Time Horizons And Emissions Trading

Roweno J.R.K. Heijmans*

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Abstract

The economic environment of this paper is a cap and trade scheme in which the cap on emissions is determined, at least in part, by conditions prevailing in the market for emissions allowances. Under a quantity rule, the cap depends on the surplus of unused allowances. Under a price rule, the cap instead depends on the market price for allowances. I show the following. Given a future ban on emissions in the form of a commonly known final period beyond which emissions are not allowed (even if firms have unused allowances left), total emissions may be higher when the final period is set earlier and the supply of allowances is governed by a quantity rule. This somewhat paradoxical result does not obtain when the supply of allowances is governed by a price rule, in which case total emissions will be less if the final period is set earlier. My results show that quantity rules are hard to combine with zero emissions pledged and thus favor price-based over quantity-based supply mechanisms.

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1 Introduction

Emissions trading is among the commonest of policies to price carbon and curb greenhouse gas emissions. In its most basic form, emissions trading – or cap and trade – fixes

*Department of Economics, Swedish University of Agricultural Sciences, Box 7013, 750 07 Uppsala, Sweden. Email: roweno.heijmans@slu.se. I thank Torben Mideska for very useful comments and great patience.

the total amount of emissions but allows covered firms to decide on the allocation of emissions under this cap, creating a market for greenhouse gases.

In contrast to textbook models, emissions trading schemes (ETSs) typically do not impose a fixed limit on emissions but instead make the cap on emissions endogenous to conditions prevailing in the market (ICAP, 2021). An endogenous emissions cap is motivated by the idea that it makes a policy more resilient to economic fluctuations and uncertainties which would otherwise render the system unstable or even ineffective (Fell, 2016; Lintunen and Kuusela, 2018; Pizer and Prest, 2020). In all existing cap and trade schemes, stabilization mechanisms are based either on the allowance price (price rules) or on the number of allowances used (quantity rules). Examples of the price rules include price floors and ceilings, used for example in California's ETS (Schmalensee and Stavins, 2017; Gerlagh et al., 2021). An example of a quantity rule is a quantity collar, used in the EU ETS (Fell, 2016; Holt and Shobe, 2016).

The first contribution of this paper is to prove a paradoxical result on the interaction between quantity rules and the time horizon of emissions trading. Suppose the policymaker fixes a point in time beyond which emissions are not allowed, even if firms have unused allowances left. I show that total emissions are higher when this final period on emissions occurs earlier rather than later if the cap and trade scheme operates a quantity rule. My second contribution is to show that the same paradox does not arise when instead the cap and trade scheme operates a price rule, in which case a shorter time horizon of emissions unambiguously reduces pollution.

I derive my results in a generic model of dynamic emissions trading. Firms produce goods and in the process pollute the environment. In each period, the policymaker dictates abatement obligations through issuing allowances which firms must surrender to cover their emissions. Temporal violations of the periodic cap are accommodated through a banking provision that facilitates the use of allowances issued in one period to meet compliance obligations pursuant to another (Kling and Rubin, 1997). Subject to this cap, firms allocate emissions allowances over time to minimize abatement costs.

The above describes a standard treatment of emissions trading. My model two important additional element meant to reflect recent real world developments. First, the supply of allowances responds to observable market outcomes through a stabilization mechanism. I limit attention to price and quantity rules because empirically these are the most relevant (ICAP, 2021). Under a price rule, the supply of allowances increases when the allowance price goes up. A quantity rule instead translates an expanding bank of unused allowances into a lower supply.

Second, the policymaker dictates a binding final period on emissions beyond which firms are not allowed to emit even if they have unused allowances left. A final period on emissions captures the idea that policymakers may try to speed up the de-carbonization of their economies. It reflects the wave of zero emission targets that governments have recently pledged to and are considered crucial in meeting the Paris Agreement temperature goals (Höhne et al., 2021).

The driving force behind my results is firms' incentive not to bank allowances beyond the final period. An allowance has value only if it can be used to cover emissions. Instead of leaving allowances unused until emissions are no longer allowed, firms had better use them prior to the final period to minimize abatement costs when they still can. An earlier final period on emissions therefore stimulates firms to use more allowances in early periods, implying less banking overall. The response of a quantity rule is to increase the supply of emissions allowances and this explains the paradox. To support the increased demand for allowances, the allowance price must go down; under a price rule, this implies fewer emissions overall and explains my second result.

A proper understanding of supply policies in cap and trade schemes is important because they form an integral part of global climate policy. According to the International Carbon Action Partnership, to date more than 30 supranational, national and local jurisdictions representing 54% of global GDP operate a cap and trade scheme for greenhouse gases while more are under way. Of these, a large majority has some kind of endogenous cap.[1] Moreover, supply policies have a major impact on emissions trading. As an example, Borenstein et al. (2019) show that the allowance price in California's cap and trade scheme is determined by the administrative price collar with 98.9 percent probability. It is hence important to study stabilization mechanisms for the design of environmental policy.

The key policy takeaway is simple. Zero emissions pledges, aimed at accelerating the de-carbonization of the economy, may backfire when combined with quantity-based cap and trade policies. On the other hand, climate neutrality goals are supported by price-based emissions trading schemes. Careful policymaking is called for when imposing additional abatement policies on cap and trade schemes governed by a quantity role.

[1]While not extensive, a list of cap and trade schemes that operate price or quantity rules of the kind studied in this paper includes California's cap and trade scheme, China's National ETS, the EU ETS, Germany's National ETS, Korea's ETS, New Zealand's ETS, the Massachusetts Limits on Emissions from Electricity Generator, the Regional Greenhouse Gas Initiative (RGGI), and Switzerland's ETS.

The paper proceeds as follows. Section 2 sets out the model and defines price- and quantity rules formally. Section 3 then studies the equilibrium of cap and trade schemes under price and quantity rules for different final periods on emissions. Emission levels are compared across scenarios, which yields my main results. Section 4 discusses the results and concludes.

2 Model

I first set up a generic model of emissions trading and derive firms' emission decisions from a dynamic abatement cost minimization problem. I then formally define and discuss the two supply policies studied. Equilibrium analyses are relegated to Section 3.

2.1 Firms' problem

Consider the dynamic abatement cost minimization problem faced by firms. In each period $t \geq 0$, abatement for firm i, with $i = 1, 2, ..., N$, is represented as $a_{it} = \bar{q}_{it} - q_{it}$ where $\bar{q}_{it}$ denotes expected business-as-usual emissions (i.e. the level emissions in the absence of any policy whatsoever) and $q_{it} \geq 0$ is the actual level of emissions in period t. The cost of abatement is determined by the abatement cost function $C_{it}(a_{it})$ which satisfies $C_{it}(0) = 0$, $C'_{it} := \frac{\partial C_{it}}{\partial a_{it}} > 0$ and $\frac{\partial^2 C_{it}}{\partial a_{it}^2} \geq 0$.

Emissions are regulated through a cap and trade scheme. Let s_{it} denote the number of allowances supplied to firm i in period t (I will elaborate on the supply path in the next section). Allowances, once supplied, may be traded on a secondary market where a firm can sell or acquire them at a price p_t which it takes as given. Hence, if a firm chooses an amount q_{it} of emissions and sells or buys a total of l_{it} allowances on the secondary market, abatement costs are $C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it}$.

It is assumed that temporal violations of the periodic cap s_{it} are facilitated through a bank and borrow provision (Kling and Rubin, 1997).[2] I define banking by firm i in period t to be $b_{it} := s_{it} + l_{it} - q_{it}$. The bank of allowances held by firm i at the start of period t is then $B_{it} := \sum_{s=0}^{t-1} b_{is} = B_{it-1} + b_{it-1} = B_{it-1} + s_{it-1} + l_{it-1} - q_{it-1}$, and the total bank of allowances at the start of period t is $B_t := \sum_i B_{it}$. The *dynamic* constraint on emissions by firm i is therefore $\sum q_{it} \leq \sum s_{it} + l_{it}$. This is the first constraint on the firm's constrained cost minimization problem.

[2]Some cap and trade schemes impose additional constraints on banking and borrowing (Shobe et al., 2014; Fell, 2016). I abstain from doing so for simplicity of the exposition.

The second constraint is a final period $T > 1$ starting from which emissions are no longer allowed even if firms have unused allowances left; that is, $q_{it} = 0$ for all i and all $t \geq T$. A situation in which allowances have an infinite lifetime can be analyzed in this model by setting $T \to \infty$. It is assumed that all firms anticipate the final period T starting from period 0.[3] The final period T on emissions reflects the future zero-emissions pledges made by governments around the globe (Höhne et al., 2021; Nature Editorial, 2021). I colloquially refer to T as the *time horizon of emissions*.

The above elements make for a straightforward constrained optimization problem:

$$\min_{q_{it}, l_{it}} \quad \sum_t \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it}\right] \tag{1}$$

$$\text{subject to} \quad \sum_t q_{it} \leq \sum_t \left[s_{it} + l_{it}\right], \tag{2}$$

$$\sum_i l_{it} = 0, \tag{3}$$

$$q_{it} = 0, \quad \text{for all} \quad t \geq T, \tag{4}$$

$$B_{it+1} = B_{it} + s_{it} + l_{it} - q_{it}, \tag{5}$$

for each i and where $\beta \in (0, 1)$ is the discount factor. Constraint (2) says that any individual firm's emissions may not exceed the total number of allowances it owns. Constraint (3) says that, on the secondary market, every allowance bought by one firm must be sold by another. Constraint (4) says that emissions are not allowed starting from period T onward. Finally, (5) is the equation of motion for the bank of allowances.

In the solution to this problem (see Appendix), marginal abatement costs grow at the discount rate, so long as firms can choose emissions:

$$C'_{it}(\bar{q}_{it} - q_{it}) = \beta C'_{it+1}(\bar{q}_{it+1} - q_{it+1}), \tag{6}$$

for all $t < T$. Moreover, each firm will emit, or abate, until marginal abatement costs roughly equal the allowance price,

$$p_t = C'_{it}(\bar{q}_{it} - q_{it}) - \mu_t, \tag{7}$$

for all $t < T$. I say that prices should roughly equal the allowances price because when

[3] This is not really an assumption as I may simply define $t = 0$ to be the first period in which T is common knowledge.

$\mu_t \neq 0$, the secondary market constraint is binding and not every firm can buy or sell the number of allowances it wants, possibly driving a wedge between the allowance price and marginal abatement costs.[4] Combining (6) and (7), prices should satisfy:

$$\frac{\partial p_{t+1}}{\partial p_t} = \frac{1}{\beta}, \tag{8}$$

which is a differential version of Hotelling's Rule. For $\mu_t = 0$, cost-minimizing prices follow Hotelling's Rule as $p_{t+1} = p_t/\beta$, implying (8). When Hotelling's Rule in its strictest interpretation is violated, it remains cost-minimizing for allowance prices to co-move.[5]

For each i and t, let $q_{it}(p_t, T)$ denote the level of emissions q_{it} that solves (7). From $C'_{it} > 0$ follows

$$\frac{\partial q_{it}(p_t, T)}{\partial p_t} < 0, \tag{9}$$

for all $t < T$. That is, the abatement cost minimizing level of emissions chosen by firm i in period t is decreasing in the prevailing allowance price in that period. For given p_t and T, define $Q_t(p_t, T) = \sum_i q_{it}(p_t, T)$ to be total emissions in period t. By (9),

$$\frac{\partial Q_t(p_t, T)}{\partial p_t} < 0. \tag{10}$$

For given price path $p = (p_1, p_2, ...)$ and final period T, cost-minimization dictates:

$$B_{iT}(p) = 0, \tag{11}$$

for all i.[6] That is, firms make sure they have no unused allowances left when the final period on emissions arrives. To see this, suppose instead that $B_{iT} > 0$. Then firm i's emissions q_{it} could be increased in some period $t < T$ without affecting the firm's abatement costs in any other period but lowering abatement costs in period t itself,

[4] When there is a large number of small firms (the price-taker assumption), the secondary market constraint will typically be fairly marginal from the individual firm's perspective and so $\mu_t \approx 0$.

[5] It is well known that factors including asymmetric information (Martimort et al., 2018), technological progress (Livernois, 2009), and arbitrage opportunities (Anderson et al., 2018) can cause violations of the rule in its canonical formulation. Nevertheless, the literature broadly supports the co-movement of prices over time (Livernois, 2009; Anderson et al., 2018; Martimort et al., 2018).

[6] An overwhelming oversupply of allowances, where the number of permits issued exceeds business as usual emissions, might lead to violations of (11). This appears to be a practically less relevant case as a large empirical literature supports the hypothesis that cap and trade schemes bring down emissions (Schmalensee et al., 1998; Fowlie, 2010; Fowlie et al., 2012; Deschenes et al., 2017; Calel, 2020; Bayer and Aklin, 2020). I hence assume that (11) is a necessary condition for cost minimization.

contradicting the hypothesis that $B_{iT} > 0$ is consistent with dynamic cost-minimization. A more formal argument is provided in the Appendix.

I conclude the discussion of firms' abatement cost minimization problem with two important notes. First, $l_{it} = 0$ for all i when $t \geq T$.[7] That is, firms neither buy nor sell allowances on the secondary market from the final period on emissions onward. Starting from period T, emissions are not allowed and so allowances are useless; hence, firms would be wasting money buying any. Second, the allowance price p_t need not satisfy (7) and/or (8) when $t \geq T$. The allowance price comes about through supply and demand on the secondary market. As there is no demand for allowances when $t \geq T$, the allowance price is not properly defined for those periods.

Firms minimize abatement costs *given* the number of emissions allowances supplied to them. The next section introduces the types of supply policies studied in this paper.

2.2 Supply policies

Let $s_t = \sum_i s_{it}$ denote the total supply of allowances in period t. I will come to the precise determination of the supply path (s_t) shortly; in any case it is assumed that $s_t < \bar{q}_t$ at all t, where $\bar{q}_t = \sum_i \bar{q}_{it}$. That is, the supply of allowances does not exceed business-as-usual emissions (Schmalensee et al., 1998; Fowlie, 2010; Fowlie et al., 2012; Deschenes et al., 2017; Calel, 2020; Bayer and Aklin, 2020).[8]

Textbook models typically treat the supply of allowances as an exogenous given quantity; in the real world, however, it often depends on conditions prevailing in the market. To stay close to reality, I will limit attention to two classes of endogenous allowance supply schemes: price and quantity rules.

Definition 1 (Price rule)**.** *A cap and trade scheme operates a* price rule *if the supply of allowances in any period t is weakly increasing in the prevailing allowance price p_t. Formally, for any period t and any two price levels p_t and p'_t it holds that $s_t(p_t) \geq s_t(p'_t)$ if and only if $p_t > p'_t$.*

Examples of price rules include price floors and ceilings (Fell et al., 2012). It is assumed that $s_t(0) \leq q_t(0, T)$ and $s_t(\infty) \geq q_t(\infty, T)$ for all t, with a stict inequality for

[7] Cost minimization forces each firm i to choose $l_{it} \leq 0$ for all $t \geq T$; all want to sell allowances if they have some. Combined with the secondary market constraint that $\sum_i l_{it} = 0$, this gives $l_{it} = 0$.

[8] Theoretically at least the precise way in which allowances are supplied does not matter. For simplicity, I will therefore be silent on the exact mechanism through which allowances reach firms. It should be mentioned, through, that studies have found market outcomes to depend on details of the supply policy (Porter et al., 2009; Goeree et al., 2010).

at least one t. While not strictly necessary for my main results, I assume that $s_t(p_t)$ is differentiable in p_t.

The other type of supply policy considered is a quantity rule.

Definition 2 (Quantity rule)**.** *A cap and trade scheme operates a* quantity rule *if the supply of allowances in period t is weakly increasing in the aggregate excess supply at the start of period t. That is, for any period t and any two B_t and B_t', it holds that $s_t(B_t) \geq s_t(B_t')$ if and only if $B_t' > B_t$.*

I assume that $s_t(B_t(p)) \leq q_t(0, T)$ and $s_t(B_t(p)) \geq q_t(\infty, T)$ for all p, with a strict inequality for at least one t. While not strictly necessary for my main results, I assume that $s_t(B_t)$ is differentiable in B_t. I also assume that $-1 < \partial s_t / \partial B_t$ for all t to avoid the counter-intuitive scenario in which polluters have an incentive to bank *less* today in order to have *more* allowances in the future – banking should happen only in the face of relative future scarcity.

It is assumed that firms take the supply of allowances as given. Under a price rule, this follows naturally from the price taking assumption. Under a quantity rule, the assumptions amounts to saying that each firm is small compared to the size of the industry regulated. In reality it is possible that a large firm finds itself in a position in which it can exercise market power (Liski and Montero, 2011); it is unclear to what extent my analysis generalizes to such environments.

3 Analysis

3.1 Dynamic price effects

Consider a price rule. Since $s_t(p_t)$ is increasing in p_t by construction while $q_t(p_t)$ is decreasing by (9), banking in period $b_t(p_t)$ is increasing in the allowance price p_t. Recall from (8) that prices co-move across periods. By implication, one has $\frac{\partial p_s}{\partial p_\tau} > 0$ for all $s, \tau \geq 0$ and therefore,

$$\begin{aligned} \frac{\partial B_t}{\partial p_\tau} &= \frac{\partial}{\partial p_\tau} \left[\sum_s^{t-1} s_s(p_s) - \sum_s^{t-1} q_s(p_s, T) \right] \\ &= \sum_s^{t-1} \frac{\partial s_s(p_s)}{\partial p_s} \frac{\partial p_s}{\partial p_\tau} - \sum_s^{t-1} \frac{\partial q_s(p_s, T)}{\partial p_s} \frac{\partial p_s}{\partial p_\tau} > 0. \end{aligned} \tag{12}$$

This establishes that the aggregate bank of allowances B_t is increasing in the allowance price p_τ for all t and τ such that $0 \leq t, \tau < T$.

Lemma 1 (Dynamic price effects under a price rule)**.** *Fix a final period on emissions T. For any two periods $\tau, t < T$, the bank of allowances $B_t(p)$ is strictly increasing in the allowance price p_τ under a price rule.*

Next, consider a quantity rule. The ffect of an increase in the allowance price on first-period banking is straightforward:

$$\frac{\partial B_1(p)}{\partial p_\tau} = \frac{\partial b_0(p_0)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = \frac{\partial [s_0 - q_0(p_0)]}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = -\frac{dq_0(p_0)}{dp_0}\frac{dp_0}{dp_\tau} \geq 0, \tag{13}$$

where the inequality is strict for all p_0 such that $q_0(p_0) > 0$ and all $\tau \geq 0$. A little more work is required to determine the sign of $\partial B_t/\partial p$ for $t > 1$. Recall that the bank of allowances evolves according to $B_t(p) = B_{t-1}(p) + s_{t-1}(B_{t-1}(p)) - q_{t-1}(p_{t-1})$. It therefore follows that

$$\begin{aligned} \frac{\partial B_t(p)}{\partial p_\tau} &= \frac{\partial B_{t-1}(p)}{\partial p_\tau} + \frac{\partial s_{t-1}(B_{t-1}(p))}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1})}{\partial p_\tau} \qquad (14)\\ &= \left(1 + \frac{\partial s_{t-1}(B_{t-1}(p))}{\partial B_{t-1}(p)}\right)\frac{\partial B_{t-1}(p)}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1})}{\partial p_{t-1}}\frac{\partial p_{t-1}}{\partial p_\tau}. \qquad (15) \end{aligned}$$

Recall now that $-1 < \partial s_t/\partial B_t$ for all t, so $1 + \partial s_t/\partial B_t > 0$. Moreover, one knows that $\frac{\partial q_{t-1}(p_{t-1})}{\partial p_{t-1}}\frac{\partial p_{t-1}}{\partial p_\tau} < 0$. Plugging these signs back into (15) and using (13), induction on t establishes that

$$\frac{\partial B_t(p)}{\partial p_\tau} \geq 0, \tag{16}$$

for all t and τ such that $0 \leq t, \tau < T$. The inequality is strict for all $p = (p_1, p_2, ...)$ that satisfy $q_t(p_t) > 0$ for at least one t.

Lemma 2 (Dynamic price effects under a quantity rule)**.** *Fix a final period on emissions T. For any two periods $\tau, t < T$, the bank of allowances $B_t(p)$ is strictly increasing in the allowance price p_τ under a quantity rule.*

3.2 Equilibrium

The market is in equilibrium when the supply of emissions allowances is equal to demand. It is assumed that the price of allowances brings about equilibrium.

Consider two periods T and $\bar{T}$ and, without loss of generality, let $\bar{T} < T$. In what follows, I will compare equilibrium emissions when the final period on emissions is T versus when final period is $\bar{T}$. The question becomes, Does a shorter time horizon of emissions help the climate? It turns out the answer to that depends on the supply policy in place. Under a price rule, a shorter time horizon of emissions reduces emissions; under a quantity rule, it increases emissions.

It is perhaps instructive to take $T \to \infty$ meaning that, at baseline, allowances have an infinite lifetime. This would correspond to important actual cap and trade schemes such at the EU ETS, California's ETS, and RGGI where allowances, once issued, do not expire.

3.2.1 Price rules

Consider first the *baseline* final period on emissions T. The market for allowances is in equilibrium if

$$\sum_{t=0}^{T} q_t(p_t^*, T) = \sum_{t=0}^{\infty} s_t(p_t^*), \tag{17}$$

where p_t^* is the *baseline* equilibrium allowance price when the scheme operates a price rule and the final period on emissions is T. Let $p^* = (p_t^*)$ denote the associated equilibrium price path.

Under a price rule, the supply of allowances depends on the prevailing allowance price. Let T^* denote the period in which the supply of emissions drops permanently to zero in the baseline equilibrium under a price rule policy, i.e. T^* is the period for which it holds that $s_t(p^*) = 0$ if and only if $t \geq T^*$. I refer to $(s_t(p^*))$ as the baseline equilibrium allowance supply path under a price rule when the time horizon of emissions is T.

Next consider the alternative, earlier final period on emissions $\bar{T}$, where $\bar{T} < T$. Hence, when the final period is $\bar{T}$ firms solve the constrained cost minimization give by (1)–(5) with constraint (4) reading $q_{it} = 0$ for all $t \geq \bar{T}$. I assume that $T^* \leq \bar{T}$, meaning that the supply of allowances in the baseline equilibrium reaches zero before the demand for emissions runs out. I also assume that the final period $\bar{T}$ is binding in the sense that the baseline equilibrium dictates strictly positive emissions in period $\bar{T}$, $q_{\bar{T}}(p_{\bar{T}}^*) > 0$.

When the time horizon of emissions is restricted to $\bar{T}$, equilibrium in the market for allowances is reached when:

$$\sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^*, \bar{T}) = \sum_{t=0}^{\infty} s_t(\bar{p}_t^*), \tag{18}$$

where $\bar{p}_t^*$ is the *restricted* equilibrium allowance price under a price rule; $\bar{p}^* = (\bar{p}_t^*)$ denotes the associated price path.

The restricted equilibrium market condition (18) implies

$$B_{\bar{T}}(\bar{p}^*) = 0. \tag{19}$$

Now recall that $T^* < \bar{T}$ and that $q_{\bar{T}}(p_{\bar{T}}^*) > 0$, which combined give

$$B_{\bar{T}}(p^*) > 0. \tag{20}$$

That is, baseline equilibrium emissions in period $\bar{T}$ are positive while the supply of allowances has already dried up, implying a positive bank of allowances at the start of period $\bar{T}$. From Lemma 1 and (8),

$$\bar{p}_t^* < p_t^*, \tag{21}$$

for all $t < \bar{T}$. By the definition of a price rule, (21) implies:

$$s_t(\bar{p}_t^*) < s_t(p_t^*), \tag{22}$$

for all $t \leq T^*$. This gives the first main result.

Proposition 1. *A shorter time horizon for emissions unambiguously* decreases *emissions in cap and trade schemes complemented with a price rule.*

Proposition 1 gives the intuitive result that a shorter time horizon for emissions unambiguously reduces aggregate emissions in cap and trade schemes complemented with a price rule. Roughly speaking, the price of an allowance is dictated by the opportunity cost of using it now rather that later. In excluding the use of allowances for a wider range of future periods, the policymaker effectively reduces the opportunity cost of using an allowance today. This decreased opportunity cost translates into a lower allowance price, which, by virtue of the price rule, reduces the aggregate supply of allowances and thus emissions.

3.2.2 Quantity rules

Consider again the baseline final period on emissions T. Equilibrium under a quantity rule is reached when

$$\sum_{t=0}^{T} q_t(p_t^{**}, T) = \sum_{t}^{\infty} s_t(B_t(p^{**})), \tag{23}$$

where p_t^{**} is the baseline equilibrium allowance price in period t under a quantity rule and $p^{**} = (p_t^{**})$ is the associated price path.

Let T^{**} be the (endogenous) period in which the supply of allowances dries up permanently under a price rule in the baseline equilibrium, i.e. T^{**} is the lowest integer such that $s_t(B_t(p^{**})) = 0$ for all $t \geq T^{**}$.

Next consider the alternative, earlier final period on emissions $\bar{T}$, where $\bar{T} < T$. Hence, when the final period is $\bar{T}$ firms solve the constrained cost minimization give by (1)–(5) with constraint (4) now reading that $q_{it} = 0$ for all $t \geq \bar{T}$. To simplify the analysis, I assume that $T^{**} \leq \bar{T}$, meaning that the supply of allowances in the baseline equilibrium reaches zero before the demand for emissions runs out. I also assume that the final period $\bar{T}$ is binding in the sense that the baseline equilibrium dictates strictly positive emissions in period $\bar{T}$, $q_{\bar{T}}(p_{\bar{T}}^{**}) > 0$.

Equilibrium under the restricted final period $\bar{T}$ requires:

$$\sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^{**}, \bar{T}) = \sum_{t}^{\infty} s_t(B_t(\bar{p}^{**})), \tag{24}$$

where $\bar{p}^{**} = (\bar{p}_t^{**})$ is the restricted equilibrium allowance price vector. Observe that (24) can be rewritten as:

$$B_{\bar{T}}(\bar{p}^{**}) = 0. \tag{25}$$

At the same time, since $T^{**} < \bar{T}$ and $q_{\bar{T}}(p_{\bar{T}}^{**}) > 0$ it holds that:

$$B_{\bar{T}}(p^{**}) > 0. \tag{26}$$

From Lemma 1, using (8),

$$\bar{p}_t^{**} < p_t^{**}, \tag{27}$$

for all $t < \bar{T}$. Using (27),

$$B_t(\bar{p}^{**}) < B_t(p^{**}), \tag{28}$$

for all $t \leq \bar{T}$. By the mechanics of a quantity rule, (28) implies that:

$$s_t(B_t(\bar{p}^{**})) > s_t(B_t(p^{**})), \tag{29}$$

for all $t < \bar{T}$. This gives the second main result.

Proposition 2. *A shorter time horizon for emissions unambiguously* increases *emissions in cap and trade schemes complemented with a quantity rule.*

Proposition 2 gives the paradoxical result that, compared to the situation in which allowances may be surrendered at any point in time, a binding final period on emissions unambiguously *increases* emissions in cap and trade schemes complemented with a quantity rule. A quantity rule is hard to combine with a pledge toward climate neutrality by, say, 2050.

At its core, Proposition 2 is the product of two effects. The first is leakage and says that any allowances firms cannot used in the future will be used today. Firms bank allowances to use them in the future. When the number of future periods in which an allowance can be used is reduced, firms will typically bank less.

With a fixed cap, leakage implies a reshuffling of emissions over time but no change in emissions overall; this is called a green paradox (Van der Ploeg and Withagen, 2012). The second effect, however, says that the reduction in banking implied by the leakage effect causes the supply of allowances to go up. This effect is immediate from the construction of the quantity rule. Since every allowance supplied will, in equilibrium, be used, aggregate emissions are unambiguously higher once the time horizon of emissions gets restricted.

4 Discussion and Conclusions

I study the effect of the time horizons of emissions trading on pollution. When the supply of emissions in a cap and trade scheme is governed by a quantity rule, a binding final period beyond which emissions are not allowed unambiguously raises aggregate emissions compared to the case in which allowances have an infinite lifetime. This paradox does not arise if instead the schemes relies on a price rule, in which case a binding final period on emissions unambiguously curbs emissions.

My result on quantity rules stabilization is related to the green paradox (Van der Ploeg and Withagen, 2012). Proposition 2 is stronger, however, as aggregate emissions *increase* when the time horizon of emissions trading is restricted. The possibility of increased emissions in response to overlapping climate policies in a two-period model was also observed by Gerlagh et al. (2021). This paper extends their work by looking at a stronger kind of environmental policy (a complete ban on emissions) for a general number periods and generic supply policies.

Considering the recent wave of zero emission pledges by governments across the global (Höhne et al., 2021), my results warrants a careful re-evaluation of quantity-based supply policies in cap and trade schemes. Superficially, the paradox could be preempted simply by not restricting the time horizon of emissions. This is an imperfect solution at best; besides the mere sum total of emissions, the natural sciences agree that the *timing* of emissions is also important for global warming (Gasser et al., 2018). A change to price-based allowance supply policies thus seems a far superior way forward.

A number of real-world cap and trade schemes operate a quantity rule or consider introducing one. The most well-known example is probably the EU ETS, the world's largest market for carbon. In addition, South Korea's ETS has a "market liquidity" provision which, in practice, functions like a quantity rule. According to the Asian Development Bank (2018) the South Korean "government applied these provisions at the end of the first year when it turned out that the overall emissions of the participants were 0.82% higher than the cap. To address this issue, the government supplied additional allowances to the market." Moreover, the UK ETS is currently designing its *Supply Adjustment Mechanism*, which may be modeled after the EU ETS; Switzerland's ETS similarly intends to adopt a quantity rule.[9] My result illustrates that policymakers of these schemes should be careful as policies to speed up the de-carbonization of the economy may backfire.

While I study stabilization mechanisms within a cap and trade scheme generally, many other types of market-based environmental policies exist, see for example Böhringer and Lange (2005), Böhringer et al. (2017), and Fowlie and Muller (2019). The critical message regarding quantity-based stabilization does not necessarily extend to other kinds of endogenous policies.

This paper makes several restrictive assumptions. First, I assume that adjustments to the final period on emissions are not accompanied by discrete supply-adjustments; changes in the supply of allowances come about entirely through the stabilization mechanism. In reality, the introduction of a final period on emission would counstitute

[9]See the *Teilrevision der Verordnung über die Reduktion der CO2-Emissionen* (in German) for details of the Swiss ETS.

a major reform which the policymaker might consider only within the context of a broader set of changes, including perhaps exogenous supply adjustments. Second, I consider a particular kind of finite time horizon in which allowances can be used at any time prior to the final period independent of when they were issued. Alternatively, policymakers could write off unused allowances depending on when they were supplied, e.g. allowances can be kept for five years at most. Third, I assume that the biding period is set after the baseline supply of allowances dries up. It could be argued, however, that truly ambitious climate policy requires emissions to end earlier.

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A Appendix

A.1 Additional details on firms' dynamic cost-minimization problem

Turning the constrained problem in (1)–(5) into an unconstrained cost minimization problem, each firm i chooses q_{it} and l_{it} to solve:[10]

$$\min_{q_{it},l_{it}} \sum_{t=0}^{T} \beta^t C_{it}(\bar{q}_{it} - q_{it}) + \sum_t \beta^t p_t l_{it} + \lambda_i \left[\sum_t q_{it} - s_{it} - l_{it} \right] + \sum_t \beta^t \mu_t \left[\sum_i l_{it} \right] \tag{30}$$
$$+ \omega_{it} \left[B_{it} - B_{it-1} - s_{it-1} - l_{it-1} + q_{it-1} \right].$$

The first-order conditions associated with the cost-minimization problem given by (30) are:

$$-\beta^t C'_{it}(\bar{q}_{it} - q_{it}) + \lambda_i + \omega_{it+1} = 0, \tag{31}$$

$$\beta^t p_t - \lambda_i + \beta^t \mu_t - \omega_{it+1} = 0, \tag{32}$$

$$\omega_{it} - \omega_{it+1} = 0. \tag{33}$$

Using (31) and (32) gives:

$$p_t + \mu_t = C'_{it}(\bar{q}_{it} - q_{it}), \tag{34}$$

implying (7). Moreover, combining (33) and (32) yields:

$$p_t + \mu_t = \beta p_{t+1} + \beta \mu_{t+1}, \tag{35}$$

so $p_{t+1} = p_t/\beta + \mu_t/\beta - \mu_{t+1}$ and this implies (8).

[10] Without loss of generality, I multiply the shadow value μ_t for the secondary market constraint (3) by β^t.