Emissions trading and the interest rate

Roweno J.R.K. Heijmans*

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Abstract

Cap and trade schemes often include measures that adjust allowance supply to stabilize otherwise volatile allowance prices. We investigate whether these measures do indeed stabilize prices in response to a changing interest rate. Informed by the global policy landscape, we focus on supply policies that rely on either the allowance price (price measures) or the surplus of unused allowances (quantity measures) to determine supply in dynamic cap and trade markets. Relative to a situation with exogenously fixed supply, price measures do stabilize allowance prices. Quantity measures, in contrast, exacerbate the sensitivity of allowance prices to the interest rate.

1 Introduction

An increasing number of cap and trade schemes has turned away from the textbook model of a fixed emissions cap in favor of variable allowance supply. Such policies are widely believed to stabilize otherwise erratic allowance prices. But do they? The answer to that question will naturally depend on both the kind of supply policy in place and the source of fluctuations in the allowance price to begin with. In this note, we study two of the most common supply policies and investigate how these affect the sensitivity of allowance prices to the interest rate. We show that the answer to our initial question depends critically on the type of supply policy in place.

At least in theory, there is an infinite number of ways in which allowance supply could respond to market conditions. To keep things informative, we restrict attention to two general classes of supply policies that dominate the policy landscape. The first class is that of price measures, containing all supply policies that relax the emissions cap (i.e. increase allowance supply) when the price of allowances increases. The second class is composed of all quantity measures, which are supply policies that tighten the emissions cap (i.e. reduce the supply of allowances) when the surplus of unused allowances grows.

Practical examples abound of feedback mechanisms that fit the descriptions just given. Price measures are used in California's ETS (Borenstein et al., 2019) and the Regional Greenhouse Gas Initiative (Friesen et al., 2022). The European Union in contrast opted

*Department of Economics, Swedish University of Agricultural Sciences, Box 7013, 750 07 Uppsala, Sweden. Email: roweno.heijmans@slu.se.

for a quantity measure to determine allowance supply in the EU Emissions Trading System (EU ETS) (Perino, 2018; Gerlagh and Heijmans, 2019), and one could argue that Korea ETS's liquidity provisions are a *de facto* quantity measure (Asian Development Bank, 2018).[1] Besides these empirical cases, the literature is replete with papers that propose, or at least discuss the merits of, price and quantity measures. Good examples are Pizer (2002), Fell et al. (2012), Fell (2016), Holt and Shobe (2016), Kollenberg and Taschini (2016), Abrell and Rausch (2017), and Pizer and Prest (2020).

This note takes to task the general proposition that price and quantity measures stabilize allowance prices. Compared to emissions trading without any sort of feedback mechanism, quantity measures make allowance prices strictly more, not less, sensitive to changes in the interest rate. Contrary to their intention, quantity measures destabilize the cap and trade market. Price measures do deliver on their promise of stabilizing allowance prices.

Why does the interest rate have such differential effects across regimes? We suggest the rationale can be found in the way the interest rate affects the development of firms' emissions incentives over time. In general, allowance prices are expected to roughly satisfy Hotelling's rule, that is, grow with the interest rate. All else equal, an increase in the interest rate thus raises the price of allowances in the future compared to their price today. This change in relative prices ignites a redistribution of emissions between periods, pushing abatement efforts toward the future when emissions have become comparatively more costly. To support the increase in emissions today, the initial allowance price has to go down. When supply is governed by a price mechanism, this downward pressure on the allowance price leads to a reduction in allowance supply, which generates additional scarcity and thus offsets part of the initial price drop. Because of this, a price measure stabilizes the allowance price. Contrast that to what happens when supply follows a quantity measure: the increase in emissions today by definition reduces the surplus of unused allowances carried over to future periods. But this, by the mechanics of a quantity measure, leads to an increase in future allowance supply, relaxing firms' abatement obligations and putting additional downward pressure on allowance prices. Rather than stabilize prices, a quantity mechanism pushes them down even further.

A policymaker could fix the destabilizing effect of quantity measures through direct interventions in the market for emissions allowances. Such a solution is imperfect at best, for at least two reasons. First, direct interventions, especially when frequent, can easily undermine trust in and thus destabilize the very market they were meant to stabilize. Second, why have a stabilization mechanism in the first place if it does not by itself do as it is intended to? Indeed, rather that stick with a quantity measure and opt for damage control, the policymaker could instead adopt a price measure and preempt the problems identified here.

[1] A more extensive list of schemes that operate price or quantity measures also includes China's National ETS, Germany's National ETS, New Zealand's ETS, the Massachusetts Limits on Emissions from Electricity Generator, and Switzerland's ETS.

2 Main Analysis

2.1 Model

Consider a cap and trade scheme that regulates emissions by a continuum $[0,1]$ of firms i over the course of two periods $t = 0, 1$. Let a_{it} denote the abatement effort of firm i in period t, and assume that $a_{it} \geq 0$. We define abatement relative to an exogenously given business-as-usual level of emissions, so $a_{it} := \bar{q}_{it} - q_{it}$ where $\bar{q}_{it}$ is the BAU level of emissions chosen by firm i in period t absent any regulation and q_{it} its actual emissions. Abatement costs are given by $C(a_{it})$ and satisfy $\frac{\partial C(a_{it})}{\partial a_{it}} > 0$ and $\frac{\partial^2 C(a_{it})}{\partial a_{it}^2} > 0$. For future reference, we denote $C'_t(q_{it}) := \frac{\partial C(\bar{q}_{it} - q_{it})}{\partial q_{it}}$. Define $q_t := \int_0^1 q_{it} di$. In each period, firms choose their emissions once and simultaneously.[2] To reduce notational clutter, the abatement cost functions C_{it} are assumed to be common knowledge. If desired, one could interpret the function C_{it} as *expected* abatement costs.[3].

The abatement obligation of firm i is determined by the number of allowances it owns. There are, in practice, various ways in which allowances are allocated to the firms, but we shall be agnostic about the precise method used. Let s_{it} allowances be supplied to firms i in period t, and $s_t := \int_0^1 s_{it} di$. Allowances, once supplied, can be traded on a secondary market against a price p_t which the firms, being small, take as given. We let l_{it} denote the number of allowances bought by player i in period t; since every allowance bought must also be sold, we have $\int l_{it} di = 0$.[4] Allowances can also be traded over time in the sense that allowances supplied but not used in period 0 are carried over to period 1 – this is called banking (Rubin, 1996). We write b_i for the amount of banking by firm i in period 0. By construction, we have $b_i = s_{i0} + l_{i0} - q_{i0}$. Total banking in period 1 is $b := \int b_i di$. While it would be realistic to assume a borrowing constraint (e.g. $b \geq 0$), we follow the literature (*c.f.* Rubin, 1996; Pizer and Prest, 2020) and allow both banking and borrowing. This assumption plays no critical role in the derivation of our results yet allows us to simplify notation.

We come to the determination of firms' behavior in the market for allowances shortly. At this point, we note that conditions prevailing in the market for allowances affect supply. To stay close with empirical reality, we limit attention to two general classes of supply policies: price and quantity measures. We define each in turn.

Definition 1 (Price measures)**.** *Under a price measure, the supply of allowances in period 1 is increasing in the allowance price p_0. Letting s_1^P denote period-1 supply under a price measure, we thus have $s_1^P(p'_0) > s_1^P(p_0)$ if and only if $p'_0 > p_0$.*

Price measures were proposed by Roberts and Spence (1976), Pizer (2002), and Abrell and Rausch (2017). Practical examples are price collars (Fell et al., 2012; Borenstein et al.,

[2] In most cap and trade schemes, firms report the number of allowances used to cover their emissions on a given date once every year. This means that firms learn others' emissions once a year only and supports our modeling hypothesis.

[3] For example, let $\tilde{C}_{it}(\cdot \mid \theta_{it})$ denote a firm's true abatement costs, where $\theta_{it} \in \Theta$ is an unobserved parameter, and define $C_{it}(a_{it}) := \int_\Theta \tilde{C}_{it}(a_{it} \mid \theta_{it}) d\theta_{it}$

[4] This is formally a constraint on firms' cost-minimization problem. Because firms are assumed to be small relative to the market, we will for convenience of notation assume this constraint is not binding.

2019). To avoid jumps in price-supply space, we assume that s_1^P is a continuous function. Because s_1^P is monotone and bounded, this implies differentiability almost everywhere.

Definition 2 (Quantity measures)**.** *Under a price measure, the supply of allowances in period 1 is decreasing in the bank of allowances b. Letting s_1^Q denote period-1 supply under a quantity measure, we thus have $s_1^Q(b') > s_1^Q(b)$ if and only if $b' < b$.*

Quantity measures were studied among others by Kollenberg and Taschini (2016), Abrell and Rausch (2017), and Pizer and Prest (2020). Examples in practice are abatement bounds (Holt and Shobe, 2016), a market stability reserve like the EU's (Gerlagh et al., 2021), or Korea ETS' liquidity provisions. We assume that s_1^Q is continuous. We also assume that $\partial s_1^Q(b)/\partial b > -1$ for all b. This ensures that firms cannot increase their emissions in both periods simultaneously.

Although supply is, to an extent, determined in the market for allowances, we assume that individual firms take the supply of allowances as given. This follows naturally from the assumption that firms are price-takers.[5] The assumption that firms operate in a competitive market where prices, and hence supply, are taken as given is common in the literature on supply measures (Fell et al., 2012; Fell, 2016; Kollenberg and Taschini, 2016; Perino and Willner, 2016).

We are interested in the sensitivity of allowance prices to the interest rate when allowance supply is based on price and quantity measures, respectively. We also want to compare these sensitivities to the case of fixed allowance supply. For future reference, let supply in period 1 under a fixed, exogenous cap be denoted s_1^f.

2.2 Equilibrium

Firms minimize the discounted sum of costs, subject to the cap and trade constraints:

$$\begin{aligned} \min_{q_{it}, l_{it}} \quad & \sum_{t=0}^{1} \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it} \right] \\ \text{subject to} \quad & q_{i0} = s_{i0} + l_{i0} - b_i, \\ & q_{i1} = s_{i1} + l_{i1} + b_i, \end{aligned} \tag{1}$$

where $\beta := 1/(1+r)$ is the discount factor and r the interest rate. In (1), the firm takes s_{i1} as given by assumption. Because both banking and borrowing are allowed, the two constraints in (1) admit the same shadow price and may therefore be combined. The Lagrangian for this problem is

$$\mathcal{L}_i = \sum_t \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it} \right] - \lambda \left[s_{i0} + s_{i1} + l_{i0} + l_{i1} - q_{i0} - q_{i1} \right],$$

[5] Recall that each individual firm is small relative to the total population. We do not claim that firms fail to understand how the supply measures work. Rather, we assume that each firm, though aware of the supply measure in place, considers its individual influence on the cap so small as to be negligible.

which has the following standard first-order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_i}{\partial q_{i0}} &= C_{i0}'(q_{i0}) + \lambda = 0, \\
\frac{\partial \mathcal{L}_i}{\partial q_{i1}} &= \beta \cdot C_{i1}'(q_{i1}) + \lambda = 0, \\
\frac{\partial \mathcal{L}_i}{\partial l_{i0}} &= p_0 - \lambda = 0, \\
\frac{\partial \mathcal{L}_i}{\partial l_{i1}} &= \beta \cdot p_1 - \lambda = 0.
\end{aligned}
$$

Cost-minimization dictates that marginal abatement costs equal the allowance price in each period, $C_{it}'(q_{it}) = p_t$. Let $q_{it}(p_t)$ denote the level of q_{it} that solves this condition. We define $q_t(p_t) := \int_0^1 q_{it}(p_t)di$. We similarly define $b(p_0) := s_0 - q_0(p_0)$. Strict convexity of C_{it} implies that q_{it} is unique and decreasing in p_t, so it follows that

$$\frac{\partial q_t(p_t)}{\partial p_t} < 0. \tag{2}$$

As banking and borrowing is allowed, dynamic arbitrage conditions imply that allowance prices should rise with the interest rate,

$$p_1 = (1 + r) \cdot p_0, \tag{3}$$

a condition known as Hotelling's rule. Had we not allowed firms to bank *and* borrow allowances, (3) might be violated if the borrowing constraint were binding, though price movements would still be positively correlated between periods.

Equilibrium is reached when supply and demand are equal; the allowance price adjusts to bring about equilibrium.

When allowance supply is fixed, $s_1 = s_1^f$, the equilibrium price p_t^f is found by solving

$$q_0(p_0^f) + q_1(p_1^f) = s_0 + s_1^f. \tag{4}$$

We define the equilibrium under fixed supply only as a point of reference to compare the equilibrium under price and quantity measures with. Let p_t^* denote the equilibrium allowance price in period t when supply is governed by a price measure. Thus, p_t^* is the solution to

$$q_0\left(p_0^*\right) + q_1\left(p_1^*\right) = s_0 + s_1^P(p_1^*). \tag{5}$$

The equilibrium price when supply is determined through a quantity measure is p_t^{**}, which solves:

$$q_0\left(p_0^{**}\right) + q_1\left(p_1^{**}\right) = s_0 + s_1^Q\left(b\left(p_0^{**}\right)\right). \tag{6}$$

Through (3), conditions (4), (5), and (6) reveal that equilibrium prices depend on the interest rate. We want to know how this dependence is affected by the supply measure in place. In what follows, we will use the equilibrium (4) – (6) to investigates how equilibrium price adjust to changes in the interest rate.

2.3 Comparative statics

We consider first the simple case in which allowance supply is fixed. To determine the effect of fluctuations in the interest rate on the equilibrium in this environment, we totally differentiate both sides of (4) with respect to the interest rate:

$$\frac{\partial q_0(p_0^f)}{\partial p_0^f}\frac{\partial p_0^f}{\partial r} + \frac{\partial q_1(p_1^f)}{\partial p_1^f}\frac{\partial p_1^f}{\partial r} = 0.$$

Using (3), we can rewrite this as:

$$\frac{\partial q_0(p_0^f)}{\partial p_0^f}\frac{\partial p_0^f}{\partial r} + \frac{\partial q_1(p_1^f)}{\partial p_1^f}\left[(1+r)\frac{\partial p_0^f}{\partial r} + p_0^f\right] = 0.$$

Collecting terms,

$$\frac{\partial p_0^f}{\partial r} = -p_0^f \cdot \frac{\frac{\partial q_1(p_1)}{\partial p_1}}{\frac{\partial q_0(p_0)}{\partial p_0} + (1+r)\frac{\partial q_1}{\partial p_1}}. \tag{7}$$

One can perform similar analyses for cap and trade schemes where supply is determined through a feedback mechanism. For the case of a price measure, totally differentiate both sides of the equilibrium condition (5):

$$\frac{\partial q_0(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r} + \frac{\partial q_1(p_1^*)}{\partial p_1^*}\frac{\partial p_1^*}{\partial r} = \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r}.$$

Using (3) again and reshuffling terms, we find:

$$\frac{\partial p_0^*}{\partial r} = -p_0^* \cdot \frac{\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}}. \tag{8}$$

Finally, when supply is governed by a quantity measure we get:

$$\frac{\partial p_0^{**}}{\partial r} = -p_0^{**} \cdot \frac{\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}}}. \tag{9}$$

We are now in a position to state our first result on the effect of the interest rate on allowance prices.

Proposition 1. *The equilibrium allowance prices p_0^f, p_0^*, and p_0^{**} are decreasing in the interest rate r:*

$$\frac{\partial p_0^f}{\partial r} < 0, \qquad \frac{\partial p_0^*}{\partial r} < 0, \qquad \frac{\partial p_0^{**}}{\partial r} < 0. \tag{10}$$

Proof. We know from (2) that $\partial q_t/\partial p_t < 0$. We also know that $s_1^P(p_0)/\partial p_0 > 0$, by definition. Plugging these signs into (7) and (8) yields the result for fixed and price-measure based allowance supply.

For the case a quantity measure, $\partial q_t/\partial p_t < 0$ is still true. Hence, the final claim is correct if and only if the denominator in (9),

$$\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}},$$

is negative. Observe that we have

$$\frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}} = \frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial b(p_0^{**})}{\partial p_0^{**}} = -\frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}},$$

where the final equality follows from $b(p_0) = s_0 - q_0(p_0)$. Now recall the assumption that $\partial s_1^Q(b)/\partial b > -1$, so $1 + \partial s_1^Q(b)/\partial b > 0$ and

$$\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}} = \underbrace{\left[1 + \frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\right]}_{>0}\underbrace{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}}}_{<0} + \underbrace{(1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}_{<0} < 0,$$

as we needed to show. *Q.E.D.*

The economist will recognize in Proposition 1 a fundamental fact of finance: the price of an emissions allowance – like that of any other asset – tends to decrease, all else equal, when the interest rate increases. While this result is well-established for the case of a fixed emissions cap (Rubin, 1996), we extend it to policies in which allowance supply is governed by a price or quantity mechanism.

2.4 (De)stabilizing price stabilization

Although Proposition 1 is informative about the direction in which allowance prices develop when the interest rate changes, one could wonder which measure yields the strongest effect. In other words, Which – if any – supply policy is better able to stabilize allowance prices when changes in monetary policy cause fluctuations in the interest rate?

Proposition 2. *For $\varepsilon > 0$ such that $|p_0^* - p_0^f| < \varepsilon$ and $|p_0^{**} - p_0^f| < \varepsilon$,*

$$\frac{1}{p_t^{**}}\frac{\partial p_t^{**}}{\partial r} > \frac{1}{p_t^f}\frac{\partial p_t^f}{\partial r} > \frac{1}{p_t^*}\frac{\partial p_t^*}{\partial r}, \tag{11}$$

for $t = 0, 1$, provided ε is sufficiently small. That is, allowance prices when supply is determined through a quantity (price) measure respond relatively more (less) strongly to changes in the interest rate than allowance prices when the cap on emissions is exogenously given.

Proof. We establish the series of inequalities for $t = 0$. This, given that (for equilibrium prices p_0 and p_1),

$$\frac{1}{p_1}\frac{\partial p_1}{\partial r} = \frac{1}{(1+r)p_0}\left[p_0 + (1+r)\frac{\partial p_1}{\partial r}\right] = \frac{1}{p_0}\frac{\partial p_0}{\partial r} + \frac{1}{1+r},$$

will imply the result for $t = 1$.

Define $R^*(p_0^*, p_0^f) := \frac{\partial p_t^*}{\partial r} / \frac{\partial p_t^f}{\partial r}$. Using (7) and (8),

$$R^*(p_0^*, p_0^f) = \frac{p_0^*}{p_f^*} \frac{\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_1(p_1^f)}{\partial p_1^f}} \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1}{\partial p_1^f}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}}.$$

We first show that $R^*(p_0^*, p_0^f) < 1$ for all (p_0^*, p_0^f) such that $p_0^* = p_0^f$. In that case, the above simplifies to

$$R^*(p_0^*, p_0^f) = \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1}{\partial p_1^f}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}} < 1,$$

where the inequality is immediate from the fact that $\partial s_1^P(p_0)/\partial p_0 > 0$.

Suppose then that $p_0^* \neq p_0^f$. Two cases can arise. i) $R^*(p_0^*, p_0^f) < 1$ for all (p_0^*, p_0^f). ii) There are (p_0^*, p_0^f) such that $R^*(p_0^*, p_0^f) \geq 1$. For any given p_0^f, let $\delta^+(p_0^f)$ denote the smallest real number such that $R^*(p_0^f + \delta(p_0^f), p_0^f) \geq 1$. Similarly, let $\delta^-(p_0^f)$ denote the smallest real number such that $R^*(p_0^f - \delta^-(p_0^f), p_0^f) \geq 1$. For all p_0^f, define $\delta(p_0^f) := \min\{\delta^+(p_0^f), \delta^-(p_0^f)\}$. Then define $\delta := \min_{p_0^f} \delta(p_0^f)$. By construction, $R^*(p_0^*, p_0^f) < 1$ for all pairs (p_0^*, p_0^f) that satisfy $|p_0^* - p_0^f| < \delta$.

Next, define $R^{**}(p_0^{**}, p_0^f) := \frac{\partial p_t^*}{\partial r} / \frac{\partial p_t^f}{\partial r}$, so

$$R^{**}(p_0^{**}, p_0^f) = \frac{p_0^{**}}{p_0^f} \frac{\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}{\frac{\partial q_1(p_1^f)}{\partial p_1^f}} \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1}{\partial p_1^f}}{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(p_0^{**})}{\partial p_0^{**}}}.$$

If $p_0^{**} = p_0^f$, then $R^{**}(p_0^{**}, p_0^f) > 1$ as $\partial s_1^Q(b(p_0))/\partial p_0 < 0$. Suppose then that $p_0^{**} \neq p_0^f$. Here, too, two cases can arise. i) $R^{**}(p_0^{**}, p_0^f) > 1$ for all (p_0^*, p_0^f). ii) There are (p_0^*, p_0^f) such that $R^{**}(p_0^{**}, p_0^f) \leq 1$. For any given p_0^f, let $\gamma^+(p_0^f)$ denote the smallest real number such that $R^{**}(p_0^f + \gamma^+(p_0^f), p_0^f) \geq 1$. Similarly, let $\gamma^-(p_0^f)$ denote the smallest real number such that $R^{**}(p_0^f - \gamma^-(p_0^f), p_0^f) \geq 1$. For all p_0^f, define $\gamma(p_0^f) := \min\{\gamma^+(p_0^f), \gamma^-(p_0^f)\}$. Then define $\gamma := \min_{p_0^f} \gamma(p_0^f)$. By construction, $R^{**}(p_0^{**}, p_0^f) > 1$ for all pairs (p_0^{**}, p_0^f) that satisfy $|p_0^{**} - p_0^f| < \gamma$. Set $\varepsilon \leq \min\{\delta, \gamma\}$. *Q.E.D.*

Proposition 2 tells us that allowance prices when supply is determined through a quantity measure responds relatively more strongly to monetary policy changes than do allowance price when supply is set through a price measure. Indeed, compared to an exogenously fixed emissions cap, quantity measures exacerbate rather than stabilize allowance price fluctuations! Price measures do properly stabilize allowances prices.

Formally, the Proposition speaks only to the *elasticity* of allowance prices with respect to the interest rate. In some cases, policymakers might be more interested in absolute fluctuations. From the condition that equilibrium prices start out sufficiently close, it is immediate the the series of inequalities in (11) also implies a slighly stronger result.

Corollary 1. *For sufficiently small $\tilde{\varepsilon} > 0$ such that $|p_0^* - p_0^f| < \tilde{\varepsilon}$ and $|p_0^{**} - p_0^f| < \tilde{\varepsilon}$, Proposition 2 generalizes to*

$$\frac{p_t^{**}}{\partial r} > \frac{\partial p_t^f}{\partial r} > \frac{\partial p_t^*}{\partial r}. \tag{12}$$

That is, starting from similar equilibrium prices, a quantity measure destabilizes the equilibrium allowance price while a price measure stabilizes prices, in relative as well as absolute terms.

Proposition 2 describes equilibrium elasticity effects for comparable baseline equilibria. Here, comparability refers to the condition that p_0^* and p_0^{**} are close enough to p_0^f (ε sufficiently small), i.e. we are considering the effect of interest rate fluctuations starting from "similar" equilibrium prices. In the important special case of quadratic abatement costs, the requirement of comparability can be relaxed.

Corollary 2. *If the abatement cost functions C_{it} are quadratic, then Proposition 2 holds true for all ε.*

Quadratic abatement costs here are an important special case in the literature on emissions trading and supply mechanisms (Kollenberg and Taschini, 2016; Pizer and Prest, 2020; Gerlagh et al., 2021). In the typical parametrized model of emissions trading, our results on the stabilizing (destabilizing) effect of price (quantity) measures hold true even when comparing dissimilar equilibrium prices.

3 Discussion and Conclusions

Most of the major cap and trade schemes, and many of the smaller ones, have adopted measures that allow for variable allowance supply. One motivation to institute these measures is to limit allowance price variability. We investigate whether that goal is achieved by the two prominent examples of adaptive supply policies. We find that price-based supply measures stabilize prices indeed. Quantity-based measures instead destabilize prices. The policy lesson is that price measures are better suited to deliver a stable emissions price.

It is often taken for granted that price and quantity measures stabilize allowance prices. There is some evidence to support those beliefs. Borenstein et al. (2019) find that the price collar in California's cap and trade scheme almost certainly stabilizes prices. Similarly, Gerlagh et al. (2020) show that the EU ETS's quantity measure is very likely to explain its success in weathering the economic downturn resulting from the COVID-19 pandemic. Our results indicate that the latter finding is not necessarily universal.

Quantity measures appear to be designed – and defended – with a surprisingly undynamic view of what drives the demand for emissions. As a means to contain price variability resulting from current or historic events, quantity measures have been demonstrated to fare reasonably well (Kollenberg and Taschini, 2016; Fell, 2016; Abrell and Rausch, 2017; Pizer and Prest, 2020; Gerlagh et al., 2020). That said, they perform much less well when fluctuations in demand derive from changing dynamic incentives. While the EU can partially justify the claim that its quantity measure "*improves the system's resilience to major shocks by adjusting the supply of allowances to be auctioned*", it certainly does not insulate the system from *all* major shocks – including changes in the interest rate.[6] This finding deepens the shadow already cast over quantity measures in several recent contributions (*c.f.* Perino and Willner, 2016; Gerlagh and Heijmans, 2019; Gerlagh et al., 2021).

We argue that our analysis has implications beyond the immediate context of changing interest rates. Changes in the interest rate leads to a recalibration of the market equilibrium is that, through Hotelling's rule, it affects the time path of allowance prices. Other events that influence the distribution of emissions over time will also interact with the stabilization mechanisms and impact equilibrium allowance supply. This suggests that any change in firms' dynamic incentives – be it through policy or some other determinant of allowance demand – that increases emissions incentives today, relative to the future, have price effects comparable to those described here.

In the wake of decades high inflation, central banks are bound to alter the course of monetary policy and increase the interest rate. Absent other policy interventions, this paper predicts that the associated correction in allowance prices will be substantially stronger in cap and trade schemes that rely on quantity measures to adjust supply. Perhaps it is time to reform the European Union's cap and trade market and, like many of its U.S. based counterparts, introduce a price measure to weather the turbulent times ahead and support the EU's ambitious climate agenda.

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[6] Quote taken European Commission's web page.

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