Definite Integrals

Examples

1. Express $\lim_{n\to\infty} [e^{-1+3/n} \cdot \frac{3}{n} + e^{-1+6/n} \cdot \frac{3}{n} + \cdots + e^2 \cdot \frac{3}{n}]$ as a definite integral on $[-1, 2]$.

2. Express $\int_{-1}^{2} \cos(x)dx$ as a limit of right endpoint Riemann sums.

3. True   False   The addition definite integration law was proved using derivative laws.

4. True   False   The addition definite integration law was proved using limit laws.

Problems

5. Express $\lim_{n\to\infty} [\tan(-1+2/n) \cdot \frac{2}{n} + \tan(-1+4/n) \cdot \frac{2}{n} + \cdots + \tan(1) \cdot \frac{2}{n}]$ as a definite integral from -1 to 1.

6. Express $\lim_{n\to\infty} [\frac{2^3}{n^3} + \frac{2 \cdot 2^3}{n^3} + \cdots + \frac{2^3 n^2}{n^3}]$ as a definite integral from 0 to 2.

7. Express $\lim_{n\to\infty} [(1 + \frac{3}{n} + \frac{9}{n^2}) \cdot \frac{3}{n} + (1 + \frac{6}{n} + \frac{36}{n^2}) \cdot \frac{3}{n} + \cdots + (1 + 3 + 3^2) \cdot \frac{3}{n}]$ as a definite integral from 0 to 3.

8. Express $\int_{-1}^{3} \cos^2(x)dx$ as a limit of right endpoint Riemann sums.

9. Express $\int_{-3}^{3} |x|dx$ as a limit of right endpoint Riemann sums.

10. Express $\int_{-2}^{0} |x^2 - x|dx$ as a limit of right endpoint Riemann sums.

Fundamental Theorem of Calculus I

Examples

11. Evaluate the integral $\int_{2}^{5} (x^2 + 1)dx$.

12. True   False   Let $F(x)$ be defined on $[a, b]$ such that $F'(x) = f(x)$ on (a, b), then $\int_{a}^{b} f(x)dx = F(b) - F(a)$.

13. True   False $\int_{-1}^{1} \frac{1}{x} dx = \ln |x| \; |_{-1}^{1} = 1 - 1 = 0.$

14. True   False $\int f'(x) dx = f(x).$

Problems

15. Evaluate the integral $\int_{0}^{4} \sqrt{x} dx.$

16. Evaluate the integral $\int_{1}^{8} \sqrt[3]{x} dx.$

17. Evaluate the integral $\int_{0}^{1} e^{x+1} dx.$

18. Find the indefinite integral $\int (4t^3 + 3t^2) dt.$

19. Find the indefinite integral $\int \frac{1}{3x} dx.$

20. Find the indefinite integral $\int 2 \sin(2\theta) d\theta.$