Global Policy Design

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Introduction

Policy in Coordination Games

How to set rewards that incentivize work in teams?

→ Winter (2004), Fischer & Huddart (2008), Halac et al. (2021, 2023)

How to raise capital from multiple investors?

→ Sákovics & Steiner (2012), Halac et al. (2020)

How to foster economic development through technology adoption?

→ Bandiera & Rasul (2006), Cai et al. (2015), Beaman et al. (2021)

How to stimulate the adoption of a beneficial social norm?

→ Ferraro et al. (2011), Lane et al. (2023)

How to induce citizens to participate in a revolution?

→ Edmond (2013), Morris & Shadmehr (2023)

Policy in Coordination Games

How to design policy in coordination games?

Ann and Bob

Ann and Bob

Ann and Bob can invest in a project

The cost of investment is c

If the project succeeds, investment yields a return $x + b$ $(b > c)$

The project succeeds only if Ann and Bob both invest

Not investing, their outside option, pays 0

Coordination problem: Ann and Bob want to invest iff the other invests

$\rightarrow$ Multiple Nash equilibria

Strategic Beliefs

Suppose it is common knowledge that $x = 0$: investment is efficient

A planner offers subsidies to induce investment

How high should these subsidies be?

If Ann expects that Bob will invest, she needs no subsidy at all

If Ann expects that Bob will *not* invest, she requires a subsidy $\geq c$

Strategic beliefs crucial for policy design

But multiple equilibria $\rightarrow$ strategic beliefs not unique

$\rightarrow$ Policy hard to pin down

A Tangle

We saw that strategic beliefs affect policy

Policy also affects strategic beliefs

If Ann receives a subsidy, investment becomes more attractive to her

This affects Bob's investment incentives...

... which then changes Ann's incentives, and so on

Strategic beliefs are both input and output of policy

A theory of policy design in coordination games should untangle this knot

Brute Force: Dominance and Discrimination

A subsidy equal to c for both Ann and Bob gets them to invest

$\rightarrow$ Makes investment strictly dominant

$\rightarrow$ Means strategic beliefs no longer matter

But... paying both their full investment cost is expensive

Idea: make investment dominant only for Ann

Ann definitely invests, so Bob wants to invest even without subsidy

Discrimination: identical players treated differently

Can go further: tax Bob's investment (extract his full surplus)

Trade-off

Seminal result: the least-cost policy discriminates

$\rightarrow$ First established by Segal (2003) and Winter (2004)

$\rightarrow$ Bernstein & Winter (2012), Eliaz & Spiegler (2015), Halac et al. (2020, 2023)

Cheapest policy subsidizes Ann by c, taxes Bob by $b - c$

$\rightarrow$ Or the other way round: policy not unique

Fundamental trade-off: equity vs. efficiency

$\rightarrow$ Either discriminate, or empty your pockets

This Paper: No Discrimination

Assume payoffs (x) are uncertain, observed with noise

→ Seems natural in a number of contexts

→ Not demanding: uncertainty can be negligibly small

Under uncertainty, trade-off between equity and efficiency **does not exist**

→ Artifact of fairly harsh assumptions about players' knowledge

Effective investment subsidies treat Ann and Bob equally...

... and cost the same as the least-cost discriminatory policy

In Ann and Bob's case, each gets a subsidy of $c - b/2$

This Paper: Different Perspective

Uncertainty about payoffs eliminates the equity-efficiency trade-off

It also fundamentally changes the relevant policy problem

The planner wants Ann and Bob to invest... if benefits are sufficiently high
$\rightarrow$ No point in stimulating investments that leave them worse off

Studying this problem ultimately permits my "no-discrimination" result...

... but also yields some new insights of independent interest

Conceptual Contribution

Core complication in coordination games is equilibrium multiplicity

$\rightarrow$ Effect of policy hard to predict, motivates extreme intervention

I deal with this complication explicitly

$\rightarrow$ Connect policy design to equilibrium selection

$\rightarrow$ Use a global games approach (Carlsson & Van Damme, 1993)

For given subsidies, the game with uncertainty has a unique equilibrium

The relationship between subsidies and equilibrium strategies is one-to-one

$\rightarrow$ Characterize (unique) policy that yields the desired equilibrium

$\rightarrow$ Cf. Sákovics & Steiner (2012)

Literature

Policy in coordination games

Segal (1999, QJE; 2003, JET), Ferraro et al. (2011, AER), Bernstein & Winter (2012, AEJ: Micro), Galeotti et al. (2020, ECTRA), Kets & Sandroni (2021, RES), Lane et al. (2023, AER)

Incentives in teams

Winter (2004, AER), Fischer & Huddart (2008, AER), Halac et al. (2020, AER; 2021, AER; 2022, AEA P&P; 2023, AEJ: Micro), Dai & Toikka (2022, ECTRA)

(Policy in) global games

Carlsson & Van Damme (1993, ECTRA), Morris & Shin (1998, AER), Frankel et al. (2003, JET), Angeletos et al. (2006, JPE), Sákovics & Steiner (2012, AER), Edmond (2013, RES), Leister et al. (2022, RES)

Coordination problems in practice

Cowan (1991, EJ), Cowan & Gunby (1996, EJ), Bandiera & Rasul (2006, EJ), Cai et al. (2015, AER), Beaman et al. (2021, AER)

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Model

Building Blocks

A game of complete information $\Gamma(x, s)$ is given by:

→ Player set $\mathscr{N} = \{1, 2, ..., N\}$

→ Actions $a_i \in \{0, 1\}$, action vectors $a = (a_i)$ and $a_{-i} = (a_j)_{j \neq i}$

→ Subsidies s_i, scheme $s = (s_i)$

→ Payoff functions (u_i)

Payoffs

Given (a, x, s), the payoff to player i is:

$$u_i(a \mid x, s_i) = \left[x + w_i(\sum a_j) + s_i - c_i\right] \cdot a_i, \tag{1}$$

where

x is a fundamental state of nature

c_i is the (opportunity) cost of playing 1

s_i subsidy to player i for playing 1

$\rightarrow$ Equivalent to a (equally sized) tax on playing 0

w_i describes the externalities players impose upon one another

$\rightarrow$ assume $w_i(n)$ is increasing in n (**coordination game**)

In first-best, net of subsidies, player i plays 1 iff $x \geq x_i^*$ (and x_i^* is unique)

Fundamental Uncertainty

I consider a perturbed information environment in which x is hidden

$\rightarrow$ Fundamental uncertainty about state of nature

$\rightarrow$ Payoff functions (u_i) not observed

Each player i receives a private and noisy signal x_i^ε of x:

$$x_i^\varepsilon = x + \varepsilon \cdot \eta_i$$

Common knowledge that $x \sim g$ on $\mathbb{R}$, $\eta_i \sim f$ on $[-1, 1]$, $\varepsilon > 0$

$\rightarrow$ Support of x can be closed interval $\mathcal{X}$ containing dominance regions

Describes a **global game** $\Gamma^\varepsilon(s)$ (Carlsson & Van Damme, 1993)

$\rightarrow$ Solve using iterated elimination if strictly dominated strategies

Timing of $\Gamma^{\varepsilon}(s)$

1. The planner publicly announces the subsidies s
2. Nature draws x
3. Each player i receives his signal x_i^{ε}
4. Players simultaneously choose their actions

Timing of $\Gamma^{\varepsilon}(s)$

N.B. planner commits to s *before* relevant uncertainties are resolved

$\rightarrow$ Sákovics & Steiner (2012), Galeotti et al.(2020), Halac et al. (2020), Leister et al. (2022), Morris & Shadmehr (2023)

$\rightarrow$ s cannot condition on x or players' signals thereof

Policies often target outcomes with uncertain returns

$\rightarrow$ nuclear energy (Cowan, 1990), pest control (Cowan & Gunby, 1996)

Or: reflects players' expertise relative to the planner (Leister et al., 2022)

Unique Implementation

Let $\tilde{x} \in \mathbb{R}$ be a **critical state**

Planner's problem is to make players coordinate on strategy

$$a_i^*(x_i^\varepsilon) = \begin{cases} 1 & \text{if } x_i^\varepsilon > \tilde{x} \\ 0 & \text{if } x_i^\varepsilon < \tilde{x} \end{cases}$$

If coordination on (a_i^*) is the unique BNE of $\Gamma^\varepsilon(\tilde{s})$, $\tilde{s}$ **implements** (a_i^*)

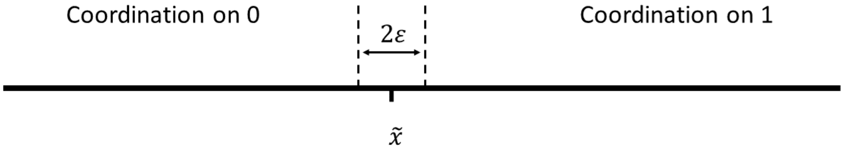
$\rightarrow$ Also results on player-specific critical states (e.g. first-best)

Implementation defined in terms of players' strategies

$\rightarrow$ Strategies map signals to outcomes

$\rightarrow$ Allow players to use their superior (private) information

Unique Implementation



Subsidies and Unique Equilibrium

For ε sufficiently small, $\Gamma^{\varepsilon}(s)$ has a unique Bayesian Nash equilibrium

Given s, there is a unique vector $(x_i(s))$ such that in equilibrium...

... player i plays 1 if and only if $x_i^{\varepsilon} \geq x_i(s)$

$\rightarrow$ Canonical global games selection result (cf. Carlsson & Van Damme, 1993)

Furthermore, the relationship between $(x_i(s))$ and s is one-to-one

$\rightarrow$ Novel extension of the classic global games result

$\rightarrow$ Cf. Sákovics & Steiner (2012), who assume common knowledge about x

Importantly, note that $x_i(s)$ depends upon entire scheme s

$\rightarrow$ Dependence on s_i intuitive (though not obvious)

$\rightarrow$ Effect of s_j indirect, through $x_j(s)$

Global Subsidies

Main Result

Let $\mathcal{B}_r(y)$ be the open ball with radius r centered at y.

Theorem

Let $\tilde{x} \in \mathbb{R}$. The following holds:

(i) *For all ε sufficiently small, there exists a unique subsidy scheme $\tilde{s} = (\tilde{s}_i)$ that implements (a_i^*);*

(ii) *Given g, for all $r > 0$, there exists $\varepsilon(r)$ such that $\tilde{s}$ is contained in $\mathcal{B}_r(s^*(\tilde{x}))$ for all $\varepsilon \leq \varepsilon(r)$, where*

$$s_i^*(\tilde{x}) = c_i - \tilde{x} - \sum_{n=0}^{N-1} \frac{w_i(n)}{N}$$

for all $i \in \mathcal{N}$.

Properties of $\tilde{s}$

Notable properties of $\tilde{s}$:

i) Unique;

ii) Continuous in model parameters;

iii) Does not make targeted strategies strictly dominant;

   $\rightarrow$ Best response to **uniform** strategic beliefs;

iv) Independent, in the limit, of prior g and noise f;

   $\rightarrow$ Planner does not need to know these

v) Symmetric for identical players;

Uniqueness

The global subsidy scheme $\tilde{s}$ is **unique**

Under complete information, optimal subsidies typically not unique

$\rightarrow$ Cf. Segal (2003), Winter (2004), Bernstein & Winter (2012), Halac et al. (2020)

Moreover, uniqueness is "global" in that *only* $\tilde{s}$ implements a^*

Stronger than uniqueness of policy *that satisfies particular properties*...

... such as implementation of targeted equilibrium at minimal cost

$\rightarrow$ Cf. Segal (2003), Winter (2004), Bernstein & Winter (2012), Sákovics & Steiner (2012), Halac et al. (2020, 2021)

Reducing the set of equilibria also reduces the set of equilibrium policies

Dominance

Global subsidies a modest relative to the planner's ambition

The scheme $\tilde{s}$ does not make a^* strictly dominant for any one player

→ Cf. Segal (2003), Winter (2004), Bernstein & Winter (2012), Sákovics & Steiner (2012), Halac et al. (2020)

→ Makes playing 1 best response to **uniform** strategic beliefs

"Too much pessimism" ruled out in $\Gamma^{\varepsilon}(\tilde{s})$...

→ For example, $\Pr_i[a_{-i} = (0, 0, ..., 0) \mid x_i^{\varepsilon} \geq \tilde{x}] \leq 1/N$

... so subsidies that make 1 strictly dominant are unnecessary

Extreme pessimism ruled out *because* subsidies work

→ Complex interplay between policy and beliefs

Dominance

Modest subsidies work thanks to a contagion effect of policy

Consider a subsidy offered to Ann

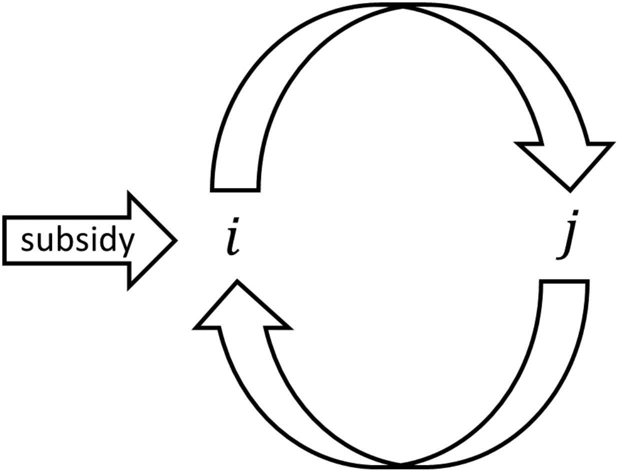
This subsidy raises Ann's incentive to invest

If Ann becomes more likely to invest, Bob's incentive to invest increases...

... increasing Ann's investment incentive yet further

Positive feedback loop/higher-order effects $\Longrightarrow$ small subsidies

Dominance



Prior and Noise Independence

In the limit as the noise in signals vanishes...

... the scheme $\tilde{s}$ is independent of the distributions g and f

Planner need not know these distributions to design her policy

Two part procedure:

1. Establish the result for (improper) uniform g
2. Show that, for small ε, posteriors "as if" g were uniform

Symmetry

Identical players receive identical subsidies

Intuitively, identical players form strategic beliefs symmetrically

→ Implies same response to given subsidy and calls for symmetric policy

→ (Presupposes that strategic beliefs get formed in the first place)

In other words, $\tilde{s}$ does not discriminate

→ Equity: ✓

→ Efficiency: to be continued...

Discrimination

Ranking Policies

Let $\bar{x} \in \mathbb{R}$ be a state

A **ranking policy** is a tuple $\langle \phi, s^R(\phi, \bar{x}) \rangle$

A ranking $\phi(\mathscr{N}) = \{i_1, i_2, ..., i_N\}$ is a permutation of the player set

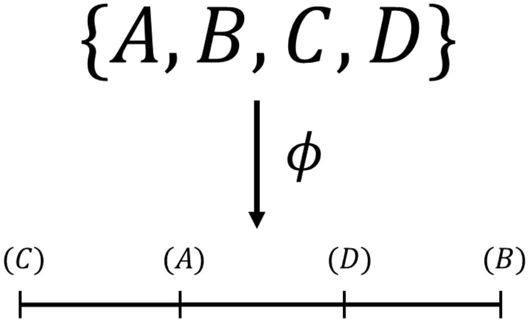
The scheme $s^R(\phi, \bar{x})$ makes $(1, 1, ..., 1)$ the unique Nash equilibrium...

... by going down the ranking in a smart way

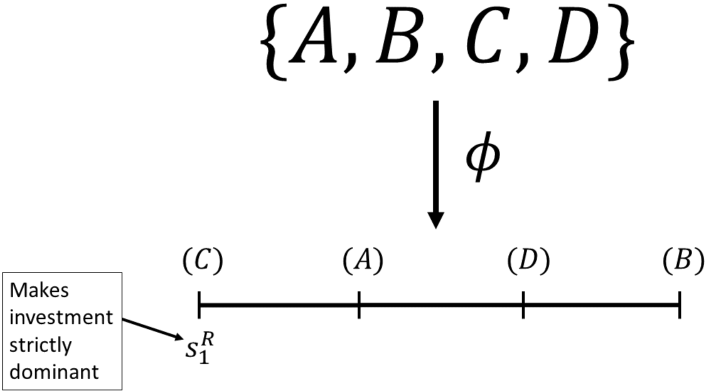
Ranking Policies

$$\{A, B, C, D\}$$

Ranking Policies



Ranking Policies



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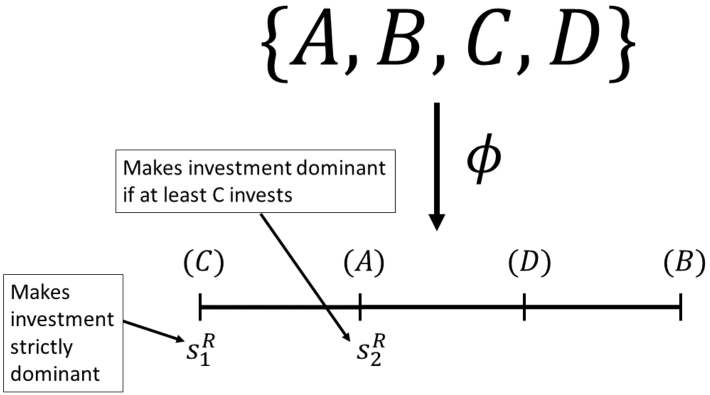

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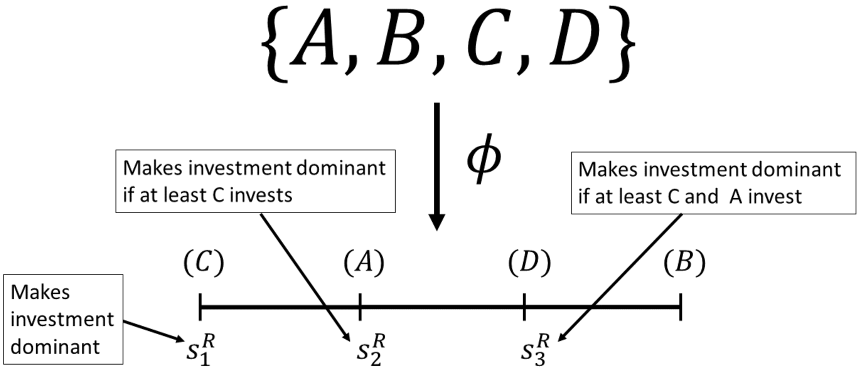
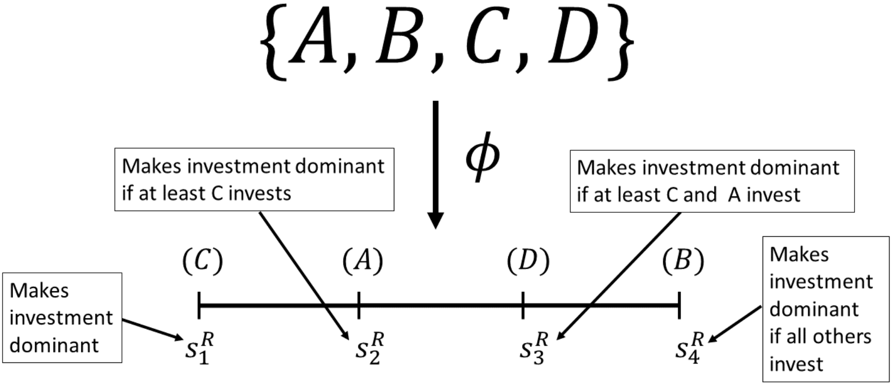


Ranking Policies



Equity vs. Efficiency

(In)equity: a ranking policy $\langle \phi, s^R(\phi, \bar{x}) \rangle$ discriminates

$\rightarrow$ Identical agents receive unequal subsidies

$\rightarrow$ Undesirable (to most people and the law, anyway)

So... why care about ranking policies?

Efficiency: the least-cost subsidy scheme that makes $(1, 1, ..., 1)$...

... the unique Nash equilibrium of $\Gamma(\bar{x}, \cdot)$ is a ranking policy

Celebrated result originally due to Segal (2003) and Winter (2004)

$\rightarrow$ Bernstein & Winter (2012), Eliaz & Spiegler (2015), Halac et al. (2020, 2023)

Cost of Ranking Policies

My goal is to compare costs between ranking policies and global subsidies

I first consider the equilibrium cost of a ranking policy

Let $K(s^R(\phi, \bar{x}) \mid \bar{x})$ be spending on subsidies in $\Gamma(\bar{x}, s^R(\phi, \bar{x}))$

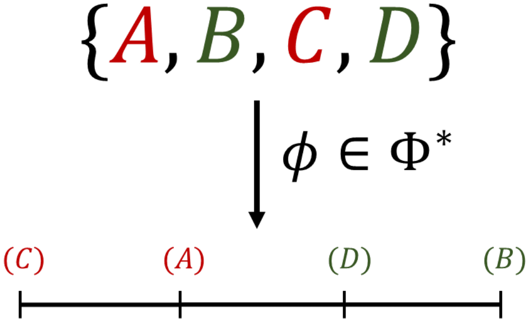
We say that the set $\Phi^*(\bar{x})$ of least-cost ranking policies,

$$\Phi^*(\bar{x}) = \arg\min_{\phi} K(s^R(\phi, \bar{x}) \mid \bar{x}),$$

is **grouping** if it contains ϕ that ranks symmetric players consecutively

(When all players are symmetric, clearly Φ^* is grouping)

Cost of Ranking Policies



Cost of Global Subsidies

Let $K^{\varepsilon}(s \mid x)$ denote (expected) equilibrium spending on subsidies...

... in $\Gamma^{\varepsilon}(s)$ when nature draws state x

One cannot directly compare $K(s^R(\phi, \bar{x}) \mid \bar{x})$ and $K^{\varepsilon}(s \mid x)$

$\rightarrow$ $\Gamma(\bar{x}, s^R)$ assumes that the state is common knowledge

$\rightarrow$ $\Gamma^{\varepsilon}(s)$ assumes that the state x is unobserved

The notion of equilibrium also varies between $\Gamma(\bar{x}, s^R)$ and $\Gamma^{\varepsilon}(s)$

$\rightarrow$ In $\Gamma(\bar{x}, s^R(\phi, \bar{x}))$, equilibrium is a vector of *actions*

$\rightarrow$ In $\Gamma^{\varepsilon}(s)$, equilibrium is a vector of *strategies* (signals $\rightarrow$ actions)

Implementation Under Uncertainty

I focus on policies s with the property that

→ The unique equilibrium strategies in $\Gamma^{\varepsilon}(s)$ render...

→ ... $(1, 1, .., 1)$ the equilibrium outcome of $\Gamma^{\varepsilon}(s)$...

→ ... when nature draws state $\bar{x}$

Let $U(\bar{x})$ denote the set of all such s

There is natural overlap between $\langle \phi, s^R(\phi, \bar{x}) \rangle$ and $s \in U(\bar{x})$

→ Both uniquely induce $(1, 1, ..., 1)$ in state $\bar{x}$

I will compare $K(s^R(\phi, \bar{x}) \mid \bar{x})$ and $K^{\varepsilon}(s \mid \bar{x})$ for $s \in U(\bar{x})$

→ Note that I evaluate K^{ε} in state $\bar{x}$

→ Cost when nature draws Segal's (2003)/Winter's (2004) payoff functions

Discrimination?

Theorem

Let $\bar{x} \in \mathbb{R}$. If Φ^* is grouping, then there exists $\bar{s} \in U(\bar{x})$ such that

(i) *For all $\phi \in \Phi^*$, $K^\varepsilon(\bar{s} \mid \bar{x}) \to K(s^R(\phi) \mid \bar{x})$ as $\varepsilon \to 0$;*

(ii) *If players $i, j \in \mathcal{N}$ are symmetric, then $\bar{s}_i = \bar{s}_j$.*

Making explicit the relationship between policy and strategic beliefs...

... discrimination is not necessary to minimize the cost of policy

The proof of the Theorem is constructive

$\rightarrow$ Means we can actually *find* $\bar{s}$

N.B. the ranking policy s^R may tax some (high-ranked) players

$\rightarrow$ If taxing not permitted (e.g. too extreme), $\bar{s}$ is cheaper

Ann and Bob

Consider again Ann and Bob from the introductory example

Suppose $\bar{x} = 0$, $c_i = c$, $w_i(0) = 0$, and $w_i(1) = b > c$, $i \in \{\text{Ann, Bob}\}$

Ranking policy

- $\rightarrow$ $s^R_{i_1} = c$ and $s^R_{i_2} = c - b (< 0)$
- $\rightarrow$ Total cost: $2c - b$

Global subsidy

- $\rightarrow$ Planners wants both to play 1 whenever $x^\varepsilon_i \geq \bar{x} - \varepsilon$, $i \in \{\text{Ann, Bob}\}$
- $\rightarrow$ Using the first Theorem, this gives $\bar{s}_i \rightarrow c - b/2$ as $\varepsilon \rightarrow 0$
- $\rightarrow$ Total cost: $2c - b$

Discussion

Common knowledge

Why such different results even as ε becomes very small?

Why $\Gamma^{\varepsilon}(s) \nrightarrow \Gamma(x, s)$ as $\varepsilon \to 0$?

Precise knowledge ($\varepsilon \to 0$) isn't comparable to common knowledge ($\varepsilon = 0$)

No such thing as "almost common knowledge"

An event E is **common knowledge** if all players know E, all know that all know E, all know that all know that all know E, and so on (Aumann, 1976)

In $\Gamma(x, s)$, the state x is common knowledge

What is common knowledge about x in $\Gamma^{\varepsilon}(s)$?

Common knowledge

Two players, i and j. Player i observes signal $x_i^\varepsilon = 1$

He therefore knows that $x \in [1-\varepsilon, 1+\varepsilon]$ and $x_j^\varepsilon \in [1-2\varepsilon, 1+2\varepsilon]$

Hence, i knows that j knows that $x \in [1-3\varepsilon, 1+3\varepsilon]$

So i knows that j knows that i knows $x \in [1-5\varepsilon, 1+5\varepsilon]$

$\vdots$

The only thing that is *common knowledge* in $\Gamma^\varepsilon(s)$ is that $x \in \mathbb{R}$

Picking Winners and Induced Coordination Failure

Subsidizing before payoffs are known is risky

Planner could end up stimulating an ex post inefficient outcome

→ nuclear energy (Cowan, 1990), pest control (Cowan & Gunby, 1996)

Not typically considered in the literature

→ Only inefficiency is budgetary: policy costs more than it has to

Calls for policy moderation

→ Planner should be careful lest she pick the wrong winner

Generalizations

I study several extensions and applications of this model

- → Games of regime change here
  - → Morris & Shin (1998), Angeletos et al. (2006, 2007), Sákovics & Steiner (2012), Basak & Zhou (2020), Halac et al. (2020)
- → Incentives in teams here
  - → Winter (2004), Halac et al. (2020, 2022, 2023)
- → Heterogeneous externalities here
  - → Subsumes games on networks Matthew & Yariv (2009), Galeotti et al. (2020), Leister et al. (2022)
- → Asymmetric policy targets (and first-best) here
- → Continuous action spaces, payoffs linear in own action here

Thank you!

Social Welfare

Let $\hat{\pi}_i(a \mid x) = \pi_i(a \mid x) - a_i \cdot s_i$ denote player i's payoff net of subsidies

Social welfare is determined by a function

$$W(\hat{\pi}_1(a \mid x), \hat{\pi}_2(a \mid x), ..., \hat{\pi}_N(a \mid x))$$

that is symmetric and increasing in each of its arguments

Proposition

There exists a unique $x^* = (x_i^*) \in \mathbb{R}^N$ such that if $(a_i(x)) = \arg\max_{a \in A} W(\cdot \mid x)$, then $a_i(x) = 1$ iff $x \geq x_i^*$. Furthermore, if players $i, j \in \mathcal{N}$ are symmetric, then $x_i^* = x_j^*$.

Back

Closed support of x

Define $\underline{x} := \sup\{x : x + w_i(N-1) + s_i - c_i \leq 0 \forall i\}$

Define $\overline{x} := \inf\{x : x + w_i(0) + s_i - c_i \geq 0 \forall i\}$

We need $\mathcal{X} \supseteq [\underline{x} - \varepsilon, \overline{x} + \varepsilon]$

Back

Imagine, for simplicity, two symmetric players
For high signals $x_i^\varepsilon \geq \overline{x}(s)$, playing 1 is a dominant strategy for each player i

Receiving a signal just below $\overline{x}(s)$, player i knows there is a strictly positive probability that $x_j^\varepsilon \geq \overline{x}_j(s_j)$, in which case j plays 1

Knowing this, player i will play 1 even for some signals below $\overline{x}(s)$ (and same for j) $\rightarrow$ new threshold $\overline{x}^1(s)$

Argument can be repeated. We obtain a sequence $(\overline{x}^k(s))_{k\in\mathbb{N}}$ where $\overline{x}(s) = \overline{x}^0(s) > \overline{x}^1(s) > \overline{x}^2(s) >$ ... The limit of this sequence is $x^*(s)$

Strategy survives iterated elimination of strictly dominated strategies iff it assigns prob. 1 to action 1 whenever $x_i^\varepsilon > x^*(s)$

Back

General strategic complementarities

Proposition: global subsidy makes players indifferent in the critical state given "double uniform strategic beliefs"

1. Uniform belief over number of players n that play 1

2. Given n, uniform belief over all $\binom{N-1}{n}$ vectors a_{-i} in which n players play 1

Back

Asymmetric policy targets

Partition the player set into $M \geq 2$ subsets [groups] $\mathcal{N}_m$

Group-specific critical states $\tilde{x}_m$, and $\tilde{x}_1 < \tilde{x}_2$

The planner wants players in group m to play 1 for all $x > \tilde{x}_m$

global subsidy $\tilde{s}$ exists and is unique

Makes $i \in \mathcal{N}_1$ indifferent in the critical state given (i) uniform beliefs about the number of players $j \in \mathcal{N}_1$ that play 1 and (ii) prob. 0 that each $j \in \mathcal{N}_2$ plays 1

Makes $i \in \mathcal{N}_2$ indifferent in the critical state given (i) uniform beliefs about the number of players $j \in \mathcal{N}_2$ that play 1 and (ii) prob. 1 that each $j \in \mathcal{N}_1$ plays 1

Back

Continuous action space

Let $a_i \in [0, 1]$

Payoffs are linear in a_i: $\pi_i(a \mid x, s_i) = a_i \cdot [x + w_i(a_{-i}) + s_i] + (1 - a_i) \cdot c_i$

E.g. per-dollar returns on investment

Main theorem applies as given to this case

Back

Joint Investment Problems

Players in $\mathscr{N}$ can invest, or not, in a project

The cost of investment to player i is c_i

If the project succeeds, player i realizes benefit $b_i + x$, $b_i > c_i$

The project succeeds iff at least a critical mass I invests

I unobserved but known to be distributed uniformly on $\{0, 1, ..., N\}$

Canonical model in the applied global games literature (with $x = 0$)

- $\rightarrow$ Morris & Shin (1998), Angeletos et al. (2006, 2007) Sákovics & Steiner (2012), Basak & Zhou (2020), Halac et al. (2020)
- $\rightarrow$ Difference: common knowledge about x/private signals about I

Unique Investment Subsidies

Planner offers subsidies $\tilde{s}$ to induce i to invest iff $x_i^\varepsilon > \tilde{x}$

Unique scheme $\tilde{s}$ that solves the planner's problem given by $(\forall N \geq 2)$

$$\tilde{s}_i = c_i - \frac{b_i + \tilde{x}}{2}$$

Literature focuses on models where $x = 0$, suggesting $\tilde{x} \nearrow 0$:

$$\tilde{s}_i \to c_i - b_i/2$$

Offer each player a subsidy less than half $(b_i > c_i)$ his investment cost

Cf. Sákovics & Steiner (2012): subsidize subset of players fully $(s_i = c_i)$

Uncertainty about payoffs matters!

Back

Incentives in Team

There is a project and a team of agents

Each agent can work toward completion of the project ($a_i = 1$), or shirk

There is a principal who **does not** observe agents' work decisions

Principal pays reward $v_i + x$ to agent i **conditional on project success**
$\rightarrow$ Common payoff x reflects e.g. profit-sharing

The probability of project success is $q(\sum_i a_i)$, increasing and supermodular

The cost of work to agent i is c_i

Equivalent to Winter (2004) and Halac et al. (2020, 2022, 2023) for $x = 0$

Incentives in Teams

Given $\tilde{x}$, the reward $\tilde{v}_i$ to player i is

$$\tilde{v}_i \to \frac{c_i}{\sum_{n=0}^{N-1}[q(n+1)-q(n)]/N} - \tilde{x}$$

Indifference between working and shirking in the critical state...

... given uniform belief about number of agents that work

$\rightarrow$ Cf. Winter (2004), Halac et al. (2020, 2023)

Back