* P1-14: Moduli space of p-div groups and period morphisms

* P15-26: Lawrence-venkatesh chapter 3

A random walk into rigid analytic Geo.

§1. generalities on formal schemes.

Defn: $I \subset A$ is called an ideal of definition, if I is an open ideal and for any open nbhd V of 0, $\exists n$, s.t. $I^n \subset V$.

Defn: A is called I-adic, if I^n form a fundamental sys. of nbhd of 0, and it is separated and complete.

$$A = \varprojlim A/I^{n+1}$$

Defn: $X = \operatorname{Spec} A$, A: I adic. We can associate a topological ringed space $(\mathfrak{X}, \mathcal{O}_{\mathfrak{X}})$.

$\mathfrak{X} = V(I)$ $\mathcal{O}_{\mathfrak{X}} = \varprojlim \mathcal{O}_X / I^{n+1}$ (affine formal scheme)

Defn: A formal scheme is a locally ringed space $(\mathfrak{X}, \mathcal{O}_{\mathfrak{X}})$ has an affine open covering.

- we can also define it as contravariant func.

$$\operatorname{Spf} A(Z) := \varinjlim_{\lambda} \operatorname{Hom}(Z, \operatorname{Spec}(A/I_\lambda))$$

§ Here A is any linear topological ring having an ideal of definition.

§2. moduli space of p-div grps.

k_0. O: cdVR. $[O:\mathbb{Z}_p]$

Nilp_O: "cat of locally noetherian $S/\mathrm{Spec}\,O$ s.t. pO_S is locally nilpotent"

$\bar{S}$: closed subscheme defined by the ideal pO_S, $\mathbb{X}/\bar{\mathbb{F}}_p$. $\mathcal{M}/\mathrm{Nilp}_{\breve{O}}$ functor.

Thm: $\mathcal{M}$ is contravariant

$$S \longrightarrow (X, \rho).$$

(i) A p-div X/S

(ii) a quasi isog. $\rho: \mathbb{X}_{\bar{S}} \to X_{\bar{S}}$.

$\mathcal{M}$ is rep. by a formal sch. locally formally of finite type $/\mathrm{Spf}\, W$.

(alternatively) $\mathrm{QIsog}_S(X,Y) \to \mathrm{QIsog}_{S'}(X_{S'}, Y_{S'})$

$\tilde{\rho}: \tilde{\mathbb{X}}_S \to X$. $/S$.

Case PE: define the functor $\breve{\mathcal{M}}$ $\lambda_X: X \to \hat{X}$

$O_B \hookrightarrow \mathbb{X}$ $\tilde{X}/O_K$. K/K_0 fin.

E: field of defn. of $\mathrm{Lie}(\tilde{X})$ as rep. of B.

$\breve{E} = EK_0$. w/ ring of integers $O_{\breve{E}}$

$\breve{\mathcal{M}} / \mathrm{Nilp}_{O_{\breve{E}}}$: $\quad S \in \mathrm{Nilp}_{O_{\breve{E}}} \rightsquigarrow (X, \rho)$

(i) X/S, $\quad O_B \hookrightarrow X$, $\quad \forall S' \to S$, $a \in O_B \otimes O_{S'}$

$\det_{O_{S'}}(a, \mathrm{Lie}(X_{S'})) = \det_K(a, \mathrm{Lie}(\mathbb{X})_K)$

(ii) an O_B-quasi-isog. $\rho: \mathbb{X}_{\bar{S}} \to X_{\bar{S}}$

(iii) $\exists\, c_X \in O_F^\times$, s.t. $\rho^\vee \circ \lambda_X \circ \rho = c_X \lambda$.

Thm: $\breve{\mathcal{M}}$ is rep. by a formal sch. locally formally of f.t. / $\mathrm{Spf}(O_{\breve{E}})$.

Eg. $\mathbb{X}$, h, dim $= 1$. Slope $1/h$

$$\mathcal{M}^{LT} \cong \coprod_{n \in \mathbb{Z}} \mathrm{Spf}\, W(\bar{\mathbb{F}}_p)[[T_1, \cdots, T_{h-1}]],$$

§3. Rigid analytic geometry.

$\mathcal{O}$: CDVR. $\mathcal{O}/\mathbb{Z}_p$ finite type.

$F = \mathrm{Frac}\,\mathcal{O}$.

Defn: (Tate alg. in n. variables).

$$F\{y_1, \cdots y_n\} := \{ f = \sum a_{i_1 \cdots i_n} y_1^{i_1} \cdots y_n^{i_n} \mid$$

$$a_i \in F,\ i_j \geq 0,\ |a_{i_1 \cdots i_n}| \to 0,\ i_1 + \cdots + i_n \to \infty \}$$

* A tate algebra is Noetherian and carries a Gauss norm $|f| := \sup\{|a_{i_1, \cdots, i_n}|\}$.

Defn: An affinoid F-alg. is an F-alg. B which is a quotient of a Tate algebra for some n,

Defn: (affinoid rigid analytic space)

Spm B w/ Grothendieck topology and a structure sheaf.

$x \in \mathrm{Spm}\,B$ has residue field B/x

- B/x is a finite extension of F.
- An F-alg. homomorphism $B \to C$ induces a map $\mathrm{Spm}\,C \to \mathrm{Spm}\,B$

Defn: A rigid analytic space $/F$ is a set X carrying a Grothendieck topology and a structure sheaf, s.t. X possesses an admissible covering by affinoid analytic space.

- we have the corresponding def of morphism, and smoothness, étaleness, ... etc, ...

※ Raynaud - Berthelot's construction:

Rigid analytic spaces can be viewed as generic fibers of formal schemes $/\mathrm{Spf}\,\mathcal{O}$ locally formally of finite type.

Defn. tfp $\mathcal{O}$-alg. is an $\mathcal{O}$-alg of the form: $A = \mathcal{O}\{x_1, \cdots, x_n\}/I$,

where I is a finitely generated ideal in the ring $\mathcal{O}\{x_1, \cdots, x_n\}$ of restricted power series:

$\sum a_J x^J \in \mathcal{O}[[x_1, \cdots, x_n]]$, $x^J = x_1^{j_1} \cdots x_n^{j_n}$

s.t. $a_J \to 0$ as $\|J\| := j_1 + \cdots + j_n \to \infty$

- A is a π-adic noetherian ring.

Defn: A tfp affine formal Scheme $/\mathcal{O}$ is a locally ringed space $\simeq \mathrm{Spf}\, A$, A is a tfp $\mathcal{O}$-algebra.

A locally of finite type tfp $\mathcal{O}$-formal Scheme is a locally ringed space w/ an open covering by tfp affine $\mathcal{O}$-formal

Raynaud's functor:

$$\mathfrak{X} \longrightarrow \mathfrak{X}^{rig}.$$

Affine:

$\mathfrak{X} = \mathrm{Spf}\, A$. $\quad A \otimes F$ is an affinoid F-alg.

$\mathfrak{X}^{rig} = \mathrm{Spm}(A \otimes F)$ $\quad x \in \mathfrak{X}^{rig}$

$$\Updownarrow$$

A/J, integral domain, f.f $/\mathcal{O}$.

specialization map: $\quad sp: \mathfrak{X}^{rig} \to \mathfrak{X}$

$x \in \mathrm{Spm}(A \otimes F) \Rightarrow A \otimes F \to k(x)$, residue field of x.

- $k(x)$ is a finite extension of F.

$A \longrightarrow \mathcal{O}(x) \subset k(x)$

$A/(\pi) \longrightarrow \tilde{k}(x)$: residue field of $k(x)$.

this gives a point $sp(x) \in \mathrm{Spec}\, A/\pi \subset \mathfrak{X}$

In general:

$\mathfrak{X}^{rig}$: the set of closed formal subscheme Y irr. red. f.f over $\mathcal{O}$.

The support of such Y is a closed pt of $\mathfrak{X}$, called the specialization of $x \in \mathfrak{X}^{rig}$ corresponding to Y.

<u>Prop.</u> (1) the rigid analytic structure on $\mathfrak{X}^{rig}$ is unique

$$\left\{ \begin{array}{l} sp^*: \mathcal{F} \to \mathcal{F}^{rig} \\ sp_{\mathfrak{X}}: (\mathcal{O}_{\mathfrak{X}^{rig}}) = \mathcal{O}_{\mathfrak{X}} \otimes F \end{array} \right.$$

(2) The specialization has universal property. Let $\mathfrak{Y}$ be any <u>rigid analytic space</u>,

$$\begin{tikzcd} & \mathfrak{X}^{rig} \arrow[d] \\ \mathfrak{Y} \arrow[ur] \arrow[r] & \mathfrak{X} \end{tikzcd}$$

(3) the functor commutes w/ products and transforms open immersions to open immersions.

In the case of locally noetherian $\mathfrak{X} = \mathrm{Spf}\, A$, $f_1 \cdots f_r$ be a system of generators of a defining ideal.

$\mathrm{Spf}(A) = \varinjlim \mathrm{Spf}(B_n)$

$B_n = \underbrace{A\{T_1, \cdots, T_r\}}_{\substack{\pi\text{-adic completion of } A[T_1, \cdots, T_r]}} / (f_1^n - \pi T_1, \cdots, f_r^n - \pi T_r)$

$\mathcal{X}^{rig} := \bigcup_n \underbrace{\mathrm{Spm}(B_n \otimes F)}$ → can be realized as closed ball with radius $\leq |\pi|^{\frac{1}{n}}$.

Eg: ① let $\mathcal{X} = \mathrm{Spf}\, \mathcal{O}\{T_1, \cdots, T_m\}$, then $\mathcal{X}^{rig}$ is the closed unit ball;

② let $\mathcal{X} = \mathrm{Spf}\, \mathcal{O}[[T_1, \cdots, T_n]]$ w/ ideal of definition $(\pi, T_1, \cdots T_n)$,

$\mathcal{X}^{rig} = \bigcup$ closed ball of radius $|\pi|^{\frac{1}{n}}$

Problem. the rigid analytic space defined in this way are not topological spaces in the classical sense.

The theory developed by Berkovich solved this issue and provides more underlying pts.

Berkovich's construction:

Let B be an affinoid F-algebra w/ F-Banach norm. $|\,|$. B is called strictly F-affinoid alg. by Berkovich.

Defn: An analytic point x of B is a semi-norm $|\cdot|_x : B \to \mathbb{R}_{\geq 0}$, satisfies

(1) $|f+g|_x \leq \max\{|f|_x, |g|_x\}$ for all $f, g \in B$

(2). $|fg|_x = |f|_x |g|_x$ for all $f, g \in B$.

(3). $|\lambda|_x = |\lambda|$ for all $\lambda \in F$

(4) $|\cdot|_x : B \to \mathbb{R}_{\geq 0}$ is continuous

Defn: the set of all analytic pts $\mathcal{M}(B)$.

top o: the coarsest top. s.t. $\forall f \in B$,

$$\begin{aligned} \mathcal{M}(B) &\to \mathbb{R}_{\geq 0} \\ x &\mapsto |f|_x \end{aligned}$$

is continuous.

call $\mathcal{M}(B)$ a strictly F-affinoid space.

- $\forall x \in \mathcal{M}(B)$ $\ker|\cdot|_x := \{b \in B \mid |b|_x = 0\}$ is prime.

Defn. Complete res field $k(x)$ of x the completof $\mathrm{Frac}(B/\ker|\cdot|_x)$ w.r.t. $|\cdot|_x$.

- $B \to k(x)$ is a continuous hom of F-alg.
- conversely, let k be a complete ext. of F w/ $|\cdot|: k \to \mathbb{R}_{\geq 0}$.

Any continuous F-alg. hom. $B \to k$ defines a semi-norm on B, hence an analytic point.

* Compare two constructions: every rigid ana. pt. $x \in \mathrm{Spm}\, B$ defines an analytic point w/ $k(x)/F$ finite.

Defn: A strictly F-analytic space is a top. space admits strictly affinoid F-analytic charts.

A good strictly F-analytic space is a strictly F-analytic space s.t. every point has a strictly F-analytic nbhd.

Defn: We call good .... Berkovich spaces.

Y. locally of f.t over F. $\rightsquigarrow Y^{an}$.

$X^{rig} = \{x \in X \mid k(x) \text{ is a fini. ext. of } F\}$

X strictly F-analytic space

*** period morphisms.**

(F, O, k, π). $\mathcal{M} / \mathrm{Spf}\, O$. formal sch.

locally formally of finite type.

$X / \mathcal{M}$ $(X / O_{\mathcal{M}})$ - p-div grp.

M_X: $\mathrm{Lie}(\overline{E(X)})$: Lie alg. of the universal ext. of X, $\mathcal{M}_0$ - k-sch def. by $I \supset (\pi)$

$\mathbb{X} / k$ fixed. ρ: $\mathbb{X}_{\mathcal{M}_0} \to X_{\mathcal{M}_0}$

rank h. dim d ✓

Thm: 2.4.1 ρ induces a canonical isom of locally free $O_{\mathcal{M}^{rig}}$ - modules of finite rank. let $E(\mathbb{X})$ be the isocrystal.

Berkovich rig $\rightsquigarrow$ an

$$\tilde{\rho}: E(\mathbb{X}) \otimes_{W(k)_\mathbb{Q}} O_{\mathcal{M}^{rig}} \simeq (M_X)^{rig}$$

Now if $\mathcal{M} = \check{\mathcal{M}}$, moduli of p-div w/ PE.

i.e. $\breve{\mathcal{M}}$ is locally formally of finite type over $\mathrm{Spf}\, O_{\breve{E}}$

let (X^{univ}, ρ^{univ}) be the universal p-div on $\breve{\mathcal{M}}$. then

$$E(\mathbb{X}) \otimes O_{\breve{\mathcal{M}}^{rig}} \overset{f}{\cong} : (M_{X^{univ}})^{rig} \quad \text{rank}: h.$$

$$\twoheadrightarrow (\mathrm{Lie}(X^{univ}))^{rig} \quad \text{rank } d.$$

$\Rightarrow$ $\ker f$ defines an $\breve{\mathcal{M}}^{rig}$-valued pt of the Grassmannian $\mathrm{Grass}_{h-d}(E(\mathbb{X}))$

$\pi : \breve{\mathcal{M}}^{rig} \to \mathrm{Grass}_{h-d}(E(\mathbb{X})^{rig})$.

☆ A question of Grothendieck:

Fix $\mathbb{X} / \overline{\mathbb{F}}_p$, height h dim d.

O_K: cdvR, $k = \overline{\mathbb{F}}_p$, $\mathrm{Frac}(O_K) = K$

$K_0 = \mathrm{Frac}(W = \overline{\mathbb{F}}_p)$.

Any p-div X / O_K lifts $\mathbb{X}$, there is an ext:

$$0 \to (\mathrm{Lie}\, X^{\vee})^{\vee}_K \to \mathbb{D}(\mathbb{X})_K \to (\mathrm{Lie}\, X)_K \to 0.$$

let $\mathcal{F} := \mathrm{Grass}_{h-d}(\mathbb{D}(\mathbb{X})_{K_0})$. $\{h-d$ dim subspace of $\mathbb{D}(\mathbb{X})_{K_0}\}$

Describe the subset of $\mathcal{F}$ formed by $(\mathrm{Lie}\, X^{*})_K^{\vee}$ for all X, K.

Answer: $\bigcap (\mathcal{F}^{wa})^{rig}$

$\mathcal{F}/_E$ is the flag variety associated to (b, μ),

$b \in G(K_0)$. φ-conjugacy class

μ: cocharacter $/E$

* this comes from the fact that

$\breve{\mathcal{X}}^{rig}$ factor through $(\breve{\mathcal{F}}_b^{wa})^{rig}$.

Faltings: Fix w.d.k.S, then up to conjugation, only finitely many semi-simple Galois representation $\rho: G_K \to GL_d(\mathbb{Q}_p)$

(a) ρ unram. outside S.

(b) pure of weight. $w. \leftrightarrow |q_p^{\frac{w}{2}}|$

(c) Frob. at $\mathfrak{p}$ has integer coefficients.

Aim: control / finiteness of fibers in a family of smooth proj. vars. w/ good reduction outside S.

3.1 setting up.

$\pi: X \to Y/_k$. proper smooth

smooth

Suppose π admits a good model over $\mathcal{O}$, i.e, it extends to a proper smooth morphism $\pi: \mathcal{X} \to \mathcal{Y}$

- the generic fiber of the relative de Rham coh $\mathcal{H}^q := R^q \pi_* \Omega^{\bullet}_{\mathcal{X}/\mathcal{Y}}$ is equipped w/ the Gauss-Manin connection

$$\mathcal{H}^q \to \mathcal{H}^q \otimes \Omega^1_{\mathcal{Y}/\mathcal{O}}.$$

- we want to bound $\mathcal{Y}(\mathcal{O})$. Study

$$\rho_y : G_K \to \mathrm{Aut}\, H^q_{et}(X_y \times_K \bar{K}, \mathbb{Q}_p).$$

s.t. for $\iota: K \hookrightarrow \mathbb{C}$

v $K \hookrightarrow K_v$,

- $\overset{v}{\underset{p}{|}}$, $p > 2$,

- K_v is unramified $/\mathbb{Q}_p$.

- no prime above p lies in S.

we want to give criteria for the finiteness of $\mathcal{U}$ in terms of period map.

$\mathcal{U} := \{ y \in \mathcal{Y}(\mathcal{O}) : y \equiv y_0 \mod \nu \}$.

it is clear that if $\mathcal{U}$ is finite for each choice of y_0, then $\mathcal{Y}(\mathcal{O})$ is finite

3.2. coh. & Gauss-Manin Connection.

Aim: to identify the coh. of nearby fibers. (K_v & $\mathbb{C}$ topo.)

Both identifications can be described as evaluations of power series w/ k-coeff, convergent both for K_v and $\mathbb{C}$ topo.

Results: $p > 2$, v unramified above p.

- $H^q_{dR}(X_{y_0}/K_v) \simeq H^q_{dR}(X_y/K_v)$.

whenever $y \in \mathcal{Y}(\mathcal{O}_v)$ satisfy $y \equiv y_0 \mod v$.

• GM: H^q_{dR}

~~$\mathcal{Y}$~~ $\in$ small nbhd of $y_0 \in \mathcal{Y}(\mathcal{O})$.

$y \in \mathcal{Y}_{\mathbb{C}}(\mathbb{C})$

comparison $\to$ compatible w/ GM.

$\phi_v \to$ semi-linear w.r.t. σ on $K_v/\mathbb{Q}_p$

$$\begin{tikzcd}
V_v = H^q_{dR}(X_{y_0}/K_v) \arrow[d, "GM"'] \arrow[dr, "\sim"] & \\
& H^q_{cris}(\bar{\mathcal{X}}_0/\mathbb{F}_v) \arrow[loop right, "\phi_v"] \\
H^q_{dR}(X_y/K_v) \arrow[ur, "\sim"'] &
\end{tikzcd}$$

$\mathcal{X}_0 = \lambda^{-1}(y_0)$, $y_0 \in \mathcal{Y}(\mathcal{O})$

$\mathcal{X}_0$ is a smooth-proper $\mathcal{O}$-model for X_0.

Recall: over $\mathbb{C}$
$V = H^q_{dR}(X_0/K)$ is equipped w/ a Hodge filtration:

$$V = F^0 V \supset F^1 V \supset \cdots\cdots \quad (*)$$

let $\mathcal{H}/_K$ be the flag variety of V.

$$h_0 \in \mathcal{H}(K) \longleftrightarrow (*).$$

By base change to K_v and $\mathbb{C}$, we get $\mathcal{H}_v$ and $\mathcal{H}_{\mathbb{C}}$. Let $h_0' \in \mathcal{H}_{\mathbb{C}}(\mathbb{C})$ be the image of h_0

GM: $t \in \Omega_{\mathbb{C}}$: contractible analytic nbhd of $y_0 \in Y^{an}_{\mathbb{C}}$

$$H_{dR}(X_t/\mathbb{C}) \simeq H_{dR}(X_0/\mathbb{C}).$$

and the Hodge filtration on $H_{dR}(X_t)$ gives a point on $\mathcal{H}_{\mathbb{C}}$

and $\Phi_{\mathbb{C}} : \Omega_{\mathbb{C}} \to \mathcal{H}_{\mathbb{C}}(\mathbb{C})$

- $\Phi_{\mathbb{C}}$ extends to a map:

$$\Phi_{\mathbb{C}} : \tilde{Y}_{\mathbb{C}}^{an} \to \mathcal{H}_{\mathbb{C}}(\mathbb{C})$$

- $\Phi_{\mathbb{C}}$: $\pi_1(Y_{\mathbb{C}}^{an}, y_0)$ equivariant

The following lemma shows that the image of the period map can be bounded below by monodromy.

- $\mu : \pi_1(Y_{\mathbb{C}}(\mathbb{C}), y_0) \to GL(V_{\mathbb{C}})$
- $\Gamma \simeq$ Zariski closure of $\mathrm{Im}\,\mu$.

Lem 3.1: given a family $X \to Y$.

$$\Gamma \cdot h_0^t \subset \overline{\Phi_{\mathbb{C}}(\Omega_{\mathbb{C}})}^{zar} \subset \mathcal{H}_{\mathbb{C}}$$

Pf: $\Phi_{\mathbb{C}}^{-1} Z$ of any alg. subvariety

$Z \supset \Phi_{\mathbb{C}}(\Omega_{\mathbb{C}})$ is a complex analytic subvar of $\tilde{Y}^{an}_{\mathbb{C}}$ containing Ω. Because complex analytic open is Zariski dense, $\Phi_{\mathbb{C}}^{-1}(Z) = \tilde{Y}^{an}_{\mathbb{C}}$

$$\cup$$

$$\gamma_i \cdot \Phi_{\mathbb{C}}^{-1}(h_0^i) = \Phi_{\mathbb{C}}^{-1}(\gamma_i \cdot h_0^i)$$

$\Rightarrow \gamma_i \cdot h_0^i \subset Z$.

$\Rightarrow \Gamma \subset Z$.

v-adic analogue.

$$H^q_{dR}(X_y/K_v) \simeq H^q_{dR}(X_{y_0}/K_v)$$

therefor we get a point under

$$\Phi_v : \Omega_v \to \mathcal{H}(K_v).$$

$$\Omega_v := \{ y \in Y(\mathcal{O}_v) : y \equiv y_0 \mod v \}$$

* Φ_v^{zar} and $\Phi_{\mathbb{C}}^{zar}$ → monodrom have the same dim.

Lemma 3.2: given power series $B_0, \dots, B_N \in K[[z_1, \dots, z_m]]$ s.t. all B_i abs. convergent, no common zero, both in v adic disc U_v, and $U_{\mathbb{C}}$.

Let. $\underline{B_v}: U_v \to \mathbb{P}^N_{K_v}$

$\underline{B_{\mathbb{C}}}: U_{\mathbb{C}} \to \mathbb{P}^N_{\mathbb{C}}$

$\exists$ K-subscheme $Z \subset \mathbb{P}^N$

$Z_{K_v} = \underline{B_v}(U_v)^{zar} \subset \mathbb{P}^N_{K_v}$,

$Z_{\mathbb{C}} = \underline{B_{\mathbb{C}}}(U_{\mathbb{C}}) \subset \mathbb{P}^N_{\mathbb{C}}$

$\dim Z_{K_v} = \dim Z_{\mathbb{C}}$

Lemma 3.1 + Lemma 3.2.

$\Rightarrow$ Lemma: 3.3

$\dim_{K_v}(\Phi_v(\Omega_v))^{zar} \geq \dim_{\mathbb{C}} \Gamma \cdot h_0^{i}$

if $\mathcal{H}_v^{bad} \subset H_v$ is Zariski closed of dimension $< \dim_{\mathbb{C}}(\Gamma \cdot h_0^{\Gamma})$

then $\Phi_v^{-1}(\mathcal{H}_v^{bad}) \subsetneq$ proper K_v analytic subset of Ω_v.

Now we relate the p-adic Galois representation ρ_y to the periods, for each $y \in U = \{y \in Y(\mathcal{O}) \mid y \equiv y_0 \mod v\}$.

ρ_y is crystalline, since we have $\mathcal{X}_y$ over X_y.

p-adic Hodge theory:

$\rho_y \in$ Crys representation of Gal_{K_v} on $\mathbb{Q}_p$ vector spaces $\hookrightarrow (W, \phi, F)$

filtered isocrystal

Crystalline comparison:

$e_y \rightsquigarrow H^q_{dR}(X_y/K_v, \phi_v, \text{Hodge fil})$

$\xrightarrow{\text{GM comparison}} \cong$

$(V_v, \phi_v, \Phi_v(y))$

Proposition 3.4. $X \to Y$ smooth proper family, $V = H^q_{dR}(X_0/K)$ $\mathcal{F}$ flags in V

$$\Phi_v : \{y \in Y(O_v) : y \equiv y_0\} \to \mathcal{H}(K_v).$$

$$\Gamma = (\pi_1 \cdot h_0)^{Zar}$$

$$h_0 = \Phi(y_0) \in K$$

suppose

$$\dim_{K_v}(C(\phi_v^{[K_v:\mathbb{Q}_p]})) < \dim_{\mathbb{C}} \Gamma \cdot h_0$$

$C(\quad)$ is the centralizer of $(\;)$

$= \mathrm{Aut}_{K_v}(V_v)$ of the K_v-linear

operator $\phi_v^{[K_v:\mathbb{Q}_p]}$.

Then $\{y \in Y(O) : y \equiv y_0, \; e_y \text{ semisimple}\}$

is contained in a proper K_v-analy subvariety of the residue disk of $Y(K_v)$ at y_0.

<u>proof</u>:

- lemma 2.3 shows that P_y belongs to a finite set of isom classes.
- By previous discussion, $(V_v, \phi_v, \Phi_v(y))$ also belongs to a finite set of isom classes, in the Category of ...

let $\{(V_v, \phi_v, h_i)\}$ be a set of representatives: we have

$$\Phi_v(y) \in \bigcup Z(\phi_v) \cdot h_i.$$

$Z(\phi_v) \subset GL_{K_v}(V_v)$ consists of elements which commute with ϕ_v,

Now $Z(\phi_v) \subset Z(\phi_v^{[K_v:\mathbb{Q}_p]})$,

and the RHS is a K_v-alg. subgrp of $GL_{K_v}(V_v)$

by assumption, $\dim_{K_v}(Z(\phi_v^{[K_v:\mathbb{Q}_p]}))$

$\leq \dim_{\mathbb{C}} \Gamma \cdot h_0^v$

so $\{y \quad\}$ is contained in the preimage of a Zar-closed subset that satisfy Lemma 3.3.

$\Rightarrow$ proper, K_v-analytic.