Adjustable allowance supply and the interest rate

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Abstract

Cap and trade schemes often use a policy of adjustable allowance supply with the intention to stabilize the market for allowances. We investigate whether these policies deliver. Our focus is on the sensitivity of allowance prices to the interest rate. We restrict attention to policies that rely on either the allowance price (price measures) or the surplus of unused allowances (quantity measures) to adjust supply in a dynamic cap and trade market. These policies are modelled after the existing policy landscape. Compared to a situation with fixed supply, we find that price measures stabilize allowance prices. Quantity measures do the opposite. Though phrased in the context of changing interest rates, our results warn more generally against the belief that quantity measures are a suitable instrument to promote a stable cap and trade market.

Keywords: emissions trading, climate change, interest rate, market-based emissions regulation, policy design

JEL codes: E61, H23, Q58

1 Introduction

An increasing number of cap and trade schemes has turned away from the textbook model of a fixed emissions cap in favor of adjustable allowance supply. One important, albeit by no means exclusive, motivation behind the move toward variable supply is that such policies are thought to stabilize otherwise erratic allowance prices. But do they? The answer to that question will naturally depend on both the supply policy in place and the source of fluctuations in the allowance price to begin with. In this note, we study the two most common adjustable supply policies and investigate how these affect the sensitivity of allowance prices to the interest rate.

There are, in theory, infinitely many ways to design a policy of variable allowance supply. We restrict attention to two classes of policies that dominate the policy landscape. The first

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class is that of price measures, which increase allowance supply when the price of allowances increases. The second class contains quantity measures, which reduce the supply of allowances when the surplus of unused allowances grows. Examples abound of supply policies that fit these descriptions. Price measures are used in California's ETS (Borenstein et al., 2019) and the Regional Greenhouse Gas Initiative (Friesen et al., 2022). The European Union instead opted for a quantity measure in its EU Emissions Trading System (Perino, 2018; Gerlagh and Heijmans, 2019), and one could argue that Korea ETS's liquidity provisions are a *de facto* quantity measure (Asian Development Bank, 2018).[1] In addition to the papers mentioned, a number of influential contributions studies the design and properties of price (Pizer, 2002; Fell et al., 2012; Pizer and Prest, 2020) or quantity (Kollenberg and Taschini, 2016; Gerlagh et al., 2021) measures to accommodate abatement cost uncertainty. Several authors also compare price and quantity measures in that context (Fell, 2016; Holt and Shobe, 2016; Abrell and Rausch, 2017).

This note takes to task the general proposition that price and quantity measures stabilize allowance prices. Compared to emissions trading under a fixed cap, quantity measures strictly increase the sensitivity of allowance prices to the interest rate. Price measures do stabilize allowance prices. The policy lesson is that quantity measures are not the one-size-fits-all tool that delivers stable prices they are hoped – and perhaps believed – to be.

We derive our results in a simple model of emissions trading in which the supply of allowances is determined through a price or quantity measure. Firms use allowances to cover their emissions. They also trade allowances between each other and over time. The allowance price adjusts to bring about equilibrium in the market so created. Free trade implies that allowance prices equal marginal abatement costs, which grow with the interest rate. All else equal, an increase in the interest rate thus raises future allowance prices. This triggers a redistribution of emissions between periods, pushing abatement toward the future. To support the increase in emissions today, today's allowance price must fall. When supply is governed by a price mechanism, this leads to a reduction in allowance supply which makes allowances scarcer and partly offsets the drop in prices. A price measure hence stabilizes the allowance price. When instead supply follows a quantity measure, the increase in emissions today implies fewer unused allowances are carried over to future periods. A quantity measure then increases allowance supply, pushing prices even further down. Quantity measures, it follows, destabilize the allowance price.

A policymaker could fix the destabilizing effect of quantity measures through direct interventions in the market for emissions allowances. Such a solution is imperfect at best, for several reasons. Direct interventions, especially when frequent, can easily undermine trust in and thus destabilize the very market they were meant to stabilize. Also, why have a stabilization mechanism in the first place if it does not by itself do as it is intended to? Indeed, rather that stick with a quantity measure and opt for damage control, the policymaker could instead adopt a price measure and preempt the problems identified here.

[1] A more extensive list of schemes that operate price or quantity measures also includes China's National ETS, Germany's National ETS, New Zealand's ETS, the Massachusetts Limits on Emissions from Electricity Generator, and Switzerland's ETS.

2 Main Analysis

2.1 Model

Consider a cap and trade scheme that regulates emissions by a continuum $[0,1]$ of firms i over the course of two periods $t = 0, 1$. Let a_{it} denote the abatement effort of firm i in period t, and assume that $a_{it} \geq 0$. We define abatement relative to an exogenously given business-as-usual level of emissions, so $a_{it} := \bar{q}_{it} - q_{it}$ where $\bar{q}_{it}$ is the BAU level of emissions chosen by firm i in period t absent any regulation and q_{it} its actual emissions. Abatement costs are given by $C(a_{it})$ and satisfy $\frac{\partial C(a_{it})}{\partial a_{it}} > 0$ and $\frac{\partial^2 C(a_{it})}{\partial a_{it}^2} > 0$. For future reference, we denote $C_t'(q_{it}) := \frac{\partial C(\bar{q}_{it} - q_{it})}{\partial q_{it}}$. Define $q_t := \int_0^1 q_{it} di$. In each period, firms choose their emissions simultaneously. To reduce notational clutter, the abatement cost functions C_{it} are assumed to be common knowledge. If desired, one could interpret C_{it} as the *expected* abatement cost function.

The abatement obligation of firm i is determined by the number of allowances it owns. There are, in practice, various ways in which allowances are allocated to the firms, but we shall be agnostic about the precise method used. Let s_{it} allowances be supplied to firms i in period t, and $s_t := \int_0^1 s_{it} di$. Allowances, once supplied, can be traded on a secondary market against a price p_t which the firms, being small, take as given. We let l_{it} denote the number of allowances bought by player i in period t; since every allowance bought must also be sold, we have $\int l_{it} di = 0$.[2] Allowances can also be traded over time in the sense that allowances supplied but not used in period 0 are carried over to period 1 – this is called banking (Rubin, 1996). We write b_i for the amount of banking by firm i in period 0. By construction, we have $b_i = s_{i0} + l_{i0} - q_{i0}$. Total banking in period 1 is $b := \int b_i di$. While it would be realistic to assume a borrowing constraint (e.g. $b \geq 0$), we follow the literature (*c.f.* Rubin, 1996; Pizer and Prest, 2020) and allow both banking and borrowing. This assumption plays no critical role in the derivation of our results yet allows us to simplify notation.

We come to the determination of firms' behavior shortly. At this point, we note that conditions in the market for allowances affect supply. To stay close with reality, we limit attention to two general classes of supply policies: price and quantity measures. We define each in turn.

Definition 1 (Price measures)**.** *Under a price measure, the supply of allowances in period 1 is increasing in the allowance price p_0. Letting s_1^P denote period-1 supply under a price measure, we thus have $s_1^P(p_0') > s_1^P(p_0)$ if and only if $p_0' > p_0$.*

Price measures were proposed by Roberts and Spence (1976), Pizer (2002), and Abrell and Rausch (2017). Practical examples are price collars. We assume that s_1^P is differentiable to keep the analysis concise.

Definition 2 (Quantity measures)**.** *Under a price measure, the supply of allowances in period 1 is decreasing in the bank of allowances b. Letting s_1^Q denote period-1 supply under a quantity measure, we thus have $s_1^Q(b') > s_1^Q(b)$ if and only if $b' < b$.*

[2] This is formally a constraint on firms' cost-minimization problem. Because firms are assumed to be small relative to the market, we will for convenience of notation assume this constraint is not binding.

Quantity measures were studied among others by Kollenberg and Taschini (2016), Abrell and Rausch (2017), and Pizer and Prest (2020). Examples in practice are abatement bounds (Holt and Shobe, 2016), a market stability reserve like the EU's (Gerlagh et al., 2021), or Korea ETS' liquidity provisions. We assume that s_1^Q is differentiable. We also assume that $\partial s_1^Q(b)/\partial b > -1$ for all b. This ensures that firms cannot increase their emissions in both periods simultaneously.

Although supply is determined in the market for allowances, we assume that individual firms take the supply of allowances as given. The assumption that firms operate in a competitive market where prices, and hence supply, are taken as given is common in the literature on adjustable supply policies (Fell et al., 2012; Fell, 2016; Kollenberg and Taschini, 2016) and follows naturally from the assumption that firms are price-takers.

We note that our definitions of price and quantity measures describe existing supply policies up to a degree of approximation. The Regional Greenhouse Gas Initiative (Friesen et al., 2022) and California's ETS (Borenstein et al., 2019), for example, use price floors and ceilings that adjust the supply of allowances only when prices threaten to move beyond those administrative boundaries. Similarly, the European Union's quantity measure adjusts supply only when banking exceeds a non-zero lower bound, and even then only indirectly through its Market Stability Reserve (Perino, 2018; Gerlagh et al., 2021). We define price and quantity measures in the abstracted ways given above for purposes of tractability. We do believe the simplified definitions used here, while abstractions, preserve the gist of the way most emissions trading systems adjust supply in response to market conditions. Claims regarding the applicability of our results to actual cap and trade schemes should be read with this proviso in mind.

We are interested in the sensitivity of allowance prices to the interest rate when allowance supply is based on price and quantity measures, respectively. We also want to compare these sensitivities to the case of fixed allowance supply. To make that comparison, let supply in period 1 under a fixed cap be denoted s_1^f.

2.2 Equilibrium

Firms minimize the discounted sum of costs, subject to the cap and trade constraints:

$$\begin{aligned} \min_{q_{it}, l_{it}} \quad & \sum_{t=0}^{1} \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it} \right] \\ \text{subject to} \quad & q_{i0} = s_{i0} + l_{i0} - b_i, \\ & q_{i1} = s_{i1} + l_{i1} + b_i, \end{aligned} \tag{1}$$

where $\beta := 1/(1+r)$ is the discount factor and r the interest rate. In (1), the firm takes s_{i1} as given by assumption. Because both banking and borrowing are allowed, the two constraints in (1) admit the same shadow price and may therefore be combined. The Lagrangian for this problem then becomes,

$$\mathcal{L}_i = \sum_t \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it} \right] - \lambda \left[s_{i0} + s_{i1} + l_{i0} + l_{i1} - q_{i0} - q_{i1} \right],$$

which has the following standard first-order conditions:

$$\frac{\partial \mathcal{L}_i}{\partial q_{i0}} = C'_{i0}(q_{i0}) + \lambda = 0,$$
$$\frac{\partial \mathcal{L}_i}{\partial q_{i1}} = \beta \cdot C'_{i1}(q_{i1}) + \lambda = 0,$$
$$\frac{\partial \mathcal{L}_i}{\partial l_{i0}} = p_0 - \lambda = 0,$$
$$\frac{\partial \mathcal{L}_i}{\partial l_{i1}} = \beta \cdot p_1 - \lambda = 0.$$

Cost-minimization dictates that marginal abatement costs equal the allowance price in each period, $C'_{it}(q_{it}) = p_t$. Let $q_{it}(p_t)$ denote the level of q_{it} that solves this condition. We define $q_t(p_t) := \int_0^1 q_{it}(p_t)di$. We similarly define $b(p_0) := s_0 - q_0(p_0)$. Strict convexity of C_{it} implies that q_{it} is unique and decreasing in p_t, so it follows that

$$\frac{\partial q_t(p_t)}{\partial p_t} < 0. \tag{2}$$

As banking and borrowing is allowed, dynamic arbitrage conditions imply that allowance prices should rise with the interest rate,

$$p_1 = (1 + r) \cdot p_0, \tag{3}$$

a condition known as Hotelling's rule. Had we not allowed both banking *and* borrowing of allowances, (3) might be violated if the borrowing constraint were binding, though price movements would still be positively correlated between periods. Hotelling's rule is commonly maintained in models of dynamic emissions trading (*c.f.* Fell et al., 2012; Fell, 2016; Kollenberg and Taschini, 2016) but can be dispensed with without invalidating our results – close inspection of the analysis reveals that $\partial[p_1/p_0]/\partial r > 0$ is the core property we use.

Equilibrium is reached when supply and demand are equal; the allowance price adjusts to bring about equilibrium. Because firms are price takers, this is the competitive market equilibrium.

When allowance supply is fixed, $s_1 = s_1^f$, the equilibrium price p_t^f is found by solving

$$q_0(p_0^f) + q_1(p_1^f) = s_0 + s_1^f. \tag{4}$$

We define the equilibrium under fixed supply only as a point of reference to compare the equilibrium under price and quantity measures with. Let p_t^* denote the equilibrium allowance price in period t when supply is governed by a price measure. Thus, p_t^* is the solution to

$$q_0\left(p_0^*\right) + q_1\left(p_1^*\right) = s_0 + s_1^P(p_1^*). \tag{5}$$

The equilibrium price when supply is determined through a quantity measure is p_t^{**}, which solves:

$$q_0\left(p_0^{**}\right) + q_1\left(p_1^{**}\right) = s_0 + s_1^Q\left(b\left(p_0^{**}\right)\right). \tag{6}$$

Through (3), conditions (4), (5), and (6) reveal that equilibrium prices depend on the interest rate. It is still unclear, though, how exactly the supply policy in place affects this relationship. In what follows, we will determine the dependence of allowance price on the interest rate more precisely.

2.3 Comparative statics

We consider first the simple case in which allowance supply is fixed. To determine the effect of fluctuations in the interest rate on the equilibrium in this environment, we totally differentiate both sides of (4) with respect to the interest rate:

$$\frac{\partial q_0(p_0^f)}{\partial p_0^f}\frac{\partial p_0^f}{\partial r} + \frac{\partial q_1(p_1^f)}{\partial p_1^f}\frac{\partial p_1^f}{\partial r} = 0.$$

Using (3), we can rewrite this as:

$$\frac{\partial q_0(p_0^f)}{\partial p_0^f}\frac{\partial p_0^f}{\partial r} + \frac{\partial q_1(p_1^f)}{\partial p_1^f}\left[(1+r)\frac{\partial p_0^f}{\partial r} + p_0^f\right] = 0.$$

Collecting terms,

$$\frac{\partial p_0^f}{\partial r} = -p_0^f \cdot \frac{\frac{\partial q_1(p_1)}{\partial p_1}}{\frac{\partial q_0(p_0)}{\partial p_0} + (1+r)\frac{\partial q_1}{\partial p_1}}. \tag{7}$$

One can perform similar analyses for cap and trade schemes where supply is determined through a feedback mechanism. For the case of a price measure, totally differentiate both sides of the equilibrium condition (5):

$$\frac{\partial q_0(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r} + \frac{\partial q_1(p_1^*)}{\partial p_1^*}\frac{\partial p_1^*}{\partial r} = \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r}.$$

Using (3) again, reshuffling and collecting terms, we find:

$$\frac{\partial p_0^*}{\partial r} = -p_0^* \cdot \frac{\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}}. \tag{8}$$

Finally, when supply is governed by a quantity measure we get:

$$\frac{\partial p_0^{**}}{\partial r} = -p_0^{**} \cdot \frac{\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}}}. \tag{9}$$

We are now in a position to state our first result on the effect of the interest rate on allowance prices.

Proposition 1. *The equilibrium allowance prices p_0^f, p_0^*, and p_0^{**} are decreasing in the interest rate r:*

$$\frac{\partial p_0^f}{\partial r} < 0, \qquad \frac{\partial p_0^*}{\partial r} < 0, \qquad \frac{\partial p_0^{**}}{\partial r} < 0. \tag{10}$$

Proof. We know from (2) that $\partial q_t/\partial p_t < 0$. We also know that $s_1^P(p_0)/\partial p_0 > 0$, by definition. Plugging these signs into (7) and (8) yields the result for fixed and price-measure based allowance supply.

For the case a quantity measure, $\partial q_t/\partial p_t < 0$ is still true. Hence, the final claim is correct if and only if the denominator in (9),

$$\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}},$$

is negative. Observe that we have

$$\frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}} = \frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial b(p_0^{**})}{\partial p_0^{**}} = -\frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}},$$

where the final equality follows from $b(p_0) = s_0 - q_0(p_0)$. Now recall the assumption that $\partial s_1^Q(b)/\partial b > -1$, so $1 + \partial s_1^Q(b)/\partial b > 0$ and

$$\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}} = \underbrace{\left[1 + \frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\right]}_{>0}\underbrace{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}}}_{<0} + \underbrace{(1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}_{<0} < 0,$$

as we needed to show. *Q.E.D.*

The economist will recognize in Proposition 1 a fundamental fact of finance: the price of an emissions allowance – like that of any other asset – tends to decrease, all else equal, when the interest rate increases. While this result is well-established for the case of a fixed emissions cap (Rubin, 1996), we extend it to policies in which allowance supply is governed by a price or quantity mechanism. The question remains how this dependence is affected by a policy of adjustable allowance supply. The next section investigates.

2.4 (De)stabilizing price stabilization

Although Proposition 1 is informative about the direction in which allowance prices develop when the interest rate changes, one could wonder which measure yields the strongest effect. In other words, Which – if any – supply policy is better able to stabilize allowance prices when changes in the face of a changing interest rate?

Proposition 2. *There exists $\varepsilon > 0$ such that if $|p_0^* - p_0^f| < \varepsilon$ and $|p_0^{**} - p_0^f| < \varepsilon$, then equilibrium allowance prices satisfy,*

$$\frac{1}{p_t^{**}}\frac{\partial p_t^{**}}{\partial r} > \frac{1}{p_t^f}\frac{\partial p_t^f}{\partial r} > \frac{1}{p_t^*}\frac{\partial p_t^*}{\partial r}, \tag{11}$$

for $t = 0, 1$, provided ε is sufficiently small. That is, price measures strictly decrease the sensitivity of equilibrium allowance prices to the interest rate. Quantity measures strictly increase equilibrium price sensitivity.

Proof. We establish the series of inequalities for $t = 0$. This, given that (for equilibrium prices p_0 and p_1),

$$\frac{1}{p_1}\frac{\partial p_1}{\partial r} = \frac{1}{(1+r)p_0}\left[p_0 + (1+r)\frac{\partial p_1}{\partial r}\right] = \frac{1}{p_0}\frac{\partial p_0}{\partial r} + \frac{1}{1+r},$$

will imply the result for $t = 1$.

Define $R^*(p_0^*, p_0^f)$ as:

$$R^*(p_0^*, p_0^f) := \frac{1}{p_t^*}\frac{\partial p_t^*}{\partial r} \Big/ \frac{1}{p_t^f}\frac{\partial p_t^f}{\partial r} = \frac{\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_1(p_1^f)}{\partial p_1^f}} \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1}{\partial p_1^f}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}},$$

where the expression for $R^*(p_0^*, p_0^f)$ follows from plugging in (7) and (8).

We first show that $R^*(p_0^*, p_0^f) < 1$ for all (p_0^*, p_0^f) such that $p_0^* = p_0^f$. In that case, the above simplifies to

$$R^*(p_0^*, p_0^f) = \frac{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}} < 1,$$

where the inequality is immediate from the fact that $\partial s_1^P(p_0)/\partial p_0 > 0$.

Suppose then that $p_0^* \neq p_0^f$. Two cases can arise. i) $R^*(p_0^*, p_0^f) < 1$ for all (p_0^*, p_0^f), in which case the second inequality in (11) is trivially true. ii) There are (p_0^*, p_0^f) such that $R^*(p_0^*, p_0^f) \geq 1$. For any given p_0^f, let $\delta^+(p_0^f)$ denote the smallest real number such that $R^*(p_0^f + \delta(p_0^f), p_0^f) \geq 1$. Similarly, let $\delta^-(p_0^f)$ denote the smallest real number such that $R^*(p_0^f - \delta^-(p_0^f), p_0^f) \geq 1$. For all p_0^f, define $\delta(p_0^f) := \min\{\delta^+(p_0^f), \delta^-(p_0^f)\}$. Then define $\delta := \min_{p_0^f} \delta(p_0^f)$. By construction, $R^*(p_0^*, p_0^f) < 1$ for all pairs (p_0^*, p_0^f) that satisfy $|p_0^* - p_0^f| < \delta$.

Next, define $R^{**}(p_0^{**}, p_0^f)$ to be

$$R^{**}(p_0^{**}, p_0^f) := \frac{1}{p_t^{**}}\frac{\partial p_t^{**}}{\partial r} \Big/ \frac{1}{p_t^f}\frac{\partial p_t^f}{\partial r} = \frac{\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}{\frac{\partial q_1(p_1^f)}{\partial p_1^f}} \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1(p_1^f)}{\partial p_1^f}}{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(p_0^{**})}{\partial p_0^{**}}}.$$

If $p_0^{**} = p_0^f$, then $R^{**}(p_0^{**}, p_0^f) > 1$ as $\partial s_1^Q(b(p_0))/\partial p_0 < 0$. Suppose then that $p_0^{**} \neq p_0^f$. Here, too, two cases can arise. i) $R^{**}(p_0^{**}, p_0^f) > 1$ for all (p_0^*, p_0^f). ii) There are (p_0^*, p_0^f) such that $R^{**}(p_0^{**}, p_0^f) \leq 1$. For any given p_0^f, let $\gamma^+(p_0^f)$ denote the smallest real number such that $R^{**}(p_0^f + \gamma^+(p_0^f), p_0^f) \geq 1$. Similarly, let $\gamma^-(p_0^f)$ denote the smallest real number such that $R^{**}(p_0^f - \gamma^-(p_0^f), p_0^f) \geq 1$. For all p_0^f, define $\gamma(p_0^f) := \min\{\gamma^+(p_0^f), \gamma^-(p_0^f)\}$. Then define $\gamma := \min_{p_0^f} \gamma(p_0^f)$. By construction, $R^{**}(p_0^{**}, p_0^f) > 1$ for all pairs (p_0^{**}, p_0^f) that satisfy $|p_0^{**} - p_0^f| < \gamma$. Set $\varepsilon \leq \min\{\delta, \gamma\}$. *Q.E.D.*

Proposition 2 tells us that allowance prices when supply is adjusted through a quantity measure are strictly more sensitive to changes in the interest rate than are allowance prices when supply is set through a price measure. Indeed, compared to an exogenously fixed emissions cap, quantity measures exacerbate rather than stabilize allowance price fluctuations. Contrary to their intention, quantity measures destabilize cap and trade markets. Price measures do properly stabilize allowances prices.

Formally, the Proposition speaks only to the *elasticity* of allowance prices with respect to the interest rate. In some cases, policymakers may instead be interested only in absolute effects. From the condition that equilibrium prices start out sufficiently close, it is immediate the series of inequalities in (11) also implies a slighly stronger result.

Corollary 1. *For $\tilde{\varepsilon} > 0$ such that $|p_0^* - p_0^f| < \tilde{\varepsilon}$ and $|p_0^{**} - p_0^f| < \tilde{\varepsilon}$, Proposition 2 generalizes to*

$$\frac{p_t^{**}}{\partial r} > \frac{\partial p_t^f}{\partial r} > \frac{\partial p_t^*}{\partial r}, \tag{12}$$

for $t = 0, 1$, provided $\tilde{\varepsilon}$ is sufficiently small.

Starting from similar equilibrium prices, a quantity measure destabilizes the allowance price while a price measure stabilizes prices, in both relative and absolute terms.

Proposition 2 describes equilibrium effects for comparable baseline equilibria. Here, comparability refers to the condition that p_0^* and p_0^{**} are close enough to p_0^f. In the prominent special case of quadratic abatement costs (*c.f.* Kollenberg and Taschini, 2016; Pizer and Prest, 2020; Gerlagh et al., 2021), comparability is redundant.

Corollary 2. *If the abatement cost functions C_{it} are quadratic, then Proposition 2 holds true for all ε.*

Quantity measures are destabilizing because an increase in the interest rate raises the relative price of emissions in the future, stimulating emissions in period 0. This suppresses the period 0 allowance price and reduces banking. By design, a reduction in banking causes an increase in supply in period 1. The increase in supply reduces firms' abatement obligations and pushes allowance prices even further down, enforcing rather than undoing the downward pressure on prices.

3 Discussion and Conclusions

Most major cap and trade schemes, and many smaller ones, have adopted measures that allow for variable allowance supply. One motivation to institute these measures is to limit allowance price variability. We investigate whether that goal is achieved by the two most prominent examples of adaptive supply policies. We find that price-based supply measures stabilize prices indeed. Quantity-based measures instead destabilize prices.

Quantity measures are insufficiently versatile to deal will *all* kinds of demand shocks. As a means to contain price variability resulting from events that mostly affect the instantaneous demand for emissions, quantity measures have been demonstrated to work well (Kollenberg and Taschini, 2016; Fell, 2016; Abrell and Rausch, 2017; Gerlagh et al., 2020; Pizer and Prest, 2020). But as we show, quantity measures perform poorly when the source of variation instead is a change in firms' relative demand over time. While the EU can partially justify the claim that its quantity measure "*improves the system's resilience to major shocks by adjusting the supply of allowances to be auctioned*", it certainly does not insulate the system from *all* major shocks.[3] This finding deepens the shadow already cast over quantity measures in several recent contributions (*c.f.* Gerlagh and Heijmans, 2019; Perino et al., 2019; Gerlagh et al., 2021).

We argue that our analysis has implications beyond the context of interest rates. Changes in the interest rate lead to a recalibration of the market equilibrium is that, through Hotelling's rule, it affects the time path of allowance prices. Other events that influence the distribution

[3] Quote taken European Commission's web page.

of emissions over time will similarly interact with the stabilization mechanisms and bear on allowance supply. This suggests that any change in firms' dynamic incentives – be it through policy or some other determinant of allowance demand – that increases emissions incentives today, relative to the future, have price effects comparable to those described here.

In the wake of decades high inflation, central banks are bound to alter the course of monetary policy and increase the interest rate. Absent additional policy interventions, we predict that the associated correction in allowance prices will be substantially stronger in cap and trade schemes that rely on quantity measures to adjust supply. Perhaps it is time to reform the European Union's cap and trade market and introduce a price measure to weather the turbulent times ahead and support the EU's ambitious climate agenda.

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