1)\binom{N-1}{m}\int\limits_0^1 q^m(1-q)^{N-m-1}dq \tag{16}$$
$$= X + d - \sum_{m=0}^{N-1} c(m+1)\frac{(N-1)!}{m!\,(N-m-1)!}\frac{m!\,(N-m-1)!}{N!} \tag{17}$$
$$= X + d - \sum_{m=0}^{N-1}\frac{c(m+1)}{N}, \tag{18}$$

as given. Equation (15) takes the expression for $\Delta_i(n \mid b)$ given in (2) and integrates over b, given $b_i = X$ and $p_{-i} = p_{-i}^X$. Equation (16) relies on integration by substitution ($q = 1/2 - (X-b)/2\varepsilon$) to rewrite the last integral in (15). Equation (17) then writes out the binomial coefficient $\binom{N-1}{m}$, and equation (18) simplifies. □

Note that the right hand side of (11) is strictly continuously increasing in X for all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$. Combined with Lemma 1, this implies that there exists a unique $X^* \in (\underline{B} + \varepsilon, \overline{B} - \varepsilon)$ such that $\Delta_i^\varepsilon(p_{-i}^{X^*} \mid X^*) = 0$. To see this, observe that Lemma 1 implies that $\Delta_i^\varepsilon(p_{-i}^{L^0} \mid L^0) < 0$ and $\Delta_i^\varepsilon(p_{-i}^{R^0} \mid R^0) > 0$. From the monotonicity and continuity of $\Delta_i^\varepsilon(p_{-i}^X \mid X)$ in X then follow both existence and uniqueness of X^* such that $\Delta_i^\varepsilon(p_{-i}^{X^*} \mid X^*) = 0$.

Proposition 2. *There exists a unique strategy vector p^* surviving iterated elimination of strictly dominated strategy in the global game G^ε. In particular, there exists a unique X^* such that $p^* = p^{X^*}$. The point X^* is given by:*

$$X^* = \sum_{n=1}^{N}\frac{c(n)}{N} - d. \tag{19}$$

I derive Proposition 2 for general $\varepsilon > 0$ provided that b and ε_i (all i) are drawn independently from the uniform distribution. For the limit as $\varepsilon \to 0$, Frankel et al. (2003) establish that Proposition 2 substantially generalizes to games in which b is drawn from any continuous density with connected support and each ε_i is drawn independently from any (possible player-specific) atomless density.[3]

The global climate game selects a unique equilibrium of the underlying coordination game with multiple equilibria. In this sense, Proposition 2 does away with a concern for coordination failure (Mielke and Steudle, 2018) and theoretically motivates the focus on a single equilibrium in the literature on IEAs (Barrett, 2006; Hoel and de Zeeuw, 2010).

Proposition 2 should not be misunderstood as saying that players will perfectly coordinate their actions (investments). An equilibrium is defined in strategies. The proposition says that there exists a unique strategy surviving iterated dominance for each player. But a strategy only yields an action once we feed it a signal. It is possible that some players receive signals above X^* while others see a signal below it. When this occurs, players will fail to coordinate their actions. The implication is that empirically observed coordination failures are not at odds with, and therefore do not by themselves invalidate the model.

Corollary 1. *Let $\varepsilon > 0$ and let p^ε be the unique Bayesian Nash equilibrium of G^ε. (i) Let $b > X^*$, then, for ε sufficiently small* $\Pr\left[p^\varepsilon(b) = (1, 1, ..., 1)\right] = 1$*. (ii) Let $b < X^*$, then, for ε sufficiently small* $\Pr\left[p^\varepsilon(b) = (0, 0, ..., 0)\right] = 1$.

The unique equilibrium can be inefficient. Players may adopt the dirty technology even if everybody's payoff were higher had they adopted the clean technology instead (and even though they know it). Intuitively, clean investment will be too risky when b is low since the noise in signals forces a player to believe that others may think that clean investment is dominated. Barrett and Dannenberg (2012) appear to share this view when they write that "players could use risk-dominance as a selection rule." For 2×2 games, their statement is backed by the theoretical literature: Carlsson and Van Damme (1993) prove that any 2×2 global game selects the risk dominant equilibrium of the underlying true game. In games with more players or actions, the statement is not generally correct.

Proposition 3.

(i) For all $b > b^* + \varepsilon$, players coordinate on $x = (1, 1, ..., 1)$

[3] In particular, the reader is referred to their Theorem 4.

(ii) For all $b < b^* - \varepsilon$, players coordinate on $x = (0, 0, ..., 0)$.

(iii) For all $b \in [b^* - \varepsilon, b^* + \varepsilon]$, players may fail to coordinate their actions.

Recall that coordination on $x = (1, 1, ..., 1)$ is Pareto dominant for all $b > \frac{c(N)-d}{N}$. It follows that players coordinate on the Pareto dominated outcome $x = (0, 0, ..., 0)$ for all $b \in (\frac{c(N)-d}{N}, b^* - \varepsilon)$, and may fail to coordinate on the Pareto dominant outcome for all $b \in [b^* - \varepsilon, b^* + \varepsilon]$. Proposition 3 thus adds the important qualification that even though the global game G^ε has a unique equilibrium, coordination on the Pareto dominant outcome of the game is by no means guaranteed. This stands in contrast to the common and often implicit assumption in the environmental literature that players can coordinate on the efficient equilibrium (Barrett, 2006; Hoel and de Zeeuw, 2010).

4 Network Subsidies

Taxes and subsidies will stimulate adoption of the clean technology whenever they cause an effective decrease of $c(m) - d$ for at least one m. The U.S. Federal Tax Credit for Solar Photovoltaics (Borenstein, 2017), California's Clean Vehicle Rebate Project (Li et al., 2017), or the U.S. National Plug-In Electric Drive Vehicle Credit (Clinton and Steinberg, 2019) are good illustrations. However, tax policies may not always be feasible, political or otherwise. For example, legislation on taxation requires unanimous agreement in the European Union, which is one reason the EU does not have a carbon tax. Subsidies and tax credits, on the other hand, come with a substantial budgetary burden. The Congressional Budget Office expects total cost from tax credits on electric vehicles to be about 7.5 billion U.S. dollars through 2019.[4] Planned spending on SDE++ subsidies in the Netherlands are 5 billion Euros in 2021.[5].

The expenditure on subsidies to stimulate clean technology adoption can be substantially mitigated without the need to compromise on efficiency through the use of *network subsidies*. A network subsidy, like any subsidy, is offered contingent on adoption of the clean technology. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number.

[4]See https://www.cbo.gov/sites/default/files/112th-congress-2011-2012/reports/electricvehiclesone-col.pdf

[5]See https://www.rvo.nl/subsidie-en-financieringswijzer/sde. SDE is an acronym for Stimulering Duurzame Energievoorziening en Klimaattransitie, or "Stimulus Sustainable Energy Supply and Climate Transition".

The strongest results obtain in the game of complete information and the global game with vanishing idiosyncratic noise, which I shall discuss in turn.

4.1 Game of Complete Information

Consider again the game of complete information (about b) discussed in Section 2. Recall from Proposition 1 that coordination on $x = (1, 1, ..., 1)$ is the Pareto dominant outcome of the game for all $b > \frac{c(N)-d}{N}$, whereas coordination on $x = (0, 0, ..., 0)$ is Pareto dominant for all $b < \frac{c(N)-d}{N}$.

My aim is to find a subsidy that incentivizes players to coordinate on the Pareto dominant outcome of the game for any b. Concretely, I want to formulate a subsidy policy such that $x_i = 1$ is dominant for all $b > \frac{c(N)-d}{N}$ while $x_i = 0$ is dominant for all $b < \frac{c(N)-d}{N}$. I say that such a subsidy *implements* the Pareto dominant outcome of the game for all b.

To this end, let the policymaker offer each player a subsidy $S(b, n)$ on adopting the clean technology. I allow the amount of subsidy an individual player receives to be conditional on n, the total number of players adopting the clean technology. Let $S(b, n)$ be given by:

$$S(b, n) = \underbrace{(N - 1) \cdot b}_{\text{environmental subsidy}} + \underbrace{c(n) - c(N)}_{\text{network subsidy}}. \tag{20}$$

I will discuss the details of (20) shortly. First I state a key result.

Proposition 4. *If the policymaker offers a conditional subsidy scheme equal to* (20)*, it implements the Pareto dominant outcome of the game for all b. Players coordinate on $x = (1, 1, ..., 1)$ for all $b > \frac{c(N)-d}{N}$, whereas they coordinate on $x = (0, 0, ..., 0)$ for all $b < \frac{c(N)-d}{N}$.*

Proof. To prove Proposition 4, one simply rewrites the gain from adopting the clean technology for any player i conditional on the subsidy $S(b, n)$, giving:

$$\Delta_i(b, m) + S(b, n) = N \cdot b - c(N) + d, \tag{21}$$

for all m. Hence, adopting the clean technology is dominant at $b > \frac{c(N)-d}{N}$, whereas adopting the dirty technology is dominant at $b < \frac{c(N)-d}{N}$. Since this is how I define implementation of the Pareto dominant outcome of the game for all b, the result follows. $\square$

The subsidy scheme $S(b,n)$ consists of two parts. One, the *environmental subsidy*, is a fixed amount $(N-1)\cdot b$ paid to any player adopting the clean technology. This subsidy is meant to correct the environmental externality deriving from investment in the clean technology. From the perspective of an individual player, the magnitude of this external effect is simply $(N-1)\cdot b$ since adoption of the clean technology yields environmental benefits equal to b and these benefits are enjoyed by exactly $N-1$ players besides the player itself. If $b > \frac{c(N)-d}{N}$, total spending on the environmental subsidy is therefore $N\cdot(N-1)\cdot b$.

The second part of $S(b,n)$ is a new type of policy. I call it a *network subsidy*. Like the environmental subsidy, a network subsidy is offered contingent on adoption of the clean technology. But the sum paid to individual investors is decreasing in the total amount of clean investments. In our present case, the policymaker offers the following network subsidy:

$$s^*(n) = c(n) - c(N), \tag{22}$$

for all $n = 1, ..., N$. In words, when player i adopts the clean technology and $n-1$ others have done so too, i is paid a network subsidy equal to $c(n)-c(N)$. We know from equation (21) that each player will adopt the clean technology if and only if $b > \frac{c(N)-d}{N}$. Remarkably, the network subsidy given by (22) therefore allows the policymaker to correct the network externality deriving from technological spillovers without requiring any payments to be made. To see this, assume that $b > \frac{c(N)-d}{N}$. Every player then chooses to adopt the clean technology, which means that $n = N$. The amount of subsidy payment due to each player is therefore:

$$s^*(N) = c(N) - c(N) = 0. \tag{23}$$

Proposition 5. *A network subsidy equal to* (22) *eliminates all equilibrium inefficiencies deriving from network externality caused by technological spillovers but does not, in equilibrium, cost the policymaker anything.*

Perhaps the easiest way to see that a network subsidy eliminates *all* equilibrium inefficiencies deriving from the network externality is to consider how the network subsidy s^* would perform in the hypothetical case that clean technology adoption does not exhibit an environmental externality. That is, suppose we could somehow cleanse the payoff function (1) from the environmental externality altogether by subtracting, for any m, an amount $m\cdot b$ players' payoffs $\pi_i(x_i \mid b_i, m)$ independently of the action they choose. We can do this without objection since such an alternative specification does not affect the gain from investment in the clean technology $\Delta_i(b, m)$ for any b or m, see (2), and the entire analysis was based on this gain function. It is easy to verify that in this alternative specification coordination on the clean technology is Pareto dominant if and only if $b > c(N) - d$. Moreover, removal of the environmental externality does not affect players' gain function Δ_i, which has the immediate implication that the network subsidy s^* incentives players to adopt the clean technology for all $b > c(N) - d$. It follows that a network subsidy equal to s^* eliminates all equilibrium inefficiencies deriving from the network externality but does not, by (23), cost the policymaker anything.

A more direct way to see that the network subsidy s^* deals with the entire network externality is to note that the environmental subsidy $(N-1) \cdot b$ offered as part of $S(b, n)$ incentives players to incorporate the entire environmental externality due to their clean investment into their payoff maximization. However, offering a subsidy of $(N-1) \cdot b$ alone will only incentive players to play $x_i = 1$ for all $b > \frac{c(n)-d}{N}$. This condition depends on players' beliefs about m – the spillovers. As these beliefs may vary in a game of complete information, when offered only an environmental subsidy equal to $(N-1) \cdot b$ players may coordinate on an inefficient outcome of the game or fail to coordinate at all. The network subsidy s^* solves all of these issues, avoiding (potential) coordination failure "caused by" a network externality.

Why does a network subsidy work so well despite the low cost? The key property of a network subsidy set at s^* is that it eliminates all *strategic uncertainty* players face, i.e. the uncertainty a player has about the actions chosen by all other players. The network subsidy thus removes the payoff uncertainty deriving from strategic uncertainty interacted with technological spillovers. In so doing, the network subsidy manages to eliminate all inefficiencies caused by players' failure to internalize the *technological* spillovers inherent in clean investments. Intuitively, the network subsidy works like an insurance. It protects individual investors against the risk of small network externalities from clean investments in case many others have adopted the dirty technology. In so doing, it impels individuals toward clean investments. The network subsidy does not have to be paid as a result, being conditional on low investments by construction.

The remarkably strong performance of a network subsidy does not rely on the assumption of complete information. I will now show that the main results derived above carry over almost exactly to the global game with vanishing idiosyncratic noise in signals.

4.2 Global Game With Vanishing Noise

Consider the global game G^ε discussed in Section 3. In this game, players do not observe b, the marginal environmental benefit of clean technology adoption. While I have so far not made any assumptions on what the policymaker knows, I will henceforth assume that the policymaker observes neither the true b nor a signal of it.

Since the policymaker does not observe b, it is impossible to offer a subsidy scheme that depends on b directly. Instead, let the policymaker offers a subsidy $S(n)$ on adopting the clean technology, given by:

$$S(n) = \underbrace{\frac{N-1}{N}\left[c(N)-d\right]}_{\text{environmental subsidy}} + \underbrace{c(n)-c(N)}_{\text{network subsidy}}. \tag{24}$$

The environmental subsidy in (24) is set at a level such that the amount of environmental subsidy received by each player is exactly equal to the total amount of environmental externality generated by an individual player's adoption of the clean technology in the hypothetical case that the (unobserved) environmental benefit b is just high enough for the policymaker to favor coordination on the clean technology. The network subsidy part in (24) is the same $s^*(n)$ given by (22).

Proposition 6. *Let ε be sufficiently small. If the policymaker offers a conditional subsidy scheme equal to* (24)*, it implements the Pareto dominant outcome of the game for all b. Players coordinate on $x = (1, 1, ..., 1)$ for all $b > \frac{c(N)-d}{N}$ and on $x = (0, 0, ..., 0)$ for all $b < \frac{c(N)-d}{N}$.*

Like Proposition 4, Proposition 6 can be established through straightforward verification. Conditional on m and $S(n)$, the expected gain to player i from investing in the clean, rather than the dirty technology, is given by:

$$\Delta_i^\varepsilon(b_i, m) + S(n) = b_i - \frac{c(N)-d}{N}. \tag{25}$$

For any m, the expected gain (25) is positive for all $b_i > \frac{c(N)-d}{N}$ and negative for all $b_i < \frac{c(N)-d}{N}$. Moreover, for any $b > \frac{c(N)-d}{N}$ we can find a ε small enough such that $b - \varepsilon > \frac{c(N)-d}{N}$ also. Provided that ε is so small, this implies that $b_i > \frac{c(N)-d}{N}$ for all

$b > \frac{c(N)-d}{N}$. Thus, players coordinate on $x = (1, 1, ..., 1)$ for all $b > \frac{c(N)-d}{N}$. By a similar argument it follows that players coordinate on $x = (0, 0, ..., 0)$ for all $b < \frac{c(N)-d}{N}$. Thus, players coordinate on the Pareto dominant outcome for all b.[6]

In the global game G^{ε} with sufficiently small ε the network subsidy s^* does not cost the policymaker anything. When players coordinate on the dirty technology, naturally the network subsidy does not cost the policymaker anything, payments being conditional on clean technology adoption in the first place. When instead players coordinate on the clean technology, then $n = N$ from the perspective of any individual player and so the amount of subsidy received by each of them is $s^*(N) = c(N) - c(N) = 0$.

Proposition 7. *Let ε be sufficiently small. The network subsidy s^* given by* (22) *eliminates all equilibrium inefficiencies deriving from network externality caused by technological spillovers but does not, in equilibrium, cost the policymaker anything.*

4.3 Global Game with Non-Vanishing Noise

The requirement that signals are sufficiently precise plays a central role in Propositions 6 and 7, but it is also a strong assumption. In this section, I investigate the expected cost of a network subsidy scheme in the global game G^{ε} when the noise in signals does not vanish. After showing that this expected cost is positive, I construct a budget-neutral network tax-subsidy scheme that restores costlessness without affecting players' incentives.

Let the policymaker again offer the subsidy scheme $S(n)$ given by (24). From (25) it is known that offering the scheme $S(n)$ turns investment in the clean technology into a dominant strategy to each player i for all $b_i > \frac{c(N)-d}{N}$.

Recall that conditional on b, each signal b_i is an i.i.d. draw from the uniform distribution on $[b-\varepsilon, b+\varepsilon]$. When $b > \frac{c(N)-d}{N} + \varepsilon$ it therefore holds that $b_i > c(N) - d$ for all i; in that case, players perfectly coordinate on $x_i = 1$. Similarly, when $b < \frac{c(N)-d}{N} - \varepsilon$ it holds that $b_i < c(N) - d$ for all i, and players coordinate on $x_i = 0$. In both cases, spending on the network subsidy s^* will be zero (i.e. either no player adopts the clean technology and therefore no one is entitled to receive the subsidy in the first place, or

[6]At this point, the reader may wonder whether an environmental subsidy equal to $(N-1)\frac{c(N)-d}{N}$ would have worked in the game of complete information as well. The answer is yes. The environmental subsidy $(N-1) \cdot b$ was chosen for expositional purposes, that is, to make clear which part of the subsidy scheme $S(b, n)$ dealt with the environmental externality and which part addresses the network externality.

all players adopt the clean technology and each receives a network subsidy equal to $s^*(N) = 0$). For true bs such that $\frac{c(N)-d}{N} - \varepsilon < b < \frac{c(N)-d}{N} + \varepsilon$, the expected performance of the subsidy scheme deteriorates. The reason is that, in these cases, some players may receive signals at which $x_i = 1$ is dominant while others may receive signals at which $x_i = 0$ is dominant, leading to coordination failure.

I will now derive the expected spending on the subsidy scheme $S(n)$ when ε is not arbitrarily small. Suppose exactly n players have chosen to adopt the clean technology, whereas the remaining $N - n$ players invested dirty. If the policymaker offers the network subsidy scheme $S(n)$ specified in (22), total spending on the subsidy is then simply $n \cdot \left[\frac{N-1}{N}(c(N) - d) + s^*(n)\right]$.

We know from (25) that an expected payoff maximizing player i adopts the clean technology for all $b_i > \frac{c(N)-d}{N}$ when offered the subsidy scheme $S(n)$ in the global game G^ε. What we want to know, therefore, is the probability that any player i's signal satisfies this inequality. Conditional on b and ε, that probability is given by:

$$\begin{aligned} &\Pr\left[b_i < \frac{c(N)-d}{N} \mid b, \varepsilon\right] = \Pr\left[\varepsilon_i < \frac{c(N)-d}{N} - b \mid b, \varepsilon\right] \\ &= \frac{1}{2} - \frac{1}{N}\frac{N \cdot b + d - c(N)}{2\varepsilon}, \end{aligned} \tag{26}$$

where the final equality holds for all $\frac{c(N)-d}{N} - \varepsilon \le b \le \frac{c(N)-d}{N} + \varepsilon$, while it is either 0 or 1 otherwise. Clearly, the probability that $b_i > \frac{c(N)-d}{N}$ is (given b and ε) simply the complement of (26):

$$\Pr[b_i > c(N) - d \mid b, \varepsilon] = 1 - \Pr[b_i < c(N) - d \mid b, \varepsilon]. \tag{27}$$

There are a total of $\binom{N}{n}$ unique ways in which n players out of N can play $x_i = 1$. Thus, the total probability that n players play $x_i = 1$ (while $N - n$ play $x_i = 0$), given b and ε, is:

$$q(n \mid b, \varepsilon) = \binom{N}{n} \cdot \left[\frac{1}{2} - \frac{1}{N}\frac{N \cdot b + d - c(N)}{2\varepsilon}\right]^n \cdot \left[\frac{1}{2} + \frac{1}{N}\frac{N \cdot b + d - c(N)}{2\varepsilon}\right]^{N-n}, \tag{28}$$

when $\frac{c(N)-d}{N} - \varepsilon \le b \le \frac{c(N)-d}{N} + \varepsilon$, and $q(n \mid b, \varepsilon) = 0$ otherwise. Note that simple multiplication of probabilities is allowed on account of the fact that conditional on b each ε_i is an i.i.d. draw from the uniform distribution on $[-\varepsilon, \varepsilon]$. The following proposition summarizes.

Proposition 8. *Let the policymaker offer the conditional subsidy scheme $S(n)$. For all $b \notin \left(\frac{c(N)-d}{N} - \varepsilon, \frac{c(N)-d}{N} + \varepsilon\right)$, spending on the subsidy is zero. For $b \in \left(\frac{c(N)-d}{N} - \varepsilon, \frac{c(N)-d}{N} + \varepsilon\right)$, expected spending on the subsidy $S(n)$ is equal to:*

$$C^{\varepsilon} = \sum_{n=0}^{N} q(n \mid b, \varepsilon) \cdot n \cdot \left[\frac{N-1}{N}(c(N) - d) + s^{*}(n)\right] > 0. \tag{29}$$

Since the prior probability that $b \in (c(N) - d - \varepsilon, c(N) - d + \varepsilon)$ is $2\varepsilon/(\overline{B} - \underline{B})$, the expected spending on the subsidy schemes $S(n)$ before b is drawn is equal to $2\varepsilon \cdot C^{\varepsilon}/(\overline{B} - \underline{B})$, which vanishes as $\varepsilon \to 0$.

The remarkably strong performance of a network subsidy discussed in Proposition 7 breaks down in the case of non-vanishing noise in signals. This begs the question whether, and how, an effective policy scheme based on the idea of network subsidies can be conceived at zero cost even when the noise in signals is not vanishing.

A straightforward way to implement an effective yet costless network subsidy on clean technological investments is to finance subsidy payments through a "network tax" levied on the dirty technology. Let this subsidy be denoted $s^{**}(n)$; the corresponding tax is denoted $t^{**}(n)$. Thus, if n players adopt the clean technology, aggregate spending on network subsidies is $n \cdot s^{**}(n)$, whereas aggregate revenues from the network tax are $(N - n) \cdot t^{**}(n)$. If we want the tax-subsidy scheme to be costless, the budget constraint for this policy is:

$$(N - n) \cdot t^{**}(n) - n \cdot s^{**}(n) = 0. \tag{30}$$

Condition (30) imposes that total spending on the network subsidy is matched by total tax income from taxing the dirty technology for any number of clean technology adopters.

Next, the tax-subsidy scheme – when offered together with the environmental subsidy equal to $\frac{N-1}{N}[c(N) - d]$, see (24) – must implement the Pareto dominant outcome of the game.[7] As we have seen before, this condition will be satisfied whenever the expected gain from clean investment, accounting for taxes and subsidies, satisfies:

$$\Delta_i^{\varepsilon}(b_i, m) + \frac{N-1}{N}[c(N) - d] + s^{**}(n) + t^{**}(n) = b_i - \frac{c(N) - d}{N}, \tag{31}$$

[7]Implementation of the Pareto dominant outcome is hard to guarantee in this case since ε can be large. I here interpret the requirement as saying that players should be incentived to play $x_i = 1$ for all $b_i > \frac{c(N)-d}{N}$ and $x_i = 0$ for all $b_i < \frac{c(N)-d}{N}$.

for all $m = 0, 1, ..., N-1$. Equation (31) represents the incentive constraint of a network tax-subsidy scheme. Combined with the budget constraint, this yields the following scheme:

$$\begin{cases} t^{**}(n) & = \frac{n}{N}\left[c(n) - c(N)\right] \\ s^{**}(n) & = \frac{N-n}{N}\left[c(n) - c(N)\right] \end{cases}. \tag{32}$$

Suppose now the policymakers offers an environmental subsidy equal to $\frac{N-1}{N}[c(N) - d]$ in addition to the network subsidy s^{**} to all players playing $x_i = 1$, taxing those who play $x_i = 0$ with a network tax given by t^{**}. That is, let the policymaker offer a subsidy scheme $S(n)$ given by:

$$S(n) = \begin{cases} \frac{N-1}{N}[c(N) - d] + s^{**}(n) & \text{if} \quad x_i = 1 \\ -t^{**}(n) & \text{if} \quad x_i = 0 \end{cases} \tag{33}$$

Since $S(n)$ solves (30) and (31), it incentivizes players to play $x_i = 1$ for all $b_i > \frac{c(N)-d}{N}$ even though net spending on the "network tax-subsidy" scheme is always zero by construction.

Proposition 9. *A conditional subsidy scheme equal to* (33) *incentivizes players to play* $x_i = 1$ *for all* $b_i > \frac{c(N)-d}{N}$ *and* $x_i = 0$ *for all* $b_i < \frac{c(N)-d}{N}$*. Moreover, it does so without requiring any net spending on "network subsidies" even when players fail to coordinate their actions.*

Finally, observe that the logic of a network subsidy does not rely on the application to climate change. Any market where (i) individual actions exhibit strategic complementarities and (ii) players do not take these into account may coordinate on an inefficient equilibrium. A network subsidy then offers an inexpensive way out of this trap. The observation suggests that network subsidies are worth studying in economics more generally.

5 Discussion and Conclusions

This paper studies climate change mitigation in a global game. The focus is on abatement through technological investment. Players invest in either of two technologies. One technology is cheap and dirty, the other expensive but clean. I consider environments in which investments are strategic complements. These could for example arise due to network effects, technological spillovers, or learning-by-doing. Consistent with the existing literature on technological investments in clean technologies, I demonstrate that the complete information version of my game has multiple equilibria (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017; Mielke and Steudle, 2018). Equilibrium multiplicity can lead to coordination failure and complicates the design of domestic policies or climate treaties.

To this well-studied framework, I add a little bit of uncertainty. I assume that the true environmental benefit of the clean technology is unobserved. Rather than observe the technology's true benefit, players receive private noisy signals of it. In this *global climate game*, I show that there exists a unique in which players adopt the clean technology if and only if their private signals exceed an endogenous threshold. For signals below the threshold, players adopt the dirty technology instead.

My first contribution is to show that the game has a unique Bayesian Nah equilibrium. This contribution directly addresses the issue of equilibrium multiplicity often encountered in this literature. Equilibrium selection is driven by the assumption of incomplete information, which seems realistic in the context of clean technologies and climate change. The analysis highlights that the precise way in which one models uncertainty is important. Although some papers conclude that "shared" uncertainty does not eliminate equilibrium multiplicity in coordination games (Barrett and Dannenberg, 2012; Barrett, 2013), this paper shows the starkly contrasting result that privately held beliefs about the shared game does force selection of a unique equilibrium. The assumed structure of uncertainty matters. My result motivates a rethink of the way environmental economists model uncertainty.

My second contribution is to introduce network subsidies. Like standard subsidies, a network subsidy provides adopters of the clean technology with a (financial) reward. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number. As I show, it is possible to construct a simple network subsidy scheme that guarantees efficient adoption of the clean technology but does not, in equilibrium, require the policymaker to pay anything. Intuitively, the network subsidy serves as an insurance against small clean technology networks. In so doing, it boosts clean investments and therefore is never claimed. Although derived in the context of technological spillovers, the notion of a network subsidy is general and applies to public economic broadly.

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