

# THE GLOBAL CLIMATE GAME\*

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## Abstract

I study contributions to a public good when contributions exhibit network effects. Network effects make the problem a coordination game with multiple Nash equilibria and open the door to coordination failures. I make two contributions. First, using the methodology of global games, I show how uncertainty about payoff functions facilitates selection of a unique equilibrium. Coordination failure occurs in well-identified cases. Second, I design a policy of network subsidies that corrects the entire externality deriving from network effects but does not, in equilibrium, cost the policymaker anything. I apply the model to climate change mitigation and the adoption renewable technologies.

## 1 Introduction

Economic theory predicts that private agents will underprovide public goods. However, this canonical result breaks down when the private benefits of the public good depend upon the network of other users (Katz and Shapiro, 1985). In such case, the agents face a coordination problem and provision can be an equilibrium of the implied coordination game. But provision is still not guaranteed as coordination games tend to have multiple equilibria. Two questions therefore arise. First, is it possible to identify conditions under which the good will be provided? And second, what does an optimal policy to guarantee provision look like? This paper provides a general answer to these questions. My results can be applied to contexts such as information technology adoption in developing economies (Jensen, 2007; Björkegren, 2019) or efforts toward disease eradication (Barrett, 2003). For concreteness and relevance, I focus on the example of climate change mitigation through the adoption of renewable technologies.

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There are at least two channels through which network effects and other strategic complementarities (Bulow et al., 1985) could arise in the context of renewables adoption. *Direct spillovers* occur if the use of renewables is directly beneficial for all other agents who also use renewables. Examples would be knowledge spillovers (Aghion and Jaravel, 2015; Aghion et al., 2016), network externalities (Barrett and Dannenberg, 2017), and – in a dynamic environment – learning-by-doing (Acemoglu et al., 2012; Van der Meijden and Smulders, 2017; Hart, 2019). Alternatively, there may be *indirect spillovers* in two-sided markets. These arise if the use of renewables stimulates some external factor which in turn make the use of renewables more attractive; the markets for electric vehicles and charging stations are typical examples (Li et al., 2017).

Strategic complementarities complicate the analysis of models of renewables adoption – and indeed of discrete public good provision generally – because they turn the decision problem into a coordination game. It is well known that coordination games tend to have multiple Nash equilibria, which raises questions about equilibrium selection and opens the door to coordination failures. This paper takes stock of both these issues.

My first contribution addresses equilibrium selection explicitly. I use the machinery of global games, first developed by Carlsson and Van Damme (1993), to show how uncertainty can help select a unique equilibrium. In well-identified cases, the unique equilibrium is inefficient. Somewhat paradoxically, rational players may maximize their expected payoffs by coordinating on an outcome (e.g. the large-scale use of fossil fuels) that is known to be inefficient. This dismal prediction is a consequence of *strategic uncertainty* – uncertainty that derives from players’ need to second-guess each other’s actions – and motivates policy intervention.

My second contribution is the design of an efficient policy that preempts coordination failure. The policy, called *network subsidies*, corrects the entire externality from technological spillovers but does not, in equilibrium, cost the policymaker anything. In contrast to standard subsidies, the amount of network subsidy a player receives is contingent not only on their own action but also on the actions pursued by all others. A policymaker can exploit this degree of freedom to offer a subsidy scheme that solves the coordination problem without requiring any payments to be made.

I derive these results in a bare-bones model of technological choices. Players choose between fossil fuels or a renewable technology. Associated with players’ choices are two externalities. The first is an environmental benefit which says that the use of renewables, rather than fossil fuels, benefits the environment and therefore everyone. The second is a network externality deriving from strategic complementarities in individual actions; one could think of technological spillovers. These externalities combined imply that players face a coordination problem with the possibility of multiple Pareto-ranked equilibria. The two externalities also imply that whenever multiple equilibria exist, coordination on renewables is efficient.

The global game adds *scientific uncertainty* about the environmental benefit from adopting renewables to the strategic uncertainty inherent to any coordination problem. From a known prior, Nature draws a true benefit  $b$  which the players do not observe. Instead, each player  $i$  receives a private noisy signal  $b_i^\varepsilon = b + \varepsilon_i$  of  $b$ , where  $\varepsilon_i$  is the error

term in player  $i$ 's observation. A strategy for player  $i$  is then a function that assigns to any signal  $b_i^\varepsilon$  a probability with which the player chooses renewables; a strategy vector is a vector of strategies for all players. I assume that the support of  $b_i^\varepsilon$  contains dominance regions: for sufficiently high [low] signals  $b_i^\varepsilon$ , using renewables [fossil fuels] is strictly dominant. This, together with some technical assumptions, allows me to show that there is a unique strategy vector that survives iterated elimination of strictly dominated strategies. It follows that the global game has a unique Bayesian Nash equilibrium.

How does adding uncertainty simplify the outcome of a game? The core of the argument is that, in the global game, a player must make probabilistic inferences about the signals received by other players. Consider then the case in which a player receives a signal just below the strict dominance threshold for renewables. Because their signal is close to a dominance region, the player assigns a strictly positive probability to the event that any of the other players received a signal *in* the dominance region; those players will definitely adopt renewables. This implies that the player, though their own beliefs would not exclude either technology per se, is forced to bound from above the probability that any other player chooses fossil fuels. Combined with the incentive to coordinate, those bounds tend to make renewables more attractive; each player can therefore extend the dominance region for renewables (and the same is true, starting from low signals, for fossil fuels). But we cannot stop here: having extended the dominance regions once, we can repeat the argument and must continue doing so indefinitely.<sup>1</sup> My first main result shows that this process of extending dominance regions for both technologies stops only when their boundaries touch.

It is immediate from the argument that the equilibrium selected is inefficient: by extending the strict dominance region for fossil fuels, coordination on renewables is preempted (with probability 1) for a range of games in which, under complete information, coordination on renewables would be an equilibrium. This motivates policy intervention. I study subsidies.

In particular, I study *network subsidies*. Network subsidies are a novel kind of policy that allow the policymaker to correct the entire externality deriving from technological spillovers but does not, in equilibrium, cost the policymaker anything. In contrast to regular subsidies, the amount of network subsidy each player is entitled to depends not only on their own action but also on the actions of all other players. I show that such a policy design allows to policymaker to implement the efficient equilibrium of the (underlying) coordination game in strictly dominant strategies. While the policy is costless in equilibrium, off equilibrium spending can be positive. To accomodate this, I also design a self-financed network tax-subsidy scheme that is always ex post budget neutral. Lastly, an alternative – though essentially equivalent – policy to network subsidies for two-sided markets characterized by indirect spillovers is also discussed.

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<sup>1</sup>Interestingly, this kind of infection argument is also found outside the realm of game theoretic models; see for example Krugman (1991) for an application to target zones for exchange rates.

## 1.1 Related literature

The coordination problem has long been recognized in the context of renewables adoption. While I treat spillovers abstractly, economists have discussed a number of factors from which such strategic complementarities could arise. These include (indirect) pure network effects (Katz and Shapiro, 1985; Greaker and Midttømme, 2016; Li et al., 2017), spillovers from R&D in “breakthrough” technologies (Barrett, 2006; Hoel and de Zeeuw, 2010), the existence of climate tipping points (Barrett and Dannenberg, 2017), political economy arguments such as climate clubs (Nordhaus, 2015, 2021), technological and knowledge spillovers (Fischer and Newell, 2008; Acemoglu et al., 2012; Aghion and Jaravel, 2015; Aghion et al., 2016; Harstad, 2016; Hart, 2019; Harstad, 2020), or even behavioral economic mechanisms such as social norms (Allcott, 2011) and reciprocity (Nyborg, 2018).

Mindful of the complications deriving from coordination incentives and equilibrium multiplicity, the profession has sought to address equilibrium selection in different ways. Two approaches are predominant. One from the outset restricts attention to a particular kind of equilibrium. Thus, players may be a priori assumed to pursue symmetric strategies (Harstad, 2012; Harstad et al., 2019) or to coordinate on the Pareto dominant outcome (Barrett, 2006; Hoel and de Zeeuw, 2010).<sup>2</sup> Another approach treats the coordination problem as theoretically indecisive and relies on laboratory experiments to make predictions (Barrett and Dannenberg, 2012, 2014, 2017; Calzolari et al., 2018). I complement these approaches in using global games to show how uncertainty about the technologies leads to equilibrium selection without the need for additional restrictions on players’ strategies or beliefs.

Subsidies to stimulate the use of renewable technologies are much discussed in the literature, for recommended examples see Joskow (2011), Murray et al. (2014), Allcott et al. (2015), Fowlie et al. (2015), Acemoglu et al. (2016), Borenstein (2017), Li et al. (2017), De Groote and Verboven (2019), Hart (2019), and Harstad (2020). My design of network subsidies complements this literature by designing a policy that stimulate renewables with different and – arguably – more desirable properties than the subsidies heretofore studied.

Network subsidies are also related to the literature on directed technical change and the environment (Acemoglu et al., 2012; Aghion and Jaravel, 2015; Aghion et al., 2016; Acemoglu et al., 2016; Hart, 2019). This literature studies the effect of policy on technology adoption when multiple and (partially) substitutable technologies co-exist with differential consequences for social welfare, the environment, and growth. Technologies are typically characterized as either clean or dirty and assumed to exhibit technology-specific positive spillovers, with the dirty technology starting off as more advanced. This literature asks how different kinds of policies – e.g. a carbon tax or R&D subsidies – can be used most efficiently to stimulate large-scale adoption of the clean technology. My contribution to this literature is to show how R&D subsidies, aimed at correcting the externality that derived from spillovers and other strategic

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<sup>2</sup>In addition, uniqueness of the equilibrium may be proven within the restricted strategy set.

comeplementarities in renewable investment, can be made substantially cheaper.

The derivation of network subsidies is also an exercise in mechanism design and implementation theory. The policymaker aims design a subsidy that makes coordination on the efficient outcome of the game a strictly dominant strategy for all players (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983). While mechanism design was applied to emissions mitigation before (Duggan and Roberts, 2002; Ambec and Ehlers, 2016; Martimort and Sand-Zantman, 2016), the focus has mostly been on policies to solve the free-rider problem. I complement this approach by designing policies that solve the coordination problem.

## 2 Basic Model

There are  $N$  players. Each player chooses between two goods, 0 and 1. Let  $x_i \in \{0, 1\}$  denote the good used by player  $i$ . If player  $i$  uses good 1, this gives a benefit  $b > 0$  to every player. Note that choosing 1 generates an externality in the amount  $b \cdot (N - 1) > 0$ ; hence, the use of good 1 is a discrete public good.<sup>3</sup> Let  $x = (x_1, x_2, \dots, x_N)$  denote the vector of actions played by all players, and let  $x_{-i} = (x_j)_{j \neq i}$  be the vector of actions by all players but  $i$ . Let  $\mathbf{1} = (1, 1, \dots, 1)$  be the action vector of all ones, and  $\mathbf{0} = (0, 0, \dots, 0)$  the action vector of all zeroes. The cost of using 0 is  $d$ , a constant. The costs of playing 1 instead depends upon the total number of players,  $n$ , that play 1 and are decreasing in  $n$ :  $c(1) > c(2) > \dots > c(N)$ . That is, the public good is also a network good (Katz and Shapiro, 1985) and the game exhibits strategic complementarities (Bulow et al., 1985). It is assumed that  $c(N) > d$ .<sup>4</sup> Note that, besides the social benefit  $b$  from using good 1, there may also be a private benefit to using either good; these benefits are henceforth considered subsumed by the cost function  $c(n)$ .

Combining these elements, the payoff to player  $i$  is:

$$\pi_i(x \mid b) = \begin{cases} b \cdot n(x) - d & \text{if } x_i = 0 \\ b \cdot n(x) - c(n(x)) & \text{if } x_i = 1 \end{cases}, \quad (1)$$

where  $n(x)$  is defined as the total number of players using good 1 in  $x$ ;  $n(x) = \sum_{i=1}^N x_i$ . I define  $n(x_{-i}) = \sum_{j \neq i} x_j$  to be the total number of players other than  $i$  that play 1 in  $x$ . The set of players  $\{1, 2, \dots, N\}$ , the set of action vectors  $x \in \{0, 1\}^N$ , and the set of payoff functions  $\{\pi_i\}$  jointly define a complete information game  $G(b)$ .<sup>5</sup> I write  $G(b)$  for the game of complete information (i.e. with common knowledge of  $b$ ) to differentiate

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<sup>3</sup>In some branches of the literature, the term “discrete public good” is used to describe public goods that are provided in some fixed quantity if and only if a minimum contribution threshold is reached, see for example Palfrey and Rosenthal (1984), Nitzan and Romano (1990), and McBride (2006). This paper uses the term discrete public good for any situation in which each player faces the discrete choice between contributing, and not contributing, to the public good.

<sup>4</sup>This assumption is of no technical importance for the analysis; it buys me the convenience of not having to discuss separately the cases where  $d < c(N)$  and  $d > c(N)$  in the welfare analysis.

<sup>5</sup>The payoff function (1) is extremely simple and one may wonder whether the results derived in

this game from the global game studied in Section 3 where players do not observe  $b$ . The choice of  $b$  as key parameter is made for convenience; one could choose other parameters instead.

It will prove convenient to analyze the game in terms of a gain function. The gain from using good 1, rather than good 0, to player  $i$  (given  $b$  and  $x_{-i}$ ) is the difference in their payoffs between playing  $x_i = 1$  and  $x_i = 0$ . For given  $x_{-i}$ , define the gain function  $\Delta_i$  as

$$\begin{aligned}\Delta_i(x_{-i} \mid b) &= \pi_i(1, x_{-i} \mid b) - \pi_i(0, x_{-i} \mid b) \\ &= b + d - c(n(x_{-i}) + 1).\end{aligned}\tag{2}$$

Moreover, if  $k = n(x_{-i})$  I write  $\Delta_i(k \mid b) = \Delta_i(x_{-i} \mid b)$ .

The action  $x_i = 1$  is strictly dominant for all  $b > c(1) - d$  as for those  $b$ s it holds that  $\Delta_i(x_{-i} \mid b) > 0$  for all  $x_{-i}$ . Alternatively,  $x_i = 0$  is strictly dominant for all  $b < c(N) - d$ . In between, the game has multiple equilibria. To smoothen notation, I shall henceforth write  $\bar{b} = \frac{c(N)-d}{N}$ . This dominance argument, combined with direct payoff comparisons, yields Proposition 1.

**Proposition 1.**

- (i)  $x = \mathbf{1}$  is a Nash equilibrium of the game for all  $b \geq c(N) - d$ . It is the unique Nash equilibrium for all  $b > c(1) - d$ .
- (ii)  $x = \mathbf{0}$  is a Nash equilibrium of the game for all  $b \leq c(1) - d$ . It is the unique Nash equilibrium for all  $b < c(N) - d$ .
- (iii)  $x = \mathbf{1}$  is strictly Pareto dominant for all  $b > \bar{b}$ .

If the social benefit from using good 1 exceeds the cost of using it, but the private benefits do not, then no player chooses 1 and one obtains the usual result that the public good gets underprovided. Similarly, if the private benefits from using good 1 are very high, the public good will be efficiently provided. Finally, when the benefit from using good 1 is moderately high, both coordination on using 1 and coordination on using 0 are Nash equilibria of the game. In this case, the players face a true coordination problem and the outcome of the game cannot be a priori predicted. The public good may or may not be provided as either outcome is an equilibrium.

Frankel et al. (2003) have observed that a game such as given by (1) is a *potential game* (Monderer and Shapley, 1996). A game in which each player has two actions is a potential game if there exists a potential function  $P : \{0, 1\}^N \rightarrow \mathbb{R}$  on action profiles such that the change in any player's payoff when switching from one action to the other is always equal to the change in the potential function; that is, for which there exists a

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this paper are due to its particular specification. Under some additional assumptions, an implication of Frankel et al. (2003) is that the results on equilibrium selection in the global game (Section 3) hold true more generally, see also the discussion following Theorem 1. A substantial theoretical generalization of my results on network subsidies (Section 4) is provided in ongoing work by Heijmans & Suetens.

function  $P$  such that  $P(x_i, x_{-i} | b) - P(1 - x_i, x_{-i} | b) = \pi_i(x_i, x_{-i} | b) - \pi_i(1 - x_i, x_{-i} | b)$  for all  $i$ . The game  $G(b)$  admits a potential function  $P(x | b)$  given by:

$$P(x | b) = \begin{cases} \sum_{k=0}^{n(x)-1} \Delta_i(k | b) & \text{if } n(x) > 0, \\ 0 & \text{if } n(x) = 0. \end{cases} \quad (3)$$

Observe that, for any  $i$  and any  $x_{-i} \in \{0, 1\}^{N-1}$ , it holds that  $P(1, x_{-i} | b) - P(0, x_{-i} | b) = \Delta_i(x_{-i} | b) = \pi_i(1, x_{-i} | b) - \pi_i(0, x_{-i} | b)$ , confirming that  $P$  is a potential function indeed.<sup>6</sup>

A *potential maximizer* is a vector  $x$  that maximizes  $P$ . One can verify that  $\mathbf{1}$  is the unique potential maximizer of  $P(x | b)$  for all  $b + d > \sum_{n=1}^N \frac{c(n)}{N}$  whereas  $\mathbf{0}$  is the unique potential maximizer of  $P(x | b)$  for all  $b + d < \sum_{n=1}^N \frac{c(n)}{N}$ . I return to this observation in the next section.

The presentation so far was technical. It relates to global warming as follows. Climate change mitigation requires large-scale reductions in the amount of greenhouse gases emitted into the atmosphere; to realize the necessary deep cuts in emissions, a coordinated switch to renewable technologies – and away from fossil fuels – is needed.

The use of renewables is hence good for the environment. Because improvements in environmental quality are beneficial for everyone, the player who uses renewables also provides a public good. It follows that good 1 can be thought of as a renewable technology; good 0, instead, is a fossil fuel technology. The problem of the players is then to decide which of the two technologies to use or, perhaps more realistically, whether to switch to renewables or not.

There are two externalities associated with using renewables. The first is an *environmental externality* and is captured by the parameter  $b$  – think of the benefits from reduced CO2 emissions. The second is a *network externality* and relates to the cost function  $c$ , i.e. it captures the fact that a player's use of renewables lowers the cost of renewables for all other players, be it through direct or indirect spillovers. The structure of multiple externalities deriving from the use of renewables is common in the literature on technological change and the environment (*c.f.* Acemoglu et al., 2012; Aghion and Jaravel, 2015; Acemoglu et al., 2016; Hart, 2019; Harstad, 2020).

### 3 The Global Climate Game

Coordination incentives drive equilibrium multiplicity under common knowledge of  $b$ . But the assumption of complete information is strong. There are large numbers of uncertainties surrounding many clean technologies's present or future potential.

*Uncertainty and signals.* For these reasons, I turn the problem into a global game. In the global game  $G^\varepsilon$ , the true parameter  $b$  is unobserved. Rather, it is assumed that  $b$

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<sup>6</sup>If  $n(x) > 1$ , the confirmation is as in the text. If, however,  $n(x) = 1$ , then one has  $P(1, x_{-i} | b) - P(0, x_{-i} | b) = \sum_{k=0}^0 \Delta_i(k | b) - 0 = \Delta_i(x_{-i} | b) = \pi_i(1, x_{-i} | b) - \pi_i(0, x_{-i} | b)$ , as desired.

is drawn from the uniform distribution on  $[\underline{B}, \overline{B}]$  where  $\underline{B} < c(N) - d$  and  $\overline{B} > c(1) - d$  and that each player  $i$  receives a private noisy signal  $b_i^\varepsilon$  of  $b$ , given by:<sup>7</sup>

$$b_i^\varepsilon = b + \varepsilon_i. \quad (4)$$

The term  $\varepsilon_i$  captures idiosyncratic noise in  $i$ 's private signal. It is common knowledge that  $\varepsilon_i$  is an i.i.d. draw from the uniform distribution on  $[-\varepsilon, \varepsilon]$ . I assume that  $\varepsilon$  is sufficiently small:  $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$ . Let  $b^\varepsilon = (b_i^\varepsilon)$  denote the vector of signals received by all players, and let  $b_{-i}^\varepsilon$  denote the vector of signals received by all players but  $j$ , i.e.  $b_{-i}^\varepsilon = (b_j^\varepsilon)_{j \neq i}$ . Note that player  $i$  observes  $b_i^\varepsilon$  but neither  $b$  nor  $b_{-i}^\varepsilon$ . I write  $\Phi^\varepsilon(\cdot \mid b_i^\varepsilon)$  for the joint probability function of  $(b, b_j^\varepsilon)_{j \neq i}$  conditional on  $b_i^\varepsilon$ . The timing of the global game  $G^\varepsilon$  is as follows:

1. Nature draws a true  $b$ ;
2. Each player  $i$  receives its private signal  $b_i^\varepsilon$  of  $b$ ;
3. All players simultaneously choose their actions;
4. Payoffs are realized according to the true  $b$  and the actions chosen by all players.

In what follows I will take  $\varepsilon > 0$  as given and introduce the concepts used to analyze the global game  $G^\varepsilon$ .

*Strategies and strict dominance.* Player  $i$  receives a signal  $b_i^\varepsilon$  prior to choosing an action. A strategy  $p_i$  for player  $i$  in  $G^\varepsilon$  is a function that assigns to any  $b_i^\varepsilon \in [\underline{B} - \varepsilon, \overline{B} + \varepsilon]$  a probability  $p_i(b_i^\varepsilon) \geq 0$  with which the player chooses action  $x_i = 1$  when they observe  $b_i^\varepsilon$ . I write  $p = (p_1, p_2, \dots, p_N)$  for a strategy vector. Similarly, I write  $p_{-i} = (p_j)_{j \neq i}$  for the vector of strategies for all players but  $i$ . Conditional on the strategy vector  $p_{-i}$  and a private signal  $b_i^\varepsilon$ , the expected gain (of choosing  $x_i = 1$  rather than  $x_i = 0$ ) to player  $i$  is given by:

$$\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) = \int \Delta_i(p_{-i}(b_{-i}^\varepsilon) \mid b) d\Phi^\varepsilon(b, b_{-i}^\varepsilon \mid b_i^\varepsilon). \quad (5)$$

I say that the action  $x_i = 1$  is strictly dominant at  $b_i^\varepsilon$  if  $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) > 0$  for all  $p_{-i}$ . Similarly, the action  $x_i = 0$  is strictly dominant (in the global game  $G^\varepsilon$ ) at  $b_i^\varepsilon$  if  $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) < 0$  for all  $p_{-i}$ . When  $x_i = x$  is strictly dominant, I say that  $x_i = 1 - x$  is strictly dominated.

**Lemma 1.** *Consider the global game  $G^\varepsilon$ . (i) For each player  $i$ , the action  $x_i = 1$  is strictly dominant at all  $b_i^\varepsilon \geq \overline{B}$ . (ii) For each player  $i$ , the action  $x_i = 0$  is strictly dominant at  $b_i^\varepsilon \leq \underline{B}$ .*

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<sup>7</sup>In game theory, it is assumed that the game (in this case  $G^\varepsilon$ ) is common knowledge; hence, the structure of the uncertainty (the joint distribution of  $b$  and all the signals  $b_j^\varepsilon$ ), the possible actions and all the payoff functions are commonly known. For a formal treatment of common knowledge, see Aumann (1976).



*Conditional dominance.* Let  $L$  and  $R$  be real numbers. The action  $x_i = 1$  is said to be dominant at  $b_i^\varepsilon$  conditional on  $R$  if  $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) > 0$  for all  $p_{-i}$  with  $p_j(b_j^\varepsilon) = 1$  for all  $b_j^\varepsilon > R$ , all  $j \neq i$ . Similarly, the action  $x_i = 0$  is dominant at  $b_i^\varepsilon$  conditional on  $L$  if  $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) < 0$  for all  $p_{-i}$  with  $p_j(b_j^\varepsilon) = 1$  for all  $b_j^\varepsilon > L$ , all  $j \neq i$ .

The concept of conditional dominance is useful for the following reason. Lemma 1 implies that, for each player  $j$ , a strategy  $p_j$  of  $G^\varepsilon$  that prescribes to play  $x_j \neq 1$  on a set  $b_j^\varepsilon > \bar{B}$  with positive measure is strictly dominated; hence, each player  $i$  can effectively assume that each player  $j$  will play  $p_j(b_j^\varepsilon) = 1$  for all  $b_j^\varepsilon \geq \bar{B}$ . Eliminating dominated strategies makes, for each player  $i$ ,  $x_i = 1$  strictly dominant for a larger set of observations and hence makes more strategies of each  $i$  strictly dominated; hence, this process can be repeated. Those strategies that survive this process (including elimination of strategies that prescribe playing 1 when that is strictly dominated) are said to survive *iterated elimination of strictly dominated strategies*. For a textbook treatment of iterated dominance, see Osborne and Rubinstein (1994).

*Increasing strategies.* For some  $X \in \mathbb{R}$ , let  $p_i^X$  denote the particular strategy such that  $p_i^X(b_i^\varepsilon) = 0$  for all  $b_i^\varepsilon < X$  and  $p_i^X(b_i^\varepsilon) = 1$  for all  $b_i^\varepsilon \geq X$ . I will call  $p_i^X$  the *increasing strategy with switching point  $X$* . By  $p^X = (p_1^X, p_2^X, \dots, p_N^X)$  I denote the strategy vector of increasing strategies with switching point  $X$ , and  $p_{-i}^X = (p_j^X)_{j \neq i}$ . Note that  $x_i = 1$  is strictly dominant at  $b_i^\varepsilon$  conditional on  $R$  if and only if  $\Delta_i^\varepsilon(p_{-i}^R \mid b_i^\varepsilon) > 0$ . Similarly, if  $x_i = 0$  is strictly dominant at  $b_i^\varepsilon$  conditional on  $L$  then it must hold that  $\Delta_i^\varepsilon(p_{-i}^L \mid b_i^\varepsilon) < 0$ .

We now have all notation in place to proceed with the core of the analysis.

*Iteration from the right.* Let  $i$  be arbitrary. Take  $p_{-i} = p_{-i}^{\bar{B}}$  and note that  $\Delta_i^\varepsilon(p_{-i}^{\bar{B}} \mid b_i^\varepsilon)$  is continuous and monotone non-decreasing in  $b_i^\varepsilon$ . Moreover, recall from Lemma 1 that  $x_i = 1$  is strictly dominant at  $b_i = \bar{B}$ , so  $\Delta_i^\varepsilon(p_{-i}^{\bar{B}} \mid \bar{B}) > 0$ . By the same Lemma, I also know that  $\Delta_i^\varepsilon(p_{-i}^{\bar{B}} \mid \underline{B}) < 0$ . Monotonicity and continuity of  $\Delta_i^\varepsilon(p_{-i}^{\bar{B}} \mid b_i^\varepsilon)$  in  $b_i$  then imply there exists a point  $R^1$  such that  $\underline{B} < R^1 < \bar{B}$  which solves:

$$\Delta_i^\varepsilon(p_{-i}^{\bar{B}} \mid R^1) = 0. \quad (6)$$

To any player  $i$ , the action  $x_i = 1$  is strictly dominant at all  $b_i^\varepsilon > R^1$  conditional on  $\bar{B}$ .

This argument can be repeated and I obtain a sequence  $\bar{B} = R^0, R^1, R^2, \dots$ . For any  $k \geq 0$  and  $R^k$  such that  $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^k) > 0$ , there exists a  $R^{k+1} < R^k$  such that  $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^{k+1}) = 0$ . Induction on  $k$  allows for the conclusion that  $(R^k)$  is a monotone sequence. Moreover, I also know that  $R^k \geq \underline{B}$  for all  $k \geq 0$  since  $x_i = 0$  is strictly dominant at  $b_i^\varepsilon < \underline{B}$ . Any bounded monotone sequence must converge. I let  $R^*$  denote the limit of sequence  $(R^k)$ . By the definition of a limit,  $R^*$  must satisfy:

$$\Delta_i^\varepsilon(p_{-i}^{R^*} \mid R^*) = 0. \quad (7)$$

It follows that a strategy  $p_i$  survives iterated elimination of strictly dominated strategies only if  $p_i(b_i^\varepsilon) = 1$  for all  $b_i^\varepsilon > R^*$ , all  $i$ .

*Iteration from the left.* Iterative elimination of strictly dominated strategies yields the point  $R^*$  when starting from the right, that is, a range of signals  $b_i^\varepsilon$  for which  $x_i = 1$

is conditionally and strictly dominant. A similar procedure can be performed starting instead from the left, from signals  $b_i^\varepsilon$  for which  $x_i = 0$  is unconditionally and strictly dominant. Because this analysis is symmetric to the procedure discussed above, I omit it in the main text. A complete proof may be found in the appendix.

**Lemma 2.** (i) *If a strategy  $p_i$  survives iterated elimination of strictly dominated strategies, then it must hold that  $p_i(b_i^\varepsilon) = 1$  for all  $b_i^\varepsilon > R^*$ .* (ii) *If a strategy  $p_i$  survives iterated elimination of strictly dominated strategies, then it must hold that  $p_i(b_i^\varepsilon) = 0$  for all  $b_i^\varepsilon < L^*$ .*

I derived two limits  $L^*$  and  $R^*$  that demarcate iterative dominance regions of the signal space. I next show that  $L^* = R^*$ . To prove this, the following result is key.

**Proposition 2.** *For all  $X$  such that  $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$ , the following holds:*

$$\Delta_i^\varepsilon(p_{-i}^X | X) = X - \sum_{m=0}^{N-1} \frac{c(m+1)}{N} + d. \quad (8)$$

*It follows that  $\Delta_i^\varepsilon(p_{-i}^X | X)$  is strictly increasing in  $X$  for all  $X$  such that  $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$ .*

From the definitions of  $R^*$  and  $L^*$ , using Proposition 2, one can conclude that  $L^* = R^*$ . I henceforth write  $B^*$  where  $B^* = L^* = R^*$ . The point  $B^*$  is given by:

$$B^* = \sum_{n=1}^N \frac{c(n)}{N} - d. \quad (9)$$

Thus, if a strategy  $p_i$  survives iterated elimination of strictly dominated strategies, then it must hold that  $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$  for all  $b_i^\varepsilon \neq B^*$ . The action prescribed by a strategy  $p_i$  that survives iterated dominance can differ from that prescribed by  $p_i^{B^*}$  only in the measure-zero event that  $b_i^\varepsilon = B^*$ . I refer to this by saying that  $G^\varepsilon$  has an *essentially unique* strategy vector surviving iterated elimination of strictly dominated strategies.

**Theorem 1.** *For all  $\varepsilon$  such that  $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$ , the strategy vector  $p^{B^*}$  is the essentially unique strategy vector surviving iterated elimination of strictly dominated strategies of the game  $G^\varepsilon$ . In particular, if, for any player  $i$ , the strategy  $p_i$  survives iterated elimination of strictly dominated strategies, then  $p_i$  must satisfy  $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$  for all  $b_i^\varepsilon \neq B^*$ .*

Theorem 1 holds for general  $\varepsilon > 0$  provided the assumption that  $b$  and  $\varepsilon_i$  (all  $i$ ) are drawn independently from the uniform distribution. For the limit as  $\varepsilon \rightarrow 0$ , Frankel et al. (2003) establish the very general result that *any* global game with strategic complementarities in which  $b$  is drawn from any continuous density with connected support and each  $\varepsilon_i$  is drawn independently from any (possible player-specific) atomless density has an essentially unique strategy vector surviving iterated elimination of

strictly dominated strategies in the limit as  $\varepsilon \rightarrow 0$ . Moreover, for potential games the equilibrium selected is noise independent and given by the vector of strategies in which each player  $i$  chooses the action that coincides with the potential maximizer of the game. This means that the strategy vector found in Theorem 1 generalizes to far more general distributions than assumed here.<sup>8</sup>

Recall that a strategy vector  $p = (p_1, p_2, \dots, p_N)$  is a Bayesian Nash equilibrium (BNE) of  $G^\varepsilon$  if for any  $p_i$  and any  $b_i^\varepsilon$  it holds that:

$$p_i(b_i^\varepsilon) \in \arg \max_{x_i \in \{0,1\}} \pi_i^\varepsilon(x_i, p_{-i}(b_{-i}^\varepsilon) \mid b_i^\varepsilon), \quad (10)$$

where  $\pi_i^\varepsilon(x_i, p_{-i}(b_{-i}^\varepsilon) \mid b_i^\varepsilon) = \int \pi_i(x_i, p_{-i}(b_{-i}^\varepsilon) \mid b) d\Phi^\varepsilon(b, b_{-i}^\varepsilon \mid b_i^\varepsilon)$ . It is therefore immediate that  $p^{B^*}$  is a BNE of  $G^\varepsilon$ . The following proposition establishes a much stronger result: if the strategy vector  $p = (p_i)$  is a BNE of  $G^\varepsilon$ , then for each  $p_i$  it must hold that  $p_i(b_i^\varepsilon) = p^{B^*}(b_i^\varepsilon)$  for all  $b_i^\varepsilon \neq B^*$ . I say that  $G^\varepsilon$  has an essentially unique BNE.

**Theorem 2.** *The strategy vector  $p^{B^*}$  is the essentially unique Bayesian Nash equilibrium of the game  $G^\varepsilon$ . In particular, any equilibrium strategy  $p_i$  satisfies  $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$  for all  $b_i^\varepsilon \neq B^*$  and all players  $i$ .*

Theorem 2 does not that players will perfectly coordinate their actions (technological choices).<sup>9</sup> For  $\varepsilon > 0$  and  $B$  sufficiently close to  $B^*$ , it is possible that some players receive signals above  $B^*$  while others see a signal below it. When this occurs, players will fail to coordinate their actions (i.e. some will choose  $x_i = 1$  while others choose  $x_i = 0$ ). Observed coordination failures hence are not necessarily at odds with, and therefore do not by themselves invalidate the model. When a coordination failure occurs, the equilibrium outcome is inefficient.

In the limit as  $\varepsilon \rightarrow 0$ , the global climate game  $G^\varepsilon$  selects an essentially unique equilibrium of the underlying coordination game with multiple equilibria. To see this, note that for any  $b > B^*$ , there is  $\varepsilon < B^* - b$  so that  $b - \varepsilon > B^*$ . Since  $b_i^\varepsilon \in [b - \varepsilon, b + \varepsilon]$  and  $p^* = p^{B^*}$  this implies that  $p_i^*(b_i^\varepsilon) = 1$  for all  $b_i^\varepsilon$  consistent with  $b$  and all  $i$ .

Even as  $\varepsilon \rightarrow 0$  and players coordinate their actions with probability 1, the unique equilibrium can be inefficient. In particular, players coordinate on **0** (use fossil fuels) for all  $b < B^*$  even though the outcome in which players coordinate on **1** (use renewables) is Pareto strictly dominant for all  $b > \bar{b}$  (and even though they know it).

The result stands in contrast to the common and often implicit assumption in the environmental literature that players can, by sheer force of will, coordinate on the efficient equilibrium (Barrett, 2006; Hoel and de Zeeuw, 2010).

**Corollary 1.** *(i) For all  $b > B^* + \varepsilon$  it holds that  $\Pr[p^{B^*}(b^\varepsilon) = \mathbf{1}] = 1$ . (ii) For all  $b < B^* - \varepsilon$  it holds that  $\Pr[p^{B^*}(b^\varepsilon) = \mathbf{0}] = 1$ .*

<sup>8</sup>In particular, the reader is referred to their result on (local) potential games with own-action quasi-concave payoffs, i.e. Theorem 4.

<sup>9</sup>Perfect coordination of actions means that all players choose the same action.

## 4 Network Subsidies

Inefficiency in both the game of complete information  $G(b)$  and the global game  $G^\varepsilon$  begs the question how a policymaker can influence the game in order to reach an efficient outcome. I consider the problem of designing policies to influence players' incentives so as to implement the Pareto efficient outcome of the game in strictly dominant strategies. That is, I seek to find policies that turn playing  $x_i = 1$  into a strictly dominant strategy whenever coordination on  $\mathbf{1}$  is also the efficient outcome of the game; similarly, I want  $x_i = 0$  to be a strictly dominant strategy when coordination on  $\mathbf{0}$  is Pareto efficient.<sup>10</sup> I assume that the policymaker is fully informed about players' possible actions as well as the parameters of the model that are common knowledge among the players. To stay with the application to climate change, I confine the set of feasible policies to subsidies and taxes.

### 4.1 Game of Complete Information

Consider again the game of complete information  $G(b)$ . Recall from Proposition 1 that coordination on  $x = \mathbf{1}$  is the Pareto dominant outcome of the game for all  $b > \bar{b}$ , whereas coordination on  $x = \mathbf{0}$  is efficient for all  $b < \bar{b}$ .

My aim is to find a subsidy that incentivizes players to coordinate on the efficient outcome of the game for any  $b$ . Concretely, I want to formulate a tax and/or subsidy policy that makes using renewables (play  $x_i = 1$ ) strictly dominant for all  $b > \bar{b}$  while leaving fossil fuels (play  $x_i = 0$ ) strictly dominant at  $b < \bar{b}$ . I say that such a subsidy *implements* the efficient outcome of the game in strictly dominant strategies for almost all  $b$ , i.e., for all  $b$  except  $\bar{b}$ .

I first show that if coordination on renewables is a Nash equilibrium of the game  $G(b)$ , then the policymaker can implement coordination on this outcome at zero cost, even if using fossil fuels is also a Nash equilibrium. The basic idea is to offer players who use renewables a subsidy that guarantees them a gain equal to that they would have realized in the hypothetical case that all other players also use renewables. To this end, let the policymaker offer a *network subsidy*  $s^*(x)$  to each  $i$  choosing to use renewables when  $x$  is played. For each  $x$ , let  $s^*(x)$  be given by:

$$s^*(x) = \Delta_i(\mathbf{1}_{-i} \mid b) - \Delta_i(x_{-i} \mid b) = c(n(x)) - c(N). \quad (11)$$

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<sup>10</sup>This question is related to the literature on mechanism design and (strictly dominant strategy) implementation. That is, I study the problem of a policymaker who seeks to change the original game studied in Section 2 and 3 with the aim of making coordination on the efficient outcome of the game a strictly dominant strategy for all players (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983; Kuzmics and Steg, 2017). For applications of mechanism design and implementation theory to pollution problems like climate change, see Duggan and Roberts (2002), Ambec and Ehlers (2016), and Martimort and Sand-Zantman (2016). As an extension for future work directly related to the mechanism design literature, I hope to explicitly compare network subsidies to the well-studied Vicker-Clarke-Groves mechanism (such a comparison is also made in Ambec and Ehlers, 2016).

If a player uses renewables (and the action vector  $x$  is played), they receive a network subsidy equal to  $s^*(x)$  as specified in (11). If instead a player uses fossil fuels, the policymaker pays them nothing. Observe that, conditional on  $s^*(\cdot)$ , an individual player's gain from using renewables, rather than fossil fuels, is:

$$\Delta_i(x_{-i} \mid b) + s^*(x) = \Delta_i(x_{-i} \mid b) + \Delta_i(\mathbf{1}_{-i} \mid b) - \Delta_i(x_{-i} \mid b) = \Delta_i(\mathbf{1}_{-i} \mid b), \quad (12)$$

for any  $x$ , confirming the claim that a network subsidy scheme  $s^*(\cdot)$  allows players to consider only the gain  $\Delta_i(\mathbf{1}_{-i} \mid b) = b - c(N) + d$  when choosing their actions.

**Theorem 3.** *Let  $G(b \mid s^*)$  denote the game  $G(b)$  in which players are offered the network subsidy  $s^*(\cdot)$  on playing 1.*

- (i) *If  $\mathbf{1}$  is a Nash equilibrium of  $G(b)$  (i.e. if  $b + d \geq c(N)$ ), then  $\mathbf{1}$  is implemented in weakly dominant strategies with  $s^*(\cdot)$  and no subsidies have to be paid.*
- (ii) *If  $\mathbf{1}$  is a strict Nash equilibrium of  $G(b)$  (i.e. if  $b + d > c(N)$ ), then  $\mathbf{1}$  is implemented in strictly dominant strategies with  $s^*(\cdot)$  and no subsidies have to be paid.*
- (iii) *If  $\mathbf{1}$  is not a Nash equilibrium of  $G(b)$  (i.e. if  $b + d < c(N)$ ), then  $\mathbf{0}$  is implemented in strictly dominant strategies with  $s^*(\cdot)$  and no subsidies have to be paid.*

Note that for all  $b > c(N) - d$ , and provided the network subsidy scheme  $s^*(\cdot)$  is offered, the policymaker may even tax playing  $x_i = 1$  yet still implement  $\mathbf{1}$  in strictly dominant strategies.

**Remark 1.** *Let  $b > c(N) - d$ , so  $\mathbf{1}$  is both a strict Nash equilibrium and the efficient outcome of the game  $G(b)$ . If the policymaker offers the network subsidy scheme  $s^*(\cdot)$ , the policymaker can impose a tax  $t(b) \leq b + d - c(N)$  on playing  $x_i = 1$  but nevertheless implement coordination on  $\mathbf{1}$  in strictly dominant strategies.*

Theorem 3 tells us that a policy of network subsidies allows the policymaker costlessly to implement the efficient Nash equilibrium of  $G(b)$  in (strictly) strictly dominant strategies if the game has multiple (strict) Nash equilibria. While this is a desirable property, it does not guarantee that a network subsidy scheme implements the efficient outcome of the game for all  $b$ . To see this, observe that  $\mathbf{0}$  is the unique strict Nash equilibrium of  $G(b)$  for all  $b < c(N) - d$ , while  $\mathbf{1}$  is the efficient outcome for all  $b > \bar{b} = (c(N) - d)/N$ . Hence, if  $c(N) > d$  the policymaker cannot implement the efficient outcome of the game for all  $b \in (\bar{b}, c(N) - d)$  using a network subsidy scheme alone.

If  $\mathbf{1}$  is not a Nash equilibrium of the game  $G(b)$ , but  $\mathbf{1}$  is the efficient outcome, then in order to implement  $\mathbf{1}$  in strictly dominant strategies, the policymaker can use a combination of Pigovian taxes and network subsidies to achieve its goal. First, let the policymaker again offer the network subsidy scheme given by  $s^*(\cdot)$ . As noted before, the net (accounting for subsidies) gain from playing 1 rather than 0 becomes

$\Delta_i(x_{-i} \mid b) + s^*(x) = \Delta_i(\mathbf{1}_{-i} \mid b)$  when players are offered  $s^*(\cdot)$ . Second, let the policymaker levy an *carbon tax*  $t(b)$  to playing 0. The purpose of the carbon tax is to make sure that  $\Delta_i(\mathbf{1}_{-i} \mid b) + t(b) > 0$  for all  $b > \bar{b}$  while  $\Delta_i(\mathbf{1}_{-i} \mid b) + t(b) < 0$  for all  $b < \bar{b}$ ; that is, the tax should make  $\mathbf{1}$  a Nash equilibrium of the game if and only if  $\mathbf{1}$  is also the efficient outcome; otherwise  $\mathbf{0}$  should be the equilibrium. A tax  $t(b)$  that achieves this is given by:

$$t(b) > \Delta(\mathbf{1}_{-i} \mid c(N) - d) - \Delta(\mathbf{1}_{-i} \mid b) = c(N) - d - b \quad \text{if } b \geq \bar{b}, \quad (13)$$

while  $t(b) = 0$  otherwise. It is easy to verify that  $t(b)$  implements coordination on  $\mathbf{1}$  as a strict Nash equilibrium for all  $b > \bar{b}$  while leaving  $x_i = 0$  strictly dominant for all  $b < \bar{b}$ .

**Theorem 4.** *Let  $G(b \mid s^*, t)$  denote the game  $G(b)$  in which the policymaker both offers the network subsidy scheme  $s^*(\cdot)$  and levies the carbon tax  $t(b)$ . If  $\mathbf{1}$  is not a Nash equilibrium of  $G(b)$ , but  $\mathbf{1}$  is the Pareto efficient outcome, then, by taxing  $x_i = 0$  through  $t(b)$  while also offering a network subsidy  $s^*(\cdot)$  to playing  $x_i = 1$ ,  $\mathbf{1}$  can be implemented in strictly dominant strategies at no cost (and tax revenues will be zero).*

The combination of a taxes and subsidies discussed in Theorem 4 resembles the recommendation by Acemoglu et al. (2012, 2016), Hart (2019) and Harstad (2020) to combine a carbon tax on the use of dirty technologies with a subsidy on (R&D in) renewables. In their setups, the tax is meant to correct the environmental externality deriving from emissions whereas the subsidy addresses the network externality, or strategic complementarity, in clean technologies. Much the same mechanism is at work here. The carbon tax is a Pigouvian tax that internalizes the environmental externality from using fossil fuels. The network subsidy instead solves the coordination problem that derives from strategic complementarities.

Why does a network subsidy work so well despite the low cost? The key property of a network subsidy set at  $s^*(\cdot)$  is that it eliminates all *strategic uncertainty*, i.e. the uncertainty a player has about the actions chosen by all other players. The network subsidy thus removes the payoff uncertainty deriving from strategic uncertainty interacted with technological spillovers – it turns the original coordination game into a simple dominance solvable game for all  $b$ . In so doing, the network subsidy manages to eliminate all inefficiencies caused by players' failure to internalize the *technological* spillovers inherent in clean investments. Intuitively, the network subsidy works like an insurance. It protects individual players against the risk of small network externalities from renewables adoption in case many others are using fossil fuels. Because of this insurance, it impels individuals toward renewables. The network subsidy does not have to be paid as a result, being conditional on low renewables adoption.

Governments may at times be reluctant to rely on taxes when trying to curb private sector emissions, for example because taxes are unpopular with voters. When this is true, the government cannot (or at least does not want to) levy the carbon tax  $t(b)$  but may rather rely on an environmental *subsidy*  $s(b)$  on playing 1.

**Remark 2.** *If  $\mathbf{1}$  is not a Nash equilibrium of  $G(b)$ , but  $\mathbf{1}$  is the Pareto efficient outcome, then, by subsidizing  $x_i = 1$  through  $s(b) = c(N) - d - b$  while also offering a network subsidy  $s(\cdot)$  to playing  $x_i = 1$ ,  $\mathbf{1}$  can be implemented in strictly dominant strategies. Total subsidy spending will be  $N \cdot s(b)$  when  $b > \bar{b}$ , and zero otherwise.*

In a global game  $G^\varepsilon$ , the logic and analysis of network subsidies is largely the same as presented here. Account must be taken of two additional complications however. First, the benefit parameter  $b$  is unobserved so the tax on using fossil fuels cannot depend on  $b$  directly. Second, perfect coordination of actions may fail due to the spread in players' signals. I take up both of these issues in the next section.

## 4.2 Global game

Consider the global game  $G^\varepsilon$  discussed in Section 3. In this game, players do not observe  $b$  but only some noisy private signal of it. I henceforth assume that the policymaker observes neither the true  $b$  nor a signal of it.

In this section I address the question of what tax-subsidy scheme suffices to implement the Pareto efficient outcome of the underlying game  $G(b)$  in strictly dominant strategies for all  $b$ . I will assume the policymaker seeks policies that, for each player  $i = 1, 2, \dots, N$ , turn  $x_i = 1$  into a strictly dominant action for all  $b_i^\varepsilon > \bar{b}$  while leaving  $x_i = 0$  strictly dominant for all  $b_i^\varepsilon < \bar{b}$ .<sup>11</sup> I will also assume that the policy scheme does not depend on the unobserved true  $b$ .<sup>12</sup>

First, let us again assume the policymaker offers each player a network subsidy  $s^*$  equal to:

$$s^*(x) = c(n(x)) - c(N), \quad (14)$$

which is the same network subsidy as in (11). One observes that for any  $b_i^\varepsilon$  it is true that  $s_i^*(x) = \Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) - \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon)$ , so in the global game the interpretation of  $s^*(x)$  is the same as in  $G(b)$ : it is the difference in the gain from using renewables between the best-case scenario with full coordination on renewables and the realized outcome  $x$ .

It is easy to verify that the network subsidy  $s^*(\cdot)$  makes playing 1 strictly dominant for all  $b_i^\varepsilon > c(N) - d$ . When players are offered  $s^*(\cdot)$  for each  $x_{-i}$ , their expected gain (the expectation is over  $b$ ) is:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon), \quad (15)$$

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<sup>11</sup>I use the word “leaving” because in the global game  $G^\varepsilon$  *without* policy intervention, playing  $x_i = 0$  is already strictly dominant for all  $i$  and all  $b_i^\varepsilon < \bar{b} < B^*$ .

<sup>12</sup>To be more precise, I assume that the only observables on which the policy scheme depends are players' actions. This is a restrictive assumption. Players possess private information (their signals) about  $b$  and this information is correlated. We thus know from the literature on mechanism design that the policymaker can (costlessly) extract the vector of signals  $b^\varepsilon = (b_1^\varepsilon, b_2^\varepsilon, \dots, b_N^\varepsilon)$  from the players (Cr  mer and McLean, 1988; McAfee and Reny, 1992). Especially when  $\varepsilon$  is small, knowing  $b^\varepsilon$  would provide an almost perfect signal of the true  $b$  to the policymaker. It seems intuitive that the policymaker might use this knowledge to its benefit (and the benefit of all players as a whole).

where  $\Delta_i^\varepsilon(x_{-i} | b) := \frac{1}{2\varepsilon} \int_{b_i^\varepsilon - \varepsilon}^{b_i^\varepsilon + \varepsilon} \Delta(x_{-i} | b) db$ . Note that  $\Delta_i^\varepsilon(\mathbf{1} | b_i^\varepsilon)$  is strictly positive for all  $b_i^\varepsilon > c(N) - d$  and strictly negative for all  $b_i^\varepsilon < c(N) - d$ . Let  $G^\varepsilon(s^*)$  denote the global game  $G^\varepsilon$  in which the policymaker offers the network subsidy scheme  $s^*(\cdot)$ .

**Lemma 3.** *Consider the global game  $G^\varepsilon$ . Let the policymaker offer a network subsidy  $s^*(\cdot)$  on playing 1. Then the action  $x_i = 0$  is strictly dominant at  $b_i^\varepsilon < c(N) - d$ ; the action  $x_i = 1$  is strictly dominant at  $b_i^\varepsilon > c(N) - d$ .*

As in the game of complete information, a network subsidy alone may not suffice to implement the efficient outcome of the game; for all  $b \in (\bar{b}, c(N) - d - \varepsilon)$ , each player  $i$  receives a signal  $b_i^\varepsilon < c(N) - d$  so playing 0 is strictly dominant despite the network subsidy. Therefore, let the policymaker – on top of the network subsidy – levy a carbon tax  $t$  on playing  $x_i = 0$  that makes  $x_i = 1$  strictly dominant, for all  $b_i^\varepsilon > \bar{b}$  and all  $i$ , constrained by the condition that  $x_i = 0$  should still be strictly dominant (despite both the subsidy and the tax) for all  $b_i^\varepsilon < \bar{b}$ . Thus, the policymaker wants to find a tax  $t$  (recall again the assumption that  $t$  does not depend on players' private knowledge of  $b$ ) that solves:

$$\begin{aligned} \Delta_i^\varepsilon(x_{-i} | b_i^\varepsilon) + s^*(x) + t &= \Delta_i^\varepsilon(\mathbf{1} | b_i^\varepsilon) + t > 0 \quad \text{for all } b_i^\varepsilon > \bar{b} \\ \Delta_i^\varepsilon(x_{-i} | b_i^\varepsilon) + s^*(x) + t &= \Delta_i^\varepsilon(\mathbf{1} | b_i^\varepsilon) + t < 0 \quad \text{for all } b_i^\varepsilon < \bar{b}, \end{aligned} \quad (16)$$

for all  $i$  and all  $x_{-i}$ . It follows that  $t$  is given by:

$$t = (N - 1) \cdot \bar{b} = (N - 1) \cdot \frac{c(N) - d}{N}. \quad (17)$$

Let  $G^\varepsilon(s^*, t)$  denote the global game  $G^\varepsilon$  in which the policymaker both offers the network subsidy scheme  $s^*(\cdot)$  and levies a carbon tax  $t$ . The following result regarding  $G^\varepsilon(s^*, t)$  is immediate.

**Theorem 5.** *Consider the global game  $G^\varepsilon(s^*, t)$ . If the policymaker offers a network subsidy  $s^*(\cdot)$  on playing  $x_i = 1$  and levies a tax  $t$  on playing  $x_i = 0$ , then, for each player  $i$ , the action  $x_i = 0$  is strictly dominant for all  $b_i^\varepsilon < \bar{b}$  and the action  $x_i = 1$  is strictly dominant for all  $b_i^\varepsilon > \bar{b}$ . Hence, for all  $b \notin [\bar{b} - \varepsilon, \bar{b} + \varepsilon]$  the policymaker can implement the efficient outcome of the game  $G(b)$  in strictly dominant actions at no cost.*

If the policymaker is reluctant to tax playing 0, it may also offer both a network subsidy  $s^*(\cdot)$  together with an environmental *subsidy* equal to  $t$  to playing 1. Such a policy is evidently equivalent with regard to players' incentives, although it differs for the policymaker's budget.

**Corollary 2.** *Consider the global game  $G^\varepsilon$ . Let the policymaker offer a network subsidy  $s^*(\cdot)$  on playing 1. In addition, let the policymaker offer an environmental subsidy (rather than a tax) equal to  $t$  on playing 1. Then the action  $x_i = 0$  is strictly dominant for all  $b_i^\varepsilon < \bar{b}$  while the action  $x_i = 1$  is strictly dominant for all  $b_i^\varepsilon > \bar{b}$ . Hence, the policymaker can implement the efficient outcome of the game  $G(b)$  for all  $b \notin [\bar{b} - \varepsilon, \bar{b} + \varepsilon]$ ; total subsidy spending is  $N \cdot \bar{b}$  if  $b > \bar{b} + \varepsilon$  and 0 if  $b < \bar{b} - \varepsilon$ .*



### 4.3 Self-financed network tax-subsidy

If Nature draws a true  $b$  in  $(\bar{b} - \varepsilon, \bar{b} + \varepsilon)$ , coordination on either  $\mathbf{0}$  or  $\mathbf{1}$  in  $G^\varepsilon(s^*, t)$  may fail. The reason is that, for those  $b$ , players' signals need not all fall in the strict dominance regions identified in Theorem 5. The network subsidy scheme  $s^*(\cdot)$  may hence not be costless; for any  $x$  not equal to  $\mathbf{0}$  or  $\mathbf{1}$ , total spending on network subsidies will be  $n(x) \cdot s^*(x) > 0$ . The strong performance of a network subsidy scheme may thus break down in a global game. For  $\varepsilon > 0$ , the event that a true  $b$  in  $(\bar{b} - \varepsilon, \bar{b} + \varepsilon)$  is drawn has prior probability  $2\varepsilon/(\bar{B} - \underline{B}) > 0$ . Only in the limit as  $\varepsilon \rightarrow 0$  will this problem disappear: players perfectly coordinate their actions (in equilibrium) save for the probability-zero event that  $b = \bar{b}$ .

To work around this problem, I now derive a network tax-subsidy scheme where subsidy payments on  $x_i = 1$  are financed through a network tax levied on choosing  $x_i = 0$ .<sup>13</sup> Let the subsidy be denoted  $s^{**}(x)$ ; the corresponding tax is denoted  $t^{**}(x)$ . Thus, when  $x$  is played, aggregate spending on network subsidies is  $n(x) \cdot s^{**}(x)$  and aggregate revenues from network taxation are  $(N - n(x)) \cdot t^{**}(x)$ . I impose the ex post budget constraint that

$$(N - n(x)) \cdot t^{**}(x) - n(x) \cdot s^{**}(x) = 0 \quad (18)$$

for all  $x$ .

Next, the tax-subsidy scheme, together with the carbon tax  $t$  given by (17), must make  $x_i = 1$  strictly dominant for all  $b_i^\varepsilon > \bar{b}$  while leaving  $x_i = 0$  strictly dominant for all  $b_i^\varepsilon < \bar{b}$ . Thus players' gains from playing 1, rather than 0, accounting for taxes and subsidies, should satisfy:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + t + s^{**}(x) + t^{**}(x) > 0 \quad \text{for all } b_i^\varepsilon > \bar{b}, \quad (19)$$

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + t + s^{**}(x) + t^{**}(x) < 0 \quad \text{for all } b_i^\varepsilon < \bar{b}, \quad (20)$$

for all  $i$  and all  $x_{-i}$ . Equations (19) and (20) represent the incentive constraints of a network tax-subsidy scheme. Combined with the budget constraint, this yields the following network tax-subsidy scheme  $(s^{**}, t^{**})$ :

$$\begin{cases} t^{**}(x) = \frac{n(x)}{N} [c(n(x)) - c(N)] \\ s^{**}(x) = \frac{N - n(x)}{N} [c(n(x)) - c(N)] \end{cases} \quad (21)$$

The policy scheme  $((s^{**}, t^{**}), t)$  can now be summarized as follows. When  $x$  is played and player  $i$  has played 1 in  $x$ , they receive a network subsidy equal to  $s^{**}(x)$ ; however,

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<sup>13</sup>An alternative approach to this problem would be to let the policymaker extract players' private signals (see footnote 12) and then construct a policy scheme such that, when the signals indicate a high  $b$ , the policymaker may tax playing 1 similarly to the way discussed in Remark 1. The policymaker could then construct this policy in such a way that ex ante, i.e. before  $b$  is drawn, the policy scheme has expected cost zero. This is different from the present analysis, which is more demanding and imposes ex post budget neutrality.

if player  $i$  played 0 in  $x$ , they pay a tax equal to  $t + t^{**}(x)$ . Let  $G^\varepsilon((s^{**}, t^{**}), t)$  denote the global game  $G^\varepsilon$  in which the policymaker both offers the network subsidy scheme  $s^*(\cdot)$  and levies a carbon tax  $t$ . The following result follows immediately from the preceding analysis.

**Theorem 6.** *Consider the global game  $G^\varepsilon((s^{**}, t^{**}), t)$ . Let the policymaker offer a network subsidy equal to  $s^{**}(\cdot)$  on playing 1 while levying a tax equal to  $t + t^{**}(\cdot)$  on playing 0. This policy makes the action  $x_i = 0$  strictly dominant for all  $b_i^\varepsilon < \bar{b}$ ; the action  $x_i = 1$  is strictly dominant for all  $b_i^\varepsilon > \bar{b}$ . Consequently, the policymaker can implement efficient outcome of the game  $G(b)$  for all  $b \notin [\bar{b} - \varepsilon, \bar{b} + \varepsilon]$ ; net spending on the policy scheme  $((s^{**}, t^{**}), t)$  is zero for all  $b$ .*

The present analysis did not make use of the fact that, without policy interventions, playing  $p^{B^*}$  is the essentially unique strategy profile surviving iterated dominance in the global game  $G^\varepsilon$ . Another approach toward ((iterative) strictly dominant strategy) implementation in  $G^\varepsilon$  would be to study what mechanisms the policymaker could design to shift the threshold  $B^*$  down toward  $\bar{b}$ . I intend to do this in future work.

## 5 Two-sided markets

When motivating the model, I mentioned that strategic complementarities in players' choices may be the consequence of indirect spillovers in a two-sided market. An example would be electric vehicles stimulating the installation of charging stations and vice versa (Li et al., 2017). How does the idea behind network subsidies pan out for two-sided markets?

Consider a simple two-sides market. Let one side of the market be composed of 'households', who choose  $x$  to maximize the payoff function:

$$\pi_i(x, I \mid b) = \begin{cases} b \cdot n(x) - d & \text{if } x_i = 0, \\ b \cdot n(x) + I - \bar{c} & \text{if } x_i = 1, \end{cases} \quad (22)$$

where (for simplicity)  $\bar{c}$  is now a constant cost of renewables and  $I$  is the spillover (a quantity of some commodity, say) from the other side of the market.

The other side of the market is populated by 'firms' who supply a commodity in the quantity  $I = I(n(x))$  to maximize some payoff function  $u(I \mid x)$  so that  $I(n(x)) = \arg \max_I u(I \mid n(x))$  and  $I(n)$  is increasing in  $n$ . It does not matter for the present discussion whether the secondary market observes  $x$  prior to choosing  $I$  or whether both markets choose  $x$  and  $I$  simultaneously. I therefore assume that  $I$  is determined *after*  $x$  was played. Note that by choosing functions such that  $\bar{c} - I(n) = c(n)$  for all  $n = 1, \dots, N$ , the payoff function (22), subject to  $I = I(x)$ , is simply (1). I will henceforth assume that  $\bar{c} - I(n) = c(n)$  for all  $n = 1, \dots, N$  so that the results derived above can be immediately applied.

For concreteness, the households may be thought of as individuals choosing between an electric vehicle (the renewable) and a fossil fuel car; the car fleet chosen is given by  $x$ .

The firms are then companies that operate charging stations; the number of charging stations is  $I$ .

Consider the following policy as an alternative to network subsidies. First, the policymaker announces that, for any  $x$  and  $I(n(x))$ , it will supply  $I(N) - I(n(x))$  whenever  $n(x) > 0$ . Then the households simultaneously choose  $x$ , after which the firms supply  $I(n(x))$ . Finally the policymaker supplies  $I(N) - I(n(x))$  (if  $n(x) > 0$ ), payoffs are realized and the game ends. Let  $K(I)$  denote the cost to the policymaker of supplying an amount  $I$  of the good in the secondary market, and  $K(0) = 0$ .

On a technical level, the following result is a corollary to Theorems 3 and 4. I label it a proposition due to its interesting policy implication.

**Proposition 3.** *Consider the game of complete information  $G(b)$  with a two-sided market. Let the policymaker offer a policy in which, for any  $x$ , it supplies  $I(N) - I(n(x))$  of the commodity in the secondary market.*

- (i) *If  $\mathbf{1}$  is a [strict] Nash equilibrium of  $G(b)$ , then  $\mathbf{1}$  is implemented in [strictly] dominant strategies. The costs to the policymaker are 0.*
- (ii) *If  $\mathbf{1}$  is not a Nash equilibrium of  $G(b)$ , then  $\mathbf{1}$  is implemented in dominant strategies provided the policymaker taxes playing  $x_i = 0$  according to (13). The costs to the policymaker are 0.*

A similar result also applies to the global game  $G^\varepsilon$ , subject to the same necessary minor alterations discussed in Section 4. The following is a corollary to Theorem 5.

**Proposition 4.** *Consider the global game  $G^\varepsilon$  with a two-sided market. Let the policymaker offer a policy in which, for any  $x$ , it supplies  $I(N) - I(n(x))$  of the commodity in the secondary market. In addition, let the policymaker tax playing  $x_i = 0$  according to (17). Then for each player  $i$ , the action  $x_i = 0$  is strictly dominant for all  $b_i^\varepsilon < \bar{b}$  and the action  $x_i = 1$  is strictly dominant for all  $b_i^\varepsilon > \bar{b}$ .*

Economically, a policy of supplying  $I(N) - I(n(x))$  of the commodity in the secondary market (e.g. building charging stations) is clearly distinct from a direct (network) subsidy on playing 1 (e.g. tax discounts on electric vehicles). Mathematically, however, the two policies are exactly equivalent for the households as  $I(N) - I(n(x)) = c(n(x)) - c(N) = s_i^*(x)$ . It follows that, at least for the households, incentives are the same whether the policymaker offers a network subsidy  $s_i^*$  or pledges to supply  $I(N) - I(n(x))$  of the secondary market commodity for all  $x$ . Combined, if necessary, with an appropriate tax on playing  $x_i = 0$ , coordination on  $x = \mathbf{1}$  is then achieved for all  $b > \bar{b}$  in  $G(b)$  ( $b > \bar{b} + \varepsilon$  in  $G^\varepsilon$ ) and firms in the secondary market supply  $I(n(x)) = I(N)$  of the commodity. The policymaker is then required to supply  $I(N) - I(N) = 0$ , and the associated cost is  $K(0) = 0$ . When instead  $b < \bar{b}$  in  $G(b)$  ( $b < \bar{b} - \varepsilon$  in  $G^\varepsilon$ ), players coordinate on  $x = \mathbf{0}$  so  $n(x) = 0$  and the policymaker does not to provide any commodity, meaning the policy is also costless.

## 6 Concluding remarks

I study discrete contributions to a public good characterized by network effects. Network effects turn the problem into a coordination game with multiple Pareto-ranked equilibria. I address equilibrium selection using the methodology of global games. Uncertainty leads to selection of a unique equilibrium that in well-identified cases is inefficient. I also design the novel policy of network subsidies that corrects the entire externality deriving from network effects but does not, in equilibrium, cost the policymaker anything. I apply the model to climate change mitigation and the adoption of renewable technologies.

Climate change is a public and its mitigation a public good. While some degree of mitigation could be achieved at the intensive margin – that is, by reducing greenhouse gas emissions given the current capital stock –, deep cuts in emissions can only be reached upon a large-scale switch to renewable technologies. Yet renewable technologies often exhibit network effects (Acemoglu et al., 2012, 2016; Hart, 2019; Harstad, 2020). In this sense, climate change mitigation is, at least in part, a coordination game of renewables adoption. The question arises whether the continued dominance of fossil fuel technologies is a coordination failure and, if so, how policies can be designed to solve this inefficiency.

My results show that a persistent reliance on fossil fuel technologies may be a rational coordination failure indeed: in a global game, expected payoff-maximizing agents may be forced to coordinate on using fossil fuels even though coordination on renewables is also a Nash equilibrium of the complete information game drawn by nature (and even though all players know it). This finding motivates policy intervention and gives rise to the design of network subsidies.

This paper fits in with the literature on directed technical change and renewable subsidies (Acemoglu et al., 2012, 2016; Hart, 2019; Harstad, 2020). My results on policy design echo the common recommendation that an efficient policy requires both a carbon tax *and* a subsidy on renewables. My results differ from the extant literature in two notable ways. If coordination on renewables is already an equilibrium of the (true) coordination game (even if coordination on fossil fuels is too), then a network subsidy alone suffices to implement the efficient outcome of the game. In those cases, the first-best can be achieved at zero cost. Moreover, even if coordination on renewables is not an equilibrium and a combination of taxes and subsidies is needed, spending on network subsidies is zero in equilibrium.

The derivation of network subsidies is also an exercise in mechanism design or implementation theory for coordination games. While mechanism design has been applied to environmental economics and climate change before (Duggan and Roberts, 2002; Ambec and Ehlers, 2016; Martimort and Sand-Zantman, 2016), extant papers construct mechanisms that solve the free-rider problem. I complement this literature by deriving a mechanism to overcome the coordination problem. Moreover, note that the logic of network subsidies depends only upon the strategic complementarities in players' actions; it does not rely on the application to climate change. In this sense, my analysis contributes to the literature on strategic policy design more generally.

The paper has a number of limitations that future work might seek to relax. First, I study a one-shot decision problem rather than a repeated game. Second, in implicitly assuming that technologies are perfectly substitutable, I shy away from discussions on the effect of imperfect substitutability on policy. Third, my results are derived for specific functional forms; while results due to Frankel et al. (2003) establish that equilibrium selection continues to occur in far more general global coordination games, such generalizations are not investigated here. Fourth, the policymaker was assumed to know all parameters of the model known to the players; it is not clear how to design a network subsidy scheme (and what its properties would be) if the policymaker knows less.

## A Proofs

### Proof of Lemma 1.

*Proof.* Observe that  $\Delta_i(x | b) > 0$  for any  $x$  given  $b \in [\bar{B} - \varepsilon, \bar{B} + \varepsilon]$ . Thus, for  $b_i^\varepsilon = \bar{B}$  the integration in (5) is over positive terms only and  $\Delta_i(p_{-i} | \bar{B}) > 0$  for all  $p_{-i}$ . This proves part (i) of the Lemma. The proof of part (ii) relies on a symmetric argument and is therefore omitted.  $\square$

### Proof for the second half of Lemma 2.

*Proof.* From Lemma 1 it is known that  $x_i = 0$  is strictly dominant at  $b_i^\varepsilon < \underline{B}$ . That is,  $\Delta_i(p_{-i}^{\underline{B}} | \underline{B}) < 0$ . Since it is common knowledge that no player plays a strictly dominated strategy, a payoff maximizing player  $i$  then finds a point  $L^1$  such that  $x_i = 0$  is strictly dominant  $b_i^\varepsilon < L^1$  conditional on  $\underline{B}$ :

$$\Delta_i(p_{-i}^{\underline{B}} | L^1) = 0. \quad (23)$$

Any expected payoff maximizing player  $i$  plays  $x_i = 0$  for all  $b_i^\varepsilon < L^1$ . Since this is common knowledge also, I can repeat the argument over and over. What I obtain is a sequence of points  $(L^k)$ ,  $k \geq 0$ , each term of which is implicitly defined by:

$$\Delta_i(p_{-i}^{L^k} | L^{k+1}) = 0. \quad (24)$$

The sequence  $(L^k)$  is monotone increasing. It is also bounded from above by  $\bar{B}$  (or, taking account of (7), by  $R^*$ ), implying that it must converge; I call its limit  $L^*$ . By construction this limit solves:

$$\Delta_i(p_{-i}^{L^*} | L^*) = 0. \quad (25)$$

It follows that a strategy  $p_i$  survives iterated elimination of strictly dominated strategies only if  $p_i(b_i^\varepsilon) = 0$  for all  $b_i^\varepsilon < L^*$ , all  $i$ .  $\square$

### Proof of Proposition 2.

*Proof.* First fix  $b \in [\underline{B} + \varepsilon, \overline{B} - \varepsilon]$ . Each player  $j \neq i$  is assumed to play  $p_j^X$ , so the probability that  $x_j = 1$  is given by

$$\Pr[b_j^\varepsilon > X \mid b] = \Pr[\varepsilon_j > X - b] = \frac{\varepsilon - X + b}{2\varepsilon}, \quad (26)$$

for all  $X \in [b - \varepsilon, b + \varepsilon]$  while  $\Pr[b_j^\varepsilon > X \mid b]$  is either 0 or 1 otherwise. Clearly,  $x_j = 0$  is played with the complementary probability (given  $b$  and  $X$ ). Since each  $\varepsilon_j$  is (conditional on  $b$ ) drawn independently, the probability that  $m$  given players  $j \neq i$  play  $x_j = 1$  while the remaining  $N - m - 1$  players play  $x_j = 0$  (given  $p_{-i}^X$  and  $b$ ) is therefore:

$$\left[ \frac{\varepsilon - X + b}{2\varepsilon} \right]^m \left[ \frac{\varepsilon + X - b}{2\varepsilon} \right]^{N-m-1}. \quad (27)$$

As there are  $\binom{N-1}{m}$  unique ways in which  $m$  out of  $N - 1$  players  $j$  can choose  $x_j = 1$ , the total probability of this happening, as a function of  $b$ , is:

$$\binom{N-1}{m} \left[ \frac{\varepsilon - X + b}{2\varepsilon} \right]^m \left[ \frac{\varepsilon + X - b}{2\varepsilon} \right]^{N-m-1}. \quad (28)$$

When  $m$  players  $j \neq i$  play  $x_j = 1$ , the cost of playing  $x_i = 1$  to player  $i$  is  $c(m + 1)$ .

The derivation so far took  $b$  as a known quantity. I now take account of the fact that player  $i$  does not observe  $b$  directly but only a noisy signal  $b_i^\varepsilon$ . Given  $p_{-i} = p_{-i}^X$  and  $b_i^\varepsilon = X$ , the expected gain to player  $i$  from playing  $x_i = 1$  rather than  $x_i = 0$  becomes:

$$\begin{aligned} \Delta_i^\varepsilon(p_{-i}^X \mid X) &= \frac{1}{2\varepsilon} \int_{X-\varepsilon}^{X+\varepsilon} b db + d \\ &\quad - \sum_{m=0}^{N-1} c(m+1) \binom{N-1}{m} \frac{1}{2\varepsilon} \int_{X-\varepsilon}^{X+\varepsilon} \left[ \frac{\varepsilon - X + b}{2\varepsilon} \right]^m \left[ \frac{\varepsilon + X - b}{2\varepsilon} \right]^{N-m-1} db \end{aligned} \quad (29)$$

$$= X + d - \sum_{m=0}^{N-1} c(m+1) \binom{N-1}{m} \int_0^1 q^m (1-q)^{N-m-1} dq \quad (30)$$

$$= X + d - \sum_{m=0}^{N-1} c(m+1) \frac{(N-1)!}{m! (N-m-1)!} \frac{m! (N-m-1)!}{N!} \quad (31)$$

$$= X + d - \sum_{m=0}^{N-1} \frac{c(m+1)}{N}, \quad (32)$$

as given. Equation (29) takes the expression for  $\Delta_i(m \mid b)$  given in (2) and integrates over  $b$  and  $m$ , given  $b_i^\varepsilon = X$  and  $p_{-i} = p_{-i}^X$ . Equation (30) relies on integration

by substitution (using  $q = 1/2 - (X - b)/2\varepsilon$ ) to rewrite the last integral in (29). Equation (31) rewrites both the integral in (30) and the binomial coefficient  $\binom{N-1}{m}$  in terms of factorials. Equation (32) simplifies.  $\square$

### Proof of Theorem 2.

*Proof.* Let  $p$  be a BNE of  $G^\varepsilon$ . For any player  $i$ , define

$$\underline{b}_i = \inf\{b_i^\varepsilon \mid p_i(b_i^\varepsilon) > 0\}, \quad (33)$$

and

$$\bar{b}_i = \sup\{b_i^\varepsilon \mid p_i(b_i^\varepsilon) < 1\}. \quad (34)$$

Observe that  $\underline{b}_i \leq \bar{b}_i$ . Now define

$$\underline{b} = \min\{\underline{b}_i\}, \quad (35)$$

and

$$\bar{b} = \max\{\bar{b}_i\}. \quad (36)$$

By construction,  $\bar{b} \geq \bar{b}_i \geq \underline{b}_i \geq \underline{b}$ . Observe that  $p$  is a BNE of  $G^\varepsilon$  only if, for each  $i$ , it holds that  $\Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \underline{b}_i) \geq 0$ . Consider then the expected gain  $\Delta_i^\varepsilon(p_{-i}^{\underline{b}}(b_{-i}^\varepsilon) \mid \underline{b}_i)$ . It follows from the definition of  $\underline{b}$  that  $p^{\underline{b}}(b^\varepsilon) \geq p(b^\varepsilon)$  for all  $b^\varepsilon$ . The implication is that, for each  $i$ ,  $\Delta_i^\varepsilon(p_{-i}^{\underline{b}}(b_{-i}^\varepsilon) \mid \underline{b}_i) \geq \Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \underline{b}_i) \geq 0$ . From Proposition 2 then follows that  $\underline{b} \geq B^*$ .

Similarly, if  $p$  is a BNE of  $G^\varepsilon$  then, for each  $i$ , it must hold that  $\Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \bar{b}_i) \leq 0$ . Consider now the expected gain  $\Delta_i^\varepsilon(p_{-i}^{\bar{b}}(b_{-i}^\varepsilon) \mid \bar{b}_i)$ . It follows from the definition of  $\bar{b}$  that  $p^{\bar{b}}(b^\varepsilon) \leq p(b^\varepsilon)$  for all  $b^\varepsilon$ . For each  $i$  it therefore holds that  $\Delta_i^\varepsilon(p_{-i}^{\bar{b}}(b_{-i}^\varepsilon) \mid \bar{b}_i) \leq \Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \bar{b}_i) \leq 0$ . Hence  $\bar{b} \leq B^*$ .

Since  $\underline{b} \leq \bar{b}$  while also  $\underline{b} \geq B^*$  and  $\bar{b} \leq B^*$  it must hold that  $\underline{b} = \bar{b} = B^*$ . Moreover, since  $p^{\underline{b}} \geq p$  while also  $p^{\bar{b}} \leq p$ , given  $\underline{b} = \bar{b} = B^*$ , it follows that  $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$  for all  $b_i^\varepsilon \neq B^*$  and all  $i$  (recall that for each player  $i$  one has  $\Delta_i^\varepsilon(p_{-i}^{B^*} \mid B^*) = 0$ , explaining the singleton exception at  $b_i^\varepsilon = B^*$ ). Thus, if  $p = (p_i)$  is a BNE of  $G^\varepsilon$  then it must hold that  $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$  for all  $b_i^\varepsilon \neq B^*$  and all  $i$ , as I needed to prove.  $\square$

### Proof of Proposition 5.

*Proof.* Strict dominance is an immediate consequence of rewriting the player  $i$ 's gain including taxes and subsidies:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) + t = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + t = b_i^\varepsilon + c(N) - d + (N-1) \cdot \frac{c(N) - d}{N}, \quad (37)$$

which is strictly positive for all  $b_i^\varepsilon > \bar{b}$  and strictly negative for all  $b_i^\varepsilon < \bar{b}$ . As to the final claim in the Proposition, observe that each  $b_i^\varepsilon$  is drawn from  $[b - \varepsilon, b + \varepsilon]$ , given  $b$ . Hence, if  $b > \bar{b} + \varepsilon$  then  $b_i^\varepsilon > \bar{b}$  for each  $i$ , so playing  $x_i = 1$  is strictly dominant and players coordinate on  $\mathbf{1}$ , the efficient outcome of the game (for those  $b$ ). In this case,

total spending on subsidies is  $s^*(\mathbf{1}) = 0$ . Similarly, if  $b < \bar{b} - \varepsilon$  then  $b_i^\varepsilon < \bar{b}$  for each  $i$ , so playing  $x_i = 0$  is strictly dominant and players coordinate on  $\mathbf{0}$ , the efficient outcome of the game (for those  $b$ ). Since no player plays 1, total subsidy spending is naturally zero.  $\square$

### Proof of Theorem 3.

*Proof.* The gain from choosing  $x_i = 1$  rather than  $x_i = 0$ , conditional on the network subsidy scheme  $s^*(\cdot)$ , given  $b$  and  $x_{-i}$  is (12) which, for all  $x_{-i}$ , is (strictly) positive if and only if  $\mathbf{1}$  is a (strict) Nash equilibrium of the game. Thus, the offering a subsidy scheme equal to  $s^*(\cdot)$  turns  $x_i = 1$  into a (strictly) strictly dominant strategy whenever  $\mathbf{1}$  is a (strict) Nash equilibrium of  $G(b)$ . When players coordinate on  $\mathbf{1}$  total spending on network subsidies is  $N \cdot s^*(\mathbf{1}) = 0$ .  $\square$

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