The Global Climate Game*
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Abstract

I study emissions abatement in a global game of technological investments. Players choose between substitutable technologies. One technology is cheap and dirty, the other clean but expensive, and technologies exhibit strategic complementarities such as technology-specific technological spillovers. The paper makes two main contributions. My first contribution is to resolve equilibrium multiplicity in games of technological investment by addressing equilibrium selection through the use of global games. In well-identified cases the unique equilibrium is inefficient, motivating policy intervention. This leads to my second contribution, the introduction of network subsidies. A network subsidy allows the policymaker to correct the externality deriving from technological spillovers (and all externalities if use of the clean technology is an equilibrium of the underlying coordination game) but does not, in equilibrium, cost anything.

1 Introduction

Climate change is a coordination failure. In order to avoid global warming from progressing every further, greenhouse gases should be reduced drastically and by everyone. These reductions in turn require investments in renewable technologies, which are costly. Economists have often and for various reasons argued that switching to renewables may pay off individually only if sufficiently many others do so too (Acemoglu et al., 2012; Aghion and Jaravel, 2015). Yet the mere fact that fighting climate change requires coordination neither necessitates a continued reliance on fossil fuels nor

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guarantees a successful transition to renewables. So, what does? How can we explain the world's generally lackluster performance on global warming? Why has coordination on clean technologies proven so hard to achieve? What economic mechanism could drive coordination on using fossil fuels despite their known disadvantages? Is it rational to continue using fossil fuels even when everyone understands them to be inferior to renewables? And is it possible to design subsidies that promote renewable technologies without being overly costly? This paper sheds a light on all of these issues.

The contribution of this paper is twofold. First, using the machinery of global games (Carlsson and Van Damme, 1993) I explicitly address equilibrium selection in a game of renewable technology adoption where players' choices exhibit strategic complementarities (Bulow et al., 1985). Even though strategic complementarities turn the decision problem into a coordination game with multiple equilibria, I demonstrate how uncertainty about players' payoffs kickstarts a process of iterated elimination of strictly dominated strategies that eliminates all but one of these equilibria. I show that, in well-identified cases, the unique equilibrium selected is inefficient. Somewhat paradoxically, fully rational players may maximize their expected payoffs by coordinating on an outcome (such as runaway climate change) that all know to be inefficient. My result does not rely on free-rider incentives.

My second contribution is to solve the inefficiency described above in a novel way. I formulate a policy scheme called network subsidies that corrects the entire externality deriving from technological spillovers but does not, in equilibrium, cost the policymaker anything. The innovation here is to make the amount of network subsidy a player receives contingent not only on their own action but also on the actions pursued by all others. As I show, a policymaker can exploit this additional degree of freedom to offer a subsidy scheme that solves the coordination problem without requiring any payments to be made. My result relies on the strategic complementarities in players' actions and is independent of the application to clean technologies, suggesting that network subsidies are a contribution to public economics more broadly.

I derive my results in a bare-bones model of technological choices. Players choose to use either dirty fossil fuels or a clean and renewable technology. Associated with players' choices are two externalities. The first is an environmental externality which says that use of the clean, rather than the dirty, technology is good for the environment and thus benefits everyone. The second is a network externality deriving from strategic complementarities in individual actions; one could think of technological spillovers, though that interpretation is strictly non-exclusive. These externalities combined imply that players face a coordination problem with the possibility of multiple Pareto-ranked equilibria. Absent additional assumptions, it is hard to make clear predictions on the eventual outcome of such a game.

The additional assumption I make is that the exact (environmental) benefit from using the clean, rather than the dirty, technology is unknown. Instead, I assume that each player receives a private noisy signal of the underlying true benefit. This information structure in which players do not observe the actual game they are playing but only some private noisy signal of it is what turns the decision problem described in the preceding paragraph into a global game (Carlsson and Van Damme, 1993; Frankel et al., 2003). As I show, uncertainty about environmental benefits drives equilibrium selection.

It may seem counterintuitive that adding uncertainty to a game simplifies the decision problem. The idea is that noisy signals force players to construct a probability distribution on the signals received by all other players. This probability distribution in turn leads to a probability distribution on other players' actions (or, more correctly, to necessary restrictions on the distribution of their actions). But a probability distribution on others' actions is extremely convenient in a coordination game where a player's payoff-maximizing behavior depends critically on what other players are doing. Using a procedure called iterated elimination of strictly dominated strategies, uncertainty thus makes the decision problem substantially easier.

To fix ideas, consider the following highly simplified example. Imagine a continuum of players. Given the clean technology's true benefit b, each player receives a private noisy signal of b which is an i.i.d. draw from the uniform distribution on $[b-1, b+1]$. If a player receives a signal of at least 10, they maximize their expected payoff by choosing the clean technology even if all other players choose dirty; formally, using the clean technology is strictly dominant for signals above 10. For signals below 1, using the dirty technology is strictly dominant. Finally, if a player receives a signal x between 1 and 10, they maximize their expected payoff from using the clean technology provided the proportion of other players choosing clean is at least $1 - \frac{x}{10}$.

Now imagine that some player who receives a signal equal to $\frac{999}{100}$. In this case, the payoff-maximizing choice of technology by the player depends on the choices made by all other players, which they do not know – they face a coordination problem. What the player does know, however, is that each opponent will definitely choose the clean technology whenever their signal exceeds 10. While the player does not know what choices the others will make, the player can hence estimate the proportion of players choosing clean: the lower bound on this proportion is simply the fraction of players whose signals are at least 10. It is easy to see that the player, having observed a signal of $\frac{999}{100}$, estimates the latter to be $\frac{999}{2000} > 1 - \frac{999}{1000}$. This means the player maximizes their expected payoff by choosing the clean technology for all signals higher than $\frac{999}{100}$. The same is true for every other player. Starting from a range of signals for which choosing clean is strictly dominant, players can in this way extend the domain of signals at which choosing clean maximizes their expected payoff.

The main force driving equilibrium selection is to essentially repeat the above reasoning. Suppose some player receives a signal equal to $\frac{998}{100}$. Knowing that all other players choose the clean technology when their signal is $\frac{999}{100}$ or higher, the player's conditional estimate on the proportion of other players choosing clean is bound from below 1/2. The player hence maximizes payoffs choosing clean for signals of $\frac{998}{100}$ and higher. The same again is true for all players so the argument should be repeated. This procedure, called iterative elimination of strictly dominated strategies, eventually yields a range of signals for which a payoff-maximizing player should use the clean technology. Starting from low signals, symmetric reasoning yields a range of signals at which choosing the dirty technology is payoff-maximizing. The first main result of the paper shows that this argument leads very far: there is only one strategy that survives iterated elimination of strictly dominated strategies.

The example serves to illustrate the argument used to proof equilibrium selection. It also highlights why iterated elimination of strictly dominated strategies does not work in a game of complete information. For, in such a game, if a player learns that b equals $\frac{999}{100}$, they know that other players know so too. There hence are no good reasons to put a lower or upper bound on the probability that any other player will use renewables, which is exactly what drives equilibrium selection in the global game.

Uncertainty about the clean technology thus eliminates equilibrium multiplicity, making the game easier the play. Unfortunately, this fact alone does not suffice to guarantee coordination on the desired, emissions-free outcome. As I show, there are well-identified conditions under which the equilibrium selected is inefficient and players coordinate on using the dirty technology even though coordination on the clean technology would benefit everyone and all know it. This observations develops our understanding of the reasons behind the continued reliance on fossil fuel despite the availability of known-to-be better renewables. It also motivates an analysis of policies to overcome equilibrium inefficiencies, such as subsidies.

Subsidies to stimulate the development and use of renewable technologies are much discussed by economists and relied upon by policymakers (Joskow, 2011; Murray et al., 2014; Allcott et al., 2015; Fowlie et al., 2015; Acemoglu et al., 2016; Borenstein, 2017; Li et al., 2017; De Groote and Verboven, 2019; Hart, 2019; Harstad, 2020). This popularity notwithstanding, subsidy schemes have an obvious budgetary disadvantage. The novel policy of network subsidies developed in this paper does away with that problem: though a true subsidy that rewards users of the clean technology with a payment, equilibrium spending on network subsidies is always zero. Network subsidies thus have a great advantage over more traditional and existing subsidy policies.

It may seem surprising that a network subsidy payment of zero suffices to solve the coordination problem. It is important to note, however, that the amount of network subsidies paid is nil only in equilibrium. If less than full coordination on the clean technology occurs, those who use it are entitled to a strictly positive subsidy payment with an intuitive interpretation. For any configuration of actions, the amount of network subsidy a user of the clean technology receives equals the difference between the payoff they actually realized (not counting subsidies) and the payoff they would have realized in the hypothetical case where full coordination on the clean technology is achieved.

By design, the network subsidy scheme does two things. First, when all players use the clean technology, the amount of network subsidy received by each is equal to zero. The natural implication is that *if* a network subsidy succeeds to let players coordinate on renewables, it does not cost the policymaker anything. Second, it guarantees each player who use the clean technology a payoff (including subsidies) as though full coordination on that technology were achieved, regardless of other players' actual choices. Due to the strategic complementarities in actions, this means that players who use the clean technology are guaranteed their maximum possible payoff from doing so. Moreover, the maximum payoff to using the clean technology is unambiguously higher than that to using the dirty technology precisely when coordination on the clean technology is also the efficient equilibrium of the game. The policy thus leads to selection of the efficient equilibrium and in so doing solves the coordination problem.

In a way, the network subsidy functions like an insurance policy. It insures users of the clean technology against the event they would enjoy little technological spillovers because other players decided to use the dirty technology – it offers protection against defection. Due to this protection, use of the clean technology becomes a safe option to the players as their payoff to using renewables is not affected by what others are doing. This motivates each player to adopt the clean technology as soon as global coordination on renewables is efficient. The event against which the network subsidy insures players is therefore not realized and payments cannot be claimed.

Technological spillovers, equilibrium multiplicity and coordination failure have long been recognized as important complications for renewable technology adoption. While I treat spillovers abstractly, economists have discussed a number of factors from which such strategic complementarities could arise. These include (indirect) pure network effects (Katz and Shapiro, 1985; Greaker and Midttømme, 2016; Li et al., 2017), spillovers from R&D in breakthrough technologies (Barrett, 2006; Hoel and de Zeeuw, 2010), the existence of climate tipping points (Barrett and Dannenberg, 2017), political economy arguments such as climate clubs (Nordhaus, 2015, 2021), technological and knowledge spillovers (Fischer and Newell, 2008; Acemoglu et al., 2012; Aghion and Jaravel, 2015; Aghion et al., 2016; Harstad, 2016; Hart, 2019; Harstad, 2020), or even behavioral economic mechanisms such as green social norms (Allcott, 2011) and reciprocity (Nyborg, 2018).

Mindful of the complications deriving from equilibrium multiplicity, the profession has sought to address equilibrium selection in different ways. Two approaches are predominant. One from the outset restricts attention to a particular kind of equilibrium. Thus, players may be a priori assumed to pursue symmetric strategies (Harstad, 2012; Harstad et al., 2019) or to coordinate on the Pareto dominant outcome (Barrett, 2006; Hoel and de Zeeuw, 2010).[1] Another approach treats the coordination problem as theoretically indecisive and relies on laboratory experiments to make predictions (Barrett and Dannenberg, 2012, 2014, 2017; Calzolari et al., 2018). I extend these approaches in using global games to show how uncertainty about the technologies leads to equilibrium selection without the need for additional restrictions on players' choices.

In a global game, equilibrium selection is catalyzed by uncertainty about the clean technology.[2] The assumption of incomplete information with player-specific idiosyncratic

[1] The global games approach also selects an equilibrium in symmetric strategies. However, this is a result rather than an assumption. Players are initially allowed to pursue any kind of strategy; it turns out they won't.

[2] Game theorists have also studied other approaches toward equilibrium selection also exist. One well-known example is Poisson games (Matsui and Matsuyama, 1995; Myerson, 1998; Makris, 2008) in which players are uncertain about the number of other players playing the game. Poisson games have a unique equilibrium as the uncertainty vanishes. Another popular approach derives equilibrium

beliefs seems innocent and hardly debatable. There are many uncertainties surrounding climate change and individual beliefs vary vastly (Hornsey et al., 2016). Within my model, one is free to interpret this uncertainty in different ways. Uncertainty could pertain to the true severity of climate change (Weitzman, 2014; Cai and Lontzek, 2019), the location of a dangerous tipping point (Lemoine and Traeger, 2014; Diaz and Keller, 2016), or the true potential of renewables. Although many authors have studied the role of incomplete information in emissions policy (Kolstad, 2007; Barrett and Dannenberg, 2012; Martimort and Sand-Zantman, 2016), my focus on private but correlated noisy signals is new to this literature. My paper therefore extends existing models of climate policy under incomplete information in a novel direction.

Inefficiency of the equilibrium selected motivates my analysis of network subsidies. To an extent, the derivation of this policy is an exercise in mechanism design or implementation theory. The policymaker aims design a subsidy that makes coordination on the efficient outcome of the game a strictly dominant strategy for all players (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983). While mechanism design was applied to emissions mitigation before (Duggan and Roberts, 2002; Ambec and Ehlers, 2016; Martimort and Sand-Zantman, 2016), the focus has mostly been on policies to solve the free-rider problem. My work develops this approach by designing policies that solve the coordination problem.

Network subsidies are also related to the literature on directed technical change and the environment (Acemoglu et al., 2012; Aghion and Jaravel, 2015; Aghion et al., 2016; Acemoglu et al., 2016; Hart, 2019). This literature studies the effect of policy on technology adoption when multiple and (partially) substituable technologies co-exist with differental consequences for social welfare, the environment, and growth. Technologies are typically characterized as either clean or dirty and assumed to exhibit technology-specific positive spillovers, with the dirty technology starting off as more advanced. This literature asks how different kinds of policies – e.g. a carbon tax or R&D subsidies – can be used most efficiently to stimulate large-scale adoption of the clean technology. My contribution to this literature is to show how R&D subsidies, aimed at correcting the externality that derived from spillovers and other strategic comeplementarities in renewable investment, can be made substantially cheaper using network subsidies.

The remainder of the paper is structured as follows. In Section 2, I present the main model and briefly discuss the game of complete information. In Section 3, I add uncertainty to the analysis and show that the global game has a unique equilibrium. Section 4 introduces network subsidies, and Section 5 concludes.

2 A Game of Complete Information

Consider a world consisting of N players. Each player chooses to invest in either of two technologies. The first, called the dirty technology, is cheap and polluting – think of

selection as a dynamic outcome of a process of evolutionary selection (Kandori et al., 1993).

fossil fuels. If a player does not invest in the dirty technology, they invest in an expensive but environmentally-friendly clean but expensive technology – think of renewables. A natural interpretation of the players would be firms or households.

Compared to investment in the dirty technology, the environmental benefit of investing in the clean technology is $b > 0$. An action for player i is a binary variable $x_i \in \{0, 1\}$ such that $x_i = 1$ corresponds to investment in the clean technology while $x_i = 0$ stands for investment in the dirty technology. Let $x = (x_1, x_2, ..., x_N)$ denote the vector of actions played by all players, and let $x_{-i} = (x_j)_{j \neq i}$ be the vector of actions by all players but i. Let $\mathbf{1} = (1, 1, ..., 1)$ be the action vector of all ones, and $\mathbf{0} = (0, 0, ..., 0)$ the action vector of all zeroes. The cost of investing in the dirty technology (play 0) is constant at d. The costs of investing in the clean technology (play 1) depend on the total number of players, n, that invest in clean and are decreasing in n: $c(1) > c(2) > ... > c(N)$. That is, the game exhibits strategic complementarities (Bulow et al., 1985). It is assumes that $c(N) > d$.[3]

Combining these elements, the payoff to player i is:

$$\pi_i(x \mid b) = \begin{cases} b \cdot n(x) - d & \text{if} \quad x_i = 0 \\ b \cdot (n(x)) - c(n(x)) & \text{if} \quad x_i = 1 \end{cases}, \tag{1}$$

where $n(x)$ is defined as the total number of players playing 1 in x; hence, $n(x) = \sum_{i=1}^{N} x_i$. I define $n(x_{-i})$ as the total number of players other than i that play 1 in x: $n(x_{-i}) = \sum_{j \neq i} x_j$. The set of players $\{1, 2, ..., N\}$, the set of action vectors $x \in \{0, 1\}^N$, and the set of payoff functions $\{\pi_i\}$ jointly define a complete information game $G(b)$.[4] I write $G(b)$ for the game of complete information (i.e. with common knowledge of b) to differentiate this game from the global game studied in Section 3 where players do not observe b. The choice of b is as key parameter is made for convenience; one could have chosen other parameters instead.

There are two externalities associated with investment in the clean technology . The first is an *environmental externality* and relates to the parameter b, the positive impact an individual player's investment in the clean technology has on the environment (and hence payoff) for all other players – think of reduced CO2 emissions. The second is a *network externality* and relates to the investment cost function c, i.e. it captures the fact that a player's investment in the clean technology lowers the cost of clean technological investment for all other players – think of technological or knowledge spillovers. The structure of multiple externalities deriving from the use of renewables is common in

[3]This assumption is of no technical importance for the analysis; it buys me the convenience of not having to discuss separately the cases where $d < c(N)$ and $d > c(N)$ in the welfare analysis.

[4]The payoff function (1) is extremely simple and one may wonder whether the results derived in this paper are due to its particular specification. Under some additional assumptions, an implication of Frankel et al. (2003) is that the results on equilibrium selection in the global game (Section 3) hold true more generally, see also the discussion following Theorem 1. A substantial theoretical generalization of my results on network subsidies (Section 4) is provided in ongoing work by Heijmans & Suetens (available upon request).

the literature on technological change and the environment (*c.f.* Acemoglu et al., 2012; Aghion and Jaravel, 2015; Acemoglu et al., 2016; Hart, 2019; Harstad, 2020)

The gain from investing in the clean rather than the dirty technology to player i (given b and x_{-i}) is the difference in payoffs between playing $x_i = 1$ and $x_i = 0$. For given x_{-i}, define

$$\begin{aligned} \Delta_i(x_{-i} \mid b) &= \pi_i(1, x_{-i} \mid b) - \pi_i(0, x_{-i} \mid b) \\ &= b + d - c(n(x_{-i}) + 1). \end{aligned} \tag{2}$$

Moreover, if $k = n(x_{-i})$ I write $\Delta_i(k \mid b) = \Delta_i(x_{-i} \mid b)$.

The action $x_i = 1$ is strictly dominant for all $b > c(1) - d$ as for those bs it holds that $\Delta_i(x_{-i} \mid b) > 0$ for all x_{-i}. Alternatively, $x_i = 0$ is strictly dominant for all $b < c(N) - d$. In between, the game has multiple equilibria. To smoothen notation, I shall henceforth write $\bar{b} = \frac{c(N)-d}{N}$.

Proposition 1.

(i) *$x = \mathbf{1}$ is a Nash equilibrium of the game for all $b \geq c(N) - d$. It is the unique Nash equilibrium for all $b > c(1) - d$.*

(ii) *$x = \mathbf{0}$ is a Nash equilibrium of the game for all $b \leq c(1) - d$. It is the unique Nash equilibrium for all $b < c(N) - d$.*

(iii) *$x = \mathbf{1}$ is Pareto strictly dominant for all $b > \bar{b}$.*

Proof. This follows from the above dominance argument, together with direct payoff comparisons. □

Environmental economists have long recognized the possibility of equilibrium multiplicity in games of technological investments (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017). It motivates the question of equilibrium selection, to which Section 3 is devoted. First, however, I offer some final remarks on the complete information game $G(b)$.

Frankel et al. (2003) have observed that a game such as given by (1) is a *potential game* (Monderer and Shapley, 1996). A game in which each player has two actions is a potential game if there exists a potential function $P : \{0, 1\}^N \to \mathbb{R}$ on action profiles such that the change in any individual player's payoff when switching from one action to the other is always equal to the change in the potential function; that is, for which there exists a function P such that $P(x_i, x_{-i} \mid b) - P(1-x_i, x_{-i} \mid b) = \pi_i(x_i, x_{-i} \mid b) - \pi_i(1-x_i, x_{-i} \mid b)$ for all i. The game $G(b)$ has a potential function $P(x \mid b)$ given by:

$$P(x \mid b) = \begin{cases} \sum_{k=0}^{n(x)-1} \Delta_i(k \mid b) & \text{if } n(x) > 0, \\ 0 & \text{if } n(x) = 0. \end{cases} \tag{3}$$

Observe that, for any i and any $x_{-i} \in \{0,1\}^{N-1}$, it holds that $P(1, x_{-i} \mid b) - P(0, x_{-i} \mid b) = \Delta_i(x_{-i} \mid b) = \pi_i(1, x_{-i} \mid b) - \pi_i(0, x_{-i} \mid b)$, confirming that P is a potential function indeed.[5]

A *potential maximizer* is a vector x that maximizes P. One can verify that $\mathbf{1}$ is the unique potential maximizer of $P(x \mid b)$ for all $b + d > \sum_{n=1}^{N} \frac{c(n)}{N}$ whereas $\mathbf{0}$ is the unique potential maximizer of $P(x \mid b)$ for all $b + d < \sum_{n=1}^{N} \frac{c(n)}{N}$. I return to this observation in the next section.

The preceding analysis was performed under the assumption that the entire game $G(b)$ is common knowledge. In the application to climate change and renewables, that assumption is unrealistic. The next section is devoted to a global games analysis of the coordination game described by (1). In particular, I will study a situation in which players do not observe the precise game $G(b)$ they are playing; rather, each receives a private noise signal of the true game played. I demonstrate that such an information structure leads to selection of a unique equilibrium out of the multiple equilibria described in Proposition 1.

3 The Global Climate Game

Strategic complementarities in clean investments drive equilibrium multiplicity under common knowledge of b, the environmental benefit of clean investment. But the assumption of complete information is strong. There are large numbers of uncertainties surrounding many clean technologies's present or future potential.

Uncertainty and signals. For these reasons, I will now study a global game (Carlsson and Van Damme, 1993). In the global game G^{ε} the true parameter b is unobserved. Rather, it is assumed that b is drawn from the uniform distribution on $[\underline{B}, \overline{B}]$ where $\underline{B} < c(N) - d$ and $\overline{B} > c(1) - d$ and that each player i receives a private noisy signal b_i^{ε} of b, given by:[6]

$$b_i^{\varepsilon} = b + \varepsilon_i. \tag{4}$$

The term ε_i captures idiosyncratic noise in i's private signal. It is common knowledge that ε_i is an i.i.d. draw from the uniform distribution on $[-\varepsilon, \varepsilon]$. I assume that ε is sufficiently small: $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$. Let $b^{\varepsilon} = (b_i^{\varepsilon})$ denote the vector of signals received by all players, and let b_{-i}^{ε} denote the vector of signals received by all players but j, i.e. $b_{-i}^{\varepsilon} = (b_j^{\varepsilon})_{j \neq i}$. Note that player i observes b_i^{ε} but neither b nor b_{-i}^{ε}. I write $\Phi^{\varepsilon}(\cdot \mid b_i^{\varepsilon})$ for the joint probability function of $(b, b_j^{\varepsilon})_{j \neq i}$ conditional on b_i^{ε}. The timing of the global game G^{ε} is as follows:

[5] If $n(x) > 1$, the confirmation is as in the text. If, however, $n(x) = 1$, then one has $P(1, x_{-i} \mid b) - P(0, x_{-i} \mid b) = \sum_{k=0}^{0} \Delta_i(k \mid b) - 0 = \Delta_i(x_{-i} \mid b) = \pi_i(1, x_{-i} \mid b) - \pi_i(0, x_{-i} \mid b)$, as desired.

[6] In game theory, it is assumed that the game (in this case G^{ε}) is common knowledge; hence, the structure of the uncertainty (the joint distrubution of b and all the signals b_j^{ε}), the possible actions and all the payoff functions are commonly known. For a formal treatment of common knowledge, see Aumann (1976).

1. Nature draws a true b;
2. Each player i receives its private signal b_i^ε of b;
3. All players simultaneously choose their actions;
4. Payoffs are realized according to the true b and the actions chosen by all players.

In what follows I will take $\varepsilon > 0$ as given and introduce the concepts used to analyze the global game G^ε.

Strategies and strict dominance. Player i receives a signal b_i^ε prior to choosing an action. A strategy p_i for player i in G^ε is a function that assigns to any $b_i^\varepsilon \in [\underline{B}-\varepsilon, \overline{B}+\varepsilon]$ a probability $p_i(b_i^\varepsilon) \geq 0$ with which the player chooses action $x_i = 1$ when they observe b_i^ε. I write $p = (p_1, p_2, ..., p_N)$ for a strategy vector. Similarly, I write $p_{-i} = (p_j)_{j \neq i}$ for the vector of strategies for all players but i. Conditional on the strategy vector p_{-i} and a private signal b_i^ε, the expected gain (of choosing $x_i = 1$ rather than $x_i = 0$) to player i is given by:

$$\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) = \int \Delta_i(p_{-i}(b_{-i}^\varepsilon) \mid b) \, d\Phi^\varepsilon(b, b_{-i}^\varepsilon \mid b_i^\varepsilon). \tag{5}$$

I say that the action $x_i = 1$ is strictly dominant at b_i^ε if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) > 0$ for all p_{-i}. Similarly, the action $x_i = 0$ is strictly dominant (in the global game G^ε) at b_i^ε if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) < 0$ for all p_{-i}. When $x_i = 1$ is strictly dominant, I say that $x_i = 0$ is strictly dominated; similarly, when $x_i = 1$ is strictly dominant, I say that $x_i = 1$ is strictly dominated.

Lemma 1. *Consider the global game G^ε. (i) For each player i, the action $x_i = 1$ is strictly dominant at all $b_i^\varepsilon \geq \overline{B}$. (ii) For each player i, the action $x_i = 0$ is strictly dominant at $b_i^\varepsilon \leq \underline{B}$.*

Proof. Observe that $\Delta_i(x \mid b) > 0$ for any x given $b \in [\overline{B} - \varepsilon, \overline{B} + \varepsilon]$. Thus, for $b_i^\varepsilon = \overline{B}$ the integration in (5) is over positive terms only and $\Delta_i^\varepsilon(p_{-i} \mid \overline{B}) > 0$ for all p_{-i}. This proves part (i) of the Lemma. The proof of part (ii) relies on a symmetric argument and is therefore omitted. □

Conditional dominance. Let L and R be real numbers. The action $x_i = 1$ is said to be dominant at b_i^ε conditional on R if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) > 0$ for all p_{-i} with $p_j(b_j^\varepsilon) = 1$ for all $b_j^\varepsilon > R$, all $j \neq i$. Similarly, the action $x_i = 0$ is dominant at b_i^ε conditional on L if $\Delta_i^\varepsilon(p_{-i} \mid b_i^\varepsilon) < 0$ for all p_{-i} with $p_j(b_j^\varepsilon) = 1$ for all $b_j^\varepsilon > L$, all $j \neq i$.

The concept of conditional dominance is useful for the following reason. Lemma 1 implies that, for each player j, a strategy p_j of G^ε that prescribes to play $x_j \neq 1$ on a set $b_j^\varepsilon > \overline{B}$ with positive measure is strictly dominated; hence, each player i can effectively assume that each player j will play $p_j(b_j^\varepsilon) = 1$ for all $b_j^\varepsilon \geq \overline{B}$. Eliminating dominated strategies makes, for each player i, $x_i = 1$ strictly dominant for a larger set of observations and hence makes more stratregies of each i strictly dominated; hence, this process can be repeated. Those strategies that survive this process (including elimination of strategies that prescribe playing 1 when that is strictly dominated) are said to survive *iterated elimination of strictly dominated strategies.* For a textbook treatment of iterated dominance, see Osborne and Rubinstein (1994).

Increasing strategies. For some $X \in \mathbb{R}$, let p_i^X denote the particular strategy such that $p_i^X(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < X$ and $p_i^X(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon \geq X$. I will call p_i^X the *increasing strategy with switching point* X. By $p^X = (p_1^X, p_2^X, ..., p_N^X)$ I denote the strategy vector of increasing strategies with switching point X, and $p_{-i}^X = (p_j^X)_{j \neq i}$. Note that $x_i = 1$ is strictly dominant at b_i^ε conditional on R if and only if $\Delta_i^\varepsilon(p_{-i}^R \mid b_i^\varepsilon) > 0$. Similarly, if $x_i = 0$ is strictly dominant at b_i^ε conditional on L then it must hold that $\Delta_i^\varepsilon(p_{-i}^L \mid b_i^\varepsilon) < 0$.

We now have all notation in place to proceed with the core of the analysis.

Iteration from the right. Let i be arbitrary. Take $p_{-i} = p_{-i}^{\overline{B}}$ and note that $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid b_i^\varepsilon)$ is continuous and monotone non-decreasing in b_i^ε. Moreover, recall from Lemma 1 that $x_i = 1$ is strictly dominant at $b_i = \overline{B}$, so $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid \overline{B}) > 0$. By the same Lemma, I also know that $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid \underline{B}) < 0$. Monotonicity and continuity of $\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid b_i^\varepsilon)$ in b_i then imply there exists a point R^1 such that $\underline{B} < R^1 < \overline{B}$ which solves:

$$\Delta_i^\varepsilon(p_{-i}^{\overline{B}} \mid R^1) = 0. \tag{6}$$

To any player i, the action $x_i = 1$ is strictly dominant at all $b_i^\varepsilon > R^1$ conditional on $\overline{B}$.

This argument can be repeated and I obtain a sequence $\overline{B} = R^0, R^1, R^2, ...$ For any $k \geq 0$ and R^k such that $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^k) > 0$, there exists a $R^{k+1} < R^K$ such that $\Delta_i^\varepsilon(p_{-i}^{R^k} \mid R^{k+1}) = 0$. Induction on k allows for the conclusion that (R^k) is a monotone sequence. Moreover, I also know that $R^k \geq \underline{B}$ for all $k \geq 0$ since $x_i = 0$ is strictly dominant at $b_i^\varepsilon < \underline{B}$. Any bounded monotone sequence must converge. I let R^* denote the limit of sequence (R^k). By the definition of a limit, R^* must satisfy:

$$\Delta_i^\varepsilon(p_{-i}^{R^*} \mid R^*) = 0. \tag{7}$$

It follows that a strategy p_i survives iterated elimination of strictly dominated strategies only if $p_i(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon > R^*$, all i.

Iteration from the left. Iterative elimination of strictly dominated strategies yields the point R^* when starting from the right, that is, a range of signals b_i^ε for which $x_i = 1$ is conditionally and strictly dominant. A similar procedure can be performed starting instead from the left, from signals b_i^ε for which $x_i = 0$ is unconditionally and strictly dominant. Because this analysis is symmetric to the procedure discussed above, I omit it in the main text. A complete proof may be found in the appendix.

Lemma 2. *(i) If a strategy p_i survives iterated elimination of strictly dominated strategies, then it must hold that $p_i(b_i^\varepsilon) = 1$ for all $b_i^\varepsilon > R^*$. (ii) If a strategy p_i survives iterated elimination of strictly dominated strategies, then it must hold that $p_i(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < L^*$.*

Proof. Follows immediately from the argument leading up to the Lemma. □

I have derived two limits L^* and R^* that demarcate iterative dominance regions of the signal space. I am going to show that $L^* = R^*$. To prove this, the following result is key.

Proposition 2. *For all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$, the following holds:*

$$\Delta_i^\varepsilon(p_{-i}^X \mid X) = X - \sum_{m=0}^{N-1} \frac{c(m+1)}{N} + d. \tag{8}$$

It follows that $\Delta_i^\varepsilon(p_{-i}^X \mid X)$ is strictly increasing in X for all X such that $\underline{B} + \varepsilon \leq X \leq \overline{B} - \varepsilon$.

Proof. First fix $b \in [\underline{B} + \varepsilon, \overline{B} - \varepsilon]$. Each player $j \neq i$ is assumed to play p_j^X, so the probability that $x_j = 1$ is given by

$$\Pr[b_j^\varepsilon > X \mid b] = \Pr[\varepsilon_j > X - b] = \frac{\varepsilon - X + b}{2\varepsilon}, \tag{9}$$

for all $X \in [b - \varepsilon, b + \varepsilon]$ while $\Pr[b_j^\varepsilon > X \mid b]$ is either 0 or 1 otherwise. Clearly, $x_j = 0$ is played with the complementary probability (given b and X). Since each ε_j is (conditional on b) drawn independently , the probability that m given players $j \neq i$ play $x_j = 1$ while the remaining $N - m - 1$ players play $x_j = 0$ (given p_{-i}^X and b) is therefore:

$$\left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1}. \tag{10}$$

As there are $\binom{N-1}{m}$ unique ways in which m out of $N-1$ players j can choose $x_j = 1$, the total probability of this happening, as a function of b, is:

$$\binom{N-1}{m} \left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1}. \tag{11}$$

When m players $j \neq i$ play $x_j = 1$, the cost of playing $x_i = 1$ to player i is $c(m+1)$.

The derivation so far took b as a known quantity. I now take account of the fact that player i does not observe b directly but only a noisy signal b_i^ε. Given $p_{-i} = p_{-i}^X$ and $b_i^\varepsilon = X$, the expected gain to player i from playing $x_i = 1$ rather than $x_i = 0$ becomes:

$$\begin{aligned}\Delta_i^\varepsilon(p_{-i}^X \mid X) =& \frac{1}{2\varepsilon} \int_{X-\varepsilon}^{X+\varepsilon} b db + d \\ & - \sum_{m=0}^{N-1} c(m+1) \binom{N-1}{m} \frac{1}{2\varepsilon} \int_{X-\varepsilon}^{X+\varepsilon} \left[\frac{\varepsilon - X + b}{2\varepsilon}\right]^m \left[\frac{\varepsilon + X - b}{2\varepsilon}\right]^{N-m-1} db\end{aligned} \tag{12}$$

$$=X + d - \sum_{m=0}^{N-1} c(m+1) \binom{N-1}{m} \int_0^1 q^n (1-q)^{N-m-1} dq \tag{13}$$

$$=X + d - \sum_{m=0}^{N-1} c(m+1) \frac{(N-1)!}{m!\,(N-m-1)!} \frac{m!\,(N-m-1)!}{N!} \tag{14}$$

$$=X + d - \sum_{m=0}^{N-1} \frac{c(m+1)}{N}, \tag{15}$$

as given. Equation (12) takes the expression for $\Delta_i(m \mid b)$ given in (2) and integrates over b and m, given $b_i^\varepsilon = X$ and $p_{-i} = p_{-i}^X$. Equation (13) relies on integration by substitution (using $q = 1/2 - (X-b)/2\varepsilon$) to rewrite the last integral in (12). Equation (14) rewrites both the integral in (13) and the the binomial coefficient $\binom{N-1}{m}$ in terms of factorials. Equation (15) simplifies. □

From the definitions of R^* and L^* given by (7) and (32), using Proposition 2, one can conclude that $L^* = R^*$. I henceforth write B^* where $B^* = L^* = R^*$. The point B^* is given by:

$$B^* = \sum_{n=1}^{N} \frac{c(n)}{N} - d. \tag{16}$$

Thus, if a strategy p_i survives iterated elimination of strictly dominated strategies, then it must hold that $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$ for all $b_i^\varepsilon \neq B^*$. The action prescribed by a strategy p_i that survives iterated dominance can differ from that prescribed by $p_i^{B^*}$ only in the measure-zero event that $b_i^\varepsilon = B^*$. I refer to this by saying that G^ε has an *essentially unique* strategy vector surviving iterated elimination of strictly dominated strategies.

Theorem 1. *For all ε such that $2\varepsilon < \min\{c(N) - d - \underline{B}, \overline{B} - c(1) + d\}$, the strategy vector p^{B^*} is the essentially unique strategy vector surviving iterated elimination of strictly dominated strategies of the game G^ε. In particular, if, for any player i, the strategy p_i survives iterated elimination of strictly dominated strategies, then p_i must satisfy $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$ for all $b_i^\varepsilon \neq B^*$.*

Theorem 1 holds for general $\varepsilon > 0$ provided the assumption that b and ε_i (all i) are drawn independently from the uniform distribution (which is symmetric for ε_i). For the limit as $\varepsilon \to 0$, Frankel et al. (2003) establish the very general result that *any* global game with strategic complementarities in which b is drawn from any continuous density with connected support and each ε_i is drawn independently from any (possible player-specific) atomless density has an essentially unique strategy vector surviving iterated elimination of strictly dominated strategies in the limit as $\varepsilon \to 0$. Moreover, for potential games the equilibrium selected is noise independent and given by the vector of strategies in which each player i chooses the action that coincides with the potential maximizer of the game. This means that the strategy vector found in Theorem 1 generalizes to far more general distributions than assumed here.[7]

Recall that a strategy vector $p = (p_1, p_2, ..., p_N)$ is a Bayesian Nash equilibrium (BNE) of G^ε if for any p_i and any b_i^ε it holds that:

$$p_i(b_i^\varepsilon) \in \arg\max_{x_i \in \{0,1\}} \pi_i^\varepsilon(x_i, p_{-i}(b_{-i}^\varepsilon) \mid b_i^\varepsilon), \tag{17}$$

where $\pi_i^\varepsilon(x_i, p_{-i}(b_{-i}^\varepsilon) \mid b_i^\varepsilon) = \int \pi_i(x_i, p_{-i}(b_{-i}^\varepsilon) \mid b)\, d\Phi^\varepsilon(b, b_{-i}^\varepsilon \mid b_i^\varepsilon)$. It is therefore immediate that p^{B^*} is a BNE of G^ε. The following proposition establishes a much stronger result: if the strategy vector $p = (p_i)$ is a BNE of G^ε, then for each p_i it must hold that $p_i(b_i^\varepsilon) = p^{B^*}(b_i^\varepsilon)$ for all $b_i^\varepsilon \neq B^*$. I say that G^ε has an essentially unique BNE.

Theorem 2. *The strategy vector p^{B^*} is the essentially unique Bayesian Nash equilibrium of the game G^ε. In particular, any equilibrium strategy p_i satisfies $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$ for all $b_i^\varepsilon \neq B^*$ and all players i.*

Proof. In the appendix. □

Theorem 2 should not be misunderstood as saying that players will perfectly coordinate their actions (investments).[8] For $\varepsilon > 0$, it is possible that some players receive signals above B^* while others see a signal below it. When this occurs, players will fail to coordinate their actions (i.e. some will choose $x_i = 1$ while others choose $x_i = 0$). The implication is that empirically observed coordination failures are not necessarily at odds with, and therefore do not by themselves invalidate the model. When a coordination failure occurs, the equilibrium outcome is inefficient. In the limit as $\varepsilon \to 0$, the global climate game G^ε selects an essentially unique equilibrium of the underlying coordination game with multiple equilibria. To see this, note that for any $b > B^*$, I can find a $\varepsilon < B^* - b$ so that $b - \varepsilon > B^*$. Since $b_i^\varepsilon \in [b - \varepsilon, b + \varepsilon]$ and $p^* = p^{B^*}$ this implies that $p_i^*(b_i^\varepsilon) = 1$ for all b_i^ε consistent with b and all i.

Even as $\varepsilon \to 0$ and players coordinate their actions with probability 1, the unique equilibrium can be inefficient. In particular, players coordinate on $\mathbf{0}$ (all adopt the dirty technology) for all $b < B^*$ even though the outcome in which players coordinate on $\mathbf{1}$ (all adopt the clean technology) is Pareto strictly dominant for all $b > \bar{b}$ (and even though they know it). The result stands in contrast to the common and often implicit assumption in the environmental literature that players generally coordinate on the efficient equilibrium (Barrett, 2006; Hoel and de Zeeuw, 2010).

Corollary 1. *(i) For all $b > B^* + \varepsilon$ it holds that* $\Pr\left[p^{B^*}(b^\varepsilon) = \mathbf{1}\right] = 1$. *(ii) For all $b < B^* - \varepsilon$ it holds that* $\Pr\left[p^{B^*}(b^\varepsilon) = \mathbf{0}\right] = 1$.

[7] In particular, the reader is referred to their result on (local) potential games with own-action quasi-concave payoffs, i.e. Theorem 4.

[8] Perfect coordination of actions means that all players choose the same action.

4 Network Subsidies

The potential of an inefficient outcome in both the game of complete information $G(b)$ and the global game G^ε begs the question how a policymaker can influence the game in order to reach an efficient outcome. In this section, I assume that there exists a policymaker who, using taxes and subsidies, has the ability to change the payoffs in the game; the players remain as assumed throughout Sections 2 and 3. I will study the policymaker's problem of finding a way to influence players' incentives so as to implement the Pareto efficient outcome of the game in strictly dominant strategies. That is, I seek to find policies that turn playing $x_i = 1$ into a strictly dominant strategy whenever coordination on $\mathbf{1}$ is also the efficient outcome of the game; similarly, I want $x_i = 0$ to be a strictly dominant strategy when coordination on $\mathbf{0}$ is Pareto efficient.[9] I assume that the policymaker is fully informed about players' possible actions as well as the parameters of the model that are common knowledge among the players. I also assume that the policymaker understands players' payoff-maximization incentives. While in the most general setup, the policymaker has a vast array of possible policies at its disposal (including outright command-and-control), to stay close with the application to climate change I shall confine the set of feasible policies to subsidies and taxes only.

Taxes and subsidies are commonly used to stimulate the adoption of clean technologies. The U.S. Federal Tax Credit for Solar Photovoltaics (Borenstein, 2017), California's Clean Vehicle Rebate Project (Li et al., 2017), or the U.S. National Plug-In Electric Drive Vehicle Credit (Clinton and Steinberg, 2019) are some illustrations. However, tax policies may not always be feasible, political or otherwise. For example, legislation on taxation requires unanimous agreement in the European Union, which is one reason the EU does not have a carbon tax. Subsidies and tax credits, on the other hand, come with a substantial budgetary burden. The Congressional Budget Office expects total cost from tax credits on electric vehicles to be about 7.5 billion U.S. dollars through 2019.[10] Planned spending on SDE++ subsidies in the Netherlands are 5 billion Euros in 2021.[11].

The expenditure on subsidies to stimulate clean technology adoption can be substantially mitigated without the need to compromise on efficiency through the use

[9]This question is related to the literature on mechanism design and (strictly dominant strategy) implementation. That is, I study the problem of a policymaker who seeks to change the original game studied in Section 2 and 3 with the aim of making coordination on the efficient outcome of the game a strictly dominant strategy for all players (Laffont and Maskin, 1982; Myerson and Satterthwaite, 1983; Kuzmics and Steg, 2017). For applications of mechanism design and implementation theory to pollution problems like climate change, see Duggan and Roberts (2002), Ambec and Ehlers (2016), and Martimort and Sand-Zantman (2016). As an extension for future work directly related to the mechanism design literature, I hope to explicitly compare network subsidies the the well-studied Vicker-Clarke-Groves mechanism (such a comparison is also made in Ambec and Ehlers, 2016).

[10]See https://www.cbo.gov/sites/default/files/112th-congress-2011-2012/reports/electricvehiclesone-col.pdf

[11]See https://www.rvo.nl/subsidie-en-financieringswijzer/sde. SDE is an acronym for Stimulering Duurzame Energievoorziening en Klimaattransitie, or "Stimulus Sustainable Energy Supply and Climate Transition".

of *network subsidies*. A network subsidy, like any subsidy, is offered contingent on adoption of the clean technology. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number.

Network subsidies provide a cheap yet effective policy instrument to stimulate the adoption of clean technologies. The literature on directed technical change has pointed out that there often exists a need for both taxes and subsidies in an efficient policy when the (clean) technologies exhibit positive external spillovers (Acemoglu et al., 2012; Aghion et al., 2016; Hart, 2019). A policy of network subsidies might provide a relatively low-cost opportunity to direct technological change toward green technologies.

4.1 Game of Complete Information

Consider again the game of complete information (about b) discussed in Section 2. Recall from Proposition 1 that coordination on $x = \mathbf{1}$ is the Pareto strictly dominant outcome of the game for all $b > \bar{b}$, whereas coordination on $x = \mathbf{0}$ is efficient for all $b < \bar{b}$.

My aim is to find a subsidy that incentivizes players to coordinate on the efficient outcome of the game for any b. Concretely, I want to formulate a tax and/or subsidy policy that makes $x_i = 1$ strictly dominant for all $b > \bar{b}$ while $x_i = 0$ becomes strictly dominant at $b < \bar{b}$. I say that such a subsidy *implements* the efficient outcome of the game in strictly dominant strategies for almost all b, i.e., for all b except $\bar{b}$.

First I will show that if $\mathbf{1}$ is a (strict) Nash equilibrium of $G(b)$, then the efficient outcome of the game can be implemented in (strictly) strictly dominant strategies at zero cost, even if $\mathbf{0}$ is also a strict Nash equilibrium. The idea will be to offer players choosing $x_i = 1$ a subsidy that guarantees them a payoff equal (when choosing 1) to what they would have realized in the hypothetical case that all other players also chose 1 – that is, to offer a subsidy that promises players a payoff as though they enjoyed the full extent of the network externality. To this end, let the policymaker offer a *network subsidy* $s^*(x)$ to each i choosing $x_i = 1$ when x is played. For each x, define $s^*(x)$ to be the function given by:

$$s^*(x) = \Delta_i(\mathbf{1}_{-i} \mid b) - \Delta_i(x_{-i} \mid b) = c(n(x)) - c(N). \tag{18}$$

Players choosing $x_i = 0$ do not receive a network subsidy. Observe that, conditional on $s^*(\cdot)$, an individual players' gain from playing 1, rather than 0, is:

$$\Delta_i(x_{-i} \mid b) + s^*(x) = \Delta_i(x_{-i} \mid b) + \Delta_i(\mathbf{1}_{-i} \mid b) - \Delta_i(x_{-i} \mid b) = \Delta_i(\mathbf{1}_{-i} \mid b), \tag{19}$$

for any x, confirming the claim that a network subsidy scheme $s^*(\cdot)$ allows players to consider only the gain $\Delta_i(\mathbf{1}_{-i} \mid b) = b - c(N) + d$ when choosing their actions.

Theorem 3. *Let $G(b \mid s^*)$ denote the game $G(b)$ in which players are offered the network subsidy $s^*(\cdot)$ on playing* 1.

(i) If $\mathbf{1}$ *is a Nash equilibrium of* $G(b)$ *(i.e. if* $b+d \geq c(N)$*), then* $\mathbf{1}$ *is implemented in weakly dominant strategies with* $s^*(\cdot)$ *and no subsidies have to be paid.*

(ii) If $\mathbf{1}$ *is a strict Nash equilibrium of* $G(b)$ *(i.e. if* $b+d > c(N)$*), then* $\mathbf{1}$ *is implemented in strictly dominant strategies with* $s^*(\cdot)$ *and no subsidies have to be paid.*

(iii) If $\mathbf{1}$ *is not a Nash equilibrium of* $G(b)$ *(i.e. if* $b+d < c(N)$*), then* $\mathbf{0}$ *is implemented in strictly dominant strategies with* $s^*(\cdot)$ *and no subsidies have to be paid.*

Proof. The gain from choosing $x_i = 1$ rather than $x_i = 0$, conditional on the network subsidy scheme $s^*(\cdot)$, given b and x_{-i} is (19) which, for all x_{-i}, is (strictly) positive if and only if $\mathbf{1}$ is a (strict) Nash equilibrium of the game. Thus, the offering a subsidy scheme equal to $s^*(\cdot)$ turns $x_i = 1$ into a (strictly) strictly dominant strategy whenever $\mathbf{1}$ is a (strict) Nash equilibrium of $G(b)$. When players coordinate on $\mathbf{1}$ total spending on network subsidies is $N \cdot s^*(\mathbf{1}) = 0$. □

Note that for all $b > c(N) - d$, and provided the network subsidy scheme $s^*(\cdot)$ is offered, the policymaker may even tax playing $x_i = 1$ yet still implement $\mathbf{1}$ in strictly dominant strategies.

Remark 1. *Let* $b > c(N) - d$*, so* $\mathbf{1}$ *is both a strict Nash equilibrium and the efficient outcome of the game* $G(b)$*. If the policymaker offers the network subsidy scheme* $s^*(\cdot)$*, the policymaker can impose a tax* $t(b) \leq b + d - c(N)$ *on playing* $x_i = 1$ *but nevertheless implement coordination on* $\mathbf{1}$ *in strictly dominant strategies.*

Theorem 3 tells us that a smart policy of network subsidies allows the policymaker costlessly to implement the efficient Nash equilibrium of $G(b)$ in (strictly) strictly dominant strategies if the game has multiple (strict) Nash equilibria. While this is a desirable property, it does not guarantee that a network subsidy scheme implements the efficient outcome of the game for all b. To see this, observe that $\mathbf{0}$ is the unique strict Nash equilibrium of $G(b)$ for all $b < c(N) - d$, while $\mathbf{1}$ is the efficient outcome for all $b > \bar{b} = (c(N) - d)/N$. Hence, if $c(N) > d$ the policymaker cannot implement the efficient outcome of the game for all $b \in (\bar{b}, c(N) - d)$ using a network subsidy scheme alone.

I next show that if $\mathbf{1}$ is not a Nash equilibrium of the game $G(b)$, but $\mathbf{1}$ is the efficient outcome, then in order to implement $\mathbf{1}$ in strictly dominant strategies, the policymaker can use a dual tax-subsidy scheme to achieve its goal. First, let the policymaker again offer the network subsidy scheme given by $s^*(\cdot)$. As I saw before, the net (accounting for subsidies) gain from playing 1 rather than 0 becomes $\Delta_i(x_{-i} \mid b) + s^*(x) = \Delta_i(\mathbf{1}_{-i} \mid b)$ when players are offered $s^*(\cdot)$. Second, let the policymaker levy an *environmental tax* $t(b)$ to playing 0. The purpose of the environmental tax is to make sure that $\Delta_i(\mathbf{1}_{-i} \mid b) + t(b) > 0$ for all $b > \bar{b}$ while $\Delta_i(\mathbf{1}_{-i} \mid b) + t(b) < 0$ for all $b < \bar{b}$; that is, the tax should make $\mathbf{1}$ a Nash equilibrium of the game if and only if $\mathbf{1}$ is also the efficient outcome; otherwise $\mathbf{0}$ should be the equilibrium. A tax $t(b)$ that achieves this is given by:

$$t(b) > \Delta(\mathbf{1}_{-i} \mid c(N) - d) - \Delta(\mathbf{1}_{-i} \mid b) = c(N) - d - b \quad \text{if} \quad b \geq \bar{b}, \tag{20}$$

while $t(b) = 0$ otherwise. It is easy to verify that $t(b)$ implements coordination on $\mathbf{1}$ as a strict Nash equilibrium for all $b > \bar{b}$ while leaving $x_i = 0$ strictly dominant for all $b < \bar{b}$.

Theorem 4. *Let $G(b \mid s^*, t)$ denote the game $G(b)$ in which the policymaker both offers the network subsidy scheme $s^*(\cdot)$ and levies the environmental tax $t(b)$. If $\mathbf{1}$ is not a Nash equilibrium of $G(b)$ (i.e. if $b < c(N) - d$), but $\mathbf{1}$ is the Pareto efficient outcome (i.e. $b > \bar{b}$), then, by taxing $x_i = 0$ through $t(b)$ while also offering a network subsidy $s^*(\cdot)$ to playing $x_i = 1$, $\mathbf{1}$ can be implemented in strictly dominant strategies at no cost (and tax revenues will be zero).*

The combination of a taxes and subsidies discussed in Theorem 4 strongly resembles the recommendation by Acemoglu et al. (2012, 2016), Hart (2019) and Harstad (2020) to combine a carbon tax on the use of dirty technologies with a subsidy on (R&D in) renewables. In their setups, tax is meant to correct the environmental externality deriving from emissions whereas the subsidy addresses the network externality, or strategic complementarity, in clean technologies. Much the same mechanism is at work here. The environmental tax is a Pigouvian tax that solves the environmental externality from use of the dirty technology. The network subsidy instead solves the coordination problem created strategic complementarities. An important distinction is that spending on renewables subsidies in Acemoglu et al. (2012, 2016) and Harstad (2020) is costly; spending on network subsidies is zero in equilibrium.

Why does a network subsidy work so well despite the low cost? The key property of a network subsidy set at $s^*(\cdot)$ is that it eliminates all *strategic uncertainty*, i.e. the uncertainty a player has about the actions chosen by all other players. The network subsidy thus removes the payoff uncertainty deriving from strategic uncertainty interacted with technological spillovers – it turns the original coordination game into a simple dominance solvable game for all b. In so doing, the network subsidy manages to eliminate all inefficiencies caused by players' failure to internalize the *technological* spillovers inherent in clean investments. Intuitively, the network subsidy works like an insurance. It protects individual investors against the risk of small network externalities from clean investments in case many others have adopted the dirty technology. Because of that insurance, it impels individuals toward clean investments. The network subsidy does not have to be paid as a result, being conditional on low investments.

Governments may at times be reluctant to rely on (carbon) taxes when trying to curb private sector emissions, for example because taxes are unpopular with voters. When this is true, the government cannot (or at least does not want to) levy the carbon tax $t(b)$ but may rather rely on an environmental *subsidy* $s(b)$ on playing 1.

Remark 2. *If $\mathbf{1}$ is not a Nash equilibrium of $G(b)$ (i.e. if $b < c(N) - d$), but $\mathbf{1}$ is the Pareto efficient outcome, then, by subsidizing $x_i = 1$ through $s(b) = c(N) - d - b$*

while also offering a network subsidy $s(\cdot)$ to playing $x_i = 1$, $\mathbf{1}$ can be implemented in strictly dominant strategies. Total subsidy spending will be $N \cdot s(b)$ when $b > \bar{b}$, and zero otherwise.

Note that a network subsidy differs from a Pigouvian subsidy on using the clean technology. A Pigouvian subsidy incentivizes each player to incorporate the effect his own choice of action has on the payoffs of all others; a network subsidy neutralizes the effect all other players' actions have on each individual player's choice of action.

4.2 Global Game

Consider the global game G^ε discussed in Section 3. In this game, players do not observe b but only some noisy private signal of it. I henceforth assume that the policymaker observes neither the true b nor a signal of it.

In this section I address the question of what tax-subsidy scheme suffices to implement the Pareto efficient outcome of the underlying game $G(b)$ in strictly dominant strategies for all b. I will assume the policymaker seeks policies that, for each player $i = 1, 2, .., N$, turn $x_i = 1$ into a strictly dominant action for all $b_i^\varepsilon > \bar{b}$ while leaving $x_i = 0$ strictly dominant for all $b_i^\varepsilon < \bar{b}$.[12] I will also assume that the policy scheme does not depend on the unobserved true b.[13].

First, let us again assume the policymaker offers each player a network subsidy s^* equal to:

$$s^*(x) = c(n(x)) - c(N), \tag{21}$$

which is the same network subsidy as in (18). It is easy to verify that the network subsidy $s^*(\cdot)$ makes playing 1 strictly dominant for all $b_i^\varepsilon > c(N) - d$. When players are offered $s^*(\cdot)$ for each x_{-i}, their expected gain (the expectation is over b) is:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon), \tag{22}$$

where $\Delta_i^\varepsilon(x_{-i} \mid b) := \frac{1}{2\varepsilon} \int_{b_i^\varepsilon - \varepsilon}^{b_i^\varepsilon + \varepsilon} \Delta(x_{-i} \mid b) db$. Note that $\Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon)$ is strictly positive for all $b_i^\varepsilon > c(N) - d$ and strictly negative for all $b_i^\varepsilon < c(N) - d$. Let $G^\varepsilon(s^*)$ denote the global game G^ε in which the policymaker offers the network subsidy scheme $s^*(\cdot)$.

[12] I use the word "leaving" because in the global game G^ε *without* policy intervention, playing $x_i = 0$ is already strictly dominant for all i and all $b_i^\varepsilon < \bar{b} < B^*$.

[13] To be more precise, I assume that the only observables on which the policy scheme depends are players' actions. This is a restrictive assumption. Players possess private information (their signals) about b and this information is correlated. We thus know from the literature on mechanism design that the policymaker can (costlessly) extract the vector of signals $b^\varepsilon = (b_1^\varepsilon, b_2^\varepsilon, ..., b_N^\varepsilon)$ from the players (Crémer and McLean, 1988; McAfee and Reny, 1992). Especially when ε is small, knowing b^ε would provide an almost perfect signal of the true b to the policymaker. It seems intuitive that the policymaker might use this knowledge to its benefit (and the benefit of all players as a whole). I hope to investigate this issue – including a direct comparison between a network subsidy and the Vickrey–Clarke–Groves mechanism – in future work.

Lemma 3. *Consider the global game G^ε. Let the policymaker offer a network subsidy $s^*(\cdot)$ on playing 1. Then the action $x_i = 0$ is strictly dominant at $b_i^\varepsilon < c(N) - d$; the action $x_i = 1$ is strictly dominant at $b_i^\varepsilon > c(N) - d$.*

As in the case of complete information, a network subsidy alone may not suffice to implement the efficient outcome of the game; for all $b \in (\bar{b}, c(N) - d - \varepsilon)$, each player i receives a signal $b_i^\varepsilon < c(N) - d$ so playing 0 is strictly dominant despite the network subsidy. Therefore, let the policymaker – on top of the network subsidy – levy an environmental tax $\bar{t}$ on playing $x_i = 0$ that makes $x_i = 1$ strictly dominant, for all $b_i^\varepsilon > \bar{b}$ and all i, constrained by the condition that $x_i = 0$ should still be strictly dominant (despite both the subsidy and the tax) for all $b_i^\varepsilon < \bar{b}$. Thus, the policymaker wants to find a tax $\bar{t}$ (recall again the restrictive assumption that $\bar{t}$ does not depend on players' private knowledge of b) that solves:

$$\begin{aligned} \Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) + \bar{t} = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t} > 0 \quad \text{for all} \quad b_i^\varepsilon > \bar{b} \\ \Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) + \bar{t} = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t} < 0 \quad \text{for all} \quad b_i^\varepsilon < \bar{b}, \end{aligned} \tag{23}$$

for all i and all x_{-i}. It follows that $\bar{t}$ is given by:

$$\bar{t} = (N-1) \cdot \bar{b} = (N-1) \cdot \frac{c(N) - d}{N}. \tag{24}$$

Let $G^\varepsilon(s^*, \bar{t})$ denote the global game G^ε in which the policymaker both offers the network subsidy scheme $s^*(\cdot)$ and levies an environmental tax $\bar{t}$. The following result regarding $G^\varepsilon(s^*, \bar{t})$ is immediate.

Proposition 3. *Consider the global game $G^\varepsilon(s^*, \bar{t})$. If the policymaker offers a network subsidy $s^*(\cdot)$ on playing $x_i = 1$ and levies a tax $\bar{t}$ on playing $x_i = 0$, then, for each player i, the action $x_i = 0$ is strictly dominant for all $b_i^\varepsilon < \bar{b}$ and the action $x_i = 1$ is strictly dominant for all $b_i^\varepsilon > \bar{b}$. Hence, for all $b \notin [\bar{b} - \varepsilon, \bar{b} + \varepsilon]$ the policymaker can implement the efficient outcome of the game $G(b)$ in strictly dominant actions at no cost.*

Proof. Strict dominance is an immediate consequence of rewriting the player i's gain including taxes and subsidies:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + s^*(x) + \bar{t} = \Delta_i^\varepsilon(\mathbf{1} \mid b_i^\varepsilon) + \bar{t} = b_i^\varepsilon + c(N) - d + (N-1) \cdot \frac{c(N) - d}{N}, \tag{25}$$

which is strictly positive for all $b_i^\varepsilon > \bar{b}$ and strictly negative for all $b_i^\varepsilon < \bar{b}$. As to the final claim in the Proposition, observe that each b_i^ε is drawn from $[b - \varepsilon, b + \varepsilon]$, given b. Hence, if $b > \bar{b} + \varepsilon$ then $b_i^\varepsilon > \bar{b}$ for each i, so playing $x_i = 1$ is strictly dominant and players coordinate on $\mathbf{1}$, the efficient outcome of the game (for those b). In this case, total spending on subsidies is $s^*(\mathbf{1}) = 0$. Similarly, if $b < \bar{b} - \varepsilon$ then $b_i^\varepsilon < \bar{b}$ for each i, so playing $x_i = 0$ is strictly dominant and players coordinate on $\mathbf{0}$, the efficient outcome of the game (for those b). Since no player plays 1, total subsidy spending is naturally zero. □

If the policymaker, for whatever reason, is reluctant to tax playing 0, it may also offer both a network subsidy $s^*(\cdot)$ together with an environmental *subsidy* equal to $\bar{t}$ to playing 1. Such a policy is evidently equivalent with regard to players' incentives, although it differs for the policymaker's budget.

Corollary 2. *Consider the global game G^ε. Let the policymaker offer a network subsidy $s^*(\cdot)$ on playing 1. In addition, let the policymaker offer an environmental subsidy (rather than a tax) equal to $\bar{t}$ on playing 1. Then the action $x_i = 0$ is strictly dominant for all $b_i^\varepsilon < \bar{b}$ while the action $x_i = 1$ is strictly dominant for all $b_i^\varepsilon > \bar{b}$. Hence, the policymaker can implement the efficient outcome of the game $G(b)$ for all $b \notin [\bar{b}-\varepsilon, \bar{b}+\varepsilon]$; total subsidy spending is $N \cdot \bar{b}$ if $b > \bar{b} + \varepsilon$ and 0 if $b < \bar{b} - \varepsilon$.*

If a true b in $(\bar{b} - \varepsilon, \bar{b} + \varepsilon)$ is drawn, players may fail to coordinate on either $\mathbf{0}$ or $\mathbf{1}$ even when the policymaker offers the network subsidy $s^*(\cdot)$ and levies the tax $\bar{t}$. The reason is that, for those b, players' signals need not all fall in the strict dominance regions identified in Proposition 3, and a coordination failure may easily arise. The network subsidy scheme $s^*(\cdot)$ may hence not be costless; for any x not equal to $\mathbf{0}$ or $\mathbf{1}$, total spending on network subsidies will be $n(x) \cdot s^*(x) > 0$. Thus, the remarkably strong performance of a network subsidy scheme may break down in the global game. For $\varepsilon > 0$, the event that a true b in $(\bar{b} - \varepsilon, \bar{b} + \varepsilon)$ is drawn occurs with strictly positive probability, namely $2\varepsilon/(\overline{B} - \underline{B}) > 0$. Only in the limit as $\varepsilon \to 0$ will this problem disappear: players perfectly coordinate their actions (in equilibrium) save for the probability-zero event that $b = \bar{b}$.

The fact that spending on network subsidies may not be zero in the global game is unfortunate. To remedy this problem, I now derive a network tax-subsidy scheme where subsidy payments on $x_i = 1$ are financed through a "network tax" levied on choosing $x_i = 0$.[14] Let the subsidy be denoted $s^{**}(x)$; the corresponding tax is denoted $t^{**}(x)$. Thus, when x is played, aggregate spending on network subsidies is $n(x) \cdot s^{**}(x)$; aggregate revenues from the network tax are $(N - n(x)) \cdot t^{**}(x)$. If I want the tax-subsidy scheme to be costless, or self-financed, the budget constraint for this scheme is given by:

$$(N - n(x)) \cdot t^{**}(x) - n(x) \cdot s^{**}(x) = 0, \tag{26}$$

which should hold for all x. Condition (26) imposes that total spending on the network subsidies to those playing 1 is matched exactly by total tax revenues from taxing those who play 0, whatever players end up playing.

Next, the tax-subsidy scheme, together with the environmental tax $\bar{t}$ given by (24), must make $x_i = 1$ strictly dominant for all $b_i^\varepsilon > \bar{b}$ while leaving $x_i = 0$ strictly dominant

[14] An alternative approach to this problem would be to let the policymaker extract players' private signals (see footnote 13) and then construct a policy scheme such that, when the signals indicate a high b, the policymaker may tax playing 1 similarly to the way discussed in Remark 1. The policymaker could then construct this policy in such a way that ex ante, i.e. before b is drawn, the policy scheme has expected cost zero. This is different from the present analysis, which is more demanding and imposes ex post budget neutrality.

for all $b_i^\varepsilon < \bar{b}$. Thus players' gains from playing 1, rather than 0, accounting for taxes and subsidies, should satisfy:

$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + \bar{t} + s^{**}(x) + t^{**}(x) > 0 \quad \text{for all} \quad b_i^\varepsilon > \bar{b}, \tag{27}$$
$$\Delta_i^\varepsilon(x_{-i} \mid b_i^\varepsilon) + \bar{t} + s^{**}(x) + t^{**}(x) < 0 \quad \text{for all} \quad b_i^\varepsilon < \bar{b}, \tag{28}$$

for all i and all x_{-i}. Equations (27) and (28) represent the incentive constraints of a network tax-subsidy scheme. Combined with the budget constraint, this yields the following network tax-subsidy scheme (s^{**}, t^{**}):

$$\begin{cases} t^{**}(x) = & \frac{n(x)}{N}\left[c(n(x)) - c(N)\right] \\ s^{**}(x) = & \frac{N-n(x)}{N}\left[c(n(x)) - c(N)\right] \end{cases} \tag{29}$$

The policy scheme $((s^{**}, t^{**}), \bar{t})$ can now be summarized as follows. When x is played and player i has played 1 in x, they receive a network subsidy equal to $s^{**}(x)$; however, if player i played 0 in x, they pay a tax equal to $\bar{t} + t^{**}(x)$. Let $G^\varepsilon((s^{**}, t^{**}), \bar{t})$ denote the global game G^ε in which the policymaker both offers the network subsidy scheme $s^*(\cdot)$ and levies an environmental tax $\bar{t}$.

Proposition 4. *Consider the global game $G^\varepsilon((s^{**}, t^{**}), \bar{t})$. Let the policymaker offer a network subsidy equal to $s^{**}(\cdot)$ on playing 1 while levying a tax equal to $\bar{t} + t^{**}(\cdot)$ on playing 0. This policy makes the action $x_i = 0$ strictly dominant for all $b_i^\varepsilon < \bar{b}$; the action $x_i = 1$ is strictly dominant for all $b_i^\varepsilon > \bar{b}$. Consequently, the policymaker can implement efficient outcome of the game $G(b)$ for all $b \notin [\bar{b} - \varepsilon, \bar{b} + \varepsilon]$; net spending on the policy scheme $((s^{**}, t^{**}), \bar{t})$ is zero for all b.*

Proof. All parts of the propositions follow immediately from the construction preceding it. □

The present analysis did not make use of the fact that, without policy interventions, playing p^{B^*} is the essentially unique strategy profile surviving iterated dominance in the global game G^ε. Another approach toward ((iterative) strictly dominant strategy) implementation in G^ε would be to study what mechanisms the policymaker could design to shift the threshold B^* down toward $\bar{b}$. I intend to do this in future work.

Finally, observe that the logic of a network subsidy does not rely on the application to climate change. Any market where (i) individual actions exhibit strategic complementarities and (ii) players do not take these into account may coordinate on an inefficient equilibrium. A network subsidy then offers an inexpensive way out of this trap. For this reason, my analysis of network subsidies is a contribution to public economics more generally.

5 Summary

This paper studies climate change mitigation in a global game. I focus on abatement through technological investment. Players invest in either of two technologies. One technology is cheap and dirty, such as fossil fuels. The other technology is clean but expensive, like renewables. My focus is on the case in which investments are strategic complements; that is, I consider the choice of technology in the context of a coordination problem. Consistent with the existing literature on technological investments in clean technologies, I demonstrate that the complete information version of my game has multiple equilibria (Barrett, 2006; Hoel and de Zeeuw, 2010; Barrett and Dannenberg, 2012, 2017). Equilibrium multiplicity can lead to coordination failure and complicates the design of domestic policies or climate treaties.

To this well-studied framework, I add a little bit of uncertainty. I assume that the true environmental benefit of the clean technology is unobserved. Rather than observe the technology's true benefit, players receive private noisy signals of it. In this *global climate game*, I show that there exists a unique equilibrium in which players adopt the clean technology if and only if their private signals exceed an endogenous threshold. For signals below the threshold, players adopt the dirty technology instead.

My first contribution is to show that the game has an essentially unique Bayesian Nah equilibrium. This contribution directly addresses the issue of equilibrium multiplicity often encountered in this literature. Equilibrium selection is driven by the assumption of incomplete information, which seems realistic in the context of clean technologies and climate change. The derivation of an essentially unique equilibrium connects this paper to the substantially more general literature on global games (Carlsson and Van Damme, 1993; Frankel et al., 2003). The analysis highlights that the precise way in which one models uncertainty is important. Although some papers conclude that "shared" uncertainty does not eliminate equilibrium multiplicity in coordination games (Barrett and Dannenberg, 2012), this paper shows the starkly contrasting result that privately held beliefs about the shared game does force selection of a unique equilibrium. The assumed structure of uncertainty matters. My result motivates a critical approach toward the modeling of uncertainty in environmental economics.

My second contribution is to introduce network subsidies. The issue of policy arises naturally in the present context where investments cause two externalities. One is an environmental externality that derives from the positive environmental effect clean investments have on all players; the other is a network externality deriving from spillovers (strategic complementarities) in clean investments. As I show, using only taxes and subsidies the policymaker can costlessly correct both of these externalities by turning the (expected) efficient action into a strictly dominant action for all players (both in the game of complete information and in the global game). Moreover, I also show that even when using only subsidies, the policymaker can correct the entire network externality at zero cost. The innovation here is what I call a network subsidy. Like standard subsidies, a network subsidy offers adopters of the clean technology a (financial) reward. In contrast to a standard subsidy scheme, however, the amount of the subsidy upon adoption also depends on how many others adopt the clean technology, and is decreasing in that number. As I show, it is possible to construct a simple network subsidy scheme that corrects the entire network externality deriving from players' ignorance of the technological spillovers caused by their clean investment but does not, in equilibrium, cost the policymaker anything. Intuitively, the network subsidy serves as an insurance against small clean technology networks. In so doing, it boosts clean investments and therefore is never claimed. Although derived in the context of technological spillovers, the notion of a network subsidy is general and applies to public economic broadly.

My derivation of network subsidies can be considered a restrictive yet simple exercise in mechanism design or implementation theory. While mechanism design has been applied to environmental economics and, in particular, emissions mitigation before (Duggan and Roberts, 2002; Ambec and Ehlers, 2016; Martimort and Sand-Zantman, 2016), these papers tend to construct mechanisms that solve the free-rider problem. In this paper, I instead derive a mechanism to overcome the coordination problem. My approach is therefore complementary to these earlier constributions.

Network subsidies also contribute to the literature on directed technical change and renewable subsidies (Acemoglu et al., 2012, 2016; Hart, 2019; Harstad, 2020). My analysis echoes the recommendation by these authors that an efficient policy generally requires a combination of a carbon tax *and* a subsidy on renewables. However, I obtain two notably different results. If coordination on renewables is already an equilibrium of the game (even if coordination on fossil fuels is too), only a network subsidy suffices to implement the efficient outcome of the game. In those cases, the first-best can be achieved at zero cost. Moreover, even if coordination on renewables is not an equilibrium and a combination of a carbon tax and a network subsidy is needed, spending on network subsidies is zero in equilibrium.

An important shortcoming of my study compared to the literature on directed technical change is the simplicity of my model. To mention some important differences, I study a one-period decision problem rather than a repeated game, and in implicitly assuming that technologies are perfectly substitutable I shy away from discussions on the effect of imperfect substitutability on policy. Thus, while a network subsidy might offer a new kind of policy to consider in discussions on directed technical change, more work is required to design network subsidy schemes for the kinds of contexts normally studied in this literature.

The results in this paper are derived for specific assumptions on functional forms, the distribution of noise, the policymaker's knowledge, and timing. While results due to Frankel et al. (2003) establish that equilibrium selection continues to occur in far more general global coordination games, such generalizations are not investigated here. The the cost-effectiveness of network subsidies would also seem to generalize; ongoing work due to Heijmans & Suetens (available upon request) confirms that intuition. Similarly, I assumed that the policymakers knows all parameters of the model known to the players; it is not clear how to design a network subsidy scheme (and what its properties would be) if the policymaker knows less. Finally, I study a static game in which decisions are taken only once. In relevant real-world cases, technology adoption and policy are inherently dynamic; this is a restrictive simplification which future work might relax.

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A Omitted proofs

Proof for the second half of Lemma 2.

Proof. From Lemma 1 it is known that $x_i = 0$ is strictly dominant at $b_i^\varepsilon < \underline{B}$. That is, $\Delta_i^\varepsilon(p_{-i}^{\underline{B}} \mid \underline{B}) < 0$. Since it is common knowledge that no player plays a strictly dominated strategy, a payoff maximizing player i then finds a point L^1 such that $x_i = 0$ is strictly dominant $b_i^\varepsilon < L^1$ conditional on $\underline{B}$:

$$\Delta_i^\varepsilon(p_{-i}^{\underline{B}} \mid L^1) = 0. \tag{30}$$

Any expected payoff maximizing player i plays $x_i = 0$ for all $b_i^\varepsilon < L^1$. Since this is common knowledge also, I can repeat the argument over and over. What I obtain is a sequence of points (L^k), $k \geq 0$, each term of which is implicitly defined by:

$$\Delta_i^\varepsilon(p_{-i}^{L^k} \mid L^{k+1}) = 0. \tag{31}$$

The sequence (L^k) is monotone increasing. It is also bound from above by $\overline{B}$ (or, taking account of (7), by R^*), implying that it must converge; I call its limit L^*. By construction this limit solves:

$$\Delta_i^\varepsilon(p_{-i}^{L^*} \mid L^*) = 0. \tag{32}$$

It follows that a strategy p_i survives iterated elimination of strictly dominated strategies only if $p_i(b_i^\varepsilon) = 0$ for all $b_i^\varepsilon < L^*$, all i. $\square$

Proof of Theorem 2.

Proof. Let p be a BNE of G^ε. For any player i, define

$$\underline{b}_i = \inf\{b_i^\varepsilon \mid p_i(b_i^\varepsilon) > 0\}, \tag{33}$$

and

$$\overline{b}_i = \sup\{b_i^\varepsilon \mid p_i(b_i^\varepsilon) < 1\}. \tag{34}$$

Observe that $\underline{b}_i \leq \overline{b}_i$. Now define

$$\underline{b} = \min\{\underline{b}_i\}, \tag{35}$$

and

$$\overline{b} = \max\{\overline{b}_i\}. \tag{36}$$

By construction, $\overline{b} \geq \overline{b}_i \geq \underline{b}_i \geq \underline{b}$. Observe that p is a BNE of G^ε only if, for each i, it holds that $\Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \underline{b}_i) \geq 0$. Consider then the expected gain $\Delta_i^\varepsilon(p_{-i}^{\underline{b}}(b_{-i}^\varepsilon) \mid \underline{b}_i)$. It follows from the definition of $\underline{b}$ that $p^{\underline{b}}(b^\varepsilon) \geq p(b^\varepsilon)$ for all b^ε. The implication is that, for each i, $\Delta_i^\varepsilon(p_{-i}^{\underline{b}}(b_{-i}^\varepsilon) \mid \underline{b}_i) \geq \Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \underline{b}_i) \geq 0$. From Proposition 2 then follows that $\underline{b} \geq B^*$.

Similarly, if p is a BNE of G^ε then, for each i, it must hold that $\Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \overline{b}_i) \leq 0$. Consider now the expected gain $\Delta_i^\varepsilon(p_{-i}^{\overline{b}}(b_{-i}^\varepsilon) \mid \overline{b}_i)$. It follows from the definition of $\overline{b}$ that $p^{\overline{b}}(b^\varepsilon) \leq p(b^\varepsilon)$ for all b^ε. For each i it therefore holds that $\Delta_i^\varepsilon(p_{-i}^{\overline{b}}(b_{-i}^\varepsilon) \mid \overline{b}_i) \leq \Delta_i^\varepsilon(p_{-i}(b_{-i}^\varepsilon) \mid \overline{b}_i) \leq 0$. Hence $\overline{b} \leq B^*$.

Since $\underline{b} \leq \overline{b}$ while also $\underline{b} \geq B^*$ and $\overline{b} \leq B^*$ it must hold that $\underline{b} = \overline{b} = B^*$. Moreover, since $p^{\underline{b}} \geq p$ while also $p^{\overline{b}} \leq p$, given $\underline{b} = \overline{b} = B^*$, it follows that $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$ for all $b_i^\varepsilon \neq B^*$ and all i (recall that for each player i one has $\Delta_i^\varepsilon(p_{-i}^{B^*} \mid B^*) = 0$, explaining the singleton exeption at $b_i^\varepsilon = B^*$). Thus, if $p = (p_i)$ is a BNE of G^ε then it must hold that $p_i(b_i^\varepsilon) = p_i^{B^*}(b_i^\varepsilon)$ for all $b_i^\varepsilon \neq B^*$ and all i, as I needed to prove. $\square$