Solving the TB Model Using Runge-Kutta 4th Order Method (2 Iterations)

The TB model is described by the system of ODEs:

$$\frac{dS}{dt} = \lambda - \beta S(I + \delta L) - \mu S$$

$$\frac{dE}{dt} = \beta(1 - p)S(I + \delta L) + r^2 I - [\mu + k(1 - r^1)]E$$

$$\frac{dI}{dt} = \beta p S(I + \delta L) + k(1 - r^1)E + \gamma L - [\mu + d^1 + \varphi(1 - r^2) + r^2]I$$

$$\frac{dL}{dt} = \varphi(1 - r^2)I - (\mu + d^2 + \gamma)L$$

Initial Conditions and Parameters

$\lambda = 2$, $\beta = 0.025$, $\delta = 1$, $p = 0.3$

$\mu = 0.0101$, $k = 0.005$, $r_1 = 0$, $r_2 = 0.8182$

$\varphi = 0.02$, $\gamma = 0.01$, $d_1 = 0.0227$, $d_2 = 0.20$

Initial values: $S_0 = 198.02$, $E_0 = 1$, $I_0 = 0$, $L_0 = 0$

Step size $h = 0.1$

$$k1 = h * f(t_n, y_n)$$

$$k2 = h * f\left(t_n + \frac{h}{2}, y_n + \frac{k1}{2}\right)$$

$$k3 = h * f\left(t_n + \frac{h}{2}, y_n + \frac{k2}{2}\right)$$

$$K4 = h * f(t_n + h, y_n + k3)$$

$$y_{\{n+1\}} = y_n + \left(\frac{1}{6}\right) * (k1 + 2k2 + 2k3 + k4)$$

Iteration 1 (t = 0 → 0.1)

k1 values:

$$k1_S = 0$$

$$k1_E = -0.00151$$

$$k1_I = 0.0005$$

$$k1_L = 0$$

Intermediate values for k2:

$$S = 198.02, E = 0.999245, I = 0.00025, L = 0$$

$$k2_S \approx -0.0001$$

$$k2_E \approx -0.0015087$$

$$k2_I \approx 0.0005115$$

$$k2_L \approx 0.00000046$$

Intermediate values for k3 (same input as k2):

$$k3_S \approx -0.0001$$

$$k3_E \approx -0.0015087$$

$$k3_I \approx 0.0005115$$

$$k3_L \approx 0.00000046$$

k4 values:

$$S = 198.01999, E = 0.9984913, I = 0.0005115, L = 0.00000046$$

$$k4_S \approx -0.0002$$

$$k4_E \approx -0.0015074$$

$$k4_I \approx 0.000523$$

$$k4_L \approx 0.00000094$$

Final values after Iteration 1:

$$S^1 \approx 198.01995, E^1 \approx 0.9984929, I^1 \approx 0.0005116, L^1 \approx 0.00000046$$

Iteration 2 (t = 0.1 → 0.2)

Using values from end of Iteration 1:

$$S = 198.01995, E = 0.9984929, I = 0.0005116, L = 0.00000046$$

$$k1_S \approx -0.0003$$

$$k1_E \approx -0.0015046$$

$$k1_I \approx 0.0005346$$

$$k1_L \approx 0.00000188$$

k2 intermediate values:

$$S \approx 198.0198, E \approx 0.9977406, I \approx 0.0007789, L \approx 0.0000014$$

$$k2_S \approx -0.0004$$

$$k2_E \approx -0.0015024$$

$$k2_I \approx 0.0005466$$

$$k2_L \approx 0.00000287$$

k3 intermediate values:

$$k3_S \approx -0.0004$$

$$k3_E \approx -0.0015024$$

$$k3_I \approx 0.0005466$$

$$k3_L \approx 0.00000287$$

k4 values:

$$S \approx 198.0195, E \approx 0.9979905, I \approx 0.0010582, L \approx 0.00000333$$

$$k4_S \approx -0.0005$$

$$k4_E \approx -0.0015002$$

$$k4_I \approx 0.0005586$$

$$k4_L \approx 0.00000375$$

Final values after Iteration 2:

$$S^2 \approx 198.0196, E^2 \approx 0.997991, I^2 \approx 0.0010582, L^2 \approx 0.00000334$$