Time Horizons and Emissions Trading

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Abstract

We study dynamic cap and trade schemes in which a policy of adjustable allowance supply determines the cap on emissions. Focusing on two common supply policies, price and quantity mechanisms, we investigate how the duration of a cap and trade scheme affects equilibrium emissions under its cap. More precisely, we quantify the reduction in equilibrium emissions realized by shortening the duration for the scheme (having the it run in fewer periods). We present four main results. First, the reduction in emissions is positive and bounded from below under a price mechanism. Second, the reduction in emissions is bounded from above under a quantity mechanism. Third, these upper and lower bounds coincide when the price and quantity mechanism are similar. Fourth, we identify sufficient conditions for which the reduction in emissions is strictly negative under a quantity mechanism. Our results show that price and quantity mechanisms are nowhere near equivalent.

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1 Introduction

Many pollution markets use a policy of adjustable allowance supply to determine the cap on emissions. The usual motivation is that unexpectedly low or high abatement costs would call for changes to the emissions cap, which adjustable supply policies can deliver. Practical examples of adjustable supply policies can be found in the California Cap-and-Trade Program (Borenstein et al., 2019), the Regional Greenhouse Gas Initiative (Friesen et al., 2022), the European Union's Emissions Trading System (Perino, 2018; Gerlagh et al., 2021), Germany's National Emissions Trading System (Traeger et al., 2020), and Québec's cap and trade system (Schmalensee and Stavins, 2017).

A unifying feature in the design of adjustable supply policies is that these are (i) based on observable conditions in the market for emissions allowances and (ii) rule-based rather than discretionary. This paper focuses on two of the most prominent adjustable supply policies that admit these features. A *price mechanism* is a policy that increases the number

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of allowances supplied when the allowance price increases. In contrast, a *quantity mechanism* is a policy that reduces the supply of allowances when the number of banked allowances goes up. Economist have long advocated both price and quantity measures as means to contain abatement costs uncertainty and variability in cap and trade schemes (Roberts and Spence, 1976; Pizer, 2002; Fell and Morgenstern, 2010; Grüll and Taschini, 2011; Stranlund et al., 2014; Abrell and Rausch, 2017; Kollenberg and Taschini, 2016, 2019; Lintunen and Kuusela, 2018; Pizer and Prest, 2020; Quemin and Trotignon, 2021).

Price and quantity mechanisms are intuitively similar. A low price or a large surplus are interpreted to indicate that abatement is cheap, motivating a tightening of the emissions cap. Both mechanisms thus aim to better align the supply and demand for emissions. Apparent similarities notwithstanding, there also is a fundamental distinction between the mechanisms. Price mechanisms use prices to update quantities, effectively turning the quantity-instrument that is cap and trade into a hybrid policy. Quantity mechanism instead uses quantities to update quantities, doubling down on the quantity aspect of emissions trading.

This paper compares price and quantity mechanisms. We investigate how the duration of a cap and trade scheme affects emissions under its cap when the supply of allowances is determined through a price or quantity mechanism. In particular, for any two possible durations of the scheme, we quantify the reduction in equilibrium emissions realized by choosing the shorter, rather than the longer, duration for the policy. We present four main results. First, the reduction in equilibrium emissions is positive and bounded from below under a price mechanism. In contrast, our second main result shows that the reduction in equilibrium emissions is bounded from above (and possibly negative) under a quantity mechanism. Third, these upper and lower bounds coincide when the price and quantity mechanism are comparable (in a way made formally precise in the model). And fourth, we identify sufficient conditions for which the reduction in equilibrium emissions is strictly negative under a quantity mechanism. Our results show that price and quantity mechanisms, though perhaps similar, are far from equivalent.

The driving force behind our results is firms' incentive not to hold any allowances once the final period arrives. An allowance has value only if it can be used to cover emissions. Rather than leave allowances unused until the scheme ends, firms use them before the final period to lower abatement costs while they still can. A shorter time horizon hence incentivizes firms to use more allowances early on, reducing the bank of allowances and exercising downward pressure on the allowance price. Because of the latter effect, a price mechanism reduces the supply of emissions. A quantity mechanism in contrast increases the early-period supply of allowances due to the reduction in banking.

We derive our results under general assumptions about abatement costs and supply mechanisms. Abatement costs should be convex and increasing, while for price and quantity mechanisms only the signs of the first derivatives are restricted. The sufficiency of such minimal assumptions, rather than specific functional forms, hints at a deep-rooted advantage of price over quantity mechanisms. The choice of generality over narrower parametric specifications naturally impedes a welfare analysis. Our results hence do not concern welfare per se bur rather the internal consistency of a set of policies; they show that specific policies cannot be combined in a straightforward way.

There can be various interpretations to what we call the duration of the scheme. First, like most policies, the scheme might simply end, such as when the planner aims to eliminate pollution over time and regulates the transitionary period using cap and trade. Second, the *effective* duration of the scheme could be dictated by a seperate ban on emissions. In this interpretation, the duration of the scheme per se does not change; rather, there is an overlapping policy, independent of the scheme, that eventually dictates the practical lifetime of the scheme. Third, the final period could be implied by a policy of retiring emissions allowances (Holland and Moore, 2013). Policymakers often do not intend to keep on supplying new allowances indefinitely. If so, a policy that retires unused allowances implies an effective end to the cap and trade scheme some years after allowences are last supplied.

The focus on the timing of emissions notwithstanding, this paper has broader implications. Any policy intended to reduce future emissions exercises a downward pressure on banking incentives and the allowance price. Price mechanisms hence reinforce such policies by supplying fewer allowances while quantity mechanisms counteract them by loosening the cap on emissions. We chose the extreme case of a final period on emissions for largely pragmatic reasons. First, as we argue above, a final period on emissions actually appears to be on the global policy agenda (Höhne et al., 2021). Second, a final period facilitates precise characterization of the upper and lower bounds on emissions reductions. In a narrow sense, this paper warns that future bans on emissions do not easily combine with cap and trade schemes that govern supply through a quantity mechanism. More broadly, it suggests that quantity mechanisms are generally harder to combine with other policies. Either way, price mechanisms have a clear edge over quantity mechanisms.

2 Model

2.1 Building blocks

Consider a dynamic market consisting of a set $N = \{1, 2, ..., n\}$ of polluters, $n > 1$, called firms for simplicity. In each period $t \geq 0$, abatement for firm i is given by $a_{it} = q_{it}^0 - q_{it}$, where q_{it}^0 denotes business-as-usual emissions (i.e. the level emissions in the absence of any policy whatsoever) and $q_{it} \geq 0$ is the actual level of emissions in period t. The cost of abatement is determined by the abatement cost function C_{it} which satisfies $C_{it}(0) = 0$, $C'_{it}(a_{it}) := \frac{\partial C_{it}(a_{it}}{\partial a_{it}} > 0$, and $\frac{\partial^2 C_{it}(a_{it})}{\partial a_{it}^2} \geq 0$. We assume perfect foresight about C_{it} throughout the main text. This assumption is harsh but inconsequential; nowhere critical does the analysis rely on perfect foresight.

Emissions in periods $0, 1, ..., T$ are regulated through a cap and trade scheme, where T is a policy variable set at the discretion of the planner. Let s_{it} denote the number of allowances supplied to firm i at the start of period t. It is allowed that $T \to \infty$ and the scheme runs indefinitely. Allowances are tradeable on a secondary market where a firm can sell or acquire them at a price p_t which it takes as given.[1] Let m_{it} denote the number of allowances bought

[1]Though the assumption of price taking behavior is strong, it is also standard in the literature on emissions trading (*c.f.* Pizer, 2002; Hasegawa and Salant, 2014; Abrell and Rausch, 2017; Pizer and Prest, 2020; Holtsmark and Midttømme, 2021). See Hintermann (2011, 2017), Liski and Montero (2011), and Stocking (2012) for discussions of market power in emissions trading schemes. Hintermann (2010); Hintermann et al. (2020) discusses the price drives or allowance prices in a prominent emissions market, the EU ETS. It should also be noted that the absolute equalization of abatement costs across firms is a theoretical ideal; Di Maria

by firm i in period t. We assume that every allowance bought must also be sold, so

$$\sum_i m_{it} = 0, \tag{1}$$

for all t. Hence, if a firm chooses an amount q_{it} of emissions and buys a total of m_{it} allowances on the secondary market, its total costs in period t are $C_{it}(q^0_{it} - q_{it}) + p_t m_{it}$.

Emissions may not, in principle, exceed the supply of allowances. Temporal violations of the periodic cap s_{it} are facilitated through a banking provision (Kling and Rubin, 1997; Hasegawa and Salant, 2014). Banking by firm i in period t is given by $b_{it} := s_{it} + m_{it} - q_{it}$. The bank of allowances held by firm i at the start of period t is therefore

$$B_{it} := \sum_{s=0}^{t-1} b_{is} = B_{it-1} + b_{it-1} = B_{it-1} + s_{it-1} + m_{it-1} - q_{it-1}, \tag{2}$$

and the total bank of allowances at the start of period t is $B_t := \sum_i B_{it}$. We also assume that borrowing is not allowed:

$$B_{it} \geq 0, \tag{3}$$

for all i and t; this assumption is not necessary, but it is realistic. The effective constraint on emissions by firm i is hence

$$\sum_{s=0}^{t} q_{is} \leq \sum_{s=0}^{t} s_{is} + m_{is}, \tag{4}$$

for all t. Allowances can only be used to cover emissions in the scheme; they are assumed to have no value after the scheme ends.

In what follows, we investigate the effect of the duration of the cap and trade scheme on equilibrium emissions. In particular, we compare emissions between a policy environment in which the final period is T and an alternative environment in which it is $\bar{T} < T$. This is equivalent to a case in which the cap and scheme does not end in period $\bar{T}$ per se but rather one in which, on top of the scheme itself, the planner imposes a zero-emissions target starting from period $\bar{T}$. Because of this equivalence our results also speak, qualitatively, to a case in which firms face a series of binding but non-zero emissions targets between periods $\bar{T}$ and T (say, a 40% reduction compared to 1990 emissions).

2.2 Firms' problem

In any period t, each firm i seeks to minimize the discounted sum of total costs

$$\sum_{\tau=t}^{T} \beta^{\tau-t} \left[C_{i\tau}(q^0_{i\tau} - q_{i\tau}) + p_\tau m_{i\tau} \right], \tag{5}$$

subject to (1)–(4). Given a vector of prices $p = (p_t)_t$, let $q_{it}(p_t)$ denote the firm's solution to this problem, the derivation of which is standard and given in the Appendix. Define $q_t(p_t) = \sum_i q_{it}(p_t)$. Convexity of C_{it} implies

$$\frac{\partial q_{it}(p_t)}{\partial p_t} \leq 0, \tag{6}$$

et al. (2020) provide evidence that it need not be perfectly realized.

for all $t \leq T$. The inequality is strict whenever $q_{it}(p_t)$ is not a corner solution. As is intuitive, the abatement cost minimizing level of emissions chosen by firm i in period t is decreasing in the prevailing allowance price in that period. For given p_t, define $q_t(p_t) = \sum_i q_{it}(p_t)$ to be total demand for emissions in period t.

Lemma 1. *In each period $t \in \{0, ..., T\}$, aggregate demand for emissions $q_{it}(p_t)$ is decreasing in the allowance price p_t.*

Lemma 2. *For all $t \in \{0, ..., T-1\}$, cost-minimizing prices co-move between periods:*

$$\frac{\partial p_{t+1}}{\partial p_t} > 0. \tag{7}$$

Lemma 2 gives a generalized version of Hotelling's rule. If neither the borrowing nor the secondary market constraint are binding, firms will bank allowances until prices rise at the discount rate β. It is well known that binding constraints and other factors cause violations of the rule in its canonical formulation. The literature nevertheless supports the positive co-movement of prices over time.

2.3 Supply mechanisms

Let $s_t = \sum_i s_{it}$ denote the total supply of allowances in period t. We will come to the precise determination of the supply vector s shortly; in any case we assume that $\sum_{s=0}^{t} s_s < \sum_{s=0}^{t} q_s^0$ for all t, where $q_t^0 = \sum_i q_{it}^0$. That is, the supply of allowances does not exceed business-as-usual emissions.[2]

The first class of supply mechanisms considered are price mechanisms. To avoid confusion, the supply of allowances under a price mechanism is denoted s_t^P.

Definition 1 (Price mechanism)**.** *A cap and trade scheme operates a* price mechanism *if the supply of allowances in any period t is increasing in the prevailing allowance price p_t. Formally, for any period t and any two price levels p_t and p_t' it holds that $s_t^P(p_t) > s_t^P(p_t')$ if and only if $p_t > p_t'$.*

Price mechanisms were proposed by Roberts and Spence (1976), Pizer (2002), and Abrell and Rausch (2017). Practical examples are price collars (Borenstein et al., 2019). We assume that $s_t^P(0) \leq q_t(0)$ and $s_t^P(\infty) \geq q_t(\infty)$ for all t, with a stict inequality for at least one t. While not strictly necessary for our main results, we assume that $s_t^P(p_t)$ is differentiable in p_t to simplify the exposition. We write B_t^P for the bank of allowances when supply is governed by a price mechanism.

Given a price vector p and two periods t_1, t_2 such that $t_1 \leq t_2$, define

$$S^P(t_1, t_2 \mid p) := \sum_{t=t_1}^{t_2} s_t^P(p_t). \tag{8}$$

[2]The case in which allowance supply exceeds BAU emissions appears empirically irrelevant (Fowlie, 2010; Calel, 2020; Bayer and Aklin, 2020).

$S^P(t_1, t_2 \mid p)$ is the number of allowances supplied between periods t_1 and t_2 under a price mechanism when the price vector is p.

The second class of supply mechanisms studied are quantity mechanisms. Let the supply of allowances under a quantity mechanism be denoted s_t^Q.

Definition 2 (Quantity mechanism)**.** *A cap and trade scheme operates a* quantity mechanism *if the supply of allowances in period t is increasing in the aggregate excess supply at the start of period t. That is, for any period t and any two B_t and B_t', it holds that $s_t^Q(B_t) > s_t^Q(B_t')$ if and only if $B_t' > B_t$.*

Quantity mechanisms were studied by Kollenberg and Taschini (2016, 2019), Abrell and Rausch (2017), Lintunen and Kuusela (2018), Pizer and Prest (2020), and Quemin and Trotignon (2021). Examples in practice are abatement bounds (Holt and Shobe, 2016; Abrell and Rausch, 2017), a market stability reserve like the EU's (Gerlagh et al., 2021), or Korea ETS' liquidity provisions (Asian Development Bank, 2018). We assume that $s_t^Q(B_t(p)) \leq q_t(0)$ and $s_t^Q(B_t(p)) \geq q_t(\infty)$ for all p, with a strict inequality for at least one t. While not strictly necessary for our main results, we assume that $s_t^Q(B_t)$ is differentiable in B_t. We also assume that $-1 < \partial s_t^Q / \partial B_t$ for all t to avoid the counter-intuitive scenario in which firms have an incentive to bank *less* today in order to have *more* allowances in the future. When supply is governed by a quantity mechanism we write B_t^Q for the bank of allowances.

Given a price vector p and two periods t_1, t_2 such that $t_1 \leq t_2$, define

$$S^Q(t_1, t_2 \mid p) := \sum_{t=t_1}^{t_2} s_t^Q(B_t^Q(p)). \tag{9}$$

$S^Q(t_1, t_2 \mid p)$ is the number of allowances supplied between periods t_1 and t_2 under a quantity mechanism when the price vector is p.

From the assumption that firms are price-takers follows that each firm, though cognizant of the supply mechanism in place, takes the supply of allowances as given.[3] We also assume that the planner is committed to its supply mechanism. This assumption is strong because it turns the supply of allowances into a mechanical rule rather than a quantity at the planner's discretion. Policy commitment is a common assumption in the literature on emissions trading and supply mechanisms.

The timing of events is as follows. At the start of period t, the planner supplies s_t allowances according to the supply mechanism in place. Firms trade allowances on the secondary market and simultaneously choose their emissions q_t; unused allowances are banked. Markets clear and period $t+1$ begins.

3 Equilibrium

The market is in equilibrium when the supply of emissions allowances is equal to demand subject to all policy constraints; prices adjust to brings about equilibrium. Because firms are price takers, we solve for the competitive market equilibrium.

[3] The same criticisms that apply to the assumption of price-taking firms apply here. See in particular Stocking (2012) Gerlagh and Heijmans (2019) for analyses of strategic agents that may try to use a supply mechanisms in their own advantage.

3.1 Price mechanisms

When supply is governed by a price mechanism, the market equilibrium is a tuple $\big(p, q(p), s^P(p), T\big)$ such that the equilibrium price vector p^P yields emissions $q(p^P)$ that solve the firms' optimization problem given supply is equal to $s^P(p^P)$ and the scheme ends in T. $f^P(p^P) \leq T$ denotes the period in which the equilibrium supply of allowances dries up permanently given the equilibrium price vector p^P, $f^P(p^P) := \min\{t : s^P_\tau(p^P) = 0 \forall \tau \geq t\}$. Recall that the final period T is set at the discretion of the planner. If instead the planner chooses to end the scheme in period $\bar{T}$, rather than T, the market equilibrium is given by $\big(\bar{p}^P, q(\bar{p}^P), s^P(\bar{p}^P), \bar{T}\big)$.

Given the notation, total equilibrium emissions when the scheme ends in T are $\sum_{t=0}^{T} q_t(p^P_t)$; similarly, total emissions when the scheme ends in $\bar{T}$ are $\sum_{t=0}^{\bar{T}} q_t(\bar{p}^P_t)$. Let R^P denote the reduction in equilibrium emissions when the scheme ends in $\bar{T}$, rather than T,

$$R^P(\bar{T}, T) := \sum_{t=0}^{T} q_t(p^P_t) - \sum_{t=0}^{\bar{T}} q_t(\bar{p}^P_t). \tag{10}$$

3.2 Quantity mechanisms

When supply is governed by a quantity mechanism, the market equilibrium is a tuple $\big(p, q(p), s^Q(B(p))\big)$ such that the equilibrium price vector p yields emissions $q(p)$ that solve the firms' optimization problem given supply is equal to $s^Q(B(p))$. To easily distinguish between cases, let $\big(p^Q, q(p^Q), s^Q(B(p^Q)), T\big)$ refer to the market equilibrium when the scheme ends in period T. $f^Q(p^Q) \leq T$ denotes the period in which the equilibrium supply of allowances dries up permanently under a price mechanism, $f^Q(p^Q) := \min\{t : s^Q_t(B(p^Q)) = 0 \forall \tau \geq t\}$. When instead the scheme ends in period $\bar{T}$, the market equilibrium is $\big(\bar{p}^Q, q(\bar{p}^Q, \bar{q}), s^Q(B(\bar{p}^Q)), \bar{T}\big)$.

Given the equilibria $\big(p^Q, q(p^Q), s^Q(B(p^Q)), T\big)$ and $\big(\bar{p}^Q, q(\bar{p}^Q, \bar{q}), s^Q(B(\bar{p}^Q)), \bar{T}\big)$, let R^Q denote the reduction in equilibrium emissions when the scheme ends in $\bar{T}$, rather than T,

$$R^Q(\bar{T}, T) := \sum_{t=0}^{T} q_t(p^Q_t) - \sum_{t=0}^{T} q_t(\bar{p}^Q_t). \tag{11}$$

The research question of this paper can now be stated concisely as follows. For any two $\bar{T}$ and T such that $\bar{T} < T$, what are the properties of $R^P(\bar{T}, T)$ and $R^Q(\bar{T}, T)$?

4 Results

This section presents the main results of the paper. All depart from an intuitive first step, formalized in Lemmas 3 and 4 below, that connects banking incentives with the lifetime of the cap and trade scheme. Profit-maximizing firms have no incentive whatsoever to bank allowances beyond the final period of the cap and trade scheme. Hence, in equilibrium firms deplete their bank of allowances before the scheme ends. It follows that banking in period $\bar{T} - 1$ will be weakly less when the scheme ends in period $\bar{T}$ compared to when it ends in T, given $\bar{T} < T$. Dynamic abatement cost minimization leads firms to spread out the reduction in banking over all periods $t < \bar{T}$: $B^P_t(\bar{p}^P) \leq B^P_t(p^P)$ and $B^Q_t(\bar{p}^Q) \leq B^Q_t(p^Q)$ for all t. To support less banking – which has to come about through increased emissions – the allowance price has to go down: $\bar{p}_t^P \leq p_t^P$ and $\bar{p}_t^Q \leq p_t^Q$ for all $t < \bar{T}$.[4]

Suppose, as a thought experiment, that we were to increase the allowance price p_τ in some period $\tau > 0$. Suppose also that this increase is anticipated by the market already in period 0. How are banking an allowance prices in all periods $t \geq 0$ affected by the increase in p_τ? That is, what can we say about $\frac{\partial B_t^P(p)}{\partial p_\tau}$ and $\frac{\partial B_t^Q(p)}{\partial p_\tau}$, for all $t \geq 0$ and $\tau > 0$?

Lemma 3 (Dynamic price effects under a price mechanism)**.** *Consider a cap and trade scheme that operates a price mechanism. Fix a final period T. For any two periods τ, t, $\tau > 0$ and $t \geq 0$, the bank of allowances B_t^P is increasing in the allowance price p_τ: $\frac{\partial B_t^P(p)}{\partial p_\tau} > 0$.*

Lemma 4 (Dynamic price effects under a quantity mechanism)**.** *Consider a cap and trade scheme that operates a quantity mechanism. Fix a final period T. For any two periods $\tau, t < T$, $\tau > 0$ and $t \geq 0$, the bank of allowances B_t is increasing in the allowance price p_τ: $\frac{\partial B_t^Q(p)}{\partial p_\tau} > 0$.*

4.1 Tight bounds

Under a price mechanism, the reduction in equilibrium emissions from having the final period at $\bar{T}$, rather than T, is positive and bounded from below.

Proposition 1. *Consider a cap and trade scheme that operates a* price mechanism*. For all $\bar{T}$ and T such that $\bar{T} < T$, the reduction in equilibrium emissions when the scheme ends in period $\bar{T}$, rather than T, satisfies*

$$R^P(\bar{T}, T) \geq S^P(\bar{T}, T \mid p^P) \geq 0. \tag{12}$$

That is, the reduction in equilibrium is bounded from below under a price mechanism. The bound is tight.

Ending the cap and trade schem earlier has two mutually reinforcing effects under a price mechanism. First, any allowances that would originally be supplied starting from period $\bar{T}$, $S^P(\bar{T}, T \mid p^P)$, are taken out of the system. Second, firms redistribute their emissions to early periods to avoid holding allowances once the scheme ends: leakage. Higher emissions in early periods suppress the allowance price in those periods. By the mechanics of a price mechanism, his translates into a reduction of supply in the periods leading up to $\bar{T}$, further reducing emissions. While the first effect is always there, the second occurs only if firms originally hold a strictly positive bank of allowances at the start of period $\bar{T}$.

Under a quantity mechanism, the reduction in equilibrium emissions from having the final period at $\bar{T}$, rather than T, is bounded from above (and possibly negative).

Proposition 2. *Consider a cap and trade scheme that operates a* quantity mechanism*. For all $\bar{T}$ and T such that $\bar{T} < T$, the reduction in equilibrium emissions when the scheme ends in period $\bar{T}$, rather than T, satisfies*

$$R^Q(\bar{T}, T) \leq S^Q(\bar{T}, T \mid p^Q). \tag{13}$$

[4]The equilibrium price is not defined in periods $t \geq \bar{T}$ for a scheme that ends in $\bar{T}$.

That is, the reduction in equilibrium is bounded from above (and possible negative) under a quantity mechanism. The bound is tight.

Shortening the time horizon of emissions has two opposing effects under a quantity mechanism. First, any allowances that would originally be supplied starting from period $\bar{T}$, $S^Q(\bar{T}, T \mid p^Q)$, are eliminated. Second, firms may redistribute their emissions to early periods to avoid holding allowances by the time the final period arrives. These two effect are exactly the same for price and quantity mechanisms. However, the mechanics of a quantity mechanism dictate that a reduction in banking prior to $\bar{T}$ results in an *increase* in allowance supply in those periods. Under a quantity mechanism, the redistribution effect offsets some (or all) of the emissions reductions achieved by eliminating supply after period $\bar{T}$. The reduction in equilibrium emissions is therefore at most $S^Q(\bar{T}, T \mid p^Q)$, implying an upper bound.

Note that an earlier final period *may* reduce emissions under a quantity mechanism. It does not have to. In the next section, we provide sufficient conditions for the emissions reduction due to an earlier final period to be negative. Under these conditions, shortening the time horizon of emissions is incompatible with strengthened climate ambitions.

4.2 Incompatibility

Fix a final period T. Posit a period $T^* < T$ at which, given the final period T, equilibrium demand is strictly positive while equilibrium supply is already (and permanently) zero. There need not be such a T^* and if it exists it need not be unique. Assuming at least one exists, set $\bar{T} = T^*$. Formally, one can verify that the conditions on T and $\bar{T}$ thus imposed are:

$$q_{\bar{T}}(p^Q_{\bar{T}}) > 0, \tag{14}$$

and

$$f^Q(p^Q) \leq \bar{T}. \tag{15}$$

If the planner advances the final period T to this $\bar{T}$, equilibrium emissions strictly increase under a quantity mechanism.

Proposition 3. *Consider a cap and trade scheme that operates a* quantity mechanism*. For all $\bar{T}$ and T such that $\bar{T} < T$ and $\bar{T}$ satisfies* (14) *and* (15)*, the reduction in equilibrium emissions when the scheme ends in period $\bar{T}$, rather than T, satisfies*

$$R^Q(\bar{T}, T) < 0. \tag{16}$$

That is, equilibrium emissions are strictly higher *when there is a series of binding emissions targets compared to when there is not.*

To understand the result, note that conditions (14) and (15) have two implications. First, there is no supply of allowances after period $\bar{T}$ even when the final period is T. Hence, bringing forward the final period to $\bar{T}$ does not eliminate any supply between $\bar{T}$ and T. Second, the fact that emissions are strictly positive in period $\bar{T}$ (when the final period is T), combined with the fact that supply reaches zero earlier, implies that emissions in $\bar{T}$ are entirely covered by banked allowances. Advancing the final period to $\bar{T}$ triggers firms to deplete their banks earlier, implying less banking overall and therefore, under a quantity mechanism, increased supply. As no supply is eliminated after period $\bar{T}$ while supply goes up before period $\bar{T}$, equilibrium emissions strictly increase when advancing the final period from T to $\bar{T}$.

4.3 Prices vs. quantities

It is possible that emissions reductions under a quantity mechanism exceed those under a price mechanism; this could happen when the lower bound for a price mechanism lies strictly below the upper bound for a quantity mechanism. Here we argue that this possibility is somewhat contrived as it relies on asymmetries in baseline equilibrium allowance supplies.

To formalize this, fix a baseline final period T. Suppose that, given T, the equilibrium supply of allowances is the same under both a price and a quantity mechanism. Formally, given the baseline final period on emissions T, for all $t \geq 0$ let:

$$s_t^P(p_t^P) = s_t^Q(B_t^Q(p^Q)), \tag{17}$$

where p^P and p^Q again denote baseline equilibrium price vectors under a price and quantity mechanism, respectively. If (17) is satisfied, we say that the baseline equilibria under a price and quantity mechanism are *symmetric* under T. The next result shows that the lower and upper bound on emission reductions under a price and quantity mechanism, respectively, coincide when the baseline equilibria are symmetric in this sense.

Proposition 4. *If* (17) *holds for all $t \leq T$, then*

$$R^Q(\bar{T}, T) \leq R^P(\bar{T}, T). \tag{18}$$

For symmetric baseline equilibria, an earlier final period leads to higher emissions reductions under a price mechanism than under a quantity mechanism. Whereas the question of prices versus quantities is as old as environmental economics itself and depends on a score of factors (Weitzman, 1974), the choice between price and quantity mechanisms is much less ambiguous. Under comparable conditions, a price mechanism outperforms a quantity mechanism when it comes to achieving environmental ambitions.

5 Discussion

The analysis entertains a number of restrictive assumptions and simplifications. In this section, we discuss how some of these influence the analysis and results.

Uncertainty. The model assumes perfect knowledge about present and future abatement costs. This is a strong but largely innocent assumption. It is straightforward to extend the model to one which incorporates asymmetric information and imperfect foresight. In such a model, the quantities $R^P(\bar{T}, T)$ and $R^Q(\bar{T}, T)$ would represent *expected* reductions in equilibrium emissions (evaluated at time $t = 0$).

To fix ideas, suppose the true abatement cost function $\tilde{C}_{it}$ depends on a parameter θ_t which is learned only at the start of period t. It is common knowledge that θ_t is drawn from a distribution function $F_t(\theta_t)$. Then one can interpret C_{it} as the expected abatement cost function, evaluated in period 0, i.e. $C_{it}(a_{it}) = \int \tilde{C}_{it}(a_{it} \mid \theta_t)\, dF_t(\theta_t)$. With this re-interpretation of C_{it}, it is clear that the analysis as carried out speaks to expected reductions in equilibrium emissions evaluated at time $t = 0$. The additional assumption one would need in such a model is that the timing of the final period itself does not affect the distribution of θ_t; that is, $F_t(\theta_t)$ remains the same whether the final periods is $\bar{T}$ or T.

Emissions targets. In the interpretation of $\bar{T}$ as the point in time at which a complementary policy, independent of the scheme, starts binding emissions to zero, another assumption requires discussion. If the planner enacts a series of binding emissions targets, those need not always be zero. In a more general environment, the planner could choose a vector of emissions targets $\hat{q} = (\hat{q}_t)_{t \geq \bar{T}}$ such that $q_t \leq \hat{q}_t$ for all $t \geq \bar{T}$. The analysis presented here is essentially the special case in which $\hat{q}_t = 0$ for all $t \geq \bar{T}$.

We argue that the main economic implications of our results remain valid under the more general assumption that the planner imposes a series of binding emissions targets $\hat{q}_t >\geq 0$ starting from period $\bar{T}$. The binding targets limit emissions in periods $\bar{T}$ and after. Anticipating this, firms will bank fewer allowances which suppresses the allowance price (compared to the case in which no binding targets are imposed). Under a quantity mechanism, the reduction in banking leads to an increase in allowance supply, increasing emissions in the periods before $\bar{T}$. Under a price mechanism, the drop in allowance prices leads to a reduction in supply, recreasing emissions in the periods before $\bar{T}$. It follows that a price mechanism can support binding future emissions targets whereas quantity mechanisms tend to work against the policy. The special case of $\hat{q}_t = 0$ facilitates precise characterization of the bound in equilibrium emissions reductions.

Net zero. A somewhat related issue is that even a zero emissions target can be ambiguous (Rogelj et al., 2021). Some argue that *net* zero emissions are the realistic target, implying that a positive amount of emissions is still allowed provided it is compensated for by an equal amount of negative emissions. In this case, even if $\bar{T}$ is interpreted as the period starting from which firms face a complementary zero emissions target, cost-minimizing firms need not necessarily reduce banking all the way to zero by the time period $\bar{T}$ arrives. But assuming negative emissions are costly, aggregate banking should still be expected to (weakly) decrease and allowance prices to drop, causing supply to go down under a price mechanism and up under a quantity mechanism.

Efficiency. The propositions provide bounds on the reduction in equilibrium emissions from having a cap and trade scheme end earlier. They do not discuss how the time horizon of emissions trading affects social welfare. In theory it may be efficient to have higher total emissions that occur earlier in time; the model is silent about this. Given the arguably reasonable assumption that a shorter time horizon of emissions trading (or a complementary emissions-reducing policy) is intended to bring down emissions, the results show how policies that explicitly target the dynamics of emissions can be inconsistent with a market-based emissions cap based on quantities.

Commitment. We assume that the planner is committed to the supply functions s^P and s^Q. If the policy functions s^P and s^Q themselves depend on the final period of the scheme (or the complementary emissions policy in place), the reduction in equilibrium emissions will naturally also depend on changes in the supply functions. For example, suppose that the supply functions are s_t^P and s_t^Q when the final period is T, but $\bar{s}_t^P$ and $\bar{s}_t^Q$ when the final period is $\bar{T}$. It is easy to verify that if $\bar{s}_t^P(p_t) < s_t^P(p_t)$ $(\bar{s}_t^Q(B_t) < s_t^Q(B_t))$ for all p_t (all B_t), then the lower (upper) bound on equilibrium emissions reductions will be weakly higher than those identified above. In contrast, if $\bar{s}_t^P(p_t) > s_t^P(p_t)$ $(\bar{s}_t^Q(B_t) > s_t^Q(B_t))$ for all p_t (all B_t), then the bounds will be weakly lower than those described in the main model. Due to the vast number of possible policy changes that could be implemented in this case, we leave the analysis of emissions reductions with a non-committed planner for future work.

6 Conclusions

We study cap and trade schemes where the cap on emissions is determined by conditions prevailing in the market for allowances. Attention is confined to two classes of empirically relevant supply policies. Under a price mechanism, the supply of allowances is increasing in the allowance price. Under a quantity mechanism, supply is reduced when the surplus of unused allowances increases. The scheme ends in a final period T, which is at the discretion of the planner. We show that the reduction in equilibrium emissions when the scheme ends in $\bar{T} < T$ rather than T is positive and bounded from below under a price mechanism, whereas it is bounded from above (and possibly negative) under a quantity mechanism. Precise characterization of these bounds are given and we identify sufficient conditions for which an earlier final period strictly increases emissions under a quantity mechanism. Our results establish that price and quantity mechanisms are not equivalent and, depending on the broader policy environment, cannot be used interchangeably.

A natural qualification to the results on quantity mechanisms is the assumed exogeneity of the quantity mechanism to policy changes. One might argue that a rational planner anticipates the effect of advancing the final period and would 'manually' reduce the supply of allowances accordingly. We concur. Even so, a clear benefit of price over quantity mechanisms remains: whereas a quantity mechanism can be made to work after additional measures are taken, a price mechanism takes care of itself.

In a sense, quantity mechanisms misinterpret market signals. They react to a reduction in banking as though it signaled an increase in the demand for emissions whereas, in reality, it is the response to a future (policy-driven) fallout of demand. This points to a more fundamental distinction between price and quantity information. While prices provide an accurate signal of the overall demand for emissions, quantities provide a signal only of *relative* demand, that is, of demand today relative to demand in the future. Being more accurate information aggregators, price signals are thus favored over quantity signals for market-based policy-updating.

This paper makes several restrictive assumptions which future work might seek to relax. First, we assume that firms are price takers. Though a common assumption in the literature on emissions trading and environmental policy, it is important to know to which extent the results presented here generalize to cases in which firms have market power (*c.f.* Hintermann, 2011, 2017). Second, the focus on price and quantity mechanisms is somewhat restrictive as other kinds of supply mechanism, while uncommon in practice and the literature, could be conceived. Third, we discuss supply mechanisms in the context of a single emissions trading scheme. When multiple cap and trade schemes are linked (Holtsmark and Midttømme, 2021), different incentives may be at work; these are not considered here.

A Appendix

A.1 Firms' dynamic cost-minimization problem

Turning the firm's constrained problem into an unconstrained cost minimization problem, each firm i chooses q_{it} and m_{it} to solve:[5]

$$\min_{q_{it},m_{it}} \sum_{t=0}^{T} \beta^t C_{it}(\bar{q}_{it} - q_{it}) + \sum_t \beta^t p_t m_{it} + \lambda_i \left[\sum_t q_{it} - s_{it} - m_{it} \right] + \sum_t \beta^t \mu_t \left[\sum_i m_{it} \right] \tag{19}$$
$$+ \omega_{it} \left[B_{it} - B_{it-1} - s_{it-1} - m_{it-1} + q_{it-1} \right] + \beta^t \psi_{it} B_{it}.$$

The first-order conditions associated with the cost-minimization problem given by (19) are:

$$-\beta^t C'_{it}(\bar{q}_{it} - q_{it}) + \lambda_i + \omega_{it+1} = 0, \tag{20}$$
$$\beta^t p_t - \lambda_i + \beta^t \mu_t - \omega_{it+1} = 0, \tag{21}$$
$$\omega_{it} - \omega_{it+1} + \beta^t \psi_{it} = 0. \tag{22}$$

Rewriting these first-order conditions gives:

$$C'_{it}(\bar{q}_{it} - q_{it}) + \psi_{it} = \beta C'_{it+1}(\bar{q}_{it+1} - q_{it+1}), \tag{23}$$

for all $t < T$. Moreover, each firm will emit, or abate, until marginal abatement costs roughly equal the allowance price,

$$p_t = C'_{it}(\bar{q}_{it} - q_{it}) - \mu_t, \tag{24}$$

for all $t < T$. We say that prices should roughly equal the allowances price because when $\mu_t \neq 0$, the secondary market constraint is binding and not every firm can buy or sell the number of allowances it wants, driving a wedge between the allowance price and marginal abatement costs.

Observe that cost minimization forces each firm i to choose $m_{it} \leq 0$ for all $t \geq T$; all want to sell allowances if they have some. Combined with the secondary market constraint that $\sum_i m_{it} = 0$ this gives $m_{it} = 0$, as stated in Remark 1.

A.2 Proofs

PROOF OF LEMMA 2

Proof. Using (20) and (21) gives:

$$p_t + \mu_t = C'_{it}(\bar{q}_{it} - q_{it}), \tag{25}$$

implying (24). Moreover, combining (22) and (21) yields:

$$p_t + \mu_t + \psi_{it} = \beta p_{t+1} + \beta \mu_{t+1}, \tag{26}$$

so $p_{t+1} = (p_t + \mu_t + \psi_{it})/\beta - \mu_{t+1}$ and this implies (7). □

[5] Without loss of generality, we multiply the shadow values μ_t for the secondary market constraint (1) and ψ_{it} for the borrowing constraint by β^t.

PROOF OF LEMMA 3

Proof. Since $s_t(p_t)$ is increasing in p_t by construction while $q_t(p_t, T)$ is decreasing by (6), banking in period $b_t(p_t)$ is increasing in the allowance price p_t. Recall from (7) that prices co-move across periods. By implication, one has $\frac{\partial p_s}{\partial p_\tau} > 0$ for all $s, \tau \in \{0, 1, ..., T\}$ and therefore,

$$\begin{aligned}\frac{\partial B_t^P}{\partial p_\tau} &= \frac{\partial}{\partial p_\tau}\left[\sum_s^{t-1} s_s^P(p_s) - \sum_s^{t-1} q_s(p_s)\right] \\ &= \sum_s^{t-1} \frac{\partial s_s^P(p_s)}{\partial p_s}\frac{\partial p_s}{\partial p_\tau} - \sum_s^{t-1} \frac{\partial q_s(p_s)}{\partial p_s}\frac{\partial p_s}{\partial p_\tau} > 0.\end{aligned} \tag{27}$$

This establishes that B_t^P is increasing in p_τ for all $t, \tau \in [0, T)$. $\square$

PROOF OF LEMMA 4

Proof. The effect of an increase in the allowance price on first-period banking is straightforward:

$$\frac{\partial B_1^Q(p)}{\partial p_\tau} = \frac{\partial b_0(p_0)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = \frac{\partial[s_0^Q - q_0(p_0)]}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = -\frac{\partial q_0(p_0)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} \geq 0, \tag{28}$$

where the inequality is strict for all p_0 such that $q_0(p_0,) > 0$ and all $\tau \geq 0$. A little more work is required to determine the sign of $\partial B_t^Q/\partial p_\tau$ for $t > 1$. Recall that the bank of allowances evolves according to $B_t^Q(p) = B_{t-1}^Q(p) + s_{t-1}^Q(B_{t-1}^Q(p)) - q_{t-1}(p_{t-1})$, where s_t depends on B_t^Q because supply is governed by a quantity mechanism. Hence,

$$\frac{\partial B_t^Q(p)}{\partial p_\tau} = \frac{\partial B_{t-1}^Q(p)}{\partial p_\tau} + \frac{\partial s_{t-1}^Q(B_{t-1}^Q(p))}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1})}{\partial p_\tau} \tag{29}$$

$$= \left(1 + \frac{\partial s_{t-1}^Q(B_{t-1}^Q(p))}{\partial B_{t-1}^Q(p)}\right)\frac{\partial B_{t-1}^Q(p)}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1})}{\partial p_{t-1}}\frac{\partial p_{t-1}}{\partial p_\tau}. \tag{30}$$

The term in parentheses, $1 + \partial s_t^Q/\partial B_t^Q$, is positive by assumption. The final term in (30) is negative by (6) and (7). The only sign left to determine in (30) is hence that of $\partial B_{t-1}^Q/\partial p_\tau$; and this we know for $t = 2$. Using (28), induction on t establishes that

$$\frac{\partial B_t^Q(p)}{\partial p_\tau} \geq 0, \tag{31}$$

for all $t, \tau \in [0, T)$. The inequality is strict for all $p = (p_1, p_2, ...)$ that satisfy $q_t(p_t, T) > 0$ for at least one t. $\square$

PROOF OF PROPOSITION 1

Proof. Two qualitatively distinct scenarios can occur: (i) $B^P_{\bar{T}}(p^P) = 0$ and (ii) $B^P_{\bar{T}}(p^P) > 0$.

In case (i), the equilibrium price vector when the ban on emissions is advanced from T to $\bar{T}$ is the same until period $\bar{T}$: $p^P_t = \bar{p}^P_t$ for all $t < \bar{T}$. This can be proven by contradiction. Suppose $\bar{p}^P \neq p^P$. Then either (a) $\bar{p}_t < p_t$ or (b) $\bar{p}_t > p_t$ for at least one $t < \bar{T}$ which, by Lemma 2, imply that (a) $\bar{p}_t \leq p_t$ or (b) $\bar{p}_t \geq p_t$ for all $t < \bar{T}$. But by Lemma 3, case (a) implies $B_{\bar{T}}(\bar{p}^P) < 0$ whereas case (b) implies $B_{\bar{T}}(\bar{p}^P) > 0$. Either of these violates the requirement that $\bar{p}^P$ is an equilibrium price vector when the final period on emissions is $\bar{p}$. Hence, $\bar{p}^P = p^P$. Equilibrium emissions when the final period is advanced to $\bar{T}$ are therefore equal to:

$$\sum_{t=0}^{\bar{T}} q_t(p^P) = \sum_{t=0}^{\bar{T}} s_t(p^P_t).$$

When the ban is at T instead, equilibrium emissions are:

$$\sum_{t=0}^{T} q_t(p^P) = \sum_{t=0}^{T} s_t(p^P_t).$$

Subtracting the former from the latter gives the reduction in equilibrium emissions:

$$R^P(\bar{T}, T) = \sum_{t=0}^{T} q_t(p^P) - \sum_{t=0}^{\bar{T}} q_t(p^P) = \sum_{t=0}^{T} s_t(p^P_t) - \sum_{t=0}^{\bar{T}} s_t(p^P_t) = S^P(\bar{T}, T \mid p^P).$$

In case (ii), firms originally hold a strictly positive bank of allowances at the start of period $\bar{T}$: $B^P_{\bar{T}}(p^P) > 0$. Equilibrium under the final period $\bar{T}$ is reached when $B_{\bar{T}}(\bar{p}^P) = 0$. By Lemma 3, this implies $p^P_t > \bar{p}^P_t$ for all $t < \bar{T}$. Equilibrium emissions when the final period is $\bar{T}$ are therefore:

$$\sum_{t=0}^{\bar{T}} q_t(\bar{p}^P) = \sum_{t=0}^{\bar{T}} s_t(\bar{p}^P_t).$$

Equilibrium emissions when the final period is T are instead:

$$\sum_{t=0}^{T} q_t(p^P) = \sum_{t=0}^{T} s_t(p^P_t) = \sum_{t=0}^{\bar{T}} s_t(p^P_t) + \sum_{t=\bar{T}+1}^{T} s_t(p^P_t).$$

Subtracting the former from the latter, the reduction in equilibrium emissions when advancing the ban from T to $\bar{T}$ is:

$$\begin{aligned} R^P(\bar{T}, T) &= \sum_{t=0}^{\bar{T}-1} s_t(p^P_t) + \sum_{t=\bar{T}}^{T} s_t(p^P_t) - \sum_{t=0}^{\bar{T}-1} s_t(\bar{p}^P_t) \\ &= S^p(\bar{T}, T \mid p^P) + \sum_{t=0}^{\bar{T}-1} s_t(p^P_t) - \sum_{t=0}^{\bar{T}-1} s_t(\bar{p}^P_t) \\ &> S^p(\bar{T}, T \mid p^P), \end{aligned}$$

where the inequality follows from the fact that $p^P_t > \bar{p}^P_t$ for all $t < \bar{T}$ and therefore, by the mechanics of a price mechanism, $s_t(p^P_t) > s_t(\bar{p}^P_t)$ for all $t < \bar{T}$.

In conclusion, either $R^P(\bar{T},T) = S^p(\bar{T},T \mid p^P)$ or $R^P(\bar{T},T) > S^p(\bar{T},T \mid p^P)$. Since $S^p(\bar{T},T \mid p^P) \geq 0$ by construction. Tightness follows from considering the case $B^P_{\bar{T}}(p^P) = 0$. $\square$

PROOF OF PROPOSITION 2

Proof. Two qualitatively distinct scenarios can occur: (i) $B^Q_{\bar{T}}(p^Q) = 0$ and (ii) $B^Q_{\bar{T}}(p^Q) > 0$. Because these scenarios, as well as their analyses, are similar to those discussed in the proof of Proposition 1, we will be short here.

In case (i), $B^Q_{\bar{T}}(p^Q) = 0$ and therefore $\bar{p}^Q_t = p^Q_t$ for all $t < \bar{T}$. The reduction in equilibrium emissions when the final period is $\bar{T}$, compared to when it is T, is therefore:

$$\begin{aligned} R^Q(\bar{T},T) &= \sum_{t=0}^{T} q_t(p^Q) - \sum_{t=0}^{\bar{T}} q_t(p^Q) \\ &= \sum_{t=0}^{T} s_t(B^Q_t(p^Q)) - \sum_{t=0}^{\bar{T}} s_t(B^Q_t(p^Q)) \\ &= S^Q(\bar{T},T \mid p^Q). \end{aligned}$$

In case (ii), $\bar{T}$: $B^Q_{\bar{T}}(p^Q) > 0$. Equilibrium under the final period $\bar{T}$ is reached when $B^Q_{\bar{T}}(\bar{p}^Q) = 0$. By Lemmas 4 and 2, this implies $p^Q_t > \bar{p}^Q_t$ for all $t < \bar{T}$. The reduction in equilibrium emissions when the final period is $\bar{T}$, compared to when it is T, is therefore:

$$\begin{aligned} R^Q(\bar{T},T) &= \sum_{t=0}^{T} q_t(p^Q) - \sum_{t=0}^{\bar{T}} q_t(\bar{p}^Q) \\ &= \sum_{t=0}^{T} s_t(B^Q_t(p^Q)) - \sum_{t=0}^{\bar{T}} s_t(B^Q_t(\bar{p}^Q)) \\ &= S^Q(\bar{T},T \mid p^Q) + \sum_{t=0}^{\bar{T}} s_t(B^Q_t(p^Q)) - \sum_{t=0}^{\bar{T}} s_t(B^Q_t(\bar{p}^Q)) \\ &< S^Q(\bar{T},T \mid p^Q), \end{aligned}$$

where the inequality is a consequence of the fact that $p^Q_t > \bar{p}^Q_t$ for all $t < \bar{T}$, so $B^Q_t(p^Q) > B^Q_t(\bar{p}^Q)$ for all $t < \bar{T}$ and therefore, by the mechanics of a quantity mechanism, $s_t(B^Q_t(p^Q)) < s_t(B^Q_t(\bar{p}^Q))$ for all $t < \bar{T}$.

The proof is now complete as we have shown that either $R^Q(\bar{T},T) = S^Q(\bar{T},T \mid p^Q)$ or $R^Q(\bar{T},T) < S^Q(\bar{T},T \mid p^Q)$, implying that $R^Q(\bar{T},T)$ is bounded from above by $S^Q(\bar{T},T \mid p^Q)$. Tightness follows from considering the case $B^Q_{\bar{T}}(p^Q) = 0$. $\square$

PROOF OF PROPOSITION 3

Proof. We know from Proposotion 2 that $R^Q(\bar{T}, T) \leq S^Q(\bar{T}, T \mid p^Q)$. Note, then, that condition (15) gives $S^Q(\bar{T}, T \mid p^Q) = 0$. Moreover, condition (14), combined with (15), gives $B_{\bar{T}}(p^Q) > 0$. The fact that $B_{\bar{T}}(p^Q) > 0$ implies that case (ii) in the proof of Proposotion 2 applies, so $R^Q(\bar{T}, T) < S^Q(\bar{T}, T \mid p^Q)$. We have already established that $S^Q(\bar{T}, T \mid p^Q) = 0$. Hence, $R^Q(\bar{T}, T) < 0$. $\square$

PROOF OF PROPOSITION 4

Proof. From Proposition 1, the reduction in emissions under a price mechanism is bounded from below by $S^P(T, \bar{T} \mid p^P)$. From Proposition 2, the reduction in emissions under a quantity mechanism is bounded from above by $S^Q(T, \bar{T} \mid p^Q)$. The condition that baseline equilibrium supply paths are symmetric means that (17) is satisfied. Now, (17) implies $S^P(T, \bar{T} \mid p^P) = \sum_{\bar{T}}^{T} s_t(p_t^P) = \sum_{\bar{T}}^{T} s_t(B_t^Q(p^Q)) = S^Q(T, \bar{T} \mid p^Q)$ Hence, $R^Q(\bar{T}, T) \leq S^Q(T, \bar{T} \mid p^Q) = S^P(T, \bar{T} \mid p^P) \leq R^P(\bar{T}, T)$, implying the result. $\square$

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