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An Improved WNNM Algorithm for Image Denoising

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Abstract. The traditional weighted nuclear norm minimization (WNNM) has excellent performance for the removal of non-sparse noise such as Gaussian noise, but attains bad performance for the removing of salt&&pepper noise and mixed noise of Gaussian noise and salt&&pepper noise. This paper proposes an improved WNNM image denoising algorithm. WNNM can effectively remove non-sparse noise such as Gaussian noise and adaptive median filtering algorithm can effectively remove sparse noise such as salt and pepper noise; the improved algorithm combines the characteristics of WNNM and adaptive median filtering. The experimental data demonstrate that the improved WNNM algorithm has better denoising effect than WNNM algorithm.

1. Introduction

In the process of image transmission, reception, etc., the noise greatly reduces the reliability and quality of the image. Therefore, image denoising is increasingly important in image processing. Image denoising can directly affect the subsequent work of image processing, and how to make the image remove noise without distortion as much as possible becomes the primary task of image denoising. In recent years, great progress has been made in image denoising, and the proposed image denoising algorithms can be roughly classified into the following categories: wavelet transform method, method based on total variation, and denoising method based on sparse representation, etc. [1]. In 2006, Micheal Elad proposed an image denoising algorithm based on the sparse representation of K-Singular Value Decomposition (K-SVD) algorithm [2].In this method, the dictionary update is atom by atom; the dictionary construction method has obvious effects and is applied extensively in image denoising; In 2007, Dabov [3] et al. proposed three-dimensional block matching (BM3D) method, joint image filtering in three-dimensional space, and finally, the denoising image is obtained by inverse linear transformation, which is one of the best image denoising algorithms in the field of image recognition. In 2013, Dong [4] et al. proposed a nonlocally centralized sparse representation image denoising method that links local and non-local information of images, using the non-local self-similarity of the image to get the sparse coding estimate of the original image under the representation dictionary, and then centralizing the sparse coding vector of the noise image to the estimate; In 2014, Gu et al. [5] proposed a weighted nuclear norm minimization (WNNM) image denoising algorithm, which introduces the nuclear norm weight coefficient based on the traditional nuclear norm minimization method (NNM),and different singular values are set different weights, increases the flexibility of the nuclear norm while taking advantage of the prior information of the image. WNNM algorithm has obvious advantages in image denoising, especially for non-sparse noise such as Gaussian noise. However, for

sparse noise, such as salt and pepper noise, the denoising effect is not ideal. As is known, the traditional median filtering replaces the value of one pixel with the middle value of the intensity value in the neighborhood of the pixel, which can achieve better effect on removing the salt and pepper noise, but only when the salt and pepper noise probability is small, when the probability of salt and pepper noise is large, the denoising effect of the original image will be greatly reduced regardless of whether the template is large or small. Adaptive median filtering is an improvement of median filtering, compared with median filtering, which can process larger-period impact noise. The size of the filtering window is automatically selected according to the statistical result of pixel grayscale in a certain area, which can better remove sparse noise.

This paper first introduces the traditional WNNM model, and demonstrates the advantages and disadvantages of the algorithm for image denoising, and then proposes an improved weighted nuclear norm minimization (IWNNM) algorithm, which is complementary to the advantages of WNNM algorithm and adaptive median filtering algorithm. Firstly, the WNNM algorithm is used to effectively remove non-sparse noise such as Gaussian noise, and then adaptive median filtering algorithm is used to remove sparse noise such as salt and pepper noise to make up for the shortcomings of WNNM algorithm. After lots of experiments, the IWNNM algorithm proposed in this paper has achieved good results both in terms of visual subjective aspect and image quality evaluation index.

2. WNNM Model

The essence of the traditional nuclear norm minimization method (NNM) is achieved by singular value decomposition combined with threshold singular values, which is more robust to noise than principal component analysis. However, the weights of all singular values in the NNM method are the same, so that the prior knowledge of the image cannot be underutilized. Based on this point, WNNM algorithm is proposed, which takes different weights for different singular values, so that the prior knowledge of the image can be underutilized, and the effect is more obvious in image denoising applications, and the following is a brief introduction to the WNNM denoising algorithm.

Assuming that y denotes the noisy observed image, x denotes the original image , then the target of image denoising is to recover x from $y=x+n$, where n is assumed to be additive Gaussian white noise with zero mean and variance σ_n^2. The groundbreaking research work of non-local mean has led to extensive research on the denoising method of nonlocal self-similarity (NSS) of image patch. Non-local self-similarity means that an image patch has many similar patches at other positions of the image, and the image patch may exist at any position of the image.

For a local patch y_j in image y , a set of non-local similar patches can be found in the image by the method of block matching, and stack these sets to form a matrix Y_j , and then the observed image can be expressed as: $Y_j=X_j+N_j$,where X_j denote the patch matrices of original image , N_j denotes noise. The low rank matrix will be transformed into a full rank matrix if affected by sparse noise. The WNNM denoising algorithm essentially decomposes the noise-affected observation matrix into the original low rank matrix X_j and noise matrix N_j . The nuclear norm minimization method (NNM) uses the method of singular value decomposition to decompose the matrix, which can effectively achieve the approximation of the low rank matrix, and the SVD method is expressed as the following formula:

$$A=U\Sigma V^T \tag{1}$$

where A represents a matrix with rank r, $A \in R^{m \times n}$, $U \in R^{m \times r}$, $V \in R^{n \times r}$, $\Sigma=\mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_r)$,and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$,where σ_i is the i-th singular value of A . The drawback of NNM method is that each singular value takes the same weight, thus ignoring the prior knowledge of the image, in order to increase the flexibility of the nuclear norm, Gu et al. proposed WNNM algorithm. The weighted nuclear norm of matrix X is defined as:

$$\|X\|_{w,*} = \Sigma_i \left| \omega_i \sigma_i (X) \right|_1 \tag{2}$$

where $w=[\omega_1,\ldots,\omega_n]$, and $\omega_i \geq 0$, ω_i is the weight of the ith singular value in matrix X. The WNNM method has the following energy function:

$$\hat{X}_j = \arg\min_{X_j} \frac{1}{\sigma_n^2}\left\|Y_j - X_j\right\|_F^2 + \left\|X_j\right\|_{w,*} \tag{3}$$

Obviously, the main problem is the solution of the weight vector. The prior knowledge of natural images is primarily represented by larger singular values, which are more important than small singular values because they represent the main components of X_j. In the applications of image denoising, the greater the singular value, the more valuable it is, therefore, the weight of the i-th singular value of X_j should be inversely proportional to $\sigma_i(X_j)$:

$$\omega_i = c\sqrt{n}\big/\left(\sigma_i(X_j)+\varepsilon\right) \tag{4}$$

where $c>0$ is a constant, n denotes the number of similar patches in Y_j. $\sigma_i(X_j)$ can be estimated as:

$$\hat{\sigma}_i(X_j) = \sqrt{\max\left(\sigma_i^2(Y_j)-n\sigma_n^2, 0\right)} \tag{5}$$

where $\sigma_i(X_j)$, $\sigma_i(Y_j)$ are the i-th singular value of X_j and Y_j, respectively. By using the above steps to each image patch and accumulating all patches together, finally the image X_j can be reconstructed.

3. Improved WNNM Algorithm

WNNM denoising algorithm proposed by Gu et al. has excellent performance to denoise Gaussian noise. But it retains a bad performance while processing salt and pepper and mixed noise, where mixed noise is the mixture of above-mentioned two kinds of noise. In order to verify this experimentally, different types of noise are added to the standard test images, here we choose boat and lena image for the test, and use WNNM algorithm for denoising. The denoising results are listed in Tables 1-2.

Table 1. Denoising results of WNNM algorithm for boat image under different noise effects

Type of noise	Gaussian	Salt & Pepper	Mixed
Noise Image PSNR	20.14	18.47	16.30
WNNM PSNR	30.51	18.60	20.53

Table 2. Denoising results of WNNM algorithm for lena image under different noise effects

Type of noise	Gaussian	Salt & Pepper	Mixed
Noise Image PSNR	20.14	18.59	16.37
WNNM PSNR	30.67	18.72	20.76

The results in Tables 1 to 2 show that the WNNM algorithm has particularly good denoising effect on Gaussian noise, but the denoising effect on salt&&pepper noise and mixed noise is not good. This is mainly due to the formation mechanism of noise and the applicability of the algorithm. Gaussian noise is non-sparse noise, and WNNM algorithm can do a really excellent job in processing it. But salt and pepper noise is sparse in the identity, WNNM algorithm has a strong denoising effect on non-sparse noise such as Gaussian noise. However, due to the existence of sparse noise such as salt and pepper noise, its shortcomings are difficult to overcome.

The median filtering is a commonly used nonlinear filtering technique, the basic principle is to select the median value of each pixel value in a neighborhood of a pixel to be processed instead of the pixel to be processed, which eliminates both noise and detail because it does not distinguish between the two, and the noise filtered by the median filter can be non-sparse. Therefore, it is suitable to use WNNM algorithm for pre-processing, and then use adaptive median filtering to remove sparse noise such as salt&&pepper noise.

In summary, this paper proposes an improved WNNM algorithm, which combines the characteristics of the traditional WNNM algorithm and adaptive median filtering. The improved WNNM algo-

rithm uses WNNM algorithm to remove non-sparse noise such as Gaussian noise in the first stage, and uses the adaptive median filtering algorithm to remove sparse noise such as salt and pepper noise in the second stage. The specific steps of the algorithm are shown in Table 3.

Table 3. The improved WNNM algorithm

Stage 1	Stage2
（I）Initialize：$\hat{x}^{(0)}$=y，$y^{(0)}$ = y	Given maximum allowed size S_{max} of the neighborhood

（II）The data is iteratively restored as follows：
 for k=1:K do
 $$y^{(k)} = \hat{x}^{(k-1)} + \delta\left(y - \hat{y}^{(k-1)}\right)$$
 for each patch y_j in $y^{(k)}$ do
 Search for similar patches Y_j
 Estimate weight vector ω
 $$[U,\Sigma,V] = SVD\left(Y_j\right)$$
 Get the estimation：$\hat{X}_j = US_w(\Sigma)V^T$
 end for
 Aggregate X_j to form the clean image $\hat{x}^{(k)}$
 end for
	For each pixel y_{ij} in the image y
	
	 Process A :
	 $$A_1 = y_{max} - y_{max}, A_2 = y_{min} - y_{max}$$
	 if $A_1 > 0$ and $A_2 < 0$, jump to process B
	 else increase the window size
	 if window size $< S_{max}$,repeat process A
	 else output y_{ij}
	
	 Process B :
	 $$B_1 = y_{ij} - y_{min}, B_2 = y_{ij} - y_{max}$$
	 if $B_1 > 0$ and $B_2 < 0$,output y_{ij}
	 else output y_{med}

4. Experimental Results

In order to test the IWNNM algorithm and prove its advantages, the WNNM algorithm and the IWNNM algorithm are applied to practical examples, and a series of comparative experiments are carried out. The computer of this experiment is ASUS, the operating system is Windows 10 family Chinese version, the configuration of the computer is CPUi5-8250U, and the memory is 8GB. To set parameters for the new algorithm, as shown: In stage 1: set patch size to $6\times6, 7\times7$ and 8×8 for $\sigma_n \leq 20$, $20 < \sigma_n \leq 40$ and $40 < \sigma_n \leq 60$ respectively. K is set to 8, 12 and 14 respectively, on these noise levels. In stage 2: the maximum allowed size of the neighborhood is $S_{max} = 5$. We use the boat and lena image for the comparison experiment. The PSNR results of the WNNM algorithm and the IWNNM algorithm are listed in Tables 4-9.

Table 4. Denoising results of boat image under the influence of Gaussian noise

Noise intensity	5	15	25	35	45
Noise Image PSNR	34.12	24.58	20.14	17.22	15.04
WNNM PSNR	39.13	33.23	30.51	28.71	27.51
IWNNM PSNR	38.72	33.17	30.49	28.71	27.51

Table 5. Denoising results of lena image under the influence of Gaussian noise

Noise intensity	5	15	25	35	45
Noise Image PSNR	34.12	24.58	20.14	17.22	15.04
WNNM PSNR	39.30	33.34	30.67	28.93	27.77
IWNNM PSNR	37.84	33.09	30.59	28.91	27.76

Table 6. Denoising results of boat image under the influence of Salt & Pepper noise

Noise intensity	0.05	0.15	0.25	0.35	0.45
Noise Image PSNR	18.47	13.67	11.49	10.01	8.88
WNNM PSNR	18.60	13.75	11.53	10.04	8.90
IWNNM PSNR	**27.58**	**18.62**	**14.65**	**12.14**	**10.28**

Table 7. Denoising results of lena image under the influence of Salt & Pepper noise

Noise intensity	0.05	0.15	0.25	0.35	0.45
Noise Image PSNR	18.59	13.72	11.45	10.01	8.94
WNNM PSNR	18.72	13.80	11.50	10.04	8.96
IWNNM PSNR	**27.61**	**18.67**	**14.61**	**12.10**	**10.34**

Table 8. Denoising results of boat image under the influence of mixed noise

			Gaussian noise of different intensities				
			5	15	25	35	45
Salt & Pepper noise of different intensities	0.05	Noise Image PSNR	18.36	17.56	16.30	14.91	13.56
		WNNM PSNR	18.54	19.15	20.53	21.95	23.77
		IWNNM PSNR	**27.27**	**26.55**	**25.76**	**25.37**	**24.83**
	0.15	Noise Image PSNR	13.64	13.38	12.91	12.28	11.57
		WNNM PSNR	13.72	14.14	15.07	16.39	17.87
		IWNNM PSNR	**18.50**	**18.59**	**18.82**	**19.41**	**19.94**
	0.25	Noise Image PSNR	11.47	11.33	11.06	10.69	10.23
		WNNM PSNR	11.52	11.77	12.34	13.25	14.45
		IWNNM PSNR	**14.60**	**14.71**	**14.98**	**15.58**	**16.44**
	0.35	Noise Image PSNR	10.00	9.91	9.75	9.50	9.20
		WNNM PSNR	10.03	10.21	10.60	11.27	12.21
		IWNNM PSNR	**12.11**	**12.22**	**12.51**	**13.04**	**13.83**
	0.45	Noise Image PSNR	8.87	8.81	8.70	8.54	8.33
		WNNM PSNR	8.89	9.03	9.33	9.85	10.61
		IWNNM PSNR	**10.28**	**10.39**	**10.64**	**11.09**	**11.80**

Table 9. Denoising results of lena image under the influence of mixed noise

			Gaussian noise of different intensities				
			5	15	25	35	45
Salt & Pepper noise of different intensities	0.05	Noise Image PSNR	18.48	17.66	16.37	14.96	13.59
		WNNM PSNR	18.66	19.26	20.76	22.40	24.35
		IWNNM PSNR	**27.27**	**26.75**	**26.30**	**25.92**	**25.31**
	0.15	Noise Image PSNR	13.69	13.43	12.95	12.32	11.59
		WNNM PSNR	13.77	14.21	15.16	16.51	18.00
		IWNNM PSNR	**18.54**	**18.62**	**18.85**	**19.48**	**20.11**
	0.25	Noise Image PSNR	11.43	11.29	11.03	10.66	10.21
		WNNM PSNR	11.48	11.73	12.29	13.20	14.38
		IWNNM PSNR	**14.55**	**14.67**	**14.95**	**15.56**	**16.40**
	0.35	Noise Image PSNR	10.00	9.91	9.74	9.50	9.20
		WNNM PSNR	10.03	10.21	10.60	11.27	12.20
		IWNNM PSNR	**12.07**	**12.17**	**12.47**	**12.99**	**13.79**
	0.45	Noise Image PSNR	8.93	8.87	8.76	8.59	8.38
		WNNM PSNR	8.95	9.09	9.40	9.92	10.70
		IWNNM PSNR	**10.33**	**10.43**	**10.67**	**11.13**	**11.86**

The tables above show the denoising results on boat and lena image with gaussian noise, salt and pepper noise, and mixture of these two kinds of noise, respectively. It can be seen from these data that the IWNNM algorithm has no advantage over the WNNM algorithm for removing Gaussian noise, and the WNNM algorithm has sufficient effective removal effect, but for salt and pepper noise and

mixture of Gaussian and salt&pepper noise, the advantages of IWNNM algorithm are extremely obvious, and denoising effect is excellent. Figure 1 shows the denoising effect after applying mixed noise to the boat image.
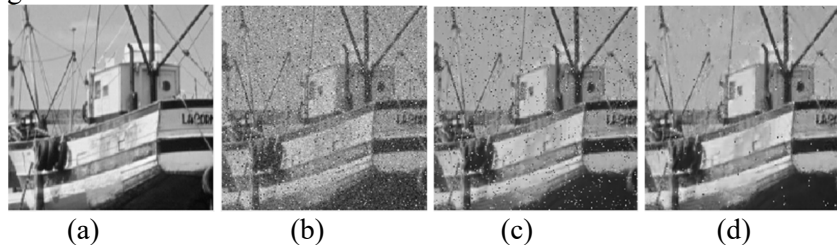

| (a) | (b) | (c) | (d) |

Fig. 1. Denoising effect after applying mixed noise to the boat image. (a) Original image. (b) Image under the influence of mixed noise, where the intensity of Gaussian noise is 25, and the intensity of salt & pepper noise is 0.05 respectively. (c) Image after denoising by WNNM algorithm. (d) Image after denoising by IWNNM algorithm.

5. Conclusion

In order to overcome the shortcoming of weighted nuclear norm minimization (WNNM) algorithm, this paper presents an improved WNNM (IWNNM) algorithm, which combines the characteristics of WNNM and adaptive median filtering and has the advantage of removing both the non-sparse noise (such as Gaussian noise) and the sparse noise (such as salt and pepper noise and mixed noise).It is not difficult to see from the above experimental results that the IWNNM algorithm has really excellent performance for image denoising in terms of visual subjective aspect and image quality evaluation index. The improved IWNNM algorithm proposed in this paper has pretty good denoising effect in some aspects, but it also has some shortcomings that we cannot handle successfully. For example, when the intensity of the mixed noise is relatively low, it is obvious to see that the denoising performance is excellent, but when the intensity of the mixed noise is relatively high, the effect could get a little worse. So how to ensure that the algorithm achieves excellent removal ability regardless of whether the mixed noise intensity is high or low is the focus of subsequent research.

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