The Global Climate Game

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Roweno J.R.K. Heijmans

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Abstract

I study emissions abatement in a global game of technological investments. Players invest in competing technologies. One technology is cheap and dirty, the other expensive but clean, and investments exhibit technological spillovers. The paper makes two main contributions. My first contribution is to resolve complications due to equilibrium multiplicity in games of technological investment by addressing equilibrium selection through the use of global games. In well-identified cases the unique equilibrium is inefficient, motivating policy intervention. This leads to my second contribution, the introduction of network subsidies. A network subsidy allows the policymaker to correct for the entire externality deriving from technological spillovers (and for all externalities if investing in the clean technology is also an equilibrium) but does not, in equilibrium, cost anything. Albeit derived in the context of climate change, the concept of a network subsidy is general and contributes to public economics more broadly.

1 Introduction

Climate change is a coordination failure of existential proportions. In order to reduce greenhouse gas emissions and prevent dangerous climate change, large-scale investments in clean technologies are necessary. These investments, however, are costly and their benefits imperfectly understood. Thus we face a strategic situation in which clean investment are required on behalf of multiple agents, leading to a coordination problem under uncertainty. In this paper, I present what is perhaps the most bare-bones model to study this type of decision problem. Players invest in competing technologies. One technology is cheap and dirty, the other expensive but clean, and investments exhibit technological spillovers.

My first contribution is to show that uncertainty about the clean technology leads to selection of a unique Bayesian Nash equilibrium. This result is derived using the machinery of global games (Carlsson and Van Damme, 1993; Frankel et al., 2003) and resolves complications caused by equilibrium multiplicity often encountered in the literature on clean technologies. As many authors have pointed out, technologies often exhibit technological spillovers or other kinds of strategic complementarities – and these may turn the game into a coordination game with multiple equilibria (Barrett, 2006; De Coninck et al., 2008; Hoel and de Zeeuw, 2010; Hong and Karp, 2012; Harstad, 2012; Battaglini and Harstad, 2016). There are several reasons why technological investments may exhibit strategic complementarities. Among those discussed in the literature on clean technologies are network effects (Katz and Shapiro, 1985; Greaker and Midttømme, 2016; Li et al., 2017; Mielke and Steudle, 2018; Clinton and Steinberg, 2019); cost sharing: (De Coninck et al., 2008); R&D in breakthrough technologies (Barrett, 2006; Hoel and de Zeeuw, 2010); climate tipping points (Barrett and Dannenberg, 2017); climate clubs (Nordhaus, 2015); technological and knowledge spillovers (Kverndokk and Rosendahl, 2007; Fischer and Newell, 2008; Gerlagh et al., 2009; Aghion and Jaravel, 2015; Harstad, 2016); social norms (Nyborg et al., 2006; Allcott, 2011; Nyborg, 2018b; Kverndokk et al., 2020; Andor et al., 2020); and reciprocity (Nyborg, 2018a).

In much of the environmental literature, equilibrium selection is treated somewhat implicitly and in a way that is not completely satisfactory. Two approaches are especially prevalent. One approach hand-picks, or at minimum focuses on a particular equilibrium. Thus, players may be a priori assumed to pursue symmetric strategies (Harstad, 2012; Harstad et al., 2019), or to coordinate on the Pareto dominant outcome (Barrett, 2006; Hoel and de Zeeuw, 2010).[1] These papers have generated many tried and tested insights, yet the question remains why we should expect real-world players to behave according to the essentially ad hoc assumptions entertained by the authors. Another approach treats the coordination problem as theoretically indecisive and relies on lab experiments

[1]In this paper, the unique equilibrium will also be in symmetric strategies. Although this is somewhat intuitive as my setting is one of symmetric players, I nonetheless allow players to pursue any strategy, including non-symmetric ones. Thus, the fact that the unique equilibrium in my game is in symmetric strategies is a result rather than an assumption.

to make predictions (for a survey of the experimental literature on coordination games, see Devetag and Ortmann (2007); for experimental studies of coordination games in the context of climate change in particular, see Barrett and Dannenberg (2012, 2014, 2017); Calzolari et al. (2018); Dengler et al. (2018)). While these papers, too, have had a lasting influence on the way we think about possible strategies to fight climate change, it is unclear whether and how to generalize their experimental findings to settings outside the laboratory. My explicit focus on equilibrium selection complements these approaches. It provides sharp conditions under which we would expect rational players to coordinate on the Pareto dominant equilibrium of the game.

The unique equilibrium of the game may be inefficient. For intermediately high clean investment benefits, players adopt the dirty technology even though they would be better off were all to adopt the clean technology instead. This result calls for policy intervention.

My second contribution is the introduction of network subsidies. The issue of taxes and subsidies, or rather policy in general, arises naturally in the present context where investments cause two externalities. One is an environmental externality that derives from the positive environmental effect clean investments have on all players; the other is a network externality deriving from spillovers (strategic complementarities) in clean investments. As I show, using only taxes and subsidies the policymaker can costlessly correct both of these externalities by turning the (expected) efficient action into a dominant action for all players (both in the game of complete information and in the global game). Moreover, I also show that even when taxes (for political or other reasons) are not possible or desirable, the policymaker can nevertheless correct the entire network externality at zero cost. The key novelty in this is what I call a network subsidy. Like standard subsidies, a network subsidy offers adopters of the clean technology a (financial) reward. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number. As I show, it is possible to construct a simple network subsidy scheme that corrects the entire network externality deriving from players' ignorance of the technological spillovers caused by their clean investment but does not, in equilibrium, cost the policymaker anything. The core property of a network subsidy driving this is the fact that the amount of subsidy paid to each individual investor is a function of aggregate clean investments. Since adoption of the clean technology is more attractive when the number of other players adopting it is higher due to the technological spillover, it is quite intuitive that the network subsidy can decrease in aggregate clean technology adoption without negatively affecting players' incentives. My result is to show that this intuition can be exploited smartly: the policymaker can offer a network subsidy scheme such that what is paid when *all* players adopt the clean technology is zero without averse affecting players' incentives.

Intuitively, the network subsidy insures adopters of the clean technology against the event they would enjoy few technological spillovers since many others adopted the dirty technology. In so doing, it boosts clean investments and therefore is never claimed. This result is independent of the application to clean technologies and suggests that network subsidies are worth studying in other contexts where strategic complementarities occur.

In my model, equilibrium selection is catalyzed by uncertainty about the clean technology.[2] The assumption seems realistic. There are many uncertainties surrounding climate change and individual beliefs vary vastly (Hornsey et al., 2016). Within the model, one is free to interpret this uncertainty in different ways. Uncertainty could pertain to the true severity of climate change, the location of a dangerous tipping point, or the true potential of a breakthrough technology. Although many authors have studied the role of incomplete information in the climate context (Kolstad, 2007; Barrett and Dannenberg, 2012; Martimort and Sand-Zantman, 2016), none consider the type of uncertainty with idiosyncratic, player-specific (posterior) beliefs studied here.

The remainder of the paper is structured as follows. In Section 2, I present the main model and briefly discuss the game of complete information. In Section 3, I add uncertainty to the analysis and show that the global game has a unique equilibrium. Section 4 introduces network subsidies, and Section 5 concludes.

2 Main Model

Consider a world consisting of N players. Each player chooses to invest in either of two technologies. The first, called the dirty technology, is a cheap and dirty technology. If a player does not invest in the dirty technology, s/he invests in the clean technology, an expensive but environmentally-friendly clean technology. One could think of the clean

[2]This type of "global uncertainty" turns the game into a global game. However, other approaches toward equilibrium selection also exist. For example, Poisson games (Matsui and Matsuyama, 1995; Myerson, 1998, 2000; Makris, 2008), i.e. games in which players are uncertain about the number of other players playing the game. Poisson coordination games also have a unique equilibrium as the uncertainty becomes arbitrarily small. Another well-known approach derives equilibrium selection as a dynamic outcome of a process of evolutionary selection (Kandori et al., 1993).

technology as a breakthrough technology (Barrett, 2006; Hoel and de Zeeuw, 2010). Compared to investment in the dirty technology, the environmental benefit of investing in the clean technology is $b > 0$. An action for player i is a binary variable $x_i \in \{0, 1\}$ such that $x_i = 1$ corresponds to investment in the clean technology while $x_i = 0$ stands for investment in the dirty technology. Let $x = (x_1, x_2, ..., x_N)$ denote the vector of actions played by all players, and let $x_{-i} = (x_j)_{j \neq i}$ be the vector of actions by all players but i. Let $\mathbf{1} = (1, 1, ..., 1)$ be the action vector of all ones, and $\mathbf{0} = (0, 0, ..., 0)$ the action vector of all zeroes.[3] The cost of investing in the dirty technology (play 0) is constant at d. The costs of investing in the clean technology (play 1) depend on the total number of players, n, that invest in clean and are decreasing in n: $c(1) > c(2) > ... > c(N)$. That is, playing $x_i = 1$ exhibits strategic complementarities Bulow et al. (1985). I assume that $c(N) > d$.

Combining these elements, the payoff to player i is:

$$\pi_i(x \mid b) = \begin{cases} b \cdot m(x_{-i}) - d & \text{if} \quad x_i = 0 \\ b \cdot (m(x_{-i}) + 1) - c(m(x_{-i}) + 1) & \text{if} \quad x_i = 1 \end{cases}, \tag{1}$$

where $m(x_{-i})$ is defined as the number of other players playing 1 in x_{-i}; hence, $m(x_{-i}) = \sum_{j \neq i} x_j$. I also define $m(x)$ as the total number of players that play 1 in x; hence, $m(x) = \sum_{i=1}^{N} x_i$. The set of players $\{1, 2, ..., N\}$, the set of action vectors $x \in \{0, 1\}^N$, and the set of payoff functions $\{\pi_i\}$ jointly define a complete information game $G(b)$.

There are two externalities associated with investment in the clean technology. The first is an *environmental externality* and relates to the parameter b, the positive impact an individual player's investment in the clean technology has on the environment (and hence payoff) for all other players – think of reduced CO2 emissions. The second is a *network externality* and relates to the investment cost function c, i.e. it captures the fact that a player's investment in the clean technology lowers the cost of clean technological investment for all other players – think of technological or knowledge spillovers.

The gain from investing in the clean rather than the dirty technology to player i (given b and x_{-i}) is the difference in payoffs between playing $x_i = 1$ and $x_i = 0$. For

[3]With a slight abuse of notation, I will use $\mathbf{1}$ and $\mathbf{0}$ both for the vectors of dimension N and dimension $N - 1$.

given x_{-i}, we define

$$\begin{aligned}\Delta_i(x_{-i} \mid b) &= \pi_i(1, x_{-i} \mid b) - \pi_i(0, x_{-i} \mid b) \\ &= b + d - c(m(x_{-i}) + 1).\end{aligned} \tag{2}$$

Moreover, for x such that $m(x) = k$ we define $\Delta_i(k \mid b) = \Delta_i(x \mid b)$.

The action $x_i = 1$ is strictly dominant for all $b > c(1) - d$ as for those bs it holds that $\Delta_i(x \mid b) > 0$ for all x. Alternatively, $x_i = 0$ is strictly dominant for all $b < c(N) - d$. In between, the game has multiple equilibria.

Proposition 1.

(i) Coordination on $x = \mathbf{1}$ is a Nash equilibrium of the game for all $b \geq c(N) - d$. It is the unique Nash equilibrium for all $b > c(1) - d$.

(ii) Coordination on $x = \mathbf{0}$ is a Nash equilibrium of the game for all $b \leq c(1) - d$. It is the unique Nash equilibrium for all $b < c(N) - d$.

(iii) The outcome $x = \mathbf{1}$ is Pareto dominant for all $b > \frac{c(N)-d}{N}$.

Proof. This follows from the above dominance argument, together with direct payoff comparisons. □

To smoothen notation, I shall henceforth write $\bar{b} = \frac{c(N)-d}{N}$.

Environmental economists have long recognized the possibility of equilibrium multiplicity in games of technological investments (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017; Mielke and Steudle, 2018). It motivates the question of equilibrium selection, to which Section 3 is devoted. First, however, I offer some final remarks on the complete information game $G(b)$.

Frankel et al. (2003) have observed that a game such as given by (1) is a *potential game* (Monderer and Shapley, 1996). A potential game is a game for which there exists a potential function $P : \{0,1\}^N \to \mathbb{R}$ on action profiles such that the change in any individual player's payoff when switching from one action to the other is always equal to the change in the potential function; that is, for which there exists a function P such that $P(x_i, x_{-i} \mid b) - P(1 - x_i, x_{-i} \mid b) = \pi_i(x_i, x_{-i} \mid b) - \pi_i(1 - x_i, x_{-i} \mid b)$ for all i. The game $G(b)$ has a potential function $P(x \mid b)$ given by:

$$P(x \mid b) = \begin{cases} \sum_{k=1}^{m(x)} \Delta_i(k \mid b) & \text{if } m(x) > 0, \\ 0 & \text{if } m(x) = 0. \end{cases} \tag{3}$$

Observe that, for any i and any $x_{-i} \in \{0,1\}^{N-1}$, it holds that $P(1, x_{-i} \mid b) - P(0, x_{-i} \mid b) = \Delta_i(x \mid b)$, confirming that P is a potential function indeed.

A *potential maximizer* is a vector x that maximizes P. One can verify that $\mathbf{1}$ is the unique potential maximizer of $P(x \mid b)$ for all $b + d > \sum_{n=1}^{N} \frac{c(n)}{N}$ whereas $\mathbf{0}$ is the unique potential maximizer of $P(x \mid b)$ for all $b + d < \sum_{n=1}^{N} \frac{c(n)}{N}$. I return to this observation in the next Section.

3 The Global Climate Game

Strategic complementarities in clean investments drive equilibrium multiplicity under common knowledge of b, the environmental benefit of clean investment. But the assumption of complete information is strong. There are large numbers of uncertainties surrounding many clean technologies's present or future potential.

Uncertainty and signals. For these reasons, I will now study a global game. In the global game G^ε the true parameter b is unobserved. Rather, it is assumed that b is drawn from the uniform distribution on $[\underline{B}, \overline{B}]$ where $\underline{B} < c(N) - d$ and $\overline{B} > c(1) - d$.[4] Each player i in addition receives a private noisy signal b_i^ε of b, given by:

$$b_i^\varepsilon = b + \varepsilon_i. \tag{4}$$

The term ε_i captures idiosyncratic noise in i's private signal. It is common knowledge that ε_i is an i.i.d. draw from the uniform distribution on $[-\varepsilon, \varepsilon]$. I assume that ε is sufficiently small: $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$. Let $b^\varepsilon = (b_i^\varepsilon)$ denote the vector of signals received by all players, and let b_{-i}^ε denote the vector of signals received by all players but j, i.e. $b_{-i}^\varepsilon = (b_j^\varepsilon)_{j \neq i}$. Note that player i observes b_i^ε but neither b nor b_{-i}^ε. Thus I write $\Phi^\varepsilon(\cdot \mid b_i^\varepsilon)$ for the joint probability function of $(b, b_j^\varepsilon)_{j \neq i}$ conditional on b_i^ε. In what follows I will take $\varepsilon > 0$ as given and introduce the concepts used to analyze the global game G^ε.

Strategies and strict dominance. Player i receives a signal b_i^ε prior to choosing an action. A strategy p_i for player i in G^ε is a function that assigns to any $b_i^\varepsilon \in [\underline{B} - \varepsilon, \overline{B} + \varepsilon]$ a probability $p_i(b_i^\varepsilon) \geq 0$ with which the player chooses action $x_i = 1$ when s/he observes

[4] In game theory, it is assumed that the game (in this case G^ε) is common knowledge; hence, the structure of the uncertainty (the joint distrubution of b and all the signals b_j^ε), the possible actions and all the payoff functions are commonly known. For a formal treatment of common knowledge, see Aumann (1976).

b_i^ε. I write $p = (p_1, p_2, ..., p_N)$ for a strategy vector. Similarly, I write $p_{-i} = (p_j)_{j \neq i}$ for the vector of strategies for all players but i. Conditional on the strategy vector p_{-i} and a private signal b_i^ε, the expected gain (of choosing $x_i = 1$ rather than $x_i = 0$) to player i is given by:

$$\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) = \int \Delta_i(p_{-i}(b_{-i}^\varepsilon) \mid b)\, d\Phi^\varepsilon(b, b_{-i}^\varepsilon \mid b_i^\varepsilon). \tag{5}$$

I say that the action $x_i = 1$ is dominant at b_i^ε if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) > 0$ for all p_{-i}. Similarly, the action $x_i = 0$ is dominant (in the global game G^ε) at b_i^ε if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) < 0$ for all p_{-i}. When $x_i = 1$ is strictly dominant, we say that $x_i = 0$ is strictly dominated; similarly, when $x_i = 1$ is strictly dominant, we say that $x_i = 1$ is strictly dominated.

Lemma 1. *Consider the global game G^ε. (i) The action $x_i = 1$ is dominant at all $b_i^\varepsilon \geq \overline{B}$. (ii) The action $x_i = 0$ is dominant at $b_i^\varepsilon \leq \underline{B}$.*

Proof. Observe that $\Delta_i(x \mid b) > 0$ for any x given $b \in [\overline{B} - \varepsilon, \overline{B} + \varepsilon]$. Thus, for $b_i^\varepsilon = \overline{B}$ the integration in (5) is over positive terms only and $\Delta_i^\varepsilon(p_{-i} \mid \overline{B}) > 0$ for all p_{-i}. This proves part (i) of the Lemma. The proof of part (ii) relies on a symmetric argument and is therefore omitted. $\square$

Recall that a strategy vector $p^* = (p_1^*, p_2^*, ..., p_N^*)$ is a Bayesian Nash equilibrium of G^ε if for any p_i^* and any b_i^ε it holds that:

$$p_i^*(b_i^\varepsilon) \in \arg\max_{x_i \in \{0,1\}} \pi_i^\varepsilon(x_i, p_{-i}^*(b_{-i}^\varepsilon) \mid b_i^\varepsilon), \tag{6}$$

where $\pi_i^\varepsilon(x_i, p_{-i}^*(b_{-i}^\varepsilon) \mid b_i^\varepsilon) = \int \pi_i(x_i, p_{-i}^*(b_{-i}^\varepsilon) \mid b)\, d\Phi^\varepsilon(b, b_{-i}^\varepsilon \mid b_i^\varepsilon)$.

Conditional dominance. Let L and R be real numbers. The action $x_i = 1$ is said to be dominant at b_i^ε conditional on R if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) > 0$ for all p_{-i} with $p_j(b_j^\varepsilon) = 1$ for all $b_j^\varepsilon > R$, all $j \neq i$. Similarly, the action $x_i = 0$ is dominant at b_i^ε conditional on L if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) < 0$ for all p_{-i} with $p_j(b_j^\varepsilon) = 1$ for all $b_j^\varepsilon > L$, all $j \neq i$.

The concept of conditional dominance will play an important role in the analysis. To see why, consider again part (i) of Lemma 1, saying that $x_i = 1$ is dominant at all $b_i^\varepsilon \geq \bar{B}$; hence, no player will play $x_i = 0$ when $b_i^\varepsilon \geq \bar{B}$. But this also means that player i knows that no $j \neq i$ will ever play $x_j = 0$ when b_j^ε either. Thus, player i can effectively disregard any strategy vector p_{-i} for which at least one $p_j(b_j^\varepsilon) = 0$ when $b_j^\varepsilon \geq \bar{B}$. Knowing this, a payoff-maximizing player i need not only exclude strictly dominated strategies from its strategy set – rationality dictates that also *conditionally* (on $\bar{B}$) dominated strategies can be written off. This again is true for all players, all know it, and so on. Repeating this reasoning over and over, players can repeatedly write off strategies as conditionally dominated (a similar procedure must also be performed starting from the observation that $x_i = 0$ is strictly dominant at $b_i^\varepsilon \leq \underline{B}$). Those strategies that survive this process are said to survive *iterated elimination of strictly dominated strategies.* For a textbook treatment of iterated dominance, see Osborne and Rubinstein (1994).

Increasing strategies. For some $X \in \mathbb{R}$, let p_i^X denote the particular strategy such that $p_i^X(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < X$ and $p_i^X(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon \geq X$. I will call p_i^X the *increasing strategy with switching point* X. By $p^X = (p_1^X, p_2^X, ..., p_N^X)$ I denote the strategy vector of increasing strategies with switching point X, and $p_{-i}^X = (p_j^X)_{j \neq i}$. Note that $x_i = 1$ is dominant at b_i^ε conditional on R if and only if $\Delta_i^\varepsilon(p_{-i}^R \mid b_i^\varepsilon) > 0$. Similarly, if $x_i = 0$ is dominant at b_i^ε conditional on L then it must hold that $\Delta_i^\varepsilon(p_{-i}^L \mid b_i^\varepsilon) < 0$.

We now have all notation in place to proceed with the core of the analysis.

Iteration from the right. Take $p_{-i} = p_{-i}^{\overline{B}}$ and note that $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid b_i^\varepsilon)$ is continuous and monotone decreasing in b_i^ε. Moreover, recall from Lemma 1 that $x_i = 1$ is strictly dominant at $b_i = \overline{B}$, so $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid \overline{B}) > 0$. By the same Lemma, we also know that $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid \underline{B}) < 0$. Monotonicity and continuity of $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid b_i^\varepsilon)$ in b_i then imply there exists a point R^1 such that $\underline{B} < R^1 < \overline{B}$ which solves:

$$\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid R^1) = 0. \tag{7}$$

To any player i, the action $x_i = 1$ is dominant at all $b_i^\varepsilon > R^1$ conditional on $\overline{B}$. Given this, player i can find a point R^1 such that it is expected payoff maximizing for player i to play $x_i = 1$ at all $b_i^\varepsilon > R^1$. Knowing that $x_j = 1$ is dominant at all $b_j^\varepsilon > R^1$, yet additional strategies may be eliminated as conditionally dominated for player i.

This argument can be repeated and we obtain a sequence $\overline{B} = R^0, R^1, R^2, ...$ For any $k \geq 0$ and R^k such that $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^k) > 0$, there exists a $R^{k+1} < R^K$ such that $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^{k+1}) = 0$. Induction on k allows for the conclusion that (R^k) is a monotone sequence. Moreover, we also know that $R^k \geq \underline{B}$ for all $k \geq 0$ since $x_i = 0$ is dominant at $b_i^\varepsilon < \underline{B}$. Any bounded monotone sequence must converge. I let R^* denote the limit of sequence (R^k). By the definition of a limit, R^* must satisfy:

$$\Delta_i^\varepsilon(p_{-i}^{R^*} \mid R^*) = 0. \tag{8}$$

It follows that a strategy p_i survives iterated elimination of strictly dominated strategies only if $p_i(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon > R^*$, all i.

Iteration from the left. Iterative elimination of strictly dominated strategies yields the point R^* when starting from the right, that is, a range of signals b_i^ε for which $x_i = 1$ is conditionally and strictly dominant. A similar procedure can be executed starting instead from the left, from signals b_i^ε for which $x_i = 0$ is unconditionally and strictly dominant. Since this analysis is symmetric to the procedure discussed above, I will only provide the key steps of the analysis.

From Lemma 1 it is known that $x_i = 0$ is strictly dominant at $b_i^\varepsilon < \underline{B}$. That is, $\Delta_i^\varepsilon(p^{-\infty} \mid \underline{B}) < 0$. Since it is common knowledge that no player plays a strictly dominated strategy, a payoff maximizing player i then finds a point L^1 such that $x_i = 0$ is dominant $b_i^\varepsilon < L^1$ conditional on $\underline{B}$:

$$\Delta_i^\varepsilon(p_{-i}^{\underline{B}} \mid L^1) = 0. \tag{9}$$

Any expected payoff maximizing player i plays $x_i = 0$ for all $b_i^\varepsilon < L^1$. Since this is common knowledge also, we can repeat the argument over and over. What we obtain is a sequence of points (L^k), $k \geq 0$, each term of which is implicitly defined by:

$$\Delta_i^\varepsilon(p_{-i}^{L^k} \mid L^{k+1}) = 0. \tag{10}$$

The sequence (L^k) is monotone increasing. It is also bound from above by $\overline{B}$ (or, taking account of (8), by R^*). It must therefore converge, and I call its limit L^*. By construction this limit solves:

$$\Delta_i^\varepsilon(p_{-i}^{L^*} \mid L^*) = 0. \tag{11}$$

It follows that a strategy p_i survives iterated elimination of strictly dominated strategies only if $p_i(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < L^*$, all i.

Lemma 2. *(i) If a strategy p_i survives iterated elimination of strictly dominated strategies, then it must hold that $p_i(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon > R^*$. (ii) If a strategy p_i survives iterated elimination of strictly dominated strategies, then it must hold that $p_i(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < L^*$.*

Proof. Follows immediately from the argument leading up to the Lemma. $\square$

I have derived two limits L^* and R^* that demarcate iterative dominance regions of the signal space. If p is a Bayesian Nash equilibrium of the game G^ε, it must hold that $p_i(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < L^*$ and $p_i(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon > R^*$. I will now show that this observation implies that the game G^ε has a unique equilibrium – in particular, the points L^* and R^* must coincide. To prove this, the following result is key.

Proposition 2. *For all X such that $\underline{B} + \varepsilon \le X \le \overline{B} - \varepsilon$, the following holds:*

$$\Delta_i^\varepsilon(p_{-i}^X \mid X) = X - \sum_{m=0}^{N-1} \frac{c(m+1)}{N} + d. \tag{12}$$

It follows that $\Delta_i^\varepsilon(p_{-i}^X \mid X)$ is strictly increasing in X for all X such that $\underline{B} + \varepsilon \le X \le \overline{B} - \varepsilon$.

Proof. First fix $b \in [\underline{B} + \varepsilon, \overline{B} - \varepsilon]$. Each player $j \neq i$ is assumed to play p_j^X, so the probability that $x_j = 1$ is given by

$$\Pr[b_j^\varepsilon > X \mid b] = \Pr[\varepsilon_j > X - b] = \frac{\varepsilon - X + b}{2\varepsilon}, \tag{13}$$

while $x_j = 0$ with the complementary probability. Since each ε_j is (conditional on b) drawn independently , the probability that m given players $j \neq i$ play $x_j = 1$ while the remaining $N - m - 1$ players play $x_j = 0$ (given p_{-i}^X and b) is therefore:

$$\left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1}. \tag{14}$$

As there are $\binom{N-1}{m}$ unique ways in which m out of $N-1$ players j can choose $x_j = 1$, the total probability of this happening, as a function of b, is:

$$\binom{N-1}{m} \left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1}. \tag{15}$$

When m players $j \neq i$ play $x_j = 1$, the cost of playing $x_i = 1$ to player i is $c(m+1)$.

The derivation so far took b as a known quantity. We now take account of the fact that player i does not observe b directly but only a noisy signal b_i^ε. Given $p_{-i} = p_{-i}^X$ and $b_i^\varepsilon = X$, the expected gain to player i from playing $x_i = 1$ rather than $x_i = 0$ becomes:

$$
\begin{aligned}
\Delta_i^\varepsilon(p_{-i}^X \mid X) =& \frac{1}{2\varepsilon}\int\limits_{X-\varepsilon}^{X+\varepsilon} b db + d \\
& - \sum_{m=0}^{N-1} c(m+1)\binom{N-1}{m}\frac{1}{2\varepsilon}\int\limits_{X-\varepsilon}^{X+\varepsilon}\left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1} db
\end{aligned} \tag{16}
$$

$$
=X + d - \sum_{m=0}^{N-1} c(m+1)\binom{N-1}{m}\int\limits_0^1 q^m(1-q)^{N-m-1}dq \tag{17}
$$

$$
=X + d - \sum_{m=0}^{N-1} c(m+1)\frac{(N-1)!}{m!\,(N-m-1)!}\frac{m!\,(N-m-1)!}{N!} \tag{18}
$$

$$
=X + d - \sum_{m=0}^{N-1} \frac{c(m+1)}{N}, \tag{19}
$$

as given. Equation (16) takes the expression for $\Delta_i(m_i \mid b)$ given in (2) and integrates over b and m_i, given $b_i^\varepsilon = X$ and $p_{-i} = p_{-i}^X$. Equation (17) relies on integration by substitution (using $q = 1/2 - (X-b)/2\varepsilon$) to rewrite the last integral in (16). Equation (18) rewrites both the integral in (17) and the the binomial coefficient $\binom{N-1}{m}$ in terms of factorials. Equation (19) simplifies. $\square$

Note that the right hand side of (12) is strictly continuously increasing in X for all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$. Combined with Lemma 1, this implies that there exists a unique $B^* \in (\underline{B} + \varepsilon, \overline{B} - \varepsilon)$ such that $\Delta_i^\varepsilon(p_{-i}^{B^*} \mid B^*) = 0$. Moreover, observe that these results hold for general $\varepsilon > 0$, not only the limit case as $\varepsilon \to 0$.

Proposition 3. *For all ε such that $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$, there exists a unique strategy vector p^* surviving iterated elimination of strictly dominated strategies in the global game G^ε. In particular, there exists a unique B^* such that $p^* = p^{B^*}$. The point B^* is given by:*

$$
B^* = \sum_{n=1}^{N} \frac{c(n)}{N} - d. \tag{20}
$$

Proof. From Lemma 2 we know that $x_i = 0$ is iteratively dominant at $b_i < L^*$ while $x_i = 1$ is iteratively dominant at $b_i > R^*$. By construction, the points L^* and R^* solve

$\Delta_i^\varepsilon(p_{-i}^{L^*} \mid L^*) = \Delta_i^\varepsilon(p_{-i}^{R^*} \mid R^*) = 0$. Proposition 2 implies that $L^* = R^* = B^*$. The result follows. $\square$

I derive Proposition 3 for general $\varepsilon > 0$ provided the assumption that b and ε_i (all i) are drawn independently from the uniform distribution (which is symmetric for ε_i). For the limit as $\varepsilon \to 0$, Frankel et al. (2003) establish the very general result that *any* global game with strategic complementarities in which b is drawn from any continuous density with connected support and each ε_i is drawn independently from any (possible player-specific) atomless density has an essentially unique strategy vector surviving iterated elimination of strictly dominated strategies in the limit as $\varepsilon \to 0$. Moreover, for potential games the equilibrium selected is noise independent, meaning that (in the limit) the strategy vector found in Proposition 3 generalizes to far more general distributions than assumed herein.[5]

Proposition 3 should not be misunderstood as saying that players will perfectly coordinate their actions (investments).[6] For $\varepsilon > 0$, it is possible that some players receive signals above B^* while others see a signal below it. When this occurs, players will fail to coordinate their actions (i.e. some will choose $x_i = 1$ while others choose $x_i = 0$). An equilibrium is defined in strategies. The proposition says that there exists a unique strategy surviving iterated dominance for each player. But a strategy only yields an action once we feed it a signal. The implication is that empirically observed coordination failures are not necessarily at odds with, and therefore do not by themselves invalidate the model. When a coordination failure occurs, the equilibrium outcome is inefficient.

Corollary 1. *Let p^ε be the unique Bayesian Nash equilibrium of G^ε. (i) For all $b > B^* + \varepsilon$ it holds that* $\Pr[p^\varepsilon(b^\varepsilon) = \mathbf{1}] = 1$. *(ii) For all $b < B^* - \varepsilon$ it holds that* $\Pr[p^\varepsilon(b^\varepsilon) = \mathbf{0}] = 1$.

In the limit as $\varepsilon \to 0$, the global climate game G^ε selects a unique equilibrium of the underlying coordination game with multiple equilibria. To see this, note that for any $b > B^*$, we can find a $\varepsilon < B^* - b$ so that $b - \varepsilon > B^*$. Since $b_i^\varepsilon \in [b - \varepsilon, b + \varepsilon]$ and $p^\varepsilon = p^{B^*}$ this implies that $p_i^\varepsilon(b_i^\varepsilon) = 1$ for all b_i^ε consistent with b and all i.

Even as $\varepsilon \to 0$ and players coordinate their actions with probability 1, the unique equilibrium can be inefficient. In particular, players coordinate on $\mathbf{0}$ (all adopt the dirty

[5] In particular, the reader is referred to their result on (local) potential games with own-action quasi-concave payoffs, i.e. Theorem 4.

[6] Perfect coordination of actions means that all players choose the same action.

technology) for all $b < B^*$ even though the outcome in which players coordinate on $\mathbf{1}$ (all adopt the clean technology) is Pareto dominant for all $b > \bar{b}$ (and even though they know it). Thus, for all b such that $\bar{b} < b < B^*$ coordination is on the Pareto dominated outcome of the underlying complete information game. Intuitively, clean investment will be too risky when b is low since the noise in signals forces a player to believe that others may think that clean investment is dominated. Barrett and Dannenberg (2012) appear to share this view when they write that "players could use risk-dominance as a selection rule." For 2×2 games, their statement is backed by the theoretical literature: Carlsson and Van Damme (1993) prove that any 2×2 global game selects the risk dominant equilibrium of the underlying true game.[7] In games with more players or actions, the statement is not generally correct. The result stands in contrast to the common and often implicit assumption in the environmental literature that players generally coordinate on the efficient equilibrium (Barrett, 2006; Hoel and de Zeeuw, 2010).

4 Network Subsidies

The potential of an inefficient outcome in both the game of complete information $G(b)$ and the global game G^{ε} begs the question how a policymaker can influence the game in order to reach an efficient outcome. In this section, we will study the policymaker's problem of finding a way to influence players' incentives so as to implement the Pareto efficient outcome of the game in strictly dominant strategies against the lowest cost. That is, we seek to find policies that turn playing $x_i = 1$ into a dominant strategy whenever coordination on $\mathbf{1}$ is also the efficient outcome of the game; similarly, we want $x_i = 0$ to be a dominant strategy when coordination on $\mathbf{0}$ is Pareto efficient.[8] While in the most general setup, the policymaker has a vast array of possible policies at its disposal (including outright command-and-control), to stay close with the application to climate change we shall confine the set of feasible policies to subsidies and taxes only.

Taxes and subsidies will stimulate adoption of the clean technology whenever they cause an effective decrease of $c(m) - d$ for at least one m. The U.S. Federal Tax Credit

[7]For a definition of risk dominance, see Harsanyi and Selten (1988).

[8]This question is related to the literature on mechanism design and (dominant strategy) implementation. That is, we study the problem of a policymaker who seeks to change the original game studied in Section 2 and 3 with the aim of making coordination on the efficient outcome of the game a dominant strategy for all players (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983; Kuzmics and Steg, 2017).

for Solar Photovoltaics (Borenstein, 2017), California's Clean Vehicle Rebate Project (Li et al., 2017), or the U.S. National Plug-In Electric Drive Vehicle Credit (Clinton and Steinberg, 2019) are good illustrations. However, tax policies may not always be feasible, political or otherwise. For example, legislation on taxation requires unanimous agreement in the European Union, which is one reason the EU does not have a carbon tax. Subsidies and tax credits, on the other hand, come with a substantial budgetary burden. The Congressional Budget Office expects total cost from tax credits on electric vehicles to be about 7.5 billion U.S. dollars through 2019.[9] Planned spending on SDE++ subsidies in the Netherlands are 5 billion Euros in 2021.[10].

The expenditure on subsidies to stimulate clean technology adoption can be substantially mitigated without the need to compromise on efficiency through the use of *network subsidies*. A network subsidy, like any subsidy, is offered contingent on adoption of the clean technology. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number.

The strongest results obtain in the game of complete information and the global game with vanishing idiosyncratic noise, which I shall discuss in turn.

4.1 Game of Complete Information

Consider again the game of complete information (about b) discussed in Section 2. Recall from Proposition 1 that coordination on $x = \mathbf{1}$ is the Pareto dominant outcome of the game for all $b > \bar{b}$, whereas coordination on $x = \mathbf{0}$ is efficient for all $b < \bar{b}$.

My aim is to find a subsidy that incentivizes players to coordinate on the efficient outcome of the game for any b. Concretely, I want to formulate a tax and/or subsidy policy that makes $x_i = 1$ dominant for all $b > \bar{b}$ while $x_i = 0$ becomes dominant at $b < \bar{b}$. I say that such a subsidy *implements* the efficient outcome of the game in dominant strategies for almost all b.

First I will show that if $\mathbf{1}$ is a (strict) Nash equilibrium of $G(b)$, then the efficient outcome of the game can be implemented in (strictly) dominant strategies at zero cost, even if $\mathbf{0}$ is also a strict Nash equilibrium. The idea will be to offer players choosing

[9]See https://www.cbo.gov/sites/default/files/112th-congress-2011-2012/reports/electricvehiclesone-col.pdf

[10]See https://www.rvo.nl/subsidie-en-financieringswijzer/sde. SDE is an acronym for Stimulering Duurzame Energievoorziening en Klimaattransitie, or "Stimulus Sustainable Energy Supply and Climate Transition".

$x_i = 1$ a subsidy that guarantees them a payoff equal (when choosing 1) to what they would have realized in the hypothetical case that all other players also chose 1 – that is, to offer a subsidy that promises players a payoff as though they enjoyed the full extent of the network externality. To this end, let the policymaker offer a *network subsidy* $s^*(x)$ to each i choosing $x_i = 1$ when x is played. For each x, define $s^*(x)$ to be the function given by:

$$s^*(x) = \Delta_i(\mathbf{1} \mid b) - \Delta_i(x \mid b) = c(m(x)) - c(N). \tag{21}$$

Players choosing $x_i = 0$ do not receive a network subsidy. Observe that, conditional on $s^*(\cdot)$, an individual players' gain from playing 1, rather than 0, is:

$$\Delta_i(x \mid b) + s^*(x) = \Delta_i(x \mid b) + \Delta_i(\mathbf{1} \mid b) - \Delta_i(x \mid b) = \Delta_i(\mathbf{1} \mid b), \tag{22}$$

for any x, confirming the claim that a network subsidy scheme $s^*(\cdot)$ allows players to consider only the gain $\Delta_i(\mathbf{1} \mid b) = b - c(N) + d$ when choosing their actions. Let $G(b \mid s^*)$ denote the game $G(b)$ in which players are offered the network subsidy $s^*(\cdot)$ on playing 1.

Proposition 4. *Consider the game $G(b \mid s^*)$.*

(i) If $\mathbf{1}$ is a Nash equilibrium of the game (i.e. if $b+d \geq c(N)$), then $\mathbf{1}$ is implemented in weakly dominant strategies with $s^*(\cdot)$ and no subsidies have to be paid.

(ii) If $\mathbf{1}$ is a strict Nash equilibrium of the game (i.e. if $b + d > c(N)$), then $\mathbf{1}$ is implemented in strictly dominant strategies with $s^*(\cdot)$ and no subsidies have to be paid.

Proof. The gain from choosing $x_i = 1$ rather than $x_i = 0$, conditional on the network subsidy scheme $s^*(\cdot)$, given b and x_{-i} is (22) which, for all x_{-i}, is (strictly) positive if and only if $\mathbf{1}$ is a (strict) Nash equilibrium of the game. Thus, the offering a subsidy scheme equal to $s^*(\cdot)$ turns $x_i = 1$ into a (strictly) dominant strategy whenever $\mathbf{1}$ is a (strict) Nash equilibrium of $G(b)$. When players coordinate on $\mathbf{1}$ total spending on network subsidies is $N \cdot s^*(\mathbf{1}) = 0$. □

Note that for all $b > c(N) - d$, and provided the network subsidy scheme $s^*(\cdot)$ is offered, the policymaker may even tax playing $x_i = 1$ yet still implement $\mathbf{1}$ in strictly dominant strategies.

Remark 1. *Let $b > c(N) - d$, so $\mathbf{1}$ is both a strict Nash equilibrium and the efficient outcome of the game $G(b)$. If the policymaker offers the network subsidy scheme $s^*(\cdot)$, the policymaker can impose a tax $t(b) \leq b + d - c(N)$ on playing $x_i = 1$ but nevertheless implement coordination on $\mathbf{1}$ in strictly dominant strategies.*

Proposition 4 tells us that a smart policy of network subsidies allows the policymaker costlessly to implement the efficient Nash equilibrium of $G(b)$ in (strictly) dominant strategies if the game has multiple (strict) Nash equilibria. While this is a desirable property, it does not guarantee that a network subsidy scheme implements the efficient outcome of the game for all b. To see this, observe that $\mathbf{0}$ is the unique strict Nash equilibrium of $G(b)$ for all $b < c(N) - d$, while $\mathbf{1}$ is the efficient outcome for all $b > \bar{b} = (c(N) - d)/N$. Hence, if $c(N) > d$ the policymaker cannot implement the efficient outcome of the game for all $b \in (\bar{b}, c(N) - d)$ using a network subsidy scheme alone.

I next show that if $\mathbf{1}$ is not a Nash equilibrium of the game $G(b)$, but $\mathbf{1}$ is the efficient outcome, then in order to implement $\mathbf{1}$ in strictly dominant strategies, the policymaker can use a dual tax-subsidy scheme to achieve its goal. First, let the policymaker again offer the network subsidy scheme given by $s^*(\cdot)$. As we saw before, the net (accounting for subsidies) gain from playing 1 rather than 0 becomes $\Delta_i(x \mid b) + s^*(x) = \Delta_i(\mathbf{1} \mid b)$ when players are offered $s^*(\cdot)$. Second, let the policymaker levy an *environmental tax* $t(b)$ to playing 0. The purpose of the environmental tax is to make sure that $\Delta_i(\mathbf{1} \mid b) + t(b) > 0$ for all $b > \bar{b}$ while $\Delta_i(\mathbf{1} \mid b) + t(b) < 0$ for all $b < \bar{b}$; that is, the tax should make $\mathbf{1}$ a Nash equilibrium of the game if and only if $\mathbf{1}$ is also the efficient outcome. A tax $t(b)$ that achieves this is given by:

$$t(b) > \Delta(\mathbf{1} \mid c(N) - d) - \Delta(\mathbf{1} \mid c(N) - d) = c(N) - d - b \quad \text{if} \quad b \geq \bar{b}, \tag{23}$$

while $t(b) = 0$ otherwise. It is easy to verify that $t(b)$ implements coordination on $\mathbf{1}$ as a strict Nash equilibrium for all $b > \bar{b}$ while leaving $x_i = 0$ dominant for all $b < \bar{b}$.

Proposition 5. *Consider the game $G(b \mid s^*, t)$. If $\mathbf{1}$ is not a Nash equilibrium of the game (i.e. if $b < c(N) - d$), but $\mathbf{1}$ is the Pareto efficient outcome (i.e. $b > \bar{b}$), then, by taxing $x_i = 0$ through $t(b)$ while also offering a network subsidy $s^*(\cdot)$ to playing $x_i = 1$, $\mathbf{1}$ can be implemented in strictly dominant strategies at no cost (and tax revenues will be zero).*

As we discussed in the introduction to this section, for various reasons governments across the globe may at times be reluctant to rely on (carbon) taxes when trying to curb private sector emissions. When this is true, the government cannot (or at least does not want to) levy the carbon tax $t(b)$ but may rather rely on an environmental *subsidy* $s(b)$ on playing 1.

Remark 2. *If* $\mathbf{1}$ *is not a Nash equilibrium of the game (i.e. if* $b < c(N) - d$*), but* $\mathbf{1}$ *is the Pareto efficient outcome, then, by subsidizing* $x_i = 1$ *through* $s(b) = c(N) - d - b$ *for* $b \geq \bar{b}$ *and* $s(b) = 0$ *otherwise while also offering a network subsidy* $s(\cdot)$ *to playing* $x_i = 1$*,* $\mathbf{1}$ *can be implemented in strictly dominant strategies. Total subsidy spending will be* $N \cdot s(b)$*.*

Why does a network subsidy work so well despite the low cost? The key property of a network subsidy set at $s^*(\cdot)$ is that it eliminates all *strategic uncertainty*, i.e. the uncertainty a player has about the actions chosen by all other players. The network subsidy thus removes the payoff uncertainty deriving from strategic uncertainty interacted with technological spillovers – it turns the original coordination game into a simple dominance solvable game for all b. In so doing, the network subsidy manages to eliminate all inefficiencies caused by players' failure to internalize the *technological* spillovers inherent in clean investments. Intuitively, the network subsidy works like an insurance. It protects individual investors against the risk of small network externalities from clean investments in case many others have adopted the dirty technology. In so doing, it impels individuals toward clean investments. The network subsidy does not have to be paid as a result, being conditional on low investments by construction.

4.2 Global Game

Consider the global game G^ε discussed in Section 3. In this game, players do not observe b. While I have so far not made any assumptions on what the policymaker knows, I will henceforth assume that the policymaker observes neither the true b nor a signal of it.

In this section I address the question of what tax-subsidy scheme suffices (at minimal cost) to implement the Pareto efficient outcome of the underlying game $G(b)$ in strictly dominant strategies for all b. However, since Pareto dominance depends on the unobserved true b, I will assume the policymaker seeks policies that turn $x_i = 1$ into a dominant action for all $b_i^\varepsilon > \bar{b} + \varepsilon$ while leaving $x_i = 0$ dominant for all $b_i^\varepsilon < \bar{b} - \varepsilon$. I will also assume that the policy scheme does not depend on the unobserved true b.[11].

[11]This evidently is a restrictive assumption. Players possess private information (their signals)

First, let us again assume the policymaker offers each player a network subsidy s^* equal to:

$$s^*(x) = c(m(x)) - c(N), \tag{24}$$

which is the same network subsidy as in (21). It is easy to verify that the network subsidy $s^*(\cdot)$ makes playing 1 dominant for all $b_i^\varepsilon > c(N) - d + \varepsilon$. When players are offered $s^*(\cdot)$, their expected gain (the expectation is over b) is:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon), \tag{25}$$

where $\Delta_i^\varepsilon(x \mid b) := \frac{1}{2\varepsilon}\int_{b_i^\varepsilon-\varepsilon}^{b_i^\varepsilon+\varepsilon} \Delta(x \mid b)db$. Note that $\Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon)$ is strictly positive for all $b_i^\varepsilon > c(N) - d + \varepsilon$ and strictly negative for all $b_i^\varepsilon < c(N) - d - \varepsilon$.

Lemma 3. *Consider the global game G^ε. Let the policymaker offers a network subsidy $s^*(\cdot)$ on playing 1. Then the action $x_i = 0$ is strictly dominant at $b_i^\varepsilon < c(N) - d - \varepsilon$; the action $x_i = 1$ is strictly dominant at $b_i^\varepsilon > c(N) - d + \varepsilon$.*

As in the case of complete information, a network subsidy alone may not suffice to implement the efficient outcome of the game; for all $b \in (\bar{b}, c(N) - d - 2\varepsilon)$, each player i receives a signal $b_i^\varepsilon < c(N) - d - \varepsilon$ so playing 0 is dominant despite the network subsidy. Therefore, let the policymaker – on top of the network subsidy – seek an environmental tax $\bar{t}$ on playing $x_i = 0$ that makes $x_i = 1$ a best response to $x_{-i} = \mathbf{1}$ for all $b_i^\varepsilon > \bar{b} + \varepsilon$ (and recall again the restrictive assumption that $\bar{t}$ does not depend on players' private knowledge of b). This $\bar{t}$ is bound from below by:

$$\Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t} > 0 \quad \text{for all} \quad b_i^\varepsilon > \bar{b} + \varepsilon \quad \implies \quad \bar{t} \geq (N-1) \cdot \bar{b} - \varepsilon. \tag{26}$$

Moreover, the policymaker does not want that playing 1 becomes attractive even when $b_i^\varepsilon < \bar{b} - \varepsilon$. Thus, $\bar{t}$ should also satisfy:

$$\Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t} < 0 \quad \text{for all} \quad b_i^\varepsilon < \bar{b} - \varepsilon \quad \implies \quad \bar{t} \leq (N-1) \cdot \bar{b} + \varepsilon. \tag{27}$$

Clearly, when the tax $\bar{t}$ on playing 0 is levied while, also, the network subsidy $s^*(\cdot)$ is

about b and this information is correlated. We thus know from the literature on mechanism design that the policymaker can (costlessly) extract the full vector of signals $b^\varepsilon = (b_1^\varepsilon, b_2^\varepsilon, ..., b_N^\varepsilon)$ from the players (Crémer and McLean, 1988; McAfee and Reny, 1992). Especially when ε is small, knowing b^ε would provide an almost perfect signal of the true b to the policymaker. It seems intuitive that the policymaker might use this knowledge to its benefit (and the benefit of all players as a whole). I hope to investigate this issue in future work.

offered to those playing 1, the gain to playing 1 for player i becomes:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + \bar{t} + s^*(x) = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t}, \tag{28}$$

for all x_{-i}. Observe that, by construction, the gain $\Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t}$ is strictly positive at $b_i^\varepsilon > \bar{b} + \varepsilon$ and strictly negative at $b_i^\varepsilon < \bar{b} - \varepsilon$, which holds for all i and x_{-i}. The following is therefore immediate.

Proposition 6. *Consider the global game G^ε. If the policymaker offers a network subsidy $s^*(\cdot)$ on playing $x_i = 1$ and levies a tax $\bar{t}$ on playing $x_i = 0$, then the action $x_i = 0$ is strictly dominant for all $b_i^\varepsilon < \bar{b} - \varepsilon$ and the action $x_i = 1$ is strictly dominant for all $b_i^\varepsilon > \bar{b} + \varepsilon$. Hence, for all $b \notin [\bar{b} - 2\varepsilon, \bar{b} + 2\varepsilon]$ the policymaker can implement the efficient outcome of the game $G(b)$ in dominant actions at no cost.*

Proof. The statements concerning strict dominance are derived in the main text and therefore not proven here. As to the final claim in the Proposition, observe that each b_i is drawn from $[b - \varepsilon, b + \varepsilon]$, given b. Hence, if $b > \bar{b} + 2\varepsilon$ then $b_i^\varepsilon > \bar{b} + \varepsilon$ for each i, so playing $x_i = 1$ is dominant and players coordinate on $\mathbf{1}$, the efficient outcome of the game (for those b). In this case, total spending on subsidies is $s^*(\mathbf{1}) = 0$. Similarly, if $b < \bar{b} - 2\varepsilon$ then $b_i^\varepsilon < \bar{b} - \varepsilon$ for each i, so playing $x_i = 0$ is dominant and players coordinate on $\mathbf{0}$, the efficient outcome of the game (for those b). Since no player plays 1, total subsidy spending is naturally zero. □

If the policymaker, for whatever reason, is reluctant to tax playing 0, it may also offer both a network subsidy $s^*(\cdot)$ together with an environmental *subsidy* equal to $\bar{t}$ to playing 1. Such a policy is evidently equivalent with regard to players' incentives, although it differs for the policymaker's budget.

Corollary 2. *Consider the global game G^ε. Let the policymaker offer a network subsidy $s^*(\cdot)$ on playing 1. In addition, let the policymaker offer an environmental subsidy equal to $\bar{t}$ on playing 1. Then the action $x_i = 0$ is dominant for all $b_i^\varepsilon < \bar{b} - \varepsilon$; the action $x_i = 1$ is dominant for all $b_i^\varepsilon > \bar{b} + \varepsilon$. Hence, the policymaker can implement the efficient outcome of the game $G(b)$ for almost all b; total subsidy spending is $N \cdot \bar{b}$ if $b > \bar{b} + 2\varepsilon$ and 0 if $b < \bar{b} - 2\varepsilon$.*

If a true b in $(\bar{b} - 2\varepsilon, \bar{b} + 2\varepsilon)$ is drawn, players may fail to coordinate on either $\mathbf{0}$ or $\mathbf{1}$ even when the policymaker offers the network subsidy $s^*(\cdot)$ and levies the tax $\bar{t}$.

The reason is that, for those b, players' signals need not all fall in the strict dominance regions identified in Proposition 6, and a coordination failure may easily arise. The network subsidy scheme $s^*(\cdot)$ may hence not be costless; for any x not equal to $\mathbf{0}$ or $\mathbf{1}$, total spending on network subsidies will be $m(x) \cdot s^*(x) > 0$. Thus, the remarkably strong performance of a network subsidy scheme may break down in the global game. For $\varepsilon > 0$, the event that a true b in $(\bar{b} - 2\varepsilon, \bar{b} + 2\varepsilon)$ is drawn occurs with strictly positive probability, namely $4\varepsilon/(\overline{B} - \underline{B}) > 0$. Only in the limit as $\varepsilon \to 0$ will this problem be almost non-existent: players perfectly coordinate their actions (in equilibrium) save for the probability-zero event that $b = \bar{b}$.

Corollary 3. *Consider the global game G^ε. Let the policymaker offer a network subsidy scheme $s^*(\cdot)$ and levy an environmental tax $\bar{t}$. (i) For all $b > \bar{b}$ it holds that* $\lim_{\varepsilon \to 0} \Pr[x = \mathbf{1} \mid b] = 1$ *if each x_i in x is the dominant action for player i given b_i^ε. (ii) For all $b < \bar{b}$ it holds that* $\lim_{\varepsilon \to 0} \Pr[x = \mathbf{0} \mid b] = 1$ *if each x_i in x is the dominant action for player i given b_i^ε. Consequently, spending on network subsidies is zero for almost all b in the limit as $\varepsilon \to 0$.*

The fact that spending on network subsidies may not be zero in the global game is unfortunate, for it implies that the policymaker's policymaking may not be without cost. To remedy this problem, I now derive a network tax-subsidy scheme where subsidy payments on $x_i = 1$ are financed through a "network tax" levied on choosing $x_i = 0$. Let the subsidy be denoted $s^{**}(x)$; the corresponding tax is denoted $t^{**}(x)$. Thus, when x is played, aggregate spending on network subsidies is $m(x) \cdot s^{**}(x)$; aggregate revenues from the network tax are $(N - m(x)) \cdot t^{**}(x)$. If we want the tax-subsidy scheme to be costless, or self-financed, the budget constraint for this scheme is given by:

$$(N - m(x)) \cdot t^{**}(x) - m(x) \cdot s^{**}(x) = 0, \tag{29}$$

which should hold for all x. Condition (29) imposes that total spending on the network subsidies to those playing 1 is matched exactly by total tax revenues from taxing those who play 0, whatever players end up playing.

Next, the tax-subsidy scheme, together with the environmental tax $\bar{t}$ bounded by (26) and (27), must make $x_i = 1$ strictly dominant for all $b_i^\varepsilon > \bar{b} + \varepsilon$ while leaving $x_i = 0$ dominant for all $b_i^\varepsilon < \bar{b} - \varepsilon$. Thus players' gains from playing 1, rather than 0, accounting for taxes and subsidies, should satisfy:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + \bar{t} + s^{**}(x) + t^{**}(x) > 0 \quad \text{for all} \quad b_i^\varepsilon > \bar{b} + \varepsilon, \tag{30}$$

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + \bar{t} + s^{**}(x) + t^{**}(x) < 0 \quad \text{for all} \quad b_i^\varepsilon < \bar{b} - \varepsilon, \tag{31}$$

for all x_{-i}. Equations (30) and (31) represent the incentive constraints of a network tax-subsidy scheme. Combined with the budget constraint, this yields the following network tax-subsidy scheme (s^{**}, t^{**}):

$$\begin{cases} t^{**}(x) = & \frac{m(x)}{N}\left[c(m(x)) - c(N)\right] \\ s^{**}(x) = & \frac{N-m(x)}{N}\left[c(m(x)) - c(N)\right] \end{cases} \tag{32}$$

Summarizing the policy scheme $(\bar{t}, (s^{**}, t^{**}))$, when x is played and player i has played 1 in x, s/he receives a network subsidy equal to $s^{**}(x)$; however, if player i played 0 in x, s/he pays a tax equal to $\bar{t} + t^{**}(x)$.

Proposition 7. *Consider the global game G^ε. Let the policymaker offer a network subsidy equal to $s^{**}(\cdot)$ on playing 1 while levying a tax equal to $\bar{t} + t^{**}(\cdot)$ on playing 0. This policy makes the action $x_i = 0$ dominant for all $b_i^\varepsilon < \bar{b} - \varepsilon$; the action $x_i = 1$ is dominant for all $b_i^\varepsilon > \bar{b} + \varepsilon$. Spending on the network tax-subsidy scheme (s^{**}, t^{**}) is zero for all b.*

Proof. All parts of the propositions follow immediately from the construction preceding it. □

Remark 3. *The network tax-subsidy scheme $(s^{**}(\cdot), t^{**}(\cdot))$ can also substitute for the pure network subsidy $s^*(\cdot)$ discussed in the game of complete information. This substitution would not affect players' incentives; nor would it have an effect on total subsidy spending, as coordination on either $\mathbf{0}$ or $\mathbf{1}$ is achieved for all b save $b = \bar{b}$ so offering $s^*(\cdot)$ was free of cost anyway.*

The present analysis did not make use of the fact that, without policy interventions, playing p^{B^*} is the essentially unique strategy profile surviving iterated dominance in the global game G^ε. Another approach toward ((iterative) dominant strategy) implementation in G^ε would be to study what mechanisms the policymaker could design to shift the threshold B^* down toward $\bar{b}$. I intend to do this in future work.

Finally, observe that the logic of a network subsidy does not rely on the application to climate change. Any market where (i) individual actions exhibit strategic complementarities and (ii) players do not take these into account may coordinate on an inefficient equilibrium. A network subsidy then offers an inexpensive way out of this trap. The observation suggests that network subsidies are worth studying in economics more generally.

5 Summary

This paper studies climate change mitigation in a global game. The focus is on abatement through technological investment. Players invest in either of two technologies. One technology is cheap and dirty, the other expensive but clean. I consider environments in which investments are strategic complements. These could for example arise due to network effects, technological spillovers, or learning-by-doing. Consistent with the existing literature on technological investments in clean technologies, I demonstrate that the complete information version of my game has multiple equilibria (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017; Mielke and Steudle, 2018). Equilibrium multiplicity can lead to coordination failure and complicates the design of domestic policies or climate treaties.

To this well-studied framework, I add a little bit of uncertainty. I assume that the true environmental benefit of the clean technology is unobserved. Rather than observe the technology's true benefit, players receive private noisy signals of it. In this *global climate game*, I show that there exists a unique equilibrium in which players adopt the clean technology if and only if their private signals exceed an endogenous threshold. For signals below the threshold, players adopt the dirty technology instead.

My first contribution is to show that the game has a unique Bayesian Nah equilibrium. This contribution directly addresses the issue of equilibrium multiplicity often encountered in this literature. Equilibrium selection is driven by the assumption of incomplete information, which seems realistic in the context of clean technologies and climate change. The analysis highlights that the precise way in which one models uncertainty is important. Although some papers conclude that "shared" uncertainty does not eliminate equilibrium multiplicity in coordination games (Barrett and Dannenberg, 2012; Barrett, 2013), this paper shows the starkly contrasting result that privately held beliefs about the shared game does force selection of a unique equilibrium. The assumed structure of uncertainty matters. My result motivates a rethink of the way environmental economists model uncertainty.

My second contribution is to introduce network subsidies. The issue of taxes and subsidies, or rather policy in general, arises naturally in the present context where investments cause two externalities. One is an environmental externality that derives from the positive environmental effect clean investments have on all players; the other is a network externality deriving from spillovers (strategic complementarities) in clean investments. As I show, using only taxes and subsidies the policymaker can costlessly correct both of these externalities by turning the (expected) efficient action into a dominant action for all players (both in the game of complete information and in the global game). Moreover, I also show that even when taxes (for political or other reasons) are not possible or desirable, the policymaker can nevertheless correct the entire network externality at zero cost. The key novelty in this is what I call a network subsidy. Like standard subsidies, a network subsidy offers adopters of the clean technology a (financial) reward. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number. As I show, it is possible to construct a simple network subsidy scheme that corrects the entire network externality deriving from players' ignorance of the technological spillovers caused by their clean investment but does not, in equilibrium, cost the policymaker anything. The core property of a network subsidy driving this is the fact that the amount of subsidy paid to each individual investor is a function of aggregate clean investments. Since adoption of the clean technology is more attractive when the number of other players adopting it is higher due to the technological spillover, it is quite intuitive that the network subsidy can decrease in aggregate clean technology adoption without negatively affecting players' incentives. My result is to show that this intuition can be exploited smartly: the policymaker can offer a network subsidy scheme such that what is paid when *all* players adopt the clean technology is zero without averse affecting players' incentives. Intuitively, the network subsidy serves as an insurance against small clean technology networks. In so doing, it boosts clean investments and therefore is never claimed. Although derived in the context of technological spillovers, the notion of a network subsidy is general and applies to public economic broadly.

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