Project Insurance
GROUP_08_SII_project

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1. Original text of the project

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:

ASSETS

- there is a unique fund made of a bond combined with an equity
- at every time step t the value of the fund (before deducting the fees) is Ft = Bt + St
- at the beginning (t=0) the value of the fund is equal to the insured capital F0 = C0 = 1000
- bond features
  - AAA corporate zero coupon bond with maturity T=10
  - B0=800, face amount N=1000
- equity features
  - listed in the regulated markets in the EEA
  - S0=200
  - No dividend yield
  - to be simulated with a Risk Neutral GBM (sigma=20%) and a time varying instantaneous risk free rate r derived from the yield curve (EIOPA IT with VA 31.03.20), supposing linear interpolation of the zero rates and using the formula DFt+dt = DFt * exp[-rt*dt]

LIABILITIES

- Term Life policy with term T=10
- the insured capital given in case of death/lapse and survivor at maturity is equal to
  - CASE A:
    * guaranteed
    * Ct = max(C0,F't)
  - CASE B:
    * not guaranteed
    * Ct = F't
  - where
    * F't = Ft − feest
    * feest = Ft-1 * 1.50%
- male insured aged x=60
- mortality rates derived from the life table SI2017 (ISTAT website)
- flat annual lapse rates lx=5%

Other specifications:

- the interest rates dynamic is deterministic, while the equity one is stochastic
- the default (credit) spread s has to be computed in the plain case (no IR stress) to match the zero coupon bond price B0=800

QUESTIONS

1. For both cases A and B, code a Matlab script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained in A and B. The risks to be considered are:
    - Market Interest
    - Market equity
    - Market spread
    - Life mortality
    - Life lapse
    - Life cat
2. Calculate the duration of the liabilities in all the cases and provide comments on the results obtained
3. Replicate the same calculations in an Excel spread sheet using a deterministic projection. Do the results differ from 1? If so, what is the reason behind?
4. Open questions:
    - what happens to the asset and liabilities when the risk free rate increases/decreases? Describe all the effects
    - what happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

2. Summary tables (one for A and one for B) with the results obtained

Case A	Assets	Liabilities	Bof	dBof	dur_L	SCR	BSCR
Base	1000	1116,069	-116,069	-	7,881		
IR_up	924,475	1037,598	-113,123	-2,946	7,837	0	
IR_down	1000	1116,069	-116,069	0	7,881	0	
Equity	942	1065,350	-123,350	7,281	7,854	7,281	
Spread	944	1108,738	-164,738	48,669	7,905	48,669	
Market tot						54,344	
Mortality	1000	1115,439	-115,439	-0,630	7,842	0	
Lapse_up	1000	1104,288	-104,289	-11,780	7,137	0	
Lapse_down	1000	1129,591	-129,591	13,522	8,714	13,522	
Lapse_mass	1000	1079,415	-79,415	-36,654	5,494	0	
CAT	1000	1116,068	-116,068	0	7,881	0	
Life tot						13,522	
TOT							59,191

Case B	Assets	Liabilities	Bof	dBof	dur_L	SCR	BSCR
Base	1000	1112,944	-112,944	-	7,896		
IR_up	924,475	1026,806	-102,330	-10,614	7,893	0	
IR_down	1000	1112,944	-112,944	0	7,896	0	
Equity	942	1055,258	-113,258	0,314	7,904	0,314	
Spread	944	1099,604	-155,604	42,660	7,949	42,660	
Market tot						42,895	
Mortality	1000	1112,259	-112,259	-0,685	7,857	0	
Lapse_up	1000	1100,013	-100,013	-12,931	7,155	0	
Lapse_down	1000	1127,729	-127,729	14,785	8,724	14,785	
Lapse_mass	1000	1071,769	-71,769	-41,175	5,523	0	
CAT	1000	1112,943	-112,943	-0,001	7,896	0	
Life tot						14,785	
TOT							48,742

3. Formulas adopted

We began the computation calculating Assets and Liabilities for the Base Scenario. Liabilities were computed taking into account the fund composed by the bond and the equity, the mortality rate of the insured person and the lapse rate. Also we compute the Basic Own Funds as:

$$BoF = Assets - Liabilities$$

Then we recompute Asset, Liabilities, BoF for the stressed scenario. The capital requirement is determined as the impact of a specified stressed scenario on the BoF:

$$d\_Bof = BoF\_Base - BoF\_Stressed$$

The d_BoF is positive when the scenario results in a loss of the BoF. If the scenario results in an increase of BoF, d_BoF is negative, it does not reflect a risk for the undertaking and so it is floored to zero:

$$SCR = \max(d\_Bof, 0)$$

We consider the Market risk and the Life risk to calculate the Basic Solvency Capital Requirement. The Basic SCR is a sum of all individual risks coming from different modules, that are aggregated through linear correlations to take into account the diversification and the mitigation effect

$$BSCR = \sqrt{\Sigma_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j}$$

with the correlation coefficients

	Market	Life
Market	1.00	0.25
Life	0.25	1.00

The exposition to the Market risk is calculated measuring the impact of the changes in equity prices, interest rates and spread. The formula is

$$SCR_{market} = \sqrt{\Sigma_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j}$$

with the correlation coefficients

	Interest	Equity	Spread
Interest	1.00	0.00	0.00
Equity	0.00	1.00	0.75
Spread	0.00	0.75	1.00

if exposed to the IR_up (SCR_up greater)

	Interest	Equity	Spread
Interest	1.00	0.50	0.50
Equity	0.50	1.00	0.75
Spread	0.50	0.75	1.00

if exposed to the IR_down (SCR_down greater).

The exposition to the Life risk is calculated measuring the impact of the changes in mortality rates, lapse rates and taking into consideration extreme or irregular events (cat). The formula is

$$SCR_{life} = \sqrt{\Sigma_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j}$$

with the correlation coefficients

	Mortality	Lapse	CAT
Mortality	1.00	0.00	0.25
Lapse	0.00	1.00	0.25
CAT	0.25	0.25	1.00

3.1. Market Interest

We took into consideration the Interest Rate risks in both the cases of an upward shock and a downward shock. The Risk Free Rates were given by EIOPA and the calculation were reproduced for both the stressed scenarios. The capital requirement for Interest Rate risk shall be equal to the larger of the SCR for the risk of an increase in the term structure of interest rates and the SCR for the risk of a decrease in the term structure of interest rates:

$$SCR_{interest} = max(SCR_{down}, SCR_{up})$$

In both cases A and B the d_BoF is negative, so it does not reflect a risk for the undertaking.

3.2. Market Equity

The Equity risk arises from the level or volatility of market prices for equities. The Equity shock scenario, for each category, is the sum of the base shock and the symmetric adjustment (that varies per each valuation date). Type 1 equities, like the one of our calculations, have an equity shock of 39% and the symmetric adjustment is -10% (31.03.2020).

$$S_{0,shocked} = S_0 * (1 - Shock - SymAdj)$$

In both cases A and B the stress resulted in a positive d_BoF, highlighting a risk, higher in case A, close to zero in case B.

3.3. Market Spread

The Spread risk results from the sensitivity of the value of Assets and Liabilities to changes in the level or in the volatility of credit spreads over the Risk Free Rates term structure. The Spread risk factor for the bond of our calculations, Credit Quality Step 0 (rating AAA) and duration more than 5 and up to 10 years, is:

$$Spread = 0.045 + 0.005 * (T - 5)$$

In both cases A and B the stress resulted in a positive d_BoF, highlighting the risk, slightly higher in case A but constituting the biggest portion of the SCR_market for both cases.

3.4. Life Mortality

The Mortality risk is the risk of loss resulting from changes in the level, trend, or volatility of mortality rates, where an increase in the mortality rate leads to an increase in the value of insurance liabilities. The mortality shock is an immediate and permanent increase of 15% in the mortality rates adopted:

$$q_{x,shocked} = 1.15 * q_x;$$

In both cases A and B the d_BoF is negative, not reflecting a risk for the undertaking.

3.5. Life Lapse

Lapse risk is the risk of loss or adverse change in liabilities due to a change in the expected exercise rates of policyholder options (contractual rights to surrender insurance cover or permit the insurance policy to lapse). The lapse rates to be used in stressed scenarios are:

- Increase of 50% in the rates in all future years that do not exceed 100%

$$l_{x,up} = min(1.5 * l_x, 1)$$

- Decrease of 50% in the rates in all future years, where the absolute variation does not exceed 20%

$$l_{x,down} = max(0.5 * l_x, l_x - 0.2)$$

- An instantaneous loss of 40% of the in-force business

$$l_{x,mass} = [0.4; l_x(2 : end)]$$

In both cases A and B, the only positive d_Bof was given by the Lapse_down stress and it is also the maximum of the three values, so the SCR_lapse is equal to that value, according to the formula:

$$SCR_{lapse} = max(SCR_{lapse,up}, SCR_{lapse,down}, SCR_{lapse,mass})$$

The value of Lapse_down represents the only portion of the Life SCR.

3.6. Life Cat

The CAT risk represents extreme or irregular events whose effects are not sufficiently captured in the other life underwriting risks, such as a pandemic event. The revision shock is an absolute 1.5 per mille increase in the rate of policyholders dying over the following year:

$$q_{x,cat} = [q_x(1) * 1.0015; q_x(2 : end)]$$

In both cases A and B the d_BoF is equal to zero, so it does not reflect a risk for the undertaking.

The BoF is actually lower than the SCR in both cases A and B, representing the fact that the company needs to take remedial actions. This can be due to the fact that we are analyzing a simplified case.

4. Deterministic calculations

For the deterministic projections, we did not consider the White Noise of the equity simulation and the formula used was:

$$S_{t+1} = S_t * e^{(zRates_t - 0.5*\sigma^2)}$$

We noticed that the BoF of the Base scenario and the Stressed scenarios, IR_up and IR_down, were higher than the stochastic simulation done in MATLAB, however the important value of the d_Bof, giving us the SCR, was very close to the not deterministic case. We made the calculations only for the Market Interest part, to see how the results differ from the stochastic problem. We noticed that the only part that influences this difference is the absence of the White Noise, that is neglected in the d_BoF, which is our important value to calculate the SCR.

5. Open questions

Calculate the duration of the liabilities in all the cases and provide comments on the results obtained

The duration values are in the table above and, since it represents the time in years required to recover the invested capital over a certain period, we can notice that the years for each risk are around 7, except for lapse_down which is around 8 and lapse_mass which is around 5, in both cases A and B.

What happens to the assets and liabilities when the risk free rate increases/decreases? Describe all the effects.

As we noticed in the interest rate up case, when the risk free rate increases the values of both the assets and the liabilities decreased. This is reasonable because as interest rates rise, asset prices fall because investors can receive a higher return on a risk-free investment. Conversely, as interest rates fall, asset prices rise.
For the interest rate down we didn't notice a difference because the rates are exactly equal to the base case, but as said above a decrease in risk free rate would increase the assets and the liabilities value.

What happens to the liabilities if the insured age increases?

It is reasonable to think that if the insured age increases the mortality rates increase too, and this would result in a higher value of the liabilities to cover ourselves from a bigger probability that the policyholder dies before the maturity.

What if there were two model points, one male and one female?

Given that the mortality rates of male and female together are lower than the male ones, we would expect a lower liabilities because we have a smaller probability to pay the insured capital.