Linking Cap And Trade Schemes

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March 10, 2021

Abstract

Recent years have witnessed a rapid increase in the number of cap and trade schemes to mitigate greenhouse gas emissions. With multiple coexisting schemes, policy discussions have turned to the topic of linking. This paper offers a theory of optimal linking under uncertainty. We show that an efficient linkage adjusts the joint cap in response to inter-scheme trades of allowances but cannot rely on simple exchange rates. Compared to standard linking, our proposal has two major advantages. First, it increases global welfare by efficiently adjusting emissions in response to private information revealed by trades. Second, post-linking price volatility is lower with an endogenous cap. The latter advantage could arguably alleviate existing political barriers to linking such as imported price volatility. An important role is played by asymmetric uncertainty. Interestingly, while asymmetric information generally harms welfare, asymmetric uncertainty compensates for part (and, in extreme cases, all) of that welfare loss when linking cap and trade schemes. The possibility to optimally link regional cap and trade schemes can vastly expand the range of model parameters for which cap and trade is favored over a carbon tax.

JEL codes: D82, D83, H23, Q52, Q54

Keywords: asymmetric information, policy updating, asymmetric uncertainty, carbon markets, linking

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1 Introduction

The number of cap and trade schemes to mitigate greenhouse gas emissions has grown steadily.[1] Reduced to its core, a cap and trade scheme caps CO_2 emissions by allocating allowances to emitters who are then allowed to trade their permits; if a firm emits CO_2, it must surrender an equivalent amount of allowances. Economists argue that such a policy combines the conservative certainty on emissions offered by direct command-and-control regimes with the efficient allocation of abatement efforts of a carbon tax.

When multiple cap and trade schemes coexist, these can be linked. A linkage between two schemes reciprocally enables the use of permits issued in one scheme to meet compliance obligations pursuant to another. Linking is seen as a promising development in cap and trade regulation (Mehling et al., 2018). Article 6 of the Paris Agreement expressly provides for the possibility of linking, and Subarticle 12 of the California Cap on Greenhouse Gas Emissions and Market-Based Compliance Mechanisms sets out guidelines for the execution of a linkage. Linking has become increasingly prominent in recent years. California's cap and trade system linked with Quebec's on 1 January 2014 and the schemes organize joint auctions.[2] On 1 January 2020, a link between the European Union's Emissions Trading System (EU ETS) and the Swiss Emisions Trading System went into force. Linkages between the Regional Greenhouse Gas Initiative (RGGI) and the Emissions Trading Systems of Virginia and Pennsylvania are currently on their way, as are implicit linkages between California's ETS and Washintong State's Clean Air Rule.[3]

Linkages can be efficient because they allow for trades of allowances that lead to an ex post equalization of marginal abatement costs across jurisdictions without affecting global emissions or climate change. An additional benefit may be that, through their increased cooperation, local planners are less likely to choose their policies noncooperatively (Mideksa and Weitzman, 2019), leading to a more efficient tradeoff between the costs of climate change and emission abatement.

This paper proposes a theory of optimal linking under abatement cost uncertainty.

[1] Some history: "The first major emissions trading system (ETS) for greenhouse gases – the European Emissions Trading System (EU ETS) – was established in 2005. To date, there are 20 ETSs in place across five continents and covering 27 jurisdictions which produce almost 40 % of global wealth (GDP). With over a dozen more governments considering or having already scheduled an ETS, emissions trading has emerged as a key instrument to cost effectively decarbonize our economies." (ICAP, 2020)

[2] Starting from 2018, the ETS of Ontatio was also part of this linked system. However, half a year after the link was formally established, the Ontario government revoked its cap and trade regulation, thus withdrawing it from the linked carbon market.

[3] The latter link is mostly hypothetical, as Washinton State's Clean Air Rule was suspended after a 2018 court ruling. Though contested, the ruling was largely upheld by the Washington State Supreme Court on Jan. 16, 2020.

For a linkage between cap and trade schemes to be optimal, it must combine two key properties. First, it should be possible to fulfill compliance obligations pursuant to one jurisdiction by means of emission allowances issued by the other. In practice this means that polluters in either jurisdiction should be allowed to trade emission allowances freely and on an one-to-one basis. Second, there must be a pre-announced rule according to which the global cap is adjusted in response trades of allowances between schemes.

Free trade of allowances ensures that firms equate marginal abatement costs both within and across schemes. This is necessary for an optimal linkage because as long as marginal abatement costs are not equal across jurisdictions, mutually beneficial exchanges of allowances can be made, thus contradicting the notion of efficiency. The intuition is essentially the same as that favoring cap and trade over more direct command and control policies in a single jurisdiction. The literature on linking has often emphasized this channel of potential efficiency gains (Carbone et al., 2009; Flachsland et al., 2009; Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). However, there is more to trading that the mere equalization of marginal abatement costs.

Allowances are traded when marginal abatement costs differ across jurisdictions. The magnitude of trade flows provides a sufficient statistic for the gap in marginal abatement costs under the initial allocation. Knowledge of this gap allows the planners to construct a posterior belief on realized abatement costs in both jurisdictions. Since this posterior will generally be different from their prior – these are confirmed only if no trades take place – the planners thus learn that the initial allocation of emission allowances was inefficient for two reasons. First, the distribution of allowances between the schemes, given the global cap, was suboptimal. This inefficiency is dealt with through trading. But second, the cap itself may turn out ex post inefficient from the point of view of global welfare.

To see how a flow of allowances between jurisdictions signals that the global cap is set inefficiently, note that a cap and trade policy prices emissions to let polluting firms incorporate the social cost of climate change into their production decisions. The planners thus try to strike a balance between the cost of climate change on the one hand, and the cost of abatement on the other. When abatement costs turn out to deviate from planners' expectations, that balance is lost. In this sense, linkages that allow for one-to-one trading implement a constrained Pareto efficient distribution of abatement efforts only. Emission levels are ex post inefficient. We therefore propose to adjust the global cap in response to trade between the jurisdictions. To our knowledge, we are the first to study such endogenous cap adjustments in the context of linked cap and trade schemes.

Interestingly, what drives cap adjustments is not the amount of uncertainty per se, but rather the degree to which uncertainty is asymmetric between jurisdictions. This has a clear intuition. Trade flows signal a wedge in jurisdictional marginal abatement costs. Learning about the *absolute* level of abatement costs occurs when planners update beliefs for the least predictable jurisdiction on their beliefs about costs in the more predictable one. In the extreme case where the planners are uncertain about abatement costs in one jurisdiction only, this allows them to pin down abatement costs in the uncertain jurisdiction exactly on the basis of trade flows alone. Since in this case there is de facto complete information about abatement costs, the planners are able to implement the first-best social optimum.

Although we propose cap adjustments in response to allowance trading, simple "exchange rates" for allowances (*c.f.* Holland and Yates, 2015) can never be optimal. An efficient allocation of abatement efforts equalizes marginal abatement costs across schemes. When emissions cannot be traded one-for-one (or, better said, ton-for-ton), firms have an incentive to trade beyond the perfect equalization of marginal abatement costs. To illustrate, suppose we were to contemplate a exchange rate such that 2 allowances issued by jurisdiction N can be traded against 1 allowance issued in jurisdiction S. Then firms in N and S will trade allowances until the marginal cost of reducing emissions in N equals *twice* the marginal cost of reducing emissions in S. A exchange rate thus drives firms' incentives away from an efficient distribution of abatement efforts and cannot be optimal.[4]

Cap adjustments based on the allowance price are another alternative to our optimal linking policy. Compared to crude quantities, prices are highly efficient information aggregators. Indeed, when the planners observe both emissions and the market price for allowances, they can perfectly back out the abatement cost functions in both jurisdictions. The possibility of using price information hence allows for implementation of the unconstrained best cap on emissions. Our first and foremost recommendation is therefore to adjust the caps in response to carbon prices. Importantly, such price-based interventions need to be continuous in allowance prices – it is unclear but seems probable that our quantity-based optimal linkage performs better than a simple fixed price collar. Our theory describes the best alternative to continuous price-based cap-adjustments.[5]

Linking cap and trade systems across jurisdictions is related to linking cap and trade

[4]The same need not apply to local pollutants like NO_X or lead pollution.

[5]An important example to which our theory applies is any linkage involving the EU ETS. Despite repeated calls for a European price floor (Flachsland et al., 2020), the European Union uses only information on quantities for cap-adjustments. Linkages between any cap and trade scheme and the EU ETS will hence benefit from our analysis, "key features for compatibility for linking" being "complications around intervention in price" (EU Commission, 2015, EU ETS Handbook)."

markets over time (Yates and Cronshaw, 2001). The latter type of dynamic integration was studied in Heutel (2020) and Pizer and Prest (2020) for flow externalities, and in Gerlagh and Heijmans (2020) for stock externalities. They show that smart dynamic instruments can (greatly) improve welfare. But it matters how the cap is endogenized. For EU ETS, Gerlagh and Heijmans (2019) and Gerlagh et al. (2021) illustrate several undesirable side-effects of endogenous intertemporal emission caps. This literature therefore offers an important lesson for linking jurisdictional cap and trade schemes: adjusting the aggregate cap to trade flows can be efficient, but the devil is in the details.

Finally, we revisit the perennial question of instrument choice (Weitzman, 1974). Clearly, the choice between a carbon tax or a cap and trade policy is tilted toward the latter when an optimal linkage is implemented. The reason is the greater expected welfare performance of a cap and trade policy with optimal linkages compared to either cap and trade with standard linking or no linking at all. Roughly speaking, the case for cap and trade is twice as strong when independent schemes are optimally linked.

The paper is organized as follows. Section 2 introduces our model and the building blocks for welfare analysis. Section 3 discusses different types of (integrated) cap and trade policies and develops our theory of optimal linking. Section 4 concludes. Proofs and lengthy derivations are in the Appendix.

2 Model

Given are two jurisdictions, North and South, each operating its own cap and trade scheme. The assumption of two jurisdictions is not restrictive. One may simply consider North a representative jurisdiction for two linked jurisdictions, East and West, where trade between North and South is essentially a reduced-form way of writing trade between East, West, and South.

Each jurisdiction i is populated by firms that produce a composite good the production of which causes emissions. We may write benefits $B_i(\tilde{e}_i; \theta_i)$ as a function of emissions $\tilde{e}_i$, given by:

$$B_i(\tilde{e}_i \mid \theta_i) = (p_i^* + \theta_i)(\tilde{e}_i - e_i^*) - \frac{b_i}{2}(\tilde{e}_i - e_i^*)^2. \tag{1}$$

Emissions yield benefits because they allow firms to produce goods and save on the cost of abatement. As an umbrella term, we refer to $B_i(\tilde{e}_|\theta_i)$ as abatement costs in jurisdiction i, but other interpretations are possible. The parameter θ_i is a fundamental of jurisdiction i's economy and is *private information* of its constituent firms, though it is common knowledge that $\mathbb{E}[\theta_i] = 0$, $\mathbb{E}[\theta_i^2] = \sigma_i^2$, and $\mathbb{E}[\theta_N \theta_S] = \rho\sigma_N\sigma_S$. We interpret σ_i^2 as a measure for the uncertainty about jurisdiction i's economy. We say that uncertainty is asymmetric if $\sigma_N \neq \sigma_S$.

Emissions accumulate in the atmosphere and cause climate change as a global externality. The cost of climate change is given by:

$$C(\tilde{e}_N + \tilde{e}_S) = p^*(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*) + \frac{c}{2}(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*)^2. \quad (2)$$

We make the simplifying assumption that emissions have local benefits but global costs. One could imagine more complicated settings where emissions also have strictly regional costs like local air pollution (Caplan and Silva, 2005). A model of this type, however, will have many moving parts which would distract us from the core question we are primarily concerned with: the optimal linking of cap and trade schemes to regulate a global externality.

Subtracting the costs of climate change from the (implicit) benefits due to emissions yields our measure of global welfare:

$$W = B_N(\tilde{e}_N \mid \theta_N) + B_S(\tilde{e}_S \mid \theta_S) - C(\tilde{e}_N + \tilde{e}_S). \quad (3)$$

We assume that policy variables in each jurisdiction are chosen to maximize (3). Thus, local emissions are set with the global social cost of carbon (Kotchen, 2018) in mind. There are four motivations to our assumption. First, we want to formulate most efficient linking policy possible in a world characterized by uncertainty. For such global efficiency, global welfare is the relevant criterion. Second, most if not all cap and trade schemes currently operative are policymakers' attempts to meet mitigation obligations implied by the Paris Agreement's pledge to try and limit climate change to 1.5°C above pre-industrial levels. Since the Paris Agreement is global in scope and intention, cap and trade schemes set up to satisfy the associated pledges operate – at least implicitly – with a measure of global climate damages in mind.[6,7] Third, the act of linking cap and trade schemes

[6] As an example, "[...] the [European] Commission proposed in September 2020 to raise the 2030 greenhouse gas emission reduction target, including emissions and removals, to at least 55% compared to 1990. [...] This will enable the EU to move towards a climate-neutral economy and implement its commitments under the Paris Agreement by updating its Nationally Determined Contribution." Moreover, "[t]o achieve the EU's overall greenhouse gas emissions reduction target for 2030, the sectors covered by the EU ETS must reduce their emissions by 43% compared to 2005 levels." Retrieved from https://ec.europa.eu/clima/policies/strategies/2030_en.

[7] As another example, the Regional Greenhouse Gas Initiative's 2005 Memorandum of Understanding explicitly mentions global damages as a key factor governing its operations, writing that "climate change poses serious potential risks to human health and terrestrial and aquatic ecosystems globally... To address global climate change and in order to do their fair share in addressing their contribution to this collective

presupposes a certain amount of cooperation or mutual agreement on the cap. Indeed, among the "essential criteria" to ensure "compatibility between the systems" mentioned up in Annex I of the *Agreement between the European Union and the Swiss Confederation on the linking of their greenhouse gas emissions trading systems* are the "ambition and stringency of the cap". Similarly, the independent but linked cap and trade schemes of California and Quebec organize joint auctions of allowances, implying a degree cooperation in determining the linked caps. Finally, we are mostly interested in the potential welfare gains of our optimal linking regime compares to standard or classic linking. Thus,

For an clear decomposition of the welfare gains due to linking when regional planners take global damages into account only once a linkage is established, the reader is referred to Doda et al. (2019).

We write $\tilde{E} = \tilde{e}_N + \tilde{e}_S$ and $E^* = e_N^* + e_S^*$. Our model is now characterized by three curves and eight parameters $(b_i, c, p_i^*, e_i^*, p^*)$. For the system to be identified, we need two parameters per curve (slope and intercept) for three curves in total (2 benefit and 1 cost). This makes for a total of six parameters. Consequently, we may take the freedom to reduce the number of parameters through defining $p^* = p_N^* = p_S^*$, with the convenient implication that (p^*, e_N^*, e_S^*) is the vector of welfare-maximizing prices and emissions for jurisdiction i, given $\theta_N = \theta_S = 0$. We label this the ex-ante optimum. This is clearly not an assumption, nor even a normalization – it is a definition.

Firms are profit maximizers. Once a policy k caps emissions at a level $\tilde{e}_i^k$, individual firms trade allowances until marginal abatement costs for all are equal to:

$$p_i^k = -b_i e_i^k + \theta_i, \tag{4}$$

which is firms' inverse demand for allowances. In a competitive market for allowances, the price at which permits are traded will be p_i^k in equilibrium.

Before proceeding to the analysis, we introduce some further notation. Superscripts will be scenario (instrument) labels for equilibrium outcomes. Moreover, let $\tilde{x}^k$ denote the value of a variable x under policy k, then let $x^k := \tilde{x}^k - x^*$ be the deviation of x under policy k from the ex-ante expected optimal value x^*, and let $\Delta^k x := \tilde{x}^k - x^{SO}$ denote the difference between the value of x under scenario k and its ex post socially optimal value (see subsection 2.1).

problem while preserving and enhancing the economic welfare to their residents, the Signatory States find it imperative to act together to control emissions of greenhouse gases, particularly carbon dioxide, into the Earth's atmosphere from within their region."

2.1 Global Social Optimum

In a perfect world without informational frictions – the Social Optimum – a welfare-maximizing planner allocates emissions between the jurisdictions in such a way that marginal benefits are the same in both, i.e. so that $MB_N^{SO} = MB_S^{SO} = MB^{SO}$. Since both jurisdictions by assumption operate a cap and trade scheme, marginal benefits of emissions are also equal to the carbon price, so $p_N^{SO} = p_S^{SO} = p^{SO} = MB^{SO}$. Next, since the level of emissions if efficient if and only if marginal climate costs equal marginal benefits in either jurisdiction, we obtain the following conditions for the socially optimal emission levels:

$$c \cdot (e_N^{SO} + e_S^{SO}) = p^{SO} = -b_i e_i^{SO} + \theta_i. \tag{5}$$

Equation (5) characterizes the Social Optimum and represents three equalities in three unknowns: e_N^{SO}, e_S^{SO}, and p^{SO}. Solving for these, we obtain:

$$p^{SO} = \frac{c(b_S\theta_N + b_N\theta_S)}{cb_N + cb_S + b_N b_S}, \tag{6}$$

$$e_i^{SO} = \frac{b_{-i}\theta_i + c(\theta_i - \theta_{-i})}{cb_N + cb_S + b_N b_S}, \tag{7}$$

$$E^{SO} = \frac{b_S\theta_N + b_N\theta_S}{cb_N + cb_S + b_N b_S}, \tag{8}$$

where $i = N, S$ and $-i$ simply means "the jurisdiction that is not i". As is intuitive, socially optimal emission levels are higher when abatement is more expensive. This basic observation will be useful later on, when we illustrate that trades of allowances between (linked) schemes signal private information about abatement costs to the planners. An optimal linkage exploits this information by adjusting the global cap in response.

for future reference, we note that the variance of prices is given by:

$$\mathbb{E}\left[\left[p^{SO}\right]^2\right] = \left(\frac{c}{c \cdot b_N + c \cdot b_S + b_N b_S}\right)^2 \left[b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho \sigma_N \sigma_S\right]. \tag{9}$$

Thus, abatement cost uncertainty translates into price volatility. We will return to this later.

2.2 Welfare Losses

We rank policies according to their expected welfare performance, which is equivalent to ranking policies according to the expected welfare loss relative to the Social Optimum.

We perform our analysis in terms of the latter for notational convenience.

Suppose a policy k induces emission levels $\tilde{e}_i^k$ in jurisdiction i. From firms' equilibrium behavior (4), we see that deviations in emissions from the social optimum scale with prices:

$$\Delta^k p_i = -b_i \Delta^k e_i. \tag{10}$$

Expected welfare losses relative to the social optimum are then given by:

$$\begin{aligned} L^k &= \mathbb{E}\left[\Delta^k B_N + \Delta^k B_S - \Delta^k C\right] \\ &= \frac{c}{2}\mathbb{E}\left[\left(\Delta^k E\right)^2\right] + \sum_i \frac{b_i}{2}\mathbb{E}\left[\left(\Delta^k e_i\right)^2\right]. \end{aligned} \tag{11}$$

Throughout the analysis, we will interpret a policy as the implementation of a constrained expected welfare maximization problem. The level of welfare in the Social Optimum, W^{SO}, is hypothetical and unaffected by the policy implemented. Thus, expected welfare maximization subject to a set of policy constraints can be treated as the dual problem of expected welfare loss minimization relative to the Social Optimum, subject to the same policy constraints. Given the equivalence between these two approaches, we may use them interchangeably for convenience.

Linking policies exhibit equal prices across jurisdictions in equilibrium, as we shall explain below. By (10), emissions (both local and global) under such policies can therefore be expressed in terms of the common price gap Δp. Plugging this into (11), expected welfare losses from a policy featuring equal prices across jurisdictions can be written as:

$$L^k = \frac{1}{2}\frac{(cb_N + cb_S + b_N b_S)(b_N + b_S)}{b_N^2 b_S^2}\mathbb{E}\left[\left(\Delta^k p\right)^2\right]. \tag{12}$$

3 Policies

3.1 Regional cap and trade

The simplest policy operates two separate cap and trade schemes. In this case, the planner of jurisdiction i sets a cap e_i to maximize (3). Equivalently, the planners minimize expected welfare losses relative to the Social Optimum, subject to the constraint that emission levels in each jurisdictions are determined on the basis of expected abatement costs. The resulting allocation is $e_N = e_S = 0$. Plugging these into (11), we obtain expected welfare losses when both jurisdictions operate their own cap and trade schemes individually:

$$L^R = \frac{1}{2}\frac{(c+b_S)\sigma_N^2 + (c+b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{13}$$

At a more formal level, regional cap and trade is characterized as the solution to the following straightforward welfare maximization problem:

$$\max_{e_N, e_S} \mathbb{E} \quad W(e_N, e_S \mid \theta_N, \theta_S). \tag{14}$$

Thus, regional cap and trade is the optimal choice of instrument under the constraint that caps must be set before any information is revealed and without using any information extracted from regulated firms' behavior. The first-order condition of (14) is that expected marginal benefits in each scheme are equal to marginal climate damages:

$$\mathbb{E}[MB_i \mid e_i] = MC. \tag{15}$$

We note that the marginal damages (the RHS of (15)) are perfectly known when caps are fixed, whereas marginal benefits are stochastic variables due to unknown fundamentals θ_i. Thus, regional cap and trade implements socially optimal emission levels if and only if abatement costs turn out exactly as expected ($\theta_N = \theta_S = 0$). If abatement costs deviate from expectations, regional cap and trade is inefficient for two reasons. First, the allocation of abatement efforts between the jurisdictions may be ex post inefficient (more precisely, this happens when abatement costs differ between them). And second, the level of emissions is ex post inefficient. The first of these is remedied by linking schemes across jurisdictions.

3.2 Linking

When North and South link their cap and trade schemes, each planner sets the expected optimal cap for its jurisdiction but allowances issued in one scheme may be used to fulfill abatement obligations pursuant to another. Thus polluters are free to trade their allowances as long as global emissions are not affected:

$$e_N + e_S = E = 0. \tag{16}$$

Linking has attracted a lot of attention in recent years (Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). Just as trade in emission allowances between firms within a jurisdiction is an efficient way to achieve a given amount of local abatement, so the trade of allowances between jurisdictions is an efficient way to achieve a pre-determined amount of global abatement. The intuition is essentially the same: while linking does not affect global emissions, and therefore climatic damages, it does lead to a closer alignment of the benefits and costs of abatement across jurisdictions. In fact, as long as marginal abatement costs in the jurisdictions are *not* the same, firms can and will profitably exchange permits. The integrated market for emission allowances therefore reaches equilibrium only once the price of an allowance is the same in North and South:

$$p_N = p_S. \tag{17}$$

Moreover, since a firm in either jurisdiction is willing to sell (buy) an allowance against the going market price as long as the price is above (below) its private marginal abatement cost, marginal benefits are also the same across firms and jurisdictions when schemes are linked:

$$-b_N e_N^L + \theta_N = -b_S e_S^L + \theta_S, \tag{18}$$

where e_N and e_S are chosen by the firms conditional on θ_N and θ_S, subject to (16). Plugging this into our welfare loss-function (11), we obtain:

$$L^L = \frac{1}{2}\frac{1}{b_N + b_S}\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{19}$$

We now state our first substantive result.

Proposition 1. *Global welfare is always higher when cap and trade schemes are linked, compared to when they are not.*

In contrast to regional cap and trade, linking guarantees that marginal abatement costs are equalized across jurisdictions. For this reason, linking is always weakly better for welfare than regional cap and trade. We can also see this more formally when taking a look at the welfare maximization problem the planners (implicitly) solve when instituting a bilateral linkage between their jurisdictions:

$$\max_G \quad \mathbb{E}\left[\max_{e_N, e_S} W(e_N, e_S \mid \theta_N, \theta_S) \quad \text{s.t.} \quad e_N + e_S = G\right]. \tag{20}$$

Here, the maximization problem consists of two steps, each with an intuitive meaning. First, the planners of North and South cap local emissions at levels that, in expectations, maximize global welfare. Since the sum of local caps is by construction equal to the global cap, this is the ulterior maximization problem in (20). When the local caps are set, permits can be traded on a one-to-one basis between schemes, as long as emissions overall remain fixed at the sum of the two jurisdictional caps. This leads to the inner maximization problem in (20). By linking their schemes, the planners of North and South effectively delay the determination of local emission caps until after θ_N and θ_S are known, guaranteeing an ex post efficiency gain through marginal abatement cost equalization, which in expectations equal marginal damages:

$$MB = MB_N = MB_S \tag{21}$$

$$\mathbb{E}[MB \mid e_N + e_S] = MC. \tag{22}$$

Importantly, though linking benefits global welfare (Proposition 1), the effects on individual jurisdictions are ambiguous. A simple thought experiment illustrates. We note that prices in equilibrium equate marginal benefits, so that the volatility of prices is equal to the volatility of marginal benefits. Hence if abatement costs in North are much less predictable than in South, $\sigma_N > \sigma_S$, North may import part of the price volatility to which South is subject. If this effect is strong enough, aggregate price volatility may increase and North may be harmed by linking with South. This mechanism has been identified by the literature.

Linking suffers from another, more implicit type on inefficiency. It disregards valuable information. To see this, suppose that after trading the planners observe emissions levels e_N^L and e_S^L in jurisdictions N and S, respectively. Since firms will trade allowances between the jurisdictions until marginal abatement costs are equal everywhere, we know that (18) must be satisfied under the observed emission levels e_N^L and e_S^L. Rewriting (18), the planners therefore learn:

$$\mu := b_N e_N^L - b_S e_S^L = \theta_N - \theta_S. \tag{23}$$

The variable μ represents the vertical gap between the marginal abatement cost curves in the jurisdictions. Conditional on μ, planners update their beliefs on the true marginal abatement cost curve in both of the jurisdictions. Since this posterior will generally deviate from their priors, the expected optimal cap on emissions should ideally respond to the observed trade of allowances. It does not under standard linking.

3.3 Optimal Linking

We will now construct our optimal linking policy. We proceed in two steps. First, we derive the expected optimal emission level conditional on the trade flows between the jurisdictions. Second, we formulate a mechanism that is known to all firms and allows the planners to implement the expected optimal cap for any observed trade of allowances.

As we saw in equation (23), when post-trading emissions levels are e_N and e_S in jurisdictions N and S, respectively, the planners can back out μ, the vertical distance between regional marginal abatement cost functions:

$$\mu = b_N e_N - b_S e_S = \theta_N - \theta_S.$$

A key observation is that μ contains more information than the relative position of jurisdictions' abatement cost functions alone – it also signals something about the *absolute* location of the curves. To see this most simply, suppose that the planners are uncertain only about θ_N whereas θ_S is perfectly known (i.e. suppose that $\sigma_S = 0$). In this hypothetical case, observing μ is clearly equivalent to observing θ_N directly. In the more general case where both θ_N and θ_S are unknown, such sharp posteriors are not possible. Still the planners know all the combinations of θ_N and θ_S consistent with μ. Depending on the regional uncertainties σ_N and σ_S, some of these combinations will be more likely than others. The planners can therefore calculate the expected marginal abatement cost in both jurisdictions, conditional on μ and post-trade emissions:

$$\begin{aligned}
\mathbb{E}[MB \mid \mu] &= \mathbb{E}[\theta_N \mid \mu] - b_N e_N = \mu \frac{\mathbb{E}[\mu\theta_N]}{\mathbb{E}[\mu^2]} - b_N e_N \\
&= \mu \frac{\rho\sigma_N\sigma_S - \sigma_N^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} - b_N e_N \\
&= -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S. \qquad (24)
\end{aligned}$$

Conditional expected marginal abatement costs in both jurisdictions, as a function of post-trade emission levels e_N and e_S, are given by (24). Since the planners' goal is to achieve that marginal abatement costs, post-trade, are equal to marginal climate damages, the planners need to solve for e_N and e_S that solve:

$$c(e_N + e_S) = -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S. \qquad (25)$$

Classic linking, as discussed in the previous section, requires that global emissions do not change in response to trading, so $e_N = -e_S$. This makes the left-hand side of (25) equal to zero by construction. But the right-hand side of (25) is not, generally, equal to zero. Thus, conditional on the information that becomes available after allowances are traded, the initial cap on emissions turns out inefficient since (25) is violated. Indeed, it is not difficult to see that an expected efficient cap, given the private information revealed through allowance trading, satisfies the following elegant condition:

$$-\frac{e_S}{e_N} = \delta, \tag{26}$$

with δ a parameter that is endogenous to the structure of our model and given by:

$$\delta = \frac{b_N[\sigma_S^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}{b_S[\sigma_N^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}. \tag{27}$$

We refer to δ as the cap-adjustment rate since it prescribes how the global cap on emissions should be adjusted in response to the information revealed by allowances trading. Interestingly, the cap-adjustment rate δ may be negative. When this happens, higher-than-expected emissions in one jurisdiction translate into higher-than-expected emissions in the other jurisdiction as well. The reason is intuitive: if abatement costs in one jurisdiction are very unpredictable yet strongly correlated to those in the other, high abatement costs in the latter are likely matched by equally high costs in the former.

Recall that (26) solves (25) when the cap-adjustment rate is set equal to (27). Adjusted emission levels therefore implement:

$$\mathbb{E}[MB \mid e_N, e_S] = MC. \tag{28}$$

Before we proceed to formulating our Optimal Linking policy, note that it by construction is the most efficient type of linkage using information on emissions only. If the policy implements (28), then it yields emission levels that are optimal conditional on the most refined information distillable from allowance trading. This, indeed, is much more precise – and therefore efficient – than simply solving for the cap that solves $\mathbb{E}[MB \mid e_N + e_S] = MC$, which is the program a classic linking policy implicitly implements, see (22).[8]

We have now completed step one of our analysis of an optimal linking policy. Conditional

[8]That is, (e_N, e_S) provides more precise information than the mere aggregate $e_N + e_S$ since there is a continuum of combinations (e_N, e_S) that add up to the same sum. Moreover, this information can only benefit the planners. Thus optimal linking in expectations should give at least as good an outcome as standard linking.

on allowance trading, the expected efficient cap satisfies:

$$e_S + e_N + (\delta - 1)e_N = 0. \tag{29}$$

Note that (29) simply rewrites (26). The term $e_N + e_S$ is the same as we have seen for classic linking, see (16), and so we can think of the optimal cap as starting from the same level as a standard linking policy. Depending on how jurisdictions split the initial number of allowances, the cap is relaxed or tightened by an amount $(\delta - 1)e_N$.

With the preceding discussion in mind, we can formulate our Optimal Linking policy. Two properties stand out:

1. Firms exchange emission allowances between jurisdictions on a one-to-one basis. Thus, there cannot be an exchange rate on allowance trading;

2. Conditional on the number of allowances traded, the planners buy/auction a total of $(\delta - 1)e_N$ allowances.

Property 1 is crucial because exchange rates according to which an allowance worth one ton of emissions in North is worth $\alpha \neq 1$ tons in South cannot be optimal – see Section 3.4. Property 2 is needed to efficiently adjust the cap in response to private information revealed through trading. Knowing that the optimal linkage implements (25), we can derive expected welfare losses from an Optimal Linking policy.

Proposition 2. *Optimal Linking is the best possible policy involving only information on quantities. Expected welfare losses under an Optimal Linking policy are given by:*

$$L^O = \frac{1}{2}\frac{b_N + b_S}{cb_N + cb_S + b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{30}$$

The first part of Proposition 2 follows from the fact that an optimal linkage implements (28) by construction.

An optimal linkage can perform remarkably well. When uncertainty about abatement costs is strongly asymmetric across jurisdictions ($\sigma_N/\sigma_S \to 0$ or $\sigma_N/\sigma_S \to \infty$), or when abatement costs a highly correlated ($\rho \to \pm 1$), optimal linking leads to welfare levels very close to the full information Social Optimum. This reflects the planners' great scope for learning in these cases.

Corollary 1. *Where the planners have perfect information about one of the two linked jurisdictions ($\sigma_i = 0$ for $i \in \{N, S\}$), or when abatement costs are perfectly correlated ($\rho = \pm 1$), Optimal Linking implements the first best levels of emissions.*

Even though asymmetric *information* leads to welfare losses, asymmetric *uncertainty* compensates for part (and, in extreme cases, all) of these losses.

Another way to see the great advantage of an optimal linkage is to compare its welfare performance with that of a classic linking policy:

$$\frac{L^O}{L^L} = \frac{(b_N^2 + b_S^2 + 2b_n b_S)(1+\rho)\sigma_N\sigma_S}{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_n b_S\rho\sigma_N\sigma_S} \cdot \frac{(1-\rho)\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{31}$$

As we expect from Corollary 1, an optimal linkage performs much better than classic linking ($L^O/L^L \to 0$) when uncertainty is highly asymmetric or when abatement costs are strongly correlated. If we believe that abatement costs are driven by macroeconomic conditions they likely are strongly correlated across jurisdictions indeed – this makes a compelling case for optimal (rather than a standard) linking of the cap and trade schemes. Interestingly, though the absolute levels of welfare losses under both classic an optimal linking depend on climate damages through c, the relative performance of an optimal linkage is independent of climate damages.

Though global welfare increases after establishing an optimal linkage, individual jurisdictions may still be worse off. A jurisdiction in which abatement costs are relatively predictable may, through linking, expose itself to the volatile abatement costs in the other jurisdiction and therefore import a variable allowance price (Holtsmark and Weitzman, 2020). To the extent that such concerns are important for real world linkages, our optimal linking policy offers some relief.

Proposition 3. *Optimally linked cap and trade schemes admit lower price volatility than classically linked cap and trade schemes:*

$$\mathbb{E}\left[\left(p^{OL}\right)^2\right] \leq \mathbb{E}\left[\left(p^{L}\right)^2\right]. \tag{32}$$

Note that with inter*temporal* trading of permits, an endogenous cap can also leads to lower price volatility, see Gerlagh et al. (2020).

Our Optimal Linking is the best the planners can do when using only information on quantities to update their beliefs about abatement costs. From a theoretical point of view, the limitation to quantity-information is arbitrary. If the planners are willing to use both (post-trade) emissions and allowance prices, the can perfectly back out the marginal abatement cost function in each jurisdiction i. To see this, rewrite firms' inverse demand function (4) to obtain:

$$\theta_i = p + b_i e_i. \tag{33}$$

An ideal policy uses information on both prices and quantities to back out θ_i in each jurisdiction $i = N, S$ and, given these, adjusts the caps so that emission levels end up in the Social Optimum.

It is important to note that the ideal instrument is continuous in prices. A simple static price collar does not do the job. Optimal Linking almost surely outperforms price-collar-based cap adjustments when uncertainty is highly asymmetric or when abatement costs are strongly correlated. In these cases, an optimal linkage implements welfare levels very close to the Social Optimum – a price collar achieves the same feat only if it is set extremely tightly around the ex post optimal price level, which seems implausible. To our knowledge, several real world cap and trade schemes introduced price floors and/or ceilings (a notable exception being EU ETS), but none operate the kind of continuous price-based cap-adjustment policy we argue is ideal. Thus, with regard to linkages a strong case can be made for our Optimal Linking instrument.

3.4 Exchange Rates

As an alternative to the kind of optimal linking introduced in Section 3.3, the planners might consider exchange rates for allowance trading. Superficially, such a policy does the same as our proposal. It also allows for free inter-scheme trade of allowances while adjusting the global cap in response to trading. On top of that, it is much simpler. Unfortunately, it does not do the job.

To see why an exchange rate for allowance trading cannot work, suppose that North and South agree on an exchange rate of α on allowances from North. Thus, if a firm in North sells 1 ton of CO_2 worth in allowances to a firm in South, the latter can increase its emissions by α tons. In that case, it is easy to see that firms will trade allowances between jurisdictions until the point where prices satisfy:

$$p_N = \alpha \cdot p_S. \tag{34}$$

Equation (34) is a no-arbitrage condition. If (34) is violated, firms could buy allowances in one jurisdiction and sell them in the other at a gain. Allowance prices must therefore satisfy (34) if the linked schemes are in equilibrium.

There is a second equilibrium condition and we have seen it before. Firms in jurisdiction i will trade allowances until marginal abatement costs are exactly equal to the allowance price. That is, condition (4) needs to be satisfied so that $p_i = MB_i$ for each jurisdiction i. If we combine this constraint with (34), we obtain the following equilibrium characterization:

$$MB_N = \alpha \cdot MB_S. \tag{35}$$

Thus, if trading of allowances between jurisdictions is subject to an exchange rate α, then the linked markets are in equilibrium if and only if regional marginal abatement costs scale by α. A policy of exchange rates creates incentives for firms to trade allowances to the point where marginal abatement costs are unequal across jurisdictions.

Proposition 4. *Exchange rates on allowance trading are inefficient.*

3.5 Prices vs. Quantities

We saw how jurisdictions can optimally link their cap and trade schemes. Alternatively, each jurisdiction might also a levy a carbon tax. We now revisit the classic question of Weitzman (1974) on the choice between instruments.

We assume that emissions are taxed at an expected optimal rate:

$$\begin{aligned} \max_{p_N, p_S} \quad & \mathbb{E} \quad B_N(e_N \mid \theta_N) + B_S(e_S \mid \theta_S) - C(e_N + e_S) \\ \text{s.t.} \quad & p_i = -b_i e_i + \theta_i, \end{aligned} \tag{36}$$

where the constraint follows from profit maximization by the firms (4). Recall from our discussion following equation (3) that $p_N^* = p_S^*$ were defined as the expected welfare-maximizing carbon price in each jurisdiction, so the solution to (36) is $p_N = p_S = 0$. We may therefore invoke (9) and (12) to derive expected welfare losses when both jurisdictions tax emissions:

$$L^{tax} = \frac{1}{2}\left(\frac{c}{b_N b_S}\right)^2 \frac{b_N + b_S}{cb_N + cb_S + b_N b_S}(b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S). \tag{37}$$

All else equal, the expected welfare loss when jurisdictions tax emissions is increasing in c, the marginal climate damage. We can now compare (37) and (30), which yields the following proposition.

Proposition 5. *Taxes are favored over optimally linked cap and trade schemes if and only if:*

$$\left(\frac{c}{b_N b_S}\right)^2 < \frac{(1+\rho)\sigma_N\sigma_S}{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}\frac{(1-\rho)\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} \tag{38}$$

When the two jurisdictions are entirely symmetric and have the same abatement cost function, i.e. $b_N = b_S$ and $\sigma_N = \sigma_S$, (38) reduces to a simple inequality:

$$2c < b. \tag{39}$$

The condition to favor regional, non-linked cap and trade schemes over a carbon tax in this symmetric case becomes:

$$4c^2 < (1-\rho)bc + b^2. \tag{40}$$

Comparing (39) and (40), we see that the minimum marginal climate damages at which optimally linked cap and trade schemes are preferred over an emissions tax may be as little as half the marginal climate damages at which standard, non-linked cap and trade performs better. Thus, if we believe that climate change is the “existential threat of our time”,[9] meaning that c is high, a policy of optimal linking between regional cap and trade schemes may vastly expand the range of (expected) abatement costs for which cap-and-trade is favored over a carbon tax. This hints at the importance of our preliminary analysis for climate mitigation debates in the real world.

4 Summary

We propose a simple theory of optimal linking. Fairly straightforward manipulations of standard linking between cap and trade schemes are shown to increase global and jurisdictional welfare. The core of our argument is that the trade of allowances between local schemes signals valuable information which an efficient policy incorporates. Practically, our proposal to adjust the cap in linked schemes in response to allowance trading. We pin down an exact analytic formulation for this endogenous cap-adjustment and discuss its properties.

Optimal linkages allow permits to be exchanged one-to-one. An “exchange rate” that differs from one, though de facto shifting the aggregate cap indeed, would only disturb individuals firms’ incentives away from the exact equalization of marginal abatement costs across schemes, which is inefficient. To implement an optimal linkage, the aggregate cap must be adjusted in response to trade, but not through trade. A straightforward way to achieve this is to either inject new permits or buy back already issued ones as

[9]The quote is taken from a speech by then-President-elect Biden on December 19, 2020, as he was nominating people to lead the EPA under his administration. See https://www.bloomberg.com/news/articles/2020-12-19/biden-calls-climate-change-existential-threat-of-our-time

called for by the observed trade flows (Hintermayer, 2020, analyzes a buyback policy in a dynamic model of EU ETS). Upward adjustment of the cap may be easy to achieve as it only requires the issuing of extra permits. Adjusting the cap downward may be more problematic, however, calling for planners to spend money on buying allowances back. This may cause distributional problems: which scheme ought to spend the money? A possible way to mitigate this concern is by having linked schemes hold joint auctions, using the revenue of the joint auction to buy back permits. Joint auctions have a political track record: they are used by the linked schemes of California and Quebec.

An important concept is asymmetric uncertainty. When two schemes are linked, information on *relative* abatement costs is revealed through trade. But when the schemes are asymmetrically uncertain, this information on relative abatement costs be used to infer (sharp) predictions about *absolute* abatement costs in either jurisdiction. The same is not possible when trade between symmetrically uncertain schemes is observed. Our results suggest that the study of asymmetric uncertainty deserves a more prominent place in environmental economics.

Cap and trade schemes have become a major policy instrument in the fight against climate change. In Europe alone, roughly 45% of greenhouse gas emissions are regulated by EU ETS, the world's largest market for carbon. With ever more cap and trade being erected, linking of regional markets has become a prominent policy development. Linkages between jurisdictional schemes already exist, and more are currently being contemplated. The are linkages between EU ETS and the Swiss ETS, between RGGI and Quebec, between Quebec and California. The up-and-coming carbon markets of China and the post-Brexit UK will create even more possibilities for linking. Given the large amount of money and CO_2 involved in cap and trade, and given the increasing prevalence of linkages between jurisdictional schemes, constructive ideas on optimal linking are needed. This paper offers some thoughts.

Although we discuss linking of cap and trade schemes at the level of a jurisdiction, our methods apply just as well to cases where an existing cap and trade scheme is expanded to cover additional sectors. Consider the aviation sector. Passenger flights outside the European Economic Area are not covered by the EU ETS. To nevertheless reduce emissions in the aviation industry, "the International Civil Aviation Organization (ICAO) agreed on a Resolution for a global market-based measure to address CO2 emissions from international aviation as of 2021. The agreed Resolution sets out the objective and key design elements of the global scheme, as well as a roadmap for the completion of the work on implementing modalities. The Carbon Offsetting and Reduction Scheme for International Aviation, or

CORSIA, aims to stabilise CO2 emissions at 2020 levels by requiring airlines to offset the growth of their emissions after 2020."[10] Though it is still unclear how exactly CORSIA and EU ETS will interact, our analysis suggests that a direct incorporation of CORSIA into the EU ETS may be suboptimal. Similarly, our results suggest that trade of allowances between sectors or clearly identified jurisdictions within an existing cap and trade scheme – think of countries in the EU ETS or states in RGGI – can be made more efficient by incorporating a policy along the lines of our optimal linking regime.

Our theory is preliminary and does not address several aspects of real life emission trading. Most large-scale cap and trade schemes are dynamic and allow covered industries to bank (and sometimes borrow) allowances across periods.

Our theory of optimal linking follows a purely quantity-based approach. We take (flows of) emissions as an input to get aggregate emissions as an output. One could instead focus on prices, or a combination of prices and emissions.

We leave an exploration of these and other aspects for future research.

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A Derivations and Proofs

DERIVATION OF (19):

Combining the definition with the firms' FOCs, (4), we find the change in permit use by jurisdiction:

$$\Delta^L e_N = \frac{\theta_N - \theta_S}{b_N + b_S} \tag{41}$$

$$\Delta^L e_S = \frac{\theta_S - \theta_N}{b_N + b_S}. \tag{42}$$

PROOF OF PROPOSITION 1:

Proof. We only need to compare welfare losses under a linked cap and trade regime, equation (19), to those under jurisdictional cap and trade, equation (13). Linking outperforms regional cap and trade iff:

$$\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{b_N + b_S} < (c + b_S)\sigma_N^2 + (c + b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S$$

$$\Longleftrightarrow$$

$$\frac{2\rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2} < \frac{cb_N + cb_S + b_N b_S}{cb_N + cb_S},$$

which is always true. □

DERIVATION OF (27):

Regional and global deviations from Socially Optimal permit use are given by:

$$\Delta^O e_N = \frac{b_S}{b_N + \delta b_S}\frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{43}$$

$$\Delta^O e_S = \frac{b_N}{b_N + \delta b_S}\frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{44}$$

$$\Delta^O Q = \frac{b_N + b_S}{b_N + \delta b_S}\frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S}. \tag{45}$$

Define

$$\xi := \frac{b_N + c(1-\delta)}{b_N + \delta b_S} \implies 1 - \xi := \frac{\delta b_S - c(1-\delta)}{b_N + \delta b_S}. \tag{46}$$

Welfare losses can now be written as:

$$\begin{aligned} L^O &= \frac{1}{2}\frac{c(b_N+b_S)^2+b_N^2b_S+b_Nb_S^2}{(cb_N+cb_S+b_Nb_S)^2}\mathbb{E}\left[(1-\xi)\theta_N+\xi\theta_S\right]^2 \\ &= \frac{b_N+b_S}{2}\frac{(1-\xi)^2\sigma_N^2+\xi^2\sigma_S^2+2\xi(1-\xi)\rho\sigma_N\sigma_S}{cb_N+cb_S+b_Nb_S}. \end{aligned} \tag{47}$$

If for notational convenience, we define:

$$\psi := \frac{1}{2}\frac{b_N+b_S}{cb_N+cb_S+b_Nb_S}, \tag{48}$$

it is straightforward to derive:

$$\frac{\partial}{\partial\xi}\frac{L^O}{\psi} = 2\xi\sigma_S^2-2(1-\xi)\sigma_N^2+2(1-\xi)\rho\sigma_N\sigma_S-2\xi\rho\sigma_N\sigma_S. \tag{49}$$

The welfare-maximizing ξ^* therefore satisfies:

$$\xi^* = \frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}. \tag{50}$$

From the definition of ξ, the optimal cap-adjustment rate δ^* follows:

$$\delta^* = \frac{(b_N+c)[\sigma_S^2-\rho\sigma_N\sigma_S]+c[\sigma_N^2-\rho\sigma_N\sigma_S]}{(b_S+c)[\sigma_N^2-\rho\sigma_N\sigma_S]+c[\sigma_S^2-\rho\sigma_N\sigma_S]}, \tag{51}$$

as given.

PROOF OF PROPOSITION 2:

Proof. Plugging (50) in (47), we find:

$$\begin{aligned} \frac{L^O}{\psi} &= \left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_N^2+\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_S^2 \\ &+\left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\rho\sigma_N\sigma_S \\ &= \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S} \\ &\Longrightarrow \\ L^O &= \frac{1}{2}\frac{b_N+b_S}{cb_N+cb_S+b_Nb_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}, \end{aligned}$$

as stated. This is strictly lower than the welfare loss under traditional Trading if and only if:

$$L^L - L^O \geq 0$$
$$\Longrightarrow$$
$$\frac{1}{b_N + b_S}\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S} - \frac{b_N + b_S}{cb_N + cb_S + b_N b_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} \geq 0$$
$$\Longrightarrow$$
$$(\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S)(b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S) - (1-\rho^2)(b_N^2 + b_S^2 + 2b_N b_S)\sigma_N^2\sigma_S^2 \geq 0$$
$$\Longrightarrow$$
$$[(b_S\sigma_N^2 - b_N\sigma_S^2) + (b_N - b_S)\rho\sigma_N\sigma_S]^2 \geq 0,$$

which is always true. □

PROOF OF PROPOSITION 3:

Proof. We derived quantity derivations under both policies. Prices are equal in both jurisdictions, so without loss of generality we can solve for price deviations in jurisdiction 1:

$$\Delta^L p_N = \frac{b_S\theta_N + b_N\theta_S}{b_N + b_S}$$
$$\Delta^{OL} p_N = \frac{\delta b_S\theta_N + b_N\theta_S}{b_N + \delta b_S}.$$

Thus:

$$\mathbb{E}\left[\left(\Delta^L p\right)^2\right] = \frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}{b_N^2 + b_S^2 + 2b_N b_S}$$
$$\mathbb{E}\left[\left(\Delta^{OL} p\right)^2\right] = \frac{\delta^2 b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2\delta b_N b_S\rho\sigma_N\sigma_S}{b_N^2 + \delta^2 b_S^2 + 2\delta b_N b_S}.$$

Writing these out, we obtain:

$$\mathbb{E}\left[\left(\Delta^{OL} p\right)^2\right] < \mathbb{E}\left[\left(\Delta^L p\right)^2\right] \iff (\delta - 1)\left[b_S\left(\sigma_N^2 - \rho\sigma_N\sigma_S\right) - b_N\left(\sigma_S^2 - \rho\sigma_N\sigma_S\right)\right] < 0.$$

Given (**??**), this condition is always satisfied. □