

Lecture 22: Dedekind Domains

Roy Zhao

2/26/24

1 Definition

Recall from last time:

Definition 1.1. A connected Shimura datum is a pair (G, u) where G/\mathbb{Q} is a semisimple algebraic group and $u: S^1 \rightarrow G_{\mathbb{R}}^{\text{ad}}$ is a homomorphism satisfying:

1. Only the characters $z, 1, z^{-1}$ occur in the representation of S^1 on $\text{Lie}(G^{\text{ad}})_{\mathbb{C}}$ defined by $\text{Ad} \circ u$
2. $\text{ad}(u(-1))$ is a Cartan involution on $G_{\mathbb{R}}^{\text{ad}}$
3. G^{ad} has no \mathbb{Q} -factor H such that $H(\mathbb{R})$ is compact.

Remark. The first condition says that we are parametrizing Hodge structures of weight 1, the second says that the u is not too strange and should behave like taking the conjugate transpose, and the last is a not so necessary condition that is used to say that arithmetic subgroups are Zariski dense.

Given a connected Shimura datum (G, u) and a level structure $\Gamma \subset G^{\text{ad}}(\mathbb{Q})^+$, which is an arithmetic subgroup, we can construct a connected Shimura variety which is

$$\Gamma \backslash D := \Gamma \backslash G_{\mathbb{R}}^{\text{ad},+} u.$$