The Global Climate Game*
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Abstract

I study the adoption of renewable technologies to mitigate climate change in a dynamic public good coordination game. Because dynamic coordination games typically have multiple equilibria, I use a global games approach to inform equilibrium selection. My first contribution is to show that the celebrated result on equilibrium selection in one-shot games generalizes to dynamic global games with both static and dynamic strategic complementarities, which have not been studied before. The unique equilibrium is determined by fundamentals of the renewable technology, not sunspots, and technological lock-in can occur. Moreover, coordination on fossil fuels may be rational even when that is inefficient and all agents would prefer to coordinate on renewables instead. Motivated by the latter observation, my second contribution is to design a policy of network subsidies. A network subsidy allows the policymaker to correct the externality deriving from technological spillovers (and all externalities if use of the clean technology is an equilibrium of the underlying coordination game) but does not, in equilibrium, cost anything To protect the policymaker against off-equilibrium behavior, I also design a network tax-subsidy scheme that is entirely self-financed, implying zero net spending on network subsidies whatever the choices made.

1 Introduction

Public good coordination games are strategic decision problems in which agents privately decide whether or not to contribute to a public good and in which the individual benefit of contributing is increasing in aggregate contributions. Because dynamic coordination games frequently have multiple equilibria, and because the private provision of public goods tends to be inefficient, two important questions arise. First, which – if any – of the equilibria in a

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dynamic public good coordination game should rational agents be expected to coordinate on, and what does that say about the real world? Second, what policies guarantee coordination on an efficient equilibrium at zero cost? This paper answers both of these questions.

While dynamic public good coordination games arise in a score on contexts, to fix ideas this paper uses the language of climate change and the choice between fossil fuel and renewable technologies. Climate change and renewable technology adoption are a prominent example of public good games with dynamic strategic complementarities. Using renewables reduces emissions and this is a public good because pollution harms the environment for everyone. On top of that, greater adoption of renewables increases the benefit to using renewables both today and in the future through a variety of possible channels: firms might target more R&D into renewable development, society moves further up along the learning curve, networks that benefit the use of renewables expand, and so on.[1] These dynamic coordination incentives complicate the decision problem compared to either static coordination games or dynamic games without dynamic strategic complementarities. Dynamic coordination incentives cut to the core of equilibrium multiplicity in models of the transition from fossil to renewables in which the equilibrium that arises is determined by essentially random expectations or sun spots (Acemoglu et al., 2012; Van der Meijden and Smulders, 2017).

To address equilibrium multiplicity, I study renewable technology adoption in a global game with static and dynamic strategic complementarities (Bulow et al., 1985), that is, where agents have an incentive to coordinate their actions instantaneously as well as over time. A global game is an incomplete information game in which agents do not observe the actual game they are playing but only some private noisy signal of it. While the literature on global games has come a long way since the pioneering contribution of Carlsson and Van Damme (1993), global games with both static and dynamic strategic complementarities have remained surprisingly unstudied.[2] Although the absence of dynamic coordination incentives can provide a reasonable approximation for specific strategic environments (see Angeletos et al., 2007, for examples), in many situations dynamic complementarities are a core feature of the decision problem at hand and hence should not be assumed away.

My first contribution takes to task the doctrine that dynamic and static strategic complementarities lead to equilibrium multiplicity in dynamic coordination games. By introducing a little bit of idiosyncratic noise into agents' knowledge of their payoff functions, turning the game into a dynamic global game, I show that a unique equilibrium gets selected. This is a twofold contribution. On an applied front, it establishes that the equilibrium to arise in dynamic games of renewable technology adoption is not random but dictated by fundamentals of the game – there are no sunspots. On a theoretical front, it extends the celebrated result on equilibrium uniqueness in one-shot global games (Carlsson and Van Damme, 1993; Frankel

[1]The channels mentioned here are purely technological and feature prominently in the literature on renewables adoption (Fischer and Newell, 2008; Acemoglu et al., 2012; Aghion and Jaravel, 2015; Aghion et al., 2016; Harstad, 2016; Greaker and Midttømme, 2016; Li et al., 2017; Hart, 2019; Harstad, 2020). There also exist other factors which may (indirectly) generate strategic complementarities. These include the existence of climate tipping points (Barrett and Dannenberg, 2017), political economy arguments such as climate clubs (Nordhaus, 2015, 2021), or even behavioral economic mechanisms like social norms (Andreoni, 1990; Allcott, 2011) and reciprocity (Nyborg, 2018).

[2]Heidhues and Melissas (2006) study a kind of global game with dynamic strategic complementarities in which the decision makers choose when to make an irreversible investment. Their setup is very different from mine, in which decision makers make decisions repeatedly.

et al., 2003) to dynamic global games with intertemporal strategic complementarities.

The literature on renewable technologies and climate change has sought to address the coordination problem and equilibrium selection in a variety of ways, neither of which is entirely satisfying from a game theoretic point of view. Some approaches admit to the existence of multiple equilibria but dot not explicitly seek to select one (Acemoglu et al., 2012; Van der Meijden and Smulders, 2017). Others pay exclusive attention to the Pareto dominant equilibrium (Barrett, 2006; Hoel and de Zeeuw, 2010) or a priori restrict the set of admissible strategies to prove equilibrium uniqueness within that set (Harstad, 2012; Harstad et al., 2019). Yet another branch of the literature treats the coordination problem as theoretically indecisive and relies on laboratory experiments to make predictions (Barrett and Dannenberg, 2012, 2014, 2017). I extend these approaches in using the methodology of global games to show how uncertainty about renewables drives to equilibrium selection without the need for additional restrictions or assumptions on agents' choices.

Uncertainty about fundamentals is necessary to catalyze equilibrium selection.[3] This assumption seems innocent and hardly debatable. There are many uncertainties surrounding climate change and individual beliefs vary vastly (Hornsey et al., 2016). Within my model, one is free to interpret this uncertainty in different ways. Uncertainty could pertain to the true severity of climate change (Weitzman, 2014; Cai and Lontzek, 2019), the location of a dangerous tipping point (Lemoine and Traeger, 2014; Diaz and Keller, 2016), or the true potential of renewables. Although many authors have studied the role of incomplete information in emissions policy (Kolstad, 2007; Barrett and Dannenberg, 2012; Martimort and Sand-Zantman, 2016), my focus on private but correlated noisy signals is new to this literature. My paper therefore extends existing models of climate policy under incomplete information in a novel direction.

There are positives as well as negatives to equilibrium uniqueness in the global game. On the plus side, rational agents manage to coordinate on a unique equilibrium; this eliminates strategic uncertainty and preempts scenarios in which some agents coordinate on one equilibrium while others coordinate on another. Moreover, the equilibrium squares well with economic intuition: better fundamentals of the renewable technology make it more likely that renewables are adopted and technological choices are partly path-dependent so technological lock-in is possible. On the downside, the unique equilibrium is inefficient. More precisely, the global game approach will in certain cases select the Pareto dominated of these equilibria. This leads to a paradox that appears to describe a dismal reality: society might coordinate on using fossil fuels even though everyone would be better off were they to coordinate on renewables instead, and even though all know it with near certainty.

My second contribution is to design a policy instrument that solves equilibrium inefficiency in a novel way. I formulate a policy scheme called network subsidies that corrects the entire externality deriving from (static and dynamic) technological spillovers but does not, in equilibrium, cost the policymaker anything. The innovation here is to make the amount of network subsidy a agent receives contingent not only on their own action but also on the

[3]Game theorists have also studied other approaches toward equilibrium selection also exist. One well-known example is Poisson games (Matsui and Matsuyama, 1995; Myerson, 1998; Makris, 2008) in which agents are uncertain about the number of other agents playing the game. Poisson games have a unique equilibrium as the uncertainty vanishes. Another popular approach derives equilibrium selection as a dynamic outcome of a process of evolutionary selection (Kandori et al., 1993).

actions pursued by all others. As I show, a policymaker can exploit this additional degree of freedom to offer a subsidy scheme that solves the coordination problem without requiring any payments to be made. My result relies on the strategic complementarities in agents' actions and is independent of the application to clean technologies, suggesting that network subsidies contribute to public economics more broadly.

Subsidies to stimulate the development and use of renewable technologies are much discussed by economists and relied upon by policymakers (Joskow, 2011; Murray et al., 2014; Allcott et al., 2015; Fowlie et al., 2015; Acemoglu et al., 2016; Borenstein, 2017; Li et al., 2017; De Groote and Verboven, 2019; Hart, 2019; Harstad, 2020). This popularity notwithstanding, subsidy schemes have an obvious budgetary disadvantage. A policy of network subsidies does away with that problem: though a true subsidy that rewards users of the clean technology with a payment, equilibrium payments are always zero. Network subsidies thus have a clear edge over more traditional subsidy policies.

In a way, the network subsidy functions like an insurance policy. It insures users of renewables against the event they would enjoy little technological spillovers because other agents decided to use fossil fuels – it offers protection against defection. Due to this insurance effect, using renewables becomes a safe option to the agents as their relative payoff from using renewables, rather than fossil fuels, is not affected by what others are doing. This motivates each agent to adopt the clean technology as soon as global coordination on renewables is efficient. The event against which the network subsidy insures agents is therefore not realized and payments cannot be claimed.

It may seem surprising that a network subsidy payment of zero suffices to solve the coordination problem. It is important to note, however, that the amount of network subsidies paid is nil only in equilibrium. If less than full coordination on the clean technology occurs, those who use it are entitled to a strictly positive subsidy payment with an intuitive interpretation. For any configuration of actions, the amount of network subsidy a user of the clean technology receives equals the difference between the payoff they actually realized (not counting subsidies) and the payoff they would have realized in the hypothetical case where full coordination on the clean technology is achieved.

In a way, the derivation network subsidies is an exercise in mechanism design or implementation theory. The policymaker aims to design a subsidy that makes coordination on the efficient outcome of the game a strictly dominant strategy for all agents (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983). While mechanism design has been applied to emissions mitigation before (Duggan and Roberts, 2002; Ambec and Ehlers, 2016; Martimort and Sand-Zantman, 2016), the focus has been on policies to solve the free-rider problem. My work develops this approach by designing policies that solve the coordination problem.

Network subsidies are also related to the literature on directed technical change and the environment (Acemoglu et al., 2012; Aghion and Jaravel, 2015; Aghion et al., 2016; Acemoglu et al., 2016; Hart, 2019). This literature studies the effect of policy on technology adoption when multiple and (partially) substituable technologies co-exist with differental consequences for social welfare, the environment, and growth. Technologies are typically characterized as either clean or dirty and assumed to exhibit technology-specific positive spillovers, with the dirty technology starting off as more advanced. This literature asks how different kinds of policies – e.g. a carbon tax or R&D subsidies – can be used most efficiently to stimulate large-scale adoption of the clean technology. My contribution to this literature is to show how R&D subsidies, aimed at correcting the externality that derived from spillovers and other strategic comeplementarities in renewable investment, can be made substantially cheaper using network subsidies.

2 A Dynamic Coordination Problem

Let there be $T < \infty$ periods and a set of agents $I = \{1, 2, ..., N\}$, where $N \geq 2$. In each period t, agent $i \in I$ must choose an action $x_{it} \in \{0, 1\}$. Here, $x_{it} = 1$ means that agent i uses the renewable technology in period t (and $x_{it} = 0$ indicates that they use fossil fuels instead). Let $x_t = (x_{it})$ denote the vector of actions chosen by all agents in period t; $x_{-it} = x_t \setminus \{x_{it}\}$. By $\mathbf{1}_t = (1, 1, ..., 1)$ I denote the particular vector of period t actions in which $x_{it} = 1$ for all $i \in I$; $\mathbf{1}_{-it}$ the vector $x_{-it} = (1, 1, ..., 1)$ such that $x_{jt} = 1$ for all $j \neq i$. In similar ways, I define $\mathbf{0}_t$ and $\mathbf{0}_{-it}$ as vectors of all zeroes. Define $X_t := \sum_{i \in I} x_{it}$ and $X_{-it} := \sum_{j \in I \setminus \{i\}} x_{jt}$. At the start of period t, let $h_t = (x_\tau)_{\tau=1}^{t-1}$ be the *history* of play; $h_1 = \emptyset$. Define $H_t = \sum_{\tau=1}^{t} \sum_{i \in I} x_{i\tau}$ to be the norm of h_t.

Given a history of play h_t, the vector of actions chosen by all other agents x_{-it}, and their own action x_{it}, the payoff to agent i in period t is:

$$u_{it}(x_{it}, x_{-it} \mid h_t, b) = a \cdot (X_t + H_t) + (b_t + f(X_{-it}, H_t)) \cdot x_{it} - c \cdot x_{it}, \tag{1}$$

where $a \geq 0$, b_t, and $c \geq 0$ are parameters and f is an increasing function in both of its arguments.[4] I assume that there exist finite lower and upper bounds $\underline{f}$ and $\overline{f}$ such that $-\infty < \underline{f} \leq f(X_t, H_t) \leq \overline{f} < \infty$ for all H_t, X_t, and t; thus, while network effects and learning-by-doing may be strong, they are bounded. The parameter b_t is singled out as an argument of the payoff function because it is assumed to be unknown in the dynamic global game. It should be clear that the choice of b_t as key parameter is not restrictive; I could also have chosen a or c and all of my results would, mutatis mutandis, carry on. The number of periods T, the agent set I, the action set $\{0, 1\}$, the payoff functions u_{it}, and the vector of parameters $b = (b_t)$ jointly define a game of complete information $G(b)$.

When a is strictly positive, using renewables is a public good.[5] I iterpret the public good as a reduction in emissions, which is good for the environment and thus benefits everyone. The constant cost c reflects the cost of using renewables, rather than fossil fuels, in a period.

The game $G(b)$ is a game with static and dynamic strategic complementarities because f is increasing in the number of agents that use renewables in a given period an all earlier periods. The implication is that the payoff from using renewables, rather than fossil fuels, is increasing in the number of past and present renewable technology users; this is the definition of static and dynamic strategic complementarities (Bulow et al., 1985).

The parameter b_t represents a fundamental of the renewable technology or the economy. At the start of period 1, Nature draws b_1 from the Normal distribution $b_1 \sim \mathcal{N}(\beta, \sigma_\beta^2)$. At the start of each period $t >$, b_t is given by:

$$b_t = b_{t-1} + \nu_t, \tag{2}$$

[4] Formally, the assumption is that i) for all H_T, X_T, and X'_t it holds that $f(X'_t, H_t) > f(X_t, H_t)$ if and only if $X'_t > X_t$; and ii) for all X_T, H_T, and H'_t it holds that $f(X_t, H'_t) > f(X_t, H_t)$ if and only if $H'_t > H_t$.

[5] When $a = 0$, renewables are a club good.

where ν_t is an i.i.d. (over time) draw from $\mathcal{N}(\nu, \sigma_\nu)$. While it is possible to let ν assume essentially any finite value, I take $\nu = 0$ to reduce notation. Hence, b_t follows a random walk. All distributions are common knowledge. In a game of complete information $G(b)$, agents observe the true b_t drawn by Nature at the start of period t before choosing their actions. for $\tau > t$, let $\Phi(b_\tau \mid b_t)$ denote the probability function of b_τ conditional on b_t.

In $G(b)$, agent i (all $i \in I$) chooses x_{it} maximize the expected discounted sum of payoffs,

$$U_{it}(x \mid h_t, b_t) = \sum_{\tau=t}^{T} \int \beta^{\tau-t} u_{i\tau}(x_{i\tau}, x_{-i\tau} \mid h_\tau, b_\tau) d\Phi(b_\tau \mid b_t), \tag{3}$$

where $\beta \in (0,1)$ is the discount factor. Let $x_{it}(h_t, b_t)$ denote a strategy for agent i in period t; that is, $x_{it} : [0, Nt] \times \mathbb{R} \to [0,1]$ is a function that dictates for any history h_t and fundamental b_t a probability $x_{it}(h_t, b_t)$ with which agent i chooses $x_{it} = 1$. By $x_t = (x_{it})_{i \in I}$ I denote a vector of strategies for all agents in period t; $x() = (x_t())$ is a vector of strategies for all agents in all periods. A strategy vector $\hat{x}$ is a perfect Bayesian Nash equilibrium (PBE) of $G(b)$ if for any possible history of play h_t and state b_t it solves,

$$\hat{x}_{it}(h_t, b_t) \in \arg\max_{x_{it}} U_{it}(x_{it}(h_t, b_t), \hat{x} \setminus \{\hat{x}_{it}\} \mid h_t, b_t), \tag{4}$$

for all $i \in I$ and all $t \geq 1$. Here, $\hat{x} \setminus \{\hat{x}_{it}\}$ denotes the vector of PBE strategies $\hat{x}$ with the particular strategy $\hat{x}_{it}$ removed. It is well known that a dynamic coordination games like $G(b)$ can have multiple PBEs (for an illustration in the application to renewables versus fossil fuels, see Van der Meijden and Smulders (2017)).

Proposition 1. *The game of complete information $G(b)$ has multiple perfect Bayesian Nash equilibria.*

Equilibrium multiplicity has clear implications for renewable technology adoption: it creates strategic uncertainty that turns the decision problem into a coordination game. This creates the possibility of coordination failure. In light of the ensuing analysis, it is important to distinguish two kinds of coordination failure in this context. One kind is a failure to coordinate strategies at all, i.e. a situation in which some agents use the renewable technology while others use fossil fuels. The other kind occurs when agents manage to coordinate their actions, but on an inefficient (or Pareto dominated) outcome. In this case, agents might use fossil fuels even though all of them would benefit were they to coordinate on renewables instead. In Section 3, I show that the first kind of coordination failure is of limited concern. The second kind, coordination on an inefficient equilibrium, remains a possibility. To remedy this kind of inefficiency, Section 4 considers questions of policy design. Before I can talk about inefficiency, however, I need to define what exactly efficiency means in a dynamic public good coordination game.

2.1 (In)efficiency

Consider a period t. Given b_t and the history of play h_t, the action vector $x_t^*(b_t, h_t)$ is *dynamic efficient* if it maximizes the discounted sum of payoffs assuming that agents choose the dynamic efficient actions $x^*_\tau(b_\tau, h_\tau)$ for all $\tau > t$. Formally, for each t the vector $x^*_t(b_t, h_t)$ is dynamic efficient if and only if:

$$x^*_t(b_t, h_t) \in \underset{x_t \in \{0,1\}^N}{\arg\max} \sum_{i \in I} \sum_{\tau=t}^{T} \int \beta^{\tau-t} u_{i\tau}(x^*_\tau(b_\tau, h_\tau) \mid b_\tau, h_\tau)\, d\Phi(b_\tau \mid b_t), \tag{5}$$

for all b_t and h_t. Because $x^*_T(b_T, h_T)$ is essentially unique for all h_T, it is easy to see that the dynamic efficient outcome in any period $t < T$ is also essentially unique. In particular, for any t and any h_t, there is a unique cutoff state $b^S_t(h_t)$ such that:

$$x^*_t(b_t, h_t) = \begin{cases} \mathbf{0}_t & \text{if} \quad b_t \le b^S_t(h_t), \\ \mathbf{1}_t & \text{if} \quad b_t \ge b^S_t(h_t). \end{cases} \tag{6}$$

The superscript S indicates that this is the level of b_t at which the social planner would switch from preferring $x_t = \mathbf{0}$ to preferring $x_t = \mathbf{1}$. Let b^S denote the vector of functions b^S_t, all t. The strategy profile x^{b^S} is the vector of strategies such that, for all i and t and $x^{b^S}_{it} \in x^{b^S}$, $x^{b^S}_{it}(b_t, h_t) = 0$ for all $b_t < b^S_t(h_t)$ and $x^{b^S}_{it}(b_t, h_t) = 1$ for all $b_t \ge b^S_t(h_t)$.

Observe that the dynamic efficient action vector $x^*_t(b_t, h_t)$ implies a dynamic efficient history of play at the start of period $t+1$, given the inherited history h_t. Let $h^*_{t+1}(b_t, h_t)$ denote this dynamic efficient history, so $h^*_{t+1}(b_t, h_t) := h_t \cup x^*_t(h_t, b_t)$. Let h^{**}_{t+1} denote the efficient history of play at the start of period $t+1$, assuming efficient choices were made in all stage $1, 2, ..., t$, so $h^{**}_{t+1} = h^*_{t+1}(b_t, h^*_t(b_{t-1}, h^*_{t-1}(...)))$. Intuitively, if a welfare maximizing policymaker could go back in time and dictate all choices made in stages $\tau < t$, the policymaker would prescribe technological choices consistent with h^{**}_τ at any $\tau < t$. Observe that, if all agents were to pursue the strategy vector x^{b^S}, then $x^*_t(b_t)$ would be played in every period t; at the start of each period t the history of play would hence be h^{**}_t. Let $x_t(b_1, ..., b_t)^{**} = x^*_t(h^{**}_t, b_t)$ denote the vector of efficient actions in period t, given b_t and given that the efficient history of play h^{**}_t has occurred.

Economist would not generally expect coordination on the efficient outcome of the game to occur. There are two main reasons for this. First, agents disregard the effect their own actions have on others; I call this the environmental externality. Second, the best action from an agent's individual payoff perspective may depend on the actions chosen by all others and so, depending on what this agent believes, they may fail to play the socially efficient action even if they would in fact prefer coordination on the efficient outcome to occur.

The precise magnitude of the environmental externality is not unambiguous. The immediate effect is clear: using renewables today reduces pollution, and this benefits all agents both today and in the future. But there can also be an indirect effect: due to technological spillovers an agent's choice of technology today may affect their and others' choice of technology in the future, which in turn has environmental consequences. The magnitude of this indirect effect naturally depends on the strategies pursued by all agents in the future and is therefore a source of uncertainty. Here, I choose the most conservative approach toward quantifying the environmental externality: it is the positive effect of an agent's using renewables now has on the discounted payoffs of all other agents, assuming none of them use renewables in any period $\tau \ge t$.

Let A_t be the environmental externality generated by an agent's use of renewables in period t. The externality A_t is given by the discounted sum of present and future environmental benefits to all agents other than i, assuming those agents not not use renewables in any period $\tau \geq t$:

$$A_t = \sum_{j \in I \setminus \{i\}} U_{jt}(1_{it}, \mathbf{0}{-it}, \mathbf{0} \mid h_t, b_t) - U_{jt}(\mathbf{0}_t, \mathbf{0} \mid h_t, b_t) = (N-1) \sum_{\tau=t}^{T} \beta^{\tau - t} a, \tag{7}$$

for all $i \in I$.

It is well-known that a policymaker can cleanse the individual agent's decision problem from the environmental externality through a Pigovian subsidy (resp. tax) s_t^p equal to A_t on using renewables (resp. fossil fuels). Even with the free-rider incentive eliminated, however, coordination on an inefficient outcome can be an equilibrium. In particular, agents may use fossil fuels even though they would be better off were all to use renewables instead.

In the next section, I extend the model by introducing idiosyncratic noise in agents' knowledge of b_t. This captures the idea of "scientific uncertainty" about the renewable technology. This additional source of uncertainty will allow me to select a unique equilibrium of the game, implying that it solves the first kind of coordination problem mentioned above. Selection of an inefficient equilibrium remains a problem, though; this problem will be addressed in Section 4.

3 Equilibrium selection

Equilibrium multiplicity obtains when agents are perfectly informed about the state b_t. This is a harsh assumption, especially in the context of renewables which typically are novel technologies. In this section, I show how uncertainty about b_t leads to selection of a unique equilibrium. I do so using the machinery of global games (Carlsson and Van Damme, 1993).

In a global game G^ε, agents do not observe b_t when choosing their actions x_t. Instead, each agent i receives a private signal b_{it}^ε of b_t, given by:

$$b_{it}^\varepsilon = b_t + \varepsilon_{it}, \tag{8}$$

where ε_{it} is an i.i.d. draw from the Normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$. The stochastic processes governing the development of b_t over time are as described in the previous section and all distributions are common knowledge.

Conditional on their signal b_{it}^ε, let $\Phi_t^\varepsilon(b, b^\varepsilon \mid b_{it}^\varepsilon)$ denote the joint probability function of $(b_t, b_{t+1}, ..., b_T, b_{-it}^\varepsilon, b_{t+1}^\varepsilon, b_t^\varepsilon)$ conditional on b_{it}^ε. The timing of the game is as follows:

1. At the start of period t, Nature draws a true b_t;

2. Each agent i receives their signal b_{it}^ε of b_t;

3. Agents simultaneously choose their actions x_{it};

4. Payoffs are realized according the the actions chosen by all agents and the true b_t;

5. Agents learn b_t, and the game moves on to stage $t+1$.

This type of game extends the literature on dynamic global games (Heidhues and Melissas, 2006; Angeletos et al., 2007) by studying *dynamic* spillovers in a model of repeated decicion making.[6]

Strategies. A strategy $p_{it} : \mathbb{R} \times [0, N \cdot t] \to [0,1]$ for agent i in period t is a function that to any b_i^ε and any h_t assigns a probability $p_{it}(b_{it}^\varepsilon, h_t) \geq 0$ with which the agent chooses action $x_{it} = 1$ when they observe b_{it}^ε and the history of play is h_t. I write $p_t = (p_{it})$ for a strategy vector in period t, and p_{-it} for the strategy vector for all agents but i. I denote by $p = (p_1, p_2, ..., p_T)$ a vector of strategy vectors.

Decreasing strategies. For some $Z \in \mathbb{R}$, I denote by p_{it}^Z the *monotone strategy* such that $p_{it}^Z(b_{it}^\varepsilon, h_t) = 0$ for all $b_{it}^\varepsilon < Z$ and $p_{it}^Z(b_{it}^\varepsilon, h_t) = 1$ for all $b_{it}^\varepsilon \geq Z$. A strategy p_{it} is *decreasing* if for any history h_t it is monotone with switching point $Z(h_t)$ and $Z(h_t)$ is non-increasing in h_t. That is, if $h_t' \geq h_t$ then $p_{it}^{Z(h_t')}(b_{it}^\varepsilon, h_t') \leq p_{it}^{Z(h_t)}(b_{it}^\varepsilon, h_t)$ for all b_{it}^ε.

Expected payoffs. The expected payoff to agent i in period t, conditional on their action x_{it}, their signal b_{it}^ε, and the vector of strategies $p \setminus \{p_{it}\}$ is denoted $u_{it}^\varepsilon(x_{it}, p \setminus \{p_{it}\} \mid h_t, b_{it}^\varepsilon)$ and equal to:

$$u_{it}^\varepsilon(x_{it}, p \setminus \{p_{it}\} \mid h_t, b_{it}^\varepsilon) = \int u_{it}(x_{it}, p \setminus \{p_{it}\}(b^\varepsilon) \mid h_t, b_t)\, d\Phi_t^\varepsilon\left(b_t, b_t^\varepsilon \mid b_{it}^\varepsilon\right). \tag{9}$$

The action x_{it} has an immediate effect on payoffs in period t. It also has an effect on payoffs in later periods, and this in two ways. Directly, because it affects the history of play h_τ for all $\tau > t$ and this influences the payoffs in periods τ. Indirectly, because it (possibly) affects action vectors x_τ for all $\tau > t$ as strategies can be conditional on the history of play.

Expected gains. In every period t, each agent i chooses their action x_{it} to maximize the discounted sum of payoffs given the signal b_{it}^ε, the history h_t, and the strategy vectors $p_{-it}, p_{t+1}, ..., p_T$. The expected gain from playing 1, rather than 0, in period t, is denoted $\Delta_{it}^\varepsilon(p \mid h_t, b_{it}^\varepsilon)$, where

$$\begin{aligned}
\Delta_{it}^\varepsilon(p \mid h_t, b_{it}^\varepsilon) = {} & u_{it}^\varepsilon(1, p \setminus \{p_{it}\} \mid h_t, b_{it}^\varepsilon) - u_{it}^\varepsilon(0, p \setminus \{p_{it}\} \mid h_t, b_{it}^\varepsilon) \\
& + \sum_{\tau=t+1}^{T} \beta^{\tau-t} u_{i\tau}^\varepsilon(p_\tau(b_\tau^\varepsilon, h_\tau(p, 1_{it})) \mid h_\tau(p, 1_{it}), b_{i\tau}^\varepsilon) \\
& - \sum_{\tau=t+1}^{T} \beta^{\tau-t} u_{i\tau}^\varepsilon(p_\tau(b_\tau^\varepsilon, h_\tau(p, 0_{it})) \mid h_\tau(p, 0_{it}), b_{i\tau}^\varepsilon),
\end{aligned} \tag{10}$$

where $h_\tau(p, x_{it})$ denotes the (expected) history of play at the beginning of period $\tau > t$ given that agent i chose action x_{it} in period t. Also note the reliance of h_τ on p, the vector of strategies. Naturally, actions chosen in periods $s < \tau$ affect the history of play at the start of period τ; since actions in period s are determined by the strategy vector p_s, this implies that the expected future history of play depends on p.[7] This is an important distinction between

[6] Note the assumption that agents learn b_t at the end of period t. This, on top of the dynamic strategic complementarities in actions, sets my dynamic global games approach apart from Angeletos et al. (2006), who assume that agents never fully learn the true fundamental.

[7] Let $\Delta_{it}^\varepsilon(x_{-it}, p \mid h_t, b_{it}^\varepsilon)$ be shorthand for $\Delta_{it}^\varepsilon(x_{-it} \cup p \setminus \{p_t\} \mid h_t, b_{it}^\varepsilon)$. That is, $\Delta_{it}^\varepsilon(x_{-it}, p \mid h_t, b_{it}^\varepsilon)$ is the expected gain from playing 1, rather than 0, in period t assuming that agents pursue the strategy p_τ in all periods $\tau > t$ and that x_{-it} is played by i's opponents in period t.

the game G^{ε} studied here and other dynamic global game models without dynamic spillovers (Angeletos et al., 2007).

The game is solved by backward induction and iterated elimination of strictly dominated strategies. I first prove two results that are of some theoretical interest in their own regard. These results combined then yield my first main result.

Proposition 2. *Let $\sigma_\varepsilon \to 0$. In stage T, there is an essentially unique strategy vector p^*_T that survives iterated elimination of strictly dominated strategies. The strategy vector p^*_T is in decreasing strategies. In particular, there exists a unique $b^*_T(h_T)$, for each h_T, such that $p^*_{iT}(b^\varepsilon_{iT}, h_T) = p^{b^*_T(h_T)}_{iT}(b^\varepsilon_{iT}, h_T)$ for all $b^\varepsilon_{iT} \neq b^*_T(h_T)$, where $b^*_T(h_T)$ is given by:*

$$b^*_T(h_T) = c - a - \sum_{m=0}^{N-1} \frac{f(m, H_T)}{N}. \tag{11}$$

Proposition 2 establishes that, when private signals are sufficiently accurate, there is a unique strategy vector that survives iterated elimination of strictly dominated strategies in the final period of the game (even when multiple equilibria exist in the complete information problem). This strategy vector is decreasing: it has the intuitive properties that a higher signal b^ε_{iT} and greater historic investments in renewables make it more likely that agent i uses the renewable technology in period T. Since the final period T can for all practical purposes be interpreted as a one-shot game, Proposition 2 is a special case of Theorem 3 on static global games in Frankel et al. (2003).

Iteratively dominant strategies are fairly straightforward to determine for period T because agents need not concern themselves with possible future effects of their choices. For periods $t < T$, the analysis is more complicated. The following Proposition simplifies this analysis substantially. It says that, assuming all agents pursue decreasing strategies in every period $\tau > t$, there is unique strategy that survives iterated elimination of strictly dominated strategies in period t and it is decreasing when the noise in signal is sufficiently small.

Proposition 3. *Let $\sigma_\varepsilon \to 0$. For all $\tau > t$, let the strategy vector p_τ be composed of decreasing strategies. Then there is an essentially unique strategy vector p^*_t that survives iterated elimination of strictly dominated strategies, and p^*_t is decreasing.*

Proposition 3 says that, assuming all agents only pursue decreasing strategies starting from next period onward, there is a unique strategy vector that survives iterated elimination of strictly dominated strategies in this period, and that strategy vector is decrearing (provided private signals are sufficiently precise). This is intuitive. If all agents play decreasing strategies in the future, using renewables today strictly increases the likelihood of renewables being used in later periods. This has two implications for each individual agent i. First, since all other agents are more likely to use renewables in those periods, the payoff to using renewables, rather than fossil fuels, then is higher in expectations. Second, since the agent uses renewables in period t, this increases the history of play h_τ in all $\tau > t$, making the use of renewables in those periods more profitable. Both effects make using renewables more attractive in period t, given b^ε_{it}. Moreover, the probability that agents in later periods adopt renewables is strictly increasing in b^ε_{it} by virtue of the strategies p_τ, $\tau > t$, being decreasing. For given x_{-it}, these are sufficient conditions for the expected gain Δ^ε_{it} to be strictly increasing in b^ε_{it} and

H_t. Under the assumption that p_τ is decreasing for all $\tau > t$, I can hence start a process of iterated elimination of strictly dominated strategies along the lines of Proposition 3, and this gives a unique strategy vector that survives iterated dominance in period t.

A priori, the assumption that all strategy vectors p_τ, for $\tau > t$, are decreasing is arbitrary. Recall then that Proposition 2 established that the unique strategy profile to survive iterated dominance in the final period T is decreasing. Plugging this result into Proposition 3 therefore establishes that, in period $T-1$ at least, the assumption that future strategies pursued are decreasing is no assumption at all; it is the necessary product of agents' rationality. Knowing there is only one decreasing strategy vector agents will pursue in period T, Proposition 3 implies that there is also only one decreasing strategy vector they will play in period $T-1$. But that implies, by Proposition 3 again, there is only one decreasing strategy vector rational agents will play in period $T-2$, and so on.

Theorem 1. *Consider the global game G^ε and let $\sigma_\varepsilon^2 \to 0$.*

i) There is an essentially unique strategy vector $p^* = (p_1^*, p_2^*, ..., p_T^*)$ that survives iterated elimination of strictly dominated strategies.

ii) There exists a unique $b_t^*(h_t)$ such that, for any agent i and any period t, the strategy p_{it}^* must satisfy $p_{it}^*(b_{it}^\varepsilon, h_t) = p_{it}^{b_t^*(h_t)}(b_{it}^\varepsilon, h_t)$ for all $b_{it}^\varepsilon \neq b_t^\varepsilon(h_t)$ and all h_t.

iii) The strategy vector p^* is decreasing. That is, $b_t^*(h_t)$ is decreasing in h_t for all t.

Theorem 1 extends the results in one-shot global games (Carlsson and Van Damme, 1993; Frankel et al., 2003) to dynamic global games with both static and dynamic strategic complementarities. Theorem 2 establishes that this is an equilibrium selection result: Any strategy vector p can be a perfect Bayesian equilibrium of G^ε if and only if it is equal to p^*. Hence, p^* is the essentially unique PBE of the game.

Theorem 2. *Consider the global game G^ε and let $\sigma_\varepsilon^2 \to 0$. The strategy profile $p^* = (p_1^*, p_2^*, ..., p_T^*)$ that survives iterated elimination of strictly dominated strategies is the essentially unique perfect Bayesian equilibrium of the game.*

By Theorem 2, uncertainty about the fundamentals b_t helps select a unique equilibrium in the dynamic coordination game of technological choices. While it is true that complete information about b_t can cause equilibrium multiplicity (Acemoglu et al., 2012; Van der Meijden and Smulders, 2017), even a vanishing amount of uncertainty eliminates that problem.[8] Theorem 2 hence takes to task the idea that static and dynamic strategic complementarities open the door for coordination on either renewables or fossil fuels based on essentially random sun spots. Allowing for a bit of uncertainty in agents' knowledge of b_t eliminates all sunspots and makes the outcome of the game essentially deterministic; determined, that is, by the true fundamental b_t and the history of play h_t.

[8] For completeness, it should be mentioned that an important assumption allowing me to prove equilibrium uniqueness is the finite time horizon of the game. Both Acemoglu et al. (2012) and Van der Meijden and Smulders (2017) consider an infinite horizon problem. It is easy to see, however, that their models will also have multiple equilibria when restricted to finite time horizons.

An interesting property of p^* is that it is decreasing, which implies that greater historic use of renewables increases the probability that renewables will be used in the future also. This is an example of technological lock-in or path-dependence, a well-documented phenomenon in the literature on rival technologies in general (David, 1985; Cowan, 1990) and renewables in particular (Aghion et al., 2016).

There is a clear downside to Theorem 2. The unique equilibrium p^* is inefficient. More precisely, there are cases in which the efficient outcome of the game (in period t) would be for agents to coordinate on renewables, but p^* prescribes coordination on fossil fuels instead.

Formally, consider the game of complete information $G(b)$. For given b_t drawn by Nature and history of play h_t, recall that $x_t^*(b_t, h_t)$ denotes the dynamic efficient, or expected welfare-maximizing, vector of technological choices x_t. I say that the equilibrium p^* is inefficient if for some (or all) t there exists a set of fundamentals $B_t(h_t)$ with strictly positive measure such that for all $b_t \in B_t(h_t)$ the vector of actions $p_t^*(b_t^\varepsilon, h_t) \neq x_t^*(b_t, h_t)$ almost surely: $\Pr\left[\lim_{\sigma_\varepsilon^2 \to 0} p_t^*(b_t^\varepsilon, h_t) = x_t^*(b_t, h_t) \mid b_t\right] = 0$.

Proposition 4. *Consider the global game G^ε and let $\sigma_\varepsilon^2 \to 0$. The unique equilibrium p^* is inefficient.*

Proposition 4 is not especially surprising. For one thing, agents do not consider the beneficial environmental effect of their renewable investments on others. This is the classic free-rider problem in private public good privision. Standard economic arguments suggest that this inefficiency can be corrected by means of a Pigovian tax/subsidy equal to the total (discounted) external effect generated by an agent's use of renewables. Henceforth, let s_t^p denote the Pigovian subsidy on renewables (tax on fossil fuels) offered to each agent i in period t. Since a Pigovian subsidy should equal the marginal external effect of using renewables, let

$$s_t^p = A_t, \tag{12}$$

for all t, where A_t is the conservative estimate of the marginal external environmental benefit from using renewables, rather than fossil fuels, in period t given by (7).

The inefficiency of p^* runs deeper than the free-rider problem described above though. Even if, for a given b_t and h_t, coordination on renewables and on fossil fuels are both equilibrium outcomes in period t of the complete information game $G(b)$, the global game G^ε may select the equilibrium in which agents coordinate on fossil fuels despite individual payoffs being higher if agents were to coordinate on renewables instead. In other words, the global games approach may eliminate Pareto dominant equilibria of the true game drawn by Nature. This phenomenon has little to do with the free-rider problem (after all, coordination on renewables is an equilibrium of $G(b)$) but rather is a consequence of the strategic uncertainty implied by (static and dynamic) strategic complementarities in agents' technological choices. In the next section, I design a policy of network subsidies that helps overcome this problem at zero cost.

4 Network subsidies

In this section, I design a policy that solves the coordination problem deriving from strategic complementarities at zero cost. In particular, I shall construct a subsidy scheme called network subsidies that makes using renewables (resp. fossil fuels) strictly dominant for all agents i and all periods t whenever b_t is such that coordination on renewables (resp. fossil fuels) is an equilibrium in the game of complete information $G(b)$. That is, the network subsidy makes coordination on the dynamic efficient *equilibrium* of the complete information game strictly dominant for each agent and in every period. Importantly, the network subsidy achieves this without costing the policymaker anything in equilibrium.

Recall that the efficient outcome of the game $G(b)$ is not necessarily an equilibrium due to free-rider incentives. But free-rider incentives are easily overcome through a Pigovian subsidy. Offering agents a Pigovian subsidy equal to A_t on using renewables, the policymaker can make the dynamic efficient outcome of the game x_t^* an equilibrium (possibly out of multiple) in each period t. This implies that a combination of a Pigovian subsidy and a network subsidy scheme implements coordination on the efficient outcome of the game in strictly dominant strategies for each individual agent. Moreover, spending on network subsidies is zero in equilibrium. Interestingly, while a Pigovian subsidy alone does not suffice to solve the coordination problem in this game, only Pigovian subsidies are paid in equilibrium.

Before discussing the details of network subsidies, a word on the informational context to which they apply. In what follows, I will discuss policies in the game of complete information $G(b)$. The policies and results derived in this context apply almost immediately to the global game G^ε. This is so because, by construction, when agents play a global game they still play a game $G(b)$ drawn by Nature – they simply do not know exactly what game that is. Hence, one way to think about policy in a global game is in terms of subsidies paid in *any* true game $G(b)$ drawn by Nature (with agents being uncertain that game). Another way to think about this is the following. A dynamic global game allows one to *select* an equilibrium, possibly out of many, of the underlying complete information game. By designing a policy scheme that makes the efficient outcome of the complete information game its unique equilibrium, there is only one equilibrium left for the global games approach to select.

Consider then the following *network subsidy* offered to agents who choose to use renewables in period t:

$$s_{it}^*(x_t \mid h_t, b_t) = \begin{cases} \Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h_t^*(h_{t-1}, b_t), b_t) - \Delta_{it}(x_{-it}, x^{b^S} \mid h_t, b_t) & \text{if} \quad x_{it} = 1, \\ 0 & \text{if} \quad x_{it} = 0, \end{cases} \tag{13}$$

for all $i \in I$ and t. I call s_{it}^* a network subsidy because the amount of subsidy a agent receives depends on technological choices made both in period t and before, which one could call the size of the network for renewables. A network subsidy scheme $s^* = (s_{it}^*)$ is a policy that offers the network subsidy s_{it}^* to every agent i in every period t.

Careful unspection of (13) reveals the core feature of a network subsidy s^*. To make things simple, consider the special case of a one-shot game ($T = 1$) so there is neither a history of play nor a vector of future strategies: $s^*(x) = \Delta_i(\mathbf{1}_{-i} \mid b) - \Delta_i(x_{-i} \mid b)$. In this case, as in the more general game, the private gain in payoffs to an agent from using renewables, rather than fossil fuels, depends on the technological choices made by all other agents. Agent i does not know these choices and hence must make a choice between fossil fuels and renewables second-guessing what others will be doing. A network subsidy scheme s^* is that it takes the need for such second-guessing away.

To see this, observe that whatever technological choices x_{-i} made by all other agents, the network subsidy s_i^* *guarantees* agent i a gain from using renewables, rather than fossil fuels, equal to that which they would realize in the hypothetical case that all other agents would adopt renewables. Because this is the gain the agent realizes irrespective of what all other agents are doing, and because this gain is positive if and only if coordination on renewables is also an equilibrium of the game,[9] the network subsidy s^* thus makes using renewables strictly dominant if and only if b_t is such that coordination on renewables is an equilibrium of the game. Being strictly dominant, the network subsidy thus turns the efficient equilibrium of the game into its unique equilibrium (even if coordination on fossil fuels would also be an equilibrium absent the subsidy). Hence, in equilibrium, agents either (i) coordinate on renewables so that $s_i^*(x) = s_i^*(\mathbf{1}) = \Delta_i(\mathbf{1}_{-i} \mid b) - \Delta_i(\mathbf{1}_{-i} \mid b) = 0$ and each agent i receives zero, or (ii) coordinate on fossil fuels in which case agents are not entitled to a subsidy to begin with. In the general game with $T > 1$, the logic is slightly more subtle but the intuition is essentially the same: a network subsidy scheme s^* guarantees each agent a best-case scenario gain from using renewables, rather than fossil fuels, stimulating coordination on the technology whenever that is dynamic efficient.

Let $G(b \mid s^p, s^*)$ denote the game in which agents are offered both the Pigovian subsidy s_t^p and the network subsidy s_t^* in every period t. I will show that i) the game $G(b \mid s^p, s^*)$ has an essentially unique equilibrium in dominant strategies, ii) in this unique equilibrium agents coordinate on the efficient outcome x^{**} of the game, and iii) spending on network series is zero in equilibrium. Hence, the network subsidy scheme s^* corrects the entire externality deriving from technological spillovers without requiring the policymaker to pay anything.

Theorem 3. *Consider the game $G(b \mid s^p, s^*)$.*

i) For all $i \in I$, the strategy $x_i^{b^S}$ is strictly dominant. Hence, $G(b \mid s^p, s^*)$ has a unique perfect Bayesian equilibrium;

ii) In the unique perfect Bayesian equilibrium of the game, agents coordinate on the efficient outcome $x_t^{**}(b_1, ..., b_t)$ for all t and $b_1, ..., b_t$;

iii) Spending on network subsidies is zero in equilibrium – hence, only the environmental subsidy needs to be paid.

In the game of complete information $G(b \mid s^p, s^*)$, the Pigovian subsidy s^p and the network subsidy scheme s^* jointly turn the efficient outcome of the game in its unique equilibrium in dominant strategies without requiring any spending on network subsidies. It stands to reason that the same is true on the global game G^ε. This statement largely true, with one exception: spending on network subsidies may (with probability zero) be strictly positive in equilibrium.

Consider the global game $G^\varepsilon(s^p, s^*)$ which is obtained from the global game G^ε by offering the network subsidy scheme s^* and the Pigovian subsidy s^p to using renewables. Because $s_t^*(x_t)$ depends on the true fundamental b_t, suppose that subsidies in each period t are paid when payoffs in that period are realized; that is, when agents learn b_t.[10] I also assume that

[9] I.e. when even the highest possible payoff to using renewables, rather than fossil fuels, is negative, using fossil fuels is a dominant strategy.

[10] This assumption is not really necessary but implies that I can use the results in Thereom 3 directly. Future work may extend the global games analysis to cases in which s_t^* cannot be conditional on the true b_t.

the policymaker observes neither the true b nor a signal of it.[11] Let $S_t^\varepsilon(b_t \mid h_t)$ denote the expected spending on network subsidies in period t in the game $G^\varepsilon(s^p, s^*)$, given a true b_t and the history of play h_t. Essentially a corollary of Theorem 3 is that, for $\sigma_\varepsilon \to 0$, the network subsidy scheme s^* makes using renewables in period t strictly dominant in $G^\varepsilon(s^p, s^*)$ for all $b_{it}^\varepsilon > b_t^S(h_t)$; fossil fuels are dominant for all $b_{it}^\varepsilon < b_t^S(h_t)$.

Corollary 1. *Consider the game $G^\varepsilon(s^p, s^*)$ and let $\sigma_\varepsilon \to 0$. For all $i \in I$, the strategy $p_{it}^{b^S}$ is strictly dominant. Hence, in the unique perfect Bayesian equilibrium of $G^\varepsilon(s^p, s^*)$ agents coordinate on the efficient outcome of the game with probability 1; expected spending on network subsidies is zero in equilibrium.*

It is important to emphasize that Corollary 1 speaks to *expected* spending on network subsidies. While agents coordinate their strategies in the equilibrium of $G^\varepsilon(s^p, s^*)$, note that a strategy is a function that prescribes a choice of tecnnology *conditional* on an agent's private signal about the fundamental b_t. Hence, if some agents receive a signal above $b_t^S(h_t)$ while others receive a signal below it, full coordination in actions fails and strictly positive network subsidies need to be paid. When $\sigma_\varepsilon \to 0$ the probability of this happening is zero (when $b_t \neq b_t^S(h_t)$); however, that merely says it is unlikely – it is not impossible.

Conditional on b_t and h_t, let expected equilibrium spending on network subsidies $S_t^\varepsilon(b_t \mid h_t)$ in period t of the global game $G^\varepsilon(s^p, s^*)$ is given by:

$$S_t^\varepsilon(b_t \mid h_t) = \int s_t^*(p_t^{b_t^S(h_t)}(b_t^\varepsilon), h_t) d\Phi(b_t^\varepsilon \mid b_t), \tag{14}$$

so that, for all $b_t \neq b_t^S(h_t)$,

$$\lim_{\sigma_\varepsilon^2 \to 0} S_t^\varepsilon(b_t \mid h_t) = 0. \tag{15}$$

Hence, expected spending on network subsidies is zero

$$\lim_{\sigma_\varepsilon^2 \to 0} \int S_t^\varepsilon(b_t \mid h_t) d\Phi_t(b_t) = 0. \tag{16}$$

Offering a network subsidy scheme s^* exposes the policymaker to some financial risk, either due to the possibility of off-equilibrium play or simply because agents' private signals may vary. In what follows, I will design a self-financed network tax-subsidy scheme that isolates the policymaker from that risk entirely.

4.1 Self-financed network subsidies

From a practical point of view, important qualifications to the costlessness of a network subsidy scheme discussed in Theorem 3 and Corollary 1 apply. First, (expected) spending on network subsidies is zero only in equilibrium. While there is no particular reason to assume agents would pursue off-equilibrium strategies, if they do network subsidy spending may be positive and substantial. Second, even if agents pursue equilibrium strategies, private signals

[11] This is necessary for the relevance of a global games analysis. For suppose that, in contrast to the agents, the policymaker does observe b. Then the policymaker might simply announce b and implement the policies discussed in the preceding section.

in the global game may be such that coordination on actions fails nonetheless and network subsidies need to be paid.

To remedy both of these concerns, I here derive a network tax-subsidy scheme where subsidy payments on the use of renewables are financed through a "network tax" levied on choosing fossil fuels. In particular, I address the question what budget-neutral tax-subsidy scheme can implement the Pareto efficient outcome of the underlying game $G(b)$ in strictly dominant strategies for all b.

As before, I assume the policymaker seeks policies that, for each agent $i \in I$, turn $x_{it} = 1$ into a strictly dominant action for all $b_t > b^S(h_t, b_t)$ while leaving $x_i = 0$ strictly dominant for all $b_t < b^S(h_t, b_t)$. Let the network subsidy offered to any agent $i \in I$ in period t be denoted $s^{**}_{it}(x_t \mid h_t, b_t)$; the network tax levied on those using fossil fuels is denoted $t^{**}_{it}(x_t)$. Thus, when x_t is played, aggregate spending on network subsidies in period t is therefore $X_t \cdot s^{**}_{it}(x_t)$ and aggregate revenues from the network tax are $(N - X_t) \cdot t^{**}_{it}(x_t)$. To say that the tax-subsidy scheme should be costless, or self-financed, is to say that:

$$(N - X_t) \cdot t^{**}_{it}(x_t) = X_t \cdot s^{**}_{it}(x_t), \tag{17}$$

for all x_t and t. Condition (17) imposes that total spending on the network subsidies to those using renewables is matched exactly by total tax revenues from taxes on the fossil fuel technology, whatever x_t agents end up playing. Next, the tax-subsidy scheme must make $x_{it} = 1$ strictly dominant for all $b_t > b^S_t(h_t)$ while leaving $x_i = 0$ strictly dominant for all $b_t < b^S_t(h_t)$. It is straightforward to see that this will hold if and only if

$$s^{**}_{it}(x_t) + t^{**}_{it}(x_t) = s^{*}_{it}(x_t), \tag{18}$$

for all i, t, and x_t; here, $s^*_{it}(x_t)$ is given by (13). Combining (17) and (18) gives one equation in one unknown,

$$N \cdot t^{**}_{it}(x_t) = X_t \cdot s^{*}_{it}(x_t),$$

which immediately identifies the unique self-financed network tax-subsidy scheme:

$$\begin{cases} t^{**}_{it}(x_t) = & \frac{X_t}{N}\left[\Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h^*_t(h_{t-1}, b_t), b_t) - \Delta_{it}(x_{-it}, x^{b^S} \mid h_t, b_t)\right], \\ s^{**}_{it}(x_t) = & \frac{N - X_t}{N}\left[\Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h^*_t(h_{t-1}, b_t), b_t) - \Delta_{it}(x_{-it}, x^{b^S} \mid h_t, b_t)\right], \end{cases} \tag{19}$$

where I plugged in the expression for $s^*_{it}(x_t)$ given by (13). Let s^{**} denote the network tax-subsidy scheme in which each agent i in every period t receives a network subsidy equal to $s^{**}_{it}(x_t)$ when x_t is played in period t and agent i uses renewables (in that period), whereas agent i must pay a network tax equal to $t^{**}_{it}(x_t)$ when x_t is played in period t and agent i uses fossil fuels (in that period). I write $G(b \mid s^p, s^{**})$ for the game $G(b)$ in which agents are offered the Pigovian tax s^p_t on using renewables as well as the network tax-subsidy scheme s^{**}_t, in every period t.

Theorem 4. *Consider the game $G(b \mid s^p, s^{**})$.*

i) For all $i \in I$, the strategy $x^{b^S}_i$ is strictly dominant. Hence, $G(b \mid s^p, s^{**})$ has a unique perfect Bayesian equilibrium;

ii) In the unique perfect Bayesian equilibrium of the game, agents coordinate on the efficient outcome $x_t^{**}(b_t)$ for all t and b_t;

iii) Net spending on network subsidies is always zero.

The elements of Theorem 4 extend to the global game $G^{\varepsilon}(s^p, s^{**})$, which one obtains by offering the Pigovian subsidy s^p and the network tax-subsidy scheme s^{**} in the global game G^{ε}.. Because the network tax-subsidy scheme s^{**} is guaranteed to be budget neutral for *any* realization of technological choices, the possibility that agents receive probability-zero signals and thus might fail to coordinate their actions bears no financial consequences for the policymaker.

Theorem 5. *Consider the game $G^{\varepsilon}(s^p, s^{**})$ and let $\sigma_{\varepsilon}^2 \to 0$.*

i) For all $i \in I$, the strategy $p_i^{b^S}$ is strictly dominant. Hence, $G^{\varepsilon}(s^p, s^{**})$ has a unique perfect Bayesian equilibrium;

ii) In the unique perfect Bayesian equilibrium of the game, agents coordinate on the efficient outcome $x_t^{**}(b_t)$ for all t and $b_t \neq b_t^S(h_h)$ with probability 1;

iii) Net spending on network subsidies is always zero.

To conclude this section on policy design, observe that nothing in the construction of network subsidies depends critically on the application to renewables. All the network subsidy exploits is the strategic complementarities in agents' actions. This suggests that network subsidies could be applied in many other coordination games and are a contribution to public economics more broadly.

5 Conclusions

I study the adoption of renewable technologies to mitigate climate change in a dynamic public goods coordination game. Because dynamic coordination typically have multiple equilibria, I use a global games approach to study equilibrium selection. My first contribution is to show that the celebrated result on equilibrium selection in one-shot games (Carlsson and Van Damme, 1993; Frankel et al., 2003) generalizes to dynamic games with both static and dynamic strategic complementarities. The unique equilibrium is determined by fundamentals of the renewable technology, not sunspots, and technological lock-in a possibility. Moreover, coordination on fossil fuels can occur even when that is inefficient and all agents would prefer to coordinate on renewables instead. My second contribution is to design a policy of network subsidies that makes coordination on the efficient equilibrium of the game strictly dominant but does not, in equilibrium, cost the policymaker anything. To protect the policymaker against off-equilibrium behavior, I also design a network tax-subsidy scheme that is entirely self-financed, implying zero net spending on network subsidies whatever choices agents make.

Though this paper uses the language of climate change and renewable technologies, dynamic coordination games arise in scores of other contexts as well. Examples include the market for houses and housing bubbles (Himmelberg et al., 2005), technological lock-in on inferior technologies such as QWERTY (David, 1985) or light water nuclear reactors (Cowan, 1990), the persistence of cultural-institutional conventions (Belloc and Bowles, 2013), peer effects in (un)healthy behaviors such as smoking (Nakajima, 2007) or drinking (Kremer and Levy, 2008), and inefficient gender norms (Field et al., 2021). To the extent my model describes elements of those and other phenomena, the analysis presented herein may help explain why an inefficient equilibrium could be selected in those applications and how cheap policies could be designed to overcome that inefficiency.

Several interesting questions remain open for future research. First, the assumption of symmetric agents is strong and future work might seek to relax it. While a generalization of the equilibrium selection result (Theorems 1 and 2) is mathematically straightforward, perfect coordination of technological choices may break down in equilibrium when agents are asymmetric. This raises questions for the design of network subsidies.

Second, my model does not incorporate an explicit spatial dimension in the decision problem. Studies show that geographic proximity can be an important factor determining the magnitude of strategic complementarities in (renewable) technology adoption (Bollinger and Gillingham, 2012; Murata et al., 2014; Aghion et al., 2016; Gillingham and Bollinger, 2021). It would be interesting to include a spatial element in renewable technology adoption. Again, the result on equilibrium uniqueness can quite easily be extended to incorporate geographic spillover effects. It would be interesting to see if such a model could endogenously generate the co-existence of local networks for renewables and fossil fuels.

Finally, a very high degree of rationality is assumed when proving equilibrium uniqueness and the efficacy of network subsidies. This raises questions of whether, and to what extent, real world decision makers can be trusted to behave according to my theoretical predictions. Laboratory experiments might test the consistency of those predictions with behavior in the lab. While experiments have shown one-shot global games to be successful in predicting behavior (Heinemann et al., 2004, 2009), whether this success extends to dynamic global games and network subsidies remains an open question. The author is currently involved in an experimental study on network subsidies.

A Proofs

Proof of Proposition 1

Proof. It suffices to identify cases in which there are multiple Nash equilibria in stage T of the game. Hence, fix a history h_T. Let $\underline{b}_T(h_T)$ and $\bar{b}_T(h_T)$, respectively, denote the solutions to:

$$u_{iT}(1, \mathbf{1}_{-iT} \mid h_T, \underline{b}_T(h_T)) = u_{iT}(0, \mathbf{1}_{-iT} \mid h_T, \underline{b}_T(h_T)),$$

and

$$u_{iT}(1, \mathbf{0}_{-iT} \mid h_T, \bar{b}_T(h_T)) = u_{iT}(0, \mathbf{0}_{-iT} \mid h_T, \bar{b}_T(h_T)).$$

Since ceteris paribus u_{it} is increasing in b_t for all t, it is clear that $\underline{b}_T < \bar{b}_T$. Now observe that:

i) $\mathbf{1}_T$ is a Nash equilibrium in stage T for all $b_T \geq \underline{b}_T(h_T)$, and;

ii) $\mathbf{0}_T$ is a Nash equilibrium in stage T for all $b_T \leq \bar{b}_T(h_T)$.

Hence, for all $b_T \in [\underline{b}_T(h_T), \bar{b}_T(h_T)]$ there are multiple Nash equilibria in stage T of the game. $\square$

Proof of Proposition 2

Proof. Fix h_T. Define $L^0(h_T)$ and $R^0(h_T)$ as the solutions to:

$$\Delta^\varepsilon_{iT}(\mathbf{1}_{-iT} \mid h_T, L^0(h_T)) = \Delta^\varepsilon_{iT}(\mathbf{0}_{-iT} \mid h_T, R^0(h_T)) = 0. \tag{20}$$

Hence, for any $j \in I$, a necessary condition for a strategy p_{jT} not to be dominated is that $p^{R^0(h_T)}_{jT}(b^\varepsilon_{jT}, h_T) \le p_{jT}(b^\varepsilon_{jT}, h_T) \le p^{L^0(h_T)}_{jT}(b^\varepsilon_{jT}, h_T)$ for all b^ε_{jT}. For $k \ge 1$, recursively define $L^k(h_T)$ and $R^k(h_T)$ such that:

$$\Delta^\varepsilon_{iT}(p^{L^{k-1}(h_T)}_{-iT} \mid h_T, L^k(h_T)) = \Delta^\varepsilon_{iT}(p^{R^{k-1}(h_T)}_{-iT} \mid h_T, R^k(h_T)) = 0. \tag{21}$$

Observe that $R^0 > L^{k+1}(h_t) > L^k(h_T)$ and $L^0 < R^{k+1}(h_t) < R^K(h_T)$ for all $k \ge 0$. Being bounded monotone sequences, $(L^k(h_T))$ and $(R^k(h_T))$ must converge; let $L^*(h_T)$ and $R^*(h_T)$ denote their limits, respectively. By construction,

$$\Delta^\varepsilon_{iT}(p^{L^*(h_T)}_{-iT} \mid h_T, L^*(h_T)) = \Delta^\varepsilon_{iT}(p^{R^*(h_T)}_{-iT} \mid h_T, R^*(h_T)) = 0. \tag{22}$$

For general σ^2_ε, given b^ε_{iT}, the conditional distribution of $(b_T, b^\varepsilon_{-iT})$ is $\mathcal{N}\left(\frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon+\sigma^2_\beta} b_{T-1} + \frac{\sigma^2_\beta}{\sigma^2_\varepsilon+\sigma^2_\beta} b^\varepsilon_{iT}, \Sigma\right)$, where the covariance matrix Σ is independent of b^ε_{iT} (these are well-known facts about the multivariate Normal distribution; see e.g. chapter 3 in Tong (1990)). In the limit as $\sigma^2_\varepsilon \to 0$, therefore, $L^*(h_T) = R^*(h_T) = b^*_T(h_T)$. This establishes that for given h_T, the unique strategy profile surviving iterated elimination of strictly dominated strategies is $p^{b^*_T(h_T)}_T$.

For any $Z \in \mathbb{R}$, observe that $\Delta^\varepsilon_{iT}(p^Z_{-iT} \mid h_T, Z)$ is increasing in h_T. Moreover, in the limit as $\sigma^2_\varepsilon \to 0$, $\Delta^\varepsilon_{iT}(p^Z_{-iT} \mid h_T, Z)$ is increasing in Z. For $\sigma^2_\varepsilon \to 0$, therefore, the unique solution $b^*_T(h_T)$ to $\Delta^\varepsilon_{iT}(p^{b^*_T(h_T)}_{-iT} \mid h_T, b^*_T(h_T)) = 0$ is decreasing in h_T. There is hence a unique strategy profile p^*_T surviving iterated elimination of strictly dominated strategies, and it is decreasing.

The point $b^*_T(h_T)$, by construction, must solve:

$$\Delta^\varepsilon_{iT}(p^{b^*_T(h_T)}_{-iT} \mid h_T, b^*_T(h_T)) = 0.$$

Using (1), for $\sigma^2_\varepsilon \to 0$ this can be written out as:

$$= c - a - \int \left(b + f\left(X_{-iT}\left(p^{b^*_T(h_T)}_{-iT}\right), H_T\right)\right) d\Phi^\varepsilon_T(b_T, b^\varepsilon_{-iT} \mid b^*_T(h_T)) \tag{23}$$

$$= c - a - b^*_T(h_T) - \int f\left(X_{-iT}\left(p^{b^*_T(h_T)}_{-iT}\right), H_T\right) d\Phi^\varepsilon_T(b^\varepsilon_{-iT} \mid b^*_T(h_T)) \tag{24}$$

$$= c - a - b^*_T(h_T) \tag{25}$$

$$- \int \sum_{m=0}^{N-1} f(m, H_T) \left[z(\varepsilon_{jT} \mid b_T, b^*_T(h_T))\right]^{N-m-1} \left[1 - z(\varepsilon_{jT} \mid b_T, b^*_T(h_T))\right]^m \phi^\varepsilon_T(b_T \mid b^*_T(h_T)) db_T \tag{26}$$

$$= c - a - b_T^*(h_T) - \int_0^1 \sum_{m=0}^{N-1} \binom{N-1}{m} q^m (1-q)^{N-m-1} dq f(m, H_T) \tag{27}$$

$$= c - a - b_T^*(h_T) - \sum_{m=0}^{N-1} \frac{(N-1)!}{m!\,(N-m-1)!} \frac{m!\,(N-m-1)!}{N!} f(m, H_T) \tag{28}$$

$$= c - a - b_T^*(h_T) - \sum_{m=0}^{N-1} \frac{f(m, H_T)}{N} = 0, \tag{29}$$

where $z(\varepsilon_{jT} \mid b_T, b_T^*(h_T)) = \Pr[\varepsilon_{jT} < b_T^*(h_T) - b_T \mid b_T]$ (given the distribution of ε_{jT}). $\square$

Proof of Proposition 3

Proof. Fix a history h_t. The core to proving this lemma is to show that $\Delta_{it}^{\varepsilon}(p^{L^*(h_T)} \mid h_t, L^*(h_t))$ is (i) increasing in x_{-it} for all b_{it}^{ε} and (ii) increasing in b_{it}^{ε} for all x_{-it} and, given that all strategies p_τ, for $\tau > t$, are decreasing.

To establish (i), note than an increase in x_{-it} clearly increases $u_{it}^{\varepsilon}(1, x_{-it} \mid h_t, b_{it}^{\varepsilon}) - u_{it}^{\varepsilon}(0, x_{-it} \mid h_t, b_{it}^{\varepsilon})$. Moreover, an increase in x_{-it} (for given x_{it}) raises h_{t+1}. Since p_τ is decreasing for all $\tau > t$, this increase in h_{t+1} causes the distribution of X_{t+1}, conditional on b_{it}^{ε} to shift rightward. The shift of this distribution implies a shift of the distribution of h_{t+1} to the right and, since p_{t+1} is decreasing, this in turn implies a shift of the distribution of X_{t+2} (conditional on b_{it}^{ε}) to the right. And so on. Hence, if p_τ is decreasing for all $\tau > t$, then an increase in x_{-it} implies aa shift to the right of the distribution of X_τ for all $\tau > t$, increasing the expected payoff in those periods. Moreover, an increase in x_{-it} increases $u_{it}^{\varepsilon}(1, p_{-it} \mid h_t, b_{it}^{\varepsilon}) - u_{it}^{\varepsilon}(0, p_{-it} \mid h_t, b_{it}^{\varepsilon})$. Therefore, p_τ is decreasing for all $\tau > t$, then an increase in x_{-it} increases $\Delta_{it}^{\varepsilon}$, given b_{it}^{ε}.

For (ii), fix x_{-it} and a vector of decreasing strategies $p_\tau^{Z(h_\tau)}$, $\tau > t$. Given h_t and x_{-it}, $u_{it}^{\varepsilon}(1, x_{-it} \mid h_t, b_{it}^{\varepsilon}) - u_{it}^{\varepsilon}(0, x_{-it} \mid h_t, b_{it}^{\varepsilon})$ is clearly increasing in b_{it}^{ε}. Moreover, note that the probability distribution $\phi_t^{\varepsilon}(b_\tau, b_\tau^{\varepsilon} \mid b_{it}^{\varepsilon})$ shifts to the right with b_{it}^{ε} (with mean exactly $(b_{it}^{\varepsilon}, b_{it}^{\varepsilon}, ..., b_{it}^{\varepsilon})$ when $\sigma_\varepsilon \to 0$). This (given the hypothesis of decreasing strategies) implies that, for any given h_τ, the conditional distribution of X_τ is increasing b_{it}^{ε}. Moreover, that increase in the distribution of X_τ immediately implies an increase in the distribution of $h_{\tau+1}$ which, for any b_{it}^{ε}, implies an increase in the distribution of $X_{\tau+1}$. Effectively, therefore, the distribution of each X_τ shifts to the right in b_{it}^{ε}. Since $u_{it}^{\varepsilon}(1, x_{-it} \mid h_t, b_{it}^{\varepsilon}) - u_{it}^{\varepsilon}(0, x_{-it} \mid h_t, b_{it}^{\varepsilon})$ is increasing in b_{it}^{ε}, this implies that $\Delta_{it}^{\varepsilon}(p \mid h_t, b_{it}^{\varepsilon})$ is increasing in b_{it}^{ε}.

I proved that $\Delta_{it}^{\varepsilon}(x_{-it}, p \mid b_t, b_{it}^{\varepsilon})$ is strictly increasing in b_{it}^{ε} and x_{it}^{ε}, assuming the vector of strategies p_τ is decreasing for all $\tau > t$. Therefore, note that there must exist an $R_t^0(h_t)$ and $L_t^0(h_t)$ which satisfy:

$$\Delta_{it}^{\varepsilon}(\mathbf{0}_{-it}, p \mid h_t, R_t^0(h_t)) = \Delta_{it}^{\varepsilon}(\mathbf{0}_{-it}, p \mid h_t, R_t^0(h_t)) = 0.$$

It follows that a strategy p_{it} is undominated iff it satisfies $p_{it}^{R^0(h_t)}(b_{it}^{\varepsilon}, h_t) \leq p_{it}(b_{it}^{\varepsilon}, h_t) \leq p_{it}^{L^0(h_t)}(b_{it}^{\varepsilon}, h_t)$, for all b_{it}^{ε}. Since this is true for every i, additional strategies become dominated, and so on. For all $k \geq 0$, define $R_t^{k+1}(h_t)$ and $L_t^{k+1}(h_t)$ as follows:

$$\Delta_{it}^{\varepsilon}(p_{-it}^{R_t^k(h_t)}, p \mid h_t, R_t^{k+1}(h_t)) = \Delta_{it}^{\varepsilon}(p_{-it}^{L_t^k(h_t)}, p \mid h_t, L_t^{k+1}(h_t)) = 0.$$

Clearly, $L_t^0(h_t) < R_t^{k+1}(h_t) < R_t^k(h_t)$ and $R_t^0(h_t) > L_t^{k+1}(h_t) > L_t^k(h_t)$. It follows that $(R_t^k(h_t))$ and $(L_t^k(h_t))$ are bounded monotone sequences; these must converge. Let $R_t^*(h_t)$ and $L_t^*(h_t)$ denote the limits of $(R_t^k(h_t))$ and $(L_t^k(h_t))$, respectively. By construction,

$$\Delta_{it}^{\varepsilon}(p_{-it}^{R_t^*(h_t)}, p \mid h_t, R_t^*(h_t)) = \Delta_{it}^{\varepsilon}(p_{-it}^{L_t^*(h_t)}, p \mid h_t, L_t^*(h_t)) = 0.$$

Hence, the strategy vector p_t survives iterated elimination of strictly dominated strategies (given p_τ for all $\tau > t$) if and only if it satisfies $p_t^{R_t^*(h_t)}(b_t^\varepsilon, h_t) \leq p_t(b_t^\varepsilon, h_t) \leq p_t^{L_t^*(h_t)}(b_t^\varepsilon, h_t)$ for all b_t^ε. Moreover, when $\sigma_\varepsilon \to 0$ the probability distribution of $(b_t, b_{-it}^\varepsilon)$ conditional on b_{it}^ε has mean $(b_{it}^\varepsilon, b_{it}^\varepsilon, ..., b_{it}^\varepsilon)$, implying that $\Delta_{it}^\varepsilon(p_{-it}^Z, p \mid h_t, Z)$ is strictly increasing in Z when $\sigma_\varepsilon \to 0$. This gives $L_t^*(h_t) = R_t^*(h_t)$, for any h_t. Hence, given the history h_t and decreasing strategy vectors p_τ for all $\tau > t$, the unique strategy vector p_t that survives iterated elimination of strictly dominated strategies is monotone. Moreover, because $\Delta_{it}^\varepsilon(x_{-it}, p \mid h_t, Z)$ is increasing in h_t for all x_{-it}, it follows that $\Delta_{it}^\varepsilon(p_{-it}^Z, p \mid h_t, Z)$ is increasing in h_t and so $R_t^*(h_t)$ (and $L_t^*(h_t)$) is decreasing in h_t. The latter implies that the unique strategy vector to survive iterated elimination of dominated strategies in period t, given that p_τ is decreasing for all $\tau > t$, is decreasing. □

Proof of Theorem 1

Proof. An immediate implication of induction on Propositions 2 and 3. □

Proof of Theorem 2

Proof. Let p be a PBE of G^ε. I prove that if p is a PBE of G^ε, then $p = p^*$. I will show that the only Bayesian Nash equilibrium in stage T is given by p_T^*. Using backward induction, one can apply the argument used to prove this to establish that p_t^* is the only PBE strategy in any stage $t < T$.

Fix a history h_T. For any agent, define (for this h_T)

$$\underline{b}_{iT} = \inf\{b_{iT}^\varepsilon \mid p_{iT}(b_{iT}^\varepsilon, h_T) > 0\}, \tag{30}$$

and

$$\overline{b}_{iT} = \sup\{b_{iT}^\varepsilon \mid p_{iT}(b_{iT}^\varepsilon, h_T) < 1\}. \tag{31}$$

Observe that $\underline{b}_{iT} \leq \overline{b}_{iT}$. Now define

$$\underline{b}_T = \min\{\underline{b}_{iT}\}, \tag{32}$$

and

$$\overline{b}_T = \max\{\overline{b}_{iT}\}. \tag{33}$$

By construction, $\overline{b}_T \geq \overline{b}_{iT} \geq \underline{b}_{iT} \geq \underline{b}_T$. Observe that p is a PBE of G^ε only if, for each i, it holds that $\Delta_{iT}^\varepsilon(p_{-iT} \mid h_T, \underline{b}_{iT}) \geq 0$. Consider then the expected gain $\Delta_{iT}^\varepsilon(p_{-iT}^{\underline{b}_T} \mid h_T, \underline{b}_{iT})$. It follows from the definition of $\underline{b}_T$ that $p^{\underline{b}_T}(b_T^\varepsilon) \geq p(b_T^\varepsilon)$ for all b_T^ε. The implication is that, for each i, $\Delta_{iT}^\varepsilon(p_{-iT}^{\underline{b}_T} \mid h_T, \underline{b}_{iT}) \geq \Delta_{iT}^\varepsilon(p_{-iT} \mid h_T, \underline{b}_{iT}) \geq 0$. Hence, $\underline{b}_T \geq b_T^*(h_T)$.

Similarly, if p is a PBE of G^ε then, for each i, it must hold that $\Delta_{iT}^\varepsilon(p_{-iT} \mid h_T, \overline{b}_{iT}) \leq 0$. Consider now the expected gain $\Delta_{iT}(p_{-iT}^{\overline{b}_t} \mid h_T, \overline{b}_{iT})$. It follows from the definition of $\overline{b}_T$ that

$p_T^{\bar{b}_t}(b_T^\varepsilon) \leq p_T(b_T^\varepsilon)$ for all b_T^ε. For each i it therefore holds that $\Delta_{iT}^\varepsilon(p_T^{\bar{b}_t} \mid h_T, \bar{b}_{iT}) \leq \Delta_{iT}^\varepsilon(p_T \mid h_T, \bar{b}_{iT}) \leq 0$. Hence, $\bar{b}_T \leq b_T^*(h_T)$.

Since $\underline{b}_T \leq \bar{b}_T$ while also $\underline{b}_T \geq b_T^*(h_T)$ and $\bar{b}_T \leq b_T^*(h_T)$, it must hold that $\underline{b}_T = \bar{b}_T = b_T^*(h_T)$. Since $p_T^{\underline{b}_T} \geq p$ while also $p_T^{\bar{b}_T} \leq p$, it follows that $p_{iT}(b_{iT}^\varepsilon, h_T) = p_{iT}^{b_T^*(h_T)}(b_{iT}^\varepsilon, h_T)$ for all $b_{iT}^\varepsilon \neq b_T^*(h_T)$ and all i (recall that for each agent i one has $\Delta_{iT}(p_{-iT}^{b_T^*(h_T)} \mid h_T, b_T^*(h_T)) = 0$, which explains the singleton exception at $b_{iT}^\varepsilon \neq b_T^*(h_T)$). Thus, if p_T is a PBE of G^ε then it must hold that $p_{iT}(b_{iT}^\varepsilon, h_T) = p_{iT}^{b_T^*(h_T)}(b_{iT}^\varepsilon, h_T)$ for all $b_{iT}^\varepsilon \neq b_T^*(h_T)$ and all i. □

Proof of Proposition 4

Proof. It suffices to show that there is at least one $p_t^* \in p^*$ that is (dynamic) inefficient. Consider p_T^*. Proposition 2 established that $p_T^* = p_T^{b_T^*(h_T)}$, where $b_T^*(h_T) = c - a - \sum_{n=0}^{N-1} \frac{f(m,H_T)}{N}$. Efficiency requires agents to coordinate on $x_T = \mathbf{1}_T$ for all $b_T > c - N \cdot a - g_T(h_T) - f(N-1)$. Let $B_t(h_t) = \left(c - N \cdot a - f(N-1, H_T), c - a - \sum_{n=0}^{N-1} \frac{f(m,h_T)}{N}\right)$. Clearly, the set $B_T(h_T)$ has strictly positive measure. Moreover, when $\sigma_\varepsilon^2 \to 0$, for all $b_T \in B_T$, one has $\Pr\left[\lim_{\sigma_\varepsilon^2 \to 0} p_T^*(b_T^\varepsilon, h_T) = \mathbf{1}_T\right] = 0$ while $x_T^*(b_T, h_T) = \mathbf{1}_T$. Hence, p_T^* is dynamic inefficient and therefore so is p^*. □

Proof of Theorem 3

Proof. The proof proceeds in several steps. (a) Using backward induction, I show that playing $x_{it}^{b^S}$ is strictly dominant for each agent i in any period t when offered both s^* and s^p. (b) Given the strict dominant of strategy $x_{it}^{b^S}$, agents coordinate on $x_t^{**}(b_t)$ in every period t.

(d) Fix a history of play h_T and consider stage T of $G(b \mid s^p, s^*)$. For any x_{-iT}, the network subsidy s_{iT}^* and environmental subsidy s_T^p make playing $x_{iT} = 1$ payoff-maximizing if and only if:

$$U_{iT}(1, x_{-iT} \mid h_T, b_T) + s_{iT}^*(x_T \mid h_T, b_T) + s_T^p \geq U_{iT}(0, x_{-iT} \mid h_T, b_T),$$

or:

$$\Delta_{iT}(x_{-iT} \mid h_T, b_T) + s_{iT}^*(x_T \mid h_T, b_T) + s_T^p \geq 0,$$

which gives:

$$\Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h_t^*, b_t) + s_t^p \geq 0, \tag{34}$$

for all x_{-iT} and h_T.

(d) Take x_{-iT} as given and fix a history of play h_T. Then $x_{it} = 1$ maximizes the sum of agents' payoffs if and only if:

$$\begin{aligned} &\sum_j U_{jT}(1_{iT}, x_{-iT} \mid h_T, b_T) + s_{jT}^*(1_{iT}, x_{-iT} \mid h_T, b_T) \\ \geq &\sum_j U_{jT}(0_{iT}, x_{-iT} \mid h_T, b_T) + s_{jT}^*(0_{iT}, x_{-iT} \mid h_T, b_T), \end{aligned}$$

or:

$$\Delta_{iT}(\mathbf{1}_{-iT} \mid h_T^*, b_T)+$$

$$\sum_{j\neq i} U_{jT}(1_{iT}, x_{-iT} \mid h_T, b_T) + s^*_{jT}(1_{iT}, x_{-iT}) - U_{jT}(0_{iT}, x_{-iT} \mid h_T, b_T) - s^*_{jT}(0_{iT}, x_{-iT}) \geq 0$$

Note that, for all action vectors x_{-jT}, one can rewrite $U_{jT}(1_{jT}, x_{-jT} \mid h_t, b_t) = \Delta_{jT}(x_{-jT} \mid h_t, b_t) + U_{jT}(0_{jT}, x_{-jT} \mid h_t, b_t)$. This allows to rewrite the above as:

$$\Delta_{iT}(\mathbf{1}_{-iT} \mid h^*_T, b_T) + \sum_{j\neq i} U_{jT}(1_{iT}, 0_{jT}, x_{-ijT} \mid h_T, b_T) - U_{jT}(0_{iT}, 0_{jT}, x_{-ijT} \mid h_T, b_T) \geq 0,$$

where $x_{-ijT} := x_T \setminus \{x_{iT}, x_{jT}\}$ is shorthand for the vector of actions x_T with both x_{iT} and x_{jT} removed. Observe that the summation uses $x_{jT} = 0$ for all $j \neq i$, implying that $x_{-iT} = (x_{jT})_{j\neq i} = \mathbf{0}_{-iT}$. The above condition hence further reduces to:

$$\Delta_{iT}(\mathbf{1}_{-iT} \mid h^*_T, b_T) + \sum_{j\neq i} U_{jT}(1_{iT}, \mathbf{0}_{-iT} \mid h_T, b_T) - U_{jT}(0_{iT}, \mathbf{0}_{-it} \mid h_T, b_T) \geq 0. \tag{35}$$

Comparing (34) and (35) immediately shows that if $s^p_T = A_T$, agent i maximizes their payoff by choosing the action x_{iT} that is dynamic efficient in period T for all h_T and all x_{-iT}. Hence, playing $x^{b^S}_{iT}$ is strictly dominant in period T, for all $i \in I$.

Suppose it is known that the policy scheme (s^*, s^p) makes playing $x^{b^S}_{i\tau}(b_\tau, h^*_\tau)$ strictly dominant for each agent i in every period $\tau > t$. What, then, are agents' incentives in period t?

Anticipating that all agents coordinate on $x^{b^S}_{\tau}(b_\tau, h^*_\tau)$ for all $\tau > t$, when offered the policy scheme (s^*, s^p) agent i chooses $x_{it} = 1$ if and only if:

$$\Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h^*_t, b_t) + s^p_t \geq 0.$$

Similarly, the action $x_{it} = 1$ maximizes the sum of agents' payoff (assuming all are offered the network subsidy scheme s^*) if and only if:

$$\begin{aligned}\Delta_{it}(\mathbf{1}_{-it}, x^{b^S}(\cdot, h^*) \mid h^*_t, b_t) &+ \sum_{j\neq i} U_{jt}(1_{it}, 0_{jt}, x_{-ijt} \mid h_t, b_t) \\ &- \sum_{j\neq i} U_{jt}(0_{iT}, 0_{jt}, x_{-ijt} \mid h_t, b_t) \geq 0.\end{aligned}$$

We already know that $x_{-ijT} = \mathbf{0}_{-ijT}$. Moreover, since each $x_{jt} = 0$ by hypothesis in the above condition, given $x_{-ijT} = \mathbf{0}_{-ijT}$, for $t = T-1$ the above condition reduces to:

$$\begin{aligned}\Delta_{iT-1}(\mathbf{1}_{-iT-1}, x^{b^S}(\cdot, h^*) \mid h^*_{T-1}, b_{T-1}) &+ \sum_{j\neq i} U_{jT-1}(1_{iT-1}, 0_{jT-1}, \mathbf{0}_{-iT-1} \mid h_{T-1}, b_{T-1}) \\ &- \sum_{j\neq i} U_{jT-1}(0_{iT}, \mathbf{0}_{-iT-1} \mid h_{T-1}, b_{T-1}) \geq 0.\end{aligned}$$

This condition in turns can be rewritten as:

$$\Delta_{iT-1}(\mathbf{1}_{-iT-1}, x^{b^S}(\cdot, h^*) \mid h^*_{T-1}, b_{T-1}) + A_{T-1} \geq 0.$$

Hence, in period $T-1$, too, the Pigovian subsidy $s^p_{T-1} = A_{T-1}$ and the network subsidy $s^*_{iT-1}(x_{T-1} \mid h_{T-1}, b_{T-1})$ make the strategy $x^{b^S}_{iT-1}(h^*_{T-1}, b_{T-1})$ strictly dominant, for all $i \in I$.

Repeated application of the above argument to preceding periods establishes that (s^p, s^*) make playing $x^{b^S}_{it}$ strictly dominant for each $i \in I$ in every period t. Now consider period $t = 1$. As there is no history of play in this period, agents coordinate on $x^*_1(b_1)$ for any b_1 drawn by Nature. At the start of period 2, therefore, the history of play is h^*_1. Since $x^{b^S}_{it}$ is strictly dominant for all t, and so in particular for $t = 2$, this implies that $x_2(b_2) = x^*_2(b_2)$, for all b_2. Hence, $h_2 = h^*_1 \cup x^*_2(b_2) = h^{**}_2$ and the history of play at the start of period 2 is h^{**}_2. Induction on t readily shows that $h_t = h^{**}_t$ for all $t \geq 1$. But by definition, $x^*_t(b_t, h^{**}_t) = x^{**}_t(b_t)$, establishing part (ii) of the Theorem.

Part (iii) is easy to see. Since in equilibrium $h_t = h^{**}_t$ and $x_t = x^{**}_t$ in every period t, for all t either $x_t = \mathbf{0}$, or $x_t = \mathbf{1}$ and therefore $s_{it}(x_t \mid h_t, b_t) = \Delta_{it}(x_{-it}, x^{b^S} \mid h_t, b_t) - \Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h^{**}_t, b_t) = \Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h^{**}_t, b_t) - \Delta_{it}(\mathbf{1}_{-it}, x^{b^S} \mid h^{**}_t, b_t) = 0$, for all i. In either scenario, spending on network subsidies is zero in equilibrium. □

Proof of Theorem 4

Proof. Observe that:

$$U_{it}(1, x_{-it}, p \mid h_t, b_t) + s^{**}_{it}(x_t \mid h_t, b_t) + s^p_t \geq U_{it}(0, x_{-it}, p \mid h_t, b_t) - t^{**}_{it}(x_t \mid h_t, b_t),$$

which is equivalent to,

$$U_{it}(1, x_{-it}, p \mid h_t, b_t) + s^{**}_{it}(x_t \mid h_t, b_t) + t^{**}_{it}(x_t \mid h_t, b_t) + s^p_t \geq U_{it}(0, x_{-it}, p \mid h_t, b_t),$$

and this, by (13), equals:

$$U_{it}(1, x_{-it}, p \mid h_t, b_t) + s^{*}_{it}(x_t \mid h_t, b_t) + s^p_t \geq U_{it}(0, x_{-it}, p \mid h_t, b_t),$$

for all $i \in I$ and all t. Because this is exactly the same condition from which the proof of Theorem 3 starts, I refer to that proof to establish that the game $G(b \mid s^p, s^{**})$ has a unique perfect Bayesian equilibrium in dominant strategies, and that this equilibrium is x^{b^S}.

Net spending on network subsidies is always zero by construction. That is, for any x_t, $s^{*}**$ guarantees that $X_t \cdot s^{**}_t(x_t \mid h_t, b_t) = (N - X_t) \cdot t^{**}_t(x_t \mid h_t, b_t)$, and therefore net spending on network subsidies (net of tax revenues, that is) is equal to: $X_t \cdot s^{**}_t(x_t \mid h_t, b_t) - (N - X_t) \cdot t^{**}_t(x_t \mid h_t, b_t) = X_t \cdot \left[\frac{N - X_t}{N}\right] s^*_{it} - (N - X_t) \cdot \left[\frac{X_t}{N}\right] s^*_{it} = 0$. □

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