ECON 672

Week 5: Panel Data and Fixed Effects (Within) Estimator

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Overview

- Takeaway
- Fixed Effects DAG
- Estimation
- Pooled OLS
- Fixed Effects or Within Estimator
- Caveats of Fixed Effects
- Example

The Takeaway

- Panel Data
  - When we observe the same i^{th} unit over time
  - With cross-sectional data, we do not observe the same i^{th} over time
  - We can use the panel structure to control for unobserved or observed time-invariant heterogeneity
  - Time-invariant heterogeneity (observed or unobserved) are covariates that vary across units but do not vary within units
- Fixed Effects (Within) Estimator
  - Is an identification strategy that can be employed with panel data
  - It works when we have unobserved time-invariant confounders

Takeaway (Fixed Effects)

- Strengths
    - We can control for unobserved confounders that do not vary over time with panel data and the fixed effects estimator
- Weaknesses
    - We cannot control for unobserved confounders that do vary over time
    - We cannot control for simultaneity or reverse causation
- Assumptions
    - Strict Exogeneity Assumption (Independence Assumption - not testable)
    - Rank Assumption (covariates must vary - testable)

Directed Acyclic Graphs: Panel Data

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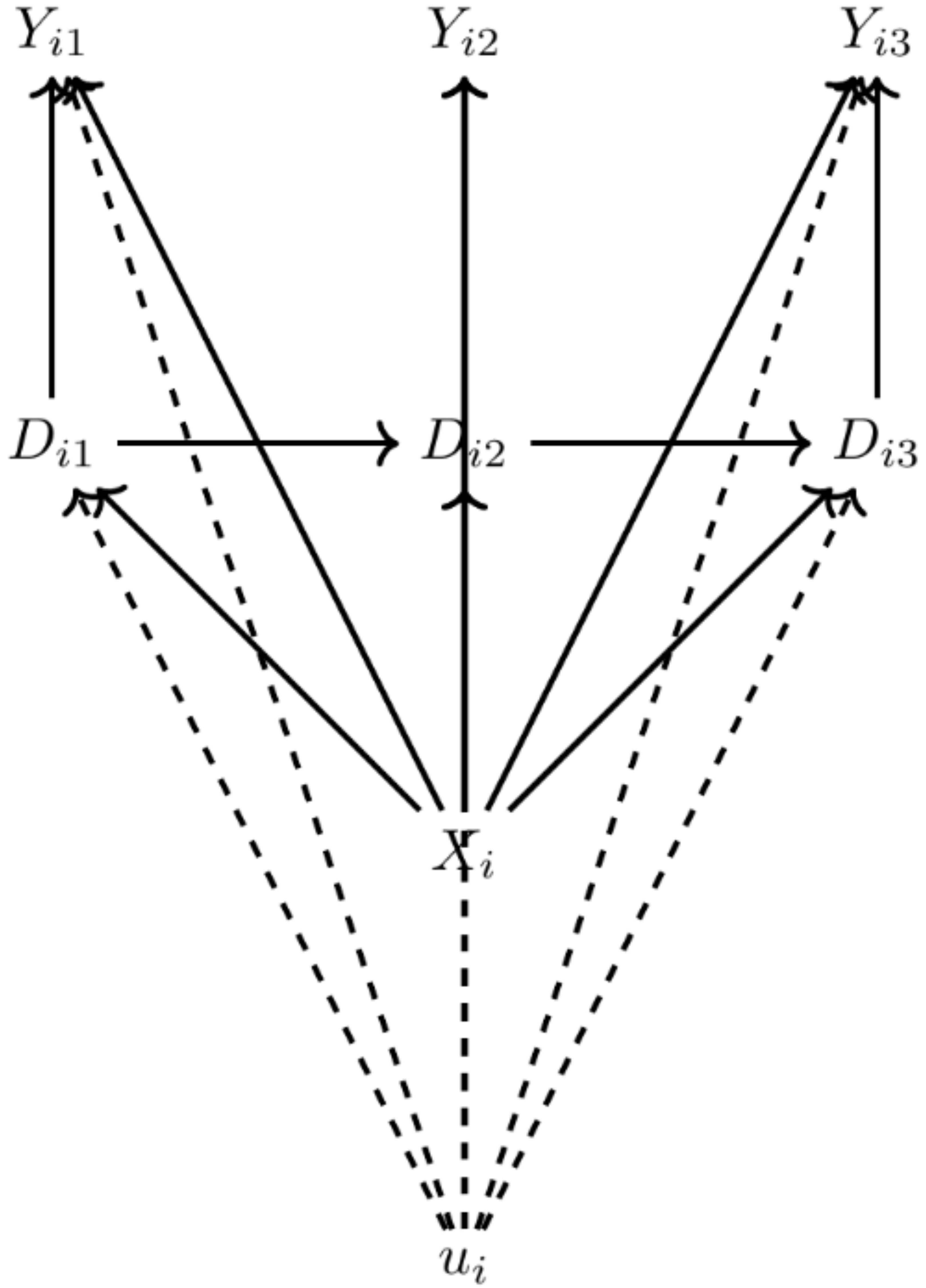
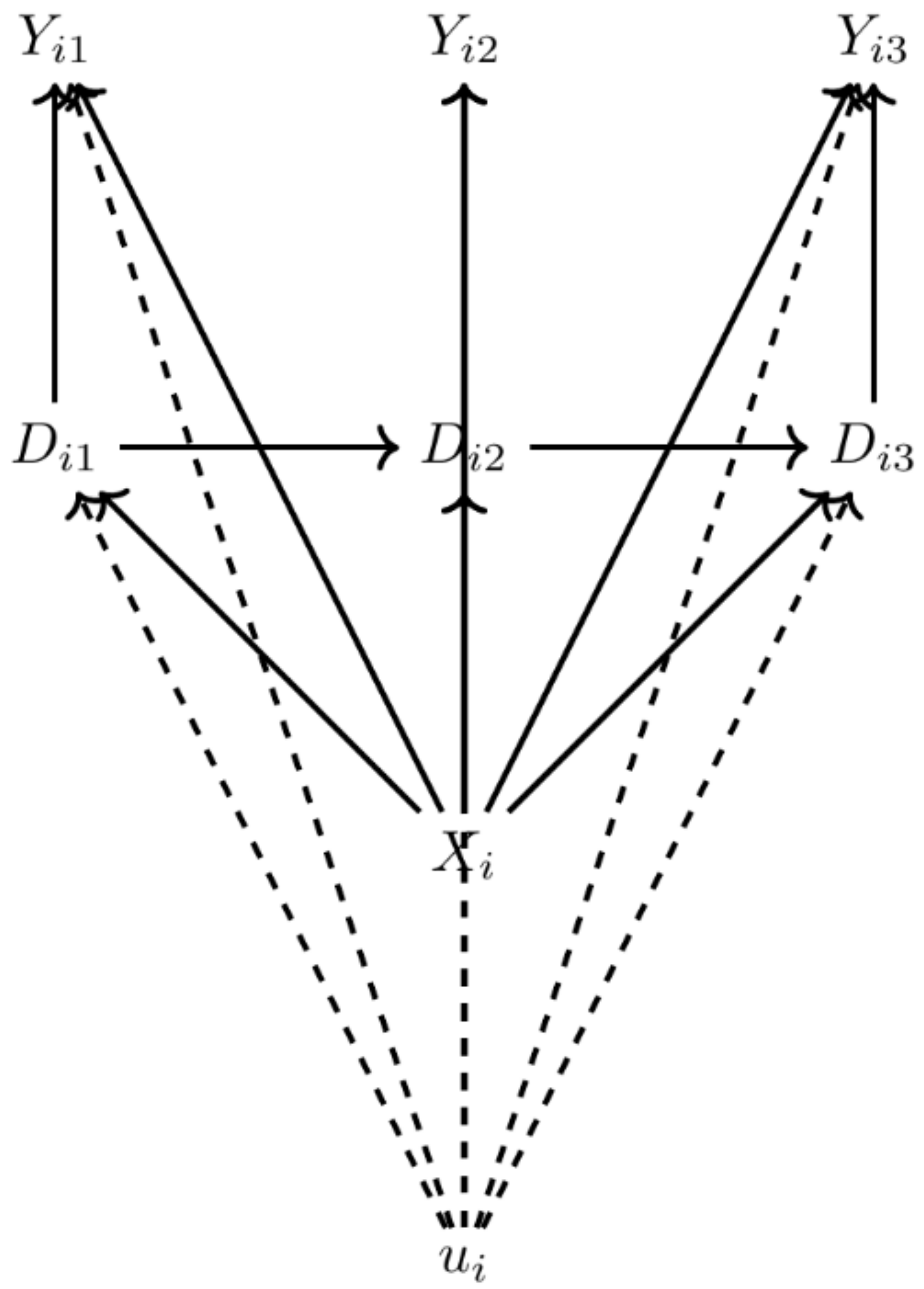


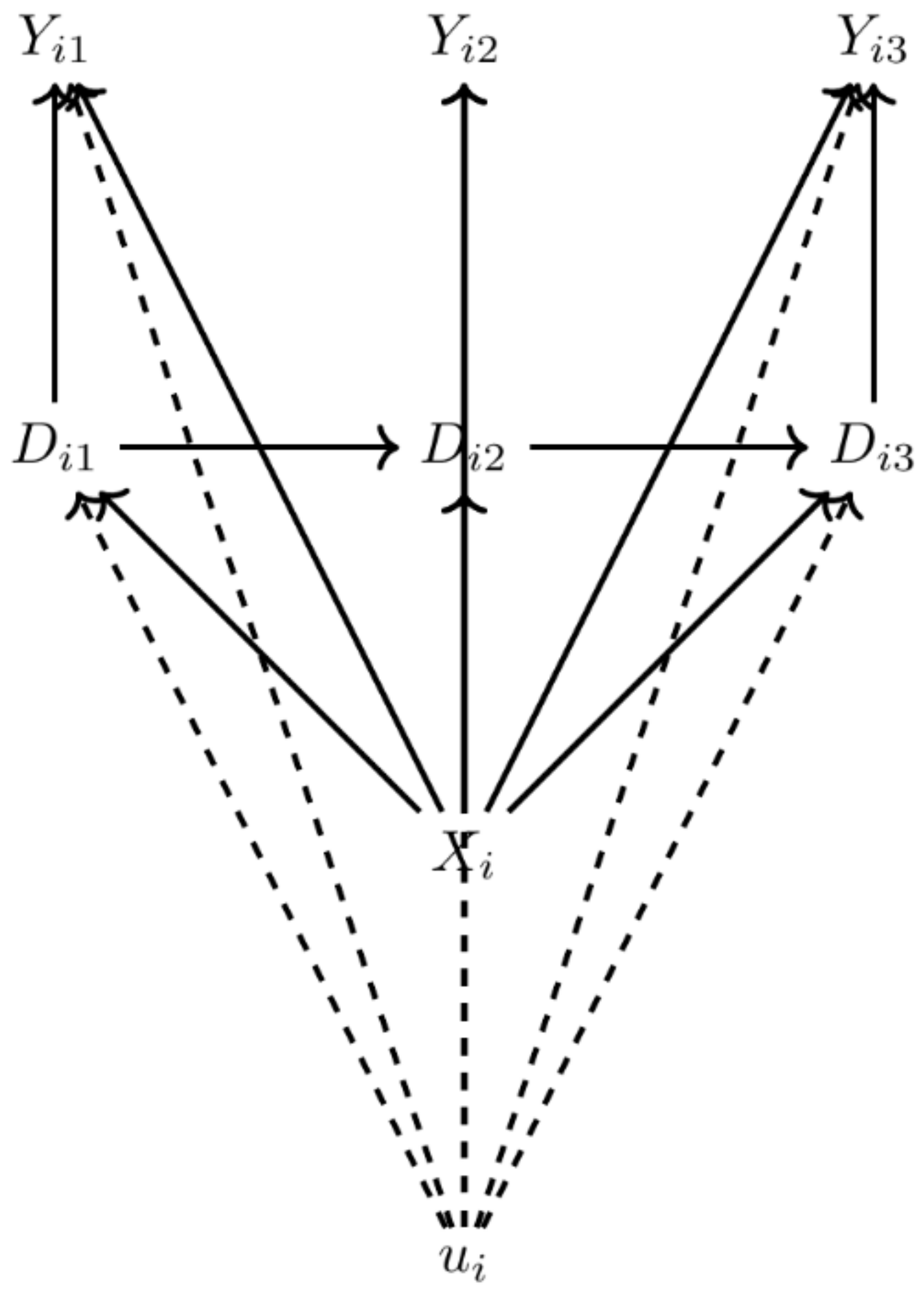
Directed Acyclic Graphs: Panel Data

- The most complex DAG yet
- D_{it} can vary over time
  - $D_{i1} \rightarrow D_{i2}$
  - $D_{i2} \rightarrow D_{i3}$
- Y_{it} can vary over time
  - $D_{i1} \rightarrow Y_{i1}$
  - $D_{i2} \rightarrow Y_{i2}$
  - $D_{i3} \rightarrow Y_{i3}$



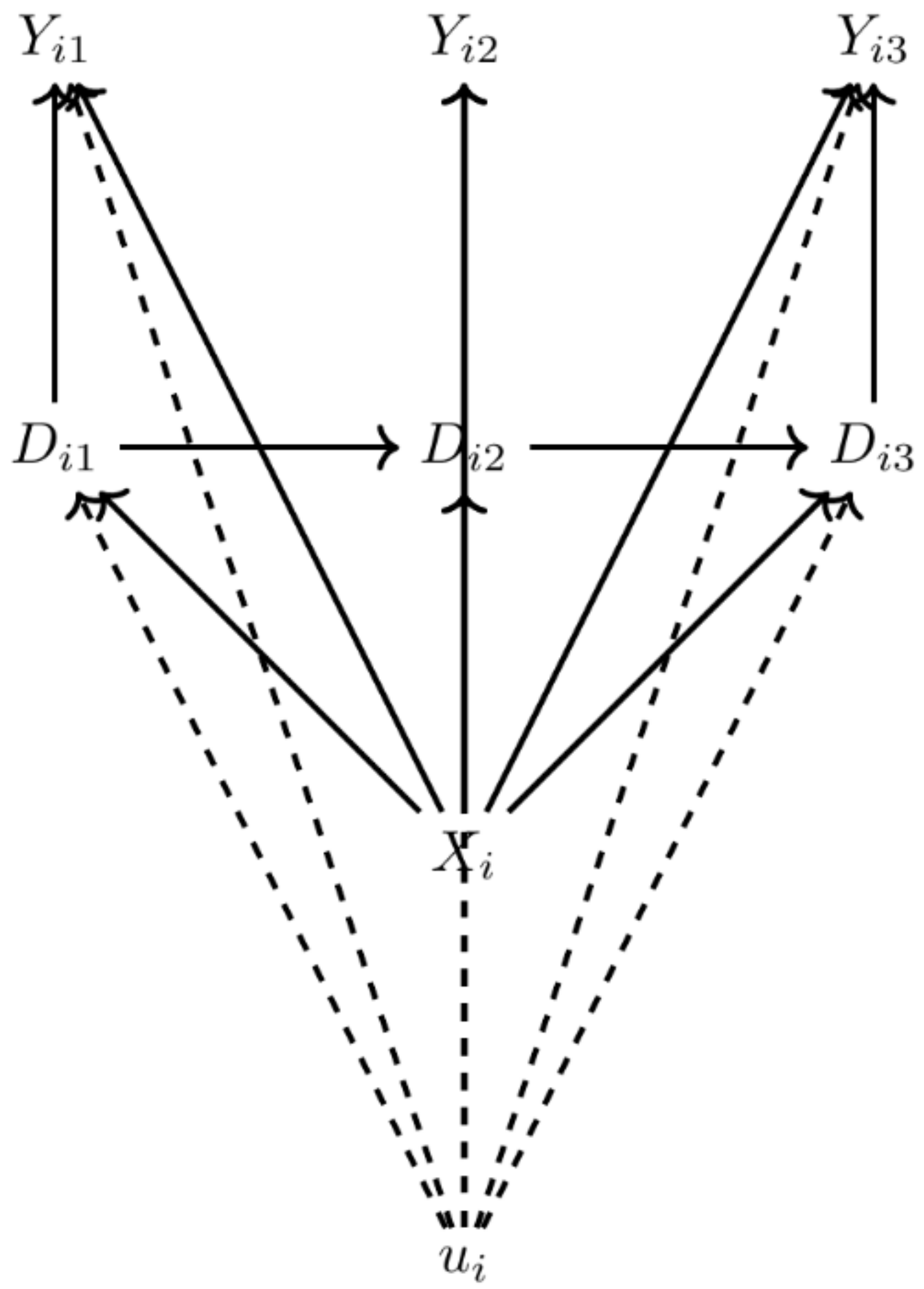
Directed Acyclic Graphs: Panel Data

- D_{it} affects Y_{it} and $D_{i(t+1)}$
    - $Y_{it} \leftarrow D_{it} \rightarrow D_{i(t+1)}$
- We **assume** that outcomes are not affected by prior outcomes
    - $Y_{it} \perp Y_{i(t+1)}$
- We assume that prior treatment does not affect current outcomes ***directly***, but mediated
    - $D_{it} \rightarrow D_{i(t+1)} \rightarrow Y_{i(t+1)}$



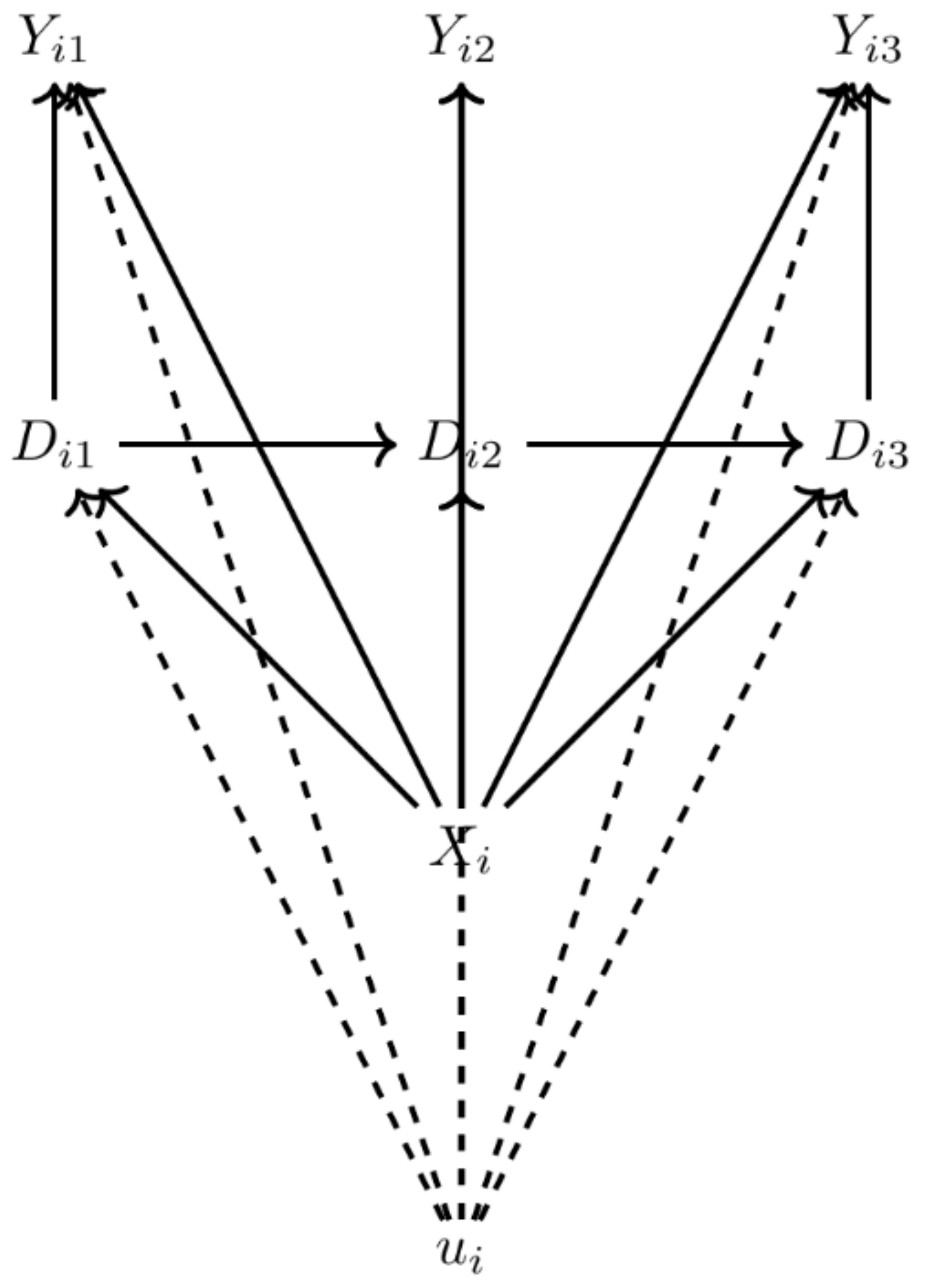
Directed Acyclic Graphs: Panel Data

- X_i are observed confounders that do not vary over time
- u_i are unobserved confounders that do not vary over time
- D_{it} and Y_{it} have t in the subscripts and X_i and u_i do not
- This means that D and Y vary over time and X and u are time-invariant



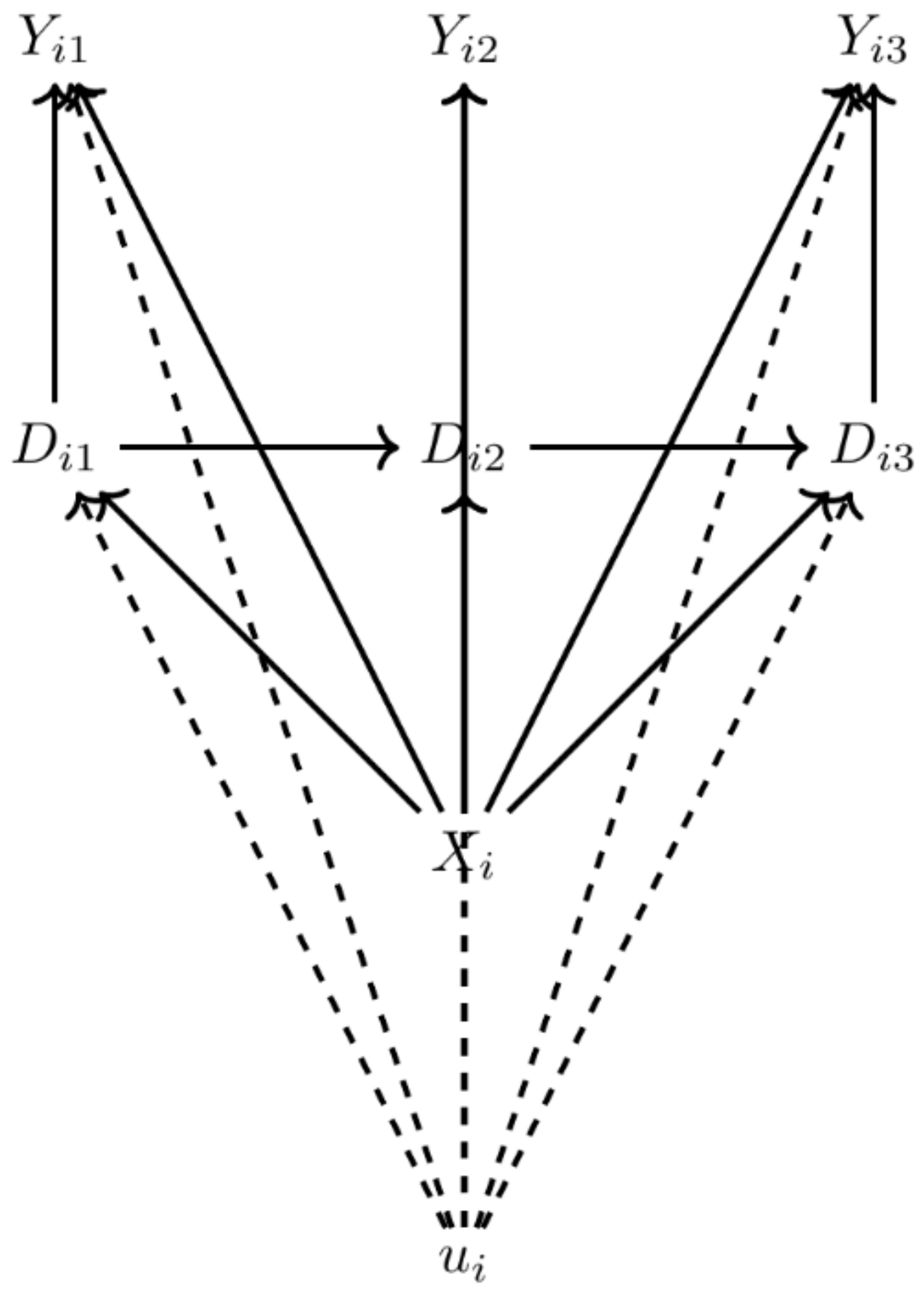
Directed Acyclic Graphs: Panel Data

- Given that X_i and u_i are time-invariant
  - $Y_{it} \leftarrow X_i \rightarrow D_{it}$
  - $Y_{it} \leftarrow - u_i - \rightarrow D_{it}$
- X_i and u_i are time-invariant confounders that affect treatment and outcome in every period



Directed Acyclic Graphs: Panel Data

- Under this scenario, we can use the Fixed Effects (Within) Estimator
- The Fixed Effects (Within) Estimator closes all of the confounding backdoors that do not vary over time
- Note: if u varied over time, then the Fixed Effects (Within) Estimator would not close all backdoor pathways



Panel Data Estimators

- Panel data refers to data where an unit of observation (individual, firm, county, etc.) is observed longitudinally or over time (more than one time period)
- If observed or unobserved confounders do not vary across time for a unit (but vary across units)
    - We can use panel data to identify the causal effect
- Estimators with Panel Data
    - Pooled OLS
    - Fixed Effects (Within) Estimator
    - First-differencing (not our focus)
    - Random Effects Estimator (requires special assumption and not our focus)

Notation

- We will utilize traditional notation instead of potential outcomes for panel data
- Let Y and D be random variables
  - Where $D \equiv D(D_i, D_2, \ldots, D_k)$
- Let u be an unobserved random variable
- We are interested in the partial effect of D_j from
  - $E[Y | D_1, D_2, \ldots, D_k, u]$

Notation

- We observe a sample $i = 1,2,...,N$ cross-sectional units for $t = 1,2,...,T$ time periods, which is a a balanced panel (no missing observations in matrix)
- Cross-sectional independence: individuals in the panel are identical and independent draws from the population $\{Y_i, D_i, u_i\}_{i=1}^{N} \sim i\,.\,i\,.\,d$
- $Y = \begin{bmatrix} Y_{11} & Y_{21} & \cdots & Y_{N1} \\ Y_{12} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1T} & Y_{2T} & \cdots & Y_{NT} \end{bmatrix}$ and $D = \begin{bmatrix} D_{11} & D_{21} & \cdots & D_{N1} \\ D_{12} & D_{22} & \cdots & D_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ D_{1T} & D_{2T} & \cdots & D_{NT} \end{bmatrix}$

Regression Notation

- $Y_{it} = \delta D_{it} + u_i + \varepsilon_{it}$ is our unobserved effects model
- Where Y_{it} is our outcome of interest for $i = 1,2,...,N$ over $t = 1,2,...,T$
- δ is our treat effect of interest
- D_{it} is our treatment for $i = 1,2,...,N$ over $t = 1,2,...,T$
- u_i is the sum of all time-invariant person-specific characteristics, such as ability
- ε_{it} is the idiosyncratic error - includes unobserved time-varying covariates

Pooled OLS

- Pooled OLS is the simplest panel data estimator
- It does not account for the panel structure
    - $Y_{it} = \delta D_{it} + \eta_{it}; \ \ t = 1,2,...,T$
- Where the composite error
    - $\eta_{it} \equiv u_i + \varepsilon_{it}$
    - Where u_i is time-invariant heterogeneity
    - ε_{it} is time-varying heterogeneity
- We need to ***assume*** that u_i is does not impact D_{it} for all time periods

Pooled OLS

- In order to identify the causal effect with Pooled OLS
  - We need to show that
  - $E[\eta_{it} | D_{i1}, D_{i2}, \ldots, D_{iT}] = E[\eta_{it} | D_{it}] = 0 \ \forall \ t = 1,2,...,T$
- TL;DR
  - We need to ignore omitted variable bias, which is unlikely to work well
  - This is likely not a credible assumption

Fixed Effects (Within) Estimator

- The Fixed Effects (FE) Estimator is also called the Within Estimator
    - It accounts for the variation ***within*** a unit of observation over time
- The FE estimator will does a better job of controlling for observed and unobserved time-invariant confounders

Fixed Effects (Within) Estimator

- Our unobserved effects model
  - $Y_{it} = \delta D_{it} + u_i + \varepsilon_{it}$
- We think of our fixed effect u_i as a covariate to be estimated
- The OLS estimation for $\hat{\delta}$ and $\hat{u}_i$ under minimizing sum of squared (over i and t)
  - $$(\hat{\delta}, \hat{u}_1, \hat{u}_2, \ldots, \hat{u}_N) = \underset{b, m_1, \ldots, m_N}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} (Y_{it} - D_{it} b - m_i)^2$$
  - This means we include N number of individual dummy variables in the regression of Y_{it} on D_{it}

Fixed Effects (Within) Estimator

- We'll use our two first order conditions
  - Recall that $E[u|x]=0$ and $E[ux]=0$
- $\sum_{i=1}^{N}\sum_{t=1}^{T} D'_{it}(Y_{it}-D_{it}\hat{\delta}-\hat{u}_i)=0$
- $\sum_{t=1}^{T}(Y_it-D_{it}\hat{\delta}-\hat{u}_i)=0$ for $i=1,2,\ldots,N$

Fixed Effects (Within) Estimator

- Therefore, for $i = 1,2,...,N$
  - $\hat{u}_i = \dfrac{1}{T}\sum_{t=1}^{T}(Y_{it} - D_{it}\hat{\delta}) = \bar{Y}_i - \bar{D}_i\hat{\delta}$
    - Where $\bar{D}_i \equiv \dfrac{1}{T}\sum_{t=1}^{T} D_{it}$ and $\bar{Y}_i \equiv \dfrac{1}{T}\sum_{t=1}^{T} Y_{it}$
- Plug in the results into the first first-order condition: $\sum_{i=1}^{N}\sum_{t=1}^{T} D'_{it}(Y_{it} - D_{it}\hat{\delta} - \hat{u}_i) = 0$

Fixed Effects (Within) Estimator

- $\hat{\delta} = \dfrac{\sum_{i=1}^{N}\sum_{t=1}^{T}(D_{it} - \bar{D}_i)'(Y_{it} - \bar{Y}_i)}{\sum_{i=1}^{N}\sum_{t=1}^{T}(D_{it} - \bar{D}_i)'(D_{it} - \bar{D}_i)} = \dfrac{\sum_{i=1}^{N}\sum_{t=1}^{T}\ddot{D}_{it}'\ddot{Y}_{it}}{\sum_{i=1}^{N}\sum_{t=1}^{T}\ddot{D}_{it}'\ddot{D}_{it}}$
  - Where $\ddot{D}_{it} \equiv D_{it} - \bar{D}_i$ and $\ddot{Y}_{it} \equiv Y_{it} - \bar{Y}_i$
- Recall $\hat{\delta} = \dfrac{C(Y, D)}{V(D)}$

Fixed Effects (Within) Estimator

- TL;DR
  - Using time-demeaned variables $\ddot{D}_{it} \equiv D_{it} - \bar{D}_i$ and $\ddot{Y}_{it} \equiv Y_{it} - \bar{Y}_i$ is equivalent to a regression of Y_{it} on D_{it} with unit-specific dummy variables
- This is why it is called the Within Estimator, since we are utilizing the variation within a specific-unit
- When we include unit-specific fixed effects and year-specific fixed effects, this is called the "two-way fixed effects" estimator
  - We'll cover this later

Fixed Effects (Within) Estimator

- Using time-demeaned variables, the time-invariant confounders zeros out
  - Time-invariant confounders do not vary, such that $E[c] = c$
  - Demeaning eliminates time-invariant observed and unobserved confounders, such that $u_i - \bar{u}_i = 0$
- $Y_{it} = \delta D_{it} + u_i + \varepsilon_{it}$
  - Demean across T: $(Y_{it} - \bar{Y}_i) = (\delta D_{it} - \delta \bar{D}_i) + (u_i - \bar{u}_i) + (\varepsilon_{it} - \bar{\varepsilon}_{it})$
- $\ddot{Y}_{it} = \delta \ddot{D}_{it} + \ddot{\varepsilon}_{it}$

Implement Fixed Effects

- There are three ways that we can implement fixed effects in our regression
- 1) Demean and regression $\ddot{Y}_{it}$ on $\ddot{D}_{it}$ (and need to correct for degrees of freedom)
- 2) Regress Y_{it} on D_{it} and unit-specific dummy variables (dummy variable regression)
- 3) Regress Y_{it} on D_{it} with canned fixed effects routine in Stata or R

Assumptions

- There are a couple of necessary identifying assumption we need in order for the Fixed Effects (Within) Estimator to identify the causal effect
- Strictly exogenous assumption (independence assumption and not testable)
  - $E[\varepsilon_{it} | D_{i1}, D_{i2}, \ldots, D_{iT}, u_i] = 0$ for $t = 1,2,...,T$
- Rank assumption (variation is required for at least some units and is testable)
  - $rank\left(\sum_{t=1}^{K} E[\ddot{D}'_{it}\ddot{D}_{it}] \right) = K$
  - Recall that this is just the $V(D_{it})$ from $\hat{\delta} = \dfrac{C(Y_{it}, D_{it})}{V(D_{it})}$

Assumptions

- Our strict exogeneity assumption will be similar to our independence assumption
  - Our time-invariant confounders can be related to D_{it} since we control for them with fixed effects
  - We need to be concerned about unobserved time-varying confounders, which will violate this assumption
- Our rank assumption requires that there be variation in treatment over time for at least some units of observation
  - Otherwise it will be 0 and violate the assumption

Assumptions

- If our two main assumption hold, then the fixed effects estimator identifies the causal effect

- The fixed effect estimate is consistent $(\underset{N \to \infty}{p \lim}\ \hat{\delta}_{FE,N} = \delta)$ and unbiased

    - This holds as long as the number of clusters is large enough

        - This will be an issue we'll run into with Diff-in-Diff

Caveats of Fixed Effects (Within) Estimator

- There are two key caveats we need to be aware of
- 1) Fixed effects cannot resolve reverse causality
  - $Y \to D$
- 2) Fixed effects cannot control for time-varying unobserved confounders
  - We have to assume that there are no time-varying unobserved confounders
  - $E[\varepsilon_{it} \mid D_{i1}, D_{i2}, \ldots, D_{iT}, u_i] = 0$ for $t = 1,2,...,T$

Caveats

- Fixed Effects Estimator cannot handle reverse causality
  - Cornwell and Trumbell (1994) find positive correlations between policing and crime rates using panel data from North Carolina

Table 8.1: Panel estimates of police on crime

Dependent variable	Between	Within	2SLS (FE)	2SLS (No FE)
Police	0.364	0.413	0.504	0.419
	(0.060)	(0.027)	(0.617)	(0.218)
Controls	Yes	Yes	Yes	Yes

North Carolina county level data. Standard errors in parenthesis.

Caveats

- Does this mean that police cause more crime?
    - It is likely a reverse causality problem and they did not identify the causal effect
    - Police spending is a function of crime rates and crimes rates
    - **Simultaneity bias** is creating bias for their estimated treatment effects
- Becker (1986) predicts the police spending per capita will theoretically reduce crime
    - What is the relationship between the outcome, treatment, and covariates of interest?

Caveats

- We need to assume no time-varying unobserved confounders
  - The presence of any time-varying unobserved confounders will prevent the backdoor criterion from being satisfied
  - This is just **omitted variable bias**
- You will need another research design to handle time-varying unobserved confounders
  - Demean time-varying confounder is just moved to the composite error term and the strict exogeneity assumption (independence assumption) is violated

Example: Returns to Marriage

- Cornwell and Rupert (1997) attempt to estimate the returns to marriage on earnings
  - The SDO shows that married men earn more than unmarried men
  - The SDO is likely biased from confounders and selection into marriage
- We'll use panel data estimators to see the returns to marriage
  - What does the Feasible Generalized Least Squared model compared to Fixed Effects?

Example: Returns to Marriage

- Let's set up the model for individuals i observed over four periods t
  - $Y_{it} = \alpha + \delta M_{it} + \beta X_{it} + A_i + \gamma_i + \varepsilon_{it}$
- Where
  - Y_{it} is earnings for individual i earnings in time period t
  - M_{it} is the outcome of interest and δ is our treatment effect of interest
  - X_{it} is a set of observable covariates for individual i earnings in time period t
  - A_i is unobserved time-invariant ability for individual i
  - γ_i is unobserved time-invariant confounders for individual i
  - ε_{it} is our idiosyncratic error or unobserved determinants of wage that are assumed to be unrelated to M_{it}

Example: Returns to Marriage

Dependent variable	FGLS	Within	Within	Within
Married	0.083	0.056	0.051	0.033
	(0.022)	(0.026)	(0.026)	(0.028)
Education controls	Yes	No	No	No
Tenure	No	No	Yes	Yes
Quadratics in years married	No	No	No	Yes

Example: Returns to Marriage

- Cornwell and Rupert (1997) find that the Feasible Generalized Least Squares model is upward biased
  - After controlling for time-varying covariates, such as education, tenure, and years of marriage
  - The treatment effect is not statistically significant

Example

- Stata Exercise (Cunningham and Kendall, 2011, 2014, 2016)

Table 8.3: POLS, FE and Demeaned OLS Estimates of the Determinants of Log Hourly Price for a Panel of Sex Workers

Depvar:	**POLS**	**FE**	**Demeaned OLS**
Unprotected sex with client of any kind	0.013	0.051*	0.051*
	(0.028)	(0.028)	(0.026)
Ln(Length)	-0.308^{***}	-0.435^{***}	-0.435^{***}
	(0.028)	(0.024)	(0.019)
Client was a Regular	-0.047^{*}	-0.037^{**}	-0.037^{**}
	(0.028)	(0.019)	(0.017)
Age of Client	-0.001	0.002	0.002
	(0.009)	(0.007)	(0.006)
Age of Client Squared	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)
Client Attractiveness (Scale of 1 to 10)	0.020***	0.006	0.006
	(0.007)	(0.006)	(0.005)

Example

- Stata Exercise (Cunningham and Kendall, 2011, 2014, 2016)

Second Provider Involved	0.055	0.113*	0.113*
	(0.067)	(0.060)	(0.048)
Asian Client	-0.014	-0.010	-0.010
	(0.049)	(0.034)	(0.030)
Black Client	0.092	0.027	0.027
	(0.073)	(0.042)	(0.037)
Hispanic Client	0.052	-0.062	-0.062
	(0.080)	(0.052)	(0.045)
Other Ethnicity Client	0.156**	0.142***	0.142***
	(0.068)	(0.049)	(0.045)
Met Client in Hotel	0.133***	0.052*	0.052*
	(0.029)	(0.027)	(0.024)
Gave Client a Massage	-0.134^{***}	-0.001	-0.001
	(0.029)	(0.028)	(0.024)

Example

- Stata Exercise (Cunningham and Kendall, 2011, 2014, 2016)

Age of provider	0.003	0.000	0.000
	(0.012)	(.)	(.)
Age of provider squared	-0.000	0.000	0.000
	(0.000)	(.)	(.)
Body Mass Index	-0.022^{***}	0.000	0.000
	(0.002)	(.)	(.)
Hispanic	-0.226^{***}	0.000	0.000
	(0.082)	(.)	(.)
Black	0.028	0.000	0.000
	(0.064)	(.)	(.)
Other	-0.112	0.000	0.000
	(0.077)	(.)	(.)
Asian	0.086	0.000	0.000
	(0.158)	(.)	(.)

Example

- Stata Exercise (Cunningham and Kendall, 2011, 2014, 2016)

Imputed Years of Schooling	0.020**	0.000	0.000
	(0.010)	(.)	(.)
Cohabitating (living with a partner) but unmarried	-0.054	0.000	0.000
	(0.036)	(.)	(.)
Currently married and living with your spouse	0.005	0.000	0.000
	(0.043)	(.)	(.)
Divorced and not remarried	-0.021	0.000	0.000
	(0.038)	(.)	(.)
Married but not currently living with your spouse	-0.056	0.000	0.000
	(0.059)	(.)	(.)
N	1,028	1,028	1,028
Mean of dependent variable	5.57	5.57	0.00

Heteroskedastic robust standard errors in parenthesis clustered at the provider level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

Concluding Remarks

- Fixed Effects (Within) Estimator can be a powerful tool
- May not be utilized enough in federal evaluations
  - Unlike PSM, fixed effects can control for some unobservable confounders
- As long as
  - 1) There are no time-varying confounders, you have a straight-forward methodology that can identify the causal effect
  - 2) There is no reverse causality
    - This will require an instrumental variable strategy