Time Horizons And Emissions Trading

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Abstract

The economic environment of this paper is a cap and trade scheme in which the cap on emissions is determined, at least in part, by conditions prevailing in the market for emissions allowances. Under a quantity mechanism, the cap depends on the surplus of unused allowances. Under a price mechanism, the cap instead depends on the market price for allowances.

I show the following. Given a future ban on emissions in the form of a commonly known final period beyond which emissions are not allowed (even if firms have unused allowances left), total emissions may be higher when the final period is set earlier and the supply of allowances is governed by a quantity mechanism. The same result does not obtain when the supply of allowances is governed by a price mechanism, in which case total emissions will be less if the final period is set earlier. My results show that quantity mechanisms are hard to combine with zero emissions pledges and thus favor price mechanisms.

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1 Introduction

Emissions trading is among the commonest of policies to price carbon and curb greenhouse gas emissions. In its most basic form, emissions trading – or cap and trade – fixes the total amount of emissions but allows covered firms to decide on the allocation of emissions under this cap, creating a market for greenhouse gases.

In contrast to textbook models, emissions trading schemes (ETSs) typically do not impose a fixed limit on emissions but instead make the cap on emissions endogenous to conditions prevailing in the market (ICAP, 2021). An endogenous emissions cap is motivated by the idea that it makes a policy more resilient to economic fluctuations and uncertainties which would otherwise render the system unstable or even ineffective (Fell, 2016; Lintunen and Kuusela, 2018; Pizer and Prest, 2020). In all existing cap and trade schemes, stabilization mechanisms are based either on the allowance price (price mechanisms) or on the number

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of allowances used (quantity mechanisms). Examples of the price mechanisms include price floors and ceilings, used for example in California's ETS (Schmalensee and Stavins, 2017; Gerlagh et al., 2021). An example of a quantity mechanism is a quantity collar, used in the EU ETS (Fell, 2016; Holt and Shobe, 2016).

The first contribution of this paper is to prove a paradoxical result on the interaction between quantity mechanisms and the time horizon of emissions trading. Suppose the policymaker fixes a point in time beyond which emissions are not allowed, even if firms have unused allowances left. I show that total emissions are higher when this final period on emissions occurs earlier rather than later if the cap and trade scheme operates a quantity mechanism. My second contribution is to show that the same paradox does not arise when instead the cap and trade scheme operates a price mechanism, in which case a shorter time horizon of emissions unambiguously reduces pollution.

I derive my results in a generic model of dynamic emissions trading. Firms produce goods and in the process pollute the environment. In each period, the policymaker dictates abatement obligations through issuing allowances which firms must surrender to cover their emissions. Temporal violations of the periodic cap are accommodated through a banking provision that facilitates the use of allowances issued in one period to meet compliance obligations pursuant to another (Kling and Rubin, 1997). Subject to this cap, firms allocate emissions allowances over time to minimize abatement costs.

The above describes a standard treatment of emissions trading. My model two important additional element meant to reflect recent real world developments. First, the cap on emissions responds to conditions prevailing in the market for allowances through a supply mechanism mechanism. I limit attention to price and quantity mechanisms because empirically these are the most relevant (ICAP, 2021). Under a price mechanism, the supply of allowances increases when the allowance price goes up. A quantity mechanism instead translates an expanding bank of unused allowances into a lower supply.

Second, the policymaker dictates a binding final period on emissions beyond which firms are not allowed to emit even if they have unused allowances left. A final period on emissions captures the idea that policymakers may try to speed up the de-carbonization of their economies. It reflects the wave of zero emission targets that governments have recently pledged to and are considered crucial in meeting the Paris Agreement temperature goals (Höhne et al., 2021).

The driving force behind my results is firms' incentive not to bank allowances beyond the final period. An allowance has value only if it can be used to cover emissions. Instead of leaving allowances unused until emissions are no longer allowed, firms had better use them prior to the final period to minimize abatement costs when they still can. An earlier final period on emissions therefore stimulates firms to use more allowances in early periods, implying less banking overall. The response of a quantity mechanism is to increase the supply of emissions allowances and this explains the paradox. To support the increased demand for allowances, the allowance price must go down; under a price mechanism, this implies fewer emissions overall and explains my second result.

A proper understanding of supply policies in cap and trade schemes is important because they form an integral part of global climate policy. According to the International Carbon Action Partnership, to date more than 30 supranational, national and local jurisdictions representing 54% of global GDP operate a cap and trade scheme for greenhouse gases while more are under way. Of these, a large majority has some kind of endogenous cap.[1] Moreover, supply policies have a major impact on emissions trading. As an example, Borenstein et al. (2019) show that the allowance price in California's cap and trade scheme is determined by the administrative price collar with 98.9 percent probability. It is hence important to study stabilization mechanisms for the design of environmental policy.

The key policy takeaway is simple. Zero emissions pledges, aimed at accelerating the de-carbonization of the economy, may backfire when combined with quantity-based cap and trade policies. On the other hand, climate neutrality goals are supported by price-based emissions trading schemes. Careful policymaking is called for when imposing additional abatement policies on cap and trade schemes governed by a quantity role.

The paper proceeds as follows. Section 2 sets out the model and defines price- and quantity mechanisms formally. Section 4 then studies the equilibrium of cap and trade schemes under price and quantity mechanisms for different final periods on emissions. Emission levels are compared across scenarios, which yields my main results. Section 6 discusses the results and concludes.

2 Model

I first set up a generic model of emissions trading and derive firms' emission decisions from a dynamic abatement cost minimization problem. I then formally define and discuss the two supply policies studied. Equilibrium analyses are relegated to Section 4.

2.1 Firms' problem

Consider the dynamic abatement cost minimization problem faced by firms. In each period $t \geq 0$, abatement for firm i, with $i = 1, 2, ..., N$, is given by $a_{it} = \bar{q}_{it} - q_{it}$, where $\bar{q}_{it}$ denotes expected business-as-usual emissions (i.e. the level emissions in the absence of any policy whatsoever) and $q_{it} \geq 0$ is the actual level of emissions in period t. The cost of abatement is determined by the abatement cost function C_{it} which satisfies $C_{it}(0) = 0$, $C'_{it}(a_{it}) := \frac{\partial C_{it}(a_{it})}{\partial a_{it}} > 0$ and $\frac{\partial^2 C_{it}(a_{it})}{\partial a_{it}^2} \geq 0$.

Emissions are regulated through a cap and trade scheme. Let s_{it} denote the number of allowances supplied to firm i in period t. Allowances, once supplied, may be traded on a secondary market where a firm can sell or acquire them at a price p_t which it takes as given. Hence, if a firm chooses an amount q_{it} of emissions and sells or buys a total of m_{it} allowances on the secondary market, its total costs are $C_{it}(\bar{q}_{it} - q_{it}) + p_t m_{it}$.

Temporal violations of the periodic cap s_{it} are facilitated through a banking provision (Kling and Rubin, 1997; Hasegawa and Salant, 2014; Shobe et al., 2014; Fell, 2016). I define banking by firm i in period t to be $b_{it} := s_{it} + m_{it} - q_{it}$. The bank of allowances held by firm i at the start of period t is then $B_{it} := \sum_{s=0}^{t-1} b_{is} = B_{it-1} + b_{it-1} = B_{it-1} + s_{it-1} + m_{it-1} - q_{it-1}$, and the total bank of allowances at the start of period t is $B_t := \sum_i B_{it}$. I also assume that

[1] While not extensive, a list of cap and trade schemes that operate price or quantity mechanisms of the kind studied in this paper includes California's cap and trade scheme, China's National ETS, the EU ETS, Germany's National ETS, Korea's ETS, New Zealand's ETS, the Massachusetts Limits on Emissions from Electricity Generator, the Regional Greenhouse Gas Initiative (RGGI), and Switzerland's ETS.

borrowing is not allowed, so $B_{it} \geq 0$ for all i and t. The *dynamic* constraint on emissions by firm i is therefore $\sum_{s=0}^{t} q_{is} \leq \sum_{s=0}^{t} s_{is} + m_{is}$ for all t. This is the first constraint on the firm's constrained cost minimization problem.

The second constraint is a final period $T > 0$ starting from which emissions are no longer allowed even if firms have unused allowances left; that is, $q_{it} = 0$ for all i and all $t \geq T$. A situation in which allowances do not expire is nested in this model by setting $T \to \infty$. I assume that all firms anticipate the final period T starting from period 0.[2] The final period T on emissions reflects the future zero-emissions pledges made by governments around the globe (Höhne et al., 2021; Nature Editorial, 2021). I colloquially refer to T as the *time horizon of emissions*.

The above elements make for a constrained optimization problem:

$$\min_{q_{it}, m_{it}} \quad \sum_t \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t m_{it} \right] \tag{1}$$

$$\text{subject to} \quad \sum_s q_{is} \leq \sum_s \left[s_{is} + m_{is} \right], \tag{2}$$

$$\sum_i m_{it} = 0, \tag{3}$$

$$q_{it} = 0, \quad \text{for all} \quad t \geq T, \tag{4}$$

$$B_{it+1} = B_{it} + s_{it} + m_{it} - q_{it}, \tag{5}$$

$$B_{it} \geq 0, \tag{6}$$

for each i and t and where $\beta > 0$ is the discount factor. Constraint (2) says that any individual firm's emissions may not exceed the total number of allowances it owns. Constraint (3) says that, on the secondary market, every allowance bought by one firm must be sold by another.[3] Constraint (4) says that emissions are not allowed starting from period T onward. Finally, (5) is the equation of motion for the bank of allowances and constraint (6) imposes that borrowing allowances from the future is not allowed.

In the solution to this problem (see Appendix), marginal abatement costs grow at the discount rate, so long as firms can choose emissions:

$$C'_{it}(\bar{q}_{it} - q_{it}) + \psi_{it} = \beta C'_{it+1}(\bar{q}_{it+1} - q_{it+1}), \tag{7}$$

for all $t < T$ and where ψ_{it} is the shadow value of the borrowing constraint (6). Moreover, each firm will emit, or abate, until marginal abatement costs roughly equal the allowance price,

$$p_t = C'_{it}(\bar{q}_{it} - q_{it}) - \mu_t, \tag{8}$$

for all $t < T$. I say that prices should roughly equal the allowances price because when $\mu_t \neq 0$, the secondary market constraint is binding and not every firm can buy or sell the number of

[2]This is not really an assumption; simply define $t = 0$ to be the first period in which T is common knowledge.

[3]If the policymaker were to put an "exchange rate" on allowances, as proposed by Holland and Yates (2015), this constraint could be violated. Because, in practice, such exchange rate are not common, I stick with constrain (3).

allowances it wants, driving a wedge between the allowance price and marginal abatement costs. Combining (7) and (8), prices should satisfy:

$$\frac{\partial p_{t+1}}{\partial p_t} = \frac{1}{\beta}, \tag{9}$$

which is a differential version of Hotelling's rule.[4] For $\mu_t = 0$, cost-minimizing prices follow Hotelling's rule as $p_{t+1} = p_t/\beta$, implying (9).

For each i and t, let $q_{it}(p_t, T)$ denote the level of emissions that solves (8). The assumptions on C_{it} imply

$$\frac{\partial q_{it}(p_t, T)}{\partial p_t} < 0, \tag{10}$$

for all $t < T$. As is intuitive, the abatement cost minimizing level of emissions chosen by firm i in period t is decreasing in the prevailing allowance price in that period. For given p_t and T, define $q_t(p_t, T) = \sum_i q_{it}(p_t, T)$ to be total demand for emissions in period t. By (10),

$$\frac{\partial q_t(p_t, T)}{\partial p_t} < 0. \tag{11}$$

I conclude this section with two remarks.

Remark 1. $m_{it} = 0$ *for all* i *and* $t \geq T$. *That is, firms neither buy nor sell allowances on the secondary market from the final period on emissions onward.*

Remark 2. *The allowance price* p_t *need not satisfy* (8) *and/or* (9) *when* $t \geq T$. *The allowance price comes about through supply and demand on the secondary market. As there is no demand for allowances when* $t \geq T$, *the allowance price is not properly defined for those periods.*

2.2 Supply mechanisms

Let $s_t = \sum_i s_{it}$ denote the total supply of allowances in period t. I will come to the precise determination of the supply path (s_t) shortly; in any case I assume that $\sum_{s=0}^{t} s_s < \sum_{s=0}^{t} \bar{q}_s$ for all t, where $\bar{q}_t = \sum_i \bar{q}_{it}$. That is, the supply of allowances does not exceed business-as-usual emissions.[5]

The exact number of allowances supplied is assumed to depend on conditions prevailing in the market for allowances. To stay close to reality, I will limit attention to two classes of endogenous allowance supply schemes: price and quantity mechanisms.

[4]It is well known that factors including asymmetric information (Martimort et al., 2018), technological progress (Livernois, 2009), arbitrage opportunities (Anderson et al., 2018), or rolling planning horizons (Spiro, 2014) can cause violations of the policy in its canonical formulation. Nevertheless, the literature broadly supports the co-movement of prices over time (Livernois, 2009; Anderson et al., 2018; Martimort et al., 2018).

[5]The case in which allowance supply exceeds BAU emissions could of course occur but appears to be largely irrelevant. A large empirical literature shows that cap and trade schemes bring down emissions (Schmalensee et al., 1998; Fowlie, 2010; Fowlie et al., 2012; Deschenes et al., 2017; Calel, 2020; Bayer and Aklin, 2020).

Definition 1 (Price mechanism)**.** *A cap and trade scheme operates a* price mechanism *if the supply of allowances in any period t is weakly increasing in the prevailing allowance price p_t. Formally, for any period t and any two price levels p_t and p'_t it holds that $s_t(p_t) \geq s_t(p'_t)$ if and only if $p_t > p'_t$.*

Examples of price mechanisms include price floors and ceilings (Fell et al., 2012). It is assumed that $s_t(0) \leq q_t(0,T)$ and $s_t(\infty) \geq q_t(\infty, T)$ for all t, with a stict inequality for at least one t; this assumption guarantees existence of a positive but finite equilibrium allowance price. While not strictly necessary for my main results, I assume that $s_t(p_t)$ is differentiable in p_t to simplify the exposition.

The other type of supply policy considered is a quantity mechanism.

Definition 2 (Quantity mechanism)**.** *A cap and trade scheme operates a* quantity mechanism *if the supply of allowances in period t is increasing in the aggregate excess supply at the start of period t. That is, for any period t and any two B_t and B'_t, it holds that $s_t(B_t) \geq s_t(B'_t)$ if and only if $B'_t > B_t$.*

Examples would be quantity collars (Holt and Shobe, 2016), a market stability reserve like the EU's (Fell, 2016), or Korea ETS-style liquidity provisions (Asian Development Bank, 2018). I assume that $s_t(B_t(p)) \leq q_t(0,T)$ and $s_t(B_t(p)) \geq q_t(\infty, T)$ for all p, with a strict inequality for at least one t. While not strictly necessary for my main results, I assume that $s_t(B_t)$ is differentiable in B_t. I also assume that $-1 < \partial s_t / \partial B_t$ for all t to avoid the counter-intuitive scenario in which firms have an incentive to bank *less* today in order to have *more* allowances in the future – banking should happen only to accommodate relative future scarcity.

I assume that firms take the supply of allowances as given. This assumptions amounts to saying that each firm is small compared to the size of the market as a whole. In some cases, it is possible that a large firms can exercise market power (Liski and Montero, 2011); I do not investigate the extent to which my analysis generalizes to such environments.

3 Dynamic price effects

In this section, I establish that the bank of allowances is increasing in the allowance price under both price and quantity mechanisms. These effects are key drivers of my main results.

Lemma 1 (Dynamic price effects under a price mechanism)**.** *Consider a cap and trade scheme that operates a price mechanism. Fix a final period on emissions T. For any two periods $\tau, t < T$, the bank of allowances B_t is increasing in the allowance price p_τ: $\frac{\partial B_t(p)}{\partial p_\tau} > 0$.*

Proof. Since $s_t(p_t)$ is increasing in p_t by construction while $q_t(p_t, T)$ is decreasing by (10), banking in period $b_t(p_t)$ is increasing in the allowance price p_t. Recall from (9) that prices co-move across periods. By implication, one has $\frac{\partial p_s}{\partial p_\tau} > 0$ for all $s, \tau \in \{0, 1, ..., T\}$ and therefore,

$$\begin{aligned}\frac{\partial B_t}{\partial p_\tau} &= \frac{\partial}{\partial p_\tau}\left[\sum_s^{t-1} s_s(p_s) - \sum_s^{t-1} q_s(p_s, T)\right] \\ &= \sum_s^{t-1} \frac{\partial s_s(p_s)}{\partial p_s}\frac{\partial p_s}{\partial p_\tau} - \sum_s^{t-1} \frac{\partial q_s(p_s, T)}{\partial p_s}\frac{\partial p_s}{\partial p_\tau} > 0.\end{aligned} \tag{12}$$

This establishes that the aggregate bank of allowances B_t is increasing in the allowance price p_τ for all t and τ such that $0 \leq t, \tau < T$. □

Lemma 2 (Dynamic price effects under a quantity mechanism)**.** *Consider a cap and trade scheme that operates a quantity mechanism. Fix a final period on emissions T. For any two periods $\tau, t < T$, the bank of allowances B_t is increasing in the allowance price p_τ: $\frac{\partial B_t(p)}{\partial p_\tau} > 0$.*

Proof. The effect of an increase in the allowance price on first-period banking is straightforward:

$$\frac{\partial B_1(p)}{\partial p_\tau} = \frac{\partial b_0(p_0)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = \frac{\partial [s_0 - q_0(p_0, T)]}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = -\frac{\partial q_0(p_0, T)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} \geq 0, \tag{13}$$

where the inequality is strict for all p_0 such that $q_0(p_0, T) > 0$ and all $\tau \geq 0$. A little more work is required to determine the sign of $\partial B_t/\partial p$ for $t > 1$. Recall that the bank of allowances evolves according to $B_t(p) = B_{t-1}(p) + s_{t-1}(B_{t-1}(p)) - q_{t-1}(p_{t-1})$, where s_t depends on B_t because supply is governed by a quantity mechanism. It therefore follows that

$$\frac{\partial B_t(p)}{\partial p_\tau} = \frac{\partial B_{t-1}(p)}{\partial p_\tau} + \frac{\partial s_{t-1}(B_{t-1}(p))}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1}, T)}{\partial p_\tau} \tag{14}$$

$$= \left(1 + \frac{\partial s_{t-1}(B_{t-1}(p))}{\partial B_{t-1}(p)}\right)\frac{\partial B_{t-1}(p)}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1}, T)}{\partial p_{t-1}}\frac{\partial p_{t-1}}{\partial p_\tau}. \tag{15}$$

The term in parentheses, $1 + \partial s_t/\partial B_t$, is positive by assumption; recall that $-1 < \partial s_t/\partial B_t$ for all t, so $1 + \partial s_t/\partial B_t > 0$. The final term in (15) is negative by (9) and (10). The only sign left to determine in (15) is hence that of $\partial B_{t-1}/\partial p_\tau$. Using (13), induction on t establishes that

$$\frac{\partial B_t(p)}{\partial p_\tau} \geq 0, \tag{16}$$

for all t and τ such that $0 \leq t, \tau < T$. The inequality is strict for all $p = (p_1, p_2, ...)$ that satisfy $q_t(p_t) > 0$ for at least one t. □

4 Equilibrium

The market is in equilibrium when the supply of emissions allowances is equal to demand. The price of allowances adjust to brings about equilibrium.

Consider two periods T and $\bar{T}$ and, without loss of generality, let $\bar{T} < T$. In what follows, I will compare equilibrium emissions when the final period on emissions is T to when the final period is $\bar{T}$. The question arises, Does a shorter time horizon of emissions benefit the climate? The answer will depend on the supply policy in place.

4.1 Price mechanisms

Consider first the *baseline* final period on emissions T. The market for allowances is in equilibrium if

$$\sum_{t=0}^{T} q_t(p_t^P, T) = \sum_{t=0}^{T} s_t(p_t^P), \tag{17}$$

where p_t^P is the *baseline* equilibrium allowance price when the scheme operates a price mechanism and the final period on emissions is T.[6] Let $p^P = (p_t^P)$ denote the associated equilibrium price path.

Under a price mechanism, the supply of allowances depends on the prevailing allowance price. For a given final period on emissions T, let $f^P(T)$ denote the period in which the supply of emissions drops permanently to zero under a price mechanism policy. Formally, $f^P(T)$ is the integer for which the statement that $s_t(p^P) = 0$ if and only if $t \geq f^P(T)$ is true. The vector $(s_t(p^P))$ represents baseline equilibrium supply path under a price mechanism when the time horizon of emissions is T.

My goal is to find out how the effect of the time horizon of emissions trading equilibrium pollution. To do so, I take an alternative final period on emissions $\bar{T} \neq T$ and compare equilibrium emissions when the final period is T to when it is $\bar{T}$. I maintain the following assumptions.

Assumption 1. *$f^P(T) \leq \bar{T} < T$.*

Assumption 2. *$q_{\bar{T}}(p_{\bar{T}}^P, T) > 0$.*

Assumption 1 says that the baseline equilibrium supply of allowances reaches zero before the demand for emissions runs out. Assumption 2 says that *at the baseline equilibrium price* $p_{\bar{T}}^P$, demand for emissions in period $\bar{T}$ is strictly positive. Combined, these assumptions imply that firms will hold a strictly positive bank of allowances at the start of period $\bar{T}$ in the baseline equilibrium when the final period is T and the allowance price path is p^P.

Proposition 1. *Consider a cap and trade scheme that operates a* price mechanism*. Consider two periods T and $\bar{T}$. If T and $\bar{T}$ satisfy Assumptions 1 and 2, aggregate emissions are* lower *when the final period is $\bar{T}$ compared to when it is T.*

Proof. When the time horizon of emissions is restricted to $\bar{T} < T$, equilibrium in the market for allowances is reached when:

$$\sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^P, \bar{T}) = \sum_{t=0}^{\bar{T}} s_t(\bar{p}_t^P), \tag{18}$$

where $\bar{p}_t^P$ is the *restricted* equilibrium allowance price under a price mechanism and $\bar{p}^P = (\bar{p}_t^P)$ denotes the associated price path. Rewrite the restricted equilibrium market condition (18) as

$$B_{\bar{T}}(\bar{p}^P) = 0. \tag{19}$$

[6] An overwhelming oversupply of allowances, where the number of permits issued exceeds business as usual emissions, might lead to violations of (17). This possibility was ruled out by assumption, and so (17) is an identifying condition for an equilibrium. A similar discussion also applies to equilibrium under a quantity mechanism, presented in the next subsection.

Assumptions 1 and 2 imply

$$B_{\bar{T}}(p^P) > 0. \tag{20}$$

From Lemma 1 and (9),

$$\bar{p}_t^P < p_t^P, \tag{21}$$

for all $t < \bar{T}$. Under a price mechanism, (21) implies:

$$s_t(\bar{p}_t^P) < s_t(p_t^P), \tag{22}$$

for all $t \leq f^P(T)$. □

Proposition 1 gives the intuitive result that a shorter time horizon for emissions reduces emissions in cap and trade schemes that operate a price mechanism. Roughly speaking, the price of an allowance is dictated by the opportunity cost of using it now rather that later. In excluding the use of allowances for a wider range of future periods, the policymaker effectively reduces the opportunity cost of using an allowance today. This decreased opportunity cost translates into a lower allowance price, which, by virtue of the price mechanism, reduces the aggregate supply of allowances and thus emissions.

4.2 Quantity mechanisms

Consider again the baseline final period on emissions T. Equilibrium under a quantity mechanism is reached when

$$\sum_{t=0}^{T} q_t(p_t^Q, T) = \sum_{t}^{T} s_t(B_t(p^Q)), \tag{23}$$

where p_t^Q is the baseline equilibrium allowance price in period t under a quantity mechanism and $p^Q = (p_t^Q)$ is the associated price path.

For a given final period on emissions T, let $f^Q(T)$ denote the period in which the supply of allowances dries up permanently under a quantity mechanism. That is, $f^Q(T)$ is the lowest integer such that $s_t(B_t(p^Q)) = 0$ for all $t \geq f^Q(T)$.

Consider a period $\bar{T} \neq T$. As for price mechanisms, I want to know how equilibrium emissions compare when the final period is T versus when it is $\bar{T}$. I maintain the following assumptions.

Assumption 3. $f^Q(T) \leq \bar{T} < T$.

Assumption 4. $q_{\bar{T}}(p_{\bar{T}}^Q, T) > 0$.

Assumptions 3 and 4 have similar interpretations as Assumptions 1 and 2: the restricted final period $\bar{T}$ bans emissions when the baseline demand for emissions is still positive but after the baseline supply of allowances has dried up.

Proposition 2. *Consider a cap and trade scheme that operates a* quantity mechanism*. Consider two periods T and $\bar{T}$. If T and $\bar{T}$ satisfy Assumptions 3 and 4, aggregate emissions are* higher *when the final period is $\bar{T}$ compared to when it is T.*

Proof. Equilibrium under the restricted final period $\bar{T}$ requires:

$$\sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^Q, \bar{T}) = \sum_{t}^{\bar{T}} s_t(B_t(\bar{p}^Q)), \tag{24}$$

where $\bar{p}^Q = (\bar{p}_t^Q)$ is the restricted equilibrium allowance price path. Rewrite (24) as

$$B_{\bar{T}}(\bar{p}^Q) = 0. \tag{25}$$

By Assumptions 3 and 4,

$$B_{\bar{T}}(p^Q) > 0. \tag{26}$$

From Lemma 1 and the differential Hotelling rule (9),

$$\bar{p}_t^Q < p_t^Q, \tag{27}$$

for all $t < \bar{T}$. Using (27),

$$B_t(\bar{p}^Q) < B_t(p^Q), \tag{28}$$

for all $t \leq \bar{T}$. By the mechanics of a quantity mechanism, (28) implies that:

$$s_t(B_t(\bar{p}^Q)) > s_t(B_t(p^Q)), \tag{29}$$

for all $t < \bar{T}$. This gives the second main result. □

Proposition 2 gives the paradoxical result that an earlier ban on emissions leads to more pollution in cap and trade schemes under a quantity mechanism. In this sense, a quantity mechanism is hard to combine with a zero-emissions pledges.

At its core, Proposition 2 is the product of two effects. The first is leakage and says that any allowances firms cannot used in the future will be used today. Firms bank allowances to use them in the future. When the number of future periods in which an allowance can be used is reduced, firms will typically bank less.

With a fixed cap, leakage implies a reshuffling of emissions over time but no change in emissions overall: the green paradox (Van der Ploeg and Withagen, 2012). The second effect, however, says that the reduction in banking implied by the leakage effect causes the supply of allowances to go up. This effect is immediate from the construction of the quantity mechanism. Since every allowance supplied will, in equilibrium, be used, aggregate emissions are unambiguously higher once the time horizon of emissions gets restricted.

5 Ambitious Climate Policy

When comparing emissions for different time horizons, I assumed that the tighter horizon $\bar{T}$ is set after the baseline equilibrium supply of emissions runs out. Yet some have argued that truly ambitious climate policy requires a faster de-carbonization of the economy (Rockström et al., 2017). What happens when the tighter horizon is set *before* baseline supply dries up? This section investigates.

5.1 Price mechanisms

Consider again a cap and trade scheme that operates a price mechanism. Given two periods T_1 and T_2 and an allowance price path p, define $\Delta^P(T_1, T_2, p)$ as

$$\Delta^P(T_1, T_2, |\, p) := \sum_{t=T_2}^{T_1} s_t(p_t). \tag{30}$$

In words, $\Delta^P(T_1, T_2, |\, p)$ is the number of emissions allowances supplied between periods T_1 and T_2 when the allowance price path is p and supply if governed by a price mechanism.

Let T again denote the baseline final period on emissions. The baseline equilibrium allowance price path p^P is defined as the solution to (17). As before, let $\bar{T}$ be an alternative, and earlier, final period on emissions.

Assumption 5. $\bar{T} < f^P(T) < T$.

Under Assumption 5, the tighter time horizon $\bar{T}$ bans emissions even before the baseline supply of emissions dries up. Compared to the baseline equilibrium, this tightening of the time horizon has two mutually re-enforcing effects. The first effect is immediate: all emissions supplied in the baseline equilibrium starting from period $\bar{T}$ are pre-empted under the tighter time horizon $\bar{T}$. The second effect is indirect and similar to that described in Section 4.1: firms adjust their use of allowances to make sure they have no unused reserves left when period $\bar{T}$ arrives. While the first effect is always present, whether or not the second affects emissions depends on the baseline bank of allowances at the start of period $\bar{T}$. Either way, banning emissions earlier causes aggregate emissions to decrease.

Proposition 3. *Consider a cap and trade scheme that operates a* price mechanism. *Consider two periods T and $\bar{T}$ that satisfy Assumption 5 and such that T is the baseline final period on emissions. Let p^P denote the equilibrium allowance price path when the final period is T. Then,*

(i) If baseline equilibrium banking is zero at the start of period $\bar{T}$, $B_{\bar{T}}(p^P) = 0$, aggregate emissions are lower *when the final period is $\bar{T}$ compared to when it is T. The reduction in aggregate emissions is given by $\Delta^P(T, \bar{T}, |\, p^P) > 0$;*

(ii) If baseline equilibrium banking is positive at the start of period $\bar{T}$, $B_{\bar{T}}(p^P) > 0$, aggregate emissions are lower *when the final period is $\bar{T}$ compared to when it is T. The reduction in aggregate emissions is greater than $\Delta^P(T, \bar{T}, |\, p^P) > 0$.*

5.2 Quantity mechanisms

Consider a cap and trade scheme that operates a quantity mechanism. Given two periods T_1 and T_2 and an allowance price path p, define $\Delta^Q(T, T_2, p)$ as

$$\Delta^Q(T_1, T_2, |\, p) := \sum_{t=T_2}^{T_1} s_t(B_t(p)) \tag{31}$$

Let T be the baseline final period on emissions, and let p^Q be the associated equilibrium price path; that is, p^Q is the vector of allowance prices that solves (23). Let $\bar{T}$ be an alternative, and earlier, final period on emissions. I make the following assumption.

Assumption 6. *$\bar{T} < f^Q(T) < T$.*

Under Assumption 6, the tighter time horizon $\bar{T}$ bans emissions even before the baseline supply of emissions runs out. The effect of an earlier ban on emissions is ambiguous under a quantity mechanism.

Proposition 4. *Consider a cap and trade scheme that operates a* quantity mechanism*. Consider two periods T and $\bar{T}$ that satisfy Assumption 6 and such that T is the baseline final period on emissions. Let p^Q denote the equilibrium allowance price path when the final period is T. Then,*

(i) If baseline equilibrium banking is zero at the start of period $\bar{T}$, $B_{\bar{T}}(p^Q) = 0$, aggregate emissions are lower *when the final period is $\bar{T}$ compared to when it is T. The reduction in aggregate emissions is equal to $\Delta^Q(T, \bar{T}, | \, p^Q) > 0$;*

(ii) If baseline equilibrium banking is positive at the start of period $\bar{T}$, $B_{\bar{T}}(p^Q) > 0$, aggregate emissions can be lower or higher *when the final period is $\bar{T}$ compared to when it is T. Either way, the reduction in aggregate emissions is less than $\Delta^Q(T, \bar{T}, | \, p^Q)$.*

6 Discussion and Conclusions

I study the effect of the time horizons of emissions trading on pollution. When the supply of emissions in a cap and trade scheme is governed by a quantity mechanism, a binding final period beyond which emissions are not allowed unambiguously raises aggregate emissions compared to the case in which allowances have an infinite lifetime. This paradox does not arise if instead the schemes relies on a price mechanism, in which case a binding final period on emissions unambiguously curbs emissions.

My result on quantity mechanisms is related to the green paradox (Van der Ploeg and Withagen, 2012). Proposition 2 is stronger, however, as aggregate emissions *increase* when the time horizon of emissions trading is restricted. The possibility of increased emissions in response to overlapping climate policies in a two-period model was also observed by Gerlagh et al. (2021). This paper extends their work by looking at a stronger kind of environmental policy (a complete ban on emissions) for a general number periods and generic supply policies.

Considering the recent wave of zero emission pledges by governments across the global (Höhne et al., 2021), my results warrants a careful re-evaluation of quantity-based supply policies in cap and trade schemes. Superficially, the paradox could be preempted simply by not restricting the time horizon of emissions. This is an imperfect solution at best; besides the mere sum total of emissions, the natural sciences agree that the *timing* of emissions is also important for global warming (Gasser et al., 2018). A change to price-based allowance supply policies thus seems a far superior way forward.

A number of real-world cap and trade schemes operate a quantity mechanism or consider introducing one. The most well-known example is probably the EU ETS, the world's largest market for carbon. In addition, South Korea's ETS has a "market liquidity" provision which, in practice, functions like a quantity mechanism. According to the Asian Development Bank (2018) the South Korean "government applied these provisions at the end of the first year when it turned out that the overall emissions of the participants were 0.82% higher than the cap. To address this issue, the government supplied additional allowances to the market." Moreover, the UK ETS is currently designing its *Supply Adjustment Mechanism*, which may be modeled after the EU ETS; Switzerland's ETS similarly intends to adopt a quantity mechanism.[7] My result illustrates that policymakers of these schemes should be careful as policies to speed up the de-carbonization of the economy may backfire.

While I study stabilization mechanisms within a cap and trade scheme generally, many other types of market-based environmental policies exist, see for example Böhringer and Lange (2005), Böhringer et al. (2017), and Fowlie and Muller (2019). The critical message regarding quantity-based stabilization does not necessarily extend to other kinds of endogenous policies.

This paper makes several restrictive assumptions. First, I assume that adjustments to the final period on emissions are not accompanied by discrete supply-adjustments; changes in the supply of allowances come about entirely through the stabilization mechanism. In reality, the introduction of a final period on emission would counstitute a major reform which the policymaker might consider only within the context of a broader set of changes, including perhaps exogenous supply adjustments. Second, I consider a particular kind of finite time horizon in which allowances can be used at any time prior to the final period independent of when they were issued. Alternatively, policymakers could write off unused allowances depending on when they were supplied, e.g. allowances can be kept for five years at most. Third, I assume that the biding period is set after the baseline supply of allowances dries up. It could be argued, however, that truly ambitious climate policy requires emissions to end earlier.

A Appendix

A.1 Additional details on firms' dynamic cost-minimization problem

Turning the constrained problem in (1)–(6) into an unconstrained cost minimization problem, each firm i chooses q_{it} and m_{it} to solve:[8]

$$\min_{q_{it},m_{it}} \sum_{t=0}^{T} \beta^t C_{it}(\bar{q}_{it} - q_{it}) + \sum_t \beta^t p_t m_{it} + \lambda_i \left[\sum_t q_{it} - s_{it} - m_{it}\right] + \sum_t \beta^t \mu_t \left[\sum_i m_{it}\right] \qquad (32)$$
$$+ \omega_{it}\left[B_{it} - B_{it-1} - s_{it-1} - m_{it-1} + q_{it-1}\right] + \beta^t \psi_{it} B_{it}.$$

The first-order conditions associated with the cost-minimization problem given by (32) are:

$$-\beta^t C'_{it}(\bar{q}_{it} - q_{it}) + \lambda_i + \omega_{it+1} = 0, \qquad (33)$$

[7] See the *Teilrevision der Verordnung über die Reduktion der CO2-Emissionen* (in German) for details of the Swiss ETS.

[8] Without loss of generality, I multiply the shadow values μ_t for the secondary market constraint (3) and ψ_{it} for the borrowing constraint by β^t.

$$\beta^t p_t - \lambda_i + \beta^t \mu_t - \omega_{it+1} = 0, \tag{34}$$

$$\omega_{it} - \omega_{it+1} + \beta^t \psi_{it} = 0. \tag{35}$$

Using (33) and (34) gives:

$$p_t + \mu_t = C'_{it}(\bar{q}_{it} - q_{it}), \tag{36}$$

implying (8). Moreover, combining (35) and (34) yields:

$$p_t + \mu_t + \psi_{it} = \beta p_{t+1} + \beta \mu_{t+1}, \tag{37}$$

so $p_{t+1} = (p_t + \mu_t + \psi_{it})/\beta - \mu_{t+1}$ and this implies (9).

Observe that cost minimization forces each firm i to choose $m_{it} \leq 0$ for all $t \geq T$; all want to sell allowances if they have some. Combined with the secondary market constraint that $\sum_i m_{it} = 0$ this gives $m_{it} = 0$, as stated in Remark 1.

A.2 Proofs of Propositions 3 and 4

PROOF OF PROPOSITION 3

Proof. Part (i). Suppose that $B_{\bar{T}}(p^P) = 0$, so the baseline equilibrium bank of allowances at the start of period $\bar{T}$ is zero. Let $\bar{p}^P$ again denote the equilibrium allowance price path when the time horizon of emissions is $\bar{T}$ (see (18)). By Lemma 1, given the co-movement of allowance prices over time (9), this implies $p_t^P = \bar{p}_t^P$ for all $t < \bar{T}$. In this case, the aggregate reduction in emissions when tightening the time horizon of emissions from T to $\bar{T}$ is therefore given by:

$$\sum_{t=\bar{T}}^{f^P(T)} s_t(p_t^P) > 0, \tag{38}$$

which by definition is equal to $\Delta^P(T, \bar{T} \mid p^P)$.

Part (ii). Suppose that $B_{\bar{T}}(p^P) > 0$, meaning that in the baseline equilibrium firms hold a strictly positive bank of allowances at the start of period $\bar{T}$. Equilibrium under the final period $\bar{T}$ is reached when $B_{\bar{T}}(\bar{p}^P) = 0$, see (19). By Lemma 1, this implies $p_t^P > \bar{p}_t^P$ for all $t < \bar{T}$. In this case, the aggregate reduction in emissions when tightening the time horizon of emissions from T to $\bar{T}$ is therefore given by:

$$\sum_{t=\bar{T}}^{f^P(T)} s_t(p_t^P) + \sum_{t=0}^{\bar{T}-1} s_t(p_t^P) - \sum_{t=0}^{\bar{T}-1} s_t(\bar{p}_t^P) > \sum_{t=\bar{T}}^{f^P(T)} s_t(p_t^P) > 0, \tag{39}$$

where the first inequality follows from the facts that $p_t^P > \bar{p}_t^P$ for all $t < \bar{T}$ and supply is governed by a price mechanism. □

PROOF OF PROPOSITION 4

Proof. Part (i). Suppose that $B_{\bar{T}}(p^Q) = 0$, so the baseline equilibrium bank of allowances at the start of period $\bar{T}$ is zero. Let $\bar{p}^Q$ again denote the equilibrium allowance price path when the final period is $\bar{T}$. In this case, the aggregate reduction in emissions when tightening the time horizon of emissions from T to $\bar{T}$ is given by:

$$\sum_{t=\bar{T}}^{f^Q(T)} s_t(B(p^P)) > 0, \tag{40}$$

which is $\Delta^Q(T, \bar{T} \mid p^Q)$.

Part (ii). Next suppose that $B_{\bar{T}}(p^Q) > 0$, so the baseline equilibrium bank of allowances at the start of period $\bar{T}$ is positive. Let $\bar{p}^Q$ again denote the equilibrium allowance price path when the final period is $\bar{T}$. Because equilibrium under the tighter horizon $\bar{T}$ requires $B_{\bar{T}}(\bar{p}^Q) = 0$, it holds that $\bar{p}_t^Q < p_t^Q$ for all $t < \bar{T}$. In this case, the aggregate reduction in emissions when tightening the time horizon of emissions from T to $\bar{T}$ is hence given by:

$$\sum_{t=\bar{T}}^{f^Q(T)} s_t(B(p^P)) + \sum_{t=0}^{\bar{T}-1} \left(s_t(B_t(p^Q)) - s_t(B_t(\bar{p}^Q)) \right) \lesseqgtr 0. \tag{41}$$

The sign of (41) is ambiguous. While tightening the time horizon of emissions from T to $\bar{T}$ causes an undisputed reduction of emissions between periods $\bar{T}$ to T, emissions may increase in periods leading up to period $\bar{T}$ when the time horizon gets tightened due to firms' incentives not to bank allowances beyond the final period. Thus, if it so happens that the baseline equilibrium bank of allowances is positive at the start of period $\bar{T}$ (Case 2), early-period emissions increase in response to the shorter time horizon. Under a quantity mechanism, this effect is further reinforced by the increase in allowance supply in those periods. If this effect, the second part on the left-hand side of (41), outweighs the reduction in emissions *after* period $\bar{T}$, emissions overall rise once the time horizon gets tightened. □

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