Linking Cap And Trade Schemes Under Uncertainty

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Abstract

Recent years have witnessed a rapid increase in the number of cap and trade schemes to mitigate greenhouse gas emissions. With multiple coexisting schemes, policy discussions have turned to the topic of linking. This paper offers a theory of optimal linking. We show that an efficient linkage adjusts the joint cap in response to inter-scheme trades of allowances but cannot rely on simple exchange rates. Compared to standard linking, our proposal has two major advantages. First, it increases global welfare by efficiently adjusting emissions in response to private information revealed by trades. Second, post-linking price volatility is lower with an endogenous cap. The latter advantage could arguably alleviate existing political barriers to linking such as imported price volatility. A key concept in our analysis is asymmetric uncertainty. Interestingly, while asymmetric information generally harms welfare, asymmetric uncertainty compensates for part (and, in extreme cases, all) of that welfare loss when linking cap and trade schemes. The possibility of optimally linking cap and trade schemes may vastly favor the range of model parameters for which cap and trade is favored over a carbon tax.

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1 Introduction

The number of cap and trade schemes to mitigate greenhouse gas emissions has grown steadily.[1] Reduced to its core, a cap and trade scheme caps CO_2 emissions by allocating allowances to emitters who are then allowed to trade their permits; if a firm emits CO_2, it must surrender an equivalent amount of allowances. Economists argue that such a policy combines the conservative certainty on emissions offered by direct command-and-control regimes with the efficient allocation of abatement efforts of a carbon tax.

When multiple cap and trade schemes coexist, these can be linked. Linking is seen as a promising development in cap and trade regulation (Mehling et al., 2018) and Article 6 of the Paris Agreement expressly provides for the possibility of linking. A linkage between two schemes reciprocally enables the use of permits issued in one scheme to meet compliance obligations pursuant to another. Linking has become increasingly prominent in recent years. California's cap and trade system linked with Quebec's on 1 January 2014 and the linked jurisdictions hold auctions together.[2] On 1 January 2020, a link between the European Union's Emissions Trading System (EU ETS) and the Swiss Emisions Trading System went into force. Linkages between the Regional Greenhouse Gas Initiative (RGGI) and the Emissions Trading Systems of Virginia and Pennsylvania are currently on their way, as are implicit linkages between California's ETS and Washintong State's Clean Air Rule.[3]

Linkages can be efficient because they allow for trades of allowances that lead to an ex post equalization of marginal abatement costs across jurisdictions without affecting global emissions or climate change. An additional benefit may be that, through their increased cooperation, local planners are less likely to choose their policies noncooperatively (Mideksa and Weitzman, 2019), leading to a more efficient tradeoff between the costs of climate change and emission abatement.

This paper proposes a theory of optimal linking under abatement cost uncertainty. For a linkage between cap and trade schemes to be optimal, it must combine two key

[1]Some history: "The first major emissions trading system (ETS) for greenhouse gases – the European Emissions Trading System (EU ETS) – was established in 2005. To date, there are 20 ETSs in place across five continents and covering 27 jurisdictions which produce almost 40 % of global wealth (GDP). With over a dozen more governments considering or having already scheduled an ETS, emissions trading has emerged as a key instrument to cost effectively decarbonize our economies." (ICAP, 2020)

[2]Starting from 2018, the ETS of Ontatio was also part of this linked system. However, half a year after the link was formally established, the Ontario government revoked its cap and trade regulation, thus withdrawing it from the linked carbon market.

[3]The latter link is mostly hypothetical, as Washinton State's Clean Air Rule was suspended after a 2018 court ruling. Though contested, the ruling was largely upheld by the Washington State Supreme Court on Jan. 16, 2020.

properties. First, it should be possible to fulfill compliance obligations pursuant to one jurisdiction by means of emission allowances issued by the other. In practice this means that polluters in either jurisdiction should be allowed to trade emission allowances freely and on an one-to-one basis. Second, there must be a pre-announced rule according to which the global cap is adjusted in response trades of allowances between schemes.

Free trade of allowances ensures that firms equate marginal abatement costs both within and across schemes. This is necessary for an optimal linkage because as long as marginal abatement costs are not equal across jurisdictions, mutually beneficial exchanges of allowances can be made, thus contradicting the notion of efficiency. The intuition is essentially the same as that favoring cap and trade over more direct command and control policies in a single jurisdiction. The literature on linking has often emphasized this channel of potential efficiency gains (Carbone et al., 2009; Flachsland et al., 2009; Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). However, there is more to trading that the mere equalization of marginal abatement costs.

Allowances are traded when marginal abatement costs differ across jurisdictions. The magnitude of trade flows then provides a sufficient statistic for the gap in marginal abatement costs under the initial allocation. Knowledge of this gap allows the planners to construct a posterior belief on realized abatement costs in both jurisdictions. Since this posterior will generally be different from their prior – these are confirmed only if no trades take place – the planners thus learn that the initial allocation of emission allowances was inefficient for two reasons. First, the distribution of allowances between the schemes, given the global cap, was suboptimal. This inefficiency is dealt with through trading. But second, the cap itself may turn out ex post inefficient from the point of view of global welfare.

To see how a flow of allowances between jurisdictions signals that the global cap is set inefficiently, note that a cap and trade policy prices emissions to let polluting firms incorporate the social cost of climate change into their production decisions. The planners thus try to strike a balance between the cost of climate change on the one hand, and the cost of abatement on the other. When abatement costs turn out to deviate from planners' expectations, that balance is lost. In this sense, linkages that allow for one-to-one trading implement a constrained Pareto efficient distribution of abatement efforts only. Emission levels are ex post inefficient. We therefore propose to adjust the global cap in response to trade between the jurisdictions. To our knowledge, we are the first to study such endogenous cap adjustments in the context of linked cap and trade schemes.

Interestingly, what drives cap adjustments is not the amount of uncertainty per se, but rather the degree to which uncertainty is asymmetric between jurisdictions. This has a clear intuition. Trade flows signal a wedge in jurisdictional marginal abatement costs. Learning about the *absolute* level of abatement costs occurs when planners update beliefs for the least predictable jurisdiction on their beliefs about costs in the more predictable one.[4] In the extreme case where the planners are uncertain about abatement costs in one jurisdiction only, this allows them to pin down abatement costs in the uncertain jurisdiction exactly on the basis of trade flows alone. Since in this case there is de facto complete information about abatement costs, the planners are able to implement the first-best social optimum.

Although we propose cap adjustments in response to allowance trading, simple exchange rates (*c.f.* Holland and Yates, 2015) are never optimal. An efficient allocation of abatement efforts equalizes marginal abatement costs across schemes. When emissions cannot be traded one-for-one (or, better said, ton-for-ton), firms have an incentive to trade beyond the perfect equalization of marginal abatement costs. To illustrate, suppose we were to contemplate a exchange rate such that 2 allowances issued by jurisdiction N can be traded against 1 allowance issued in jurisdiction S. Then firms in N and S will trade allowances until the marginal cost of reducing emissions in N equals *twice* the marginal cost of reducing emissions in S. A exchange rate thus drives firms' incentives away from an efficient distribution of abatement efforts and cannot be optimal.[5]

Linking cap and trade systems across jurisdictions is related to linking cap and trade markets over time (Yates and Cronshaw, 2001). The latter type of dynamic integration was studied in Heutel (2020) and Pizer and Prest (2020) for flow externalities, and in Gerlagh and Heijmans (2020) for stock externalities. They show that smart dynamic instruments can (greatly) improve welfare. But it matters how the cap is endogenized. For EU ETS, Gerlagh and Heijmans (2019) and Gerlagh et al. (2021) illustrate several undesirable side-effects of endogenous intertemporal emission caps. This literature therefore offers an important lesson for linking jurisdictional cap and trade schemes: adjusting the aggregate cap to trade flows can be efficient, but the devil is in the details.

[Countries within EU ETS/States within RGGI?]

Finally, we revisit the perennial question of instrument choice (Weitzman, 1974). Clearly, the choice between a carbon tax or a cap and trade policy is tilted toward the latter when an optimal linkage is implemented. The reason is the greater expected welfare

[4] The same intuition underlies statistical filters like the Kalman filter.

[5] The same need not apply to local pollutants like NO_X or lead pollution. That said, the case for linking also seems much weaker when regulating local pollutants.

performance of a cap and trade policy with optimal linkages compared to either cap and trade with standard linking or no linking at all. Roughly speaking, the case for cap and trade is twice as strong when independent schemes are optimally linked.

The paper is organized as follows. Section 2 introduces our model and the building blocks for welfare analysis. Section 3 discusses different types of (integrated) cap and trade policies and develops our theory of optimal linking. Section 4 concludes. Proofs and lengthy derivations are in the Appendix.

2 Model

Given are two jurisdictions, North and South, each operating its own cap and trade scheme. The assumption of two jurisdictions is not restrictive. One may simply consider North a representative jurisdiction for two linked jurisdictions, East and West, where trade between North and South is essentially a reduced-form way of writing trade between East, West, and South.

Each jurisdiction i is populated by firms that produce a composite good and production causes emissions. We may thus write benefits $B_i(\tilde{e}_i;\theta_i)$ as a function of emissions $\tilde{e}_i$, given by:

$$B_i(\tilde{e}_i \mid \theta_i) = (p_i^* + \theta_i)(\tilde{e}_i - e_i^*) - \frac{b_i}{2}(\tilde{e}_i - e_i^*)^2. \tag{1}$$

Emissions yield benefits because they allow firms to produce goods and save on the cost of abatement. As an umbrella term, we refer to $B_i(\tilde{e}_|\theta_i)$ as abatement costs in jurisdiction i, but other interpretations are possible. The parameter θ_i is a fundamental of jurisdiction i's economy and is *private information* of its constituent firms, though it is common knowledge that $\mathbb{E}[\theta_i] = 0$, $\mathbb{E}[\theta_i^2] = \sigma_i^2$, and $\mathbb{E}[\theta_N\theta_S] = \rho\sigma_N\sigma_S$. We interpret σ_i^2 as a measure for the uncertainty about jurisdiction i's economy. We say that uncertainty is asymmetric if $\sigma_N \neq \sigma_S$.

Emissions accumulate in the atmosphere and cause climate change as a global externality. We let the cost of climate change be given by:

$$C(\tilde{e}_N + \tilde{e}_S) = p^*(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*) + \frac{c}{2}(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*)^2. \tag{2}$$

We make the simplifying assumption that emissions have local benefits but global costs. One could imagine more complicated settings where emissions also have strictly regional costs like local air pollution (Caplan and Silva, 2005). A model of this type, however, will have many moving parts which would distract us from the core question we are primarily concerned with: the optimal linking of cap and trade schemes to regulate a global externality.

Subtracting the costs of climate change from the benefits due to emissions yields global welfare:

$$W = B_N(\tilde{e}_N \mid \theta_N) + B_S(\tilde{e}_S \mid \theta_S) - C(\tilde{e}_N + \tilde{e}_S). \tag{3}$$

We assume that policy variables in each jurisdiction are chosen to maximize (3). Thus, local emissions are set with the global social cost of carbon (Kotchen, 2018) in mind. There are three motivations to our assumption. First, most if not all cap and trade schemes currently operative are policymakers' attempts to meet mitigation obligations implied by the Paris Agreement's pledge to try and limit climate change to 1.5°C above pre-industrial levels. Since the Paris Agreement is global in scope and intention, cap and trade schemes set up to satisfy the associated pledges operate – at least implicitly – with a measure of global climate damages in mind.[6,7] Second, the act of linking cap and trade schemes presupposes a certain amount of cooperation or mutual agreement on the cap. Indeed, the "essential criteria" to ensure "compatibility between the systems" summed up in Annex I of the *Agreement between the European Union and the Swiss Confederation on the linking of their greenhouse gas emissions trading systems* explicitly include the "ambition and stringency of the cap". Similarly, the independent but linked cap and trade schemes of California and Quebec organize joint auctions of allowances, implying a degree cooperation in determining the linked caps. Finally, we are mostly interested in the potential welfare gains of our optimal linking regime compares to standard or classic linking. Thus,

For an clear decomposition of the welfare gains due to linking when regional planners take global damages into account only once a linkage is established, the reader is referred to Doda et al. (2019).

We write $\tilde{E} = \tilde{e}_N + \tilde{e}_S$ and $E^* = e^*_N + e^*_S$. Our model is now characterized by three

[6] As an example, "[...] the [European] Commission proposed in September 2020 to raise the 2030 greenhouse gas emission reduction target, including emissions and removals, to at least 55% compared to 1990. [...] This will enable the EU to move towards a climate-neutral economy and implement its commitments under the Paris Agreement by updating its Nationally Determined Contribution." Moreover, "[t]o achieve the EU's overall greenhouse gas emissions reduction target for 2030, the sectors covered by the EU ETS must reduce their emissions by 43% compared to 2005 levels." Retrieved from https://ec.europa.eu/clima/policies/strategies/2030_en.

[7] As another example, the Regional Greenhouse Gas Initiative's 2005 Memorandum of Understanding explicitly mentions global damages as a key factor governing its operations, writing that "climate change poses serious potential risks to human health and terrestrial and aquatic ecosystems globally... To address global climate change and in order to do their fair share in addressing their contribution to this collective problem while preserving and enhancing the economic welfare to their residents, the Signatory States find it imperative to act together to control emissions of greenhouse gases, particularly carbon dioxide, into the Earth's atmosphere from within their region."

curves and eight parameters $(b_i, c, p_i^*, e_i^*, p^*)$. For the system to be identified, we need two parameters per curve (slope and intercept) for three curves in total (2 benefit and 1 cost). This makes for a total of six parameters. Consequently, we may take the freedom to reduce the number of parameters through defining $p^* = p_N^* = p_S^*$, with the convenient implication that (p^*, e_N^*, e_S^*) is the vector of welfare-maximizing prices and emissions for jurisdiction i, given $\theta_N = \theta_S = 0$. We label this the ex-ante optimum. This is clearly not an assumption, nor even a normalization – it is a definition.

Firms are profit maximizers. Once a policy k caps emissions at a level $\tilde{e}_i^k$, individual firms trade allowances until marginal abatement costs for all are equal to:

$$p_i^k = -b_i e_i^k + \theta_i, \tag{4}$$

which is firms' inverse demand for allowances. In a competitive for allowances, the price at which permits are traded will be p_i^k in equilibrium.

Before proceeding to the analysis, we introduce some further notation. Superscripts will be scenario (instrument) labels for equilibrium outcomes. Moreover, let $\tilde{x}^k$ denote the value of a variable x under policy k, then let $x^k := \tilde{x}^k - x^*$ be the deviation of x under policy k from the ex-ante expected optimal value x^*, and let $\Delta^k x := \tilde{x}^k - x^{SO}$ denote the difference between the value of x under scenario k and its ex post socially optimal value (see subsection 2.1).

2.1 Global Social Optimum

In a perfect world without informational frictions – the Social Optimum – a welfare-maximizing planner allocates emissions between the jurisdictions in such a way that marginal benefits are the same in both, i.e. so that $MB_N^{SO} = MB_S^{SO} = MB^{SO}$. Since both jurisdictions by assumption operate a cap and trade scheme, marginal benefits of emissions are also equal to the carbon price, so $p_N^{SO} = p_S^{SO} = p^{SO} = MB^{SO}$. Next, since the level of emissions if efficient if and only if marginal climate costs equal marginal benefits in either jurisdiction, we obtain the following conditions for the socially optimal emission levels:

$$c \cdot (e_N^{SO} + e_S^{SO}) = p^{SO} = -b_i e_i^{SO} + \theta_i. \tag{5}$$

Equation (5) characterizes the Social Optimum and represents three equalities in three unknowns: e_N^{SO}, e_S^{SO}, and p^{SO}. Solving for these, we obtain:

$$p^{SO} = \frac{c(b_S\theta_N + b_N\theta_S)}{cb_N + cb_S + b_N b_S}, \tag{6}$$

$$e_i^{SO} = \frac{b_{-i}\theta_i + c(\theta_i - \theta_{-i})}{cb_N + cb_S + b_N b_S}, \tag{7}$$

$$E^{SO} = \frac{b_S\theta_N + b_N\theta_S}{cb_N + cb_S + b_N b_S}, \tag{8}$$

where $i = N, S$ and $-i$ simply means "the jurisdiction that is not i". As is intuitive, socially optimal emission levels are higher when abatement is more expensive. This basic observation will be useful later on, when we illustrate that trades of allowances between (linked) schemes signal private information about abatement costs to the planners. An optimal linkage exploits this information by adjusting the global cap in response.

The variance of prices is given by:

$$\mathbb{E}\left[\left[p^{SO}\right]^2\right] = \left(\frac{c}{c \cdot b_N + c \cdot b_S + b_N b_S}\right)^2 \left[b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho \sigma_N \sigma_S\right]. \tag{9}$$

Note that uncertainty translates into price volatility. We will return to this later.

2.2 Welfare Losses

We rank policies according to their expected welfare performance, which is equivalent to ranking policies according to the expected welfare loss relative to the Social Optimum. We perform our analysis in terms of the latter for notational convenience.

Suppose a policy k induces emission levels $\tilde{e}_i^k$ in jurisdiction i. From firms' equilibrium behavior (4), we see that deviations in emissions from the social optimum scale with prices:

$$\Delta^k p_i = -b_i \Delta^k e_i. \tag{10}$$

Expected welfare losses relative to the social optimum are then given by:

$$\begin{aligned} L^k &= \mathbb{E}\left[\Delta^k B_N + \Delta^k B_S - \Delta^k C\right] \\ &= \frac{c}{2}\mathbb{E}\left[\left(\Delta^k E\right)^2\right] + \sum_i \frac{b_i}{2}\mathbb{E}\left[\left(\Delta^k e_i\right)^2\right]. \end{aligned} \tag{11}$$

Throughout the analysis, we will interpret a policy as the implementation of a constrained expected welfare maximization problem. The level of welfare in the Social Optimum, W^{SO}, is hypothetical and unaffected by the policy implemented. Thus, expected welfare maximization subject to a set of policy constraints can be treated as the dual problem of expected welfare loss minimization relative to the Social Optimum, subject to the same policy constraints.

Linking policies exhibit equal prices across jurisdictions in equilibrium, as we shall explain below. By (10), emissions (both local and global) under such policies can therefore be expressed in terms of the common price gap Δp. Plugging this into (11), expected welfare losses from a policy featuring equal prices across jurisdictions can be written as:

$$L^k = \frac{1}{2}\frac{(cb_N + cb_S + b_N b_S)(b_N + b_S)}{b_N^2 b_S^2}\mathbb{E}\left[\left(\Delta^k p\right)^2\right]. \tag{12}$$

3 Policies

3.1 Regional cap and trade

The simplest policy operates the two cap and trade schemes individually. In this case, the planner in jurisdiction i sets a cap e_i to maximize (3). Equivalently, the planners minimize expected welfare losses relative to the Social Optimum, subject to the constraint that emission levels in each jurisdictions are determined on the basis of expected abatement costs. The resulting allocation is $e_N = e_S = 0$. Plugging these into (11), we obtain expected welfare losses when both jurisdictions operate their own cap and trade schemes individually:

$$L^R = \frac{1}{2}\frac{(c + b_S)\sigma_N^2 + (c + b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{13}$$

At a more formal level, regional cap and trade is characterized as the solution to the following straightforward welfare maximization problem:

$$\max_{e_N, e_S} \mathbb{E} \quad W(e_N, e_S \mid \theta_N, \theta_S). \tag{14}$$

Thus, regional cap and trade is the optimal choice of instrument under the constraint that caps must be set before any information is revealed and without using any information extracted from regulated firms' behavior. The first-order condition of (14) is that expected marginal benefits in each scheme are equal to marginal climate damages:

$$\mathbb{E}[MB_i \mid e_i] = MC. \tag{15}$$

We note that the marginal damages (the RHS of (15)) are perfectly known when caps are fixed, whereas marginal benefits are stochastic variables due to unknown fundamentals θ_i. Thus, regional cap and trade implements socially optimal emission levels if and only if abatement costs turn out exactly as expected ($\theta_N = \theta_S = 0$). If abatement costs deviate from expectations, regional cap and trade is inefficient for two reasons. First, the allocation of abatement efforts between the jurisdictions may be ex post inefficient (more precisely, this happens when abatement costs differ between them). And second, the level of emissions is ex post inefficient. The first of these is remedied by linking schemes across jurisdictions.

3.2 Linking

When North and South link their cap and trade schemes, each planner sets the expected optimal cap for its jurisdiction but allowances issued in one scheme may be used to fulfill abatement obligations pursuant to another. Thus polluters are free to trade their allowances as long as global emissions are not affected:

$$e_N + e_S = E = 0. \tag{16}$$

Linking has attracted much academic interest in recent years (Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). Just as trade in emission allowances between firms within a jurisdiction is an efficient way to achieve a given amount of local abatement, so the trade of allowances between jurisdictions is an efficient way to achieve a pre-determined amount of global abatement. The intuition is essentially the same: while linking does not affect global emissions, and therefore climatic damages, it does lead to a closer alignment of the benefits and costs of abatement across jurisdictions. In fact, as long as marginal abatement costs in the jurisdictions are *not* the same, firms can and will profitably exchange permits. The integrated market for emission allowances therefore reaches equilibrium only once the price of an allowance is the same in North and South:

$$p_N = p_S. \tag{17}$$

Moreover, since a firm in either jurisdiction is willing to sell (buy) an allowance against the going market price as long as the price is above (below) its private marginal abatement cost, marginal benefits are also the same across firms and jurisdictions when the schemes are linked:

$$-b_N e_N^L + \theta_N = -b_S e_S^L + \theta_S, \tag{18}$$

where e_N and e_S are chosen by the firms conditional on θ_N and θ_S, subject to (16). Plugging this into our welfare loss-function (11), we obtain:

$$L^L = \frac{1}{2}\frac{1}{b_N + b_S}\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{19}$$

We now state our first substantive result.

Proposition 1. *Global welfare is always higher when cap and trade schemes are linked, compared to when they are not.*

In contrast to regional cap and trade, linking guarantees that marginal abatement costs are equalized across jurisdictions. For this reason, linking is always weakly better for welfare than regional cap and trade. We can also see this more formally when taking a look at the welfare maximization problem the planners (implicitly) solve when instituting a bilateral link between jurisdictions:

$$\max_G \quad \mathbb{E}\left[\max_{e_N, e_S} W(e_N, e_S \mid \theta_N, \theta_S) \quad \text{s.t.} \quad e_N + e_S = G\right]. \tag{20}$$

Here, the

Practically, the planners of North and South cap local emissions at levels that, in expectations, maximize global welfare. When issued, permits can be traded on a one-to-one basis between schemes, as long as emissions overall remain fixed at the sum of the two jurisdictional caps. By linking their schemes, the planners of North and South effectively delay the determination of local emission caps until after jurisdictional abatement costs are known, guaranteeing an ex post efficiency gain through the equalization of marginal benefits, which in expectations equal marginal damages:

$$MB = MB_N = MB_S \tag{21}$$

$$\mathbb{E}[MB \mid e_N + e_S] = MC. \tag{22}$$

Though linking benefits global welfare (Proposition 1), the effects on individual jurisdictions are ambiguous. A simple thought experiment illustrates. We note that prices in equilibrium equate marginal benefits, so that the volatility of prices is equal to the volatility of marginal benefits. Hence if abatement costs in North are much less predictable than in South, $\sigma_N > \sigma_S$, North may import part of the price volatility to which South is subject. If this effect is strong enough, aggregate price volatility may increase and North may be harmed by linking with South (there are papers on this, cite here).

Linking suffers from another, more implicit type on inefficiency. It disregards valuable information. Remember that allowances will be traded if, and only if, there is a wedge in jurisdictional marginal benefits under the local caps. A flow of allowances from one scheme to another therefore reveals information about θ_N and θ_S. The global cap on emissions ideally responds to these ex post observations, but does not under standard linking.

3.3 Optimal Linking

When two schemes are linked, a flow of allowances from one scheme to the other reveals information. Our goals is to develop a linking policy that optimally incorporates this information. In particular, we propose that if a total of e_N allowances issued in North are sold to firms in South, then the global cap on emissions is changed by an amount $f(e_N)$, where f is a cap-adjustment function chosen to maximize global welfare. In our linear framework, the function f is simply linear in e_N:

$$(1-\delta)e_N = E \implies \delta e_N + e_S = 0, \tag{23}$$

where the cap-adjustment parameter δ is chosen to maximize welfare and given by:

$$\delta = \frac{b_N[\sigma_S^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}{b_S[\sigma_N^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}. \tag{24}$$

We will discuss the technical derivation of (23) and (24) shortly. For now, we stress that our proposal really has two key properties. First, and most obviously, the global cap responds to inter-jurisdictional trade of allowances. This is done to make sure the information revealed in inter-scheme trades is incorporated into the global cap. Second, and more subtle though just as important, *jurisdictional schemes exchange allowances one-to-one*. Linking creates efficiency gains because it incentives firms to equate marginal abatement costs globally. This desirable property would be lost if permits are not traded one-to-one between schemes, for example if allowance are traded against an exchange rate. We elaborate on this in Section 3.5.

While free trade between jurisdictions stimulates the equalization of marginal abatement costs across all emitters, the global cap is adjusted to bring marginal abatement costs more in line with marginal climate costs. If we plug (24) into (23), noting (**??**), one can show that the policy we propose implements the solution to the following program:

$$\mathbb{E}[MB \mid e_N, e_S] = MC. \tag{25}$$

To see this, note that from the observed flow of permits between jurisdictions we can construct the difference in marginal abatement cost innovations:

$$\mu \equiv \theta_S - \theta_N = b_N e_N - b_S e_S.$$

Using the demand equation (4) and plugging in μ, we find:

$$\begin{aligned}
\mathbb{E}[MB \mid e_N, e_S] &= \mathbb{E}[\theta_N|\mu] - b_N e_N \\
&= \mu\frac{\mathbb{E}[\mu\theta_N]}{\mathbb{E}[\mu^2]} - b_N e_N \\
&= \mu\frac{\rho\sigma_N\sigma_S - \sigma_N^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} - b_N e_N \\
&= -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S
\end{aligned}$$

This can be equated to marginal damages:

$$c(e_N + e_S) = -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S, \tag{26}$$

which for convenience we rewrite as

$$\delta e_N + e_S = 0,$$

which is (23), as claimed. Lastly, solving (23) for δ, we obtain (24), which shows that our optimal linking policy indeed implements the solution to (25).

Note that if $\delta = 1$, optimal linking coincides with standard linking of schemes. Since the planners are free to set $\delta = 1$ but not required to do so, it is clear that optimal linking outperforms standard linking. In fact, looking at (24), we see that δ will generally *not* be equal to 1:

$$\delta \lesseqgtr 1 \iff \frac{b_N}{b_S} \lesseqgtr \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_S^2 - \rho\sigma_N\sigma_S}. \tag{27}$$

Only in specific parametric conditions do we find $\delta = 1$. Since δ is chosen to maximize welfare it immediately follows that linking with an endogenous cap always outperforms standard linking, in most cases strictly.

Proposition 2. *Optimal linking is strictly welfare-superior to both jurisdictional cap and trade and standard linking. The expected welfare loss is given by:*

$$L^O = \frac{1}{2}\frac{b_N + b_S}{cb_N + cb_S + b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{28}$$

All else equal, the expected welfare loss when jurisdictions optimally link their cap and trade scheme is decreasing in c, the marginal climatic damage due to emissions.

Looking at the relative welfare performance of an optimal linkage compared to classic linking, it is easy to derive that:

$$\frac{L^O}{L^L} = \frac{(b_N^2 + b_S^2 + 2b_n b_S)(1+\rho)\sigma_N\sigma_S}{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_n b_S\rho\sigma_N\sigma_S}\cdot\frac{(1-\rho)\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}, \tag{29}$$

which cannot exceed 1. Interestingly, though the absolute levels of welfare losses under both classic an optimal linking depend on climate damages through c, the relative performance of an optimal linkage is independent of climate damages.

Another way to see that optimal linking is better than standard linking is to think in terms of efficient use of information. This can be done by looking at the mathematical program either policy solves. Endogenous cap and trade implements the solution to $\mathbb{E}[MB \mid e_N, e_S] = MC$, which is different $\mathbb{E}[MB \mid e_N + e_S] = MC$, the program for linking. Because (e_N, e_S) provides more precise information than the mere aggregate $e_N + e_S$, optimal linking should give at least as good an outcome as standard linking. From this discussions follows an important lessen. Optimal linking not 'just another' cap and trade policy. It is *the* most efficient implementable quantity-based regulation across jurisdictions. It equalizes marginal costs and expected marginal benefits given the finest information available in observed trades.

The fact that optimal linking increases global welfare compared to standard linking an regional cap and trade still does not that it increases welfare in both regions. Though the welfare of North and South jointly increases, individual jurisdictions may be worse off after linking.[8] The reason is that a relatively stable jurisdiction may, through linking, expose itself to the volatile abatement costs in the other jurisdiction, importing a variable

[8] Of course, since the sum of welfares has gone up, it is possible to define a system of transfers so that both jurisdictions are still strictly better off in the end. Such a system of transfers may be politically infeasible.

allowance price (Holtsmark and Weitzman, 2020). However, our optimal linking policy mitigates this concern by lowering price volatility compared to standard linking:[9]

Proposition 3. *Endogenous cap and trade admits lower price volatility than linked cap and trade schemes:*

$$\mathbb{E}\left[\left(p^{OL}\right)^2\right] \leq \mathbb{E}\left[\left(p^{L}\right)^2\right]. \tag{30}$$

3.4 Asymmetric Uncertainty

Looking at the expression for δ given in (24), we observe that the cap-adjustment rate may be negative. This means that higher-than-expected emissions in one jurisdiction may translate into higher-than-expected emissions in the other jurisdiction too. This occurs for strongly positively correlated innovations and very asymmetric uncertainty: if benefits in one jurisdiction are very unpredictable but strongly correlated to those in the other, predictable jurisdiction, an increase in the value of emissions in the latter is likely to be matched by an equally strong increase in the former. A negative cap-adjustment rate bears some resemblance with putting negative weights on observations in making (econometric) predictions.

We also observe that the share of global emission reductions absorbed by a jurisdiction is decreasing in its responsiveness of marginal benefits to emissions, that is, in β. For any adaption of global emissions to shocks, marginal costs change accordingly. Since trade leads to the ex-post equality of jurisdictional marginal benefits, and since an optimal mechanism equates jurisdictional marginal benefits to global marginal costs, for any realized pair (θ_N, θ_S) emissions change relatively less in the jurisdiction with steeper marginal benefits.

Our potentially most interesting observation is that the cap-adjustment rate tends to increase, all else equal, if the uncertainty about benefits in South (σ_S) increases. Intuitively, this means that the endogenous emissions cap should be anchored on the demand for emissions in the most predictable jurisdiction. The opportunity for learning is greatest for jurisdiction we know least about. Asymmetric uncertainty should be used as an important input into policy making.

An extreme example can illustrate the power of asymmetric uncertainty. Suppose we know marginal benefits in North perfectly, so $\sigma_N = 0$, but we are uncertain about benefits in South, $\sigma_S > 0$. The planners clearly face an environment with asymmetric uncertainty.

Suppose now that we observe an interjurisdictional trade of allowances. Since we know the exact marginal benefit curve in North, we also know marginal benefits in North for

[9]With inter*temporal* trading of permits, an endogenous cap can also leads to lower price volatility, see Gerlagh et al. (2020).

any e_N. And since permits will be traded until the point where marginal benefits in both jurisdictions are equal, we in fact know them for South as well. But this implies we in fact observe θ_S. Asymmetric uncertainty allows us to extract valuable information from the market.

Our extreme example to illustrate the power of asymmetric uncertainty is reflected in equation (28). If the planners have perfect knowledge about North (or South), then all abatement costs are revealed through trade and no welfare losses occur, $L^O = 0$.

Corollary 1. *In the extreme case with perfect information about one of the two linked schemes, optimal linking implements the ex post optimal level of emissions in both jurisdictions.*

Without paying attention to asymmetric uncertainty, a flow of allowances from one scheme to another merely indicates that there is a wedge in marginal abatement costs under the initial allocation of allowances. Efficiency is gained by linking since marginal benefits will be equated. The allocation is only constrained Pareto optimal, though: given a potentially suboptimal cap, emissions are allocated in an efficient way. Our optimal linking policy additionally adjusts the cap in response to the information revealed in allowance trading, moving it closer to unconstrained optimal levels. We have shown that a key ingredient for smart cap-adjustments is asymmetric uncertainty. Intuitively, there is *more* scope for learning about agents whose abatement costs is *more* uncertain. If the planners want to use as much information as possible, they must allows themselves to learn differentially about either scheme in response to one and the same trade flow.

3.5 Exchange Rates

As an alternative to the kind of optimal linking introduced in Section 3.3, the planners might consider exchange rates for allowance trading. Superficially, such a policy does the same as our proposal. It also allows for inter-scheme trade of allowances while adjusting the global cap in response to trading. On top of that, it is much simpler.

To see why an exchange rate for allowance trading cannot work, suppose that North and South agree on an exchange rate of α on allowances from North. Thus, if a firm in North sells 1 ton of CO_2 worth in allowances to a firm in South, the latter can increase its emissions by α tons. In that case, it is easy to see that firms will trade allowances between jurisdictions until the point where prices satisfy:

$$p_N = \alpha \cdot p_S. \tag{31}$$

Equation (31) is a no-arbitrage condition. If (31) is violated, firms could buy allowances in one jurisdiction and sell them in the other at a gain. Allowance prices must therefore satisfy (31) if the linked schemes are in equilibrium.

There is, however, a second equilibrium condition. Firms in jurisdiction i will trade allowances until marginal abatement costs are exactly equal to the allowance price. That is, condition (4) needs to be satisfied so that $p_i = MB_i$ for each jurisdiction i. If we combine this constraint with (31), we obtain the following equilibrium characterization:

$$MB_N = \alpha \cdot MB_S. \tag{32}$$

Thus, if trading of allowances between jurisdictions is subject to an exchange rate α, then the linked markets are in equilibrium if and only if regional marginal abatement costs scale by α. A policy of exchange rates creates incentives for firms to trade allowances to the point where marginal abatement costs are unequal across jurisdictions.

Proposition 4. *A policy of exchange rates on allowance trading between jurisdictions is never efficient.*

3.6 Prices vs. Quantities

We saw how jurisdictions can optimally link their cap and trade schemes. Alternatively, each jurisdiction might also a levy a carbon tax. We now revisit the classic question of Weitzman (1974) on the choice between instruments.

We assume that emissions are taxed at an expected optimal rate:

$$\begin{aligned} \max_{p_N, p_S} \quad & \mathbb{E} \quad B_N(e_N \mid \theta_N) + B_S(e_S \mid \theta_S) - C(e_N + e_S) \\ \text{s.t.} \quad & p_i = -b_i e_i + \theta_i, \end{aligned} \tag{33}$$

where the constraint follows from profit maximization by the firms (4). Recall from our discussion following equation (3) that $p_N^* = p_S^*$ were defined as the expected welfare-maximizing carbon price in each jurisdiction, so the solution to (33) is $p_N = p_S = 0$. We may therefore invoke (9) and (12) to derive expected welfare losses when both jurisdictions tax emissions:

$$L^{tax} = \frac{1}{2}\left(\frac{c}{b_N b_S}\right)^2 \frac{b_N + b_S}{cb_N + cb_S + b_N b_S}(b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S). \tag{34}$$

All else equal, the expected welfare loss when jurisdictions tax emissions is increasing in

c, the marginal climate damage. We can now compare (34) and (28), which yields the following proposition.

Proposition 5. *Taxes are favored over optimally linked cap and trade schemes if and only if the following inequality is satisfied:*

$$\left(\frac{c}{b_N b_S}\right)^2 < \frac{(1+\rho)\sigma_N\sigma_S}{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}\frac{(1-\rho)\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} \tag{35}$$

When the two jurisdictions are entirely symmetric and have the same abatement cost function, i.e. $b_N = b_S$ and $\sigma_N = \sigma_S$, (35) reduces to a very simple condition:

$$2c < b. \tag{36}$$

For comparison, the condition to favor regional, non-linked cap and trade schemes over a carbon tax in this symmetric case becomes:

$$4c^2 < (1-\rho)bc + b^2. \tag{37}$$

When we compare (36) and (37), we see that the minimum marginal climate damages at which optimally linked cap and trade schemes are preferred over an emissions tax may be as little as half the marginal climate damages at which standard, non-linked cap and trade performs better. Thus, if we believe that climate change is the "existential threat of our time",[10] meaning that c is high, a policy of optimal linking between regional cap and trade schemes may vastly expand the range of (expected) abatement costs for which cap-and-trade is favored over a carbon tax. This indicates the importance of our preliminary analysis for climate mitigation debates in the real world.

4 Summary

We propose a simple theory of optimal linking. Fairly straightforward manipulations of standard linking between cap and trade schemes are shown to increase global and jurisdictional welfare. The core of our argument is that the trade of allowances between local schemes signals valuable information which an efficient policy incorporates. Practically,

[10]The quote is taken from a speech by then-President-elect Biden on December 19, 2020, as he was nominating people to lead the EPA under his administration. See https://www.bloomberg.com/news/articles/2020-12-19/biden-calls-climate-change-existential-threat-of-our-time

our proposal to adjust the cap in linked schemes in response to allowance trading. We pin down an exact analytic formulation for this endogenous cap-adjustment and discuss its properties.

Optimal linkages allow permits to be exchanged one-to-one. An “exchange rate” that differs from one, though de facto shifting the aggregate cap indeed, would only disturb individuals firms’ incentives away from the exact equalization of marginal abatement costs across schemes, which is inefficient. To implement an optimal linkage, the aggregate cap must be adjusted in response to trade, but not through trade. A straightforward way to achieve this is to either inject new permits or buy back already issued ones as called for by the observed trade flows (Hintermayer, 2020, analyzes a buyback policy in a dynamic model of EU ETS). Upward adjustment of the cap may be easy to achieve as it only requires the issuing of extra permits. Adjusting the cap downward may be more problematic, however, calling for planners to spend money on buying allowances back. This may cause distributional problems: which scheme ought to spend the money? A possible way to mitigate this concern is by having linked schemes hold joint auctions, using the revenue of the joint auction to buy back permits. Joint auctions have a political track record: they are used by the linked schemes of California and Quebec.

A key concept we exploit is asymmetric uncertainty. When two schemes are linked, information on *relative* abatement costs is revealed through trade. But when the schemes are asymmetrically uncertain, this information on relative abatement costs be used to infer (sharp) predictions about *absolute* abatement costs in either jurisdiction. The same is not possible when trade between symmetrically uncertain schemes is observed. Our results suggest that the study of asymmetric uncertainty deserves a more prominent place in environmental economics.

Cap and trade schemes have become a major policy instrument in the fight against climate change. In Europe alone, roughly 45% of greenhouse gas emissions are regulated by EU ETS, the world’s largest market for carbon. With ever more cap and trade being erected, linking of regional markets has become a prominent policy development. Linkages between jurisdictional schemes already exist, and more are currently being contemplated. The are linkages between EU ETS and the Swiss ETS, between RGGI and Quebec, between Quebec and California. The up-and-coming carbon markets of China and the post-Brexit UK will create even more possibilities for linking. Given the large amount of money and CO_2 involved in cap and trade, and given the increasing prevalence of linkages between jurisdictional schemes, constructive ideas on optimal linking are needed. This paper offers some thoughts.

Although we discuss linking of cap and trade schemes at the level of a jurisdiction, our methods apply just as well to cases where an existing cap and trade scheme is expanded to cover additional sectors. Consider the aviation sector. Passenger flights outside the European Economic Area are not covered by the EU ETS. To nevertheless reduce emissions in the aviation industry, "the International Civil Aviation Organization (ICAO) agreed on a Resolution for a global market-based measure to address CO2 emissions from international aviation as of 2021. The agreed Resolution sets out the objective and key design elements of the global scheme, as well as a roadmap for the completion of the work on implementing modalities. The Carbon Offsetting and Reduction Scheme for International Aviation, or CORSIA, aims to stabilise CO2 emissions at 2020 levels by requiring airlines to offset the growth of their emissions after 2020."[11] Though it is still unclear how exactly CORSIA and EU ETS will interact, our analysis suggests that a direct incorporation of CORSIA into the EU ETS may be suboptimal. Similarly, our results suggest that trade of allowances between sectors or clearly identified jurisdictions within an existing cap and trade scheme – think of countries in the EU ETS or states in RGGI – can be made more efficient by incorporating a policy along the lines of our optimal linking regime.

Our theory is preliminary and does not address several aspects of real life emission trading. Most large-scale cap and trade schemes are dynamic and allow covered industries to bank (and sometimes borrow) allowances across periods.

Our theory of optimal linking follows a purely quantity-based approach. We take (flows of) emissions as an input to get aggregate emissions as an output. One could instead focus on prices, or a combination of prices and emissions.

We leave an exploration of these and other aspects for future research.

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A Derivations and Proofs

DERIVATION OF (19):
Combining the definition with the firms' FOCs, (4), we find the change in permit use by jurisdiction:

$$\Delta^L e_N = \frac{\theta_N - \theta_S}{b_N + b_S} \tag{38}$$

$$\Delta^L e_S = \frac{\theta_S - \theta_N}{b_N + b_S}. \tag{39}$$

PROOF OF PROPOSITION 1:

Proof. We only need to compare welfare losses under a linked cap and trade regime, equation (19), to those under jurisdictional cap and trade, equation (13). Linking outperforms regional cap and trade iff:

$$\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{b_N + b_S} < (c + b_S)\sigma_N^2 + (c + b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S$$

$$\Longleftrightarrow$$

$$\frac{2\rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2} < \frac{cb_N + cb_S + b_N b_S}{cb_N + cb_S},$$

which is always true. □

DERIVATION OF (24):

Regional and global deviations from Socially Optimal permit use are given by:

$$\Delta^{OL} e_N = \frac{b_S}{b_N + \delta b_S} \frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{40}$$

$$\Delta^{OL} e_S = \frac{b_N}{b_N + \delta b_S} \frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{41}$$

$$\Delta^{OL} Q = \frac{b_N + b_S}{b_N + \delta b_S} \frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S}. \tag{42}$$

Define

$$\xi := \frac{b_N + c(1-\delta)}{b_N + \delta b_S} \implies 1 - \xi := \frac{\delta b_S - c(1-\delta)}{b_N + \delta b_S}. \tag{43}$$

Welfare losses can now be written as:

$$\begin{aligned} L^O &= \frac{1}{2} \frac{c(b_N + b_S)^2 + b_N^2 b_S + b_N b_S^2}{(cb_N + cb_S + b_N b_S)^2} \mathbb{E}\left[(1-\xi)\theta_N + \xi\theta_S\right]^2 \\ &= \frac{b_N + b_S}{2} \frac{(1-\xi)^2\sigma_N^2 + \xi^2\sigma_S^2 + 2\xi(1-\xi)\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \end{aligned} \tag{44}$$

If for notational convenience, we define:

$$\psi := \frac{1}{2} \frac{b_N + b_S}{cb_N + cb_S + b_N b_S}, \tag{45}$$

it is straightforward to derive:

$$\frac{\partial}{\partial \xi} \frac{L^O}{\psi} = 2\xi\sigma_S^2 - 2(1-\xi)\sigma_N^2 + 2(1-\xi)\rho\sigma_N\sigma_S - 2\xi\rho\sigma_N\sigma_S. \tag{46}$$

The welfare-maximizing ξ^* therefore satisfies:

$$\xi^* = \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{47}$$

From the definition of ξ, the optimal cap-adjustment rate δ^* follows:

$$\delta^* = \frac{(b_N + c)[\sigma_S^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 - \rho\sigma_N\sigma_S]}{(b_S + c)[\sigma_N^2 - \rho\sigma_N\sigma_S] + c[\sigma_S^2 - \rho\sigma_N\sigma_S]}, \tag{48}$$

as given.

PROOF OF THEOREM 2:

Proof. Plugging (47) in (44), we find:

$$
\begin{aligned}
\frac{L^O}{\psi} &= \left[\frac{\sigma_S^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\right]^2 \sigma_N^2 + \left[\frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\right]^2 \sigma_S^2 \\
&+ \left[\frac{\sigma_S^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\right]\left[\frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\right] \rho\sigma_N\sigma_S \\
&= \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} \\
&\Longrightarrow \\
L^O &= \frac{1}{2}\frac{b_N + b_S}{cb_N + cb_S + b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S},
\end{aligned}
$$

as stated. This is strictly lower than the welfare loss under traditional Trading if and only if:

$$
\begin{gathered}
L^L - L^O \geq 0 \\
\Longrightarrow \\
\frac{1}{b_N + b_S}\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S} - \frac{b_N + b_S}{cb_N + cb_S + b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} \geq 0 \\
\Longrightarrow \\
(\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S)(b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S) - (1-\rho^2)(b_N^2 + b_S^2 + 2b_N b_S)\sigma_N^2\sigma_S^2 \geq 0 \\
\Longrightarrow \\
[(b_S\sigma_N^2 - b_N\sigma_S^2) + (b_N - b_S)\rho\sigma_N\sigma_S]^2 \geq 0,
\end{gathered}
$$

which is always true. □

PROOF OF PROPOSITION 3:

Proof. We derived quantity derivations under both policies. Prices are equal in both jurisdictions, so without loss of generality we can solve for price deviations in jurisdiction 1:

$$
\begin{aligned}
\Delta^L p_N &= \frac{b_S\theta_N + b_N\theta_S}{b_N + b_S} \\
\Delta^{OL} p_N &= \frac{\delta b_S\theta_N + b_N\theta_S}{b_N + \delta b_S}.
\end{aligned}
$$

Thus:

$$\mathbb{E}\left[\left(\Delta^{L}p\right)^{2}\right] = \frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}{b_N^2 + b_S^2 + 2b_N b_S}$$

$$\mathbb{E}\left[\left(\Delta^{OL}p\right)^{2}\right] = \frac{\delta^2 b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2\delta b_N b_S\rho\sigma_N\sigma_S}{b_N^2 + \delta^2 b_S^2 + 2\delta b_N b_S}.$$

Writing these out, we obtain:

$$\mathbb{E}\left[\left(\Delta^{OL}p\right)^{2}\right] < \mathbb{E}\left[\left(\Delta^{L}p\right)^{2}\right] \iff (\delta - 1)\left[b_S\left(\sigma_N^2 - \rho\sigma_N\sigma_S\right) - b_N\left(\sigma_S^2 - \rho\sigma_N\sigma_S\right)\right] < 0.$$

Given (27), this condition is always satisfied. □