THE GLOBAL CLIMATE GAME

Job Market Paper

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Abstract

I study emissions abatement in a global game of technological investments. Players invest in competing technologies. One technology is cheap and dirty, the other expensive but clean, and investments exhibit technological spillovers. The paper makes two main contributions. My first contribution is to resolve complications due to equilibrium multiplicity in games of technological investment by addressing equilibrium selection through the use of global games. In well-identified cases the unique equilibrium is inefficient, motivating policy intervention. This leads to my second contribution, the introduction of network subsidies. A network subsidy allows the policymaker to correct for the entire externality deriving from technological spillovers but does not, in equilibrium, cost anything. Albeit derived in the context of climate change, the concept of a network subsidy is general and contributes to public economics broadly.

1 Introduction

Climate change is a coordination failure of existential proportions. In order to reduce greenhouse gas emissions and prevent dangerous climate change, large-scale investments in clean technologies are necessary. These investments, however, are costly and their benefits imperfectly understood. Thus we face a strategic situation in which clean investment are required on behalf of multiple agents, leading to a coordination problem under uncertainty. In this paper, I present what is perhaps the most bare-bones model

to study this type of decision problem. Players invest in competing technologies. One technology is cheap and dirty, the other expensive but clean, and investments exhibit technological spillovers.

My first contribution is to show that uncertainty about the clean technology leads to selection of a unique Bayesian Nash equilibrium. This result is derived using the machinery of global games (Carlsson and Van Damme, 1993; Frankel et al., 2003) and resolves complications caused by equilibrium multiplicity often encountered in the literature on clean technologies. As many authors have acknowledged, a focus on clean technologies often causes equilibrium multiplicity (Barrett, 2006; De Coninck et al., 2008; Hoel and de Zeeuw, 2010; Hong and Karp, 2012; Harstad, 2012; Battaglini and Harstad, 2016). The intuition is that technological investments can exhibit positive spillovers or other kinds of strategic complementarities (Bulow et al., 1985), turning the game into a coordination game with multiple equilibria. There are several reasons why technological investments may exhibit strategic complementarities. Among those discussed in the literature on clean technologies are network effects (Katz and Shapiro, 1985; Greaker and Midttømme, 2016; Li et al., 2017; Mielke and Steudle, 2018; Clinton and Steinberg, 2019); cost sharing: (De Coninck et al., 2008); R&D in breakthrough technologies (Barrett, 2006; Hoel and de Zeeuw, 2010); climate tipping points (Barrett and Dannenberg, 2017); climate clubs (Nordhaus, 2015); technological and knowledge spillovers (Kverndokk and Rosendahl, 2007; Fischer and Newell, 2008; Gerlagh et al., 2009; Aghion and Jaravel, 2015; Harstad, 2016); social norms (Nyborg et al., 2006; Allcott, 2011; Nyborg, 2018b; Kverndokk et al., 2020; Andor et al., 2020); and reciprocity (Nyborg, 2018a).

In much of the environmental literature, equilibrium selection is treated somewhat implicitly and in a way that is not completely satisfactory. Two approaches are especially prevalent. One approach hand-picks, or at minimum focuses on a particular equilibrium. Thus, players may be a priori assumed to pursue symmetric strategies (Harstad, 2012; Harstad et al., 2019), or to coordinate on the Pareto dominant outcome (Barrett, 2006; Hoel and de Zeeuw, 2010). These papers have generated many tried and tested insights, yet the question remains why we should expect real-world players to behave according to the essentially ad hoc assumptions entertained by the authors. Another approach

¹In this paper, the unique equilibrium will also be in symmetric strategies. Although this is somewhat intuitive as my setting is one of symmetric players, I nonetheless allow players to pursue any strategy, including non-symmetric ones. Thus, the fact that the unique equilibrium in my game is in symmetric strategies is a result rather than an assumption.

treats the coordination problem as theoretically indecisive and relies on lab experiments to make predictions (for a survey of the experimental literature on coordination games, see Devetag and Ortmann (2007); for experimental studies of coordination games in the context of climate change in particular, see Barrett and Dannenberg (2012, 2014, 2017); Calzolari et al. (2018); Dengler et al. (2018)). While these papers, too, have had a lasting influence on the way we think about possible strategies to fight climate change, it is unclear whether and how to generalize their experimental findings to settings outside the laboratory. My explicit focus on equilibrium selection complements these approaches. It provides sharp conditions under which we would expect rational players to coordinate on the Pareto dominant equilibrium of the game.

The unique equilibrium of the game may be inefficient. For intermediately high clean investment benefits, players adopt the dirty technology even though they would be better off were all to adopt the clean technology instead. This result calls for policy intervention.

My second contribution is the introduction of network subsidies. Like standard subsidies, a network subsidy offers adopters of the clean technology a (financial) reward. Yet the amount paid to an individual investor is contingent on total adoption. As I show, it is possible to construct a simple network subsidy scheme that corrects the entire network externality deriving from players' ignorance of the technological spillovers caused by their clean investment but does not, in equilibrium, cost the policymaker anything. Key to this result is the property that the amount of subsidy paid to each individual investor is a function of aggregate clean investments. Since adoption of the clean technology is more attractive when the number of other players adopting it is higher due to the technological spillover, it is quite intuitive that the network subsidy can decrease in aggregate clean technology adoption without negatively affecting players' incentives. My result is to show that this intuition can be exploited smartly: the policymaker can offer a network subsidy scheme such that what is paid when all players adopt the clean technology is zero while at the same time creating incentives to adopt the clean technology whenever that is Pareto dominant (in expectations). Intuitively, the network subsidy insures adopters of the clean technology against the event they would enjoy few technological spillovers since many others adopted the dirty technology. In so doing, it boosts clean investments and therefore is never claimed. This result is independent of the application to clean technologies and suggests that network subsidies are worth studying in other contexts where strategic complementarities occur. In my model, equilibrium selection is catalyzed by uncertainty about the clean technology.² The assumption seems realistic. There are many uncertainties surrounding climate change and individual beliefs vary vastly (Hornsey et al., 2016). Within the model, one is free to interpret this uncertainty in different ways. Uncertainty could pertain to the true severity of climate change, the location of a dangerous tipping point, or the true potential of a breakthrough technology. Although many authors have studied the role of incomplete information in the climate context (Kolstad, 2007; Barrett and Dannenberg, 2012; Martimort and Sand-Zantman, 2016), none consider the type of uncertainty with idiosyncratic, player-specific (posterior) beliefs studied here.

The remainder of the paper is structured as follows. In Section 2, I present the main model and briefly discuss the game of complete information. In Section 3, I add uncertainty to the analysis and show that the global game has a unique equilibrium. Section 4 introduces network subsidies, and Section 5 concludes.

2 Main Model

Consider a world consisting of N players. Each player chooses to invest in either of two technologies. The first, called the dirty technology, is a cheap and dirty technology. If a player does not invest in the dirty technology, s/he invests in the clean technology, an expensive but environmentally-friendly clean technology. One could think of the clean technology as a breakthrough technology (Barrett, 2006; Hoel and de Zeeuw, 2010). Compared to investment in the dirty technology, the environmental benefit of investing in the clean technology is b > 0. An action for player i is a binary variable $x_i \in \{0,1\}$ such that $x_i = 1$ corresponds to investment in the clean technology while $x_i = 0$ stands for investment in the dirt technology. Let $x = (x_1, x_2, ..., x_N)$ denote the vector of actions played by all players, and let $x_{-i} = (x_j)_{j \neq i}$ be the vector of actions by all players but i. Let $\mathbf{1} = (1, 1, ..., 1)$ be the action vector of all ones, and $\mathbf{0} = (0, 0, ..., 0)$ the action vector of all zeroes. The costs of investing in the dirty technology (play 0) is constant at d. The costs of investing in the clean technology (play 1) depend on the total number of players, n, that invest in clean and are decreasing in n: c(1) > c(2) > ... > c(N).

²This type of "global uncertainty" turns the game into a global game. However, other approaches toward equilibrium selection also exist. For example, Poisson games (Matsui and Matsuyama, 1995; Myerson, 1998, 2000; Makris, 2008), i.e. games in which players are uncertain about the number of other players playing the game. Poisson coordination games also have a unique equilibrium as the uncertainty becomes arbitrarily small. Another well-known approach derives equilibrium selection as a dynamic outcome of a process of evolutionary selection (Kandori et al., 1993).

Combining these elements, the payoff to player i is:

$$\pi_i(x \mid b) = \begin{cases} b \cdot m_i(x) - d & \text{if } x_i = 0 \\ b \cdot (m_i(x) + 1) - c(m_i(x) + 1) & \text{if } x_i = 1 \end{cases}, \tag{1}$$

where $m_i(x)$ is defined as the number of other players playing 1 in x_{-i} ; hence, $m_i(x) = \sum_{x_{-i}} x_j$. I also define m(x) as the total number of players that play 1 in x; hence, $m(x) = \sum_x x_i$. The set of players $\{1, 2, ..., N\}$, the set of action vectors $x \in \{0, 1\}^N$, and the set of payoff functions $\{\pi_i\}$ jointly define a complete information game G(b).

There are two externalities associated with investment in the clean technology. The first is an environmental externality and relates to the parameter b, the positive impact an individual player's investment in the clean technology has on the environment (and hence payoff) for all other players – think of reduced CO2 emissions. The second is a network externality and relates to the investment cost function c, i.e. it captures the fact that a player's investment in the clean technology lowers the cost of clean technological investment for all other players – think of technological or knowledge spillovers.

Define the gain from investing in the clean rather than the dirty technology to player i (given b and x) as the difference in payoffs between playing $x_i = 1$ and $x_i = 0$:

$$\Delta_i(x \mid b) = \pi_i(1, x_{-i} \mid b) - \pi_i(0, x_{-i} \mid b)$$

= $b + d - c(m_i(x))$. (2)

The action $x_i = 1$ is strictly dominant for all b > c(1) - d as for those bs it holds that $\Delta_i(x \mid b) > 0$ for all x. Alternatively, $x_i = 0$ is strictly dominant for all b < c(N) - d. In between, the game has multiple equilibria.

Proposition 1.

- (i) Coordination on x = 1 is a Nash equilibrium of the game for all $b \ge c(N) d$. It is the unique Nash equilibrium for all b > c(1) d.
- (ii) Coordination on $x = \mathbf{0}$ is a Nash equilibrium of the game for all $b \le c(1) d$. It is the unique Nash equilibrium for all b < c(N) d.
- (iii) The outcome x = 1 is Pareto dominant for all $b > \frac{c(N)-d}{N}$.

Proof. This follows from the above dominance argument, together with direct payoff comparisons. \Box

To smoothen notation, I shall henceforth write $\bar{b} = \frac{c(N)-d}{N}$.

Environmental economists have long recognized the possibility of equilibrium multiplicity in games of technological investments (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017; Mielke and Steudle, 2018). It motivates the question of equilibrium selection, to which Section 3 is devoted. First, however, I offer some final (and somewhat technical) remarks on the complete information game G(b).

Frankel et al. (2003) have observed that a game such as given by (1) is a potential game (Monderer and Shapley, 1996). A potential game is a game for which there exists a potential function $P: \{0,1\}^N \to \mathbb{R}$ on action profiles such that the change in any individual player's payoff when switching from one action to the other is always equal to the change in the potential function. The game G(b) has a potential function $P(x \mid b)$ given by:

$$P(x \mid b) = \begin{cases} \sum_{k=1}^{m(x)-1} \Delta_i(k \mid b) & \text{if } m(x) > 0, \\ 0 & \text{if } m(x) = 0. \end{cases}$$
 (3)

Observe that, for any i and any $x_{-i} \in \{0,1\}^{N-1}$, it holds that $P(1, x_{-i} \mid b) - P(0, x_{-i} \mid b) = \Delta_i(x_{-i} \mid b)$, confirming that P is a potential function indeed.

A potential maximizer is a vector x that maximizes P. One can verify that $\mathbf{1}$ is the unique potential maximizer of $P(x \mid b)$ for all $b+d > \sum_{n=1}^{N} \frac{c(n)}{N}$ whereas $\mathbf{0}$ is the unique potential maximizer of $P(x \mid b)$ for all $b+d < \sum_{n=1}^{N} \frac{c(n)}{N}$. I return to this observation in the next Section.

3 The Global Climate Game

Strategic complementarities in clean investments drive equilibrium multiplicity under common knowledge of b, the environmental benefit of clean investment. But the assumption of complete information is strong. There are large numbers of uncertainties surrounding many clean technologies's present or future potential.

Uncertainty and signals. For these reasons, I will now study a global game. In the global game G^{ε} the true parameter b is unobserved. Rather, it is assumed that b is drawn from the uniform distribution on $[\underline{B}, \overline{B}]$ where $\underline{B} < c(N) - d$ and $\overline{B} > c(1) - d$.

³In game theory, it is assumed that the game (in this case G^{ε}) is common knowledge; hence, the structure of the uncertainty (the joint distribution of b and all the signals b_{j}^{ε}), the possible actions and all the payoff functions are commonly known. For a formal treatment of common knowledge, see Aumann (1976).

Each player i in addition receives a private noisy signal b_i^{ε} of b, given by:

$$b_i^{\varepsilon} = b + \varepsilon_i. \tag{4}$$

The term ε_i captures idiosyncratic noise in i's private signal. It is common knowledge that ε_i is an i.i.d. draw from the uniform distribution on $[-\varepsilon, \varepsilon]$. I assume that ε is sufficiently small: $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$. Let $b^{\varepsilon} = (b_i^{\varepsilon})$ denote the vector of signals received by all players, and let b_{-i}^{ε} denote the vector of signals received by all players but j, i.e. $b_{-i}^{\varepsilon} = (b_j^{\varepsilon})_{j\neq i}$. Note that player i observes b_i^{ε} but neither b nor b_{-i}^{ε} . Thus I write $\Phi^{\varepsilon}(\cdot \mid b_i^{\varepsilon})$ for the joint probability function of $(b, b_j^{\varepsilon})_{j\neq i}$ conditional on b_i^{ε} . In what follows I will take $\varepsilon > 0$ as given and introduce the concepts used to analyze the global game G^{ε} .

Strategies and strict dominance. Player i receives a signal b_i^{ε} prior to choosing an action. A strategy p_i for player i in G^{ε} is a function that assigns to any $b_i^{\varepsilon} \in [\underline{B} - \varepsilon, \overline{B} + \varepsilon]$ a probability $p_i(b_i^{\varepsilon}) \geq 0$ with which the player chooses action $x_i = 1$ when s/he observes b_i^{ε} . I write $p = (p_1, p_2, ..., p_N)$ for a strategy vector. Similarly, I write $p_{-i} = (p_j)_{j \neq i}$ for the vector of strategies for all players but i. Conditional on the strategy vector p_{-i} and a private signal b_i^{ε} , the expected gain (of choosing $x_i = 1$ rather than $x_i = 0$) to player i is given by:

$$\Delta_i^{\varepsilon}(p_{-i} \mid b_i^{\varepsilon}) = \int \Delta_i(p_{-i}(b_{-i}^{\varepsilon}) \mid b) d\Phi^{\varepsilon}(b, b_{-i}^{\varepsilon} \mid b_i^{\varepsilon}). \tag{5}$$

I say that the action $x_i = 1$ is dominant at b_i^{ε} if $\Delta_i^{\varepsilon}(p_{-i} \mid b_i^{\varepsilon}) > 0$ for all p. Similarly, the action $x_i = 0$ is dominant (in the global game G^{ε}) at b_i^{ε} if $\Delta_i^{\varepsilon}(p_{-i} \mid b_i^{\varepsilon}) < 0$ for all p.

Lemma 1. Consider the global game G^{ε} . (i) The action $x_i = 1$ is dominant at all $b_i^{\varepsilon} \geq \overline{B}$. (ii) The action $x_i = 0$ is dominant at $b_i^{\varepsilon} \leq \underline{B}$.

Proof. Observe that $\Delta_i(x \mid b) > 0$ for any x given $b \in [\overline{B} - \varepsilon, \overline{B} + \varepsilon]$. Thus, for $b_i^{\varepsilon} = \overline{B}$ the integration in (5) is over positive terms only and $\Delta_i^{\varepsilon}(p_{-i} \mid \overline{B}) > 0$ for all p_{-i} . This proves part (i) of the Lemma. The proof of part (ii) relies on a symmetric argument and is therefore omitted.

Recall that a strategy vector $p^* = (p_1^*, p_2^*, ..., p_N^*)$ is a Bayesian Nash equilibrium of G^{ε} if for any p_i^* and any b_i^{ε} it holds that:

$$p_i^*(b_i^{\varepsilon}) \in \operatorname*{arg\,max}_{x_i \in \{0,1\}} \pi_i^{\varepsilon}(x_i, p_{-i}^*(b_{-i}^{\varepsilon}) \mid b_i^{\varepsilon}), \tag{6}$$

where $\pi_i^{\varepsilon}(x_i, p_{-i}^*(b_{-i}^{\varepsilon}) \mid b_i^{\varepsilon}) = \int \pi_i(x_i, p_{-i}^*(b_{-i}^{\varepsilon}) \mid b) d\Phi^{\varepsilon}(b, b_{-i}^{\varepsilon} \mid b_i^{\varepsilon}).$

Conditional dominance. Let L and R be real numbers. The action $x_i = 1$ is dominant at b_i^{ε} conditional on R if $\Delta_i^{\varepsilon}(p_{-i} \mid b_i^{\varepsilon}) > 0$ for all p_{-i} such that (for all $j \neq i$) $p_j(b_j^{\varepsilon}) = 1$ for all $b_j^{\varepsilon} > R$. Similarly, the action $x_i = 0$ is dominant at b_i^{ε} conditional on L if $\Delta_i^{\varepsilon}(p_{-i} \mid b_i^{\varepsilon}) < 0$ for all p_{-i} such that (for all $j \neq i$) $p_j(b_j^{\varepsilon}) = 1$ for all $b_j^{\varepsilon} > L$.

Increasing strategies. For some $X \in \mathbb{R}$, let p_i^X denote the particular strategy such that $p_i^X(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < X$ and $p_i^X(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon \ge X$. I will call p_i^X the increasing strategy with switching point X. By $p^X = (p_1^X, p_2^X, ..., p_N^X)$ I denote the strategy vector of increasing strategies with switching point X, and $p_{-i}^X = (p_j^X)_{j \ne i}$. Note that if $x_i = 1$ is dominant at b_i^ε conditional on R then it must hold that $\Delta_i^\varepsilon(p_{-i}^R \mid b_i^\varepsilon) > 0$. Similarly, if $x_i = 0$ is dominant at b_i^ε conditional on L then it must hold that $\Delta_i^\varepsilon(p_{-i}^L \mid b_i^\varepsilon) < 0$.

We now have all notation in place to proceed with the core of the analysis.

Iteration from the right. Take $p_{-i}=p_{-i}^{\overline{B}}$ and note that $\Delta_i^{\varepsilon}(p_{-i}^{\overline{B}}\mid b_i^{\varepsilon})$ is continuous and monotone decreasing in b_i^{ε} . Moreover, recall from Lemma 1 that $x_i=1$ is strictly dominant at $b_i=\overline{B}$, so $\Delta_i^{\varepsilon}(p_{-i}^{\overline{B}}\mid \overline{B})>0$. By the same Lemma, we also know that $\Delta_i^{\varepsilon}(p_{-i}^{\overline{B}}\mid \underline{B})<0$. Monotonicity and continuity of $\Delta_i^{\varepsilon}(p_{-i}^{\overline{B}}\mid b_i^{\varepsilon})$ in b_i then imply there exists a point R^1 such that $\underline{B}< R^1<\overline{B}$ which solves:

$$\Delta_i^{\varepsilon}(p_{-i}^{\overline{B}} \mid R^1) = 0. \tag{7}$$

To any player i, the action $x_i=1$ is dominant at all $b_i^{\varepsilon}>R^1$ conditional on \overline{B} . Any undominated strategy p_i must satisfy $p_i(b_i^{\varepsilon})=1$ for all $b_i^{\varepsilon}>\overline{B}$; that is, a necessary condition for the strategy p_i to be undominated is that $p_i(b_i^{\varepsilon})\geq p_i^{\overline{B}}(b_i^{\varepsilon})$ for all b_i^{ε} . Conditional on this, player i can find a point R^1 such that it is expected payoff maximizing for player i to play $x_i=1$ at all $b_i^{\varepsilon}>R^1$. Knowing that $x_j=1$ is dominant at all $b_j^{\varepsilon}>R^1$, yet additional strategies may be eliminated as conditionally dominated for player i.

This argument can be repeated and we obtain a sequence $\overline{B} = R^0, R^1, R^2, \ldots$ For any $k \geq 0$ and R^k such that $\Delta_i^{\varepsilon}(p_{-i}^{R^k} \mid R^k) > 0$, there exists a $R^{k+1} < R^K$ such that $\Delta_i^{\varepsilon}(p_{-i}^{R^k} \mid R^{k+1}) = 0$. Induction on k allows for the conclusion that (R^k) is a monotone sequence. Moreover, we also know that $R^k \geq \underline{B}$ for all $k \geq 0$ since $x_i = 0$ is dominant at $b_i^{\varepsilon} < \underline{B}$. Any bounded monotone sequence must converge. I let R^* denote the limit of sequence (R^k) . By the definition of a limit, R^* must satisfy:

$$\Delta_i^{\varepsilon}(p_{-i}^{R^*} \mid R^*) = 0. \tag{8}$$

We say that $x_i = 1$ is iteratively dominant at $b_i^{\varepsilon} > R^*$, for all i.

Iteration from the left. Iterative elimination of strictly dominated strategies yields the point R^* when starting from the right, that is, a range of signals b_i^{ε} for which $x_i = 1$ is unconditionally and strictly dominant. A similar procedure can be executed starting instead from the left, from signals b_i^{ε} for which $x_i = 0$ is unconditionally and strictly dominant. Since this analysis is symmetric to the procedure discussed above, I will only provide the key steps of the analysis.

From Lemma 1 it is known that $x_i = 0$ is strictly dominant at $b_i^{\varepsilon} < \underline{B}$. That is, $\Delta_i^{\varepsilon}(p^{-\infty} \mid \underline{B}) < 0$. Since it is common knowledge that no player plays a strictly dominated strategy, a payoff maximizing player i then finds a point L^1 such that $x_i = 0$ is dominant $b_i^{\varepsilon} < L^1$ conditional on \underline{B} :

$$\Delta_i^{\varepsilon}(p_{-i}^{\underline{B}} \mid L^1) = 0. \tag{9}$$

Any expected payoff maximizing player i plays $x_i = 0$ for all $b_i^{\varepsilon} < L^1$. Since this is common knowledge also, we can repeat the argument over and over. What we obtain is a sequence of points (L^k) , $k \geq 0$, each term of which is implicitly defined by:

$$\Delta_i^{\varepsilon}(p_{-i}^{L^k} \mid L^{k+1}) = 0. \tag{10}$$

The sequence (L^k) is monotone increasing. It is also bound from above by \overline{B} (or, taking account of (8), by R^*). It must therefore converge, and I call its limit L^* . By construction this limit solves:

$$\Delta_i^{\varepsilon}(p_{-i}^{L^*} \mid L^*) = 0. \tag{11}$$

Given $L^0 = \underline{B}$, a boundary for strict dominance of $x_i = 0$, we say that $x_i = 0$ is iteratively dominant at $b_i^{\varepsilon} < L^*$.

Lemma 2. (i) The action $x_i = 0$ is iteratively dominant at $b_i < L^*$. (ii) The action $x_i = 1$ is iteratively dominant at $b_i > R^*$.

Proof. Follows immediately from the argument leading up to the Lemma. \Box

I have derived two limits L^* and R^* that demarcate iterative dominance regions of the signal space. If p is a Bayesian Nash equilibrium of the game G^{ε} , it must hold that $p_i(b_i^{\varepsilon}) = 0$ for all $b_i^{\varepsilon} < L^*$ and $p_i(b_i^{\varepsilon}) = 1$ for all $b_i^{\varepsilon} > R^*$. I will now show that

this observation implies that the game G^{ε} has a unique equilibrium – in particular, the points L^* and R^* must coincide. To prive this, the following result is key.

Proposition 2. For all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$, the following holds:

$$\Delta_i^{\varepsilon}(p_{-i}^X \mid X) = X - \sum_{m=0}^{N-1} \frac{c(m+1)}{N} + d.$$
 (12)

It follows that $\Delta_i^{\varepsilon}(p_{-i}^X \mid X)$ is continuous and strictly increasing in X for all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$.

Proof. First fix $b \in [\underline{B} + \varepsilon, \overline{B} - \varepsilon]$. Each player $j \neq i$ is assumed to play p_j^X , so the probability that $x_j = 1$ is given by

$$\Pr[b_j^{\varepsilon} > X \mid b] = \Pr[\varepsilon_j > X - b] = \frac{\varepsilon - X + b}{2\varepsilon},\tag{13}$$

while $x_j = 0$ with the complementary probability. Since each ε_j is (conditional on b) drawn independently, the probability that m given players $j \neq i$ play $x_j = 1$ while the remaining N - m - 1 players play $x_j = 0$ (given p_{-i}^X and b) is therefore:

$$\left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N - m - 1}.$$
 (14)

As there are $\binom{N-1}{m}$ unique ways in which m out of N-1 players j can choose $x_j=1$, the total probability of this happening, as a function of b, is:

$$\binom{N-1}{m} \left[\frac{\varepsilon - X + b}{2\varepsilon} \right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon} \right]^{N-m-1}.$$
 (15)

When m players $j \neq i$ play $x_j = 1$, the cost of playing $x_i = 1$ to player i is c(m+1).

The derivation so far took b as a known quantity. We now take account of the fact that player i does not observe b directly but only a noisy signal b_i^{ε} . Given $p_{-i} = p_{-i}^X$ and $b_i^{\varepsilon} = X$, the expected gain to player i from playing $x_i = 1$ rather than $x_i = 0$ becomes:

$$\Delta_i^{\varepsilon}(p_{-i}^X\mid X) = \frac{1}{2\varepsilon}\int\limits_{X-\varepsilon}^{X+\varepsilon}bdb + d$$

$$-\sum_{m=0}^{N-1} c(m+1) \binom{N-1}{m} \frac{1}{2\varepsilon} \int_{X-\varepsilon}^{X+\varepsilon} \left[\frac{\varepsilon - X + b}{2\varepsilon} \right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon} \right]^{N-m-1} db$$
(16)

$$=X + d - \sum_{m=0}^{N-1} c(m+1) {N-1 \choose m} \int_{0}^{1} q^{m} (1-q)^{N-m-1} dq$$
 (17)

$$=X+d-\sum_{m=0}^{N-1}c(m+1)\frac{(N-1)!}{m!(N-m-1)!}\frac{m!(N-m-1)!}{N!}$$
(18)

$$=X + d - \sum_{m=0}^{N-1} \frac{c(m+1)}{N},\tag{19}$$

as given. Equation (16) takes the expression for $\Delta_i(m_i \mid b)$ given in (2) and integrates over b and m_i , given $b_i^{\varepsilon} = X$ and $p_{-i} = p_{-i}^X$. Equation (17) relies on integration by substitution (using $q = 1/2 - (X - b)/2\varepsilon$) to rewrite the last integral in (16). Equation (18) rewrites both the integral in (17) and the binomial coefficient $\binom{N-1}{m}$ in terms of factorials. Equation (19) simplifies.

Note that the right hand side of (12) is strictly continuously increasing in X for all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$. Combined with Lemma 1, this implies that there exists a unique $X^* \in (\underline{B} + \varepsilon, \overline{B} - \varepsilon)$ such that $\Delta_i^{\varepsilon}(p_{-i}^{X^*} \mid X^*) = 0$. To see this, observe that Lemma 1 implies that $\Delta_i^{\varepsilon}(p_{-i}^{L^0} \mid L^0) < 0$ and $\Delta_i^{\varepsilon}(p_{-i}^{R^0} \mid R^0) > 0$. From the monotonicity and continuity of $\Delta_i^{\varepsilon}(p_{-i}^X \mid X)$ in X then follow both existence and uniqueness of X^* such that $\Delta_i^{\varepsilon}(p_{-i}^{X^*} \mid X^*) = 0$. Moreover, observe that these results hold for general $\varepsilon > 0$ (provided appropriate bounds on X are imposed), not only the limit case as $\varepsilon \to 0$.

Proposition 3. For all ε such that $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$, there exists a unique strategy vector p^* surviving iterated elimination of strictly dominated strategies in the global game G^{ε} . In particular, there exists a unique B^* such that $p^* = p^{B^*}$. The point B^* is given by:

$$B^* = \sum_{n=1}^{N} \frac{c(n)}{N} - d. \tag{20}$$

Proof. From Lemma 2 we know that $x_i = 0$ is iteratively dominant at $b_i < L^*$ while $x_i = 1$ is iteratively dominant at $b_i > R^*$. By construction, the points L^* and R^* solve

 $\Delta_i^{\varepsilon}(p_{-i}^{L^*} \mid L^*) = \Delta_i^{\varepsilon}(p_{-i}^{R^*} \mid R^*) = 0$. Proposition 2 implies that $L^* = R^* = B^*$. The result follows.

I derive Proposition 3 for general $\varepsilon > 0$ provided the (strong) assumption that b and ε_i (all i) are drawn independently from the uniform distribution. For the limit as $\varepsilon \to 0$, Frankel et al. (2003) establish that Proposition 3 substantially generalizes to games in which b is drawn from any continuous density with connected support and each ε_i is drawn independently from any (possible player-specific) atomless density.⁴

Proposition 3 should not be misunderstood as saying that players will perfectly coordinate their actions (investments). An equilibrium is defined in strategies. The proposition says that there exists a unique strategy surviving iterated dominance for each player. But a strategy only yields an action once we feed it a signal. For $\varepsilon > 0$, it is possible that some players receive signals above B^* while others see a signal below it. When this occurs, players will fail to coordinate their actions (i.e. some will choose $x_i = 1$ while others choose $x_i = 0$). The implication is that empirically observed coordination failures are not necessarily at odds with, and therefore do not by themselves invalidate the model. When a coordination failure occurs, the equilibrium outcome is inefficient.

Corollary 1. Let p^{ε} be the unique Bayesian Nash equilibrium of G^{ε} . For $\varepsilon > 0$, let $b \in [X^* - \varepsilon, X^* + \varepsilon]$. Then (i) $\Pr[p^{\varepsilon}(b^{\varepsilon}) = \mathbf{1}] < 1$ and (ii) $\Pr[p^{\varepsilon}(b^{\varepsilon}) = \mathbf{0}] < 1$. In other words, players may fail to coordinate their actions.

In the limit as $\varepsilon \to 0$, the global climate game G^{ε} selects a unique equilibrium of the underlying coordination game with multiple equilibria. To see this, note that for any $b > B^*$, we can find a $\varepsilon < B^* - b$ so that $b - \varepsilon > X^*$. Since $b_i^{\varepsilon} \in [b - \varepsilon, b + \varepsilon]$ and $p^{\varepsilon} = p^{B^*}$ this implies that $p_i^{\varepsilon}(b_i^{\varepsilon}) = 1$ for all b_i^{ε} consistent with b and all i.

Corollary 2. Let p^{ε} be the unique Bayesian Nash equilibrium of G^{ε} .

- (i) Let $b > X^*$, then, if ε is sufficiently small, $\Pr[p^{\varepsilon}(b^{\varepsilon}) = \mathbf{1}] = 1$.
- (ii) Let $b < X^*$, then, if ε is sufficiently small, $\Pr[p^{\varepsilon}(b^{\varepsilon}) = \mathbf{0}] = 1$.

Even as $\varepsilon \to 0$ and players coordinate their actions with probability 1, the unique equilibrium can be inefficient. In particular, players coordinate on **0** (all adopt the dirty

⁴In particular, the reader is referred to their result on (local) potential games with own-action quasi-concave payoffs, i.e. Theorem 4.

technology) for all $b < X^*$ even though the outcome in which players coordinate on 1 (all adopt the clean technology) is Pareto dominant for all $b > \bar{b}$ (and even though they know it). Thus, for all b such that $\bar{b} < b < B^*$ coordination is on the Pareto dominated outcome of the underlying complete information game. Intuitively, clean investment will be too risky when b is low since the noise in signals forces a player to believe that others may think that clean investment is dominated. Barrett and Dannenberg (2012) appear to share this view when they write that "players could use risk-dominance as a selection rule." For 2×2 games, their statement is backed by the theoretical literature: Carlsson and Van Damme (1993) prove that any 2×2 global game selects the risk dominant equilibrium of the underlying true game. In games with more players or actions, the statement is not generally correct. The result stands in contrast to the common and often implicit assumption in the environmental literature that players generally coordinate on the efficient equilibrium (Barrett, 2006; Hoel and de Zeeuw, 2010).

4 Network Subsidies

Taxes and subsidies will stimulate adoption of the clean technology whenever they cause an effective decrease of c(m) - d for at least one m. The U.S. Federal Tax Credit for Solar Photovoltaics (Borenstein, 2017), California's Clean Vehicle Rebate Project (Li et al., 2017), or the U.S. National Plug-In Electric Drive Vehicle Credit (Clinton and Steinberg, 2019) are good illustrations. However, tax policies may not always be feasible, political or otherwise. For example, legislation on taxation requires unanimous agreement in the European Union, which is one reason the EU does not have a carbon tax. Subsidies and tax credits, on the other hand, come with a substantial budgetary burden. The Congressional Budget Office expects total cost from tax credits on electric vehicles to be about 7.5 billion U.S. dollars through 2019.⁶ Planned spending on SDE++ subsidies in the Netherlands are 5 billion Euros in 2021.⁷.

The expenditure on subsidies to stimulate clean technology adoption can be substantially mitigated without the need to compromise on efficiency through the use

⁵For a definition of risk dominance, see Harsanyi and Selten (1988).

 $^{^6} See\ https://www.cbo.gov/sites/default/files/112 th-congress-2011-2012/reports/electric$ vehicles one-col.pdf

⁷See https://www.rvo.nl/subsidie-en-financieringswijzer/sde. SDE is an acronym for Stimulering Duurzame Energievoorziening en Klimaattransitie, or "Stimulus Sustainable Energy Supply and Climate Transition".

of *network subsidies*. A network subsidy, like any subsidy, is offered contingent on adoption of the clean technology. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number.

The strongest results obtain in the game of complete information and the global game with vanishing idiosyncratic noise, which I shall discuss in turn.

4.1 Game of Complete Information

Consider again the game of complete information (about b) discussed in Section 2. Recall from Proposition 1 that coordination on x = 1 is the Pareto dominant outcome of the game for all $b > \bar{b}$, whereas coordination on x = 0 is efficient for all $b < \bar{b}$.

My aim is to find a subsidy that incentivizes players to coordinate on the efficient outcome of the game for any b. Concretely, I want to formulate a subsidy policy that makes $x_i = 1$ is dominant for all $b > \bar{b}$ while $x_i = 0$ becomes dominant at $b < \bar{b}$. I say that such a subsidy *implements* the efficient outcome of the game in dominant strategies for almost all b.⁸

First I will show that if **1** is a (strict) Nash equilibrium of G(b), then the efficient outcome of the game can be implemented in (strictly) dominant strategies at zero cost, even if **0** is also a strict Nash equilibrium. To this end, define $s^*(x)$ to be the function given by:

$$s^*(x) = \Delta_i(\mathbf{1} \mid b) - \Delta_i(x \mid b) = c(m(x)) - c(N).$$
 (21)

I call $s^*()$ a network subsidy scheme.

Proposition 4. Consider the game G(b).

- (i) If 1 is a Nash equilibrium of the game (i.e. if $b + c \ge c(N)$), then 1 can be implemented in weakly dominant strategies with $s^*(\cdot)$ and no subsidies have to be paid.
- (ii) If **1** is a strict Nash equilibrium of the game (i.e. if b + c > c(N)), then **1** can be implemented in strictly dominant strategies with $s^*(\cdot)$ and no subsidies have to be paid.

⁸This question is related to the literature on (dominant strategy) implementation. That is, we study the problem of a policymaker who seeks to change the original game studied in Section 2 and 3 with the aim of making coordination on the efficient outcome of the game a dominant strategy for all players (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983; Kuzmics and Steg, 2017).

Proof. The gain from choosing $x_i = 1$ rather than $x_i = 0$, conditional on the network subsidy scheme $s^*(\cdot)$, given b and x_{-i} is:

$$\Delta_i(x \mid b) + s^*(x) = \Delta_i(x \mid b) + \Delta_i(\mathbf{1} \mid b) - \Delta_i(x \mid b) = \Delta_i(\mathbf{1} \mid b), \tag{22}$$

which, for all x_{-i} , is (strictly) positive if and only if **1** is a (strict) Nash equilibrium of the game. Thus, the offering a subsidy scheme equal to $s^*(\cdot)$ turns $x_i = 1$ into a (strictly) dominant strategy whenever **1** is a (strict) Nash equilibrium of G(b). When players coordinate on **1** total spending on network subsidies is $N \cdot s^*(\mathbf{1}) = 0$.

Proposition 4 shows that a smart policy of network subsidies allows the policymaker to costlessly implement the efficient outcome of the game in (strictly) dominant strategies if the efficient outcome is a (strict) Nash equilibrium of the game. The Pareto efficient outcome can be implemented without cost in the case of multiplicity.

Note that for all b > c(N) - d, it holds that $\Delta_i(\mathbf{1} \mid b) > 0$, where the inequality is strict. It follows that, provided the network subsidy scheme $s^*(\cdot)$ is offered, the policymaker may even tax playing $x_i = 1$ yet still implement $\mathbf{1}$ in strictly dominant strategies.

Remark 1. Let b > c(N) + d, so **1** is the efficient outcome of the game G(b). If the policymaker offers the network subsidy scheme $s^*(\cdot)$, the policymaker can impose a tax $t(b) \le b + d - c(N)$ on playing $x_i = 1$ but nevertheless implement coordination on **1** in strictly dominant strategies.

I next show that if **1** is not a Nash equilibrium of the game G(b), i.e. if b < c(N) - d, then it can be implemented as a strict Nash equilibrium through offering an environmental subsidy equal to s(b) on playing $x_i = 1$, where it holds that

$$s(b) > c(N) - d - b \quad \text{if} \quad b \ge \bar{b}, \tag{23}$$

while s(b) = 0 otherwise. Observe that s(b) may be negative for b > c(N) - d, which is related to Remark 1. Of course, a tax equal to s(b) on playing $x_i = 0$ is equivalent in terms of players' incentives (though not for the policymakers' budget). It is easy to verify that s(b) implements coordination on 1 as a strict Nash equilibrium for all $b > \bar{b}$. The gain from playing $x_i = 1$ rather than $x_i = 0$, given the environmental subsidy

scheme s(b), is given by:

$$\Delta_{i}(x \mid b) + s(b) > \Delta_{i}(x \mid c(N) - d) \quad \text{for all} \quad b \ge \bar{b}$$

$$\Delta_{i}(x \mid b) + s(b) < \Delta_{i}(x \mid c(N) - d) \quad \text{for all} \quad b < \bar{b}$$
(24)

Since $\Delta_i(\mathbf{1} \mid c(N) - d) = 0$, it follows that s(b) implements the efficient outcome of the game as a strict Nash equilibrium for all b. We may therefore invoke Proposition 4 and obtain the following result.

Proposition 5. Consider the game G(b).

- (i) If $\mathbf{1}$ is not a Nash equilibrium of the game (i.e. if b < c(N) d), but $\mathbf{1}$ is the Pareto efficient outcome, then, by subsidizing $x_i = 1$ through s(b), $\mathbf{1}$ can be implemented in strictly dominant strategies; total subsidy spending will be $N \cdot s(b)$.
- (ii) If $\mathbf{1}$ is not a Nash equilibrium of the game (i.e. if b < c(N) d), but $\mathbf{1}$ is the Pareto efficient outcome, then, by taxing $x_i = 0$ through s(b), $\mathbf{1}$ can be implemented in strictly dominant strategies; total tax revenues will be 0.

Observe that, although implementation in strictly dominant strategies is a much stronger requirement than implementation in Nash equilibrium, both can be achieved at the same cost.

Remark 2. If 1 can be implemented as a Nash equilibrium, then it can be implemented in strictly dominant strategies at no additional cost through a network subsidy $s^*(\cdot)$.

The formal argument that underlies Remark 2 is that an environmental subsidy $s(b) > \Delta_i(\mathbf{1} \mid c(N) - d) - \Delta_i(\mathbf{1} \mid b)$ for all $b > \bar{b}$ (and s(b) = 0 otherwise) implements $\mathbf{1}$ as a strict Nash equilibrium whenever this is efficient. Moreover, if $\mathbf{1}$ is a strict Nash equilibrium for all $b > \bar{b}$, then a network subsidy $s^*(x) = \Delta_i(\mathbf{1} \mid b) - \Delta_i(x \mid b) = c(m(x)) - c(N)$ turns $x_i = 1$ into a strictly dominant strategy for all $b > \bar{b}$ so that, given s(b), total spending on the network subsidy is $s^*(\mathbf{1}) = 0$. It follows that total spending required to implement $\mathbf{1}$ in strictly dominant strategies is $N \cdot [s(b) + s^*(\mathbf{1})] = N \cdot s(b)$, the sum total of subsidy payments required to implement $\mathbf{1}$ as a strict Nash equilibrium.

Why does a network subsidy work so well despite the low cost? The key property of a network subsidy set at $s^*(\cdot)$ is that it eliminates all *strategic uncertainty*, i.e. the uncertainty a player has about the actions chosen by all other players. The network subsidy thus removes the payoff uncertainty deriving from strategic uncertainty

interacted with technological spillovers – it turns the original coordination game into a simple dominance solvable game for all b. In so doing, the network subsidy manages to eliminate all inefficiencies caused by players' failure to internalize the technological spillovers inherent in clean investments. Intuitively, the network subsidy works like an insurance. It protects individual investors against the risk of small network externalities from clean investments in case many others have adopted the dirty technology. In so doing, it impels individuals toward clean investments. The network subsidy does not have to be paid as a result, being conditional on low investments by construction.

4.2 Global Game With Vanishing Noise

Consider the global game G^{ε} discussed in Section 3. In this game, players do not observe b. While I have so far not made any assumptions on what the policymaker knows, I will henceforth assume that the policymaker observes neither the true b nor a signal of it.

Since b is unobserved, the policymaker cannot rely on the environmental subsidy s(b) given by (23). In this section I therefore address the modified question of what subsidy scheme suffices (at minimal cost) to implement 1 in dominant strategies for all $b > \bar{b}$ and 0 for all $b < \bar{b}$ given that the subsidy scheme itself cannot be a function of b.

Note that, in the limit as $\varepsilon \to 0$, we know that $b > \bar{b}$ implies $b_i^{\varepsilon} > \bar{b}$ for all i, and similarly for the reversed inequality. Our approach toward the policymaker's problem is therefore to find a subsidy scheme $s(\cdot)$ that implements $p_i(b_i^{\varepsilon}) = 1$ as a dominant strictly strategy for all $b_i^{\varepsilon} > \bar{b}$ and $p_i(b_i^{\varepsilon}) = 0$ as a strictly dominant strategy for all $b_i^{\varepsilon} < \bar{b}$.

If he subsidy $s(\cdot)$ should achieve that $p_i(b_i) = 1$ is dominant for all $b_i^{\varepsilon} > \bar{b}$, then it must solve:

$$\Delta_i^{\varepsilon}(x \mid b_i^{\varepsilon}) > 0, \tag{25}$$

for all x_{-i} if $b_i^{\varepsilon} > \bar{b}$. For all $b_i^{\varepsilon} > c(N) - d$, I first show that the policymaker can offer a network subsidy s^* such that this decision criterion has to hold only for $x_{-i} = 1$, at no cost. Let the policymaker offer a network subsidy s^* equal to:

$$s^* = \Delta_i^{\varepsilon}(\mathbf{1} \mid b_i^{\varepsilon}) - \Delta_i^{\varepsilon}(x \mid b_i^{\varepsilon}) = c(m(x)) - c(N), \tag{26}$$

which is the same network subsidy as in (21) and where $\Delta_i^{\varepsilon}(x \mid b) := \frac{1}{2\varepsilon} \int_{b_i^{\varepsilon} - \varepsilon}^{b_i^{\varepsilon} - \varepsilon} \Delta(x \mid b) db$.

Proposition 6. Consider the global game G^{ε} and let $\varepsilon \to 0$. If the policymaker offers

a network subsidy equal to $s^*(\cdot)$, then $p_i(b_i^{\varepsilon}) = 1$ is a strictly dominant strategy for all $b_i > c(N) - d$ and all i; similarly, $p_i(b_i^{\varepsilon}) = 0$ is a strictly dominant strategy for all $b_i < c(N) - d$ and all i. Spending on network subsidies is hence zero for almost all b.

Proof. Let $\varepsilon \to 0$ and $b_i^{\varepsilon} > c(N) - d$, so $b_i^{\varepsilon} - \varepsilon > c(N) - d$. This implies that $\Delta_i^{\varepsilon}(p_{-i} \mid b_i^{\varepsilon}) + s^*(x) = \Delta_i^{\varepsilon}(\mathbf{1} \mid b_i^{\varepsilon}) > 0$ and $p_i(b_i^{\varepsilon}) = 1$ is dominant. Dominance of $p_i(b_i^{\varepsilon}) = 0$ for all $b_i < c(N) - d$ is proven symmetrically. Unless b = c(N) - d it therefore follows that spending on $s^*(\cdot)$ is zero; either players coordinate on $\mathbf{0}$ and no subsidies are paid, or players coordinate on $\mathbf{1}$ and each is paid $s^*(\mathbf{1}) = 0$.

If $p_i(b_i^{\varepsilon})=1$ is a strict best response to $x_{-i}=\mathbf{1}$ for some b_i^{ε} , then a network subsidy $s^*(\cdot)$ implements $p_i(b_i^{\varepsilon})=1$ (for this same b_i^{ε}) as a strictly dominant strategy at no cost. Note, also, that if $p_i(b_i^{\varepsilon})=0$ is a strictly best response to $x_{-i}=\mathbf{1}$ for some b_i^{ε} , then $p_i(b_i^{\varepsilon})=0$ is dominant for this b_i^{ε} . By Proposition 6 the problem of finding a subsidy scheme $S(\cdot)$ that implements, in dominant strategies, $p_i(b_i^{\varepsilon})=1$ for all $b_i>\bar{b}$ and $p_i(b_i^{\varepsilon})=0$ for all $b_i<\bar{b}$ can therefore be reduced to finding an environmental subsidy (or tax) \bar{s} for which it holds that $\Delta_i^{\varepsilon}(\mathbf{1}\mid b_i^{\varepsilon})+\bar{s}>0$ for all $b_i^{\varepsilon}>\bar{b}$ while also $\Delta_i^{\varepsilon}(\mathbf{1}\mid b_i^{\varepsilon})+\bar{s}<0$ for all $b_i^{\varepsilon}<\bar{b}$. This is easy enough:

$$\Delta_i^{\varepsilon}(\mathbf{1} \mid b_i^{\varepsilon}) + \bar{s} > 0 \quad \text{for all} \quad b_i^{\varepsilon} > \bar{b} \quad \Longrightarrow \quad \bar{s} \ge (N - 1) \cdot \bar{b}, \tag{27}$$

which is true since $\bar{b} = \frac{c(N)-d}{N}$ by definition, so $\Delta_i^{\varepsilon}(\mathbf{1} \mid b_i^{\varepsilon}) + (N-1) \cdot \bar{b} = b_i^{\varepsilon} + d - c(N) + (N-1) \cdot \bar{b} > N \cdot \bar{b} + d - c(N) = 0$ for all $b_i^{\varepsilon} > \bar{b}$. An environmental subsidy (or tax) lower than $(N-1) \cdot \bar{b}$ does not make $p_i(b_i^{\varepsilon}) = 1$ a strict best response for all $b_i^{\varepsilon} > \bar{b}$; hence, $\bar{s} = (N-1) \cdot \bar{b}$ is the cheapest environmental subsidy the policymaker might offer to implement $p_i(b_i^{\varepsilon}) = 1$ as a dominant strategy for all $b_i^{\varepsilon} > \bar{b}$ while leaving $p_i(b_i^{\varepsilon}) = 0$ dominant for all $b_i^{\varepsilon} < \bar{b}$.

Combining the network subsidy with the environmental subsidy (or tax), let the policymaker thus offer the following subsidy scheme:

$$S(x) = \underbrace{(N-1) \cdot \bar{b}}_{\text{environmental subsidy}} + \underbrace{s^*(x)}_{\text{network subsidy}}. \tag{28}$$

Proposition 7. Consider the global game G^{ε} and let $\varepsilon \to 0$. If the policymaker offers the conditional subsidy scheme $S(\cdot)$, it implements $p_i(b_i^{\varepsilon}) = 0$ as a dominant strategy for

 $[\]frac{9 \text{Let } \bar{s} = (N-1) \cdot \bar{b} - \delta. \text{ Then } \Delta_i^{\varepsilon}(\mathbf{1} \mid b_i^{\varepsilon}) + \bar{s} > 0 \iff b_i^{\varepsilon} > \bar{b} + \delta, \text{ so if } \Delta_i^{\varepsilon}(\mathbf{1} \mid b_i^{\varepsilon}) > 0 \text{ must hold for all } b_i^{\varepsilon} > \bar{b} \text{ then it must hold that } \bar{s} \ge (N-1) \cdot \bar{b}.$

all $b_i < \bar{b}$ and $p_i(b_i^{\varepsilon}) = 1$ as a dominant strategy for all $b_i > \bar{b}$. Hence, the policymaker can implement the efficient outcome of the game G(b) in strictly dominant strategies for almost all b; total subsidy spending is $N \cdot (N-1) \cdot \bar{b}$ is $b > \bar{b}$ and 0 if $b < \bar{b}$.

Proof. If $b_i^{\varepsilon} > \bar{b}$, then $p_i(b_i^{\varepsilon}) = 1$ is a best response to $x_{-i} = \mathbf{1}$ given the environmental subsidy $\bar{s} = (N-1) \cdot \bar{b}$. We can thus invoke Proposition 6 and see that, adding the network subsidy $s^*(\cdot)$, $p_i(b_i^{\varepsilon}) = 1$ is a dominant strategy for all $b_i^{\varepsilon} > \bar{b}$. Since $b_i^{\varepsilon} > \bar{b}$ if and only if $b > \bar{b}$ when $\varepsilon \to 0$, this implies that players coordinate on $\mathbf{1}$ for all $b > \bar{b}$ and total subsidy spending is $S(\mathbf{1}) = (N-1) \cdot \bar{b} + s^*(\mathbf{1}) = (N-1) \cdot \bar{b}$. The proof for $b < \bar{b}$ is similar and omitted.

The policymaker might also offer a network subsidy $s^*(\cdot)$ to those playing $x_i = 1$ buy levy an environmental tax equal to $(N-1) \cdot \bar{b}$ on playing $x_i = 0$. Such a policy is evidently equivalent with regard to players' incentives, although it differs for the policymaker's budget.

Corollary 3. Consider the global game G^{ε} and let $\varepsilon \to 0$. Let the policymaker offer a network subsidy $s^*(\cdot)$ on playing $x_i = 1$. In addition, let the policymaker levy a tax $\bar{s} = (N-1) \cdot \bar{b}$ on playing $x_i = 0$. This policy implements $p_i(b_i^{\varepsilon}) = 0$ as a dominant strategy for all $b_i < \bar{b}$ and $p_i(b_i^{\varepsilon}) = 1$ as a dominant strategy for all $b_i > \bar{b}$. Hence, the policymaker can implement the efficient outcome of the game G(b) in strictly dominant strategies for almost all b; total revenues are $N \cdot (N-1) \cdot \bar{b}$ is $b < \bar{b}$ and 0 if $b > \bar{b}$.

Note that for $\varepsilon \to 0$ it holds that $b > \bar{b} \implies b_i > \bar{b}$ and $b < \bar{b} \implies b_i < \bar{b}$ for all i. An immediate consequence of Proposition 7 and the preceding discussion follows.

Corollary 4. In the limit as $\varepsilon \to 0$, the lowest subsidy on $x_i = 1$ (tax on $x_i = 0$) required to implement the efficient outcome of the game G(b) in dominant strategies for almost all b is $(N-1) \cdot \bar{b} + s^*(\cdot)$.

The present analysis did not make use of the fact that, without policy interventions, playing p^{B^*} is the essentially unique strategy profile surviving iterated dominance in the global game G^{ε} . Much like for the game of complete information, what I did here was to seek policies that turn $x_i = 1$ into a dominant action for all i and all $b > \bar{b}$ without any regard for other players' strategies. Another approach toward ((iterative) dominant strategy) implementation in G^{ε} would be to study what mechanisms the policymaker could design to shift the threshold B^* down toward \bar{b} . I intend to do this in future work.

4.3 Global Game with Non-Vanishing Noise

The requirement that signals are sufficiently precise played a central role in the previous analysis. It is also a strong assumption. In this section, I investigate the expected cost of the subsidy scheme $S(\cdot)$ given by (28) in the global game G^{ε} when the noise in signals does not vanish (though I shall continue to assume that $2\varepsilon > \min\{c, c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$). I show that spending on both the environmental and the network subsidy part of $S(\cdot)$ is positive in expectations. I then construct a budget-neutral network tax-subsidy scheme that restores costlessness of the network-subsidy-component of $S(\cdot)$ without adversely affecting players' incentives.

Let the policymaker again offer the subsidy scheme S(x) given by $S(x) = (N-1) \cdot \bar{b} + s^*(x)$. From Proposition 7, we know that offering the scheme S(x) turns $p_i(b_i^{\varepsilon}) = 1$ into a dominant strategy to each player i for all $b_i^{\varepsilon} > \bar{b}$.

Recall that conditional on b, each signal b_i^{ε} is an i.i.d. draw from the uniform distribution on $[b-\varepsilon,b+\varepsilon]$. When $b>\bar{b}+\varepsilon$ it therefore holds that $b_i^{\varepsilon}>c(N)-d$ for all i; in that case, players perfectly coordinate on $x_i=1$. Similarly, when $b<\bar{b}-\varepsilon$ it holds that $b_i^{\varepsilon}< c(N)-d$ for all i, and players coordinate on $x_i=0$. In both cases, spending on the network subsidy s^* will be zero (i.e. either no player adopts the clean technology and therefore no one is entitled to receive the subsidy in the first place, or all players adopt the clean technology and each receives a network subsidy equal to $s^*(N)=0$). For true bs such that $\bar{b}-\varepsilon< b<\bar{b}+\varepsilon$, the expected performance of the subsidy scheme deteriorates. The reason is that, in these cases, some players may receive signals at which $x_i=1$ is dominant while others may receive signals at which $x_i=0$ is dominant, leading to coordination failure.

I will now derive the expected spending on the subsidy scheme $S(\cdot)$ when ε is not arbitrarily small. For any true b we know the probability distribution over signals $b^{\varepsilon} = (b_1^{\varepsilon}, b_2^{\varepsilon}, ..., b_N^{\varepsilon})$. Given the incentive to play $x_i = 1$ for all $b_i^{\varepsilon} > \bar{b}$ and $x_i = 0$ for all $b_i^{\varepsilon} < \bar{b}$ generated by the subsidy scheme, this implies a probability distribution over the number of players n that play 1, allowing us to calculate expected subsidy spending. What follows is mostly an exercise in probability theory.

Suppose x was played and exactly n players in x chose to play 1, so the remaining N-n players chose 0. If the policymaker offers the network subsidy scheme $s^*(x)$, total spending on the subsidy is then simply $n \cdot \lceil (N-1) \cdot \bar{b} + s^*(m(x)) \rceil$.

We know from Proposition 7 that an expected payoff maximizing player i adopts the clean technology for all $b_i^{\varepsilon} > \bar{b}$ when offered the subsidy scheme $S(\cdot)$ in the global game G^{ε} . What we want to know, therefore, is the probability that any player *i*'s signal satisfies this inequality. Conditional on *b* and ε , that probability is given by:

$$\Pr\left[b_i^{\varepsilon} < \bar{b} \mid b, \varepsilon\right] = \Pr\left[\varepsilon_i < \bar{b} - b \mid b, \varepsilon\right]$$

$$= \frac{1}{2} - \frac{1}{N} \frac{N \cdot b + d - c(N)}{2\varepsilon},$$
(29)

where the final equality holds for all $\bar{b} - \varepsilon \leq b \leq \bar{b} + \varepsilon$, while it is either 0 or 1 otherwise. Clearly, the probability that $b_i^{\varepsilon} > \bar{b}$ is (given b and ε) simply the complement of (29):

$$\Pr[b_i^{\varepsilon} > c(N) - d \mid b, \varepsilon] = 1 - \Pr[b_i^{\varepsilon} < c(N) - d \mid b, \varepsilon]. \tag{30}$$

There are a total of $\binom{N}{n}$ unique vectors x for which m(x) = n. Thus, the total probability that n players play $x_i = 1$ (while N - n play $x_i = 0$), given b and ε , is:

$$q^{\varepsilon}n \mid b) = \binom{N}{n} \cdot \left[\frac{1}{2} - \frac{1}{N} \frac{N \cdot b + d - c(N)}{2\varepsilon}\right]^{n} \cdot \left[\frac{1}{2} + \frac{1}{N} \frac{N \cdot b + d - c(N)}{2\varepsilon}\right]^{N-n}, \quad (31)$$

when $\bar{b} - \varepsilon \leq b \leq \bar{b} + \varepsilon$, and $q^{\varepsilon}(n \mid b) = 0$ otherwise. Note that simple multiplication of probabilities is allowed on account of the fact that conditional on b each ε_i is an i.i.d. draw from the uniform distribution on $[-\varepsilon, \varepsilon]$. The following proposition summarizes.

Proposition 8. Let the policymaker offer the conditional subsidy scheme $S(\cdot)$. For all $b \notin (\bar{b} - \varepsilon, \bar{b} + \varepsilon)$, spending on the subsidy is zero. For $b \in (\bar{b} - \varepsilon, \bar{b} + \varepsilon)$, expected spending on the subsidy S(n) is equal to:

$$C^{\varepsilon} = \sum_{n=0}^{N} q^{\varepsilon}(n \mid b, \varepsilon) \cdot n \cdot \left[(N-1) \cdot \bar{b} + s^{*}(n) \right] > 0.$$
 (32)

From the preceding analyses of both the game of complete information and the global game with vanishing noise, we know that the policymaker cannot, generally, implement coordination on 1 (if this is deemed desired) either in Nash equilibrium or in dominant strategies without paying out environmental subsidies (or levying taxes on $x_i = 0$). The fact that in this global game with non-vanishing noise the policymaker has positive expected spending on environmental subsidies $(N-1) \cdot \bar{b}$ should therefore not come as a surprise. What is new, however, is that in this global game of non-vanishing noise, expected spending on network subsidies too is positive.

The remarkably strong performance of a network subsidy $s^*(\cdot)$ breaks down in the

case of non-vanishing noise. The reason is that players' signals may lie far apart, making perfect coordination of actions unlikely. Yet by construction the amount of network subsidy received by each player is zero if and only if $x = \mathbf{0}$, in which case players are not entitled to receive a subsidy in the first place, or $x = \mathbf{1}$ so that $s^*(\mathbf{1}) = c(N) - c(N) = 0$. If players fail to perfectly coordinate their actions, spending on the network subsidy is strictly positive. This begs the question whether, and how, a policy scheme based on the idea of network subsidies can be conceived at zero cost even when players may fail to coordinate with strictly positive probability.

A straightforward way to implement an effective yet costless network subsidy on clean technological investments is to finance subsidy payments on $x_i = 1$ through a "network tax" levied on choosing $x_i = 0$. Let this subsidy be denoted $s^{**}(x)$; the corresponding tax is denoted $t^{**}(x)$. Thus, if n players play 1 in x, aggregate spending on network subsidies is $n \cdot s^{**}(x)$, whereas aggregate revenues from the network tax are $(N-n) \cdot t^{**}(x)$. If we want the tax-subsidy scheme to be costless, the budget constraint for this policy is:

$$(N-n) \cdot t^{**}(n) - n \cdot s^{**}(n) = 0. \tag{33}$$

Condition (33) imposes that total spending on the network subsidy is matched by total tax income from taxing the dirty technology for any number n of players playing $x_i = 1$.

Next, the tax-subsidy scheme – when offered together with the environmental subsidy equal to $(N-1) \cdot \bar{b}$ – must make $x_i = 1$ dominant for all $b_i^{\varepsilon} > \bar{b}$ and $x_i = 0$ dominant for all $b_i^{\varepsilon} < \bar{b}$. As we have seen before, this condition will be satisfied whenever the expected gain from clean investment, accounting for taxes and subsidies, satisfies:

$$\Delta_i^{\varepsilon}(b_i^{\varepsilon}, m) + (N - 1) \cdot \bar{b} + s^{**}(n) + t^{**}(n) = b_i^{\varepsilon} - \bar{b}, \tag{34}$$

for all m = 0, 1, ..., N - 1. Equation (34) represents the incentive constraint of a network tax-subsidy scheme. Combined with the budget constraint, this yields the following scheme:

$$\begin{cases} t^{**}(x) &= \frac{n}{N} \left[c(m(x)) - c(N) \right] \\ s^{**}(x) &= \frac{N-n}{N} \left[c(m(x)) - c(N) \right] \end{cases}$$
 (35)

Having derived (35), assume now the policymaker offer a subsidy scheme S(n) given by:

$$S(n) = \begin{cases} (N-1) \cdot \bar{b} + s^{**}(n) & \text{if } x_i = 1\\ -t^{**}(n) & \text{if } x_i = 0 \end{cases}$$
(36)

Since S(n) solves (33) and (34), it incentivizes players to play $x_i = 1$ for all $b_i^{\varepsilon} > \bar{b}$ even though net spending on the "network tax-subsidy" scheme is always zero by construction.

Proposition 9. A conditional subsidy scheme equal to (36) incentivizes players to play $x_i = 1$ for all $b_i^{\varepsilon} > \bar{b}$ and $x_i = 0$ for all $b_i^{\varepsilon} < \bar{b}$. Net spending on the network tax-subsidy scheme $(s^{**}(\cdot), t^{**}(\cdot))$ is always zero.

Proof. Both statements in the propositions are true by construction. \Box

Remark 3. The network tax-subsidy scheme $(s^{**}(\cdot), t^{**}(\cdot))$ can also substitute for the pure network subsidy $s^*(\cdot)$ discussed in the game of complete information and the global game with vanishing noise. This substitution would not affect players' incentives in these games; nor would it have an effect on total subsidy spending, as coordination on either $\mathbf{0}$ or $\mathbf{1}$ is achieved for all b save $b = \bar{b}$ so offering $s^*(\cdot)$ is free of cost anyway.

Finally, observe that the logic of a network subsidy does not rely on the application to climate change. Any market where (i) individual actions exhibit strategic complementarities and (ii) players do not take these into account may coordinate on an inefficient equilibrium. A network subsidy then offers an inexpensive way out of this trap. The observation suggests that network subsidies are worth studying in economics more generally.

5 Summary

This paper studies climate change mitigation in a global game. The focus is on abatement through technological investment. Players invest in either of two technologies. One technology is cheap and dirty, the other expensive but clean. I consider environments in which investments are strategic complements. These could for example arise due to network effects, technological spillovers, or learning-by-doing. Consistent with the existing literature on technological investments in clean technologies, I demonstrate that the complete information version of my game has multiple equilibria (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017; Mielke and Steudle, 2018). Equilibrium multiplicity can lead to coordination failure and complicates the design of domestic policies or climate treaties.

To this well-studied framework, I add a little bit of uncertainty. I assume that the true environmental benefit of the clean technology is unobserved. Rather than observe

the technology's true benefit, players receive private noisy signals of it. In this *global climate game*, I show that there exists a unique equilibrium in which players adopt the clean technology if and only if their private signals exceed an endogenous threshold. For signals below the threshold, players adopt the dirty technology instead.

My first contribution is to show that the game has a unique Bayesian Nah equilibrium. This contribution directly addresses the issue of equilibrium multiplicity often encountered in this literature. Equilibrium selection is driven by the assumption of incomplete information, which seems realistic in the context of clean technologies and climate change. The analysis highlights that the precise way in which one models uncertainty is important. Although some papers conclude that "shared" uncertainty does not eliminate equilibrium multiplicity in coordination games (Barrett and Dannenberg, 2012; Barrett, 2013), this paper shows the starkly contrasting result that privately held beliefs about the shared game does force selection of a unique equilibrium. The assumed structure of uncertainty matters. My result motivates a rethink of the way environmental economists model uncertainty.

My second contribution is to introduce network subsidies. Like standard subsidies, a network subsidy provides adopters of the clean technology with a (financial) reward. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number. As I show, it is possible to construct a simple network subsidy scheme that guarantees efficient adoption of the clean technology but does not, in equilibrium, require the policymaker to pay anything. Intuitively, the network subsidy serves as an insurance against small clean technology networks. In so doing, it boosts clean investments and therefore is never claimed. Although derived in the context of technological spillovers, the notion of a network subsidy is general and applies to public economic broadly.

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