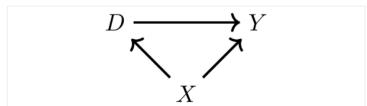
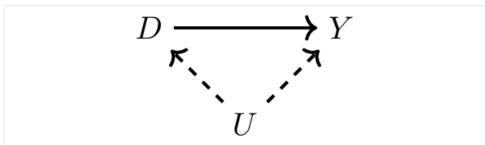
Directed Acyclic Graphs

- 1) Directed Acyclic Graphs (DAGs) are graphical forms of chains of causal effects
 - a. DAGs shows all causal relationship that are relevant to the effect of D on Y
 - b. Developed by theory, literature, or deep institutional knowledge
- 2) Causal effects occur in two ways:
 - a. Direct Pathway: $D \rightarrow Y$
 - b. Indirect (mediated by a third variable): $X \to D \to Y$, $D \leftarrow X \to Y$, etc.
 - c. Assumptions are not displayed as causal relationship in the DAG
- 3) Simple DAG



- a.
- b. We have 3 random variables: X, D, and Y
 - i. D affects Y
 - ii. X affects Y and D
- c. There are two paths affecting Y:
 - i. Direct Pathway: $D \rightarrow Y$
 - ii. Backdoor Pathway: $D \leftarrow X \rightarrow Y$
 - iii. The direct path is causal while the backdoor pathway is not causal but a process that creates a spurious correlation between D and Y that is driven by fluctuations in X
- 4) Backdoor Criterion
 - a. Backdoor pathways matter because they create systematic noncausal correlations between the causal variables of interest and the outcome of interest
 - b. Our Goal: Close all of the backdoor pathways and identify the direct pathway of D on Y using an identification strategy or credible research design
 - i. Conditioning or controlling for the confounder or the appearance of a collider closes a backdoor pathway
- 5) Confounders
 - a. In the Simple DAG, X is an observed confounder
 - b. When X mediates the values D and Y, X is considered a confounder
 - c. Unobserved confounders



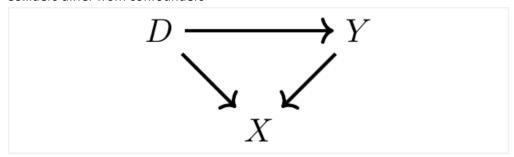
i.

ii. Since U is unobserved, there remains an open backdoor pathway from D to Y

a.

6) Colliders

a. Colliders differ from confounders



b.

c. There are two pathways

i. Direct Pathway: $D \rightarrow Y$

ii. Backdoor Pathway: $D \rightarrow X \leftarrow Y$

d. When two variables cause a third variable along some path, it is a collider

e. D and Y collide at X and fluctuations in D and Y affect X

f. Colliders mean that the backdoor pathway is closed; when left alone the backdoor pathway is closed in the presence of a collider

Regression Review

1) The expected value of beta-hat is beta if OLS is unbiased if assumptions hold

a.
$$E(\hat{\beta}) = \beta$$

b. First Assumption: Linear in Parameters

i. In the population model, the dependent variable y is related to the independent variable and the error term u: $y = \beta_0 + \beta_1 x + u$

c. Second Assumption: Random Sampling

i. We have a random sample of size n with a set of number $\{(x_i, y_i): i = 1, 2, ..., n\}$ following the population model $y = \beta_0 + \beta_1 x + u$

ii. Where i is a random sampling draw from the population

iii. Such that $y_i = \beta_0 + \beta_1 x_i + u_i$, i = 1, 2, ..., n

d. Third Assumption: Sample Variation in Explanatory Variables

i. The sample outcomes on x, $\{x, i = 1, 2, ..., n\}$ are not the same value

ii. If there is no variation in x then this assumption fails

e. Fourth Assumption: Zero Conditional Mean

i. The most crucial assumption in causal inference and the one most likely to fail

ii. E(u|x) = 0

iii. We can still calculate beta-hat even if the fourth assumption fails, but it means that beta-hat will not be an unbiased estimator

2) Bias of Expected Value of OLS

a. If there is an omission of a confounding variable or the model is underspecified we will have omitted variable bias: When $E[U|X] \neq 0$

i.
$$\hat{\beta}_1 = \beta_1 + \beta_2 \frac{C(U,X)}{V(X)} = \beta_1 + \frac{C(Y,U)}{V(U)} \frac{C(U,X)}{V(X)}$$

b. The direction of the bias depends on the direction of the covariance among Y, X, and U

Direction of Bias	C(X,U) > 0	C(X,U) < 0
C(Y,U) > 0	Upward Bias	Downward Bias
C(Y,U) < 0	Downward Bias	Upward Bias

- 3) Robust Standard Errors
 - a. Whenever we assume homoskedasticity

i.
$$V(u|x) = E(u^2|x) = \sigma^2 = E(u^2)$$

- b. When the fifth assumption is violated, the variance of the estimator beta is biased and no longer has the minimum mean squared errors
 - i. We need robust standard errors $V\left(\hat{eta}_1
 ight) = rac{\sum_{i=1}^n (x_i \vec{x})^2 \widehat{u}_i^2}{SST_x^2}$
- 4) Clustered Robust Standard Errors
 - a. In addition to heteroskedastic errors, we need to be considered if errors are correlated at an aggregate level even if they don't affect individuals directly
 - b. When use the cluster option in the regression by the cluster_id, the standard errors in the clustered data are similar to their distribution in the unclustered data set

c.