Global Policy Design

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Introduction

Policy in Coordination Games

How to incentivize work in teams?

$\longrightarrow$ Winter (2004), Fischer & Huddart (2008), Halac et al. (2021, 2023)

How to raise capital from multiple investors?

$\longrightarrow$ Sákovics & Steiner (2012), Halac et al. (2020)

How to stimulate network technology adoption?

$\longrightarrow$ Bandiera & Rasul (2006), Cai et al. (2015), Beaman et al. (2021)

How to shift social norms?

$\longrightarrow$ Ferraro et al. (2011), Lane et al. (2023)

Policy in Coordination Games

How to design policy in coordination games?

Ann and Bob

Ann and Bob

Ann and Bob can invest in a project

The cost of investment is c

If the project succeeds, investment yields a return w ($w > c$)

The project succeeds if and only if Ann and Bob both invest

Not investing, their outside option, pays 0

Coordination problem: invest iff the other invests ♝♜

→ Both investing is a Nash equilibrium 🤩

→ Neither investing is also a Nash equilibrium 😱

Strategic Beliefs

A planner offers subsidies to induce investment

How high should these subsidies be?

→ If Ann thinks Bob will invest, she needs no subsidy at all

→ If Ann thinks Bob won't invest, she requires a subsidy $\geq c$

Strategic beliefs crucial for policy design

But... multiple Nash equilibria: strategic beliefs not unique

Straightforward solution: make investment strictly dominant for both

→ Effective: almost by definition, strategic beliefs no longer matter

→ But costly

Discrimination

A subsidy to Ann has two (mutually reinforcing) effects

→ **Direct**: subsidy reduces Ann's effective cost of investment

→ **Indirect**: Bob becomes more optimistic about investment by Ann

Clever idea: leverage indirect effect to reduce cost of policy

→ Make adoption dominant for Ann, tax Bob to indifference

→ Such a policy **discriminates**: treats "identical" players differently

Seminal result: discrimination minimizes costs (Segal, 2003; Winter, 2004)

→ Bernstein & Winter (2012), Eliaz & Spiegler (2015), Halac et al. (2020, 2023)

→ Assumes that payoff functions (w, c) are **common knowledge**

Fundamental **trade-off**: equity vs. efficiency

This Paper

I show that the trade-off between equity and and efficiency disappears...

... when agents possess noisy private information about payoffs

Under uncertainty, discrimination is not imperative for efficiency

Study policy design in connection to the problem of equilibrium selection

$\rightarrow$ Global games approach (Carlsson & Van Damme, 1993)

$\rightarrow$ Methodological contribution that drives my no-discrimination result

Literature

Policy in coordination games

Segal (1999, QJE; 2003, JET), Ferraro et al. (2011, AER), Bernstein & Winter (2012, AEJ: Micro), Galeotti et al. (2020, ECTRA), Kets & Sandroni (2021, RES), Lane et al. (2023, AER), Boucher et al. (2024, ECTRA)

Incentives in teams

Winter (2004, AER), Fischer & Huddart (2008, AER), Halac et al. (2020, AER; 2021, AER; 2022, AEA P&P; 2023, AEJ: Micro), Dai & Toikka (2022, ECTRA)

(Policy in) global games

Carlsson & Van Damme (1993, ECTRA), Morris & Shin (1998, AER), Frankel et al. (2003, JET), Angeletos et al. (2006, JPE), Sákovics & Steiner (2012, AER), Edmond (2013, RES), Leister et al. (2022, RES)

Coordination problems in practice

Cowan (1991, EJ), Cowan & Gunby (1996, EJ), Bandiera & Rasul (2006, EJ), Cai et al. (2015, AER), Beaman et al. (2021, AER)

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Game

Building Blocks

A game of complete information $\Gamma(x, s)$ is given by:

- $\rightarrow$ Player set $\mathscr{N} = \{1, 2, ..., N\}$
- $\rightarrow$ Actions $a_i \in \{0, 1\}$, action vectors $a = (a_i)$
- $\rightarrow$ Subsidies s_i, scheme $s = (s_i)$
- $\rightarrow$ Payoff functions (u_i)

Payoffs

Given (a, x, s), the payoff to player i is:

$$u_i(a \mid x, s_i) = \left[x + w_i(\sum a_j) + s_i - c_i\right] \cdot a_i, \quad (1)$$

where

- x is a fundamental state of nature
- c_i is the (opportunity) cost of playing 1
- s_i subsidy to player i for playing 1
  - $\rightarrow$ Equivalent to a (equally sized) tax on playing 0
- w_i describes the externalities players impose upon one another
  - $\rightarrow$ assume $w_i(n)$ is increasing in n (**coordination game**)

Paper covers extensions and generalizations of u_i

Welfare

Let $\hat{u}_i(a \mid x) = u_i(a \mid x, s_i) - a_i \cdot s_i$ be agent i's payoff net of subsidies

Social welfare is determined by an increasing function

$$W(\hat{u}_1(a \mid x), \hat{u}_2(a \mid x), ..., \hat{u}_N(a \mid x))$$

Proposition

There exists a unique $x^* = (x_i^*) \in \mathbb{R}^N$ such that if $(a_i^*(x)) = \arg\max_{a \in A} W(\cdot)$, then $a_i^*(x) = 1$ iff $x \geq x_i^*$. Furthermore, if W is symmetric in its arguments, then $x_i^* = x_j^*$ for any two symmetric agents $i, j \in \mathcal{N}$.

Fundamental Uncertainty

I consider a perturbed information environment in which x is hidden

$\rightarrow$ Fundamental uncertainty about state of nature

$\rightarrow$ Payoff functions (u_i) not observed

Each player i receives a private and noisy signal x_i^ε of x:

$$x_i^\varepsilon = x + \varepsilon \cdot \eta_i$$

Common knowledge that $x \sim g$ on $\mathbb{R}$, $\eta_i \sim f$ on $[-1, 1]$, $\varepsilon > 0$

Gives a **global game** $\Gamma^\varepsilon(s)$ (Carlsson & Van Damme, 1993)

Timing of $\Gamma^{\varepsilon}(s)$

1. The planner publicly announces the subsidies s
2. Nature draws x
3. Each player i receives his signal x_i^{ε}
4. Players simultaneously choose their actions

Concepts

A **strategy** p_i maps signals to probability distributions over actions

→ $p_i(x_i^\varepsilon)$ is the probability that i plays 1 [e.g. invests]

→ $p = (p_i)$ is a vector of strategies

For $c \in \mathbb{R}$, an **increasing strategy** p_i^c prescribes 1 if $x_i^\varepsilon \geq c$, 0 otherwise

→ c called **switching point**

An **equilibrium** p is a fixed point of the best-reply correspondence of $\Gamma^\varepsilon(s)$

A scheme s **implements** p if p is the unique equilibrium of $\Gamma^\varepsilon(s)$

p is **implementable** if there exists a scheme s that implements it

→ ... and **uniquely implementable** if s is unique

Characterizations

Implementable Strategies

Proposition

Let ε be sufficiently small.

(i) *A strategy vector p is implementable iff p is increasing;*

(ii) *If p is implementable, then p is uniquely implementable.*

Implementable Strategies

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Corollary

Let $p^* = (p_i^{x_i^*})$ denote the vector of increasing strategies in which each agent i has switching point x_i^*. There is a unique subsidy scheme s^* that implements p^*. As $\varepsilon \to 0$, the scheme s^* induces the first-best/efficient outcome of the game almost surely.

To Each Their Own (Switching Point): Planner's Problem

Partition $\mathscr{N}$ into $M \leq N$ subsets $\mathscr{N}_1, \mathscr{N}_2, ..., \mathscr{N}_M$

- $\rightarrow$ $n_m = |\mathscr{N}_m|$ is the number of agents in $\mathscr{N}_m$
- $\rightarrow$ $N_m = N_{m-1} + n_{m-1}$, $N_1 := 0$

Pick a vector of critical states $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_M) \in \mathbb{R}^M$, $M \leq N$

- $\rightarrow$ Without loss, $\tilde{x}_1 < \tilde{x}_2 < ... < \tilde{x}_M$
- $\rightarrow$ Assume $\varepsilon < \tilde{x}_{m+1} - \tilde{x}_m$ (else, consider only $\varepsilon \rightarrow 0$)

Let $p^{\tilde{x}} = (p_i^{\tilde{x}_m})$ for all $i \in \mathscr{N}_m$, $m = 1, ..., M$

The planner seeks the unique subsidy scheme $\tilde{s} = (\tilde{s}_i)$ that implements $p^{\tilde{x}}$

Global Subsidies

For all $i \in \mathscr{N}_m$, $m = 1, ..., M$, let

$$s_i^*(\tilde{x}) = c_i - \tilde{x}_m - \sum_{n=0}^{n_m - 1} \frac{w_i(N_m + n)}{n_m}$$

Let $\mathcal{B}_r(y)$ be the open ball with radius r centered at y

Theorem

Let $\tilde{x} \in \mathbb{R}^M$. The following holds:

(i) *For all ε sufficiently small, there exists a unique* **global subsidy scheme** $\tilde{s} = (\tilde{s}_i)$ *that implements $p^{\tilde{x}}$;*

(ii) *For all $r > 0$, there exists $\varepsilon(r)$ such that $\tilde{s} \in \mathcal{B}_r(s^*(\tilde{x}))$ for all $\varepsilon \leq \varepsilon(r)$.*

If g uniform and f symmetric, Theorem holds for all $\varepsilon > 0$.

Discrimination

State-Contingent Implementation

Pick some state $\overline{x} \in \mathbb{R}$

I want to find subsidy schemes that uniquely induce $(1, 1, ..., 1)$ in $\bar{x}$

$\rightarrow$ All agents should, in equilibrium, invest in state $\bar{x}$

Well-studied problem for the case of common knowledge of state $\bar{x}$

$\rightarrow$ Yields equity-efficiency trade-off/discrimination results

I'll explain the case of common knowledge first...

... and then move on to implementation under uncertainty

Common Knowledge: Ranking and Discrimination

Discrimination results build upon ranking policies

A **ranking policy** is a tuple $\langle \phi, s^R(\phi, \bar{x}) \rangle$

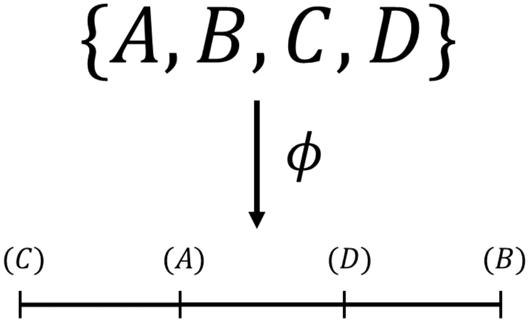
A ranking $\phi(\mathscr{N}) = \{i_1, i_2, ..., i_N\}$ is a permutation of the player set

$s^R(\phi, \bar{x})$ is a subsidy scheme conditional on the ranking $\phi(\mathscr{N})$

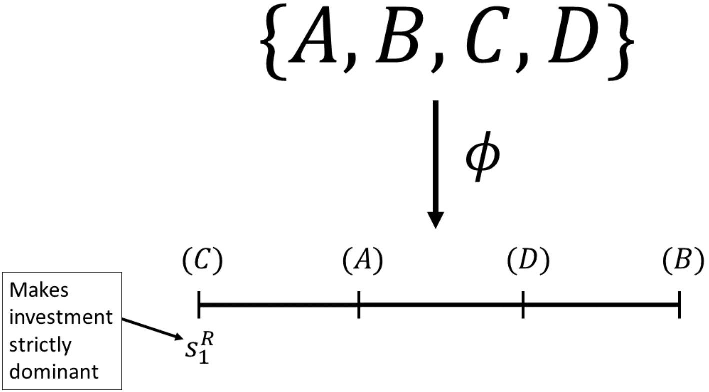
Common Knowledge: Ranking and Discrimination

$$\{A, B, C, D\}$$

Common Knowledge: Ranking and Discrimination



Common Knowledge: Ranking and Discrimination



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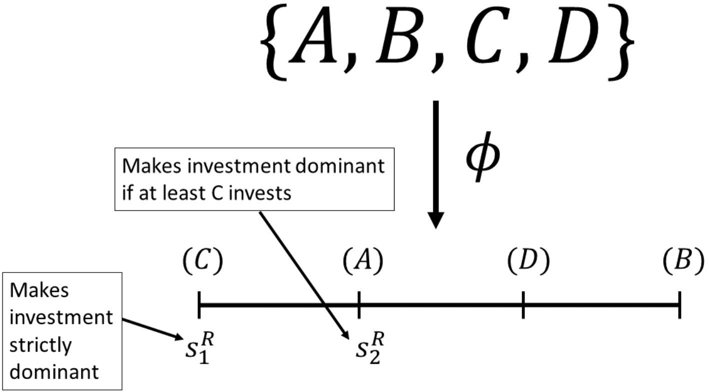

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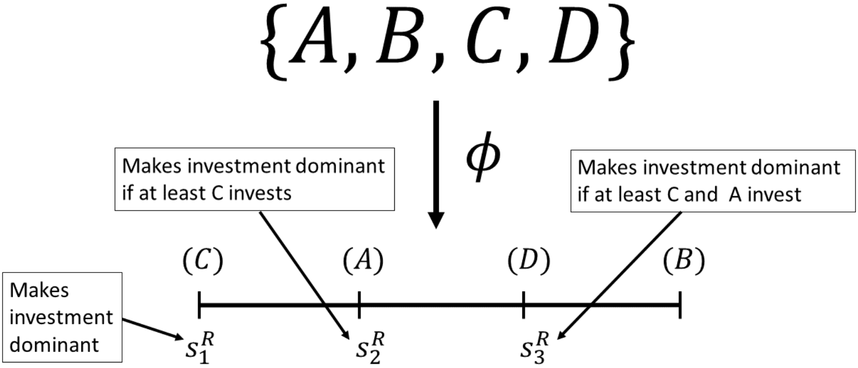

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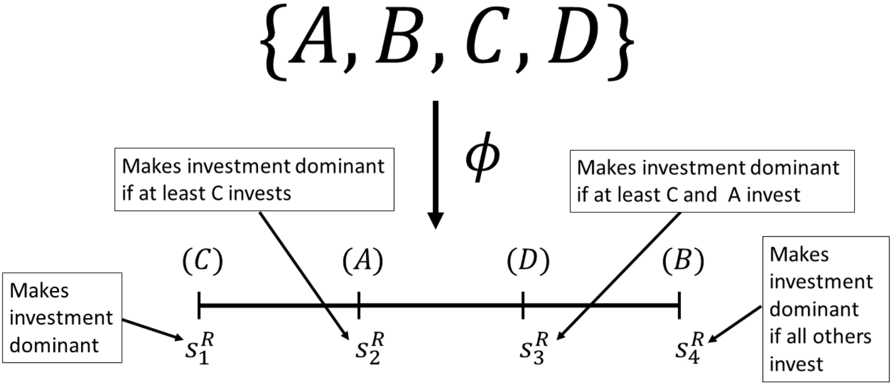

Common Knowledge: Ranking and Discrimination

Canonical result: a ranking policy is strictly optimal in $\Gamma(\bar{x}, \cdot)$

→ Minimizes sum of subsidies that uniquely induce $(1, 1, ..., 1)$

→ Segal (1999, 2003), Winter (2004), Bernstein & Winter (2012), Halac et al. (2020, 2023)

→ N.B. result applies under common knowledge that state is $\bar{x}$

Let $K(s^R(\phi, \bar{x}) \mid \bar{x})$ be spending on subsidies in $\Gamma(\bar{x}, s^R(\phi, \bar{x}))$

→ I.e. $K(s^R(\phi, \bar{x}) \mid \bar{x}) = \sum_{n=1}^{N} s^R_{i_n}(\phi, \bar{x})$

The set of least-cost rankings is

$$\Phi^*(\bar{x}) = \arg\min_{\phi} K(s^R(\phi, \bar{x}) \mid \bar{x})$$

Implementation Under Uncertainty

How to go about state-contingent implementation under uncertain x?

In $\Gamma^{\varepsilon}(s)$, choose policies s such that $x_i(s) \leq \bar{x} - \varepsilon$ for all i

- $\rightarrow$ Equilibrium *outcome* of $\Gamma^{\varepsilon}(s)$ is $(1, 1, .., 1)$ in state $\bar{x}$
- $\rightarrow$ Define $U^{\varepsilon}(\bar{x}) = \{s : x_i(s) \leq \bar{x} - \varepsilon \forall i\}$
- $\rightarrow$ For $\phi \in \Phi^*(\bar{x})$, observe that $s^R(\phi, \bar{x}) \in U^{\varepsilon}(\bar{x})$ as $\varepsilon \rightarrow 0$

For $s \in U^{\varepsilon}(\bar{x})$, define $K^{\varepsilon}(s \mid \bar{x}) = \sum_{s_i \in s} s_i$

- $\rightarrow$ The equilibrium cost of s in state $\bar{x}$ in $\Gamma^{\varepsilon}(s)$

N.B. I evaluate $K^{\varepsilon}(s \mid \bar{x})$ in state $\bar{x}$

- $\rightarrow$ Cost when nature draws Segal's (2003)/Winter's (2004) payoff functions

Convergence

Theorem

Let $\bar{x} \in \mathbb{R}$. Under some mild technical conditions on Φ^*, as $\varepsilon \to 0$, there exists $\bar{s} \in U^\varepsilon(\bar{x})$ such that

(i) *If players $i, j \in \mathscr{N}$ are symmetric, then $\bar{s}_i = \bar{s}_j$;*

Convergence

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(i) *If players $i, j \in \mathscr{N}$ are symmetric, then $\bar{s}_i = \bar{s}_j$;*

(ii) *For all $\phi \in \Phi^*$, $K^\varepsilon(\bar{s} \mid \bar{x}) \to K(s^R(\phi, \bar{x}) \mid \bar{x})$.*

Convergence

Theorem

Let $\bar{x} \in \mathbb{R}$. Under some mild technical conditions on Φ^*, as $\varepsilon \to 0$, there exists $\bar{s} \in U^\varepsilon(\bar{x})$ such that

(i) *If players $i, j \in \mathcal{N}$ are symmetric, then $\bar{s}_i = \bar{s}_j$;*

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Discrimination is not necessary to minimize the cost of policy

- $\rightarrow$ Equity-efficiency trade-off is an artifact of certainty about payoffs...
- $\rightarrow$ ... and the implied inability of players to form strategic beliefs

Ann and Bob

Consider again Ann and Bob from the introduction

- → Cost of investment c
- → Return given project success w, $w > c$
- → Equivalent to $x = 0$: choose $\bar{x} = 0$

Ranking policy

- → $s_1^R = c$ and $s_2^R = c - w (< 0)$
- → Total cost: $2c - w$

Global subsidy

- → Planner wants both to play 1 whenever $x_i^\varepsilon \geq \bar{x} - \varepsilon$, $i \in \{\text{Ann, Bob}\}$
- → Using the first Theorem, this gives $\bar{s}_i \to c - w/2$ as $\varepsilon \to 0$
- → Total cost: $2c - w$

Generalizations

I study several extensions and applications of the model presented today

→ Games of regime change here
  → Morris & Shin (1998), Angeletos et al. (2006, 2007), Sákovics & Steiner (2012), Basak & Zhou (2020), Halac et al. (2020)

→ Incentives in teams here
  → Winter (2004), Halac et al. (2020, 2022, 2023)

→ Heterogeneous externalities/games on networks here
  → Matthew & Yariv (2009), Galeotti et al. (2020), Leister et al. (2022)

→ Continuous action spaces, payoffs linear in own actions here

Closed support of x

Define $\underline{x} := \sup\{x : x + w_i(N-1) + s_i - c_i \leq 0 \forall i\}$

Define $\overline{x} := \inf\{x : x + w_i(0) + s_i - c_i \geq 0 \forall i\}$

We need $\mathcal{X} \supseteq [\underline{x} - \varepsilon, \overline{x} + \varepsilon]$

Back

Imagine, for simplicity, two symmetric players
For high signals $x_i^\varepsilon \geq \overline{x}(s)$, playing 1 is a dominant strategy for each player i

Receiving a signal just below $\overline{x}(s)$, player i knows there is a strictly positive probability that $x_j^\varepsilon \geq \overline{x}_j(s_j)$, in which case j plays 1

Knowing this, player i will play 1 even for some signals below $\overline{x}(s)$ (and same for j) $\rightarrow$ new threshold $\overline{x}^1(s)$

Argument can be repeated. We obtain a sequence $(\overline{x}^k(s))_{k\in\mathbb{N}}$ where $\overline{x}(s) = \overline{x}^0(s) > \overline{x}^1(s) > \overline{x}^2(s) >$ ... The limit of this sequence is $x^*(s)$

Strategy survives iterated elimination of strictly dominated strategies iff it assigns prob. 1 to action 1 whenever $x_i^\varepsilon > x^*(s)$

Back

General strategic complementarities

Proposition: global subsidy makes players indifferent in the critical state given "double uniform strategic beliefs"

1. Uniform belief over number of players n that play 1

2. Given n, uniform belief over all $\binom{N-1}{n}$ vectors a_{-i} in which n players play 1

Back

Continuous action space

Let $a_i \in [0, 1]$

Payoffs are linear in a_i: $\pi_i(a \mid x, s_i) = a_i \cdot [x + w_i(a_{-i}) + s_i] + (1 - a_i) \cdot c_i$

E.g. per-dollar returns on investment

Main theorem applies as given to this case

Back

Joint Investment Problems

Players in $\mathscr{N}$ can invest, or not, in a project

The cost of investment to player i is c_i

If the project succeeds, player i realizes benefit $b_i + x$, $b_i > c_i$

The project succeeds iff at least a critical mass I invests

I unobserved but known to be distributed uniformly on $\{0, 1, ..., N\}$

Canonical model in the applied global games literature (with $x = 0$)

→ Morris & Shin (1998), Angeletos et al. (2006, 2007) Sákovics & Steiner (2012), Basak & Zhou (2020), Halac et al. (2020)

→ Difference: common knowledge about x/private signals about I

Unique Investment Subsidies

Planner offers subsidies $\tilde{s}$ to induce i to invest iff $x_i^\varepsilon > \tilde{x}$

Unique scheme $\tilde{s}$ that solves the planner's problem given by ($\forall N \geq 2$)

$$\tilde{s}_i = c_i - \frac{b_i + \tilde{x}}{2}$$

Literature focuses on models where $x = 0$, suggesting $\tilde{x} \nearrow 0$:

$$\tilde{s}_i \to c_i - b_i/2$$

Offer each player a subsidy less than half ($b_i > c_i$) his investment cost

Cf. Sákovics & Steiner (2012): subsidize subset of players fully ($s_i = c_i$)

Uncertainty about payoffs matters!

Back

Incentives in Team

There is a project and a team of agents

Each agent can work toward completion of the project ($a_i = 1$), or shirk

There is a principal who **does not** observe agents' work decisions

Principal pays reward $v_i + x$ to agent i **conditional on project success**
$\rightarrow$ Common payoff x reflects e.g. profit-sharing

The probability of project success is $q(\sum_i a_i)$, increasing and supermodular

The cost of work to agent i is c_i

Equivalent to Winter (2004) and Halac et al. (2020, 2022, 2023) for $x = 0$

Incentives in Teams

Given $\tilde{x}$, the reward $\tilde{v}_i$ to player i is

$$\tilde{v}_i \to \frac{c_i}{\sum_{n=0}^{N-1}[q(n+1)-q(n)]/N} - \tilde{x}$$

Indifference between working and shirking in the critical state...

... given uniform belief about number of agents that work

$\rightarrow$ Cf. Winter (2004), Halac et al. (2020, 2023)

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