FISI4997: Quantum Teleportation Simulation

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1 What is Quantum Teleportation?

Quantum Teleportation (QT) is a protocol which consist of the transfer of information of the state of a message qubit from the sender, Ana, to the receiver, Bob. The initial state of the message qubit is:

$$|m\rangle = a\,|0\rangle_A + b\,|1\rangle_A \tag{1}$$

where the subscripts denote who possesses the qubits. However, note that Ana, although in possession of the message qubit, does not know the probability amplitudes a and b. The information of the state is sent through classical means by Ana and the reconstruction of the initial state of the message qubit by Bob is enabled through the use of a pair of entangled qubits. These qubits are in the Bell state:

$$|\lambda\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right) \tag{2}$$

Ana is in possession of the first of these entangled qubits and Bob has the second one, as is denoted by the subscripts. This means that the initial state of the system is given by:

$$|\psi_0\rangle = |m\rangle \otimes |\lambda\rangle \tag{3}$$

The quantum circuit which performs the QT with the state $|\psi_0\rangle$ as the initial state is shown the following figure.

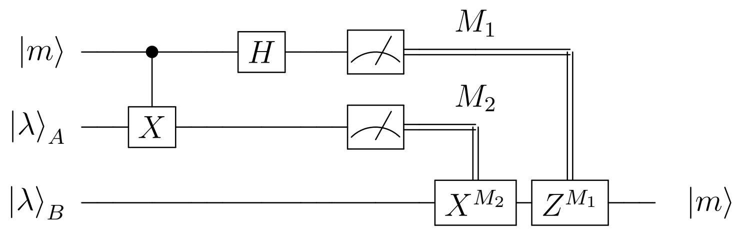

Figure 1: Quantum circuit diagram for QT protocol

The first two gates, the CNOT and the H-gate, set the system to a very particular state which is shown below.

$$\begin{aligned}|\psi_1\rangle = \frac{1}{2}[&|00\rangle_A \otimes (a\,|0\rangle_B + b\,|1\rangle_B) + |10\rangle_A \otimes (a\,|0\rangle_B - b\,|1\rangle_B) \\ &+ |01\rangle_A \otimes (a\,|1\rangle_B + b\,|0\rangle_B) + |11\rangle_A \otimes (a\,|1\rangle_B - b\,|0\rangle_B)] \quad (4)\end{aligned}$$

In this state, the information about the state the message qubit was in is stored in Bob's entangled qubit. When Ana makes measurements on her pair of qubits, Bob's qubits will be projected unto states that require very simple operations to reconstruct the original state of the message qubit. In order for Bob to know what operations he will need to do, Ana sends him through classical means bits denoting the results of her measurements. This part can be seen in the X^{M_2} and Z^{M_1} gates which Bob applies to his qubits. As an example, let's say Ana measures her qubits to be in the state $|10\rangle_A$. In such a case, $M_1 = 1$ and $M_2 = 0$. So, when Bob applies his set of gates, he will only really be applying the Z-gate. Because Bob's qubit is now projected unto the state $a\,|0\rangle_B - b\,|1\rangle_B$, the application of the Z-gate would change the state of his qubit to the one the message qubit was originally in, thus completing the QT protocol.

2 QT Simulation

The QT simulation built with Cirq works with the message qubit's state using its Bloch representation and its general expression in the computational basis. As such the initial state of the message qubit can be expressed as:

$$|m\rangle = e^{-\frac{1}{2}i\phi}\cos\frac{1}{2}\theta\,|0\rangle + e^{\frac{1}{2}i\phi}\sin\frac{1}{2}\theta\,|1\rangle \tag{5}$$

This means that to specify the state to be sent, the angles ϕ and θ need to be given. In the program, these are stored in the variables named phi and theta respectively.

After the qubits to be used are declared and named Q0, Q1 and Q2 for the messenger qubits, and Ana and Bob's entangled qubits respectively, and after the variables phi and theta are given a value, a Circuit object is created and stored in the variable circ using the function called qteleport_circuit_construction. This function takes as input phi, theta and the three declared qubits.

By default, Cirq starts every qubit in the state $|0\rangle$. The function qteleport_circuit_construction starts off by defining a pair of moments, inM_0 and inM_1 ("initialization Moment" 0 and 1). These, in conjunction with the values of phi and theta and when appended to the Circuit object, are used to initialize the state of the message qubit by applying the following sub-circuit:

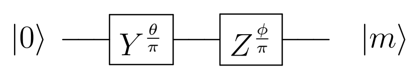

Figure 2: Message qubit initialization subcircuit

The Y and Z-gates respectively correspond to rotations about the y and z axis of the Bloch sphere representation by π. So, the $Y^{\frac{\theta}{\pi}}$ and $Z^{\frac{\phi}{\pi}}$ represent rotations about the same axis, but by the angles ϕ and θ. For example, if θ is $\pi/2$, then $Y^{\frac{\theta}{\pi}} = Y^{\frac{1}{2}}$, and so the rotation would be about the y axis by $\pi/2$.

After this, the function defines another pair of Moment objects, enM_0 and enM_1 ("entanglement Moment" 0 and 1), which set Ana and Bob's auxiliary qubits to the state shown in (1). When appended to the Circuit object stored in the vairable circuit, these two moments form the following subcircuit.

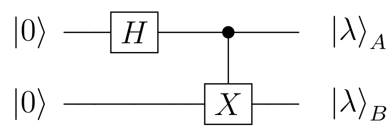

Figure 3: Auxiliary qubit entanglement subcircuit

Finally, the function defines a trio of Moment objects, M_0, M_1 and M_2 ("Moment" 0, 1 and 2), which, when appended to the circuit, form and analogous circuit to the QT circuit shown in Figure 1. The reason for it being analogous is that the X^{M_2} and Z^{M_1}-gates are switched to the equivalent CNOT and CZ-gates respectively.

All of the aforementioned Moments are then appended to the circuit variable, giving the following end product:

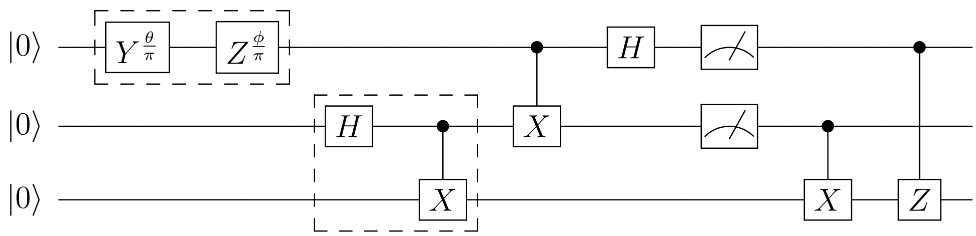

Figure 4: Quantum circuit diagram for QT simulation

The function exits by returning the circuit variable. After this, A's message qubit state is printed and then the simulation of the circuit is run, the results of which are stored in the sim variable. The results are then printed and the program ends.