

Flexible emissions caps and the green paradox

Reyer Gerlagh*, Roweno J.R.K. Heijmans[†], Knut Einar Rosendahl[‡]

October 18, 2022

Abstract

We give a substantial generalization of a result due to Gerlagh et al. (2021). Any cap and trade scheme in which the aggregate cap responds to the use of allowances necessarily suffers from a Green Paradox: if information on quantities is used to determine the effective cap on emission, there always exists an emissions-reducing policy complementary to the scheme that increases emissions overall.

JEL codes: D59; E61; H23; Q50; Q54; Q58

Keywords: Emissions trading; green paradox; environmental policy; dynamic modeling

1 Introduction

Cap and trade is one of the most prominent examples of environmental policy across the globe. Important pollution problems such as acid rain (caused by high atmospheric concentrations of SO_2 and NO_X) or global warming (the result of greenhouse gas emissions like CO_2) are often regulated through a cap and trade policy.

In its most basic form, a cap and trade scheme issues a fixed number of allowances and distributes these to firms covered by the scheme. If a firm wants to emit, it must surrender the corresponding number of allowances. By limiting the supply of allowances, the policymaker reduces emissions. A simple quota or other types of direct command and control policy could of course achieve the same reduction; under a cap and trade policy, however, firms are allowed to trade their allowances. Many cap and trade schemes in addition permit the use of allowances issued in one period to meet compliance obligations at another (future) point in time. The unifying idea is that firms know better how much abatement efforts will cost them than the policymaker does;

*Department of Economics, Tilburg University, the Netherlands. Email: r.gerlagh@uvt.nl.

[†]Department of Economics, Swedish University of Agricultural Sciences, Sweden. Email: roweno.heijmans@slu.se.

[‡]School of Economics and Business of the Norwegian University of Life Sciences, Norway and Statistics Norway, Norway. Email: knut.einar.rosendahl@nmbu.no.

allowing for the free exchange of allowances both within and across periods, dynamic cap and trade hence achieves a given emission reduction at lowest cost.

As cap and trade stimulates firms to use their private information, the resulting market signals provide indicators of firms' private information (Kwerel, 1977; Dasgupta et al., 1980). If the cap is set to strike an (approximate) balance between the marginal benefits and costs of emissions, this information is relevant to pin down the total amount of emissions an efficient policy should allow for. Thus, a recent literature promotes the idea of cap adjustments in response to firms' emissions decisions (Abrell and Rausch, 2017; Kollenberg and Taschini, 2016, 2019; Pizer and Prest, 2020; Heutel, 2020; Gerlagh and Heijmans, 2020).

Though a policy of cap adjustments may concord well with economic intuition, the devil is in the details. As was first shown by Gerlagh et al. (2021), cap adjustments based on quantities – that is, on the number of used/unused emissions allowances – can backfire. In particular, Gerlagh et al. show that anticipated overlapping policies, aimed at reducing emissions in the future, can lead to an overall increase in emissions. Comparable result were also proven by Jarke-Neuert and Perino (2020), Perino et al. (2022), and Heijmans (2022).

Our main contribution in this paper is to offer a substantial generalization of the result due to Gerlagh et al. (2021). We show that any cap and trade scheme in which the cap on emissions responds to the use of allowances necessarily suffers from a green paradox. If information on quantities is used to adjust the cap, there always exists an emissions-reducing policy complementary to the scheme that increases emissions overall.

To establish our result in full generality, we present a technical analysis. Such a presentation notwithstanding, there is a solid economic intuition for our green paradox. Consider the simple example of a two-period dynamic cap and trade scheme with an endogenous emissions cap. This example is illustrated in the figure below. When the amount of unused allowances in period 1 goes down, from the equilibrium we move to the left along the solid curve.

Demand and supply of new allowances in period (region) 2 is increased so much that total emissions go up.¹ This paper establishes this feature of an endogenous cap in a general setting with multiple linked periods, regions or sectors. It thus provides reason for caution when linking cap-and trade over space, or over time. Adding flexibility can greatly improve efficiency (Karp and Traeger, 2021), but it is essential to use price information for adjusting the cap. Flexibility based on quantity information only will always lead to a green paradox.

¹Those familiar with European climate policy will recognize a similarity to the Emissions Trading Scheme (EU ETS) in these mechanics (Perino, 2018; Gerlagh and Heijmans, 2019).

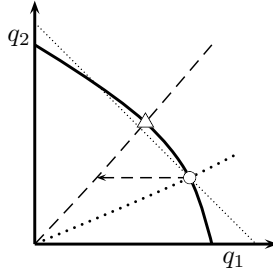


Figure 1: This figure presents allowances in two periods, regions or sectors, with an endogenous cap depicted through the solid curved line. The dotted line presents allocations with constant aggregate allowances; the circle represents an equilibrium. If demand for allowances in period (region/sector) 1 decreases (arrow to left), total emissions increase when prices adjust in the new equilibrium (triangle).

2 Analysis

Consider a cap and trade scheme that regulates emissions in a finite number of periods, regions, or sectors. We will use index $i = 1, 2, \dots$ and for convenience refer to a specific i as a “period”. We assume the ratio of prices between any two periods to be exogenously fixed by a (possibly time-varying) constant; for example, the canonical version of Hotelling’s rule would fix p_{i+1}/p_i at $1 + r$, with r the interest rate.² This assumption buys one the convenience of capturing all price information by a single scalar variable p . The equilibrium, to be formally defined below, is hence uniquely identified by a scalar p^* and equilibrium effect can summarily be described by changes in p^* .

Let $\mathbf{d}(p, \boldsymbol{\lambda})$ denote the demand vector for emissions, which depends upon the price p and the vector of emissions policies $\boldsymbol{\lambda} = (\lambda_i)_i$. We normalize $\boldsymbol{\lambda}$ so that $\partial \mathbf{d} / \partial \boldsymbol{\lambda} = \mathbf{u}$, where $\mathbf{u} = (1, 1, \dots, 1)^T$ is the transposed all-ones-vector with 1 everywhere.³ We assume that the demand for allowances decreases in prices, $\mathbf{d}' \equiv \partial \mathbf{d} / \partial p < 0$. Aggregate demand is denoted $D(p, \boldsymbol{\lambda}) = \mathbf{u}^T \mathbf{d}$.

We allow for the number of periods with strictly positive emissions to be endogenous, say T . We assume that demand has a (period-specific) finite choke price \bar{p}_i . It follows immediately that demand drops to zero when prices rise permanently above the maximum choke price. We abstract from negative emission technologies, meaning that emissions in every period are at least 0.

We are interested in quantity-based flexible emissions caps, i.e. cap and trade schemes in which the aggregate supply of allowances S depends on the demand for allowances \mathbf{d} .

Definition 1 (Flexible emissions cap). *Under a flexible emissions cap, the aggregate*

²Alternatively, when indices i represent sectors of regions, free trade will imply that allowance prices are equal for all i and j , $i \neq j$, in which case the ratio p_j/p_i is fixed at 1.

³That is, the set of (linearly) independent policies is equal to the number of periods.

supply S depends on the demand for emissions \mathbf{d} :

$$S(\mathbf{d}(p^*, \boldsymbol{\lambda})) = \mathbf{u}^T \mathbf{s}(\mathbf{d}(p, \boldsymbol{\lambda})). \quad (1)$$

The rate of adjustment of the emissions cap with respect to emissions is given by the gradient $\mathbf{s}' \equiv \nabla \mathbf{u}^T \mathbf{s}(\mathbf{d})$. The emissions cap is fixed, or exogenous, if the supply gradient is either 0, $\mathbf{s}' = 0$, or more generally proportional (but not equal) to the all-ones-vector, $\mathbf{s}' \propto \mathbf{u}$. The latter, more general, definition follows from the fact that for a given gradient $\mathbf{s}' \propto \mathbf{u}$, aggregate emissions must be constant under the cap.⁴ A cap is flexible if it is not fixed.

The market is in equilibrium when excess demand, $D - S$, equals zero. It is assumed that polluters are sufficiently small to take prices as given. The price of allowances adjusts to bring about equilibrium.

Definition 2 (Equilibrium). *The equilibrium price p^* is the price level p that equates aggregate demand and supply:*

$$D(p, \boldsymbol{\lambda}) = \mathbf{u}^T \mathbf{d}(p^*, \boldsymbol{\lambda}) = \mathbf{u}^T \mathbf{s}(\mathbf{d}(p^*, \boldsymbol{\lambda})) = S(\mathbf{d}(p^*, \boldsymbol{\lambda})). \quad (2)$$

It follows that if the cap is fixed, then equilibrium emissions $D(p^*, \boldsymbol{\lambda})$ are also fixed.

A natural approach would be to study properties of the equilibrium through the response of the equilibrium condition with respect to prices p^* . However, as the flexible emissions cap depends on the demand vector \mathbf{d} directly, the present analysis is better served by considering the response of the equilibrium condition (a demand vector) with respect to the demand vector \mathbf{d} itself. To this end, note that by definition, the equilibrium is characterized by the condition $\mathbf{u}^T \mathbf{d}(p^*, \boldsymbol{\lambda}) - \mathbf{s}(\mathbf{d}(p^*, \boldsymbol{\lambda})) = 0$. The gradient of the equilibrium in demand space is hence $\mathbf{u}^T - \mathbf{s}'$.

From the equilibrium definition, a change in demand $\Delta \mathbf{d}$ is consistent with equilibrium if and only if $(\mathbf{u} - \mathbf{s}')^T \Delta \mathbf{d} = 0$. Let $\Delta \mathbf{d} \geq 0$ denote the event that changes in demand are non-negative in all periods i and strictly positive in at least one i . If there exists a $\Delta \mathbf{d} \geq 0$ that is consistent with the equilibrium changes in emissions, i.e. that satisfies $(\mathbf{u} - \mathbf{s}')^T \Delta \mathbf{d} = 0$, then this would imply a “free lunch” for the polluters whose emissions are covered by the scheme. We rule out free lunches.

Assumption 1 (No free lunch). *There does not exist a non-negative change in emissions $\Delta \mathbf{d} \geq 0$ such that $(\mathbf{u} - \mathbf{s}')^T \Delta \mathbf{d} = 0$.*

Assumption 1 is equivalent to a condition on the equilibrium gradient, namely $\mathbf{u} - \mathbf{s}' \neq 0$.

We are interested in the effects of policy-induced changes in demand. Recall that $\boldsymbol{\lambda} = (\lambda_i)_i$ denotes the vector of emissions policies. Let α_i denote the response of the equilibrium price p^* to a change in λ_i :

$$\alpha_i \equiv dp^*/d\lambda_i. \quad (3)$$

⁴To see this, consider the case $S = S_0 + \beta \mathbf{u}^T \mathbf{d}$, so that $\mathbf{s}' = \beta \mathbf{u}$. This is equivalent to fixed aggregate emissions $D(p, \boldsymbol{\lambda}) = S_0/(1 - \beta)$.

Similarly, let $\boldsymbol{\gamma}^i$ denote the change in the vector of equilibrium emissions in response to a change in λ_i :

$$\boldsymbol{\gamma}^i \equiv d\mathbf{d}^*/d\lambda_i, \quad (4)$$

where γ_j^i denotes the change in demand in period j from a policy-induced demand change in period i . We adopt the notational convention that \mathbf{x}_i (subscript i) denotes the i^{th} element of a vector \mathbf{x} , so a scalar, whereas \mathbf{x}^i (superscript i) denotes the i^{th} column vector of a matrix \mathbf{x} . Hence, \mathbf{x}_j^i the j^{th} element of the i^{th} vector of \mathbf{x} .

We refer to the matrix of all policy-induced changes as Γ so that with a slight abuse of notation we can write $\Gamma \mathbf{e}^i \equiv \boldsymbol{\gamma}^i$, where \mathbf{e}^i is the unit vector with zeros everywhere but 1 at the i^{th} place. Taking the derivative of (2) with respect to λ_i immediately yields the following lemma:

Lemma 1. *All policy-induced demand changes are orthogonal to the demand space gradient for all i :*

$$(\mathbf{u} - \mathbf{s}')^T \boldsymbol{\gamma}^i = 0. \quad (5)$$

An immediate implication of Lemma 1 is that any linear combination of the set $\{\boldsymbol{\gamma}^i\}_i$, that is, any combination of demand policies, will also satisfy the orthogonality property, which implies

$$(\mathbf{u} - \mathbf{s}')^T \Gamma = \mathbf{0}. \quad (6)$$

We now show that the equilibrium satisfies intuitive conditions on price and demand responses to policy shifts.

Lemma 2. *Prices increase with demand-increasing policies,*

$$\alpha = -\frac{(\mathbf{u} - \mathbf{s}')^T}{(\mathbf{u} - \mathbf{s}')^T \mathbf{d}'} > 0, \quad (7)$$

and own-period demand increases, while other-period demand decreases:

$$\gamma_i^i > 0, \quad (8)$$

$$\gamma_j^i < 0 \text{ for } j \neq i. \quad (9)$$

Proof. We write the full derivatives for the equilibrium equation (2):

$$-(\mathbf{u} - \mathbf{s}')^T \mathbf{d}' dp^* = (\mathbf{u} - \mathbf{s}')^T d\boldsymbol{\lambda} \quad (10)$$

We note that the term $-(\mathbf{u} - \mathbf{s}')^T \mathbf{d}'$ on the LHS is a positive scalar (due to Assumption 1 and $\mathbf{d}' < 0$); hence (7) follows. The positive price effect implies a negative demand response in all periods $j \neq i$, and because no free lunch is possible, there must be a positive demand response in the own period i . \square

Our aim is to establish that for any system with a quantity-based endogenous cap, there exists a policy that induces a green paradox. Loosely speaking, a green paradox occurs when a policy-induced reduction in demand in some period i , $d\lambda_i < 0$ causes an increase in aggregate emissions, $dD > 0$.

Definition 3 (Green Paradox). *There is a green paradox if a demand-decreasing policy, $d\lambda < 0$, leads to increasing aggregate emissions, $dD = d(\mathbf{u}^T \mathbf{d}^*) > 0$.*

Theorem 1. *For every quantity-based endogenous cap system without a free lunch, there exists a policy $d\lambda < 0$ that induces a green paradox, $d(\mathbf{u}^T \mathbf{d}^*) > 0$.*

3 Discussion and Conclusions

We have shown that any emissions trading scheme in which the cap on emissions is updated in response to information on quantities necessarily suffers from a green paradox. In particular, it is always possible to implement a complementary climate policy aimed at reducing emissions that causes an increase in emissions overall through the endogenous emissions cap. This is a substantial generalization of recent results due to Gerlagh et al. (2021) and Osorio et al. (2021).

Perhaps it is useful to emphasize what this paper does *not* say. We do not argue that complementary emissions policies necessarily harm the environment. Rather, we prove that certain policies aimed at reducing emissions may harm the environment in some cases. Neither do we claim that complementary climate policies can never be combined with a cap and trade scheme. Our result only establishes that certain complementary climate policies increase emissions when complementing a cap and trade scheme which uses quantity information to update its cap. Relatedly, this paper does not say that an endogenous emissions cap is always a bad idea. Our Theorem specifically pertains to the use of quantity information to endogenize the emissions cap. If the policymaker were to use prices instead, our results cease to hold.

That final remark brings us to the policy implications of our work. There is no obvious advantage of using quantity information over allowance prices to update an emissions cap; however, this paper demonstrates that there are clear disadvantages. Unless a cap and trade scheme operates in absolute isolation from other climate policies – a claim hard to defend – it is a risky endeavor to combine emissions trading with complementary emissions policies. Given the enormous complexity of the climate problem, however, a combination of multiple and different policies is almost certainly called for. All in all, these observations would strongly favor the use of price signals over quantity signals to update an emissions cap.

Our policy implications also have policy relevance. Cap and trade schemes to regulate greenhouse gas emissions are used in most industrialized economies across the globe. Some of these schemes, including the Regional Greenhouse Gas Initiative (RGGI), California’s ETS, and the ETS in Quebec, rely on allowance prices to update the cap on emissions. Our somewhat worrying result does not speak to those cap

and trade schemes. That said, other – and important – cap and trade schemes do currently use quantity information to endogenize their caps. Examples include the European Union’s Emissions trading scheme (the world’s largest market for carbon), Switzerland’s ETS, and South Korea’s ETS.⁵ The key policy takeaway of this paper is that complementary climate policies may be hard to combine with these cap and trade schemes. It is up to policymakers to formulate an appropriate policy-response to our findings. Perhaps the simplest possible strategy, though, is to abandon quantity-based cap adjustment and start using price signals instead. The examples in RGGI, California, and Quebec prove that to be possible.

ACKNOWLEDGEMENTS

We acknowledge funding from The Research Council of Norway through CREE (grant 209698; Gerlagh and Rosendahl) and the NorENS project (grant 280987; Rosendahl). Heijmans was supported by a Jan Wallanders och Tom Hedelius stiftelse program grant (P22-0229).

A Proof of the Theorem

Proof. We prove the result by contradiction. Assume there is no green paradox. Our proof relies on showing that, if so, then it is necessarily possible to construct a demand-reducing policy $d\lambda < 0$ that decreases emissions in all periods $dd < 0$, which violates Assumption 1.

If there is no green paradox, then specifically all policies that reduce demand in some period i decrease aggregate demand; the matrix Γ is diagonally dominant over columns: $\forall i : \mathbf{u}^T \boldsymbol{\gamma}^i \geq 0$.

Define normalized policies and responses. We write $\boldsymbol{\kappa}^i = d\lambda^i / \gamma_i^i < 0$, and $\boldsymbol{\eta}^i = \boldsymbol{\gamma}^i / \gamma_i^i$, for the policy in period i and the vector demand response over all periods, respectively, such that if $\boldsymbol{\kappa}^i = -\mathbf{e}^i$, the policy reduces demand by one unit in period i . Let H be the matrix of normalized responses, $(H\mathbf{e}^i)_j = \eta_j^i$. The matrix H is also diagonally dominant over its columns (inherited property from Γ), with unity elements on the diagonal and negative numbers everywhere else. In this notation, the effect of a policy vector $d\lambda < 0$ on demand can be described through $dd = H\boldsymbol{\kappa}$. Let A be chosen such that any policy directly reducing demand in period i by one unit will reduce aggregate demand by at least A units: $A = \min_i \{\mathbf{u}^T H\mathbf{e}^i\}$. That is, A is the lower bound for the cumulative effectiveness of a policy in any period i . Absence of a green paradox implies $A > 0$. We now have all notation in place.

We construct a series of vectors \mathbf{z}^k , with $k = 1, \dots, \infty$, recursively, so that the series converges to $\mathbf{z}^k \rightarrow \boldsymbol{\kappa} < 0$, and $H\boldsymbol{\kappa} < 0$.

⁵South Korea does not formally use quantity information to update its cap, although it has historically done so according to the Asian Development Bank (2018).

We start for $k = 1$ with $\mathbf{z}^1 = -\mathbf{e}^1$. That is, the policy \mathbf{z}^1 decreases demand in the first period by one unit, $(H\mathbf{z}^1)_1 = -1$, increasing demand in all other periods, $\forall j \neq 1 : (H\mathbf{z}^1)_j > 0$, but aggregate demand is decreased, $\mathbf{u}^T H\mathbf{z}^1 < -A < 0$. This property also implies that the sum of all positive elements is bound from above: $\sum_i \max\{0, (H\mathbf{z}^1)_i\} < (1 - A)$.

We now describe the inductive step. We assume that in step k , we have (i) $\mathbf{u}^T H\mathbf{z}^k < 0$, and (ii) the sum of all positive elements is bound from above by $\sum_i \max\{0, (H\mathbf{z}^k)_i\} < (1 - A)^k$. We can then construct the next element in the sequence, making sure that the properties (i) $\mathbf{u}^T H\mathbf{z}^{k+1} < 0$, and (ii) $\sum_i \max\{0, (H\mathbf{z}^{k+1})_i\} < (1 - A)^{k+1}$ are transferred to the next inductive step.

In the inductive step, consider all positive elements of $\mathbf{u}^T H\mathbf{z}^k$, that we want to neutralize. Thus, let \mathbf{z}^{k+1} be defined by $(\mathbf{z}^{k+1} - \mathbf{z}^k)_i = -\max\{0, (H\mathbf{z}^k)_i\} < 0$. The required properties follow immediately from this construction:

$$\mathbf{u}^T H\mathbf{z}^{k+1} = \mathbf{u}^T H\mathbf{z}^k + \mathbf{u}^T H(\mathbf{z}^{k+1} - \mathbf{z}^k) < 0 \quad (11)$$

$$\sum_i \max\{0, (H\mathbf{z}^{k+1})_i\} < (1 - A) \sum_i \max\{0, (H\mathbf{z}^k)_i\} < (1 - A)^{k+1} \quad (12)$$

Finally, we show that $\mathbf{z}^k \rightarrow \boldsymbol{\kappa} < 0$ is well defined, because from the construction, we see that we have a Cauchy sequence:

$$\mathbf{u}^T (\mathbf{z}^{k+1} - \mathbf{z}^k) = \sum_i \max\{0, (H\mathbf{z}^k)_i\} < (1 - A)^k \quad (13)$$

Any Cauchy sequence defined on a compact set converges to a point in the set. Thus, we want to establish compactness. To this end, recall that in any step κ , the sum of all positive demand-changes, i.e. the increase in aggregate in each step κ , was bound from above by $(1 - A)^\kappa$. The aggregate increase in demand is therefore never larger than $\sum_{\kappa=1}^\infty (1 - A)^\kappa = 1/A < \infty$, where the last inequality follows from the fact that $A > 0$. But this means the series of vectors \mathbf{z}^k is defined on a closed and bounded set. By the Heine-Borel Theorem, a closed and bounded set is compact.

Thus, we have $\mathbf{z}^k \rightarrow \boldsymbol{\kappa} < 0$, and $H\boldsymbol{\kappa} < 0$. We can rewrite before normalization that we have constructed a strict demand-reducing policy $d\boldsymbol{\lambda} < 0$ and associated strict negative emissions response in all periods, $\boldsymbol{\gamma} = \Gamma d\boldsymbol{\lambda} < 0$. But combined with Assumption 1, $(\mathbf{u} - \mathbf{s}')^T \neq 0$, this implies $(\mathbf{u} - \mathbf{s}')^T \boldsymbol{\gamma} < 0$, which contradicts (5). \square

References

- Abrell, J. and Rausch, S. (2017). Combining price and quantity controls under partitioned environmental regulation. *Journal of Public Economics*, 145:226–242.
- Dasgupta, P., Hammond, P., and Maskin, E. (1980). On imperfect information and optimal pollution control. *The Review of Economic Studies*, 47(5):857–860.

- Gerlagh, R. and Heijmans, R. (2020). Regulating stock externalities.
- Gerlagh, R. and Heijmans, R. J. (2019). Climate-conscious consumers and the buy, bank, burn program. *Nature Climate Change*, 9(6):431–433.
- Gerlagh, R., Heijmans, R. J., and Rosendahl, K. E. (2021). An endogenous emissions cap produces a green paradox. *Economic Policy*.
- Heijmans, R. (2022). Adjustable emissions caps and the price of pollution. *Working paper*.
- Heutel, G. (2020). Bankability and information in pollution policy. *Journal of the Association of Environmental and Resource Economists*, 7(4):779–799.
- Jarke-Neuert, J. and Perino, G. (2020). Energy efficiency promotion backfires under cap-and-trade. *Resource and Energy Economics*, 62:101189.
- Karp, L. and Traeger, C. P. (2021). Smart cap. *CESifo WP*, 8917-2021.
- Kollenberg, S. and Taschini, L. (2016). Emissions trading systems with cap adjustments. *Journal of Environmental Economics and Management*, 80:20–36.
- Kollenberg, S. and Taschini, L. (2019). Dynamic supply adjustment and banking under uncertainty in an emission trading scheme: the market stability reserve. *European Economic Review*.
- Kwerel, E. (1977). To tell the truth: Imperfect information and optimal pollution control. *The Review of Economic Studies*, 44(3):595–601.
- Osorio, S., Tietjen, O., Pahle, M., Pietzcker, R. C., and Edenhofer, O. (2021). Reviewing the market stability reserve in light of more ambitious eu ets emission targets. *Energy Policy*, 158:112530.
- Perino, G. (2018). New eu ets phase 4 rules temporarily puncture waterbed. *Nature Climate Change*, 8(4):262–264.
- Perino, G., Ritz, R. A., and Van Benthem, A. (2022). Overlapping climate policies. Technical report, National Bureau of Economic Research.
- Pizer, W. A. and Prest, B. C. (2020). Prices versus quantities with policy updating. *Journal of the Association of Environmental and Resource Economists*, 7(3):483–518.