Trading Emission Allowances: An endogenous cap always comes with a Green Paradox

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Abstract

We establish a substantial generalization of a result due to Gerlagh et al. (2021). Any spatial or dynamic cap and trade scheme where the aggregate cap responds to the use of allowances necessarily suffers from a Green Paradox; that is, if information on quantities is used to update the cap, there must exists an emissions-reducing policy complementary to the scheme that increases emissions overall.

JEL codes: D59; E61; H23; Q50; Q54; Q58

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1 Introduction

Cap and trade is one of the most prominent examples of environmental policy across the globe. Important pollution problems such as acid rain (caused by high atmospheric atmospheric concentrations of SO_2 and NO_X) or global warming (the result of greenhouse gas emissions like CO_2) are often regulated through a cap and trade policy.

In its most basic form, a cap and trade scheme issues a fixed number of allowances and distributes these to firms covered by the scheme. If a firm wants to emit, it

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must surrender the corresponding number of allowances. By limiting the supply of allowances, the policymaker reduces emissions. A simple quota or other types of direct command and control policy could of course achieve the same reduction; under a cap and trade policy, however, firms are allowed to trade their allowances. Many cap and trade schemes in addition permit the use of allowances issued in one period to meet compliance obligations at another (later) point in time. The underlying idea is that firms know better how much abatement efforts will cost them than the policymaker does. In allowing for the free exchange of allowances both within and across periods, cap and trade therefore achieves a given emission reduction at lowest cost.

Cap and trade stimulates firms to use their private information about, say, abatement costs; the resulting market signals provide indicators of firms' private information (Kwerel, 1977; Dasgupta et al., 1980). If the cap is set to strike an (approximate) balance between the marginal benefits and costs of emissions, this information is relevant to pin down the total amount of emissions an efficient policy should allow for. Thus, a recent literature promotes the idea of cap adjustments in response to firms' emissions decisions (Pizer and Prest, 2020; Heutel, 2020; Heijmans and Gerlagh, 2020; Gerlagh and Heijmans, 2020).

The main result of this paper offers a substantial generalization of Proposition 1 in Gerlagh et al. (2021). We show that any spatial or dynamic cap and trade scheme where the aggregate cap responds to the use of allowances necessarily suffers from a green paradox; that is, if information on quantities is used to update the cap, there always exists an emissions-reducing policy complementary to the scheme that increases emissions overall.[1]

To establish our result in full generality, we present a technical analysis. This presentation notwithstanding, there is a solid economic intuition for our green paradox. Consider the simple example of a two-period dynamic cap and trade scheme, or a two-region linked cap and trade scheme, with an endogenous emissions cap, as presented in the figure below. When the amount of unused allowances in period (region) 1 goes down, from the equilibrium we move to the left along the solid curve. Demand and supply of new allowances in period (region) 2 is increased so much that total emissions go up.[2] This paper establishes this feature of an endogenous cap in a general setting

[1]See also Gerlagh et al. (2021), Jarke-Neuert and Perino (2020), and Perino et al. (2020) for particular examples of this general result.

[2]Those familiar with European climate policy will recognize a similarity to the Emissions Trading Scheme (EU ETS) in these mechanics (Perino, 2018; Gerlagh and Heijmans, 2019).

with multiple linked periods, regions or sectors. It thus provides reason for caution when linking cap-and trade over space, or over time. Adding flexibility can greatly improve efficiency (Karp and Traeger, 2021), but it is essential to use price information for adjusting the cap. Flexibility based on quantity information only will always lead to a green paradox.

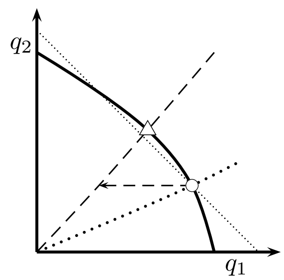

Figure 1: This figure presents allowances in two periods, regions or sectors, with an endogenous cap depicted through the solid curved line. The dotted line presents allocations with constant aggregate allowances; the circle represents an equilibrium. If demand for allowances in period (region/sector) 1 decreases (arrow to left), total emissions increase when prices adjust in the new equilibrium (triangle).

2 Analysis

Consider a cap and trade scheme that regulates emissions in a finite number of periods, regions, or sectors. For notational convenience, here we will use index i as if we consider a dynamic cap-and-trade. We assume the ratio of prices for allowances between the periods, regions, or sectors to be fixed by an exogenous exchange or interest rate. Thus, all price information is captured in a scalar variable p. Let $\boldsymbol{d}(p, \boldsymbol{\lambda})$ denote the demand vector for allowances, which depends on price p and demand shifting policies $\boldsymbol{\lambda}$ normalized so that $\partial \boldsymbol{d}/\partial \boldsymbol{\lambda} = \boldsymbol{u}$, where $\boldsymbol{u} = (1, 1, ..., 1)^T$ is the transposed all-ones-vector with 1 everywhere.[3] We assume that the demand for allowances decreases in prices, $\boldsymbol{d}' \equiv \partial \boldsymbol{d}/\partial p < 0$. Aggregate demand is denoted $D = \boldsymbol{u}^T \boldsymbol{d}$.

We allow for the number of periods (regions, sectors) with strictly positive emissions to be endogenous, say T. We assume that demand has a (period/region/sector-specific) finite choke price. It follows immediately that demand drops to zero when prices rise

[3] That is, the set of (linearly) independent policies is equal to the number of periods.

above the maximum choke price. We abstract from negative emission technologies, meaning that emissions in every period are at least 0.

We study a quantity-based endogenous emissions cap, i.e. a cap and trade scheme where the aggregate supply of allowances S depends on the demand for allowances $\boldsymbol{d}$.

Definition 1 (Quantity-based cap)**.** *With a quantity-based emissions cap, aggregate supply S depends on the demand for emissions:*

$$S = s(\boldsymbol{d}). \tag{1}$$

The market is in equilibrium when excess demand, aggregated over all periods i, equals zero.

Definition 2 (Equilibrium)**.** *In equilibrium, prices adjust so that aggregate demand equals aggregate supply:*

$$D = \boldsymbol{u}^T \boldsymbol{d}(p, \boldsymbol{\lambda}) = s(\boldsymbol{d}(p, \boldsymbol{\lambda})) = S. \tag{2}$$

We denote equilibrium values by a superscript asterisk. It is natural to study the equilibrium through the response of the equilibrium condition with respect to prices p^*, but for the present analysis it is more useful to take one step back and consider the response of the equilibrium condition with respect to the demand vector $\boldsymbol{d}$. By construction, the equilibrium in demand space $\boldsymbol{d}$ is characterized by $\boldsymbol{u}^T \boldsymbol{d} - s(\boldsymbol{d}) = 0$. The gradient of the equilibrium demand space (in short: demand space gradient) is therefore $\boldsymbol{u} - \boldsymbol{s}'$. We say that a cap is exogenous, or fixed, if $\boldsymbol{s'} = 0$, or more generally, if supply is proportional (but not equal) to the all-ones-vector, $\boldsymbol{s'} \propto \boldsymbol{u}$.[4] When the cap is exogenous, aggregate demand D is also fixed in equilibrium. A cap is endogenous if it is not exogenous.

By $\Delta \boldsymbol{d} \geq 0$ we denote the event that demand changes are at least zero in all periods and strictly positive in at least one period. We consider non-trivial cap systems that do not allow such a 'free lunch'.

Assumption 1 (No free lunch)**.** *There does not exist a non-zero increase (or decrease) in emissions $\Delta \boldsymbol{d} \geq 0$ that is feasible, that is, for which $(\boldsymbol{u} - \boldsymbol{s}')^T \Delta \boldsymbol{d} = 0$ holds.*

[4]To see this, consider the case $S = S_0 + \beta \boldsymbol{u}^T \boldsymbol{d}$, so that $\boldsymbol{s'} = \beta \boldsymbol{u}$. This is equivalent to fixed aggregate supply $D = S_0/(1-\beta)$.

Observe that Assumption 1 can be restated as a gradient condition: $\boldsymbol{u} - \boldsymbol{s}' > 0$. We are interested in the effects of policy-induced demand changes $\boldsymbol{\lambda}$.

Definition 3 (Policy-induced price and demand change)**.** *We define changes in the equilibrium price due to demand shifting policies in period i, $d\boldsymbol{\lambda}_i$:*

$$\boldsymbol{\alpha}_i = dp^*/d\boldsymbol{\lambda}_i. \tag{3}$$

Similarly, we define changes in the equilibrium demand due to demand shifting policies:

$$\boldsymbol{\gamma}^i = d\boldsymbol{d}^*/d\boldsymbol{\lambda}_i, \tag{4}$$

wiih $\boldsymbol{\gamma}^i_s$ denoting the demand change in period s from a policy-induced demand change in period i.

As a notational convention, we let $\boldsymbol{x}_i$ (subscript i) denote the i^{th} element of a vector $\boldsymbol{x}$, so a scalar, while $\boldsymbol{x}^i$ (superscript i) denotes a vector of a matrix $\boldsymbol{x}$. Hence, $\boldsymbol{x}^i_j$ denotes element j of the vector $\boldsymbol{x}^i$.

We refer to the matrix of all policy-induced changes as Γ so that with a slight abuse of notation we can write $\Gamma \boldsymbol{e}^i \equiv \boldsymbol{\gamma}^i$, where $\boldsymbol{e}^i$ is the unit vector with zeros everywhere but 1 at the i^{th} place. Taking the derivative of (2) with respect to $\boldsymbol{\lambda}_i$ immediately yields the following lemma:

Lemma 1. *Any policy-induced demand change is orthogonal to the demand space gradient for all i:*

$$(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{\gamma}^i = 0. \tag{5}$$

An immediate implication of Lemma 1 is that any linear combination of the set $\{\boldsymbol{\gamma}^i\}_i$, that is, any combination of demand policies, will also satisfy the orthogonality property, or $(\boldsymbol{u} - \boldsymbol{s}')^T \Gamma = \boldsymbol{0}$.

We now show that the equilibrium satisfies intuitive conditions on price and demand responses to policy shifts.

Lemma 2. *Prices increase with demand-increasing policies,*

$$\boldsymbol{\alpha} = -\frac{(\boldsymbol{u} - \boldsymbol{s}')^T}{(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{d}'} > 0, \tag{6}$$

and own-period demand increases, while other-period demand decreases:

$$\boldsymbol{\gamma}_i^i > 0, \tag{7}$$

$$\boldsymbol{\gamma}_j^i < 0 \text{ for } j \neq i. \tag{8}$$

Proof. We write the full derivatives for the equilibrium equation (2):

$$-(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{d}' dp^* = (\boldsymbol{u} - \boldsymbol{s}')^T d\boldsymbol{\lambda} \tag{9}$$

We note that the term $-(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{d}'$ on the LHS is a positive scalar (due to Assumption 1 and $\boldsymbol{d}' < 0$); hence (6) follows. The positive price effect implies a negative demand response in all jurisdictions $j \neq i$, and because no free lunch is possible, there must be a positive demand response in the own jurisdiction i. □

Our aim is to establish that for any system with a quantity-based endogenous cap, there exists a policy that induces a green paradox. Loosely speaking, a green paradox occurs when a policy-induced reduction in demand in some period i, $d\boldsymbol{\lambda}_i < 0$ causes an increase in aggregate emissions, $dD > 0$.

Definition 4 (Green Paradox)**.** *There is a green paradox if a demand-decreasing policy, $d\boldsymbol{\lambda} < 0$, leads to increasing aggregate emissions, $dD = d(\boldsymbol{u}^T \boldsymbol{d}^*) > 0$.*

Theorem 1. *For every quantity-based endogenuous cap system without a free lunch, there exists a policy $d\boldsymbol{\lambda} < 0$ that induces a green paradox, $d(\boldsymbol{u}^T \boldsymbol{d}^*) > 0$.*

Proof. We prove the result by contradiction. Assume there is no green paradox. We show that we can then construct a demand-reducing policy $d\boldsymbol{\lambda} < 0$ such that demand decreases in all periods $d\boldsymbol{d} < 0$, and then we show that such an outcome violates the 'no free lunch' condition.

If there is no green paradox, then specifically all policies that reduce demand in some jurisdiction i decrease aggregate demand; the matrix Γ is diagonally dominant over columns: $\forall i : \boldsymbol{u}^T \boldsymbol{\gamma}^i \geq 0$.

Define normalized policies and responses. We write $\boldsymbol{\kappa}^i = d\boldsymbol{\lambda}^i / \boldsymbol{\gamma}_i^i < 0$, and $\boldsymbol{\eta}^i = \boldsymbol{\gamma}^i / \boldsymbol{\gamma}_i^i$, for the policy in jurisdiction i and the vector demand response over all jurisdictions, respectively, such that if $\boldsymbol{\kappa}^i = -\boldsymbol{e}^i$, the policy reduces demand by one unit in jurisdiction i. Let H be the matrix of normalized responses, $(H\boldsymbol{e}^i)_j = \boldsymbol{\eta}_j^i$. The matrix H is also diagonally dominant over its columns (inherited property from Γ), with unity elements on the diagonal and negative numbers everywhere else. In this notation, the effect of a policy vector $d\boldsymbol{\lambda} < 0$ on demand can be described through $d\boldsymbol{d} = H\boldsymbol{\kappa}$. Let A be chosen such that any policy directly reducing demand in jurisdiction i by one unit will reduce aggregate demand by at least A units: $A = \min_i\{\boldsymbol{u}^T H\boldsymbol{e}^i\}$. That is, A is the lower bound for the cumulative effectiveness of a policy in any jurisdiction i. Absence of a green paradox implies $A > 0$. We now have all notation in place.

We construct a series of vectors $\boldsymbol{z}^k$, with $k = 1, ..., \infty$, recursively, so that the series converges to $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$, and $H\boldsymbol{\kappa} < 0$.

We start for $k = 1$ with $\boldsymbol{z}^1 = -\boldsymbol{e}^1$. That is, the policy $\boldsymbol{z}^1$ decreases demand in the first jurisdiction by one unit, $(H\boldsymbol{z}^1)_1 = -1$, increasing demand in all other periods, $\forall j \neq 1 : (H\boldsymbol{z}^1)_j > 0$, but aggregate demand is decreased, $\boldsymbol{u}^T H\boldsymbol{z}^1 < -A < 0$. This property also implies that the sum of all positive elements is bound from above: $\sum_i \max\{0, (H\boldsymbol{z}^1)_i\} < (1 - A)$.

We now describe the inductive step. We assume that in step k, we have (i) $\boldsymbol{u}^T H\boldsymbol{z}^k < 0$, and (ii) the sum of all positive elements is bound from above by $\sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1-A)^k$. We can then construct the next element in the sequence, making sure that the properties (i) $\boldsymbol{u}^T H\boldsymbol{z}^{k+1} < 0$, and (ii) $\sum_i \max\{0, (H\boldsymbol{z}^{k+1})_i\} < (1-A)^{k+1}$ are transferred to the next inductive step.

In the inductive step, consider all positive elements of $\boldsymbol{u}^T H\boldsymbol{z}^k$, that we want to neutralize. Thus, let $\boldsymbol{z}^{k+1}$ be defined by $(\boldsymbol{z}^{k+1} - \boldsymbol{z}^k)_i = -\max\{0, (H\boldsymbol{z}^k)_i\} < 0$. The required properties follow immediately from this construction:

$$\boldsymbol{u}^T H\boldsymbol{z}^{k+1} = \boldsymbol{u}^T H\boldsymbol{z}^k + \boldsymbol{u}^T H(\boldsymbol{z}^{k+1} - \boldsymbol{z}^k) < 0 \tag{10}$$

$$\sum_i \max\{0, (H\boldsymbol{z}^{k+1})_i\} < (1 - A) \sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1 - A)^{k+1} \tag{11}$$

Finally, we show that $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$ is well defined, because from the construction, we see that we have a Cauchy sequence:

$$\boldsymbol{u}^T(\boldsymbol{z}^{k+1} - \boldsymbol{z}^k) = \sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1 - A)^k \tag{12}$$

Any Cauchy sequence defined on a compact set converges to a point in the set. Thus, we want to establish compactness. To this end, recall that in any step κ, the sum of all positive demand-changes, i.e. the increase in aggregate in each step κ, was bound from above by $(1 - A)^\kappa$. The aggregate increase in demand is therefore never larger than

$\sum_{\kappa=1}^{\infty}(1-A)^{\kappa} = 1/A < \infty$, where the last inequality follows from the fact that $A > 0$. But this means the series of vectors $\boldsymbol{z}^k$ is defined on a closed and bounded set. By the Heine-Borel Theorem, a closed and bounded set is compact.

Thus, we have $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$, and $H\boldsymbol{\kappa} < 0$. We can rewrite before normalization that we have constructed a strict demand-reducing policy $d\boldsymbol{\lambda} < 0$ and associated strict negative emissions response in all jurisdictions, $\boldsymbol{\gamma} = \Gamma d\boldsymbol{\lambda} < 0$. But combine this with the 'no free lunch' assumption, $(\boldsymbol{u} - \boldsymbol{s}')^T > 0$, and we conclude $(\boldsymbol{u} - \boldsymbol{s}')^T\boldsymbol{\gamma} < 0$, which contradicts (5). □

3 Discussion and Conclusions

We have shown that any emissions trading scheme in which the cap on emissions is updated in response to information on quantities necessarily suffers from a green paradox. In particular, it is always possible to implement a complementary climate policy aimed at reducing emissions that causes an increase in emissions overall through the endogenous emissions cap. This is a substantial generalization of recent results due to Gerlagh et al. (2021) and Osorio et al. (2021).

Perhaps it is useful to emphasize what this paper does *not* say. We do not argue that complementary emissions policies necessarily harm the environment. Rather, we prove that certain policies aimed at reducing emissions may harm the environment in some cases. Neither do we claim that complementary climate policies can never be combined with a cap and trade scheme. Our result only establishes that certain complementary climate policies increase emissions when complementing a cap and trade scheme which uses quantity information to update its cap. Relatedly, this paper does not say that an endogenous emissions cap is always a bad idea. Our Theorem specifically pertains to the use of quantity information to endogenize the emissions cap. If the policymaker were to use prices instead, our results cease to hold.

That final remark brings us to the policy implications of our work. There is no obvious advantage of using quantity information over allowance prices to update an emissions cap; however, this paper demonstrates that there are clear disadvantages. Unless a cap and trade scheme operates in absolute isolation from other climate policies – a claim hard to defend – it is a risky endeavor to combine emissions trading with complementary emissions policies. Given the enormous complexity of the climate problem, however, a combination of multiple and different policies is almost certainly called for. All in all, these observations would strongly favor the use of price signals over quantity signals to update an emissions cap.

Our policy implications also have policy relevance. Cap and trade schemes to regulate greenhouse gas emissions are used in most industrialized economies across the globe. Some of these schemes, including the Regional Greenhouse Gas Initiative (RGGI), California's ETS, and the ETS in Quebec, rely on allowance prices to update the cap on emissions. Our somewhat worrying result does not speak to those cap and trade schemes. That said, other – and important – cap and trade schemes do currently use quantity information to endogenize their caps. Examples include the European Union's Emissions trading scheme (the world's largest market for carbon), Switzerland's ETS, and South Korea's ETS.[5] The key policy takeaway of this paper is that complementary climate policies may be hard to combine with these cap and trade schemes. It is up to policymakers to formulate an appropriate policy-response to our findings. Perhaps the simplest possible strategy, though, is to abandon quantity-based cap adjustment and start using price signals instead. The examples in RGGI, California, and Quebec prove that to be possible.

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[5]South Korea does not formally use quantity information to update its cap, although it has historically done so according to the Asian Development Bank (2018).

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